Machine Learning for Stock Index Predictions

This code is an implementation of a Bachelor's thesis titled Machine Learning for Stock Index Predictions.

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Importing libraries and data

```
# Importing essential data manipulation and visualization libraries
import pandas as pd
import numpy as np
import seaborn as sns
import matplotlib.pyplot as plt
from matplotlib.colors import ListedColormap
import time

# Libraries for acquiring financial data
import yfinance as yf
from yahoofinancials import YahooFinancials
import pandas_ta as ta

# Machine learning libraries
from sklearn.neighbors import KNeighborsClassifier
from sklearn import neighbors
from sklearn.ensemble import RandomForestClassifier
from sklearn.linear_model import LogisticRegression
from sklearn.svm import SVC
from sklearn.model_selection import train_test_split, cross_val_score
from sklearn.metrics import confusion_matrix, roc_curve,
accuracy_score
from tensorflow.keras.models import Sequential
from tensorflow.keras.layers import LSTM, Dense, Dropout
import statsmodels.api as sm
import tensorflow as tf
from sklearn.preprocessing import MinMaxScaler
from numpy.random import seed

# Importing data of S&P 500 index from 15.1.2001 until 24.1.2024
sp500 = yf.download('^GSPC', period='max')
sp500_data = yf.download('^GSPC', period='max')
sp500 = sp500.loc['2001-01-15':'2024-01-24']
sp500.head(5)
```

```
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[*********************100%***********************]  1 of 1 completed

                                  Open         High          Low
Close  \
Date

2001-01-16 00:00:00-05:00  1318.319946  1327.810059  1313.329956
1326.650024
2001-01-17 00:00:00-05:00  1326.650024  1346.920044  1325.410034
1329.469971
2001-01-18 00:00:00-05:00  1329.890015  1352.709961  1327.410034
1347.969971
2001-01-19 00:00:00-05:00  1347.969971  1354.550049  1336.739990
1342.540039
2001-01-22 00:00:00-05:00  1342.540039  1353.619995  1333.839966
1342.900024

                             Adj Close      Volume
Date
2001-01-16 00:00:00-05:00  1326.650024  1205700000
2001-01-17 00:00:00-05:00  1329.469971  1349100000
2001-01-18 00:00:00-05:00  1347.969971  1445000000
2001-01-19 00:00:00-05:00  1342.540039  1407800000
2001-01-22 00:00:00-05:00  1342.900024  1164000000

# Changing the index format to 'YYYY-MM-DD'
sp500.index = sp500.index.strftime('%Y-%m-%d')

# Converting index to datetime format
sp500.index = pd.to_datetime(sp500.index)

# Creating a new column: close next day
sp500['Close day before'] = sp500['Close'].shift(1)

# Calculating percantage change
sp500['pct_change'] = (sp500['Close'] - sp500['Close day before'])  /
sp500['Close day before'] * 100

# Defining a function to convert an integer to a boolean value
def bool_to_int(val):
    if val:
        return 1
    else:
        return 0

# Creating a Direction column where a value of 1 indicates an upward
movement in the index on that day,
# while a value of 0 indicates a downward movement
sp500['Direction'] = sp500['pct_change'].apply(lambda x:
```

```
bool_to_int(x>0))

sp500[-5:]
```

```
                   Open         High          Low        Close     Adj
Close  \
Date

2024-01-18  4760.100098  4785.790039  4740.569824  4780.939941
4780.939941
2024-01-19  4796.279785  4842.069824  4785.870117  4839.810059
4839.810059
2024-01-22  4853.419922  4868.410156  4844.049805  4850.430176
4850.430176
2024-01-23  4856.799805  4866.479980  4844.370117  4864.600098
4864.600098
2024-01-24  4888.560059  4903.680176  4865.939941  4868.549805
4868.549805

                Volume  Close day before  pct_change  Direction
Date
2024-01-18  4019000000       4739.209961    0.880526          1
2024-01-19  4287200000       4780.939941    1.231350          1
2024-01-22  4297610000       4839.810059    0.219433          1
2024-01-23  3912800000       4850.430176    0.292137          1
2024-01-24  4330030000       4864.600098    0.081193          1
```

Defining functions

When repeated code patterns occur frequently, it's typically more efficient to write that code into a function. By doing so, we eliminate the need to rewrite the code each time and can instead simply call the function when needed.

Functions for Visualizing Results

```
def c_matrix(y_test, y_pred_class):
    """
    Prints a confusion matrix.

    Parameters:
    y_pred_class (pandas Series) : Series that contains the predicted
label of each day.
    y_test (pandas Series) : Series that contains the actual label of
each day.

    Returns:
    Confusion matrix.
