Zadatak 3

Zadaća 4, Matej Jurec

$$\left.\begin{aligned} r_i &= (1-u)p_i + u p_{i+1}, \quad i = 0,1,2 \\ s_i &= (1-u)r_i + u r_{i+1}, \quad i = 0,1 \\ t_0 &= (1-u)s_0 + u s_1 \\ f(u) &= t_0 \end{aligned}\right\} (*)$$

$$b_{i,n}(u) = \binom{n}{i}(1-u)^{n-i} u^i$$

$n = 3$:

$$b_{i,3}(u) = \binom{3}{i}(1-u)^{3-i} u^i$$

Bernstein polinomi stupnja $n=3$:

$$\left.\begin{aligned} b_0(u) &= (1-u)^3 \\ b_1(u) &= 3u(1-u)^2 \\ b_2(u) &= 3u^2(1-u) \\ b_3(u) &= u^3 \end{aligned}\right\} \text{zbrojimo: } T(u) = (1-u)^3 p_0 + 3u(1-u)^2 p_1 + 3u^2(1-u)p_2 + u^3 p_3$$

Za $u \in [0,1]$ i $(*) \Rightarrow f(u) = t_0$

$$\begin{aligned} t_0 &= (1-u)s_0 + u s_1 = (1-u)\left((1-u)r_0 + u r_1\right) + u\left((1-u)r_1 + u r_2\right) = \\ &= (1-u)^2 r_0 + 2u(1-u) r_1 + u^2 r_2 = \\ &= (1-u)^2\left((1-u)p_0 + u p_1\right) + 2u(1-u)\left((1-u)p_1 + u p_2\right) + u^2\left((1-u)p_2 + u p_3\right) = \\ &= (1-u)^3 p_0 + 3u(1-u)^2 p_1 + 3u^2(1-u) p_2 + u^3 p_3 = \\ &= T(u) \quad \blacksquare \end{aligned}$$