***BIOSTATISTICS\_Daniel\_C12***

C12\_*CHI-SQUARE DISTRIBUTION*

*THE CHI-SQUARE DISTRIBUTION AND THE ANALYSIS OF FREQUENCIES*

This chapter explores techniques that are commonly used in the analysis of count or frequency data

epidemiological studies

After studying this chapter, the student will

1. understand the mathematical properties of the chi-square distribution.
2. be able to use the chi-square distribution for goodness-of-fit tests.
3. be able to construct and use contingency tables to test independence and  
      
   homogeneity.
4. be able to apply Fisher’s exact test for 2 􏰇 2 tables.
5. understand how to calculate and interpret the epidemiological concepts of  
      
   relative risk, odds ratios, and the Mantel-Haenszel statistic.

**12.1 INTRODUCTION**

hypotheses concerning a population variance.

For example, we may know for a sample of hospitalized patients how many are male and how many are female.

For the same sam ple we may also know how many have private insurance coverage, how many have Medicare insurance, and how many are on Medicaid assistance.

We may wish to know, for the population from which the sample was drawn, if the type of insurance coverage differs according to gender.

type of insurance coverage - tipo de cobertura de seguro

For another sample of patients, we may have frequencies for each diagnostic category represented and for each geographic area represented. We might want to know if, in the population from which the same was drawn, there is a relationship between area of residence and diagnosis

**12.2 THE MATHEMATICAL PROPERTIES OF THE CHI-SQUARE DISTRIBUTION**

The chi-square distribution may be derived from normal distributions.

The mathematical form of

, *u* 7 0 (12.2.3)

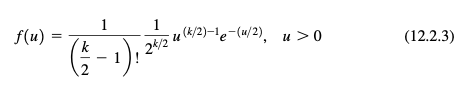
where *e* is the irrational number 2.71828 . . . and *k* is the number of degrees of free- dom. The variate *u* is usually designated by the Greek letter chi 1x2 and, hence, the distribution is called the chi-square distribution. As we pointed out in Chapter 6, the chi- square distribution has been tabulated in Appendix Table F. Further use of the table is demonstrated as the need arises in succeeding sections.

The mean and variance of the chi-square distribution are *k* and 2*k*, respectively. The modal value of the distribution is *k* - 2 for values of *k* greater than or equal to 2 and is zero for *k* = 1.

The shapes of the chi-square distributions for several values of *k* are shown in Fig- ure 6.9.1. We observe in this figure that the shapes for *k* = 1 and *k* = 2 are quite differ- ent from the general shape of the distribution for *k* 7 2. We also see from this figure that chi-square assumes values between 0 and infinity. It cannot take on negative values, since it is the sum of values that have been squared. A final characteristic of the chi-square dis- tribution worth noting is that the sum of two or more independent chi-square variables also follows a chi-square distribution.

**Types of Chi-Square Tests** As already noted, we make use of the chi-square distribution in this chapter in testing hypotheses where the data available for analysis are in the form of frequencies. These hypothesis testing procedures are discussed under the topics of *tests of goodness-of-fit, tests of independence,* and *tests of homogeneity.* We will discover that, in a sense, all of the chi-square tests that we employ may be thought of as goodness-of-fit tests, in that they test the goodness-of-fit of observed frequencies to frequencies that one would expect if the data were generated under some particular the- ory or hypothesis. We, however, reserve the phrase “goodness-of-fit” for use in a more

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**Types of Chi-Square Tests**

These hypothesis testing procedures are discussed under the topics of *tests of goodness-of-fit, tests of independence,* and *tests of homogeneity.*

We, however, reserve the phrase “goodness-of-fit” for use in a more

restricted sense. We use it to refer to a comparison of a sample distribution to some the- oretical distribution that it is assumed describes the population from which the sample came.

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**Observed Versus Expected Frequencies**

The chi-square statistic is most appropriate for use with categorical variables

There are two sets of frequencies with which we are con- cerned, *observed frequencies* and *expected frequencies.*

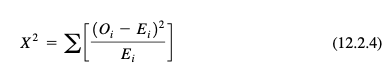
if some null hypothesis about the variable is true.

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**The Chi-Square Test Statistic**

The test statistic for the chi-square tests we

discuss in this chapter is



When the null hypothesis is true, *X*2 is distributed approximately as x2 with *k* - *r* degrees of freedom.

In determining the degrees of freedom, *k* is equal to the number of groups for which observed and expected frequencies are available, and *r* is the number of restrictions or constraints imposed on the given comparison.

The quantity *X*2 is a measure of the extent to which, in a given situation, pairs of observed and expected frequencies agree.

disagreement

the poorer the agreement

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**The Decision Rule**

>> Small Expected Frequencies

In the literature the point is frequently made that the approximation of *X*2 to x2 is not strictly valid when some of the expected frequencies are small.

the earlier ones

Some writ- ers, especially the earlier ones, suggest lower limits of 10, whereas others suggest that all expected frequencies should be no less than 5.

adjacent categories may be combined to achieve the suggested min- imum

**12.3 TESTS OF GOODNESS-OF-FIT**

As we have pointed out, a goodness-of-fit test is appropriate when one wishes to decide if an observed distribution of frequencies is incompatible with some preconceived or hypothesized distribution.

