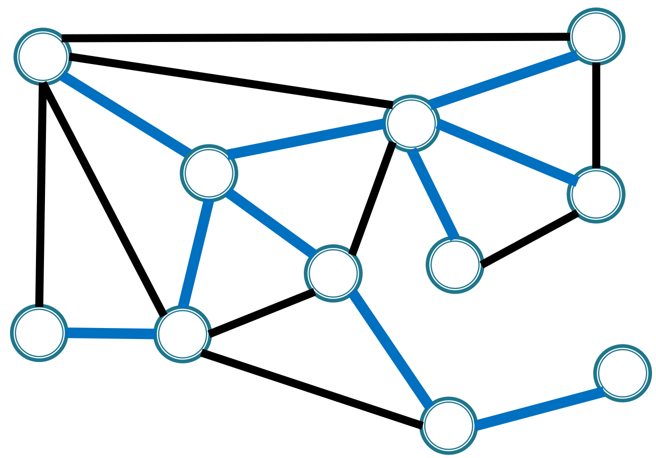
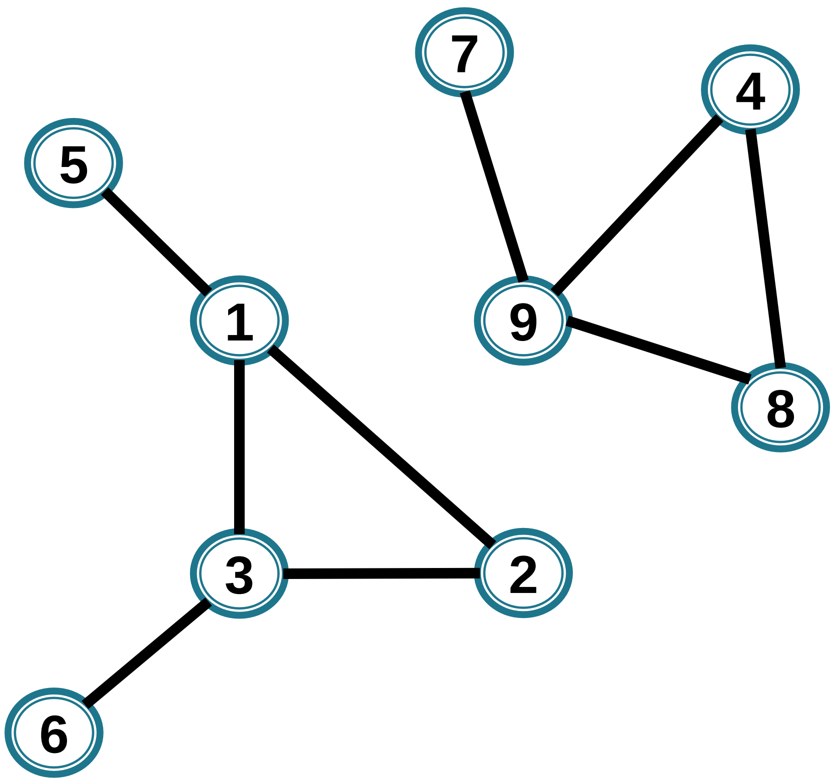
Arbori parțiali de cost minim

Arbori parțiali



- “Scheletul” grafului
- Transmiterea de mesaje în rețea astfel încât mesajul să ajungă o singură dată în fiecare vârf
- Conectare fără redundanță + cu cost minim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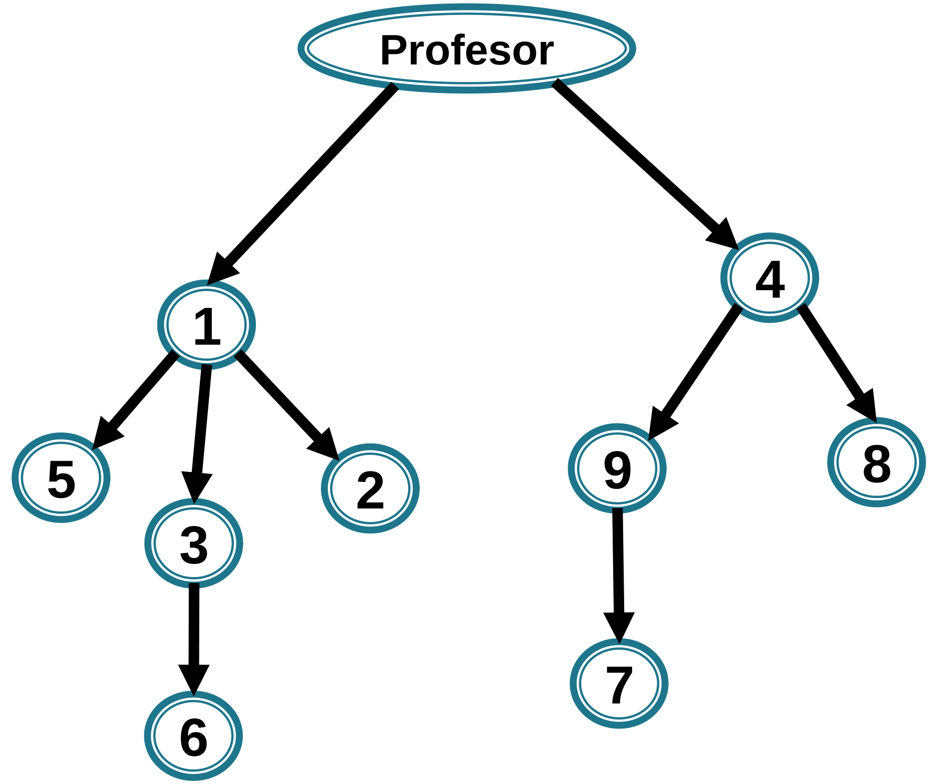
Profesor
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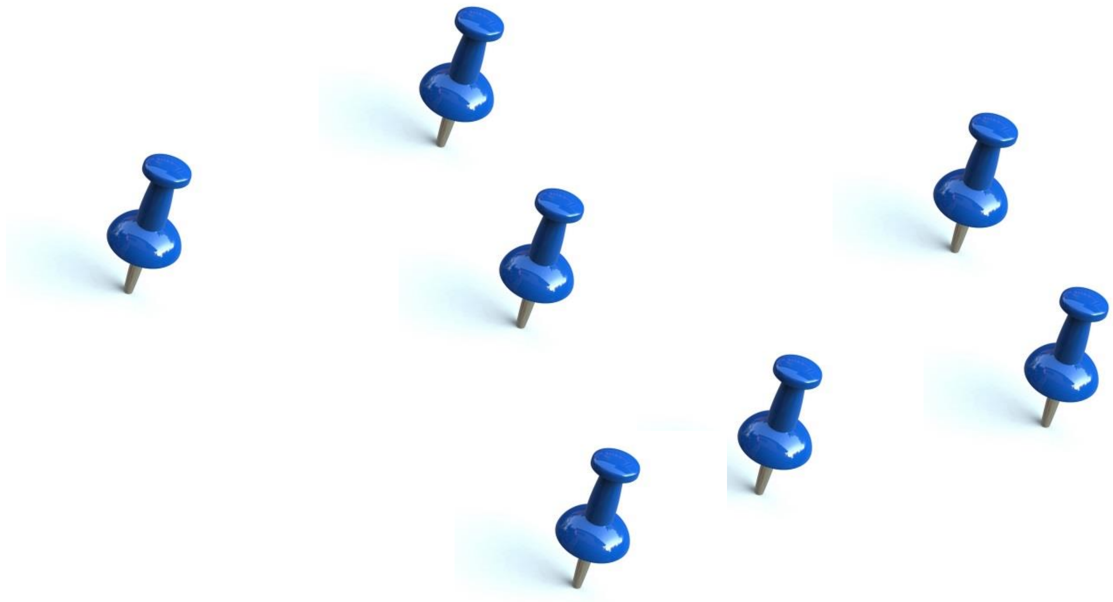
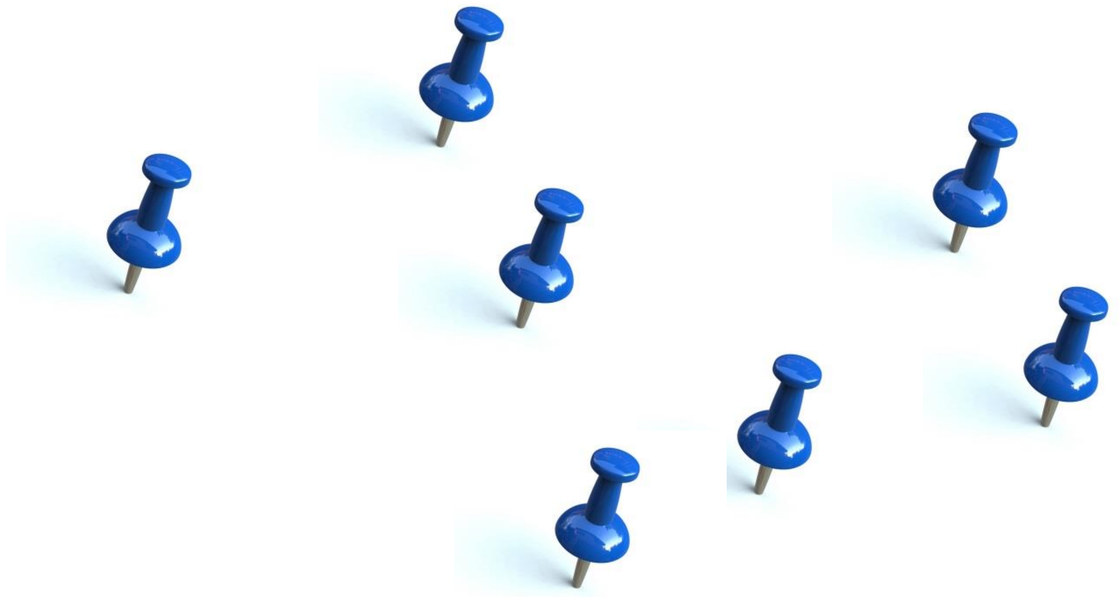
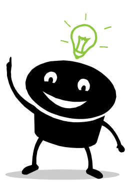
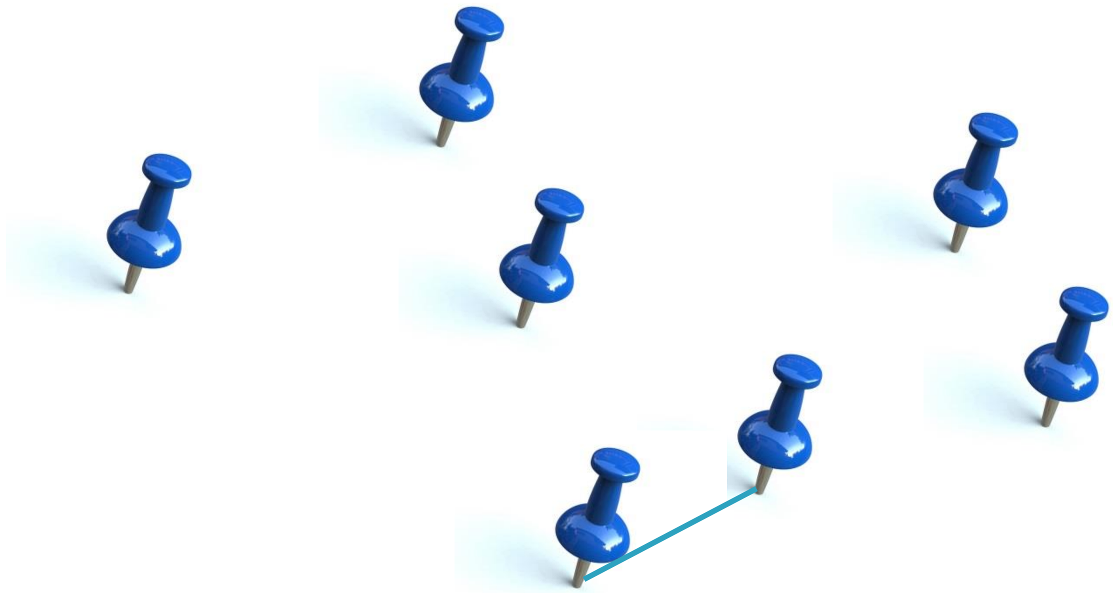


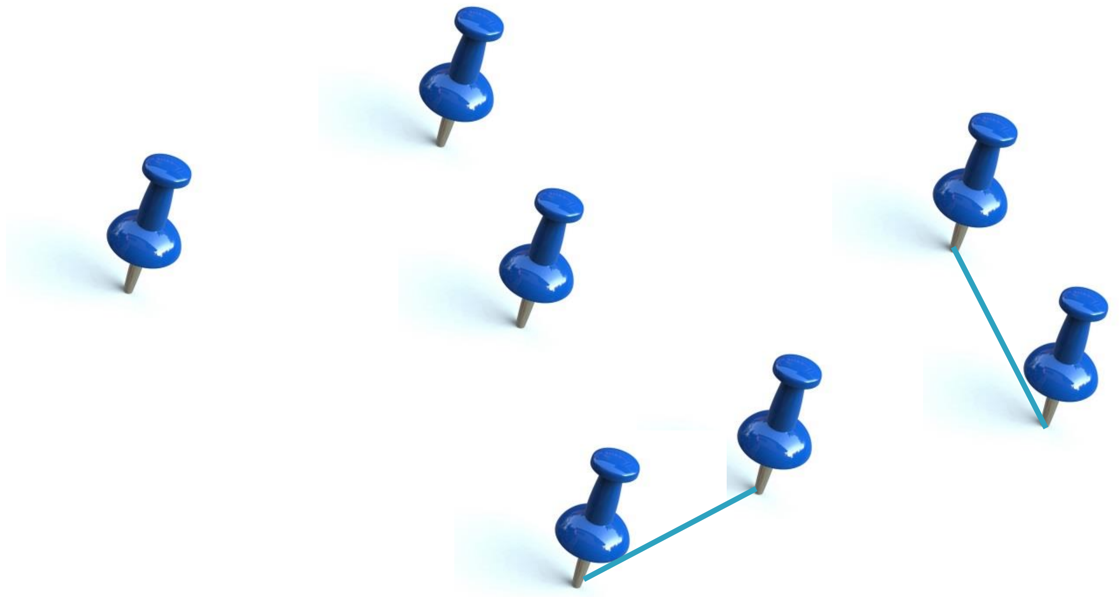
Conectați pinii astfel încât să folosiți cât mai puțin cablu

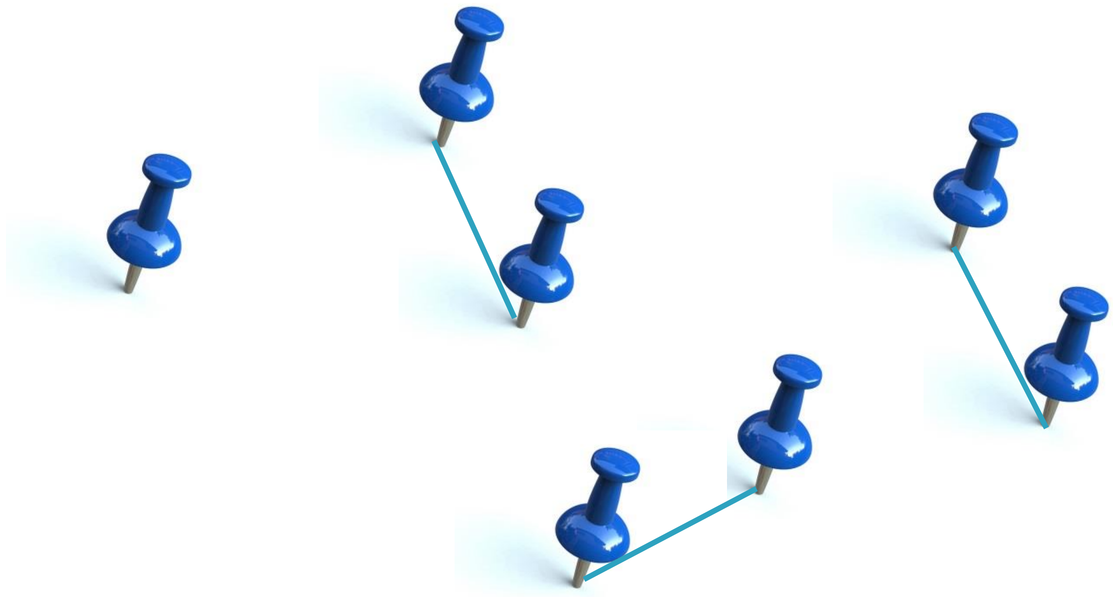


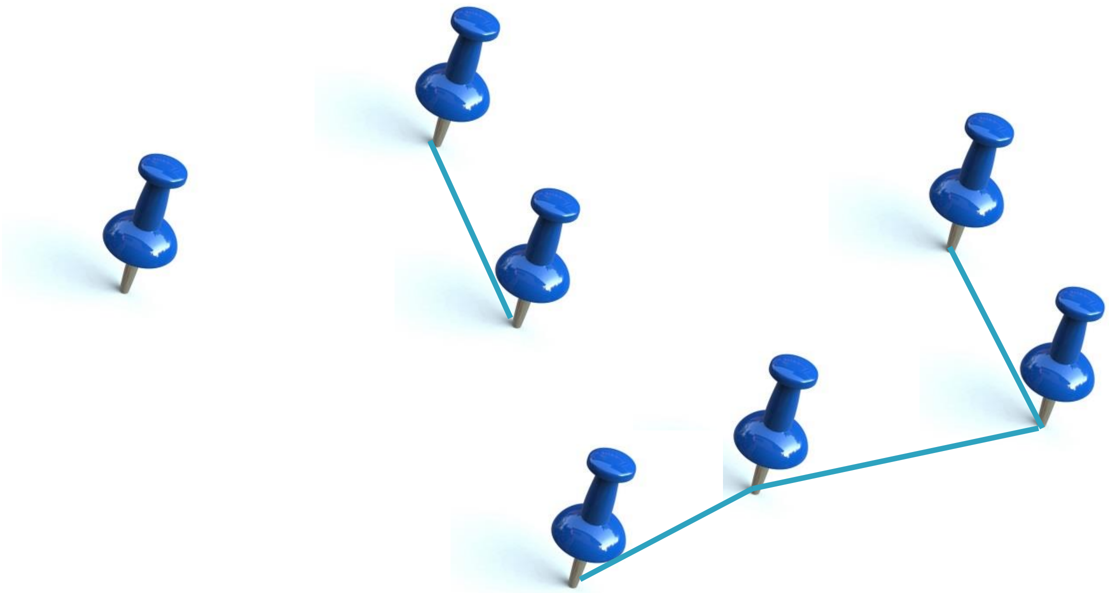


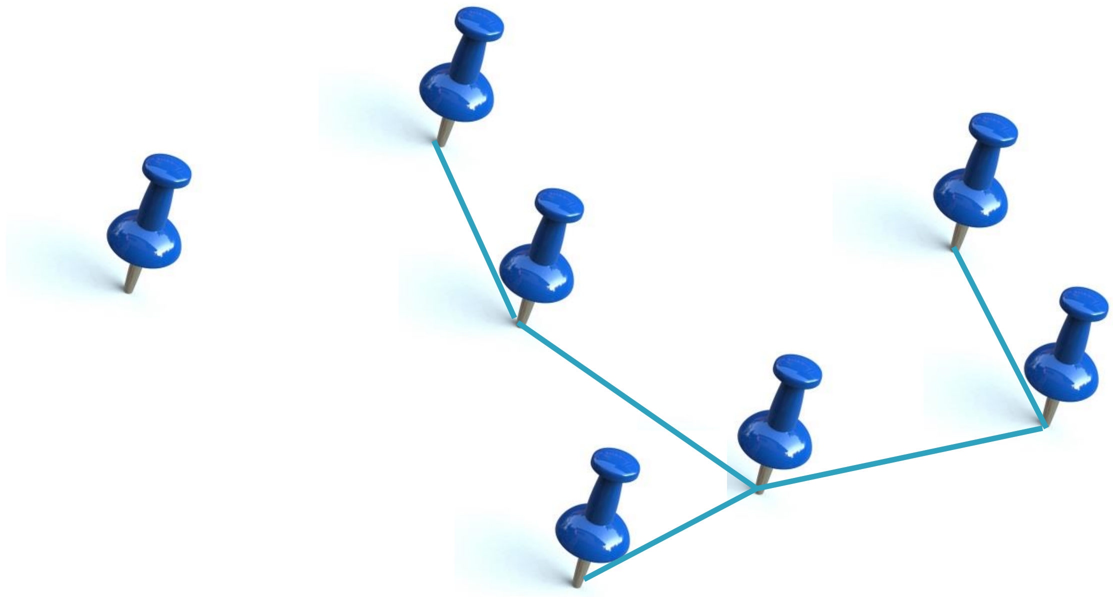
- Legăm pini apropiați
- Nu închidem cicluri

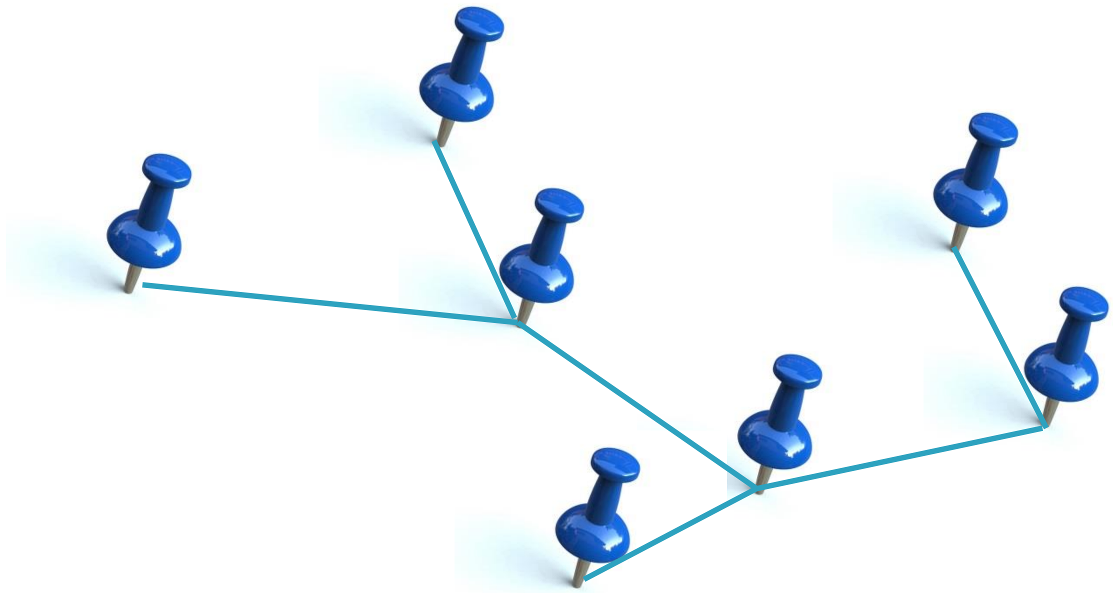


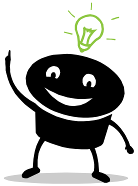












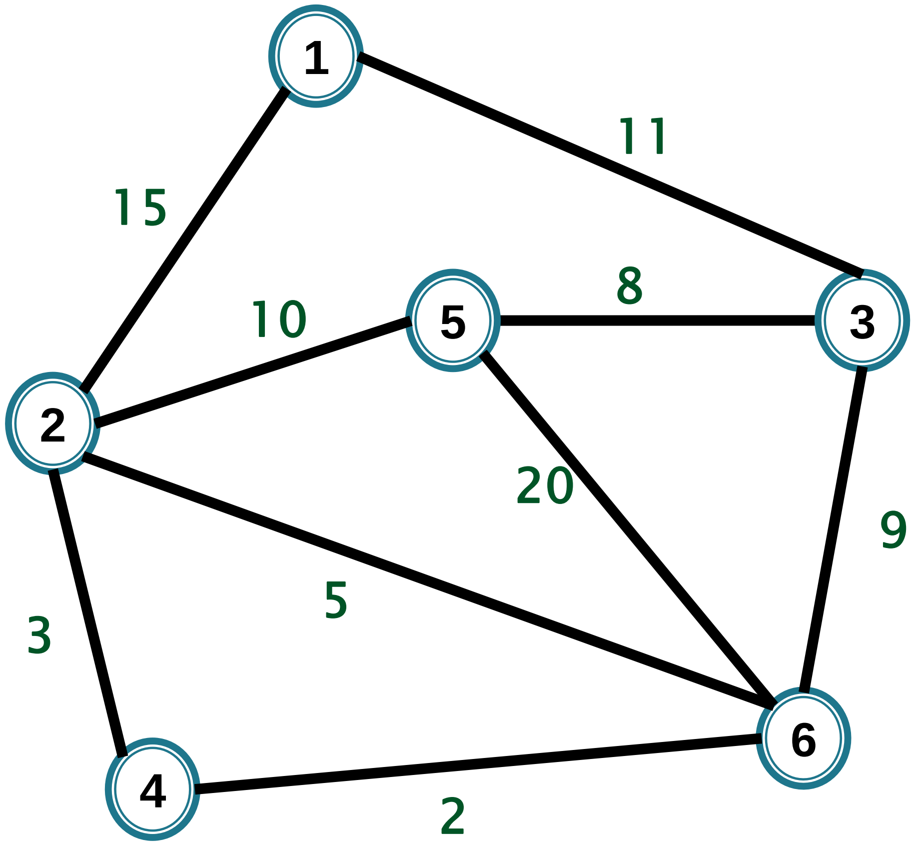
conectare cu cost minim $\Rightarrow$ evităm ciclurile

