Seminar 3 - Exerciții rezolvate

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Cuprins

Teorema lui Master pentru "subtract-and-conquer"

$T(n) = a \cdot T(n-b) + f(n), a, b \in \mathbb{N}^*$
$T(i) \in \Theta(1), \forall i \in \mathbb{N}^*, i < b$[a]
$f(n) \in O(n^p)$

[a]Pentru simplitatea formulelor am folosit și cazul de bază pentru $n = 0$.

Notăm $h = \left\lfloor \frac{n}{b} \right\rfloor$.

$$\begin{aligned}
T(n) &= a \cdot T(n-b) & &+ f(n) \\
a \cdot T(n-b) &= a^2 \cdot T(n-2\cdot b) & &+ a \cdot f(n-b) \\
a^2 \cdot T(n-2\cdot b) &= a^3 \cdot T(n-3\cdot b) & &+ a^2 \cdot f(n-2b) \\
&\dots \\
a^k \cdot T(n-k\cdot b) &= a^{k+1} \cdot T(n-(k+1)\cdot b) & &+ a^k \cdot f(n-k\cdot b) \\
&\dots \\
a^{h-1} \cdot T(n-(h-1)\cdot b) &= a^h \cdot \underbrace{T(n-h\cdot b)}_{\Theta(1)} & &+ a^{h-1} \cdot f(n-(h-1)b)
\end{aligned}$$

$$\boxed{\Rightarrow T(n) = a^{\lfloor \frac{n}{b} \rfloor} \cdot \Theta(1) + \sum_{k=0}^{h-1} a^k \cdot f(n-k\cdot b)}$$

$$Q(n) = \sum_{k=0}^{h-1} a^k \cdot f(n-k\cdot b) = \sum_{k=0}^{h-1} a^k O((n-k\cdot b)^p) = \sum_{k=0}^{h-1} a^k O(n^p)$$

$$\sum_{i=0}^{j-1} a^i = \frac{a^j - 1}{a-1}$$

$$\sum_{k=0}^{h-1} a^k = \begin{cases} O(1) & a < 1 \text{ deoarece } \lim_{n\to\infty} a^{\lfloor \frac{n}{b} \rfloor} = 0 \\ O(n) & a = 1 \\ O(a^h) & a > 1 \end{cases}$$

$$Q(n) = \begin{cases} O(n^p) & a < 1 \\ O(n^{p+1}) & a = 1 \\ O(a^{\lfloor \frac{n}{b} \rfloor} \cdot n^p) & a > 1 \end{cases}$$

$$a^{\lfloor \frac{n}{b} \rfloor} \cdot \Theta(1) = \begin{cases} O(1) & a < 1 \\ O(n) & a = 1 \\ O(a^{\lfloor \frac{n}{b} \rfloor}) & a > 1 \end{cases}$$

$$\boxed{T(n) = \begin{cases} O(n^p) & a < 1 \\ O(n^{p+1}) & a = 1 \\ O(a^{\lfloor \frac{n}{b} \rfloor} \cdot n^p) & a > 1 \end{cases}}$$