$\mathbb{P}(A_1 \cup \dots \cup A_n) = \sum_{i=1}^{n} \mathbb{P}(A_i) - \sum_{i<j} \mathbb{P}(A_i \cap A_j) + \dots + (-1)^{n+1} \mathbb{P}(A_1 \cap \dots \cap A_n)$

$C_n^r = \frac{n!}{r!(n-r)!}$

$A_n^k = \frac{n!}{(n-k)!}$

	ordine	fără
întoarcere	n^k	C_{n+k-1}^{k}
fără	$\frac{n!}{(n-k)!}$	C_n^k

$\binom{n}{n_1}\binom{n-n_1}{n_2} \dots \binom{n-n_1-\dots-n_{k-1}}{n_k} = \frac{n!}{n_1! n_2! \dots n_k!} = \binom{n}{n_1, n_2, \dots, n_k}$

$\mathbb{P}(A|B) = \frac{\mathbb{P}(A \cap B)}{\mathbb{P}(B)}$; $\mathbb{P}(A|B) \cdot \mathbb{P}(B) = \mathbb{P}(A)\,\mathbb{P}(B|A)$

'A|B' ≠ eveniment

Fie $A_1, \dots, A_n \in \mathcal{F}$
$\Rightarrow \mathbb{P}(A_1 \cap \dots \cap A_n) = \mathbb{P}(A_1) \cdot \mathbb{P}(A_2|A_1) \cdot \mathbb{P}(A_3|A_1 \cap A_2) \cdot \dots \cdot \mathbb{P}(A_n | A_1 \cap \dots \cap A_{n-1})$

Formula prob. totale
$A_1, \dots, A_n$ partiție a lui Ω
$\Rightarrow \mathbb{P}(X) = \sum_{i=1}^{n} \mathbb{P}(X|A_i) \cdot \mathbb{P}(A_i)$

[redacted]

$A, B \in \mathcal{F}$ indep.
$\Leftrightarrow \mathbb{P}(A|B) = \mathbb{P}(A)$
sau $\mathbb{P}(A \cap B) = \mathbb{P}(A)\,\mathbb{P}(B)$

$A_1, \dots, A_n$ indep dacă $\mathbb{P}(\bigcap_{i \in I} A_i) = \prod_{i \in I} \mathbb{P}(A_i)$ $I \subseteq \{1, \dots, n\}$ finite

$A, B \perp\!\!\!\perp$ condiționat de C dacă $\mathbb{P}(A \cap B | C) = \mathbb{P}(A|C)\,\mathbb{P}(B|C)$

V.a. discrete

Def Fct de repartiție (cumulativă)
$F: \mathbb{R} \to [0,1]$, $F(x) = \mathbb{P}(X \leq x)$ $\forall x \in \mathbb{R}$
$F(x) = (\mathbb{P} \circ X^{-1})((-\infty, x])$
$F(x) = \sum_{y \leq x} f(y)$

Def Fct de masă pt $X: \Omega \to \mathbb{R}$ v.a.d.
$f(x) = \mathbb{P}(X = x)$ $\forall x \in X(\Omega)$ și
a) $f(x) \geq 0$ $\forall x$
b) $\sum_{x \in X(\Omega)} f(x) = 1$

Scheme

1. v.a. ct $X = c$, $c \in \mathbb{R}$
$f(x) = \begin{cases} 1, & x = c \\ 0, & x \neq c \end{cases}$

2. **Bernoulli**
$X: \Omega \to \{0,1\}$
$\mathbb{P}(X=1) = p$ și $\mathbb{P}(X=0) = 1-p$

3. **Binomială** Repetăm de n ori și ne int. realizarea unui eveniment A cu $\mathbb{P}(A) = p$.
$X \sim Bin(n,p)$ $\mathbb{P}(X=k) = C_n^k \cdot p^k \cdot (1-p)^{n-k}$
$E[X] = np$
$Var(X) = np(1-p)$

4. **HG** n extrageri fără întoarcere
N bile, M negre, N-M albe
X = nr bile negre $\sim HG(n, N, M)$
$\mathbb{P}(X=k) = \frac{C_M^k \cdot C_{N-M}^{n-k}}{C_N^n}$ $E[X] = \frac{n \cdot M}{N}$

Uniform
$X \sim U(D)$
$\mathbb{P}(X=x) = \frac{1}{|D|}$ $\forall x \in D$

Geometric
experiment Sansă succes p.
X = # experimente până la primul succes $\sim Geom(p)$
$\mathbb{P}(X=x) = (1-p)^{x-1} \cdot p$
$E[X] = \frac{1}{p}$
$Var(X) = \frac{1-p}{p}$