    """
    conf_matrix = confusion_matrix(y_test, y_pred_class)
    sns.heatmap(conf_matrix, annot=True, fmt='d', cmap='Blues')
```

```
    plt.xlabel('Predicted labels')
    plt.ylabel('True labels')
    plt.show()

def roc(y_test, y_pred_class):
    """
    Prints a ROC curve.

    Parameters:
    y_pred_class (pandas Series) : Series that contains the predicted
label of each day.
    y_test (pandas Series) : Series that contains the actual label of
each day.

    Returns:
    The ROC curve.
    """
    fpr, tpr, thresholds = roc_curve(y_test, y_pred_class)
    plt.plot(fpr, tpr, label='ROC curve')
    plt.plot([0, 1], [0, 1], 'k--', label='Random guess')
    plt.xlabel('False positive rate')
    plt.ylabel('True positive rate')
    plt.title('Receiver operating characteristic (ROC) curve')
    plt.legend()
    plt.show()

def investment_return(y_pred_class, sp500_data):
    """
    Calculates the NORD value.

    Parameters:
    y_pred_class (pandas Series) : Series that contains the predicted
label of each day.
    sp500_data (pandas DataFrame): Stock market data for the given
time period.

    Returns:
    investment_returns (pandas DataFrame): DataFrame containing the
cummulative NORD value over time
    and S&P 500 returns.
    """

    # Calculating the daily returns of the S&P 500
    sp500_returns = sp500_data['pct_change']

    # Creating a new DataFrame to track the investment performance.
    investment_returns = pd.DataFrame({'SP500': sp500_returns,
'Investment': y_pred_class * sp500_returns})

    # Creating new columns
```

```
    investment_returns['dollars_SP500'] = 0
    investment_returns['dollars'] = 0

    # Tracking the daily change in value for one dollar - cummulative
NORD
    investment_returns.iloc[0,
investment_returns.columns.get_loc('dollars_SP500')] = 1 +
investment_returns.iloc[0]["SP500"] / 100
    for i in range(1, len(investment_returns)):
        investment_returns.iloc[i,
investment_returns.columns.get_loc('dollars_SP500')] = (1 +
investment_returns.iloc[i]["SP500"] / 100) * investment_returns.iloc[i
- 1]["dollars_SP500"]

    # Same for the SP500
    investment_returns.iloc[0,
investment_returns.columns.get_loc('dollars')] = 1 +
investment_returns.iloc[0]["Investment"] / 100
    for i in range(1, len(investment_returns)):
        investment_returns.iloc[i,
investment_returns.columns.get_loc('dollars')] = (1 +
investment_returns.iloc[i]["Investment"] / 100) *
investment_returns.iloc[i - 1]["dollars"]
    return investment_returns

def plot_investment_performance(y_pred_class, sp500_data):
    """
    This function generates a graphical comparison of our model's
performance with the S&P 500.

    Parameters:
    y_pred_class (pandas Series) : Series that contains the predicted
label of each day.
    sp500_data (pandas.DataFrame): A DataFrame containing the daily
prices of the S&P 500.

    Returns:
    Graph.
    """

    investment_returns = investment_return(y_pred_class, sp500_data)

    # Plotting the cumulative returns of the S&P 500 and the NORD
    plt.plot(investment_returns["dollars_SP500"].values, label='S&P
500')
    plt.plot(investment_returns["dollars"].values, label='Our Model')
    plt.legend()
    plt.title('Performance of our model compared to S&P 500')
    plt.xlabel('Number of Days')
```

```
    plt.ylabel('Cumulative Return')
    plt.show()

def plot_knn(X, y, n_neighbors=12):
    """
    Visualization of K-Nearest Neighbors algorithm.

    Parameters:
    X (pandas Dataframe): DataFrame containing the values of features.
    y (array-like): Array containing the labels for each day.
    n_neighbors (int): Number of neighbors used in K-Nearest Neighbors
algorithm.

    Returns:
    A plot with Feature 1 on the x-axis and Feature 2 on the y-axis,
where each dot corresponds to a
    specific day.The colors in the figure have the following meanings:
    • Blue dots: Data points belonging to class zero.
    • Red dots: Data points belonging to class one.
    • Light Red: Represents an area in which, if a new data point is
assigned, its predicted
    class will be one.
    • Light Blue: Represent a prediction of a fall in the index S&P
500 in this area, classi-
    fying a new point as 0.

    The function also returns the execution time.
