Statistical mechanics HW1

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Problem 2

c)

$$\rho(y, y') = (\alpha/\pi)^{1/2} e^{-\alpha/2(y^2+y'^2)}$$

$$\rho(Q + r/2, Q - r/2) = (\alpha/\pi)^{1/2} e^{-\alpha/2(2Q^2+r^2/4)}$$

$$W(Q, P) = (\alpha/\pi)^{1/2} \int dr e^{ipr/\hbar} e^{-\alpha(Q^2+r^2/4)}$$

$$W(Q, P) = (\alpha/\pi)^{1/2} e^{-\alpha Q^2} e^{-p^2/\alpha\hbar^2} \int dr e^{-\alpha/4(r-2ip/\alpha\hbar)^2}$$

$$W(Q, P) = 2e^{-\alpha Q^2} e^{-p^2/\alpha\hbar^2}$$

Problem 6

$$Z = \sum_n e^{-\hbar\omega(n+1/2)/T} = e^{-\hbar\omega/2T} \frac{1}{1 - e^{-\hbar\omega/T}}$$

$$F(T) = -T\ln(Z) = \hbar\omega/2 + T\ln(1 - e^{-\hbar\omega/T})$$

$$S(T) = -\frac{dF}{dT} = -\ln(1 - e^{-\hbar\omega/T}) + \frac{\hbar\omega}{T} \frac{1}{e^{\hbar\omega/T} - 1}$$

$$C_V(T) = -T\frac{dS}{dT} = -\frac{\hbar\omega}{T} \frac{2}{e^{\hbar\omega/T} - 1} + \left(\frac{\hbar\omega}{T}\right)^2 e^{\hbar\omega/T} \frac{1}{(e^{\hbar\omega/T} - 1)^2}$$

$$E(T) = \hbar\omega \left(\frac{1}{2} + \frac{1}{e^{\hbar\omega/T} - 1}\right)$$

Problem 8

a)

$$\Gamma(x) = \int_{-\infty}^{\infty} dy y^{x-1} e^{-y} = \int_{-\infty}^{\infty} dy e^{-y+(x-1)\ln y}$$

$$f'(y) = (y - (x-1)\ln(y))' = 1 - \frac{x-1}{y} \Rightarrow y_0 = x - 1$$

$$f''(y_0) = \left.\frac{x-1}{y^2}\right|_{y_0} = \frac{1}{x-1}$$

$$\Gamma(x) = e^{-(x-1)}(x-1)^{(x-1)}\sqrt{2\pi(x-1)}$$

b)

$$K_0(x) = \int_0^{\infty} dy e^{-x\cosh(y)}$$

$$f'(y) = \sinh(y) = 0 \Rightarrow y_0 = 0$$

$$K_0(x) = e^{-x}\sqrt{2\pi/x}$$

c)