22.5.2024.

Temă

Tema 1

1) $B = \{f_1 = (1,1,1),\ f_2 = (1,1,-1),\ f_3 = (1,-1,-1)\}$

$$\begin{cases} e_1 = \dfrac{f_1}{\|f_1\|} \\ e_i = \dfrac{e_i'}{\|e_i'\|} \end{cases} \qquad e_i' = f_i - \sum_{j=1}^{i-1} \langle f_i, e_j \rangle \cdot e_j \quad \forall i = \overline{2,n}$$

$e_1 = \frac{1}{\sqrt{3}}(1,1,1)$

$e_2 = \frac{e_2'}{\|e_2'\|}$ $\quad e_2' = f_2 - \langle f_2, e_1 \rangle e_1 =$

$= (1,1,-1) - \frac{1}{\sqrt{3}} \cdot 1 \cdot \frac{1}{\sqrt{3}}(1,1,1) =$

$= (1,1,-1) - \frac{1}{3}(1,1,1) =$

$= \frac{2}{3}(1,1,-2)$

$e_2 = \frac{2}{3}(1,1,-2) \cdot \frac{3}{2} \cdot \frac{1}{\sqrt{6}} = \frac{1}{\sqrt{6}}(1,1,-2)$

$e_3 = \frac{e_3'}{\|e_3'\|}$ $\quad e_3' = f_3 - \langle f_3, e_1 \rangle e_1 - \langle f_3, e_2 \rangle e_2 =$

$= (1,-1,-1) - \frac{-1}{\sqrt{3}} \cdot \frac{1}{\sqrt{3}}(1,1,1) - \frac{1}{\sqrt{6}} \cdot 2 \frac{1}{\sqrt{6}}(1,1,-2)$

$= (1,-1,-1) + \frac{1}{3}(1,1,1) - \frac{1}{3}(1,1,-2) =$

$= (1,-1,0)$

$e_3 = (1,-1,0) \cdot \frac{1}{\sqrt{2}}$

2) $B = \{ f_1 = (0,1,1),\ f_2 = (1,0,1),\ f_3 = (1,1,0) \}$

$e_1 = \frac{1}{\sqrt{2}}(0,1,1)$

$e_2 = \frac{e_2'}{\|e_2'\|}$ $\quad e_2' = f_2 - \langle f_2, e_1 \rangle e_1 =$

$= (1,0,1) - \frac{1}{\sqrt{2}} \cdot 1 \cdot \frac{1}{\sqrt{2}}(0,1,1) =$

$= (1,0,1) - \frac{1}{2}(0,1,1) =$

$= \left(1, -\frac{1}{2}, \frac{1}{2}\right) = \frac{1}{2}(2,-1,1)$

$e_2 = \frac{1}{2}(2,-1,1) \cdot 2 \cdot \frac{1}{\sqrt{6}} = \frac{1}{\sqrt{6}}(2,-1,1)$

$e_3 = \frac{e_3'}{\|e_3'\|}$ $\quad e_3' = f_3 - \langle f_3, e_1 \rangle e_1 - \langle f_3, e_2 \rangle e_2 =$

$= (1,1,0) - \frac{1}{\sqrt{2}} \cdot 1 \cdot \frac{1}{\sqrt{2}}(0,1,1) - \frac{1}{\sqrt{6}} \cdot 1 \cdot \frac{1}{\sqrt{6}}(2,-1,1) =$

$= (1,1,0) - \frac{1}{2}(0,1,1) - \frac{1}{6}(2,-1,1) =$

$= \frac{6}{6}(1,1,0) - \frac{3}{6}(0,1,1) - \frac{1}{6}(2,-1,1) =$

$= \left(\frac{4}{6}, \frac{4}{6}, \frac{-4}{6}\right) = \frac{2}{3}(1,1,-1)$

$e_3 = \frac{2}{3}(1,1,-1) \cdot \frac{3}{2} \cdot \frac{1}{\sqrt{3}} = \frac{1}{\sqrt{3}}(1,1,-1)$

3) $B' = \left\{ e_1 = \frac{1}{\sqrt{2}}(0,1,1),\ e_2 = \frac{1}{\sqrt{6}}(2,-1,1),\ e_3 = \frac{1}{\sqrt{3}}(1,1,-1) \right\}$

bază ortonormată

det coord vectorului:

$w = (-1, 1, 2)$

$[w]_{B'} = (\alpha, \beta, \gamma)$

$\exists!\ \alpha, \beta, \gamma \in \mathbb{R}$ a.î. $w = \alpha e_1 + \beta e_2 + \gamma e_3 \ \Big|\ \langle , e_1\rangle,\ \langle , e_2\rangle,\ \langle , e_3\rangle$

$\langle w, e_1\rangle = \langle \alpha e_1 + \beta e_2 + \gamma e_3, e_1\rangle =$

$= \alpha \underbrace{\langle e_1, e_1\rangle}_{=1} + \beta \underbrace{\langle e_2, e_1\rangle}_{=0} + \gamma \underbrace{\langle e_3, e_1\rangle}_{=0} = \alpha$

B' bază ortonormată

$\Rightarrow \alpha = \langle w, e_1\rangle = \frac{1}{\sqrt{2}} \cdot 3$

analog $\beta = \langle w, e_2\rangle = \frac{1}{\sqrt{6}} \cdot (-1)$

$\gamma = \langle w, e_3\rangle = \frac{1}{\sqrt{3}} \cdot (-2)$

$[w]_{B'} = \left(\frac{3}{\sqrt{2}}, \frac{-1}{\sqrt{6}}, \frac{-2}{\sqrt{3}}\right)$