    """
    start_time = time.time()

    # Defining the step size for meshgrid
    h = .02

    # Creating a color map for red and blue colors
    cmap_light = ListedColormap(['#FFAAAA', '#AAAAFF'])
    cmap_bold = ListedColormap(['#FF0000', '#0000FF'])

    # Creating a KNN classifier
    clf = neighbors.KNeighborsClassifier(n_neighbors)

    # Fitting the classifier on the data
    clf.fit(X, y)

    # Plotting the decision boundaries
    x_min, x_max = X.iloc[:, 0].min() - 1, X.iloc[:, 0].max() + 1
    y_min, y_max = X.iloc[:, 1].min() - 1, X.iloc[:, 1].max() + 1
    xx, yy = np.meshgrid(np.arange(x_min, x_max, h), np.arange(y_min,
y_max, h))
    Z = clf.predict(np.c_[xx.ravel(), yy.ravel()])
```

```
    # Plotting the mesh grid
    Z = Z.reshape(xx.shape)
    plt.figure(figsize=(15, 15))
    plt.pcolormesh(xx, yy, Z, cmap=cmap_light)

    # Plotting the training points
    plt.scatter(X.iloc[:, 0], X.iloc[:, 1], c=y, cmap=cmap_bold,
edgecolor='k', s=20)
    plt.xlim(xx.min(), xx.max())
    plt.ylim(yy.min(), yy.max())

    # Setting labels and title
    plt.xlabel('Day 1 Before')
    plt.ylabel('Day 2 Before')
    plt.title(f"kNN (k = {n_neighbors})")

    # Adding legend
    plt.legend(['Positive', 'Negative'])

    plt.show()

    end_time = time.time()
    elapsed_time = end_time - start_time
    print(f"The code took {elapsed_time:.5f} seconds to run.")

def plot_random_forest(X, y):
    """
    Visualization of Random Forest algorithm.

    Parameters:
    X (pandas Dataframe): DataFrame containing the values of features.
    y (array-like): Array containing the labels for each day.

    Returns:
    A plot with Feature 1 on the x-axis and Feature 2 on the y-axis,
where each dot corresponds to a
    specific day.The colors in the figure have the following meanings:
    • Blue dots: Data points belonging to class zero.
    • Red dots: Data points belonging to class one.
    • Light Red: Represents an area in which, if a new data point is
assigned, its predicted
    class will be one.
    • Light Blue: Represent a prediction of a fall in the index S&P
500 in this area, classi-
    fying a new point as 0.

    The function also returns the execution time.
    """
    start_time = time.time()
```

```
    # Defining the step size for meshgrid
    h = .02

    # Creating a color map for red and blue colors
    cmap_light = ListedColormap(['#FFAAAA', '#AAAAFF'])
    cmap_bold = ListedColormap(['#FF0000', '#0000FF'])

    # Creating a RF classifier
    clf = RandomForestClassifier(max_depth=6, random_state=42)

    # Fitting the classifier on the data
    clf.fit(X, y)

    # Plotting the decision boundaries
    x_min, x_max = X.iloc[:, 0].min() - 1, X.iloc[:, 0].max() + 1
    y_min, y_max = X.iloc[:, 1].min() - 1, X.iloc[:, 1].max() + 1
    xx, yy = np.meshgrid(np.arange(x_min, x_max, h), np.arange(y_min,
y_max, h))
    Z = clf.predict(np.c_[xx.ravel(), yy.ravel()])

    # Plotting the mesh
    Z = Z.reshape(xx.shape)
    plt.figure(figsize=(15, 15))
    plt.pcolormesh(xx, yy, Z, cmap=cmap_light)

    # Plotting the training points
    plt.scatter(X.iloc[:, 0], X.iloc[:, 1], c=y, cmap=cmap_bold,
edgecolor='k', s=20)
    plt.xlim(xx.min(), xx.max())
    plt.ylim(yy.min(), yy.max())

    # Setting labels and title
    plt.xlabel('Day 1 Before')
    plt.ylabel('Day 2 Before')
    plt.title("Random forest model boundries")

    # Adding legend
    plt.legend(['Positive', 'Negative'])

    plt.show()

    end_time = time.time()
    elapsed_time = end_time - start_time
    print(f"The code took {elapsed_time:.5f} seconds to run.")
```

Algorithms

```
def fit_random_forest(X, Y, threshold=0.5, depth=None,
n_estimators=100):
    """
```

```
    Generates predictions of labels on the validation set by using the
Random Forest algorithm.

    Parameters:
    X (pandas.DataFrame): DataFrame containing the values of features
used in our Random Forest algorithm.
    Y (pandas.Series): Direction column.
    threshold (float): The threshold used by the algorithm to convert
probabilities to labels.
    depth (int): Max depth of the trees.
    n_estimators (int): Number of estimators used in the algorithm.

    Returns:
    accuracy (float): Accuracy of our algorithm on the validation set.
    y_validation (pandas Series): The true labels of each day in the
validation set.
    y_pred_class (pandas Series): Predicted labels for each day in the
validation set.
    y_pred_prob (pandas Series): Predicted probabilities of each day
belonging to label 1 in the
    validation set.