Deci trebuie să construim

graf conex + fără cicluri $\Rightarrow$ arbore

cu **suma costurilor muchiilor minimă**

Grafuri ponderate



Grafuri ponderate

- **G = (V, E) ponderat =**
  - $w : E \to \mathbb{R}$ funcție **pondere** (**cost**)
- notat $G = (V, E, w)$

Grafuri ponderate

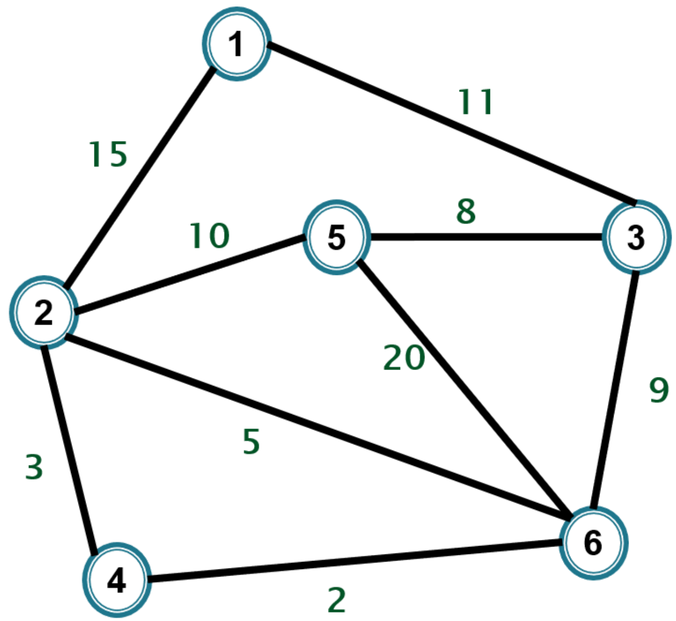
- G = (V, E, w) graf ponderat

- Pentru A ⊆ E

$$w(A) = \sum_{e \in A} w(e)$$

- Pentru T subgraf al lui G

$$w(T) = \sum_{e \in E(T)} w(e)$$



$$w([1, 2, 5]) = w(1,2) + w(2,5) = 15 + 10 = 25$$

Grafuri ponderate

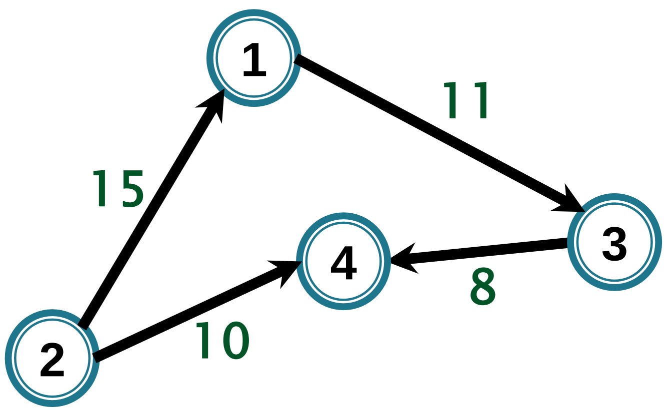
Reprezentarea grafurilor ponderate

Grafuri ponderate

Reprezentarea grafurilor ponderate

- **Matrice de costuri (ponderi)** $W = (w_{ij})_{i,j=1..n}$

$$w_{ij} = \begin{cases} 0, \text{ daca } i = j \\ w(i,j), \text{ daca } ij \in E \\ \infty, \text{ daca } ij \notin E \end{cases}$$

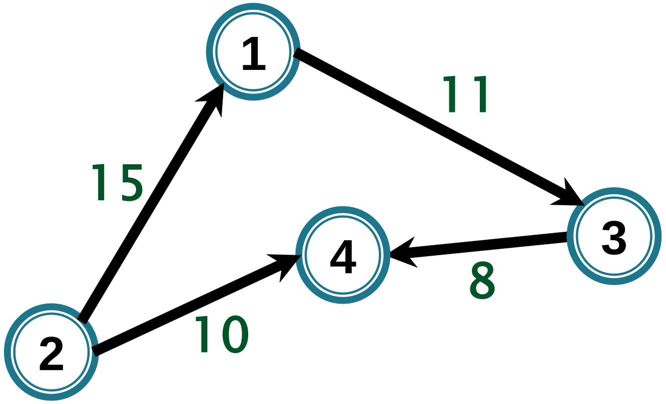


0	∞	11	∞
15	0	∞	10
∞	∞	0	8
∞	∞	∞	0

Grafuri ponderate

Reprezentarea grafurilor ponderate

- Liste de adiacență



1: 3 / 11

2: 1 / 15, 4 / 10

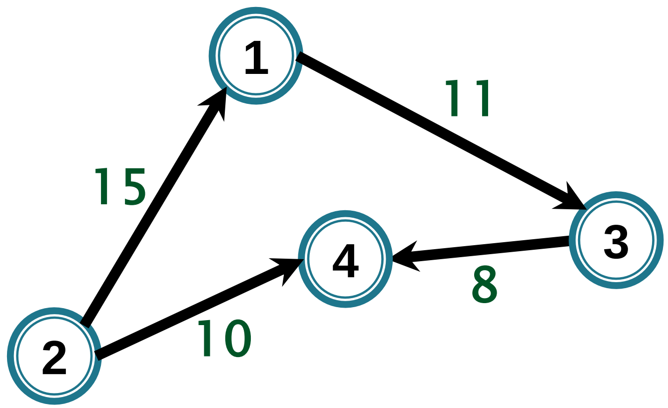
3: 4 / 8

4:

Grafuri ponderate

Reprezentarea grafurilor ponderate

- Liste de muchii/arce



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2 4 10

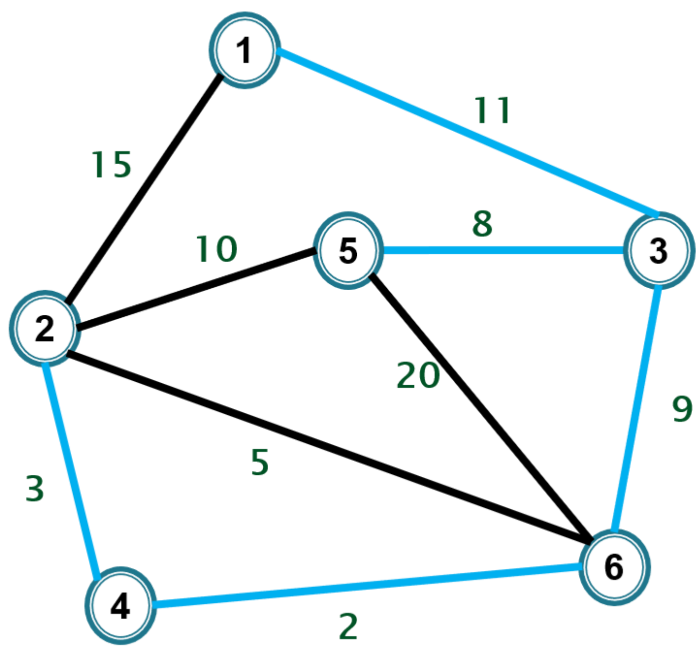
3 4 8

A.p.c.m

- G = (V, E, w) conex ponderat

- **Arbore parțial de cost minim** al lui G = un arbore parțial T_{min} al lui G cu

$$w(T_{min}) = \min \{ w(T) \mid T \text{ arbore partial al lui } G\}$$



A.p.c.m.

graf 250 noduri

apcm

Imagine din
R. Sedgewick, K. Wayne – **Algorithms, 4th edition, Pearson** Education, 2011

Aplicații a.p.c.m.

- **Construcția/renovarea unui sistem de căi ferate a.î.:**
  - oricare două stații să fie conectate (prin căi renovate)
  - sistem economic (costul total minim)

- **Proiectarea de rețele, circuite electronice**
  - conectarea pinilor cu cost minim/ fără cicluri

- **Clustering**

- **Subrutină în alți algoritmi (trasee hamiltoniene)**

- **Proiectarea de rețele**

- **Protocoale de rutare cu evitarea ciclurilor, analiza imaginilor...**

https://www.ics.uci.edu/~eppstein/gina/mst.html

Aplicații a.p.c.m.

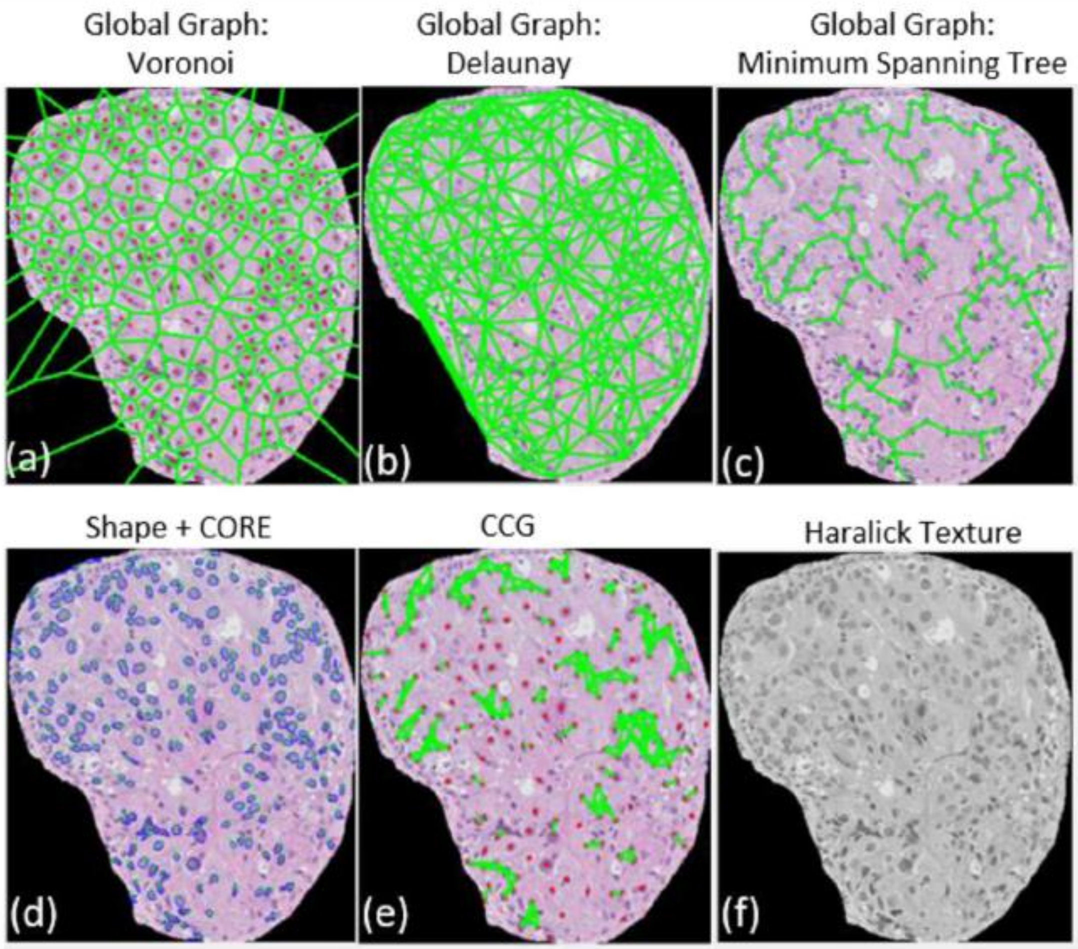


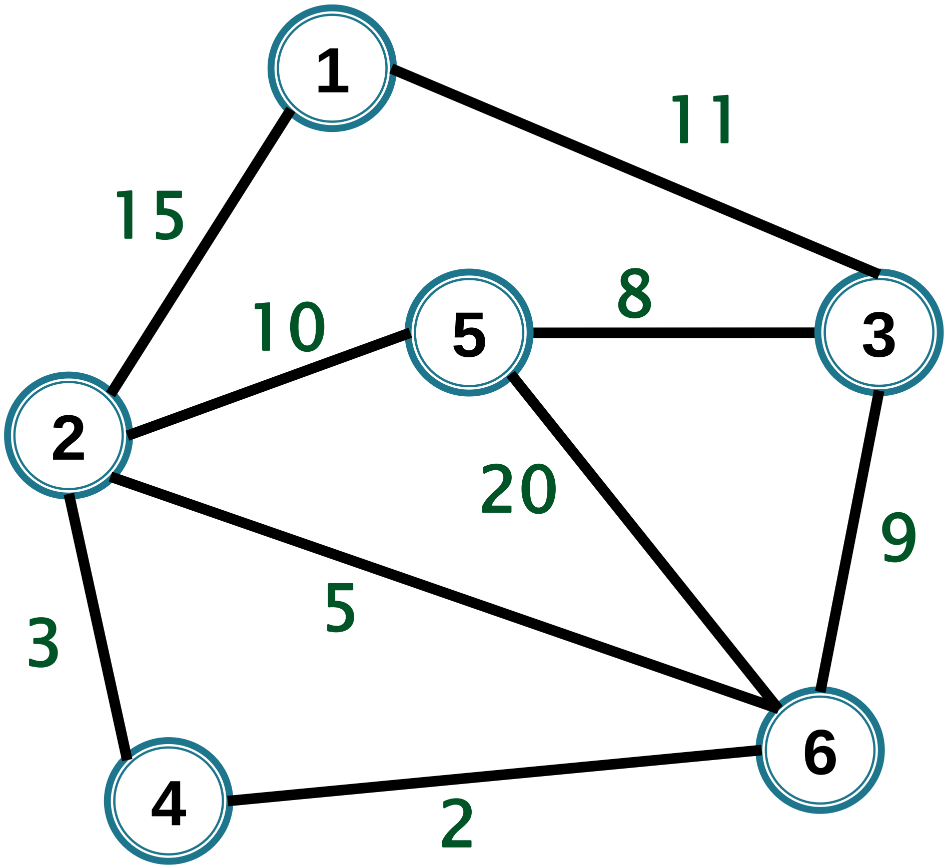
Fig. 4 Illustration of the feature maps corresponding to global graph (Voronoi (**a**), Delaunay (**b**), and Minimum Spanning Tree (**c**)), Shape (**d**), CORE (**d**), CCG (**e**), and Haralick Texture (**f**) features, capturing respectively spatial arrangement, shape, orientation, local arrangement, and heterogeneity of nuclei within a tissue image of a DCIS patient corresponding to the high ODx risk category the feature maps corresponding to the intermediate- and low-risk categories were included in S.3 as Figure 2 (II) and Figure 2 (III) respectively

https://d-nb.info/1204628343/34

Algoritmi de determinare a unui arbore parțial de cost minim

Arbori parțiali de cost minim

Cum determinăm un arbore parțial de cost minim al unui graf conex ponderat?



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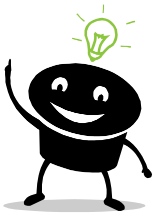
Arbori parțiali de cost minim



Idee: **Prin <u>adăugare</u> succesivă de muchii,** astfel încât mulțimea de muchii selectate

- să aibă costul cât mai mic
- să fie submulțime a mulțimii muchiilor unui arbore parțial de cost minim (apcm)

Arbori parțiali de cost minim

După ce criteriu selectăm muchiile?

Arbori parțiali de cost minim

După ce criteriu selectăm muchiile?

$\Rightarrow$ diverși algoritmi

Arbori parțiali de cost minim

Kruskal

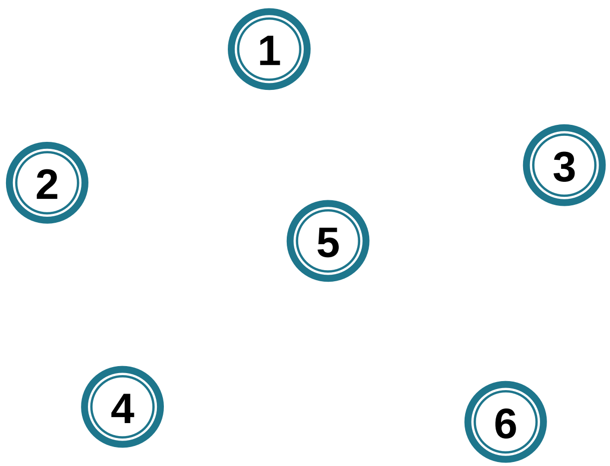
- **Inițial T= (V; ∅)**
- **pentru i = 1, n-1**
  - alege o muchie uv cu **cost minim** a.î. u,v sunt în **componente conexe diferite** **(T+uv aciclic)**
  - $E(T) = E(T) \cup uv$

Prim

- **s- vârful de start**
- **Inițial T= ({s}; ∅)**
- **pentru i = 1, n-1**
  - alege o muchie uv cu **cost minim** a.î. $u \in V(T)$ și $v \notin V(T)$
  - $V(T) = V(T) \cup \{v\}$
  - $E(T) = E(T) \cup uv$

Kruskal

- **Inițial**: cele n vârfuri sunt izolate, fiecare formând o componentă conexă



- **Se încearcă unirea acestor componente prin muchii de cost minim**

Prim

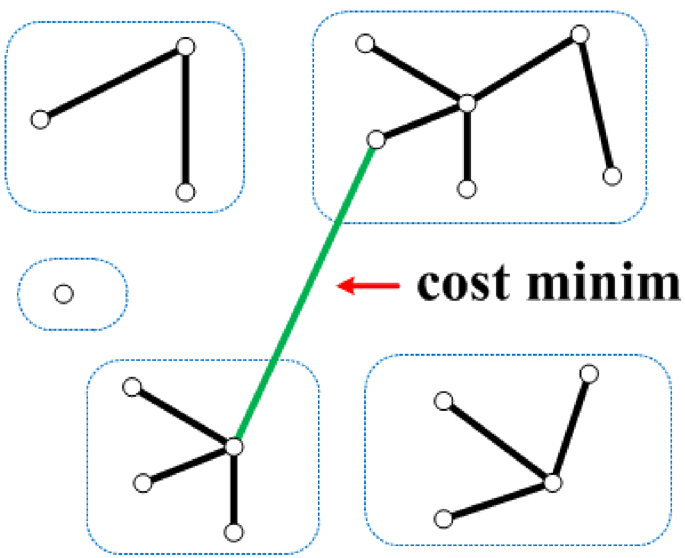
- **Inițial**: se pornește de la un vârf de start

- **Se adăugă pe rând câte un vârf la arborele deja construit, folosind muchii de cost minim**

Kruskal

- <u>La un pas</u>:

Muchiile selectate formează o **pădure**

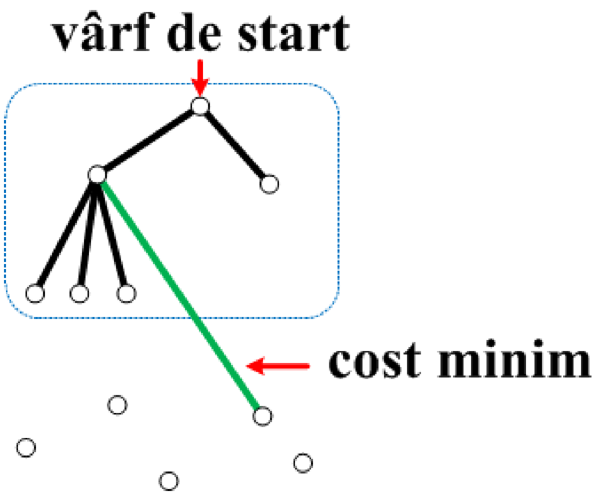


Este selectată o muchie de cost minim care unește doi arbori din pădurea curentă (două componente conexe)

Prim

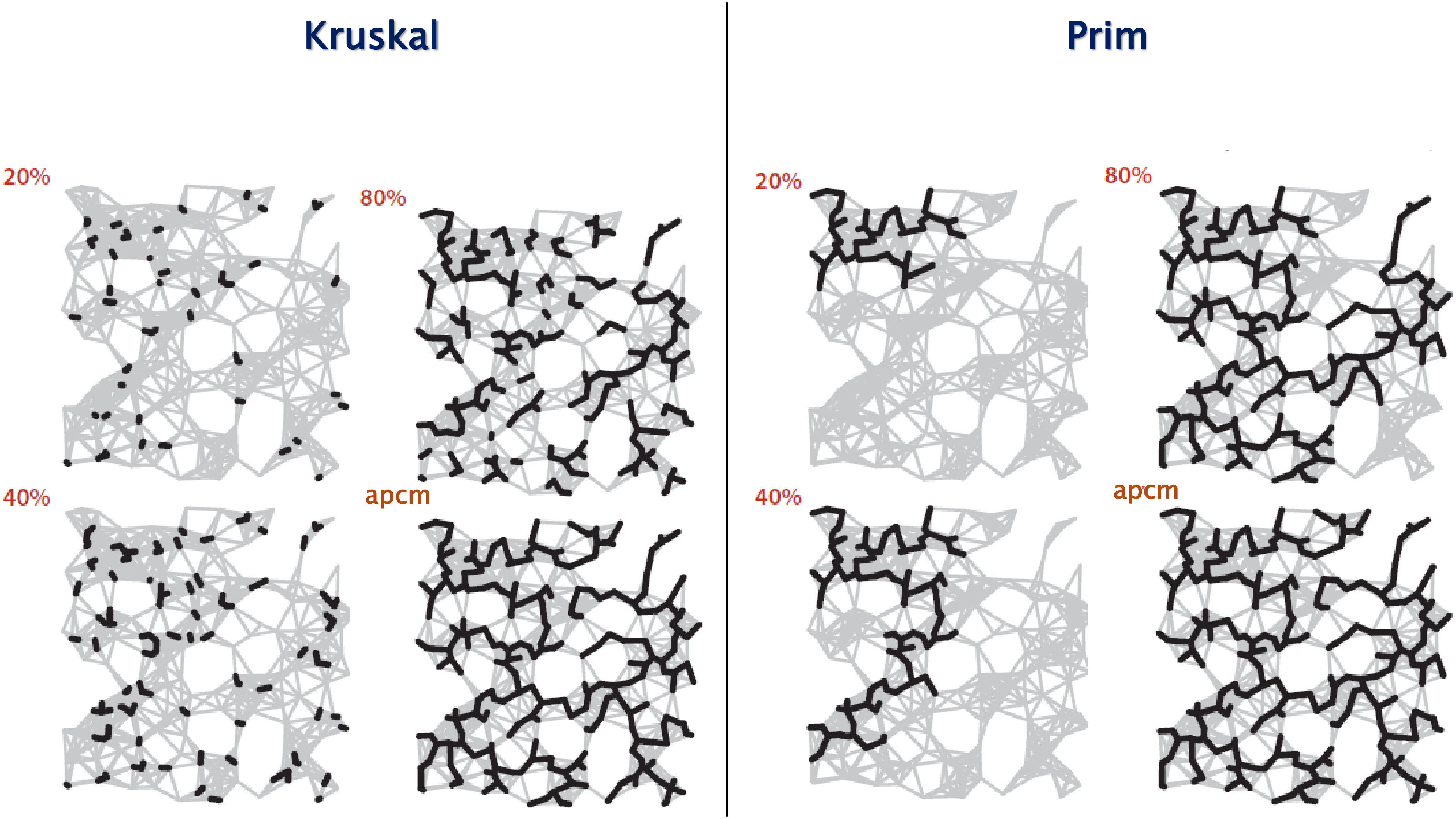
- <u>La un pas</u>:

Muchiile selectate formează un **arbore**



Este selectată o muchie de cost minim care unește un vârf din arbore cu unul care nu este în arbore(neselectat)

Arbori parțiali de cost minim



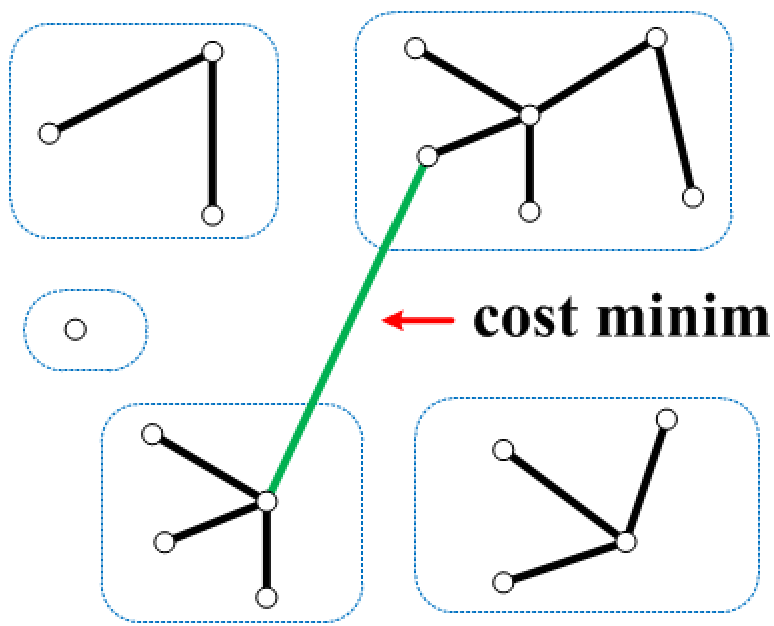
Imagine din

R. Sedgewick, K. Wayne – **Algorithms, 4th edition, Pearson** Education, 2011

Algoritmul lui Kruskal

Algoritmul lui Kruskal

- La un pas este selectată o muchie de cost minim din G care nu formează cicluri cu muchiile deja selectate (care unește două componente conexe din graful deja construit)



