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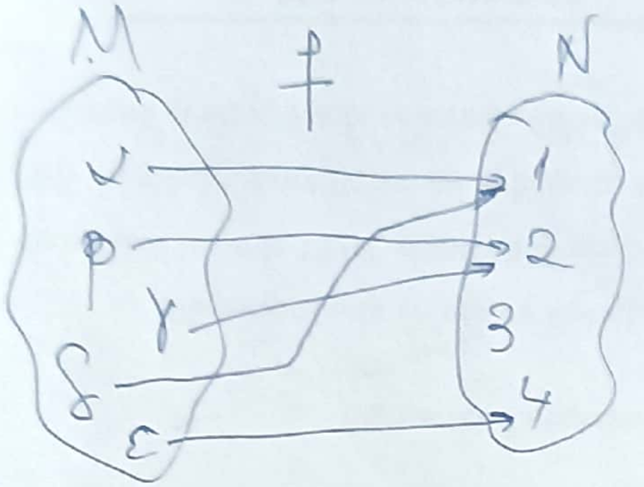
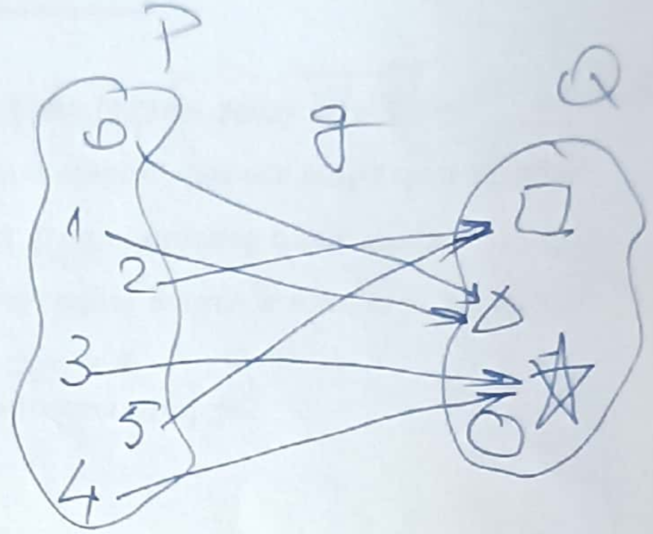
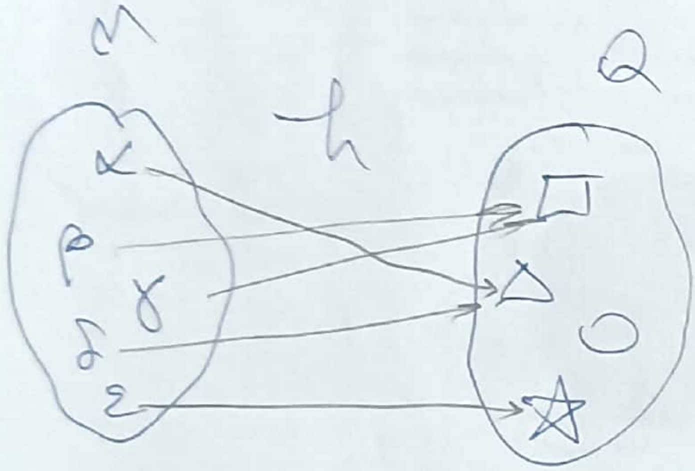
10.10.23

SEMINAR 2 _ 132







$h : M \longrightarrow Q$,

$h = g \circ f$

$h(\alpha) = g(f(\alpha))$

$h(\beta) = g(f(\beta))$

$h(\gamma) = \square = g(f(\gamma))$

$h(\delta) = \triangle = g(f(\delta))$

$h(\varepsilon) = ☆ = g(f(\varepsilon))$

$h(\xi) = g(f(\xi)) \quad \forall\, \xi \in M$

Cele de mai sus se concretizează așa:

dată fiind $f: A \to B$, $g: C \to D$, cu $B \subset C$, definim funcția

$$g \circ f : A \longrightarrow D, \quad \boxed{(g \circ f)(a) = g(f(a))} \quad \forall a \in A.$$

Def. $g \circ f$ s.n. COMPUSA funcțiilor g și f

$$\begin{tikzcd} A \arrow[r, "f"] \arrow[ddr, dashed, "g \circ f"'] & B \subset C \arrow[dd, "g"] \\ & \\ & D \end{tikzcd}$$

condiția să se poată face $g \circ f$.