    """

    # Splitting the data chronologically
    X_train = X.loc[:'2017-9-25']
    X_validation = X.loc['2017-9-26':'2021-11-22']
    y_train = Y.loc[:'2017-9-25']
    y_validation = Y.loc['2017-9-26':'2021-11-22']

    # Creating a RandomForestClassifier object
    model = RandomForestClassifier(n_estimators=n_estimators,
max_depth=depth, random_state=1, min_samples_split=100)

    # Fitting the model to the training data
    model.fit(X_train, y_train)

    # Making predictions on the testing data
    y_pred_prob = model.predict_proba(X_validation)[:, 1]
    y_pred_class = np.where(y_pred_prob > threshold, 1, 0)

    accuracy = accuracy_score(y_validation, y_pred_class)
    return accuracy, y_validation, y_pred_class, y_pred_prob

def fit_logistic_regression(X, Y, threshold = 0.5):
    """
    Generates predictions of labels on the validation set by using
Logistic regression algorithm.

    Parameters:
    X (pandas.DataFrame): DataFrame containing the values of features
```

```
used in our Logistic regression
    algorithm.
    Y (pandas.Series): Direction column.
    threshold (float): The threshold used by the algorithm to convert
probabilities to labels.

    Returns:
    accuracy (float): Accuracy of our algorithm on the validation set.
    y_validation (pandas Series): The labels of each day in the
validation set.
    y_pred_class (pandas Series): Predicted labels for each day in the
validation set.
    y_pred_prob (pandas Series): Predicted probabilities of each day
belonging to label 1 in the
    validation set.
    """

    # Splitting the data chronologically
    X_train = X.loc[:'2017-9-25']
    X_validation = X.loc['2017-9-26':'2021-11-22']
    y_train = Y.loc[:'2017-9-25']
    y_validation = Y.loc['2017-9-26':'2021-11-22']

    # Creating a LogisticRegression object
    X_train = sm.add_constant(X_train)
    model = sm.Logit(y_train, X_train)
    result = model.fit()

    # Making predictions on the validation data
    X_validation = sm.add_constant(X_validation)
    y_pred_prob = result.predict(X_validation)
    y_pred_class = np.where(y_pred_prob > threshold, 1, 0)

    # Calculating and returning the accuracy of the model
    accuracy = accuracy_score(y_validation, y_pred_class)
    return accuracy, y_validation, y_pred_class, y_pred_prob

def fit_knn(X, Y, threshold=0.5, n_neighbors=5):
    """
    Generates predictions of labels on the validation set by using the
K-Nearest Neighbors algorithm.

    Parameters:
    X (pandas.DataFrame): DataFrame containing the values of features
used in our K-Nearest Neighbors
    algorithm.
    Y (pandas.Series): Direction column.
    threshold (float): The threshold used by the algorithm to convert
probabilities to labels.
    n_neighbors (int): Number of neighbors used in the K-Nearest
```

```
Neighbors algorithm.

    Returns:
    accuracy (float): Accuracy of our algorithm on the validation set.
    y_validation (pandas Series): The labels of each day in the
validation set.
    y_pred_class (pandas Series): Predicted labels for each day in the
validation set.
    y_pred_prob (pandas Series): Predicted probabilities of each day
belonging to label 1
    in the validation set.
    """

    # Splitting the data chronologically
    X_train = X.loc[:'2017-9-25']
    X_validation = X.loc['2017-9-26':'2021-11-22']
    y_train = Y.loc[:'2017-9-25']
    y_validation = Y.loc['2017-9-26':'2021-11-22']

    # Creating a KNeighborsClassifier object
    model = KNeighborsClassifier(n_neighbors=n_neighbors)

    # Fitting the model to the training data
    model.fit(X_train, y_train)

    # Making predictions on the testing data
    y_pred_prob = model.predict_proba(X_validation)[:, 1]
    y_pred_class = np.where(y_pred_prob > threshold, 1, 0)

    # Calculating and returning the accuracy of the model
    accuracy = accuracy_score(y_validation, y_pred_class)
    return accuracy, y_validation, y_pred_class, y_pred_prob

def fit_svm(X, Y, threshold=0.5, C=1.0, kernel='rbf', gamma='scale'):
    """
    Generates predictions of labels on the validation set by using the
Support Vector Classifier.

    Parameters:
    X (pandas.DataFrame): DataFrame containing the values of features
used in our Support Vector
    Classifier algorithm.
    Y (pandas.Series): Direction column.
    threshold (float): The threshold used by the algorithm to convert
probabilities to labels.
    kernel (string): Specifies the type of kernel function used in the
algorithm.
    gamma (string): Parameter for the kernel, influencing the
flexibility of the decision boundary
    in the Support Vector Classifier.
```

```
    Returns:
    accuracy (float): Accuracy of our algorithm on the validation set.
    y_validation (pandas Series): The labels of each day in the
validation set.
    y_pred_class (pandas Series): Predicted labels for each day in the
validation set.
    y_pred_prob (pandas Series): Predicted probabilities of each day
belonging to label 1 in
    the validation set.
