QFT Lecture 13

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Recall loop stuff.

$$\frac{1}{i}\tilde{\Delta}_{full}(k^2) = \frac{1}{i}\frac{1}{k^2+m^2-\pi(k^2)-i\epsilon}$$

$$\frac{(ig)^2}{2i^2}\int\frac{d^dl}{(2\pi)^d}\frac{1}{l^2+m^2-i\epsilon}\frac{1}{(l+k)^2+m^2-i\epsilon} - i(Ak^2+Bm^2)$$

Impose:
i) $\pi(-m^2)=0$ such that m is the physical mass
ii) $\pi'(-m^2)=0$ such that the propagator has the correct normalization as $k^2 \to -m^2$.

We did the loop integral above by applying the Feynman trick, and then Wick rotation (to Euclidean time). The answer that we got was:

$$\pi(k^2) = \left[\frac{g^2}{2}\int_0^1 dx \int\frac{d^d\bar{q}}{(2\pi)^d}\frac{1}{(\bar{q}^2+D)^2}\right] - (Ak^2+Bm^2)$$

$$\int\frac{d^d\bar{q}}{(2\pi)^d}\frac{(\bar{q}^2)^a}{(\bar{q}^2+D)^b} = \frac{\Gamma(b-a-d/2)\Gamma(a+d/2)}{(2\pi)^{d/2}\Gamma(b)\Gamma(d/2)}D^{-(b-a-d/2)}$$

The gamma function at special points:

$$\Gamma(n) = (n-1)!$$

$$\Gamma(n+1/2) = \frac{(2n)!}{n!4^n}\sqrt(\pi)$$

Poles at $x=0$ or $x=-n$; asymptotic dependence:

$$\Gamma(x)_{x\to 0} \sim \frac{1}{x} - \gamma + O(x)$$

$$\Gamma(-n+x)_{x\to 0} \sim \frac{(-1)^n}{n!}\left(\frac{1}{x} - \gamma + 1 + \frac{1}{2} + ... + \frac{1}{n} + O(x)\right)$$

Using these formulas, the result of our loop integral is:

$$\frac{\Gamma(2-d/2)}{(4\pi)^{d/2}}D^{-(2-d/2)}$$

We are interested in $d = 6$, where this diverges. We will pretend therefore that $d = 6 - \epsilon$:

$$\frac{\Gamma(-1+\epsilon/2)}{(4\pi)^{3-\epsilon/2}} D^{1-\epsilon/2}$$

We will therefore say that:

$$\int \frac{d^d\bar{q}}{(2\pi)^d} \frac{1}{(\bar{q}^2+D)^2} = \frac{1}{(4\pi)^3}\left(1+\frac{\epsilon}{2}\mathrm{ln}(4\pi)+\mathrm{O}(\epsilon^2)\right)(-1)\left(\frac{2}{\epsilon}-\gamma+1+O(\epsilon)\right)$$

$$D\left(1-\frac{\epsilon}{2}\mathrm{lnD}+\mathrm{O}(\epsilon^2)\right) =$$

$$= \frac{D}{(4\pi)^3}\left(\frac{2}{\epsilon}-\gamma+1+\mathrm{ln}4\pi-\mathrm{lnD}+\mathrm{O}(\epsilon)\right)$$

This is called method of dimensional regularization. We can rewrite the answer as:

$$\frac{D}{(4\pi)^3}\left(\frac{2}{\epsilon}+1+\mathrm{ln}(4\pi/\mathrm{e}^\gamma)-\mathrm{lnD}+\mathrm{O}(\epsilon)\right)$$

$$\pi(k^2) = -\frac{g^2}{2(2\pi)^3}\left[\int_0^1 dxD(2/\epsilon+1+\mathrm{ln}(4\pi/\mathrm{e}^\gamma)-\int_0^1 \mathrm{dxDlnD}\right]-(Ak^2+Bm^2)$$

The first integral gives:

$$\left(\frac{k^2}{6}+m^2\right)\left(\frac{2}{\epsilon}+1+\mathrm{ln}\frac{4\pi}{\mathrm{e}^\gamma}\right)$$

Which explains the fact that we want to substract terms proportional to k^2 and m^2. This tells us that A and B should contain terms:

$$A = Z_\phi - 1 = -\frac{1}{6}\frac{g^2}{(4\pi)^3}\frac{1}{\epsilon}+...$$

$$B = Z_m - 1 = -\frac{g^2}{(4\pi)^3}\frac{1}{\epsilon}+...$$

Where the ... represent non-divergent terms. Call these a and b, and the leftovers are:

$$\pi(k^2) = \frac{q^2}{2(2\pi)^3}\int_0^1 dxD\mathrm{lnD}+(\mathrm{ak}^2+\mathrm{bm}^2)$$

In order to satisfy $\pi(-m^2) = 0, \pi'(-m^2) = 0$ we get:

$$\pi(k^2) = \frac{g^2}{2(4\pi)^3}\int_0^1 dxD\mathrm{ln}\frac{\mathrm{D}}{\mathrm{D}_0}-\frac{1}{12}\frac{\mathrm{g}^2}{(4\pi)^3}(\mathrm{k}^2+\mathrm{m}^2)$$

Where $D_0 = D(k^2 = -m^2)$.

Read chapter 14. On Thursday we do chapter 16.