$X \sim Geom(p) \Rightarrow \mathbb{P}(X = n+k | X > n) = \mathbb{P}(X=k)$

NB
X = nr de aruncări până obținem pt prima oară $r \geq 1$ succese $\sim NB(r, p)$
$\mathbb{P}(X=k) = C_{k-1}^{r-1} \cdot (1-p)^{k-r} \cdot p^r$
$X = X_1 + \dots + X_r$ unde $X_i \sim Geom(p)$ indep

Poisson
$X \sim Pois(\lambda)$
$\mathbb{P}(X=k) = \frac{\lambda^k}{k!} e^{-\lambda}$
obs $X \sim Bin(n,p) \sim Pois(n \cdot p)$
$np \to \lambda \Rightarrow p \simeq \frac{\lambda}{n}$

$E(X) = \lambda$
$Var(X) = \lambda$

$X \sim Pois(\lambda_1)$ $Y \sim Pois(\lambda_2)$
$\Rightarrow X+Y \sim Pois(\lambda_1 + \lambda_2)$

$X \perp\!\!\!\perp Y \Rightarrow \mathbb{P}(X=x, Y=y) = \mathbb{P}(X=x)\,\mathbb{P}(Y=y)$
$X \sim B(n,p)$ $Y \sim B(m,p)$ $X \perp\!\!\!\perp Y \Rightarrow X+Y \sim B(n+m, p)$

(momentul de ordin k) $E[X^k]$
(mom centrat ... ordin k) $E[(X-c)^k]$
(mom centrat de ordin k) $E[(X-E[X])^k]$

$E[X] = \sum_x x\,\mathbb{P}(X=x)$
$X \perp\!\!\!\perp Y \Rightarrow E[XY] = E[X]E[Y]$
$Y = g(X) \Rightarrow E[Y] = \sum_x g(x)\,\mathbb{P}(X=x)$

- $Cov(X,Y) = E[(X-E[X])(Y-E[Y])] = E[XY] - E[X]E[Y]$
- $Cov(aX+bY, cX+dY) = ac\,Cov(X,X) + ad\,Cov(X,Y) + bc\,Cov(Y,X) + bd\,Cov(Y,Y)$
- $Cov(X,X) = Var(X)$ $Cov(X,Y) = Cov(Y,X)$

$Var(X) = E[(X-E[X])^2] = E[X^2] - E[X]^2$

$Var(aX \pm bY) = a^2 Var(X) + b^2 Var(Y) \pm 2ab\,Cov(X,Y)$

Va continue

X v.a cont dacă ∃ f pozitivă a.î.

$\mathbb{P}(X\in A)=\int_A f(x)\,dx \quad \forall x\in\mathbb{R}$

f = densitate de rep. dacă 1) $f(x)\geq 0$ 2) $\int_{-\infty}^{\infty} f(x)dx=1$

Fct de rep:

$F:\mathbb{R}\to[0,1]$

$F(x)=\mathbb{P}(X\leq x)=\int_{-\infty}^{x} f(t)dt$

$\Rightarrow f(x)=F'(x)$

$E[X]=\int_{-\infty}^{\infty} x f(x)dx$

$V(X)=\int_{-\infty}^{\infty}(x-E[X])^2 f(x)dx$

prob supraviețuire: $\mathbb{P}(X>a)=e^{-\lambda a}$

Rep uniformă

$U\sim\mathcal{U}([a,b])\to f_U(x)=\begin{cases}\frac{1}{b-a}, & x\in[a,b]\\ 0, & \text{altfel}\end{cases}$

$E[U]=\frac{b+a}{2} \quad Var(U)=\frac{(b-a)^2}{12}$

$U=a+(b-a)V \Rightarrow U\sim([a,b])$

$V\sim\mathcal{U}([0,1])$

Rep exp

$X\sim Exp(\lambda) \quad f(x)=\lambda e^{-\lambda x}, x>0, \lambda>0$

$F(x)=\begin{cases}0, & x<0\\ 1-e^{-\lambda x}, & x\geq 0\end{cases}$

$E[X]=\frac{1}{\lambda}, \; Var(X)=\frac{1}{\lambda^2}$

$P(X\geq s+t \mid X\geq s)=\mathbb{P}(X\geq t)\Leftrightarrow X\sim Exp(a)$

Rep normală: $X\sim\mathcal{N}(0,1)\Rightarrow$ densitatea $\varphi(x)=\frac{1}{\sqrt{2\pi}}e^{-\frac{x^2}{2}}$ și fct rep: $\Phi(x)=\int_{-\infty}^{x}\varphi(t)dt$