    """

    # Splitting the data chronologically
    X_train = X.loc[:'2017-9-25']
    X_validation = X.loc['2017-9-26':'2021-11-22']
    y_train = Y.loc[:'2017-9-25']
    y_validation = Y.loc['2017-9-26':'2021-11-22']

    # Creating a Support Vector Machine object
    model = SVC(C=C, kernel=kernel, gamma=gamma, probability=True)

    # Fitting the model to the training data
    model.fit(X_train, y_train)

    # Making predictions on the testing data
    y_pred_prob = model.predict_proba(X_validation)[:, 1]
    y_pred_class = np.where(y_pred_prob > threshold, 1, 0)

    # Calculating and returning the accuracy of the model
    accuracy = accuracy_score(y_validation, y_pred_class)
    return accuracy, y_validation, y_pred_class, y_pred_prob

def fit_lstm(X, Y, threshold=0.5, n_steps=5, n_features=1):
    """
    Generates predictions of labels on the validation set by using the
Long-Short Term Memory algorithm.

    Parameters:
    X (pandas.DataFrame): DataFrame containing the values of features
used in our Long-Short Term Memory
    algorithm.
    Y (pandas.Series): Direction column.
    threshold (float): The threshold used by the algorithm to convert
probabilities to labels.
    n_steps (int): Refers to the number of time steps or lag
observations Long-Short Term Memory should
    consider.
    n_features (int): Refers to the number of input features or
variables used in the model.
```

```
    Returns:
    accuracy (float): Accuracy of our algorithm on the validation set.
    y_validation (pandas Series): The labels of each day in the
validation set.
    y_pred_class (pandas Series): Predicted labels for each day in the
validation set.
    y_pred_prob (pandas Series): Predicted probabilities of each day
belonging to label 1 in
    the validation set.
    """

    # Splitting the data chronologically
    X_train = X.loc[:'2017-9-25']
    X_validation = X.loc['2017-9-26':'2021-11-22']
    y_train = Y.loc[:'2017-9-25']
    y_validation = Y.loc['2017-9-26':'2021-11-22']

    # Reshaping the input data for the Long-Short Term Memory model
    X_train = X_train.values.reshape((X_train.shape[0], n_steps,
n_features))
    X_validation = X_validation.values.reshape((X_validation.shape[0],
n_steps, n_features))

    # Creating an Long-Short Term Memory model
    seed(42)
    tf.random.set_seed(42)
    model = Sequential()
    model.add(LSTM(50, input_shape=(n_steps, n_features)))
    model.add(Dense(1, activation='sigmoid'))
    model.compile(loss='binary_crossentropy', optimizer='adam',
metrics=['accuracy'])

    # Fitting the model to the training data
    model.fit(X_train, y_train, epochs=50, batch_size=72, verbose=0)

    # Making predictions on the testing data
    y_pred_prob = model.predict(X_validation)
    y_pred_class = np.where(y_pred_prob > threshold, 1, 0)

    # Calculating and returning the accuracy of the model
    accuracy = accuracy_score(y_validation, y_pred_class)
    return accuracy, y_validation, y_pred_class, y_pred_prob

def fit_lstm_improved(X, Y, threshold=0.5, n_steps=5, n_features=1):
    """
    Generates predictions of labels on the validation set by using the
Long-Short Term Memory Improved
    algorithm. Long-Short Term Memory Improved alogirthm compared to
Long-Short Term Memory Improved
```

```
    algorithm has more sequences and also contains dropouts.

    Parameters:
    X (pandas.DataFrame): DataFrame containing the values of features
used in our Long-Short Term Memory
    Improved algorithm.
    Y (pandas.Series): Direction column.
    threshold (float): The threshold used by the algorithm to convert
probabilities to labels.
    n_steps (int): Refers to the number of time steps or lag
observations Long-Short Term Memory Improved
    should consider.
    n_features (int): Refers to the number of input features or
variables used in the model.

    Returns:
    accuracy (float): Accuracy of our algorithm on the validation set.
    y_validation (pandas Series): The labels of each day in the
validation set.
    y_pred_class (pandas Series): Predicted labels for each day in the
validation set.
    y_pred_prob (pandas Series): Predicted probabilities of each day
belonging to label 1 in the
    validation set.