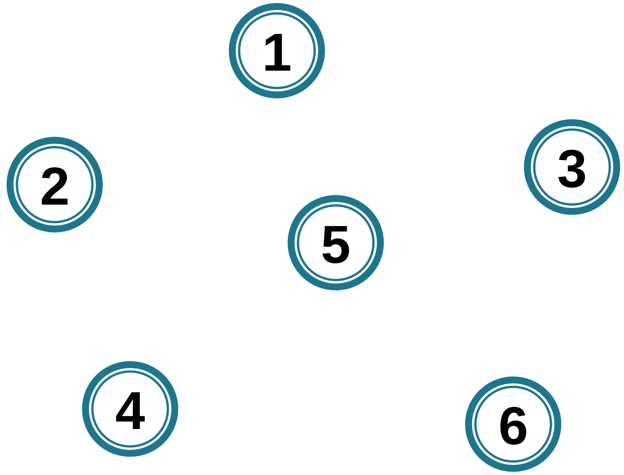
O primă formă a algoritmului

Kruskal

- **Inițial T= (V; ∅)**
- **pentru i = 1, n-1**
  - alege o muchie uv cu **cost minim din G** a.î. u,v sunt în **componente conexe diferite** (**T+uv aciclic**)
  - $E(T) = E(T) \cup \{uv\}$

Kruskal

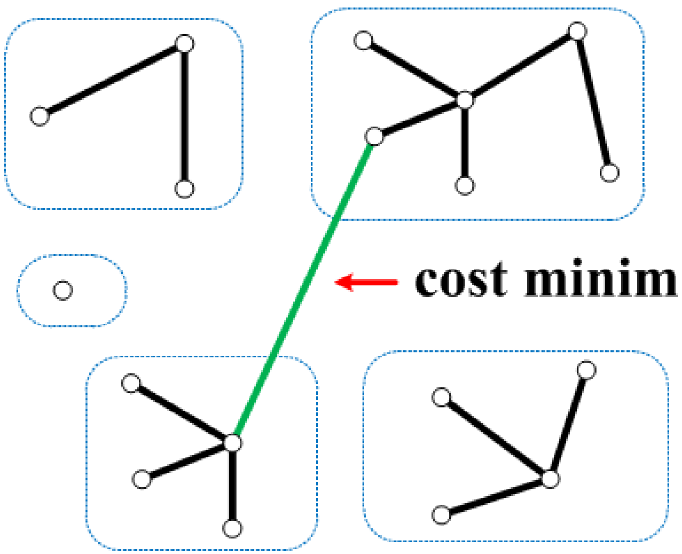
- **<u>Inițial</u>**: cele n vârfuri sunt **izolate**, fiecare formând o componentă conexă



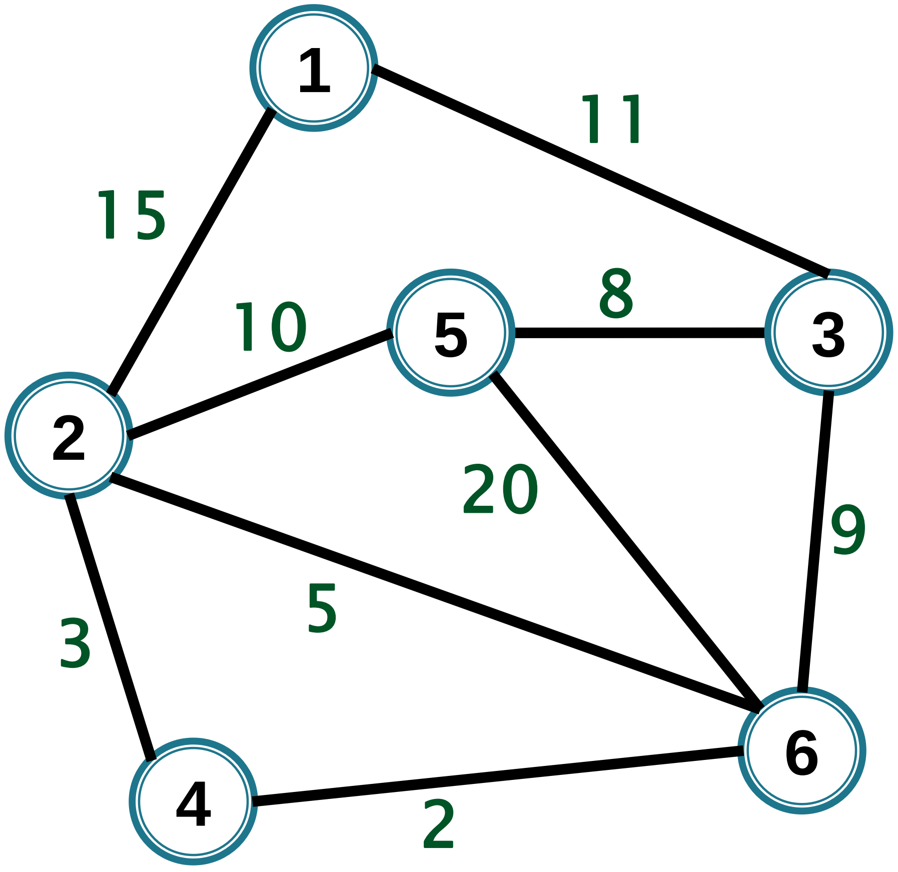
Kruskal

- <u>La un pas</u>:

Muchiile selectate formează o **pădure**



Este selectată o muchie de cost minim care unește doi arbori din pădurea curentă (două componente conexe)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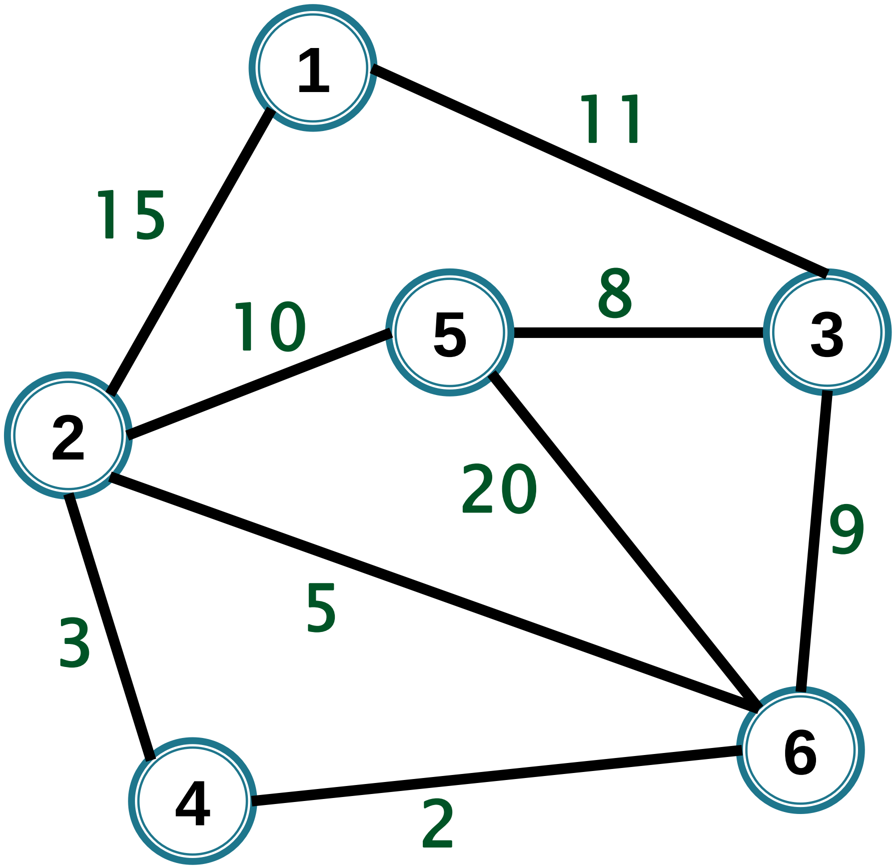
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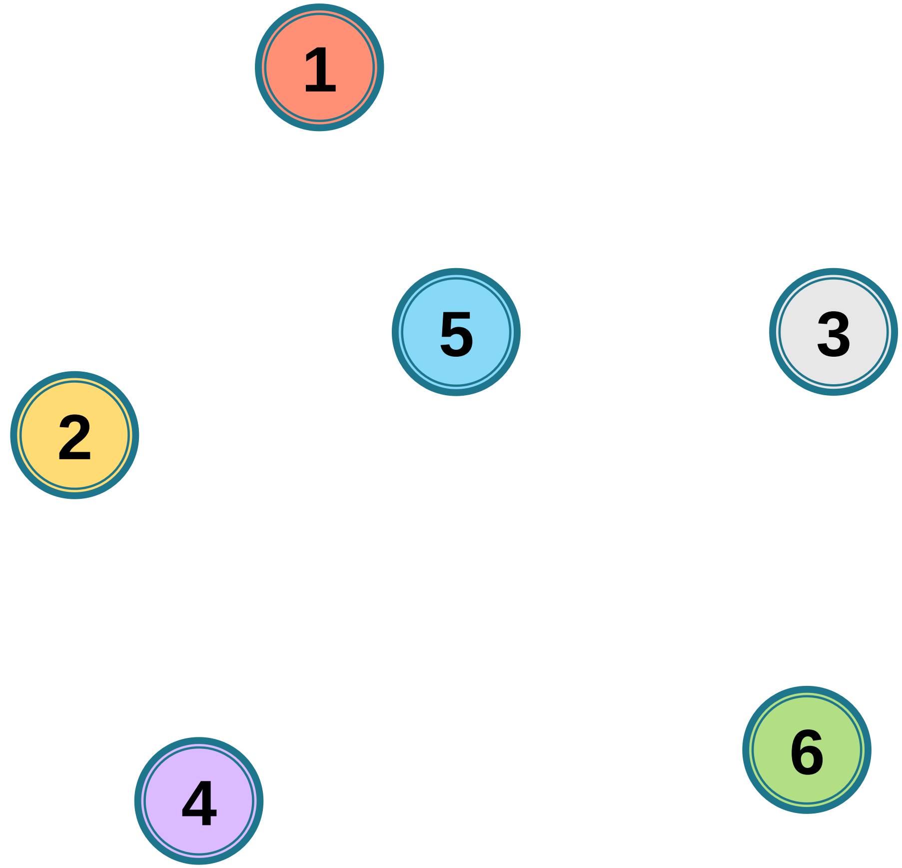
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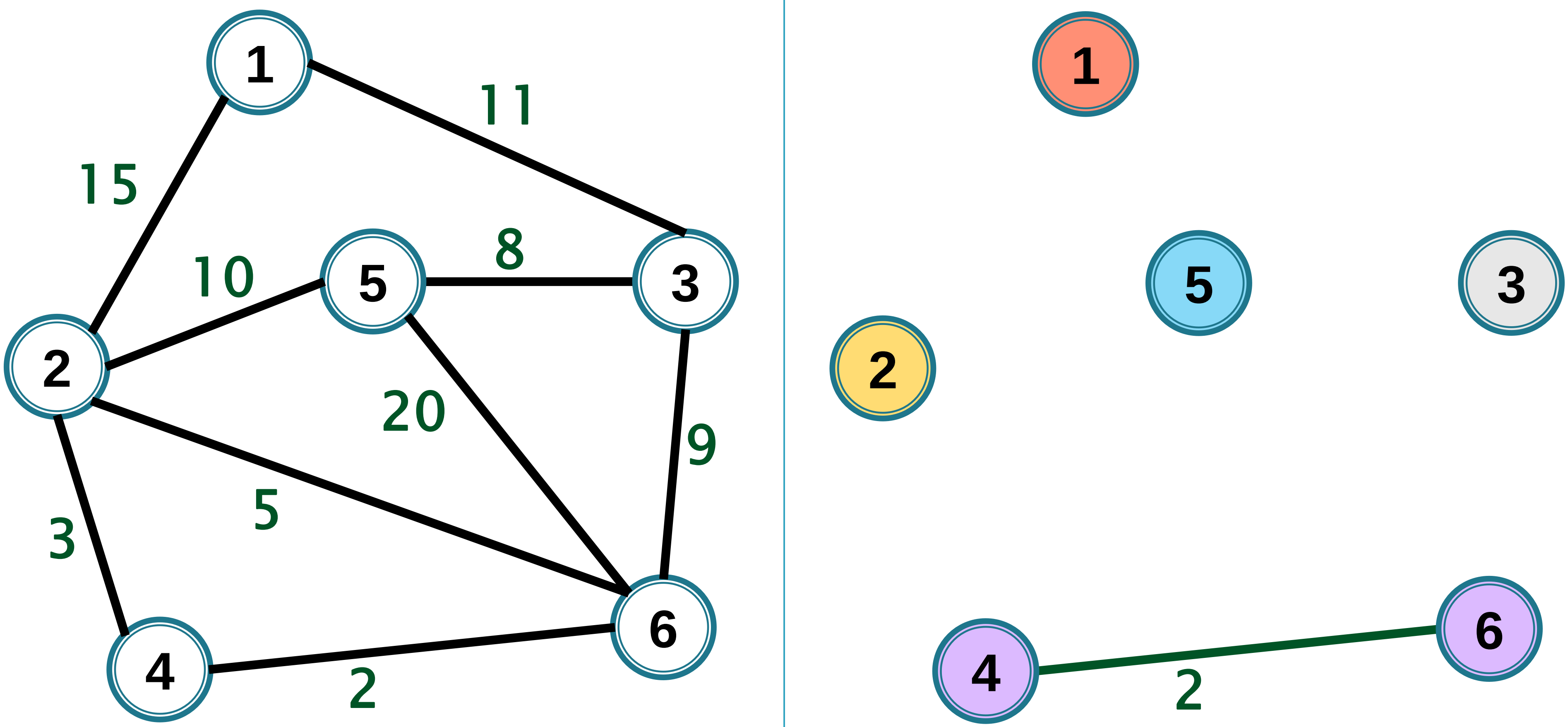
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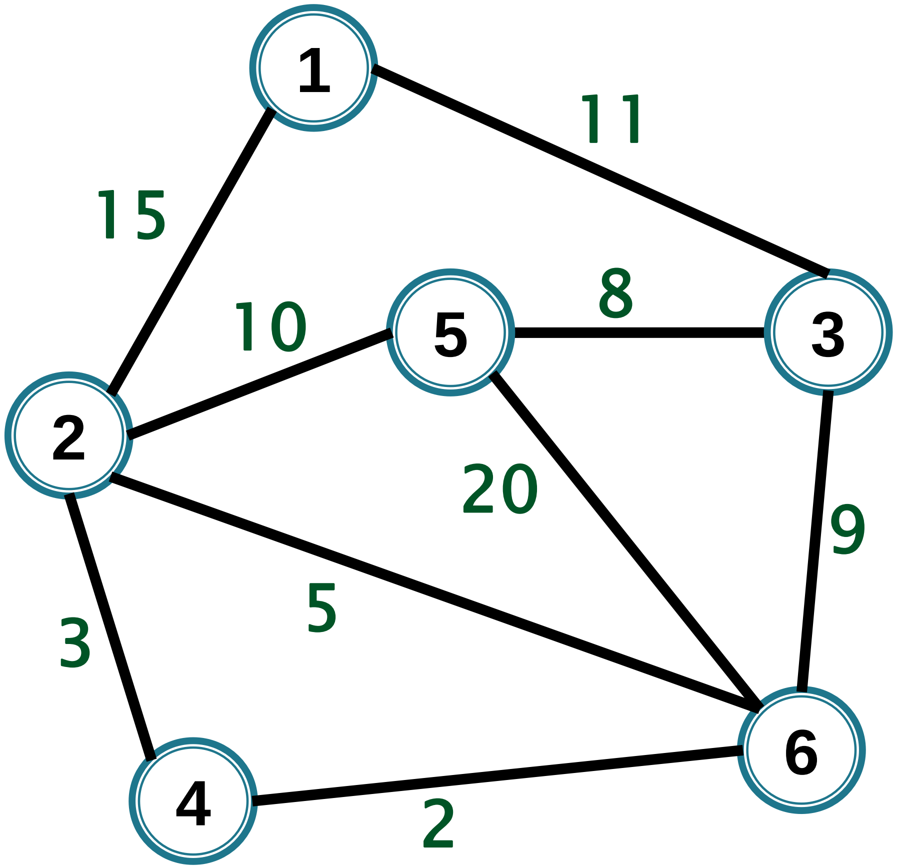
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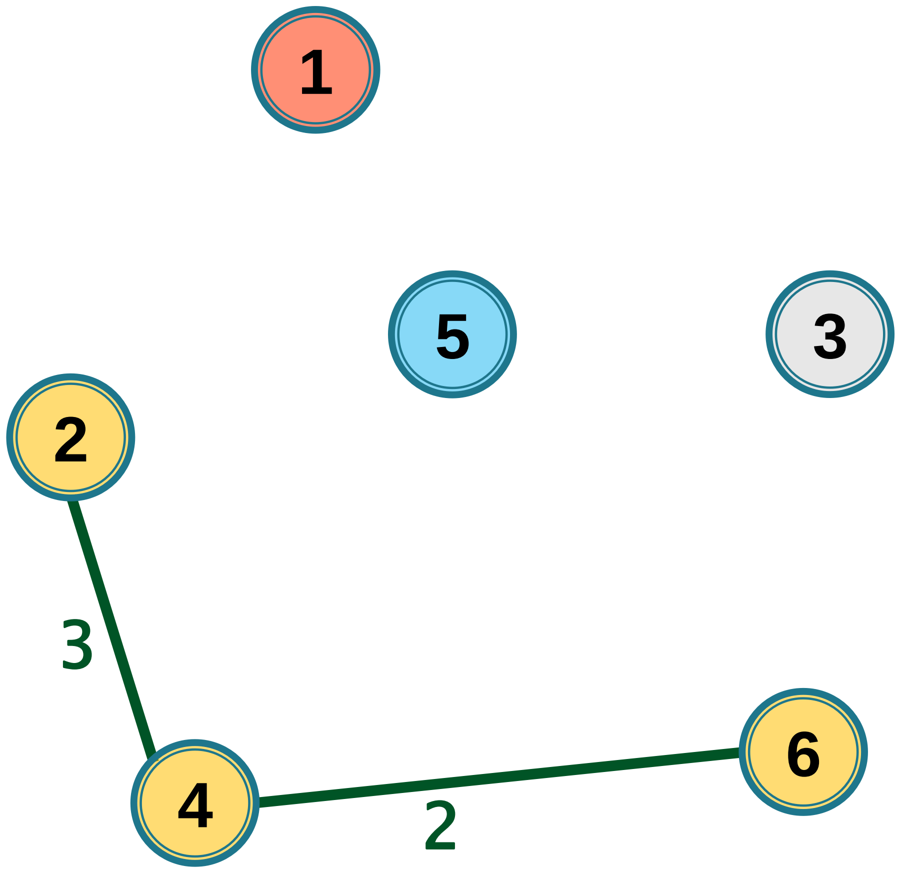
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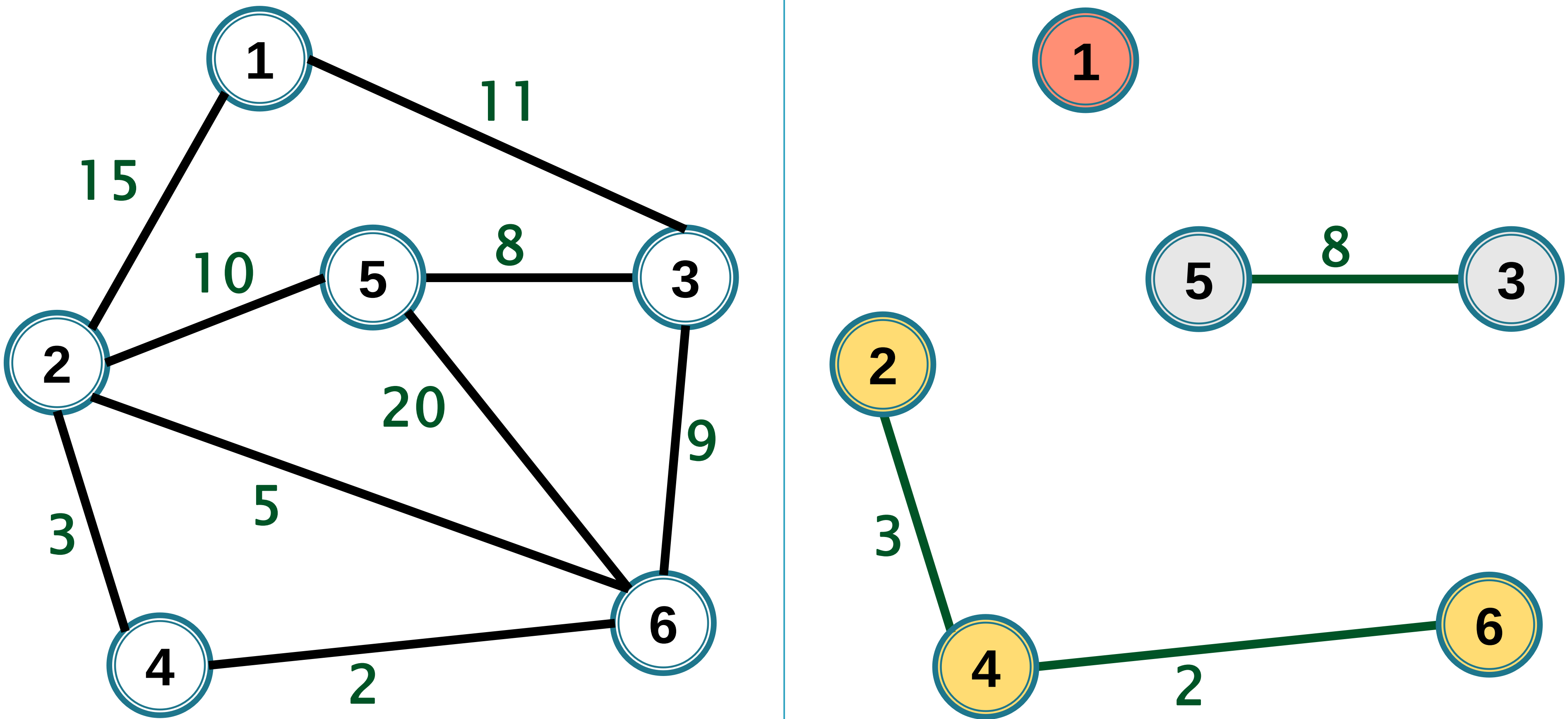
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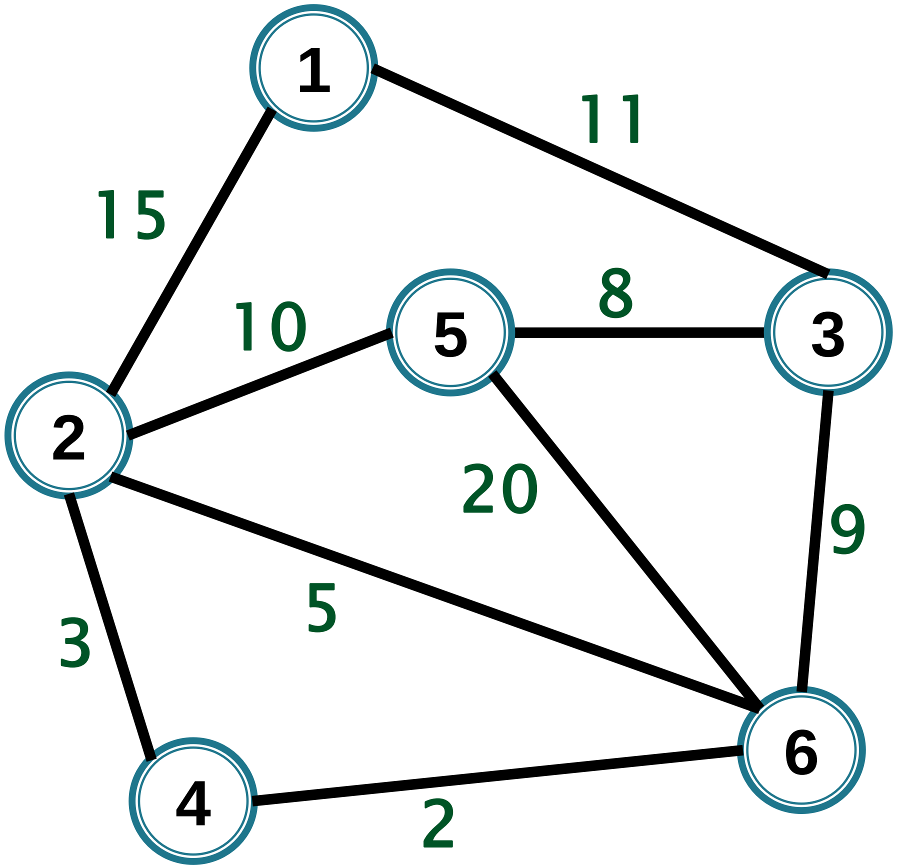
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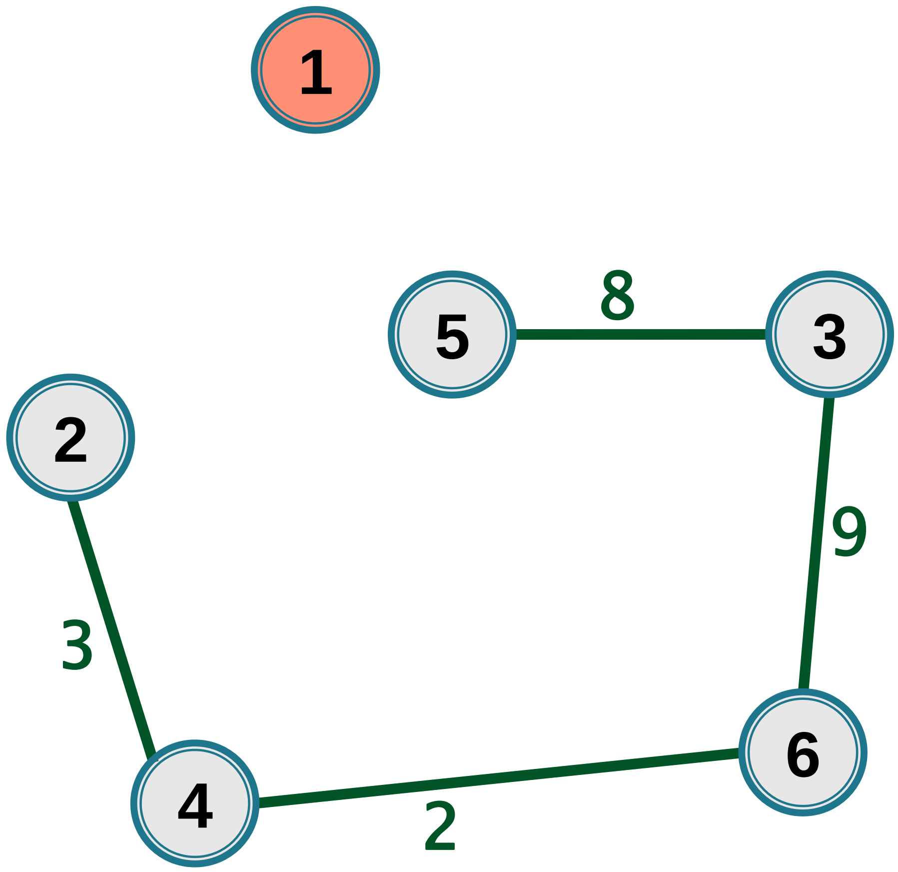
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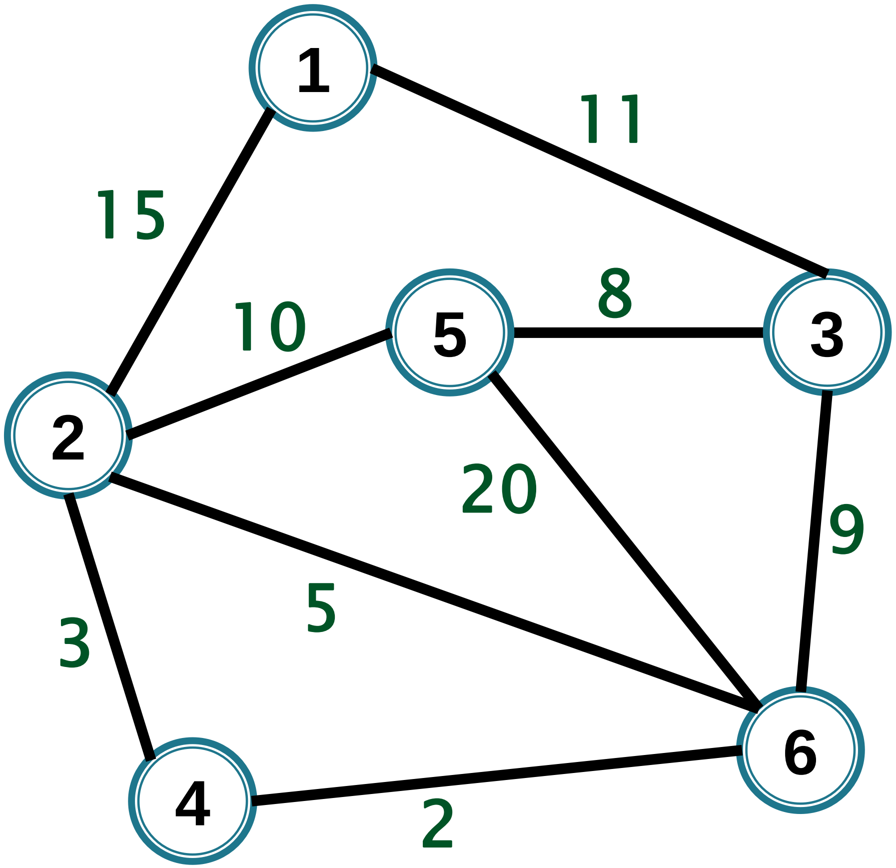
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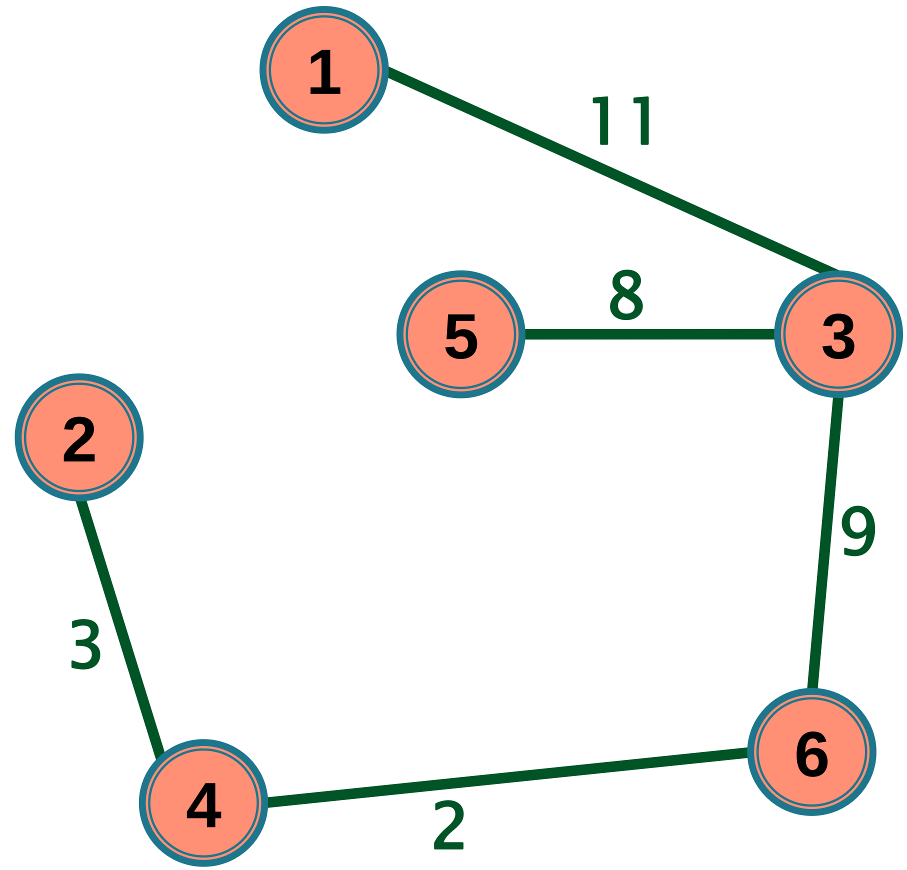
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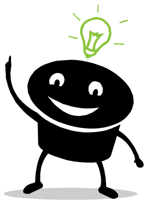
Kruskal – Implementare