$\hookrightarrow \Phi(-x)=1-\Phi(x)$

$Z\sim\mathcal{N}(\mu,\sigma^2)\Rightarrow f(x)=\frac{1}{\sqrt{2\pi\sigma^2}}e^{-\frac{(x-\mu)^2}{2\sigma^2}} \quad Z=\mu+\sigma N\Rightarrow E[Z]=\mu+\sigma E[X]=\mu$

$Var(X)=\sigma^2 Var(Z)=\sigma^2$

$\mathbb{P}(|Z-\mu|\leq\sigma)\simeq 68\%$; $\mathbb{P}(|Z-\mu|\leq 2\sigma)\simeq 95\%$ $\mathbb{P}(|Z-\mu|\leq 3\sigma)\simeq 99.7\%$

Rep comună

Discret

$f(x,y)=\mathbb{P}(X=x,Y=y)$

$f_X(x)=\sum_y f(x,y)=\sum_y f_{X|Y}(x|y)\cdot f_Y(y)$

$f_{X|Y}(x|y)=\mathbb{P}(X=x|Y=y)=\frac{f(x,y)}{f_Y(y)}$ | $E(X|Y)$ v.a. def prin $g(y)=E(X|Y=y)$

$E[g(X,Y)]=\sum_{x,y} g(x,y)f(x,y)$; $E(g(X)|A)=\sum_x g(x) f_{X|A}(x)$

Continue $\mathbb{P}((X,Y)\in A\times B)=\iint_{A\times B} f_{(X,Y)}(x,y)dxdy$

f dens $\Rightarrow \iint_{\mathbb{R}^2} f(x,y)dxdy=1$

$f_X(x)=\int_{\mathbb{R}} f(x,y)dy$ și $\mathbb{P}(X\in B)=\int_B f_X(x)dx$

$F_{X,Y}(x,y)=\mathbb{P}(X\leq x, Y\leq y)=\int_{-\infty}^{x}\int_{-\infty}^{y} f(u,v)dvdu$

$f_{X|X\in A}(x)=\frac{f(x)}{\mathbb{P}(X\in A)} \quad x\in A$

$f_X(x)=\sum_{i=1}^{n} f_{X|A_i}(x)\,\mathbb{P}(A_i)\to A_1,\dots,A_n$ partiție

$f_{X|Y}(x|y)=\frac{f(x,y)}{f_Y(y)}$ | $X\perp Y\Rightarrow F(x,y)=F(x)F(y)$, $f(x,y)=f(x)\cdot f(y)$

$E[g(X,Y)]=\iint_{\mathbb{R}^2} g(x,y)f_{X,Y}(x,y)dxdy$

$E(X|A)=\int x f_{X|A}(x)dx$

$\Rightarrow E(X|Y=y)=\int x f_{X|Y}(x|y)dx$

$E[X]=\int E(X|Y=y)f_Y(y)dy$

$Var(X)=Var(E(X|Y))+E(Var(X|Y))$

Hibrid X\Y	discret	cont
discret	$f(x)=\sum_y f_{X|Y}(x|y)\cdot f_Y(y)$	$\mathbb{P}(X=x)=\int \mathbb{P}(X=x|Y=y)f_Y(y)dy$
cont	$f(x)=\sum_y f_{X|Y}(x|y)f_Y(y)$	$f(x)=\int f_{X|Y}(x|y)\cdot f_Y(y)dy$

$\rho=\frac{Cov(X,Y)}{\sqrt{Var X}\sqrt{Var Y}}\in[-1,1]$

Ⓟ CBS X,Y, $Vr(X)<\infty$, $Vr(Y)<\infty$

$|E[XY]|\leq\sqrt{E[X^2]}\sqrt{E[Y^2]}$

$|Cov(X,Y)|\leq\sqrt{Var X}\sqrt{Var Y}$

Inegalități:

Ⓟ Jensen

f convexă dacă $\forall x,y$, $\forall\lambda\in(0,1)$

$f(\lambda x+(1-\lambda)y)\leq\lambda f(x)+(1-\lambda)f(y)$

Ⓟ. X va și g o fct. Atunci

→ g convexă:

$E[g(X)]\geq g(E[X])$

→ g concavă

$E[g(X)]\leq g(E[X])$

Ⓟ Markov

X va ≥ 0, $a>0\Rightarrow\mathbb{P}(X\geq a)\leq\frac{E[X]}{a}$

Ⓟ Chernoff

X va >0, $a>0$, $t>0$

$\mathbb{P}(X\geq a)\leq\frac{E[e^{tX}]}{e^{ta}}$

Ⓟ Ineg Cebîșev:

X va cu $E[X]=\mu$ și $Var(X)=\sigma^2$

$\mathbb{P}(|X-\mu|\geq a)\leq\frac{Vr(X)}{a^2} \quad \forall a\geq 0$