QFT Lecture 8

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How to think about the scalar field?

Let's look for a second about an ocean wave. The e.o.m. is:

$$\left(-\frac{\partial^2}{\partial t^2}+c_0^2\frac{\partial^2}{\partial x^2}\right)\phi(\mathbf{x},t)$$

The action, which is NOT Lorentz invariant:

$$S=\int dtd^3x\left(\frac{1}{2}(\partial_t\phi)^2-\frac{c_0^2}{2}(\partial_x\phi)^2\right)$$

This wave does not have an abvious particle interpretation. In the quantum case, it's important that we cannot have a simply flat wave, not even in the vacuum state:

$$\langle 0|T\hat{\phi}_1\hat{\phi}_2|0\rangle$$

Eigenvalues and eigenstates of the field:

$$\hat{\phi}(x)|f(x)\rangle = f(x)|f(x)\rangle$$

ANY 4D configuration is an eigenstate of the field, as long as it evolves according to the Klein-Gordon eq. Notice that it evolves backward in time:

$$\hat{\phi}(\mathbf{x},t)=e^{iHt}\hat{\phi}(\mathbf{x},0)e^{-iHt}$$

$$\hat{\phi}(\mathbf{x},t)|f(\mathbf{x}),t\rangle = f(\mathbf{x})|f(\mathbf{x}),t\rangle$$
$$e^{iHt}\hat{\phi}(\mathbf{x},0)e^{-iHt}|f(\mathbf{x}),t\rangle = f(\mathbf{x})|f(\mathbf{x}),t\rangle$$
$$\hat{\phi}(\mathbf{x},0)|f(\mathbf{x}),0\rangle = f(\mathbf{x})|f(\mathbf{x}),0\rangle$$

So:

$$|f(\mathbf{x}),t\rangle = e^{iHt}|f(\mathbf{x}),0\rangle$$

Bubble diagrams

$$\mathcal{L}_{int} = -\frac{1}{2}(Z_\phi - 1)(\partial\phi)^2 - \frac{1}{2}(Z_m - 1)m^2\phi^2 + \frac{1}{3!}Z_g g\phi^3 + Y\phi + \Lambda$$

Λ is related to the cosmological constant. Somehow, quantum mechanics forces all these counter-terms on us whenever we try to introduce the ϕ^3 interaction. Last time we studied:

$$Z_1(J) = \int D\phi e^{i\int d^4x(\mathcal{L}_0 + \frac{1}{3!}Z_g g + J\phi)}$$

There's a theorem saying that:

$$Z_1(J) = e^{\sum connected\ diagrams}$$

To prove it, consider the sum:

$$\sum_{\{n_I\}} D^{\{n_I\}}$$

The collection of n_I's shows how many of each connected bubbles are in D, so $D = \Pi_I(C_I)^{n_I}$. Introduce a symmetry factor $s_D = \Pi_I(n_I!)$.

$$\sum_{\{n_I\}} D^{\{n_I\}} = \sum_{\{n_I\}} \Pi_I \frac{C_I^{n_I}}{n_I!} = \Pi_I \sum_{\{n_I\}} \frac{C_I^{n_I}}{n_I!} = \Pi_I e^{C_I} = e^{\sum C_I}$$

Think about why we can interchange the sum and product!