1. **Cum reprezentăm graful în memorie?**

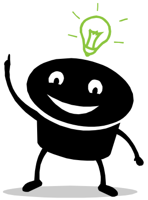
2. **Cum selectăm ușor o muchie:**
   - de cost minim
   - care unește două componente (nu formează cicluri cu muchiile deja selectate)

Kruskal



Pentru a selecta ușor o muchie de cost minim cu proprietatea dorită **ordonăm crescător muchiile după cost şi considerăm muchiile în această ordine**

Kruskal



Reprezentarea grafului ponderat

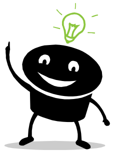
- **Listă de muchii:** memorăm pentru fiecare muchie extremitățile și costul

Kruskal

Cum testăm dacă muchia curentă unește două componente (⇔ nu formează cicluri cu muchiile deja selectate) ?

Kruskal

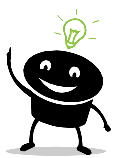
Cum testăm dacă muchia curentă unește două componente ($\Leftrightarrow$ nu formează cicluri cu muchiile deja selectate) ?



Verificăm printr-o parcurgere dacă extremităţile muchiei sunt deja unite printr-un lanţ

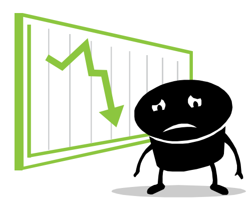
Kruskal

Cum testăm dacă muchia curentă unește două componente ($\Leftrightarrow$ nu formează cicluri cu muchiile deja selectate) ?

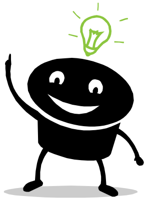


Verificăm printr-o parcurgere dacă extremitățile muchiei sunt deja unite printr-un lanț

$\Rightarrow$ O(mn) - ineficient



Kruskal



La un pas componentele conexe din T sunt mulțimi disjuncte din V (partiţie a lui V)

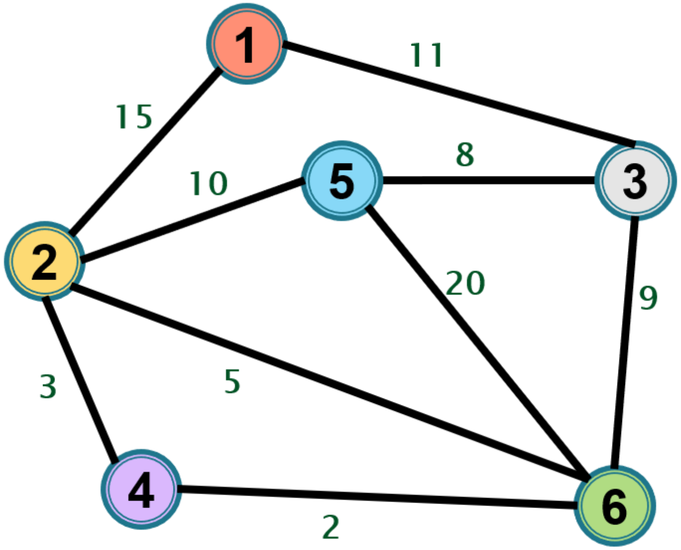
$\Rightarrow$ **structuri pentru mulțimi disjuncte**

Kruskal

La un pas componentele conexe din T sunt mulțimi disjuncte din V (partiţie a lui V)

$\Rightarrow$ **structuri pentru mulțimi disjuncte**

- **asociem fiecărei componente un reprezentant (o culoare)**



Kruskal

- **Operații necesare**:
  - **Initializare**(u) –
  - **Reprez**(u) –
  - **Reuneste**(u,v) –

Kruskal

- **Operații necesare**:

  - **Initializare(u)** – creează o componentă cu un singur vârf, u

  - **Reprez(u)** – returnează reprezentantul (culoarea) componentei care conține pe u

  - **Reuneste(u,v)** – unește componenta care conține u cu cea care conține v

Kruskal

- O muchie uv uneşte două componente dacă și numai dacă

Kruskal

- O muchie uv uneşte două componente dacă şi numai dacă

$$\textbf{Reprez}(u) \neq \textbf{Reprez}(v)$$

Kruskal

- O muchie uv uneşte două componente dacă și numai dacă

$$\textbf{Reprez}(u) \neq \textbf{Reprez}(v)$$

Kruskal

```
sorteaza(E)

for(v=1;v<=n;v++)
    Initializare(v);
```

Kruskal

```
sorteaza(E)

for(v=1;v<=n;v++)
    Initializare(v);

nrmsel=0

for(uv ∈ E)
   if(Reprez(u)!=Reprez(v))
   {


   }
```

Kruskal

```
sorteaza(E)

for(v=1;v<=n;v++)
    Initializare(v);

nrmsel=0

for(uv ∈ E)
   if(Reprez(u)!=Reprez(v))
   {
     E(T) = E(T) ∪ {uv};






   }
```

Kruskal

```
sorteaza(E)

for(v=1;v<=n;v++)
    Initializare(v);

nrmsel=0

for(uv ∈ E)
   if(Reprez(u)!=Reprez(v))
   {
      E(T) = E(T) ∪ {uv};
      Reuneste(u,v);
      nrmsel=nrmsel+1;
      if(nrmsel==n-1)
          STOP;
   }
```

Kruskal

Complexitate

De câte ori se execută fiecare operație?

Kruskal

Complexitate

- Sortare -> O(m log m) = O(m log n)
- ? * Initializare
- ? * Reprez
- ? * Reuneste

Kruskal

Complexitate

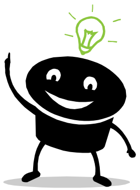
- Sortare -> O(m log m) = O(m log n)
- n * Initializare
- 2m * Reprez
- (n-1) * Reuneste

Depinde de modalitatea de memorare a componentelor

Kruskal

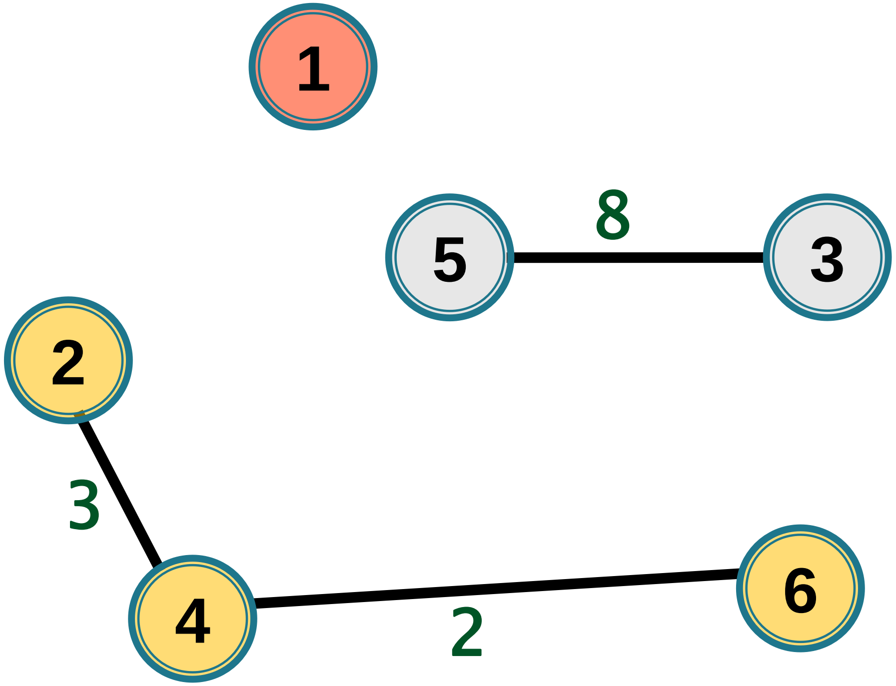
Cum memorăm componentele + reprezentantul / culoarea componentei în care se află un vârf?

Kruskal

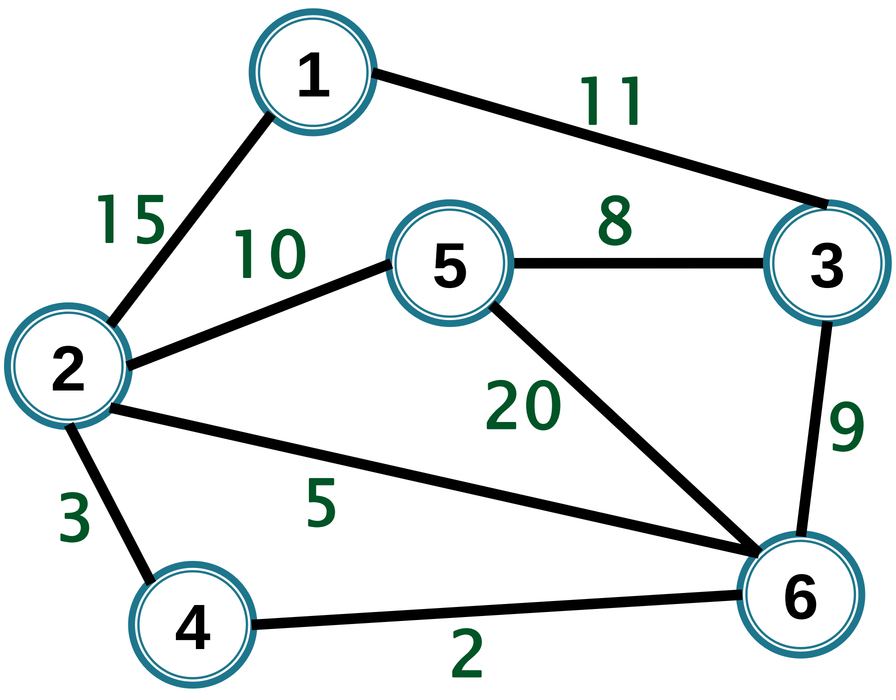


Varianta 1 – memorăm într-un vector pentru fiecare vârf reprezentantul/culoarea componentei din care face parte

`r[u]` = **culoarea (reprezentantul) componentei care conține vârful u**



$r = [1, 2, 3, 2, 3, 2]$



Kruskal

- Initializare
- Reprez
- Reuneste

Kruskal

- Initializare   – O(1)

```
void Initializare(int u){
    r[u]=u;
}
```

- Reprez

- Reuneste

Kruskal

- Initializare – O(1)

```
void Initializare(int u){
    r[u]=u;
}
```

- Reprez – O(1)

```
int Reprez(int u){
    return r[u];
}
```

- Reuneste

Kruskal

- Initializare – O(1)

```
void Initializare(int u){
    r[u]=u;
}
```

- Reprez – O(1)

```
int Reprez(int u){
    return r[u];
}
```

- Reuneste – O(n)

```
void Reuneste(int u,int v)
{
  r1 = Reprez(u);//r1=r[u]
  r2 = Reprez(v);//r2=r[v]
  for(k=1;k<=n;k++)
    if(r[k] == r2)
       r[k] = r1;
}
```

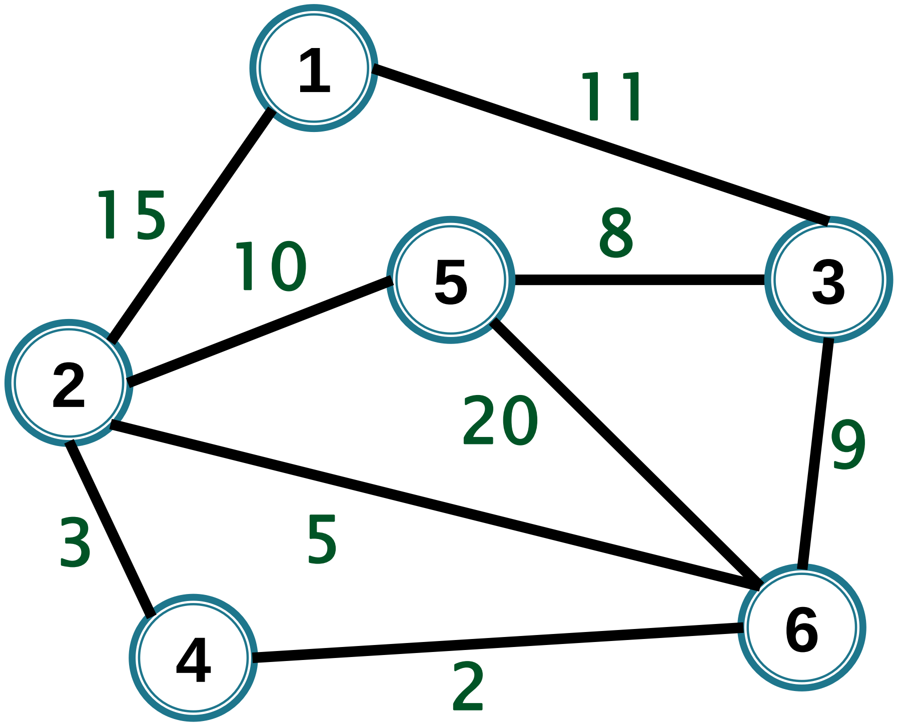
Kruskal

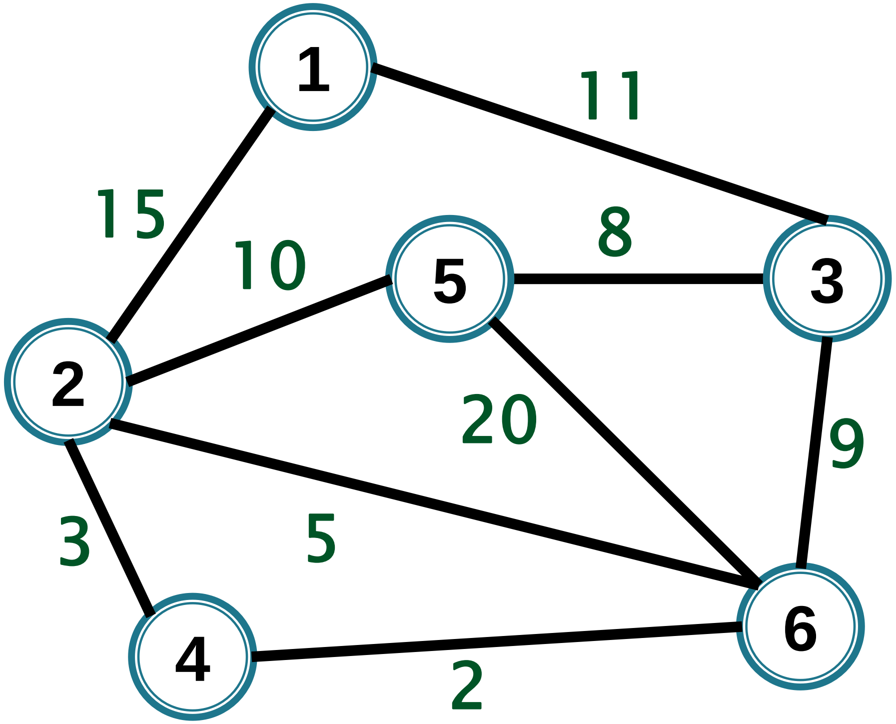
Complexitate

Varianta 1– dacă folosim vector de reprezentanți

- **Sortare** -> $O(m \log m) = O(m \log n)$
- **n * Initializare** -> $O(n)$
- **2m * Reprez** -> $O(m)$
- **(n-1) * Reuneste** -> $O(n^2)$

$$O(m \log n + n^2)$$


1
2
3
4
5
6
11
15
10
8
20
9
3
5
2



(4,6)

(2,4)

(2,6)

(3,5)

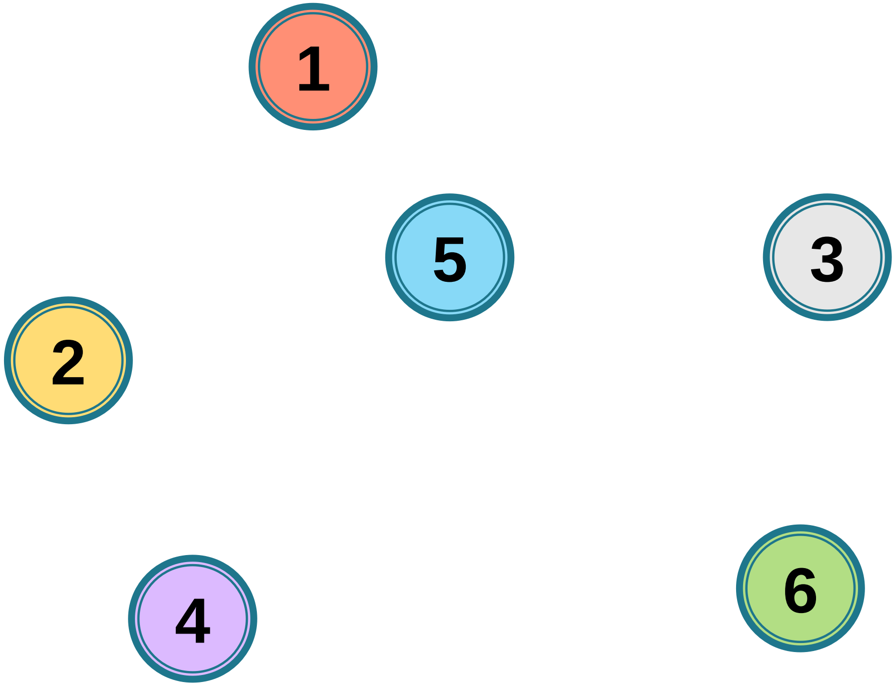
(3,6)

