# QFT Lecture 18

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## Non-abelian (internal) symmetries

Reference: ch. 24

$$S = \int d^4x \sum \left( -\frac{1}{2}(\partial \phi_i)^2 - \frac{1}{2}m^2\phi_i^2 \right) + \lambda \left( \sum \phi_i^2 \right)^2$$

Invariant under $\phi_i = R_{ij}\phi_j$, where $R_{ij} \in O(n)$. Lie algebra of $O(n)$ given by $T^a$ such that:

$$[T^a, T^b] = if^{abc}T^c$$

## Renormalization schemes and renormalization group

Reference: chapter 27, 28

Define $\alpha = g^2/4\pi$. Then:

$$\pi(k^2) = -\frac{\alpha}{2}\left(\frac{k^2}{6} + m^2\right)\left(\frac{2}{\epsilon} + 1 - \gamma + \ln(4\pi)\right) - (Ak^2 + Bm^2) + \frac{\alpha}{2}\int dx D \ln\frac{D}{\mu^2}$$

Renormalization scheme so far is called the on-shell (OS) scheme: $\pi(-m^2) = 0$ and $\pi'(-m^2) = 0$. This is one way of fixing A and B. Sometimes we don't want to do this. Consider the case $m = 0$, for which the two above conditions hold automatically (independent of A and B). For example, look at modified minimal substraction scheme: $\overline{MS}$. Minimal substraction means removing only the diverging term. Modified if also remiving the $\gamma$ and logarithm. Thus, for $\overline{MS}$:

$$\pi(k^2) = -\frac{\alpha}{2}\left(\frac{k^2}{6} + m^2\right) + \frac{\alpha}{2}\int dx D \ln\frac{D}{\mu^2}$$

Let's see what this implies about m and the physical mass. The physical mass is the pole $\tilde{\Delta}(-m^2_{\text{phys}}) = 0$. So:

$$m^2_{\text{phys}} = m^2 - \pi(m^2_{\text{phys}})$$

Notice that the last term is of order $\alpha$, thus small. So we can use $m = m_{\text{phys}}$ in the expression of $\pi$. (And then proceed iteratively if we have nothing better to do.)

$$m_{\text{phys}}^2 = m^2 \left[1 - \frac{\alpha}{2}\int dx(1-x+x^2)\text{ln}\frac{m^2}{\mu^2} - \frac{\alpha}{2}\{\text{some numbers}\}\right]$$

$$m_{\text{phys}}^2 = m^2 \left[1 + \frac{5}{12}\alpha\text{ln}\frac{\mu^2}{m^2} + \frac{5}{12}\alpha c' + O(\alpha^2)\right]$$

$$\text{ln}\frac{m_{\text{phys}}}{m} = \frac{5}{12}\alpha\text{ln}\frac{\mu}{m} + \frac{5}{24}\alpha c' + O(\alpha^2)$$

Since we introduced $\mu$ just for the purposes of dimensional regularization, the physical mass at low energies should be independent of $\mu$ at high energies. (HW shows that $\mu$ is a sort of ultraviolet cutoff.)

$$0 = \frac{d(\text{ln}m)}{d(\text{ln}\mu)} + \frac{5\alpha}{12} + \frac{5}{12}\frac{d\alpha}{d(\text{ln}\mu)} - \frac{5\alpha}{12}\frac{d(\text{ln}m)}{d(\text{ln}\mu)} + \frac{5}{24}c'\frac{d\alpha}{d(\text{ln}\mu)}$$

Will show at some point that the last 3 terms are $O(\alpha^2)$. Therefore the anomalous dimension of mass is:

$$\gamma_m(\alpha) = \frac{d(\text{ln}m)}{d(\text{ln}\mu)} = -\frac{5\alpha}{12}$$

Note that in 2→2 scattering we have eq. 27.22:

$$|\mu|^2 \sim \left(1 - \frac{3}{2}\alpha\text{ln}\frac{s}{\mu^2} + ...\right)$$

One way to think about this: $\alpha$ has a logarithmic dependence of energy. Another way to think about this is that we need to have $s \sim \mu^2$ such that the expansion doesn't diverge.

Let's briefly talk about the renormalization group:

$$\mathcal{L} = \frac{1}{2}Z_\phi(\partial\phi)^2 - \frac{1}{2}Z_m m^2\phi^2 + \frac{1}{6}Z_g g\mu^{\epsilon/2}\phi^3 + Y\phi + \Lambda$$

$$A = Z_\phi - 1 = -\frac{\alpha}{6\epsilon} + \{\text{finite}\} + O(\alpha^2)$$

$$B = Z_m - 1 = -\frac{\alpha}{\epsilon} + \{\text{finite}\} + O(\alpha^2)$$

$$C = Z_g - 1 = -\frac{\alpha}{6\epsilon} + \{\text{finite}\} + O(\alpha^2)$$

The idea is that all renormalization schemes remove the divergence, so they give basically the same result, up to the finite terms.