    """

    # Splitting the data chronologically
    X_train = X.loc[:'2017-9-25']
    X_validation = X.loc['2017-9-26':'2021-11-22']
    y_train = Y.loc[:'2017-9-25']
    y_validation = Y.loc['2017-9-26':'2021-11-22']

    # Applying MinMaxScaler
    sc = MinMaxScaler(feature_range=(0, 1))
    X_train = sc.fit_transform(X_train)
    X_validation = sc.transform(X_validation)

    # Reshaping the input data for the Long-Short Term Memory model
    X_train = X_train.reshape((X_train.shape[0], n_steps, n_features))
    X_validation = X_validation.reshape((X_validation.shape[0],
n_steps, n_features))

    # Creating an Long-Short Term Memory model with dropout layers
    seed(42)
    tf.random.set_seed(42)
    model = Sequential()
    model.add(LSTM(units=50, return_sequences=True,
input_shape=(n_steps, n_features)))
    model.add(Dropout(0.2))
    model.add(LSTM(units=50, return_sequences=True))
```

```
    model.add(Dropout(0.2))
    model.add(LSTM(units=50, return_sequences=True))
    model.add(Dropout(0.2))
    model.add(LSTM(units=50))
    model.add(Dropout(0.2))
    model.add(Dense(units=1, activation='sigmoid'))
    model.compile(loss='binary_crossentropy', optimizer='adam',
metrics=['accuracy'])

    # Fitting the model to the training data
    model.fit(X_train, y_train, epochs=100, batch_size=32, verbose=0)

    # Making predictions on the testing data
    y_pred_prob = model.predict(X_validation)
    y_pred_class = np.where(y_pred_prob > threshold, 1, 0)

    # Calculating and returning the accuracy of the model
    accuracy = accuracy_score(y_validation, y_pred_class)
    return accuracy, y_validation, y_pred_class, y_pred_prob

def fit_logistic_regression_test_data(X, Y, threshold = 0.5):
    """
    Generates predictions of labels on the test set by using Logistic
regression algorithm.

    Parameters:
    X (pandas.DataFrame): DataFrame containing the values of features
used in our Logistic regression
    algorithm.
    Y (pandas.Series): Direction column.
    threshold (float): The threshold used by the algorithm to convert
probabilities to labels.

    Returns:
    accuracy (float): Accuracy of our algorithm on the test set.
    y_validation (pandas Series): The labels of each day in the test
set.
    y_pred_class (pandas Series): Predicted labels for each day in the
test set.
    y_pred_prob (pandas Series): Predicted probabilities of each day
belonging to label 1 in the
    test set.
    """

    # Splitting the data chronologically
    X_train = X.loc['2003-12-03':'2021-11-22']
    X_test = X.loc['2021-11-23':'2024-1-24']
    y_train = Y.loc['2003-12-03':'2021-11-22']
    y_test = Y.loc['2021-11-23':'2024-1-24']
```

```
    # Creating a LogisticRegression object
    X_train = sm.add_constant(X_train)
    model = sm.Logit(y_train, X_train)
    result = model.fit()

    # Making predictions on the testing data
    X_test = sm.add_constant(X_test)
    y_pred_prob = result.predict(X_test)
    y_pred_class = np.where(y_pred_prob > threshold, 1, 0)

    # Calculating and returning the accuracy of the model
    accuracy = accuracy_score(y_test, y_pred_class)
    return accuracy, y_test, y_pred_class, y_pred_prob
```

Matrices

```
def treshold_matrix(X, Y, sp500_data, n_features, n_steps=1,
neighbors=5, n_estimators=100, max_depth=None):
    """
    Calculates the NORD for different algorithms and thresholds on the
validation set.
    Plots the result as matrix.

    Parameters:
    X (pandas.DataFrame): DataFrame containing the values of features
used in our algorithms.
    Y (pandas.Series): Direction column.
    sp500_data (pandas.DataFrame): A DataFrame containing the daily
prices of the S&P 500.
    n_features (int): Refers to the number of input features or
variables used in the model.
    n_steps (int): Refers to the number of time steps or lag
observations Long-Short Term memory
    should consider.
    n_neighbors (int): Number of neighbors used in K-Nearest Neighbors
algorithm.
    n_estimators (int): Number of estimators used in the Random Forest
algorithm.

    Returns:
    Matrix that contains the NORD values for different algorithms and
thresholds.
    The function also returns the execution time.