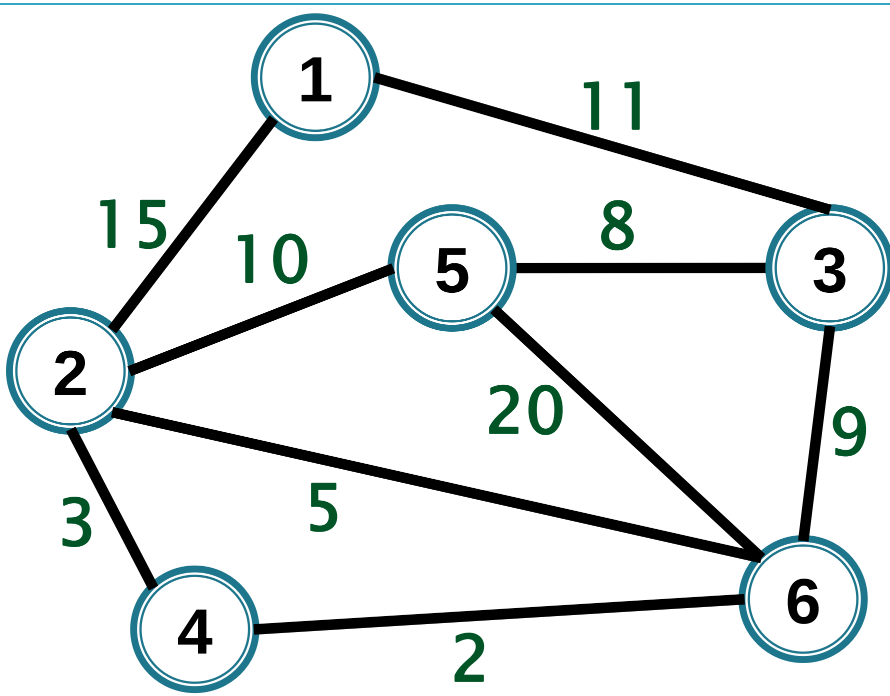
(2,5)

(1,3)

(1,2)

(5,6)





r = [1,2,3,4,5,6]

(4,6)

(2,4)

(2,6)

(3,5)

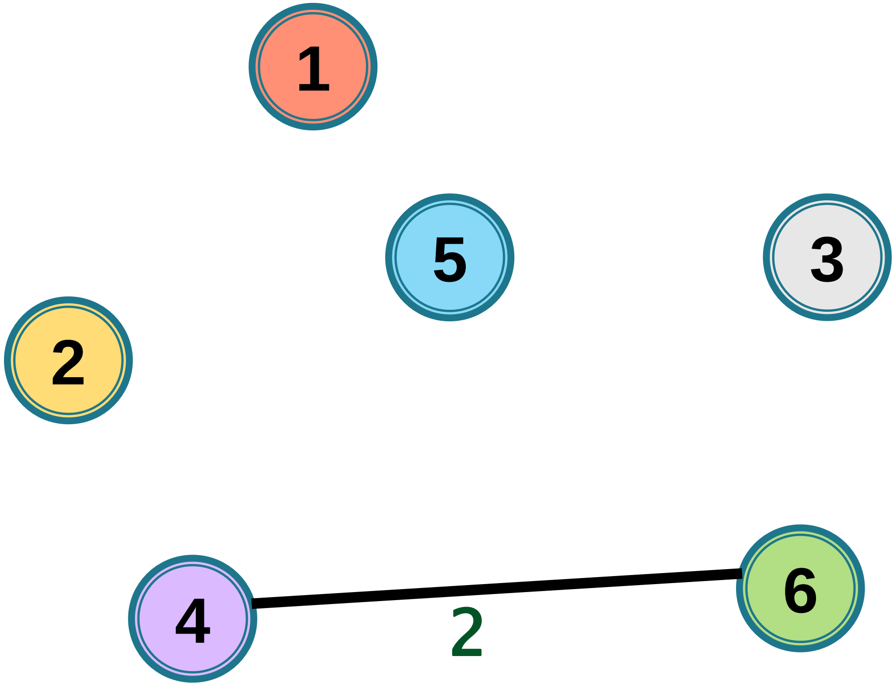
(3,6)

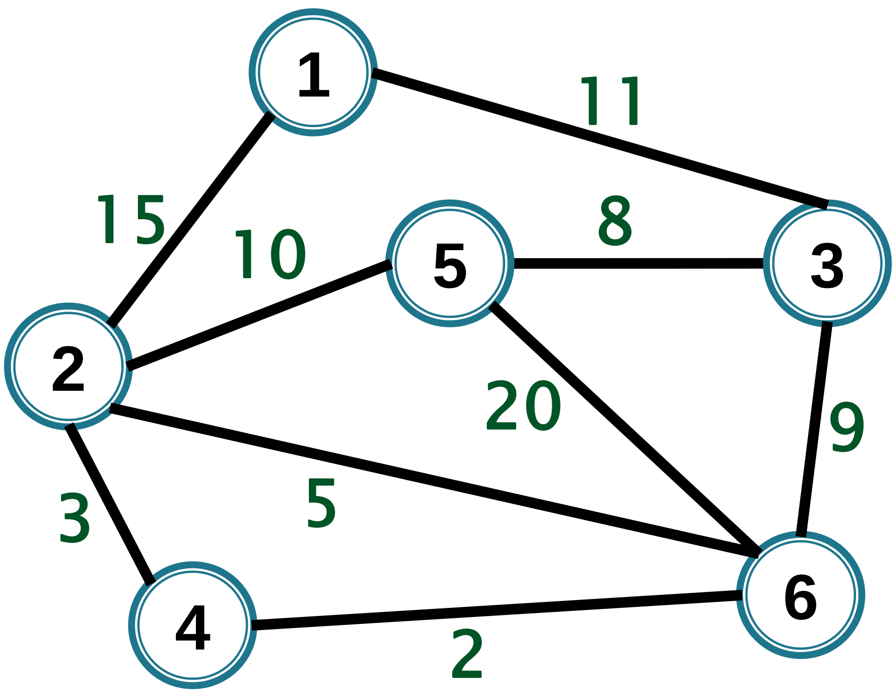
(2,5)

(1,3)

(1,2)

(5,6)





$r = [1,2,3,\underline{4},5,\underline{6}]$

(4,6) $r(4) \neq r(6)$

(2,4)

(2,6)

(3,5)

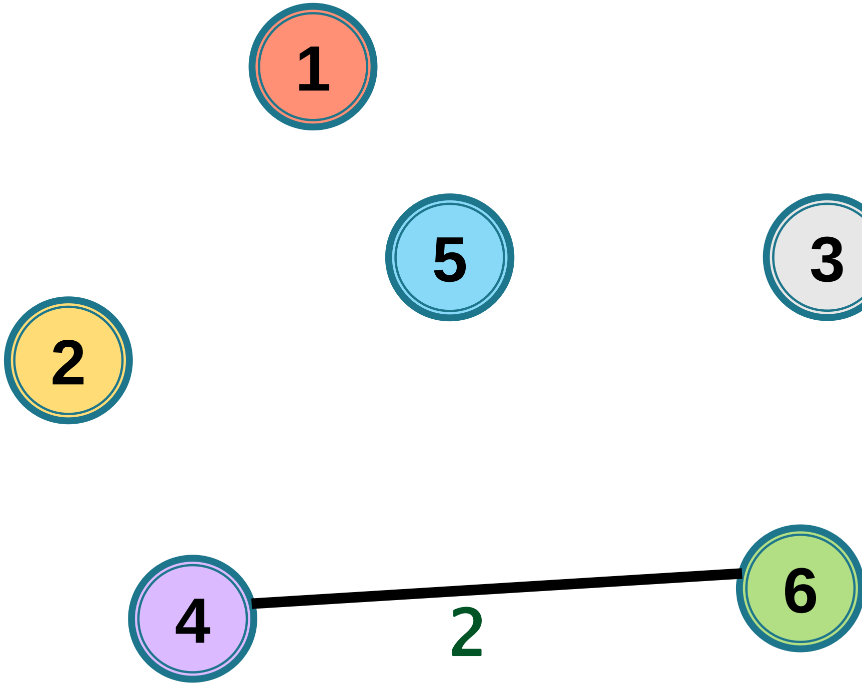
(3,6)

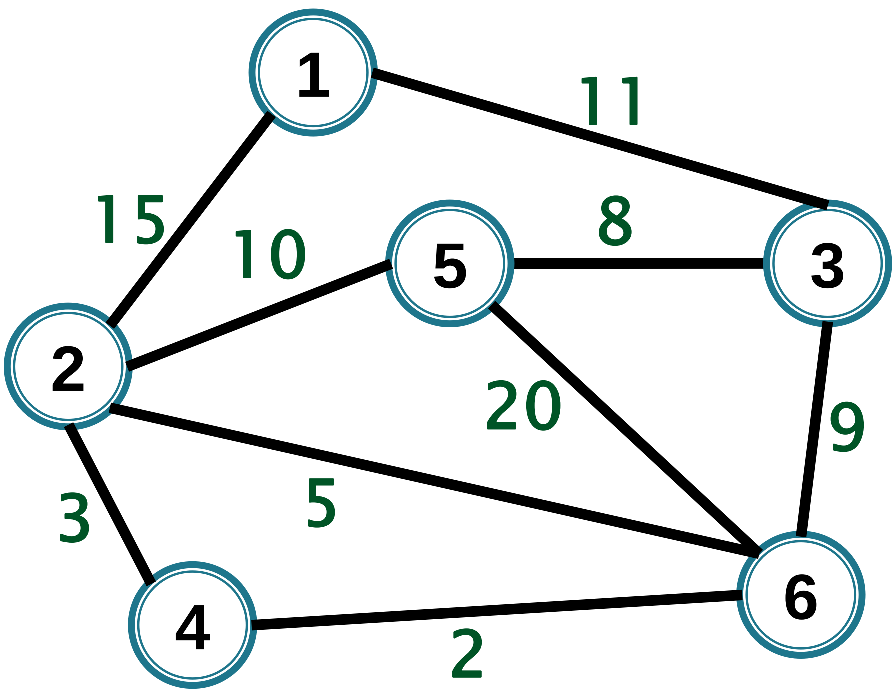
(2,5)

(1,3)

(1,2)

(5,6)





r = [1,2,3,<u>4</u>,5,<u>6</u>]

(4,6) Reuneste(4, 6)

(2,4)

(2,6)

(3,5)

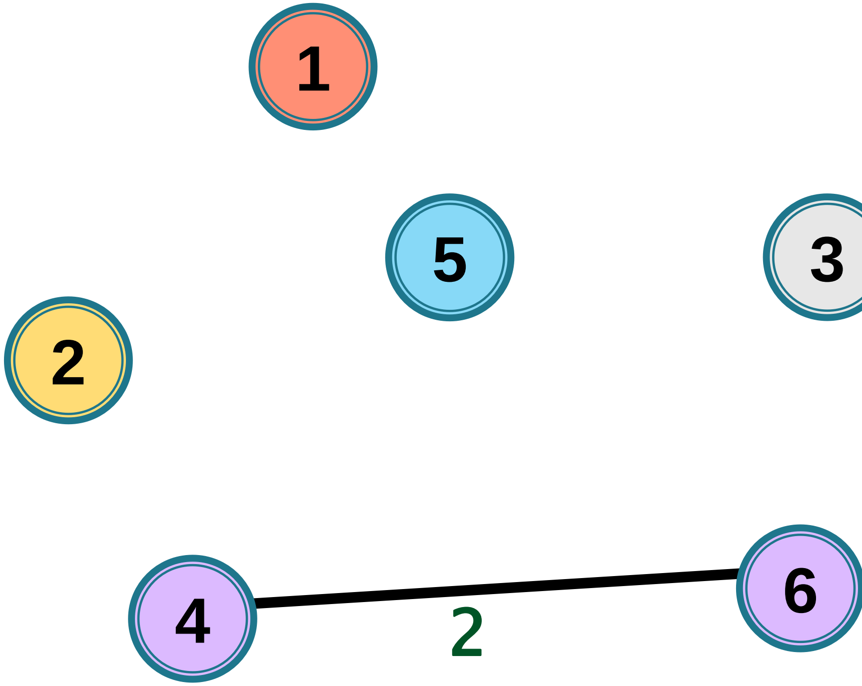
(3,6)

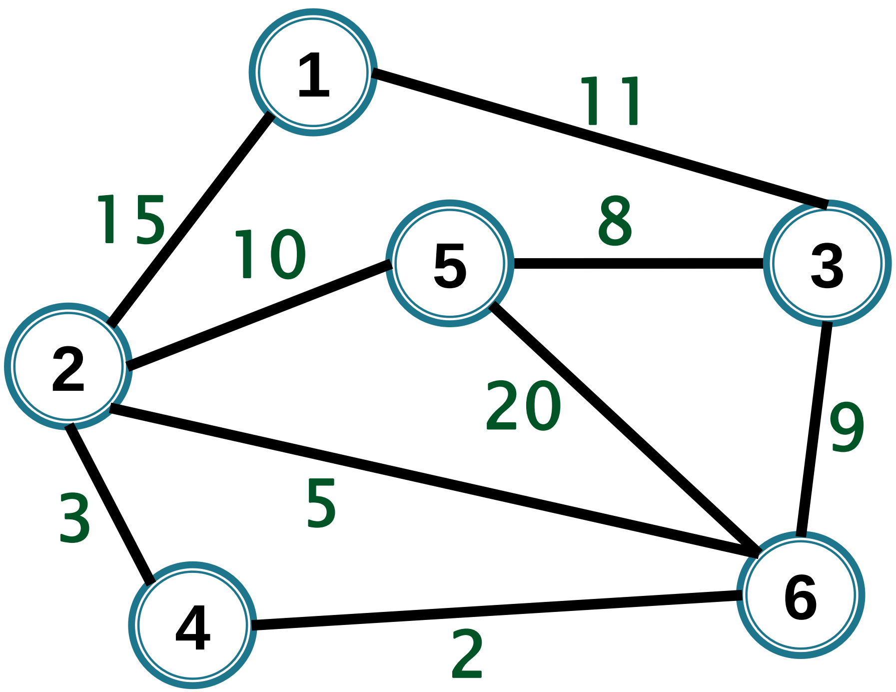
(2,5)

(1,3)

(1,2)

(5,6)





r = [1,2,3,<u>4</u>,5,<u>6</u>]

(4,6) r = [1,2,3,4,5,4]

(2,4)

(2,6)

(3,5)

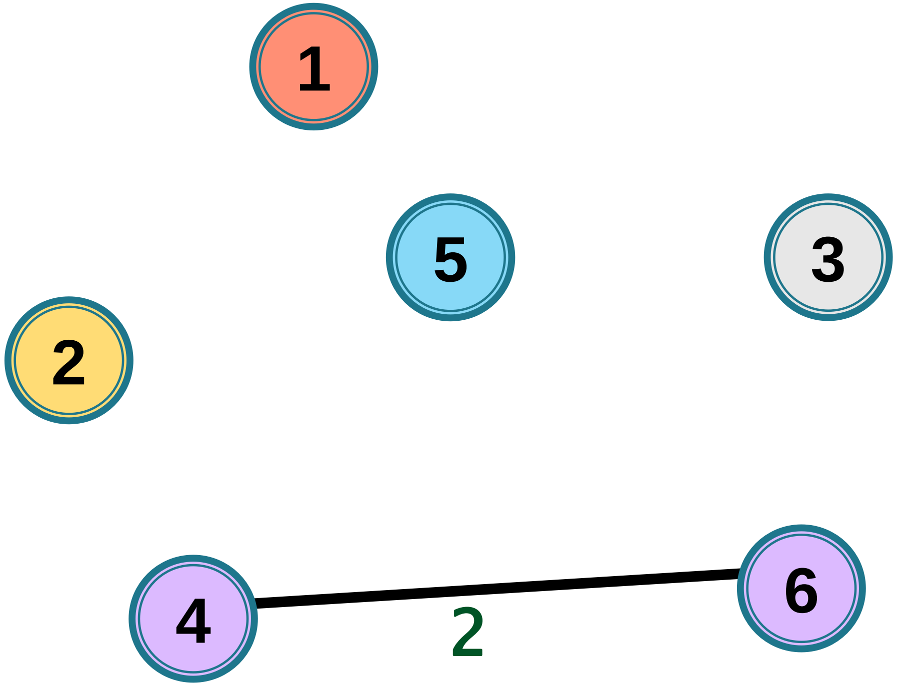
(3,6)

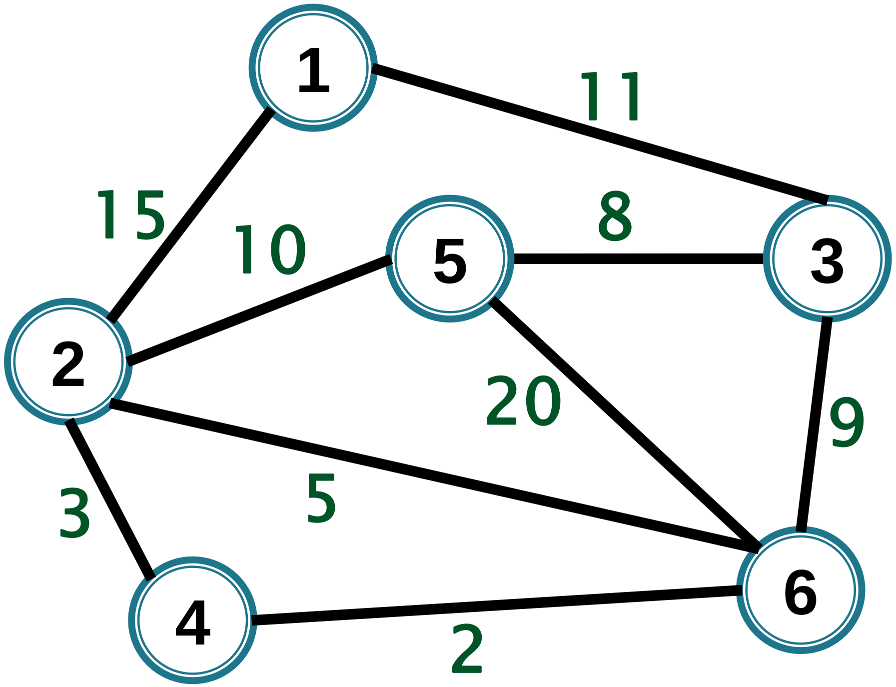
(2,5)

(1,3)

(1,2)

(5,6)





r = [1,2,3,4,5,6]

(4,6) r = [1,2,3,4,5,4]

(2,4)

(2,6)

(3,5)

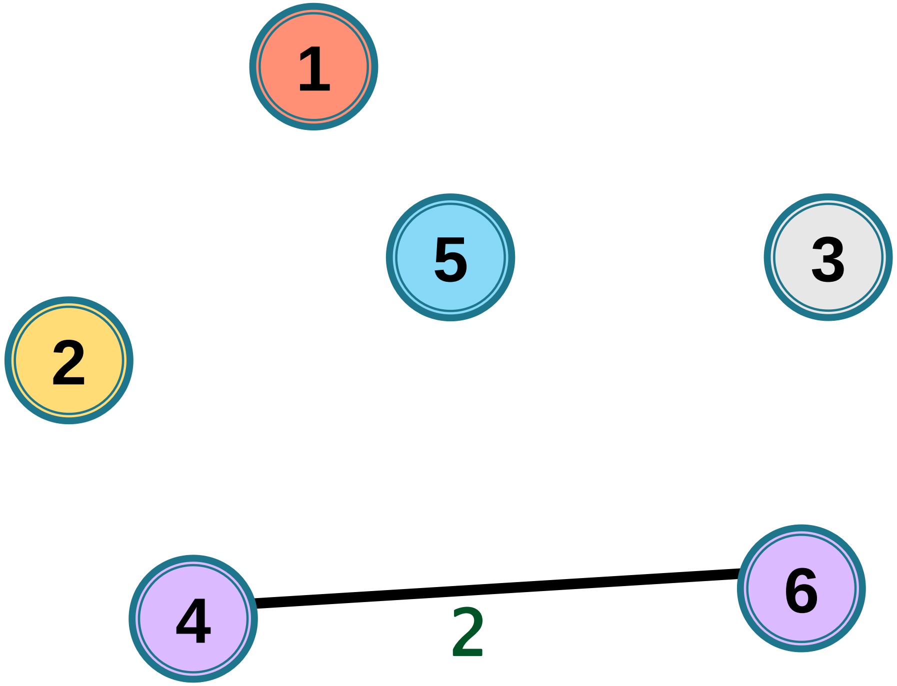
(3,6)

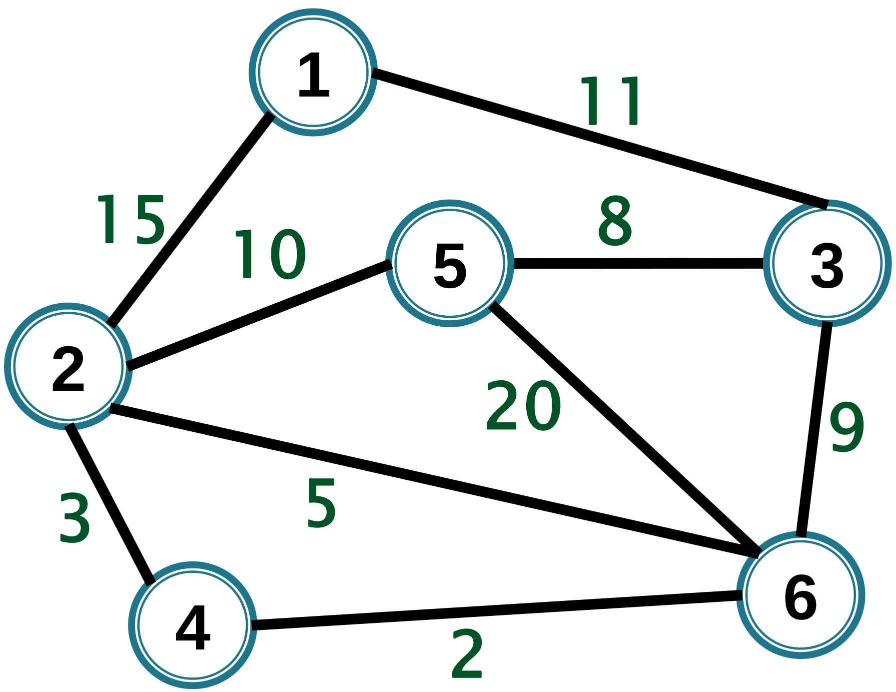
(2,5)

(1,3)

(1,2)

(5,6)





$r = [1,2,3,4,5,6]$

(4,6) $r = [1,\underline{2},3,\underline{4},5,4]$

(2,4) $r(2) \neq r(4)$

(2,6)

(3,5)

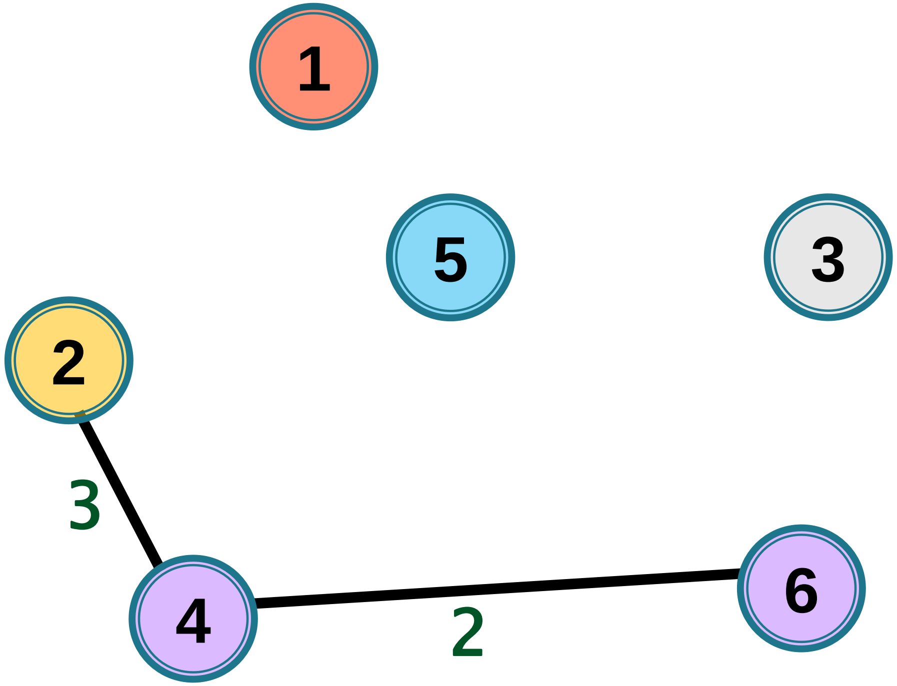
(3,6)

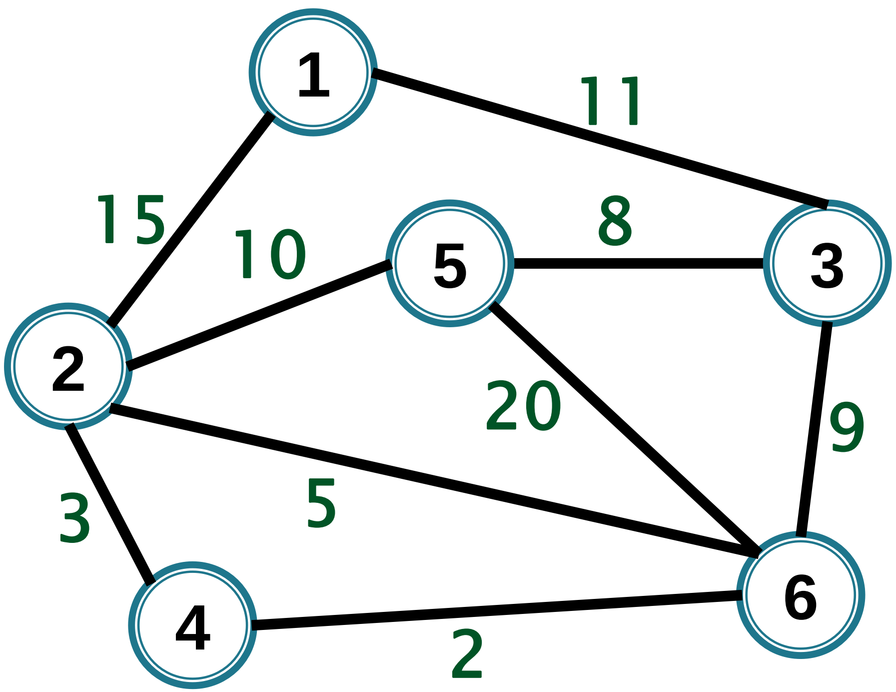
(2,5)

