QFT Lecture 17

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Continuous symmetries

Reference: Srednicki 22

For a symmetry that is path connected to the identity, we know: Classically, Noether's theorem, $\partial_\mu j^\mu = 0$ on the e.o.m. Quantum, Ward identity:

$$\partial_\mu \langle j^\mu(x_0)\mathcal{O}(x_1)\rangle = \delta(x_0 - x_1)\langle \delta\mathcal{O}(x_1)\rangle$$

From this we can deduce:

$$[\hat{Q}, \mathcal{O}] = \delta\mathcal{O}$$

Classically, we have $\frac{\delta S}{\delta \phi} = 0$ on the e.o.m. In quantum, $\langle \frac{\delta S}{\delta\phi(x_0)}, \phi(x_1)\rangle =$? To compute this, first note that, by a twisted analog of the fundamental theorem of calculus:

$$\int D\phi \frac{\delta(\phi(x_1)e^{iS})}{\delta\phi(x_0)} = 0$$

Where we have used the fact that the fields decay to 0 at ∞. Schwinger - Dyson eq.:

$$\left\langle \frac{\delta S}{\delta\phi(x_0)}\phi(x_1)...\phi(x_n)\right\rangle = i\delta(x_1 - x_0)\langle\phi(x_2)...\phi(x_n)\rangle + ...$$

Continuous symmetries of two types:
a) internal. ex. $\phi \to \phi + \text{const}$
b) spacetime, ex. translations $\delta\phi = a^\mu \delta_\mu \phi$

Discrete symmetries

Reference: Srednicki 23

Look at $\phi \to -\phi$. As a consequence, if we have an even number of incoming particles, we should have an even number of outgoing.

Let $\hat{Z}$ be the operator that generates the symmetry transformation: $\hat{Z}^{-1}\hat{\phi}\hat{Z} = -\hat{\phi}$. Let's suppose our theory is symmetric, i.e. $[\hat{Z}, \hat{H}] = 0$. Also suppose $\hat{Z}|0\rangle = |0\rangle$. Then $\langle 0|\hat{\phi}|0\rangle = 0$.

Recall that for a Lorentz transformation:

$$U^{-1}(\Lambda)\hat{\phi}(x)U(\Lambda) = \hat{\phi}(\Lambda^{-1}x)$$

Parity:

$$U^{-1}(\mathbb{P})\hat{\phi}(x)U(\mathbb{P}) = \hat{\phi}(\mathbb{P}^{-1}x)$$

$$\mathbb{P} = \begin{pmatrix} 1 & & & \\ & -1 & & \\ & & -1 & \\ & & & -1 \end{pmatrix}$$

Let $P = U(\mathbb{P})$, and define a pseudoscalar as: $P^{-1}\phi(x)P = -\phi(\mathbb{P}^{-1}x)$. For interactions with only even powers of ϕ, it doesn't matter if ϕ is scalar or pseudoscalar, but if we have odd terms, need it to be scalar to have symmetry.