QFT Lecture 19

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Renormalization group (Srednicki 28); Wilsonian effective field theory (Srednicki 29, PS 12.1, Zee VI.8)

Renormalization group

bare quantities:

$$\phi_0 = Z_\phi^{-1/2}\phi$$

$$m_0 = Z_\phi^{-1/2} Z_m^{1/2} m$$

$$g_0 = Z_\phi^{-3/2} Z_g g \mu^{\epsilon/2}$$

$$\alpha_0 = Z_\phi^{-3} Z_g^2 \mu^\epsilon \alpha$$

Taking the log of the last eq.:

$$ln\alpha_0 = -3[\alpha(-1/6\epsilon + \text{finite}) + O(\alpha^3)] + 2[\alpha(-1/\epsilon + \text{finite}) + O(\alpha^3)] + \epsilon ln\mu + ln\alpha$$

LHS should be independent of μ:

$$\frac{dln\alpha_0}{\d{l}n\mu} = 0 = \frac{d\alpha}{d\mu}(-3/2\epsilon + \text{finite}) + \epsilon + \frac{dln\alpha}{dln\mu}$$

$$\frac{d\alpha}{dln\mu} = a\alpha + b\alpha^2 + ...$$

$$0 = (a\alpha + b\alpha^2 + ...)(-3/2\epsilon + finite) + O(\alpha^2) + \epsilon + 1/\alpha(a\alpha + b\alpha^2 + ...)$$

To order α_0:

$$a = -\epsilon$$

$$b = -3/2$$

$$\frac{d\alpha}{dln\mu} = -\epsilon\alpha - 3/2\alpha^2$$

Important that this is < 0, so the theory is asymptotically free.

Wilsonian EFT

UV cutoff:

$$Z = \int_{|k|<\Lambda_0} D\phi e^{-S_0}$$

$$S_0 = \int d^d x \left(\frac{1}{2}(\partial \phi)^2 + \frac{1}{2} m^2(\Lambda_0)\phi^2 + \frac{1}{4!}\lambda(\Lambda_0)\phi^4 \right)$$

Goal: integrate out $\Lambda < |k| < \Lambda_0$. We first split the field into low momentum and high momentum modes:

$$\phi(x) \to \phi(x) + \hat{\phi}(x)$$

Now:

$$Z = \int D\phi_{|k|<\Lambda} \exp\left[\int d^d x \big(\frac{1}{2}(\partial \phi)^2 + \frac{1}{2} m^2(\Lambda_0)\phi^2 + \frac{1}{4!}\lambda(\Lambda_0)\phi^4\big)\right] \cdot$$

$$\cdot \int D\phi_{\Lambda<|k|<\Lambda_0} \exp\left[\int d^d x \big(\frac{1}{2}(\partial \phi)^2 + \frac{1}{2} m^2(\Lambda_0)\phi^2 + \frac{1}{4!}\lambda(\Lambda_0)\phi^4\big)\right]$$