Exemplu: $f: \mathbb{N} \to \mathbb{Z}$, $f(n) = 5 - 3n$

$g: \mathbb{Q} \to \mathbb{Q}$, $g(h) = \frac{h}{h^2+1}$.

$f \circ g$? $\mathbb{Q} \xrightarrow{g} \mathbb{Q} \not\subset \mathbb{N} \xrightarrow{f} \mathbb{Z}$

Cf. diagramei, $f \circ g$ nu are sens.

$$\begin{tikzcd} \mathbb{N} \arrow[r, "f"] & \mathbb{Z} \subset \mathbb{Q} \arrow[d, "g"] \\ & \mathbb{Q} \end{tikzcd}$$

...f. diagramei, $g \circ f : \mathbb{N} \to \mathbb{Q}$.

...n plus, $(g\circ f)(n) = g(f(n)) =$

$$= \frac{f(n)}{f(n)^2+1} = \frac{5-3n}{(5-3n)^2+1} = \frac{5-3n}{9n^2-30n+26}$$

$f : \mathbb{R} \to \mathbb{R}$, $f(x) = 7-3x$

$g : \mathbb{R} \to \mathbb{R}$, $g(x) = \begin{cases} x^2+1, & x > 3 \\ 2-x, & x \le 3 \end{cases}$

$h : \mathbb{R} \to \mathbb{R}$, $h(x) = \begin{cases} 5x-2, & x > -1 \\ x+1, & x \le -1 \end{cases}$.

$g\circ f : \mathbb{R} \to \mathbb{R}$

$$(g\circ f)(x) = g(f(x)) = \begin{cases} f(x)^2+1, & f(x) > 3 \\ 2-f(x), & f(x) \le 3 \end{cases} =$$

$$= \begin{cases} 9x^2-42x+50, & 7-3x > 3 \\ 3x-5, & 7-3x \le 3 \end{cases} =$$

$$= \begin{cases} 9x^2-42x+50, & x < \frac{4}{3} \\ 3x-5, & x \ge \frac{4}{3} \end{cases}.$$

$f \circ g : \mathbb{R} \to \mathbb{R}$,

$(f \circ g)(x) = f(g(x)) = 7 - 3g(x) = \cancel{7 - 3x^2 - 5},$

$$7 - 3 \cdot \begin{cases} x^2+1, & x > 3 \\ 2 - x, & x \leq 3 \end{cases} =$$

$$= \begin{cases} 4 - 3x^2, & x > 3 \\ 3x + 1, & x \leq 3 \end{cases}$$

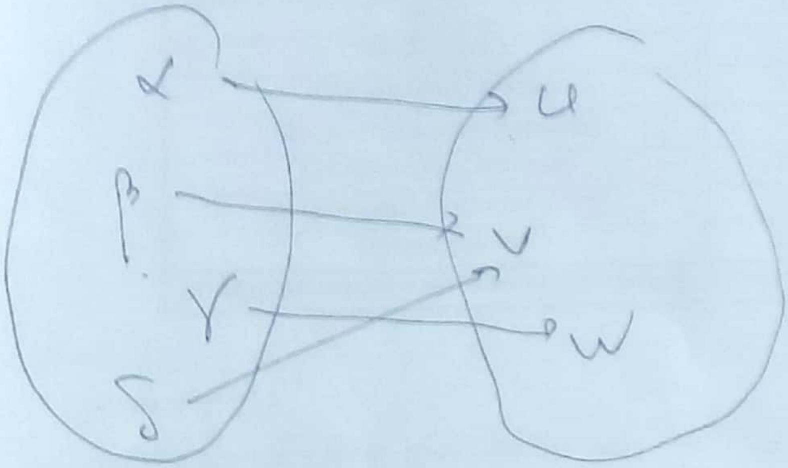
$h \circ g : \mathbb{R} \to \mathbb{R}$, $(h \circ g)(x) = h(g(x)) =$