(1,3)

(1,2)

(5,6)





$r = [1,2,3,4,5,6]$

(4,6) $r = [1,\underline{2},3,\underline{4},5,4]$

(2,4)

(2,6)

(3,5)

(3,6)

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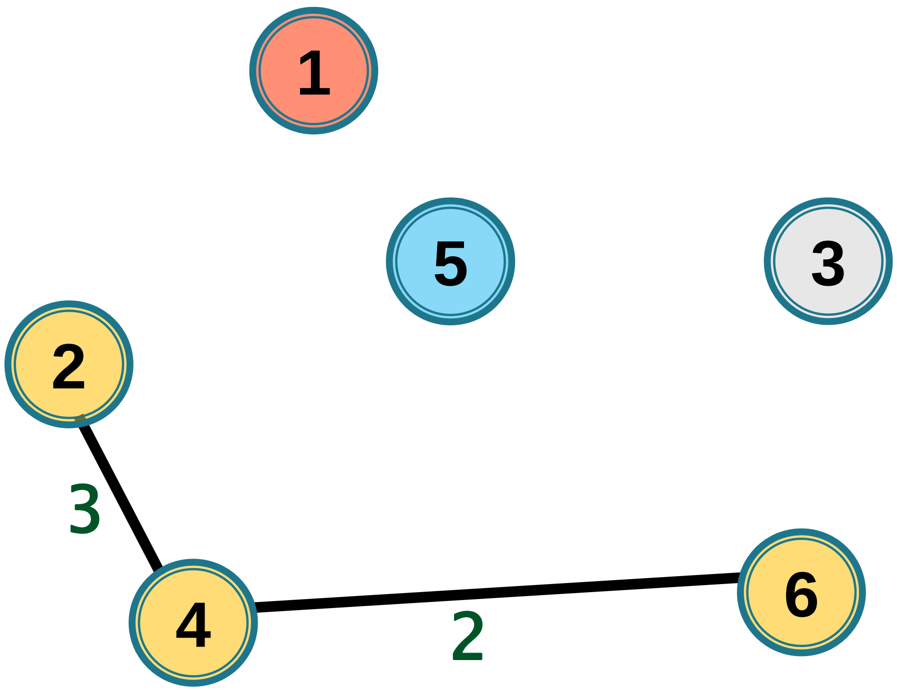
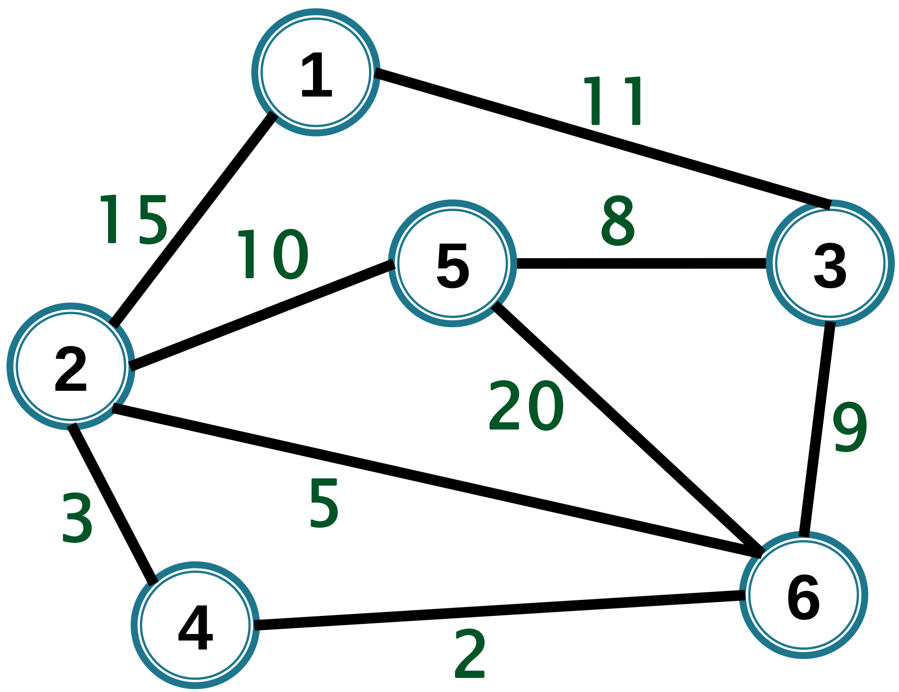
(1,3)

(1,2)

(5,6)





r = [1,2,3,4,5,6]

(4,6) r = [1,2,3,4,5,4]

(2,4) r = [1,2,3,2,5,2]

(2,6)

(3,5)

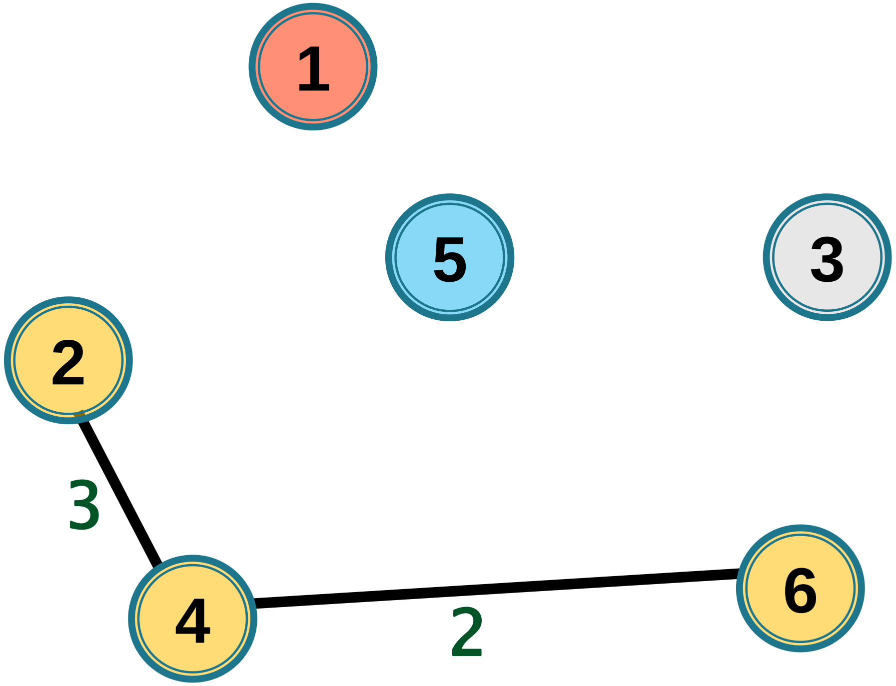
(3,6)

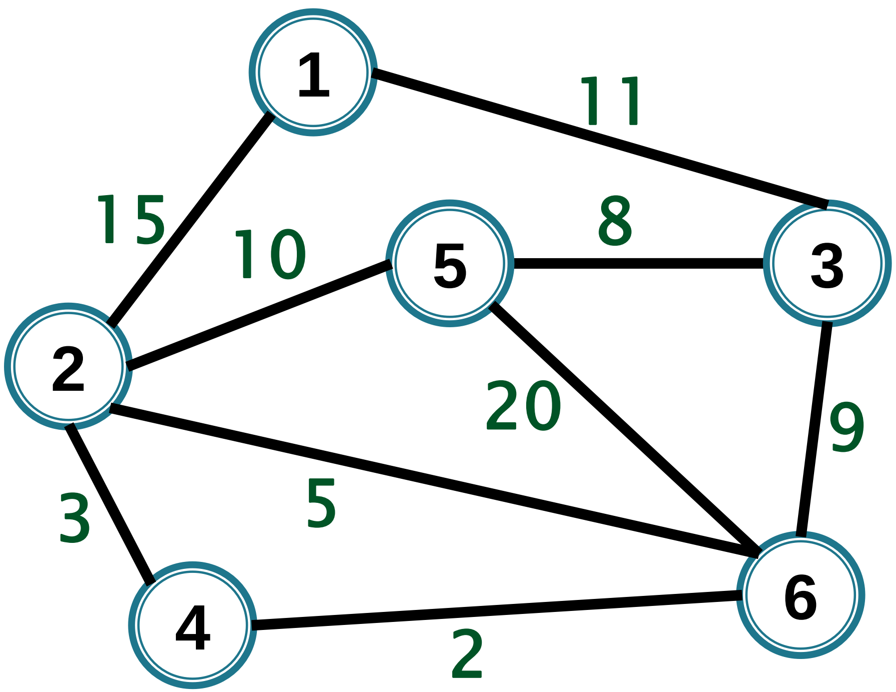
(2,5)

(1,3)

(1,2)

(5,6)





r = [1,2,3,4,5,6]

(4,6) r = [1,2,3,4,5,4]

(2,4) r = [1,2,3,2,5,2]

(2,6)

(3,5)

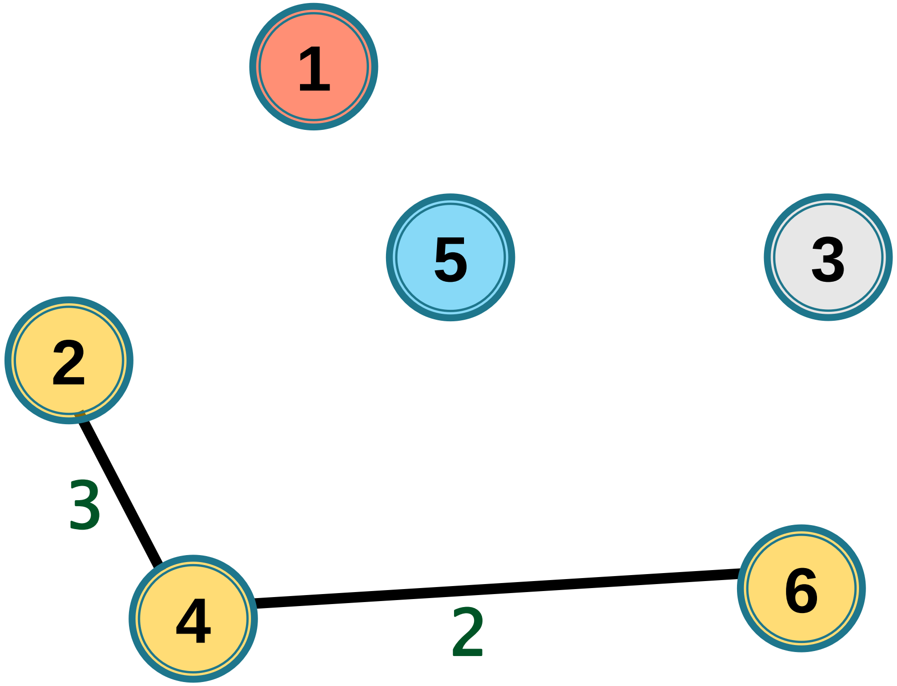
(3,6)

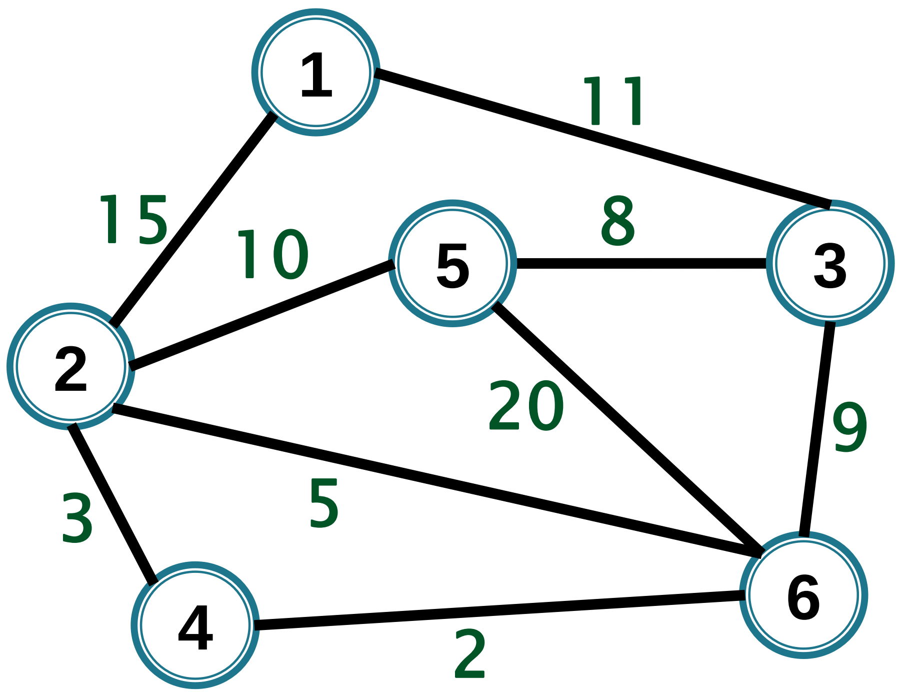
(2,5)

(1,3)

(1,2)

(5,6)





r = [1,2,3,4,5,6]

(4,6) r = [1,2,3,4,5,4]

(2,4) r = [1,2,3,2,5,2]

(2,6) r(2) =r(6) -> NU

(3,5)

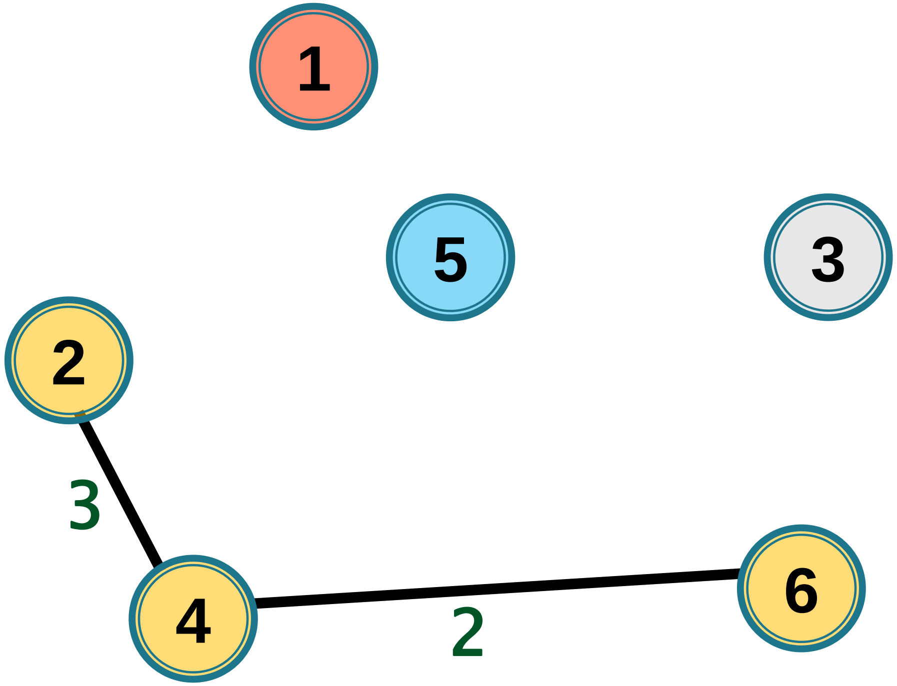
(3,6)

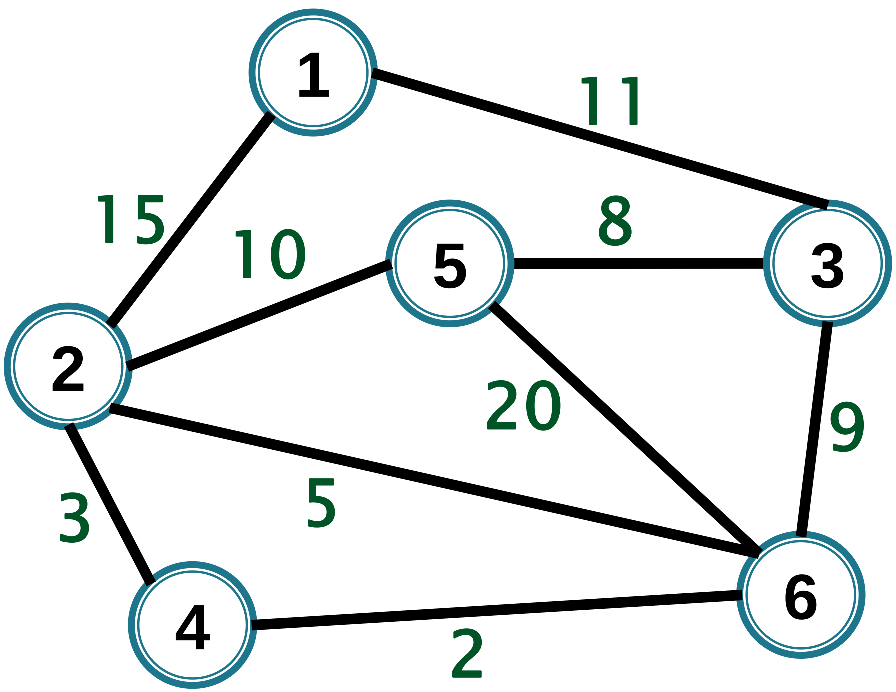
(2,5)

(1,3)

(1,2)

(5,6)





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(2,4) r = [1,2,3,2,5,2]

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(3,6)

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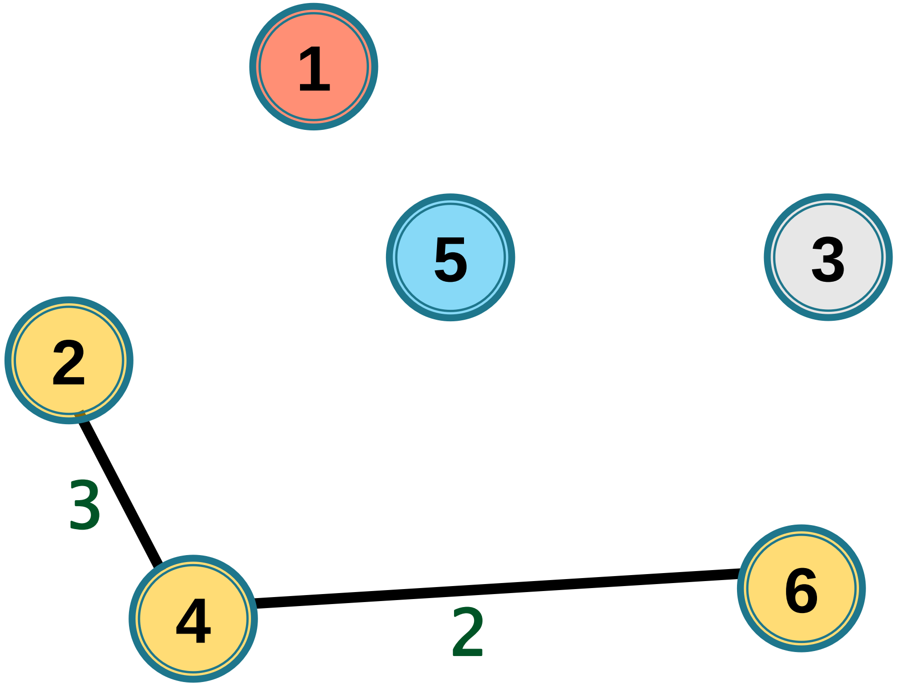
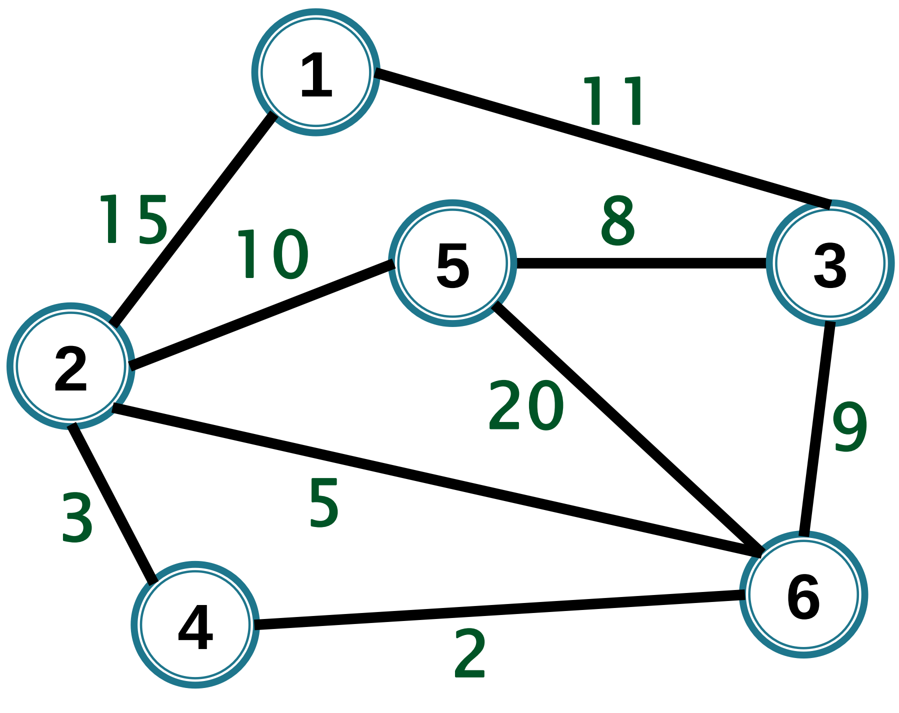
(1,3)

(1,2)

(5,6)





r = [1,2,3,4,5,6]

(4,6) r = [1,2,3,4,5,4]

(2,4) r = [1,2,<u>3</u>,2,<u>5</u>,2]

(2,6) r(2) =r(6) -> NU

(3,5) $r(3) \neq r(5)$

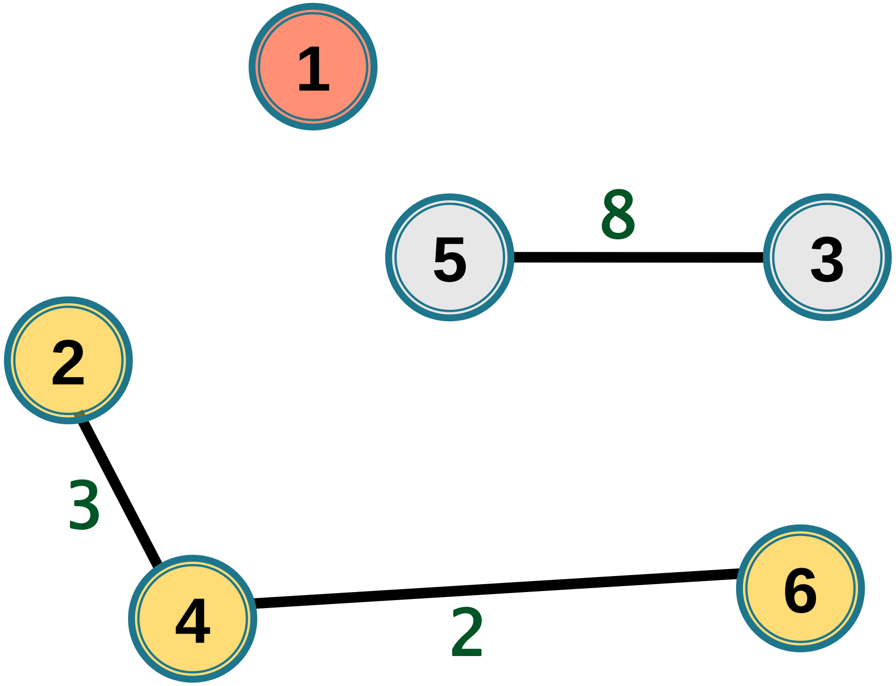
(3,6)

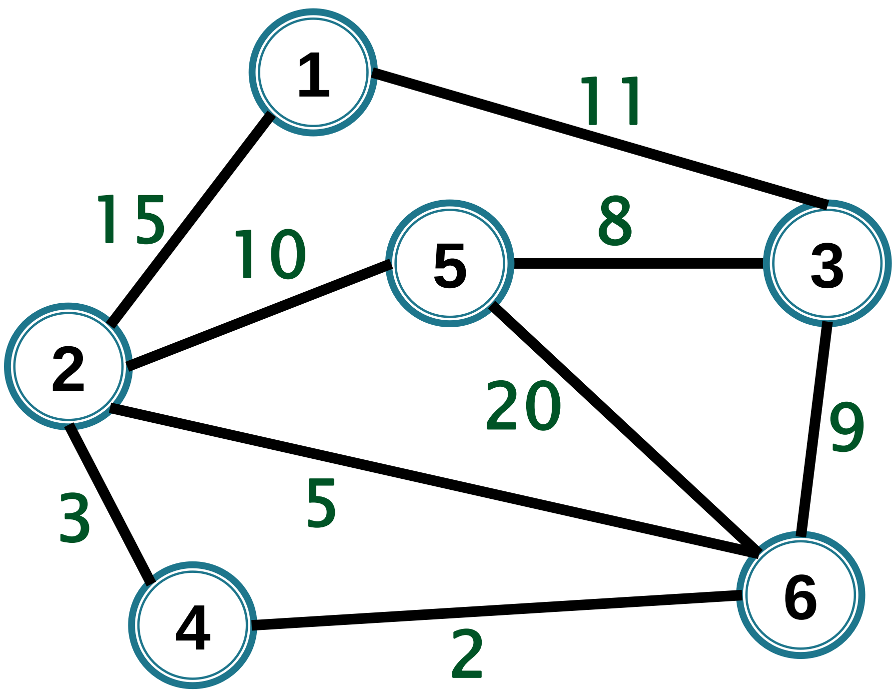
(2,5)

