Tema 4) 2)

$$\begin{cases} \alpha_1 + \beta_1 - \gamma_1 = 0 \\ \alpha_1 - \beta_1 + \gamma_1 = 1 \\ -\alpha_1 + \beta_1 + \gamma_1 = 2 \end{cases} \Rightarrow \begin{pmatrix} 1 & 1 & -1 & 0 \\ 1 & -1 & 1 & 1 \\ -1 & 1 & 1 & 2 \end{pmatrix} \xrightarrow[L_3 \leftarrow L_3 + L_1]{L_2 \leftarrow L_2 - L_1}$$

$$\rightarrow \begin{pmatrix} 1 & 1 & -1 & 0 \\ 0 & -2 & 2 & 1 \\ 0 & 2 & 0 & 2 \end{pmatrix} \xrightarrow{L_3 \rightarrow L_3 + L_2} \begin{pmatrix} 1 & 1 & -1 & 0 \\ 0 & -2 & 2 & 1 \\ 0 & 0 & 2 & 3 \end{pmatrix} \rightarrow$$

$$\xrightarrow[L_3 \leftarrow L_3 \cdot \frac{1}{2}]{L_2 \leftarrow L_2 \cdot -\frac{1}{2}} \begin{pmatrix} 1 & 1 & -1 & 0 \\ 0 & 1 & -1 & -\frac{1}{2} \\ 0 & 0 & 1 & \frac{3}{2} \end{pmatrix} \Rightarrow \begin{cases} \gamma_1 = \frac{3}{2} \\ \beta_1 = -\frac{1}{2} + \frac{3}{2} = 1 \\ \alpha_1 = \frac{3}{2} - 1 = \frac{1}{2} \end{cases}$$

$$(\alpha_1, \beta_1, \gamma_1) = \left(\frac{1}{2}, 1, \frac{3}{2}\right)$$

pt $v = e_2$

$$(1, 0, 2) = \alpha_2 (1, 1, -1) + \beta_2 (1, -1, 1) + \gamma_2 (-1, 1, 1)$$

$$\begin{cases} \alpha_2 + \beta_2 - \gamma_2 = 1 \\ \alpha_2 - \beta_2 + \gamma_2 = 0 \\ -\alpha_2 + \beta_2 + \gamma_2 = 2 \end{cases} \Rightarrow \text{analog } e_1 \Rightarrow$$

$$\rightarrow \begin{pmatrix} 1 & 1 & -1 & 1 \\ 0 & 1 & -1 & \frac{1}{2} \\ 0 & 0 & 1 & 1 \end{pmatrix} \Rightarrow \begin{cases} \gamma_2 = 1 \\ \beta_2 = \frac{1}{2} + 1 = \frac{3}{2} \\ \alpha_2 = 1 + 1 - \frac{3}{2} = \frac{1}{2} \end{cases}$$

$\Rightarrow (\alpha_2, \beta_2, \gamma_2) = \left(\frac{1}{2}, \frac{3}{2}, 1\right)$

analog $f_2 \Rightarrow \begin{pmatrix} 1 & 1 & -1 & 2 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 1 & 2 \end{pmatrix} \Rightarrow \begin{cases} \gamma_3 = 2 \\ \beta_3 = 2 \\ \alpha_3 = 2+2-2=2 \end{cases}$

$\Rightarrow (\alpha_3, \beta_3, \gamma_3) = (2, 2, 2)$

3) $f : \mathbb{R}^2 \to \mathbb{R}^3$

$f(x, y) = (x+y, x, -y)$

$\operatorname{Ker} f = \{0_{\mathbb{R}^2}\} \Rightarrow \dim_{\mathbb{R}} \operatorname{Ker} f = 0$

$f(x,y) = (0,0,0) \Rightarrow A = \begin{pmatrix} 1 & 0 \\ 1 & 1 \\ 0 & -1 \end{pmatrix} \quad rg A = 2$

$\Rightarrow \dim_{\mathbb{R}} \mathbb{R}^2 = 2$

~~dim~~ $\dim_{\mathbb{R}} \operatorname{Im} f = 2$

$\operatorname{Im} f = \{(x', y', z') \in \mathbb{R}^3 \mid x' - y' + z' = 0\} \subset \mathbb{R}^3$

~~c~~ c) f nu e surj $\Rightarrow \operatorname{Im} f \neq \mathbb{R}^3$

$\Rightarrow \dim_{\mathbb{R}} \operatorname{Im} f = 2$

$\Rightarrow$ teorema rang-defect e validă

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