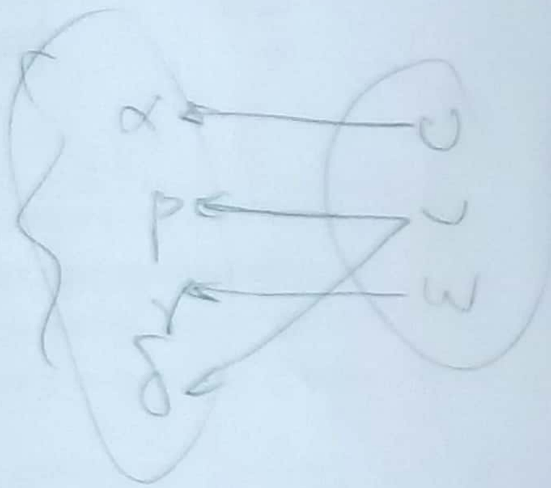
$$= \begin{cases} 5g(x) - 2, & g(x) > -1 \\ g(x) + 1, & g(x) \leq -1 \end{cases}$$

$$= \begin{cases} 5(x^2+1) - 2, & \left.\begin{cases} x^2+1 > -1 \Leftrightarrow x^2 > -2 \Leftrightarrow x \in \mathbb{R} \\ x > 3 \end{cases}\right\} \Leftrightarrow x > 3 \\ 5(2-x) - 2, & \left.\begin{cases} 2 - x > -1 \Leftrightarrow x < 3 \\ x \leq 3 \end{cases}\right\} \Leftrightarrow x < 3 \\ (x^2+1) + 1, & \left.\begin{cases} x^2+1 \leq -1 \Leftrightarrow x^2 \leq -2 \Leftrightarrow x \in \emptyset \\ x > 3 \end{cases}\right\} \Leftrightarrow x \in \emptyset \\ (2-x) + 1, & \left.\begin{cases} 2 - x \leq -1 \Leftrightarrow x \geq 3 \\ x \leq 3 \end{cases}\right\} \Leftrightarrow x = 3 \end{cases} =$$

$$= \begin{cases} 5x^2+3, & x>3 \\ 8-5x, & x<3 \\ 0, & x=3 \end{cases}$$

$\boxed{\text{TD: } 9.4}$





INTUITIV

„o fct s.n. INJECTIVĂ dacă duce (orice două) elemente diferite în elemente diferite".

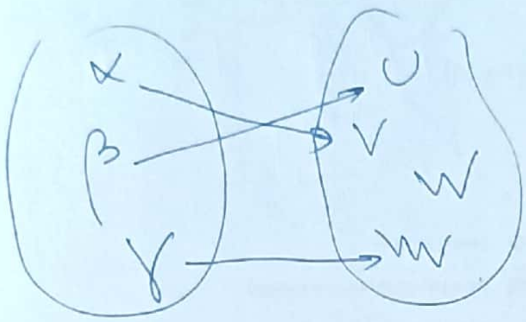
RIGUROS!

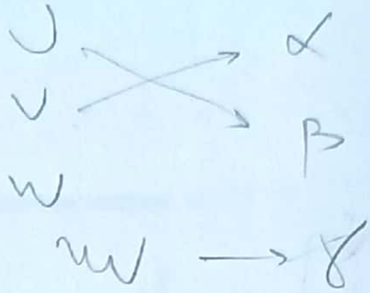
Funcția $f: A \to B$ s.n. INJECTIVĂ dacă

$$\forall a_{1,2} \in A \quad a_1 \neq a_2 \Rightarrow f(a_1) \neq f(a_2)$$

Prop Funcția $f: A \to B$ e injectivă ddacă

$\forall a_{1,2} \in A \quad f(a_1) = f(a_2) \Rightarrow a_1 = a_2$





Nu e funcție.