(1,3)

(1,2)

(5,6)





r = [1,2,3,4,5,6]

(4,6) r = [1,2,3,4,5,4]

(2,4) r = [1,2,3,2,5,2]

(2,6) r(2) =r(6) -> NU

(3,5) r = [1,2,3,2,3,2]

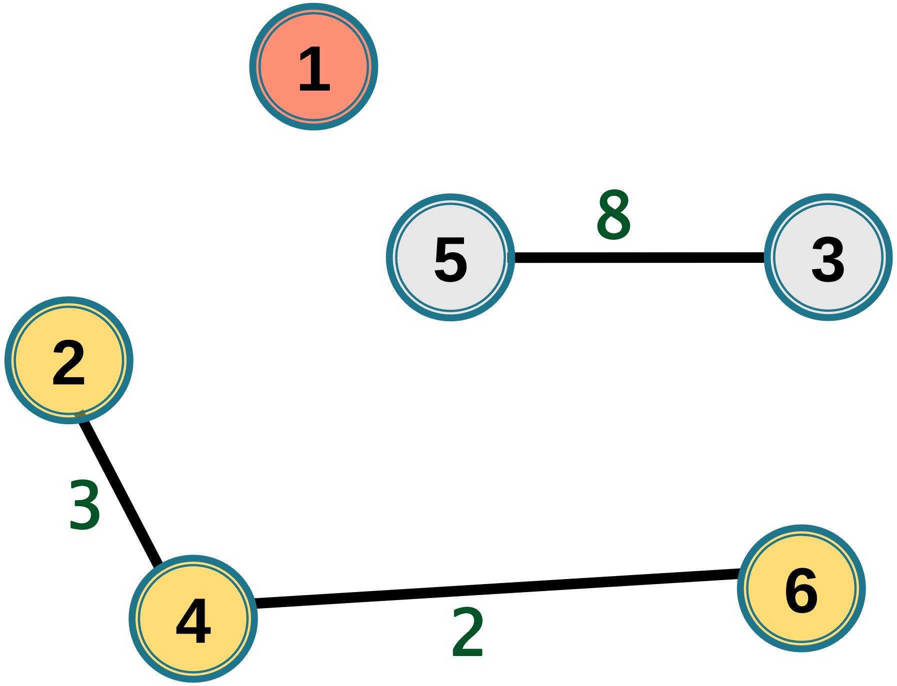
(3,6)

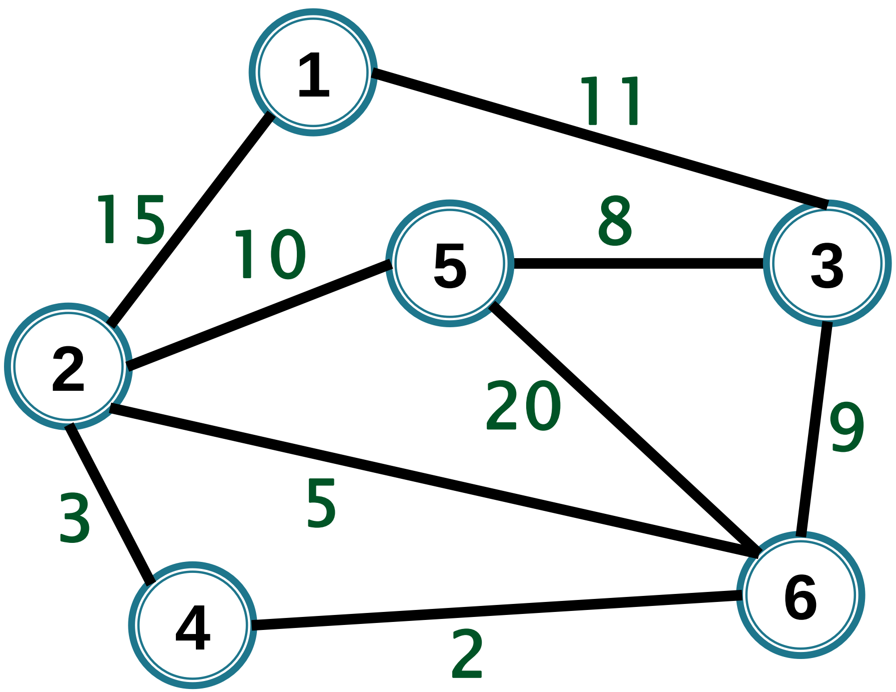
(2,5)

(1,3)

(1,2)

(5,6)





r = [1,2,3,4,5,6]

(4,6) r = [1,2,3,4,5,4]

(2,4) r = [1,2,3,2,5,2]

(2,6) r(2) =r(6) -> NU

(3,5) r = [1,2,3,2,3,2]

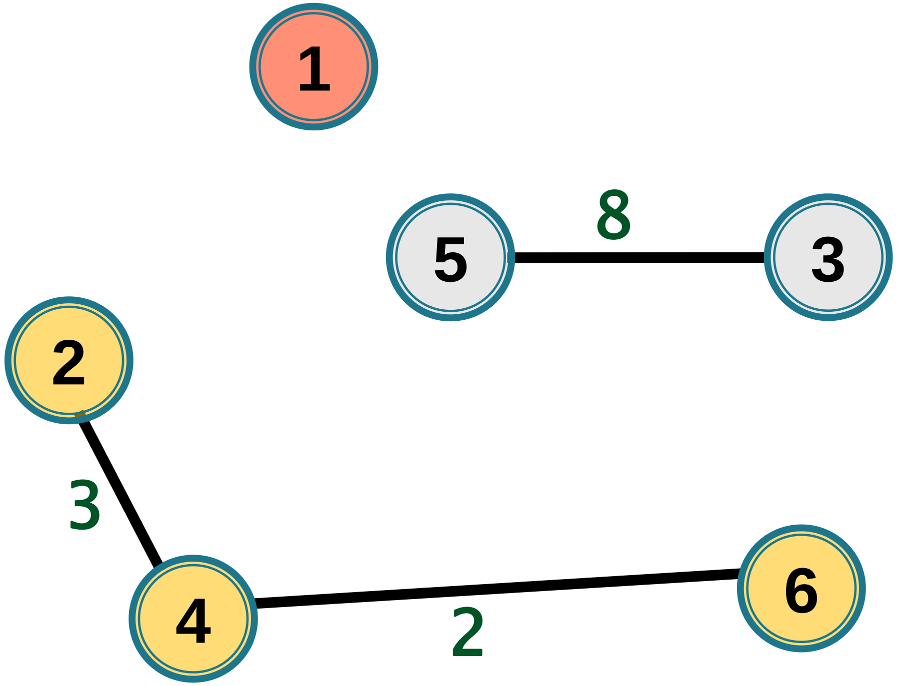
(3,6)

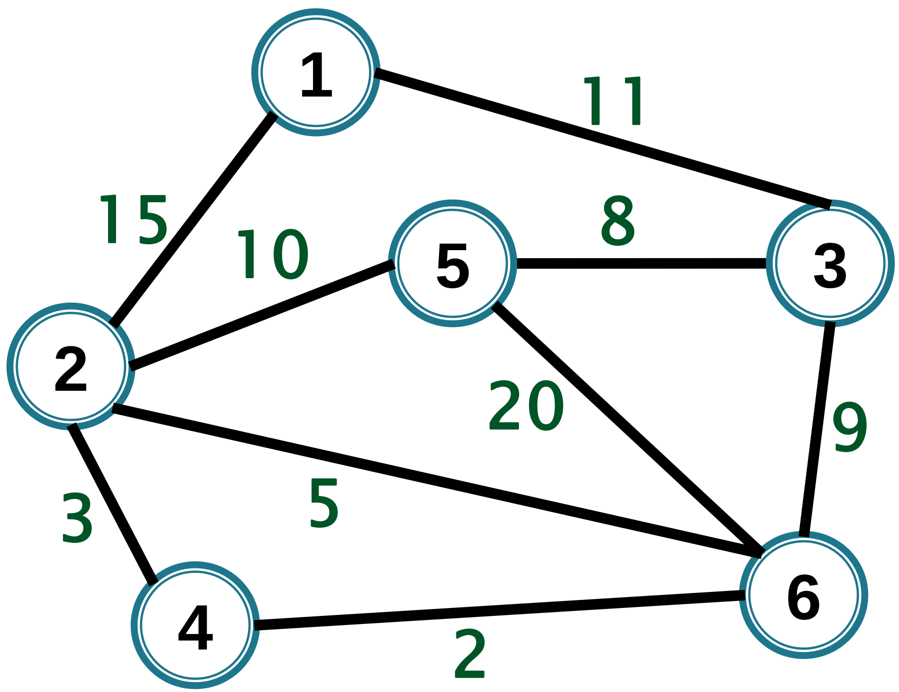
(2,5)

(1,3)

(1,2)

(5,6)





r = [1,2,3,4,5,6]

(4,6) r = [1,2,3,4,5,4]

(2,4) r = [1,2,3,2,5,2]

(2,6) r(2) =r(6) -> NU

(3,5) r = [1,2,<u>3</u>,2,3,<u>2</u>]

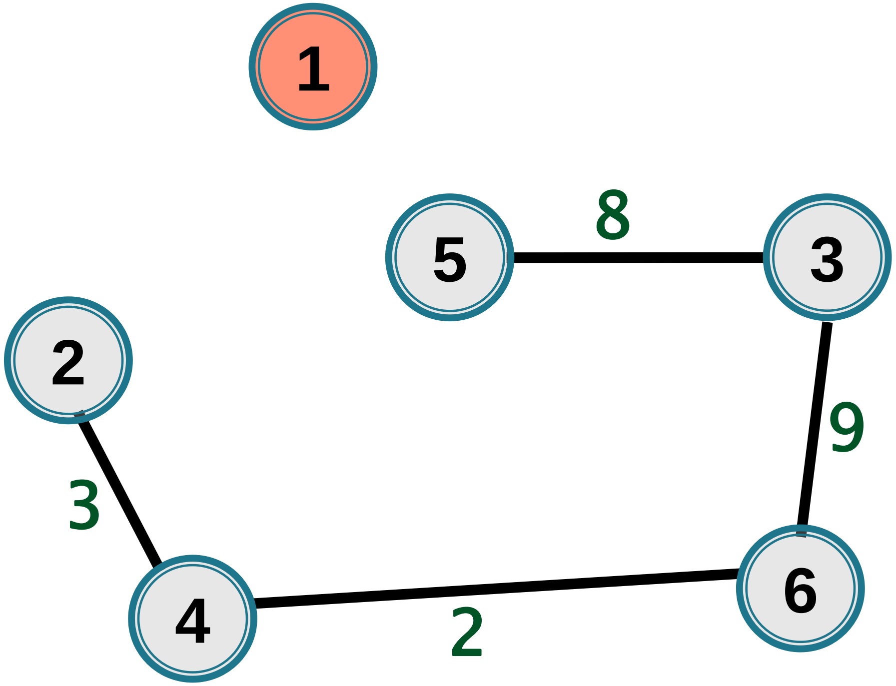
(3,6) r(3) ≠ r(6)

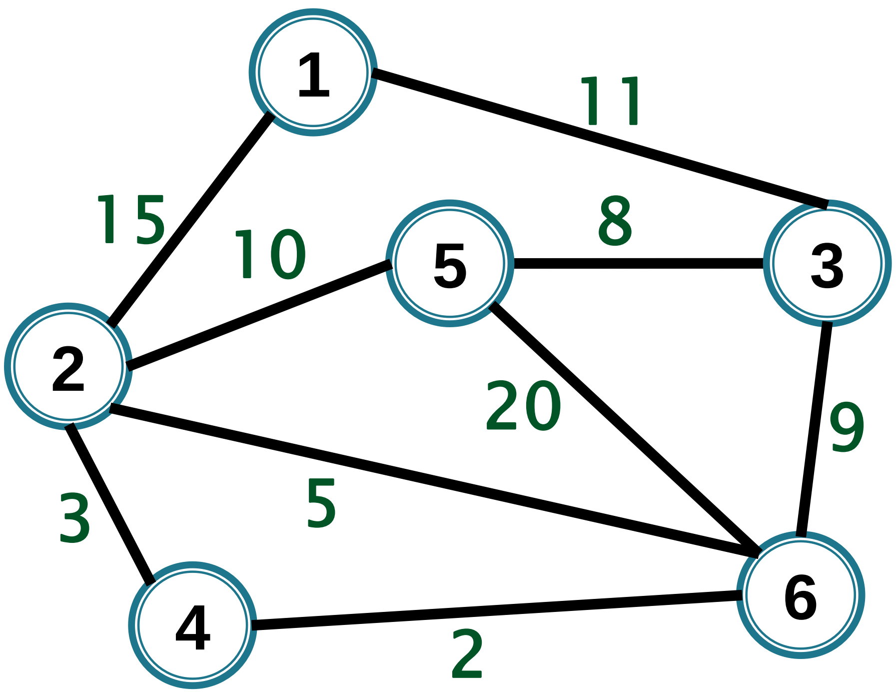
(2,5)

(1,3)

(1,2)

(5,6)





r = [1,2,3,4,5,6]

(4,6) r = [1,2,3,4,5,4]

(2,4) r = [1,2,3,2,5,2]

(2,6) r(2) =r(6) -> NU

(3,5) r = [1,2,<u>3</u>,2,3,<u>2</u>]

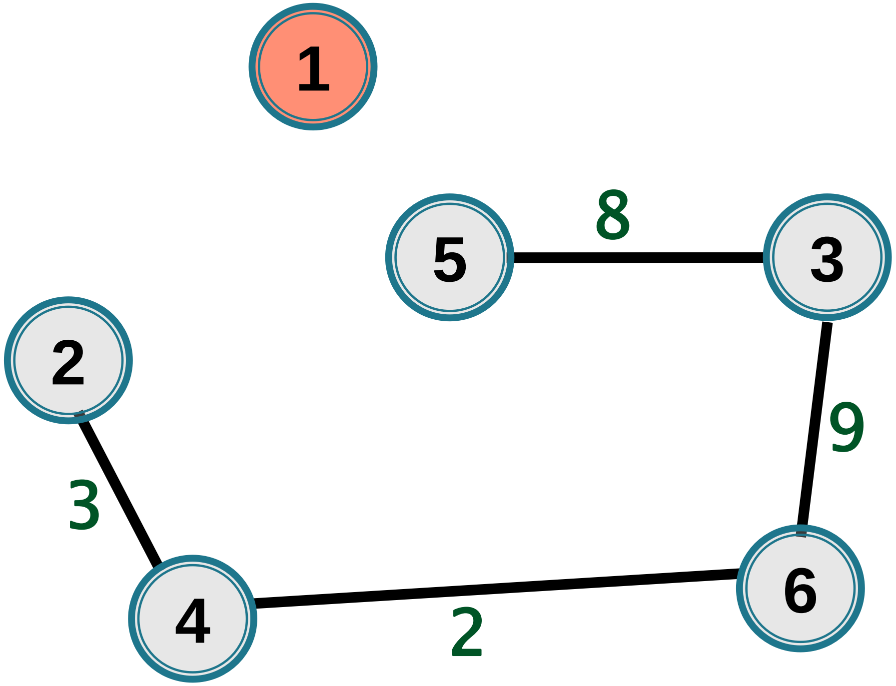
(3,6) r = [1,3,3,3,3,3]

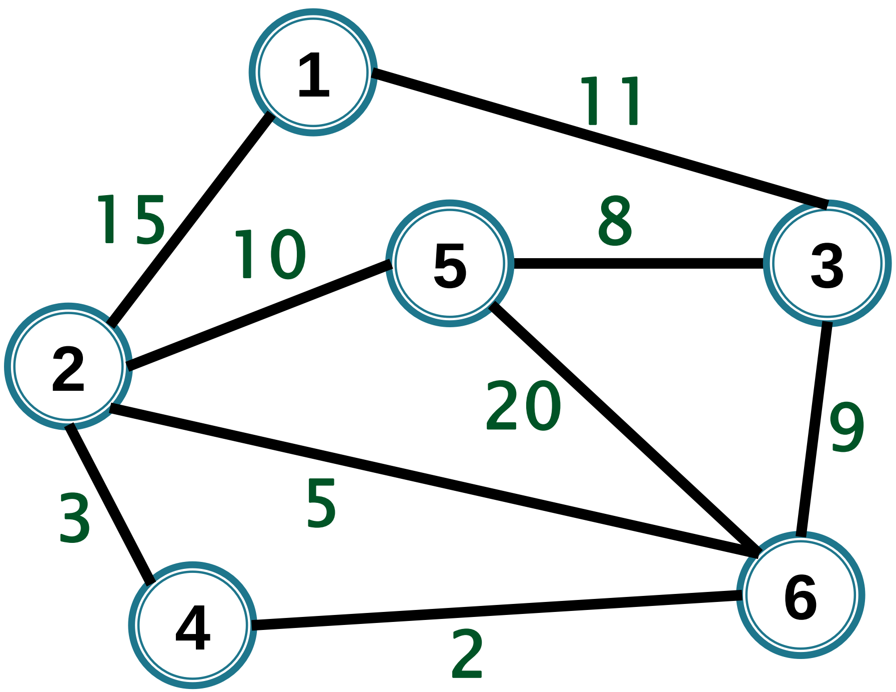
(2,5)

(1,3)

(1,2)

(5,6)





r = [1,2,3,4,5,6]

(4,6) r = [1,2,3,4,5,4]

(2,4) r = [1,2,3,2,5,2]

(2,6) r(2) =r(6) -> NU

(3,5) r = [1,2,3,2,3,2]

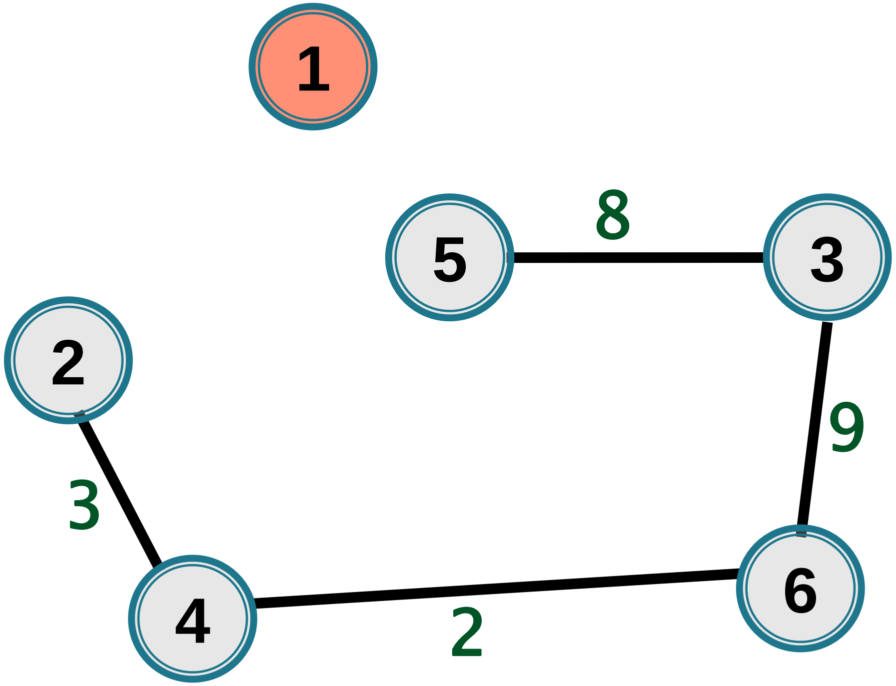
(3,6) r = [1,3,3,3,3,3]

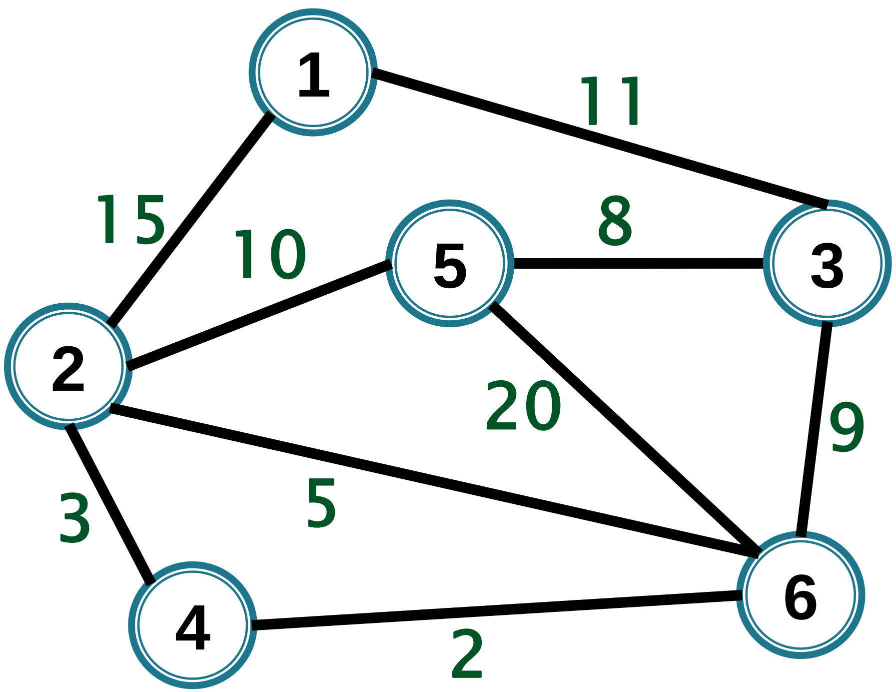
(2,5)

(1,3)

(1,2)

(5,6)





r = [1,2,3,4,5,6]

(4,6) r = [1,2,3,4,5,4]

(2,4) r = [1,2,3,2,5,2]

(2,6) r(2) =r(6) -> NU

(3,5) r = [1,2,3,2,3,2]

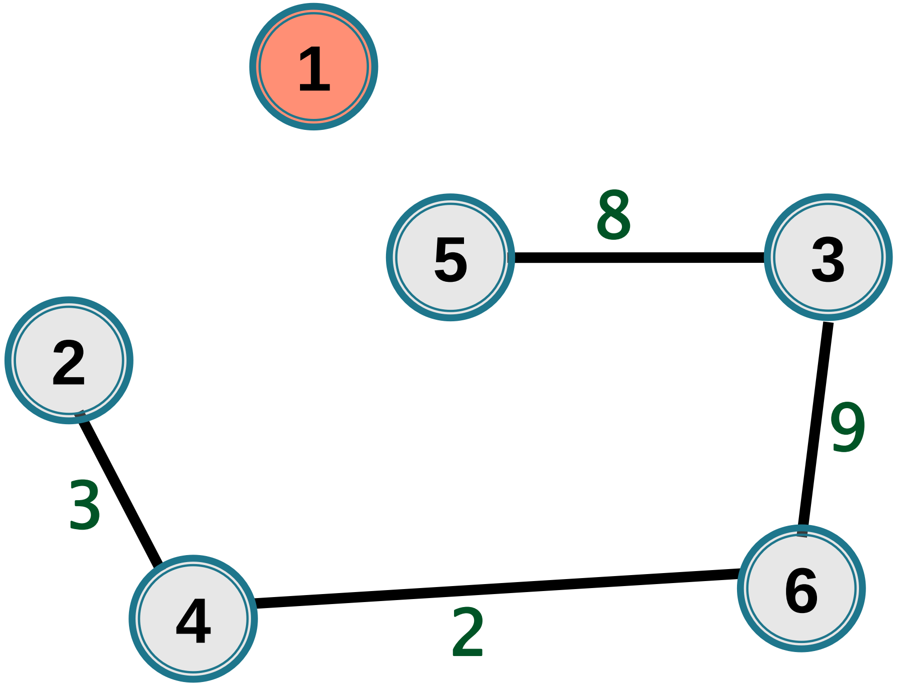
(3,6) r = [1,3,3,3,3,3]

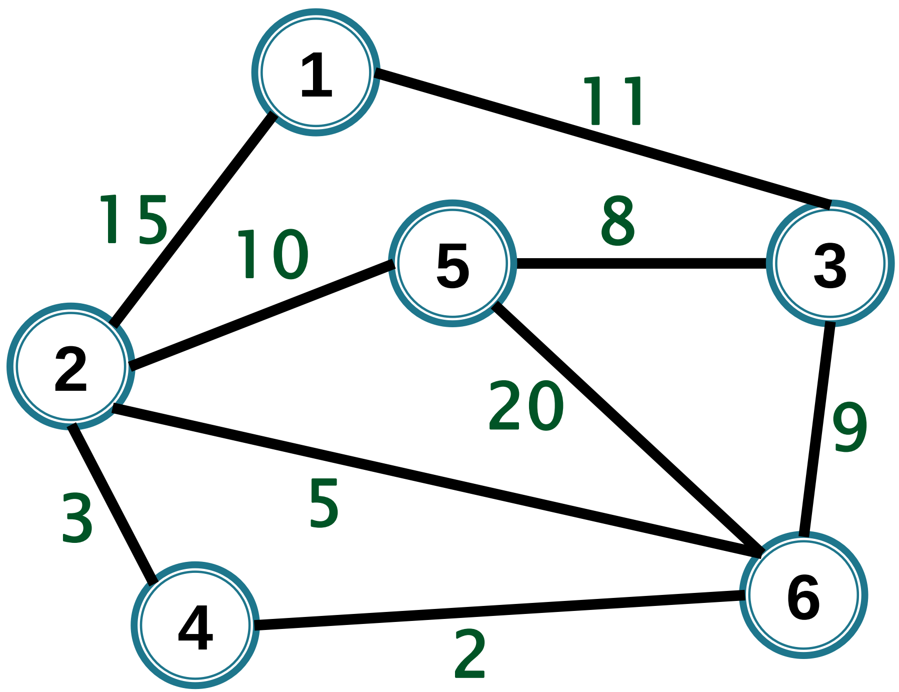
(2,5) r(2) =r(5) -> NU

(1,3)