    """
    start_time = time.time()

    # Creating matrix that consists of zeros
    matrix = [[0 for j in range(8)] for i in range(7)]

    thresholds = [0.2, 0.3, 0.4, 0.45, 0.5, 0.6, 0.7, 0.8]
```

```
    # Fitting the models
    [accuracy, y_test, y_pred_class, y_pred_prob1] =
fit_random_forest(X, Y, threshold=0.5, depth=max_depth,
n_estimators=n_estimators)
    [accuracy, y_test, y_pred_class, y_pred_prob2] =
fit_logistic_regression(X, Y, threshold = 0.5)
    [accuracy, y_test, y_pred_class, y_pred_prob3] = fit_knn(X, Y,
threshold=0.5, n_neighbors=neighbors)
    [accuracy, y_test, y_pred_class, y_pred_prob4] = fit_svm(X, Y,
threshold=0.5, C=1.0, kernel='rbf', gamma='scale')
    [accuracy, y_test, y_pred_class, y_pred_prob5] = fit_lstm(X, Y,
threshold=0.5, n_steps=n_steps, n_features=n_features)
    [accuracy, y_test, y_pred_class, y_pred_prob6] = fit_lstm(X, Y,
threshold=0.5, n_steps=n_steps, n_features=n_features)

    # Random Forest
    j = 0
    for i in thresholds:
        y_pred_class = np.where(y_pred_prob1 > i, 1, 0)
        investment_returns = investment_return(y_pred_class,
sp500_data)
        matrix[0][j] = round(investment_returns["dollars"].iloc[-1],
2)
        matrix[6][j] =
round(investment_returns["dollars_SP500"].iloc[-1], 2)
        j = j + 1

    # logistic regression
    j = 0
    for i in thresholds:
        y_pred_class = np.where(y_pred_prob2 > i, 1, 0)
        investment_returns = investment_return(y_pred_class,
sp500_data)
        matrix[1][j] = round(investment_returns["dollars"].iloc[-1],
2)
        j = j + 1

    # K-Nearest Neighbors
    j = 0
    for i in thresholds:
        y_pred_class = np.where(y_pred_prob3 > i, 1, 0)
        investment_returns = investment_return(y_pred_class,
sp500_data)
        matrix[2][j] = round(investment_returns["dollars"].iloc[-1],
2)
        j = j + 1

    # Support Vector Classifier
```

```
    j = 0
    for i in thresholds:
        y_pred_class = np.where(y_pred_prob4 > i, 1, 0)
        investment_returns = investment_return(y_pred_class,
sp500_data)
        matrix[3][j] = round(investment_returns["dollars"].iloc[-1],
2)
        j = j + 1

    # Long-Short Term Memory
    j = 0
    for i in thresholds:
        y_pred_class = np.where(y_pred_prob5 > i, 1, 0)
        investment_returns = investment_return(y_pred_class.reshape(-
1), sp500_data)
        matrix[4][j] = investment_returns["dollars"].iloc[-1]
        j = j + 1

    # Long-Short Term Memory Improved
    j = 0
    for i in thresholds:
        y_pred_class = np.where(y_pred_prob6 > i, 1, 0)
        investment_returns = investment_return(y_pred_class.reshape(-
1), sp500_data)
        matrix[5][j] = investment_returns["dollars"].iloc[-1]
        j = j + 1


    x_labels = thresholds
    y_labels = ['fit_random_forest', 'fit_logistic_regression',
'fit_knn', 'fit_svm', 'lstm', 'lstm_improved','sp500']
    plt.figure(figsize=(10, 10))
    matrix = np.around(matrix, decimals=2)

    # Creating heatmap using Seaborn
    sns.heatmap(matrix, cmap='coolwarm', annot=True,
xticklabels=x_labels, yticklabels=y_labels)

    # Setting plot title and axis labels
    plt.title('Threshold matrix')
    plt.xlabel('Threshold')
    plt.ylabel('Model')

    # Displaying the plot
    plt.show()

    end_time = time.time()
    elapsed_time = end_time - start_time
```

```python
    print(f"The code took {elapsed_time:.5f} seconds to run.")
    return matrix

# Defining intervals used in interval matrix

# Interval 1 is our whole validation set
interval1 = ('2017-9-26', "2021-11-22")

# Splitting the validation set into two subsets
# Interval 2 is first subset of the validation set
interval2 = ("2020-2-18", "2020-3-23")

# Interval 3 is second subset of the validation set
interval3 = ("2020-3-24", "2021-11-22")

def interval_matrix(X, Y, interval1, interval2, interval3, sp500_data,
n_features, thresholds, n_steps=1, neighbors=5, n_estimators=100,
max_depth=None):
    """
    Calculates the NORD for each algorithm and interval. Plots the
result as matrix.

    Parameters:
    X (pandas.DataFrame): DataFrame containing the values of features
used in our algorithms.
    Y (pandas.Series): Direction column.
    interval1 (tuple): Train set used by algorithms.
    interval2 (tuple): Validation set used by algorithms. First
subset.
    interval3 (tuple): Validation set used by algorithms. Second
subset.
    sp500_data (pandas.DataFrame): A DataFrame containing the daily
prices of the S&P 500.
    n_features (int): Refers to the number of input features or
variables used in the model.
    thresholds (list): A collection of threshold values used by the
algorithms.
    n_steps (int): Refers to the number of time steps or lag
observations Long-Short Term memory should consider.
    n_neighbors (int): Number of neighbors used in K-Nearest Neighbors
algorithm.
    n_estimators (int): Number of estimators used in the algorithm.

    Returns:
    Matrix that contains the NORD values for different algorithms and
intervals.
    The function also returns the execution time.