INTUITIV

O funcție s.n. SURJECTIVĂ dacă „își umple codomeniul"

RIGUROS:

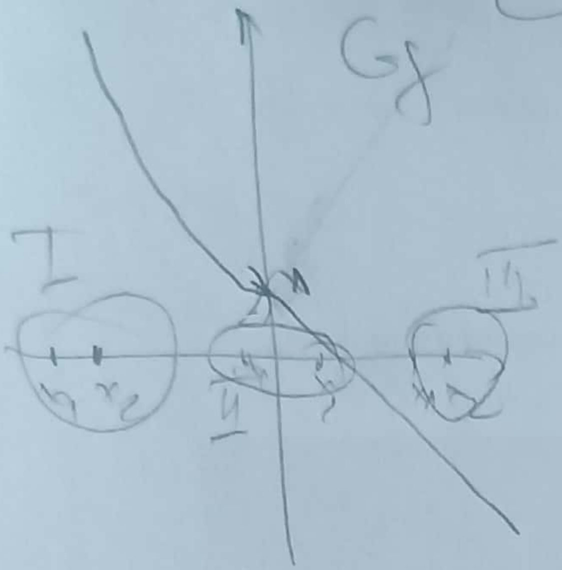
Funcția $f: A \to B$ s.n. SURJECTIVĂ dacă

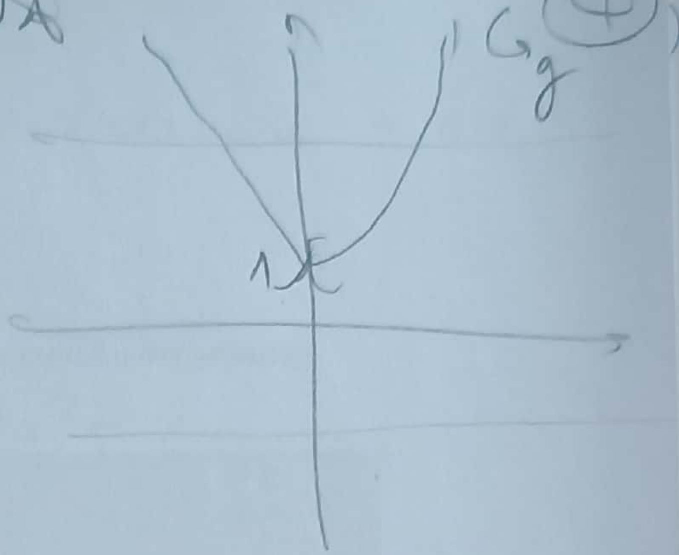
$$\forall b \in B \quad \exists a \in A \quad f(a) = b$$

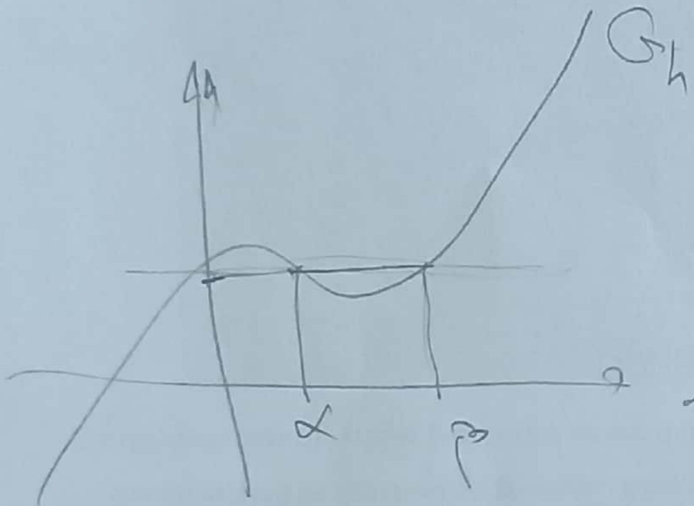
$f, g: \mathbb{R} \to \mathbb{R}, \quad f(x) = \begin{cases} x^2+1, & x < 0 \\ 1-x, & x \geq 0 \end{cases},$