(1,2)

(5,6)





r = [1,2,3,4,5,6]

(4,6) r = [1,2,3,4,5,4]

(2,4) r = [1,2,3,2,5,2]

(2,6) r(2) =r(6) -> NU

(3,5) r = [1,2,3,2,3,2]

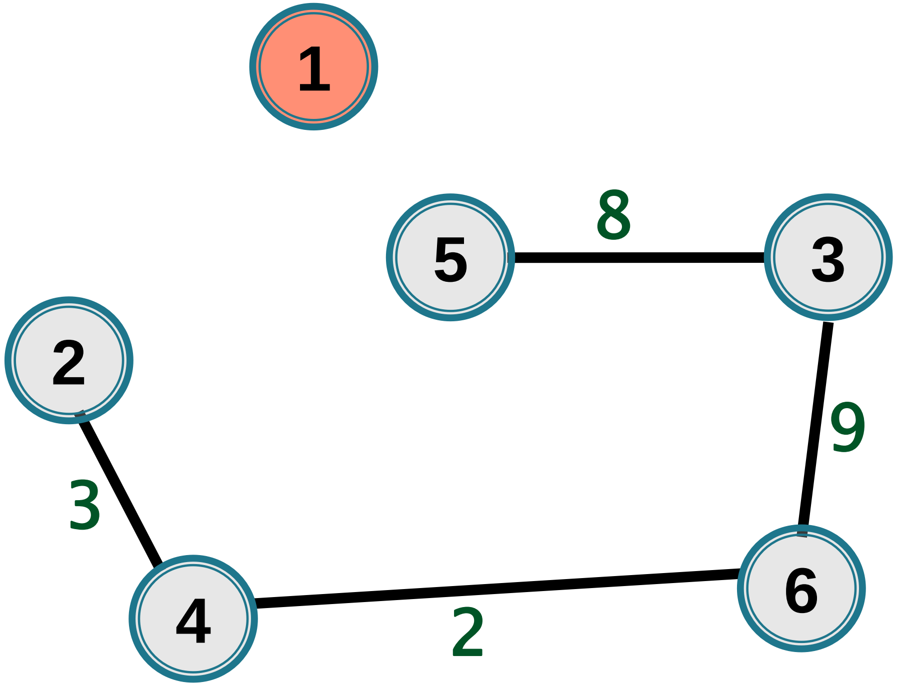
(3,6) r = [1,3,3,3,3,3]

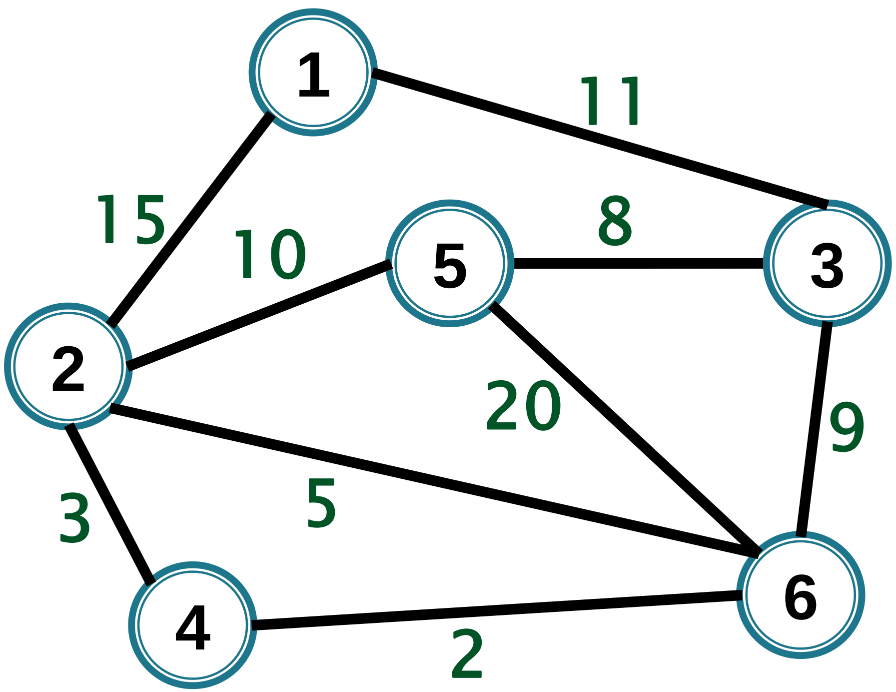
(2,5) r(2) =r(5) -> NU

(1,3)

(1,2)

(5,6)





r = [1,2,3,4,5,6]

(4,6) r = [1,2,3,4,5,4]

(2,4) r = [1,2,3,2,5,2]

(2,6) r(2) =r(6) -> NU

(3,5) r = [1,2,3,2,3,2]

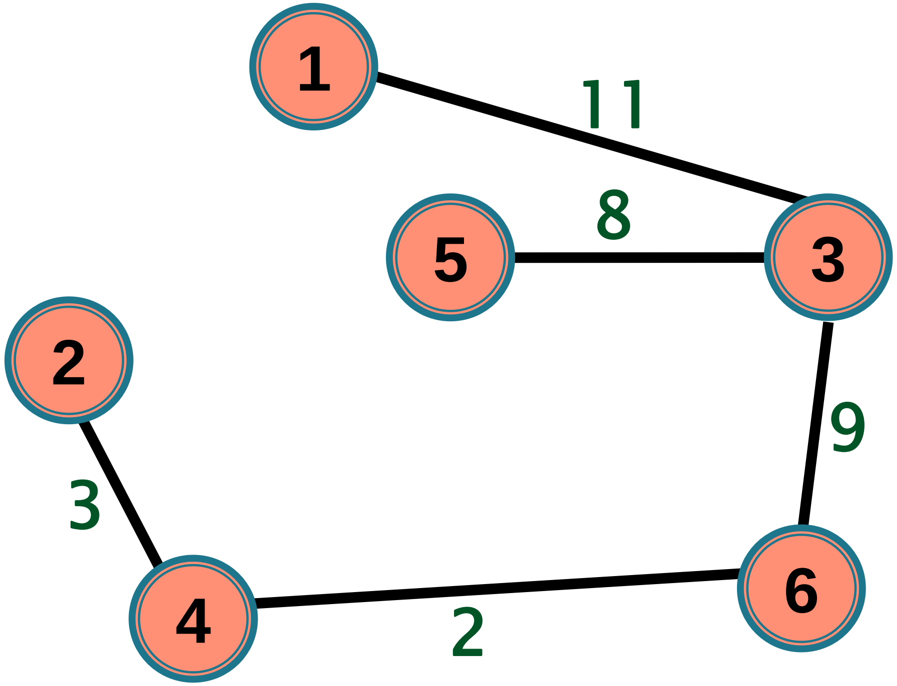
(3,6) r = [1,3,3,3,3,3]

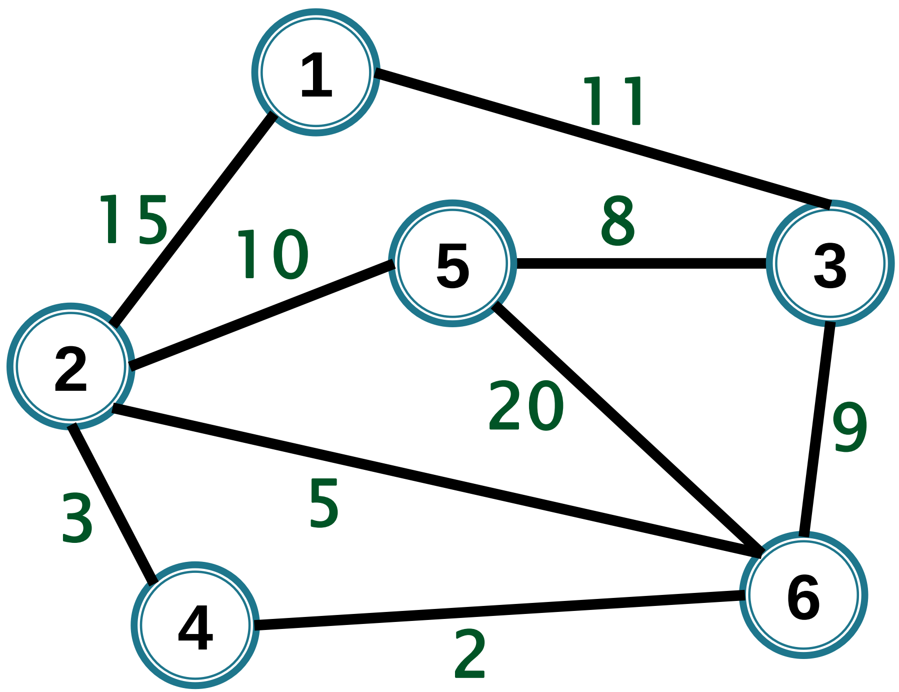
(2,5) r(2) =r(5) -> NU

(1,3) r(1) ≠ r(3)

(1,2)

(5,6)





r = [1,2,3,4,5,6]

(4,6) r = [1,2,3,4,5,4]

(2,4) r = [1,2,3,2,5,2]

(2,6) r(2) =r(6) -> NU

(3,5) r = [1,2,3,2,3,2]

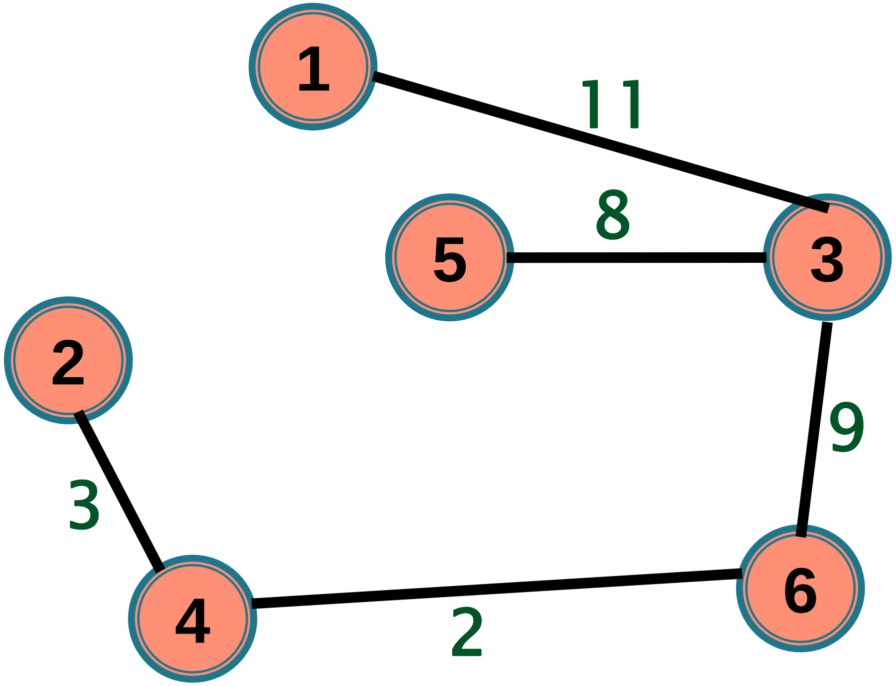
(3,6) r = [1,3,3,3,3,3]

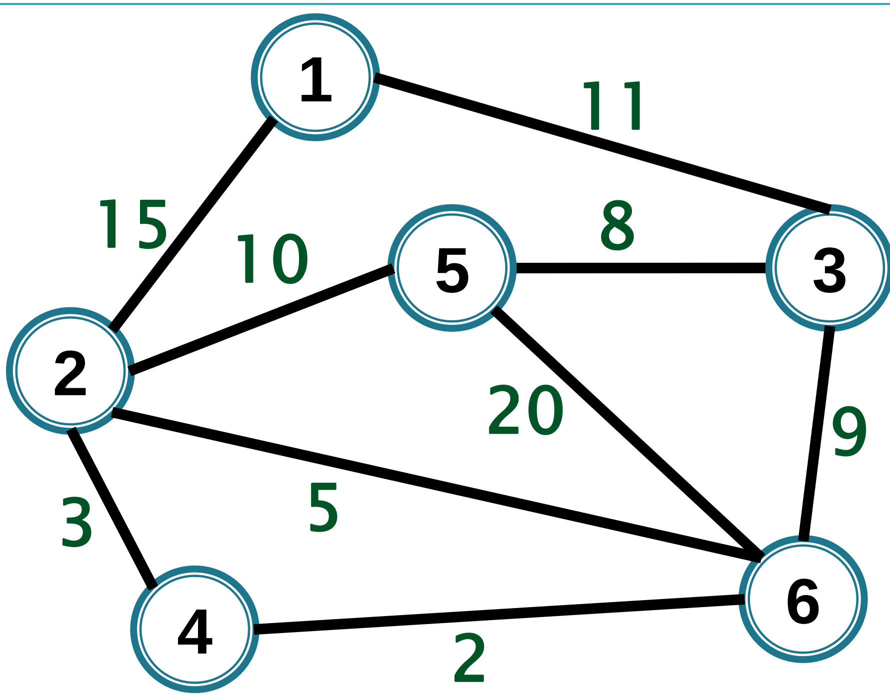
(2,5) r(2) =r(5) -> NU

(1,3) r = [1,1,1,1,1,1]

(1,2)

(5,6)





r = [1,2,3,4,5,6]

(4,6) r = [1,2,3,4,5,4]

(2,4) r = [1,2,3,2,5,2]

(2,6) r(2) =r(6) -> NU

(3,5) r = [1,2,3,2,3,2]

(3,6) r = [1,3,3,3,3,3]

(2,5) r(2) =r(5) -> NU

(1,3) r = [1,1,1,1,1,1]

STOP

(1,2)

(5,6)

Kruskal

Varianta 2 – Structuri pentru mulțimi disjuncte Union/Find

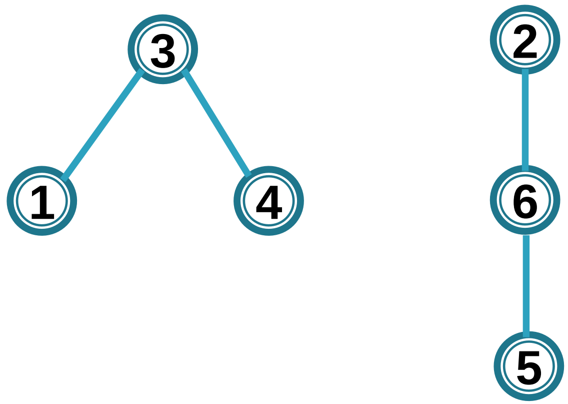
Kruskal

Varianta 2 – Structuri pentru mulțimi disjuncte Union/Find – arbori cu rădăcină

- memorăm componentele conexe ca arbori,

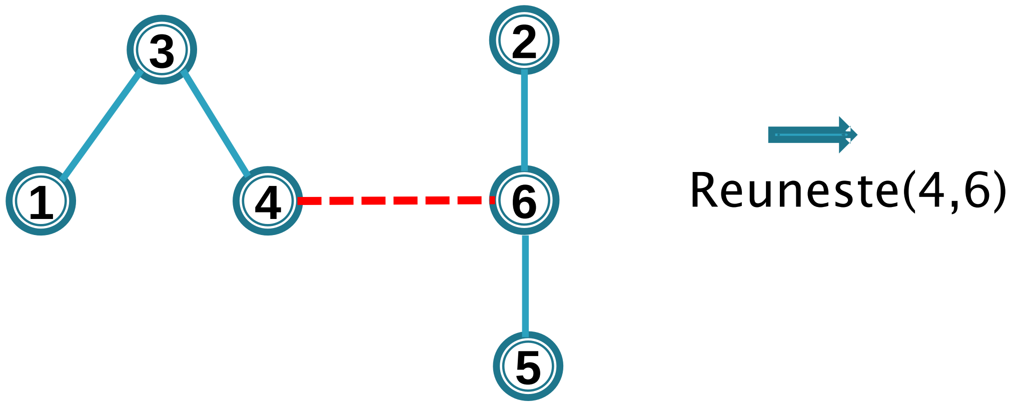
  folosind **vectorul tata**;

- **reprezentantul componentei va fi rădăcina arborelui**



Kruskal

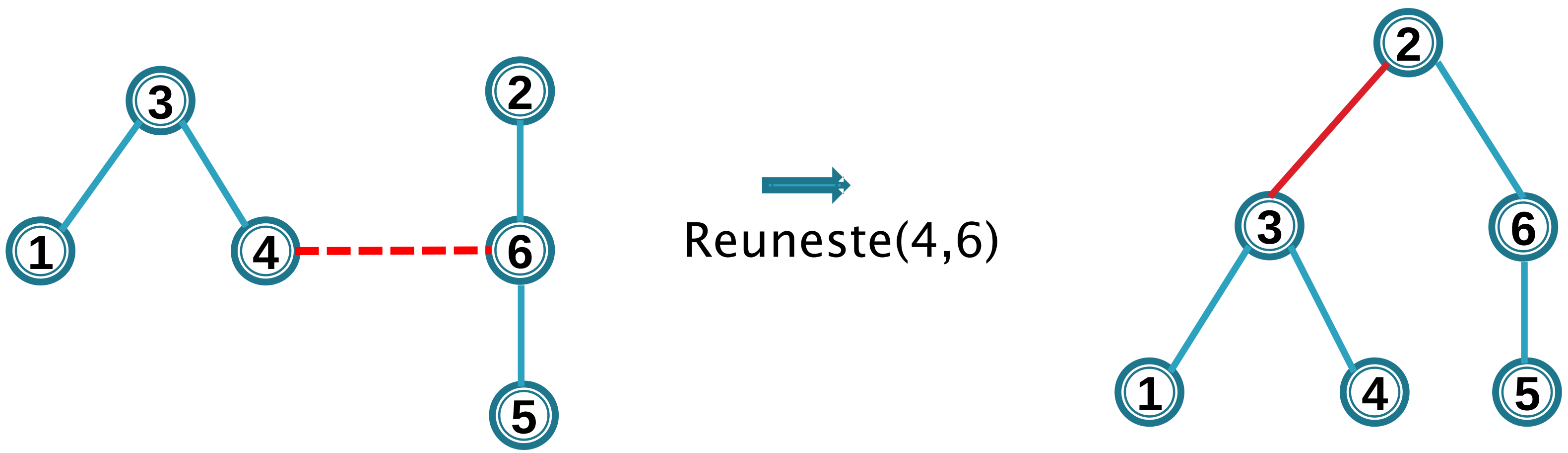
- **Reuniunea** a doi arbori ⇒ rădăcina unui arbore devine fiu al rădăcinii celuilalt arbore



Kruskal

- Reuniunea se va face în funcţie de înălţimea/dimensiunea arborilor (reuniune ponderată), pentru a obține arbori de înălțime mică

$\Rightarrow$ arbori de înălțime logaritmică



- arborele cu înălţimea mai mică devine subarbore al rădăcinii celuilalt arbore

Kruskal

Detalii de implementare operații cu structuri Union/Find pentru mulțimi disjuncte:

- **Initializare**
- **Reprez(u)** $\Rightarrow$ determinarea rădăcinii arborelui care conține u

  \+ **compresie de cale** (v. seminar+laborator)
- **Reuneste(u,v)** $\Rightarrow$ reuniune ponderată

Kruskal

```
void Initializare(int u){
    tata[u]=h[u]=0;
}

int Reprez(int u){
    while(tata[u]!=0)
       u = tata[u];
    return u;
}
```

Kruskal

```
void Initializare(int u){
    tata[u]=h[u]=0;
}

int Reprez(int u){
    while(tata[u]!=0)
        u = tata[u];
    return u;
}
```

```
void Reuneste(int u,int v)
{
    int ru,rv;
    ru = Reprez(u);
    rv = Reprez(v);
    if (h[ru] > h[rv])
        tata[rv] = ru;
    else{
        tata[ru] = rv;



    }
}
```

Kruskal

```
void Initializare(int u){
    tata[u]=h[u]=0;
}

int Reprez(int u){
    while(tata[u]!=0)
       u = tata[u];
    return u;
}
```

```
void Reuneste(int u,int v)
{
   int ru,rv;
   ru = Reprez(u);
   rv = Reprez(v);
   if (h[ru] > h[rv])
       tata[rv] = ru;
   else{
       tata[ru] = rv;
       if(h[ru] == h[rv])
           h[rv] = h[rv]+1;
   }
}
```

Kruskal

Complexitate

Varianta 2 – dacă folosim arbori Union/Find

- Sortare -> $O(m \log m) = O(m \log n)$
- n * Initializare -> $O(n)$
- 2m * Reprez ->
- (n-1) * Reuneste ->

Kruskal

Complexitate

Varianta 2 – dacă folosim arbori Union/Find

- Sortare -> O(m log m) = O(m log n)
- n * Initializare -> O(n)
- 2m * Reprez -> O(m log n)
- (n-1) * Reuneste -> O(n log n)

Kruskal

Complexitate

Varianta 2 – dacă folosim arbori Union/Find

- **Sortare** -> O(m log m) = O(m log n)
- n * **Initializare** -> O(n)
- 2m * **Reprez** -> O(m log n)
- (n-1) * **Reuneste** -> O(n log n)

mai mică dacă folosim și compresie de cale

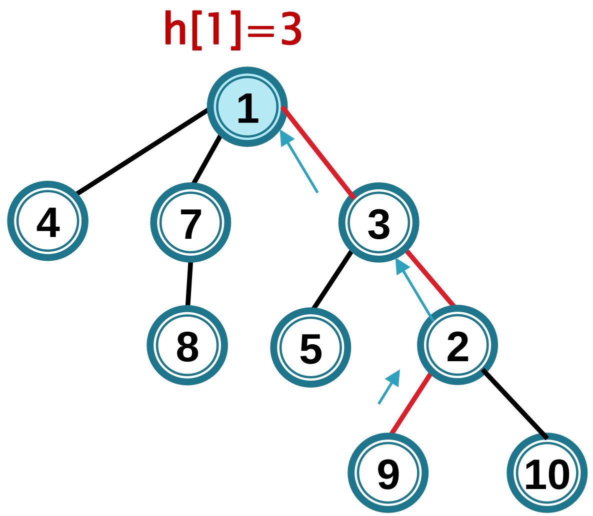
O(m log n)

Kruskal

Compresie de cale

```
int Reprez(int u){
  if (tata[u]==0)
        return u;
  tata[u]=Reprez(tata[u]);
  return tata[u];
}
```

După apelul Reprez(9) pentru arborele



rezultatul va fi 1, iar arborele devine

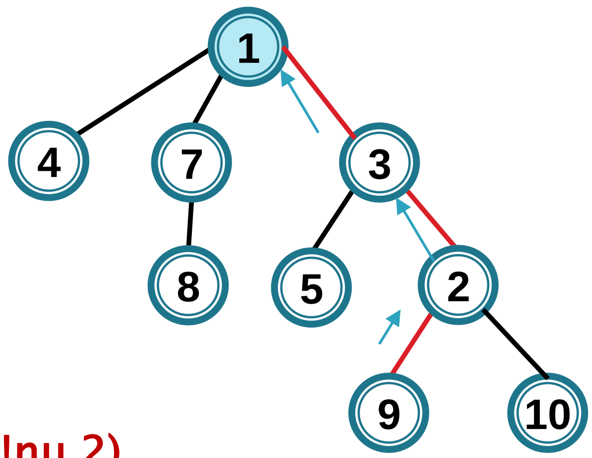
Kruskal

Compresie de cale

```
int Reprez(int u){
   if (tata[u]==0)
          return u;
   tata[u]=Reprez(tata[u]);
   return tata[u];
}
```

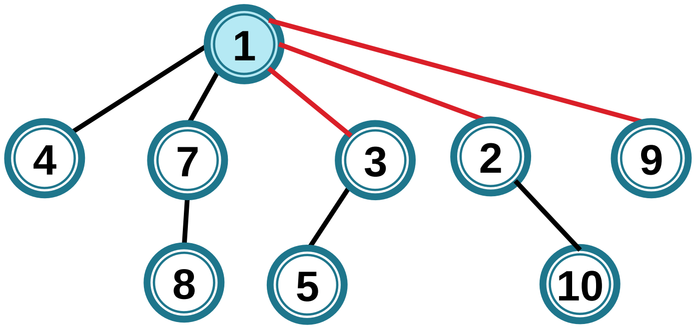
h[1]=3

După apelul Reprez(9) pentru arborele



h[1]=3 (!!nu 2)

rezultatul va fi 1, iar arborele devine



Kruskal

Concluzii complexitate

- Vector de culori $O(m \log n + n^2)$
- Structuri union/find $O(m \log n)$

Kruskal

Temă

- Dacă ponderile sunt in mulțimea {1, …, k} , k<100, ce complexitate are algoritmul lui Kruskal?
  - se pot sorta muchiile cu o complexitate mai mică decât O(m log(n))?
- Dacă ponderile sunt in mulțimea {1,…, |V|} ce complexitate are algoritmul lui Kruskal?