**EXAMPLE 12.3.1 The Normal Distribution**

Cranor and Christensen (A-1) conducted a study to assess short-term clinical, economic, and humanistic outcomes of pharmaceutical care services for patients with diabetes in community pharmacies. For 47 of the subjects in the study, cholesterol levels are sum- marized in Table 12.3.1.

We wish to know whether these data provide sufficient evidence to indicate that the sample did not come from a normally distributed population. Let a = .05

**Alternatives** Although one frequently encounters in the literature the use of chi- square to test for normality, it is not the most appropriate test to use when the hypoth- esized distribution is continuous. The Kolmogorov–Smirnov test, described in Chapter 13, was especially designed for goodness-of-fit tests involving continuous distributions.

**EXAMPLE 12.3.2 The Binomial Distribution**

In a study designed to determine patient acceptance of a new pain reliever, 100 physicians each selected a sample of 25 patients to participate in the study. Each patient, after trying the new pain reliever for a specified period of time, was asked whether it was preferable to the pain reliever used regularly in the past.

The results of the study are shown in Table 12.3.4.

new pain reliever - Nueva analgésico

We are interested in determining whether or not these data are compatible with the hypothesis that they were drawn from a population that follows a binomial distribution. Again, we employ a chi-square goodness-of-fit test.

**EXAMPLE 12.3.3 The Poisson Distribution**

A hospital administrator wishes to test the null hypothesis that emergency admissions follow a Poisson distribution with l = 3. Suppose that over a period of 90 days the num- bers of emergency admissions were as shown in Table 12.3.6.

**EXAMPLE 12.3.4 The Uniform Distribution**

The flu season in southern Nevada for 2005–2006 ran from December to April, the coldest months of the year. The Southern Nevada Health District reported the numbers of vaccine-preventable influenza cases shown in Table 12.3.9. We are interested in knowing whether the numbers of flu cases in the district are equally distributed among the five flu season months. That is, we wish to know if flu cases follow a uniform distribution.

**EXAMPLE 12.3.5**

A certain human trait is thought to be inherited according to the ratio 1:2:1 for homozygous dominant, heterozygous, and homozygous recessive. An examination of a simple random sample of 200 individuals yielded the following distribution of the trait: dominant, 43; heterozygous, 125; and recessive, 32. We wish to know if these data provide sufficient evidence to cast doubt on the belief about the distribution of the trait.

**12.4 TESTS OF INDEPENDENCE**

Another, and perhaps the most frequent, use of the chi-square distribution is to test the null hypothesis that two criteria of classification, when applied to the same set of enti- ties, are independent.

We say that two criteria of classification are independent if the distribution of one criterion is the same no matter what the distribution of the other cri- terion.

For example, if socioeconomic status and area of residence of the inhabitants of a certain city are independent, we would expect to find the same proportion of families in the low, medium, and high socioeconomic groups in all areas of the city.

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**The Contingency Table**

the *r* rows represent the var- ious levels of one criterion of classification and the *c* columns represent the various levels of the second criterion. Such a table is generally called a *contingency table*.

We will be interested in testing the null hypothesis that in the population the two criteria of classification are independent. If the hypothesis is rejected, we will conclude that the two criteria of classification are not independent.

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**Calculating the Expected Frequencies**

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**Observed Versus Expected Frequencies**

**EXAMPLE 12.4.1**

In 1992, the U.S. Public Health Service and the Centers for Disease Control and Prevention recommended that all women of childbearing age consume 400 mg of folic acid daily to reduce the risk of having a pregnancy that is affected by a neural tube defect such as spina bifida or anencephaly. In a study by Stepanuk et al. (A-3), 693 pregnant women called a teratology information service about their use of folic acid supplementation. The researchers wished to determine if preconceptional use of folic acid and race are independent. The data appear in Table 12.4.3.

of childbearing age - en edad fértil

pregnancy - el embarazo

as spina bifida or anencephaly - como espina bífida o anencefalia

teratology - teratología

Race - Raza

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**Small Expected Frequencies**

may be encountered

He suggests that for contingency tables with more than 1 degree of freedom a minimum expectation of 1 is allowable if no more than 20 percent of the cells have expected frequencies of less than 5. To meet this rule, adjacent rows and/or adjacent columns may be com- bined when to do so is logical in light of other considerations. If *X*2 is based on less than 30 degrees of freedom, expected frequencies as small as 2 can be tolerated. We did not experience the problem of small expected frequencies in Example 12.4.1, since they were all greater than 5.

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**The 2** : **2 Contingency Table**

**EXAMPLE 12.4.2**

According to Silver and Aiello (A-4), falls are of major concern among polio survivors. Researchers wanted to determine the impact of a fall on lifestyle changes. Table 12.4.6 shows the results of a study of 233 polio survivors on whether fear of falling resulted in lifestyle changes.

**Small Expected Frequencies** The problems of how to handle small expected frequencies and small total sample sizes may arise in the analysis of 2 \* 2 contingency tables. Cochran (5) suggests that the x2 test should not be used if *n* 6 20 or if 20 6 *n* 6 40 and any expected frequency is less than 5. When *n* = 40, an expected cell frequency as small as 1 can be tolerated.

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**Yates’s Correction**

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**Tests of Independence: Characteristics**

**12.5 TESTS OF HOMOGENEITY**

A characteristic of the examples and exercises presented in the last section is that, in each case, the total sample was assumed to have been drawn before the entities were classified according to the two criteria of classification.

That is, the observed number of entities falling into each cell was determined after the sample was drawn.

reliever -