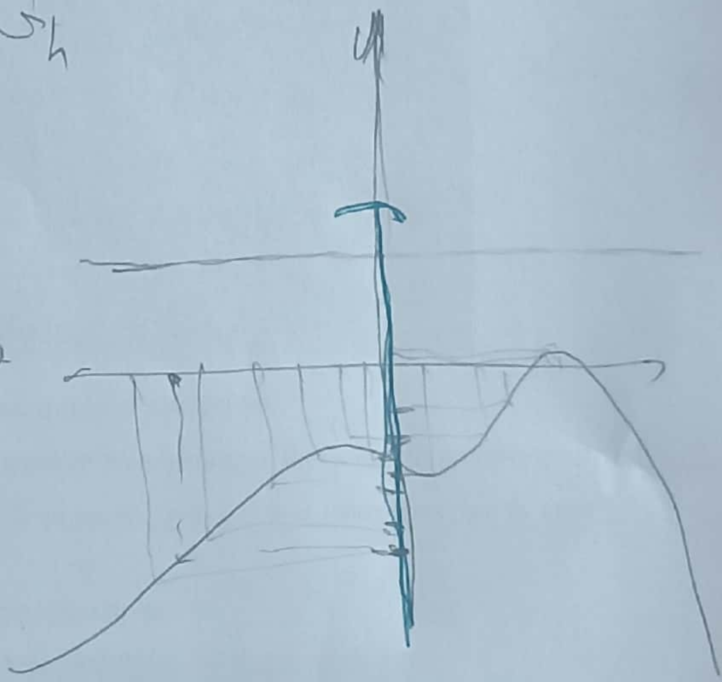
$$g(x) = \begin{cases} x^2+1, & x \geq 0 \\ 1-x, & x < 0 \end{cases}$$

CIORNĂ









$x^2+1=y$

$x^2=\sqrt{y-1}$

inj

$\forall a_{1,2} \quad a_1 \neq a_2 \rightarrow f(a_1) \neq f(a_2)$

non-inj: $\exists a_{1,2} \quad a_1 \neq a_2 \wedge f(a_1) = f(a_2)$

Fie $x_{1,2} \in \mathbb{R}$, $x_1 \neq x_2$

pt a fixa ideile, considerăm $x_1 < x_2$.

I $x_1 < x_2 < 0$.

Atunci $-x_1 > -x_2 > 0 \Leftrightarrow$

$x_1^2 > x_2^2 > 0 \Rightarrow x_1^2+1 > x_2^2+1 \Rightarrow$

$f(x_1) > f(x_2) \Rightarrow f(x_1) \neq f(x_2)$.

II $x_1 < 0 \leq x_2$.

Atunci $\left.\begin{array}{l} f(x_1) = x_1^2+1 > 1 \\ f(x_2) = 1 - x_2 \leq 1 \end{array}\right\} \Rightarrow f(x_1) > f(x_2) \Rightarrow$

$f(x_1) \neq f(x_2)$

III $0 \leq x_1 < x_2$.

Atunci $-x_1 > -x_2$,

deci $f(x_1) = 1 - x_1 > 1 - x_2 = f(x_2) \Rightarrow$

$f(x_1) \neq f(x_2)$

Deci f e injectivă $\square$

Fie $y \in \mathbb{R}$. Vreau: $\exists x \in \mathbb{R}$ $f(x) = y$

Dacă $y > 1$, luăm $x = -\sqrt{y-1} < 0$.

Atunci $f(x) = x^2+1 = (-\sqrt{y-1})^2+1 = y-1+1 = y$ ✓

Dacă $y \leq 1$, luăm $x = 1 - y \geq 0$.

Atunci $f(x) = 1 - x = 1 - (1-y) = y$

Deci f e surjectivă □.

$g(1) = 2 = g(-1)$

Evident, $1 \neq -1$.

Deci, g nu e injectivă □

CAPCANĂ !

g surj: $\forall y \in \mathbb{R}$ $\exists x \in \mathbb{R}$ $g(x) = y$

g nesurj: $\exists y \in \mathbb{R}$ $\forall x \in \mathbb{R}$ $g(x) \neq y$

Luăm $y = 0$.

Fie $x \in \mathbb{R}$

Dacă $x < 0$, $g(x) = 1 - x > 1 > 0 = y \Rightarrow$
$g(x) \neq y$.

Dacă $x \geq 0$,

$g(x) = x^2 + 1 \geq 1 > 0 = y \Rightarrow g(x) \neq y$.

Ca urmare, g nu e surjectivă □