    """
    start_time = time.time()
```

```
    # Splitting the data chronologically
    X_train = X.loc[:'2017-9-25']
    X_validation = X.loc['2017-9-26':'2021-11-22']
    y_train = Y.loc[:'2017-9-25']
    y_validation = Y.loc['2017-9-26':'2021-11-22']

    # Fitting the models

    # Random Forest
    model1 = RandomForestClassifier(n_estimators=n_estimators,
max_depth=max_depth, random_state=1, min_samples_split=100)
    model1.fit(X_train, y_train)

    # logistic regression
    X_train1 = sm.add_constant(X_train)
    model2 = sm.Logit(y_train, X_train1)
    result = model2.fit()

    # K-Nearest Neighbors
    model3 = KNeighborsClassifier(n_neighbors=neighbors)
    model3.fit(X_train, y_train)

    # Support Vector Classifier
    model4 = SVC(C=1.0, kernel='rbf', gamma='scale', probability=True)
    model4.fit(X_train, y_train)

    # Long-Short Term Memory
    X_train2 = X_train.values.reshape((X_train.shape[0], n_steps,
n_features))
    X_validation2 =
X_validation.values.reshape((X_validation.shape[0], n_steps,
n_features))
    X_validation2_df = pd.DataFrame(X_validation2.squeeze(),
index=X_validation.index)
    seed(42)
    tf.random.set_seed(42)
    model5 = Sequential()
    model5.add(LSTM(50, input_shape=(n_steps, n_features)))
    model5.add(Dense(1, activation='sigmoid'))
    model5.compile(loss='binary_crossentropy', optimizer='adam',
metrics=['accuracy'])
    model5.fit(X_train2, y_train, epochs=50, batch_size=72, verbose=0)

    # Long-Short Term Memory improved
    sc = MinMaxScaler(feature_range=(0, 1))
    X_train3 = sc.fit_transform(X_train)
    X_validation3 = sc.transform(X_validation)
    X_train3 = X_train3.reshape((X_train.shape[0], n_steps,
n_features))
```

```
    X_validation3 = X_validation3.reshape((X_validation.shape[0],
n_steps, n_features))
    X_validation3_df = pd.DataFrame(X_validation3.squeeze(),
index=X_validation.index)
    seed(42)
    tf.random.set_seed(42)
    model6 = Sequential()
    model6.add(LSTM(units=50, return_sequences=True,
input_shape=(n_steps, n_features)))
    model6.add(Dropout(0.2))
    model6.add(LSTM(units=50, return_sequences=True))
    model6.add(Dropout(0.2))
    model6.add(LSTM(units=50, return_sequences=True))
    model6.add(Dropout(0.2))
    model6.add(LSTM(units=50))
    model6.add(Dropout(0.2))
    model6.add(Dense(units=1, activation='sigmoid'))
    model6.compile(loss='binary_crossentropy', optimizer='adam',
metrics=['accuracy'])
    model6.fit(X_train3, y_train, epochs=100, batch_size=32,
verbose=0)

    matrix = [[0 for j in range(3)] for i in range(7)]

    intervals = [interval1, interval2, interval3]

    # Calculating each investment return for each interval

    # Random Forest
    j = 0
    for interval in intervals:
        y_pred_prob =
model1.predict_proba(X_validation.loc[interval[0]:interval[1]])[:,1]
        y_pred_class = np.where(y_pred_prob > thresholds[0], 1, 0)
        sp500_data_interval = sp500_data.loc[interval[0]:interval[1]]
        investment_returns = investment_return(y_pred_class,
sp500_data_interval)
        matrix[0][j] = round(investment_returns["dollars"].iloc[-1],
2)
        matrix[6][j] =
round(investment_returns["dollars_SP500"].iloc[-1], 2)
        j = j + 1

    # K-Nearest Neighbors
    j = 0
    for interval in intervals:
        y_pred_prob =
model3.predict_proba(X_validation.loc[interval[0]:interval[1]])[:, 1]
        y_pred_class = np.where(y_pred_prob > thresholds[2], 1, 0)
```

```
        sp500_data_interval = sp500_data.loc[interval[0]:interval[1]]
        investment_returns = investment_return(y_pred_class, 
sp500_data_interval)
        matrix[2][j] = round(investment_returns["dollars"].iloc[-1], 
2)
        j = j + 1

    # Support Vector Classifier
    j = 0
    for interval in intervals:
        y_pred_prob = 
model4.predict_proba(X_validation.loc[interval[0]:interval[1]])[:, 1]
        y_pred_class = np.where(y_pred_prob > thresholds[3], 1, 0)
        sp500_data_interval = sp500_data.loc[interval[0]:interval[1]]
        investment_returns = investment_return(y_pred_class, 
sp500_data_interval)
        matrix[3][j] = round(investment_returns["dollars"].iloc[-1], 
2)
        j = j + 1

    # Logistic regression
    j = 0
    for interval in intervals:
        X_validation1 = 
sm.add_constant(X_validation.loc[interval[0]:interval[1]])
        y_pred_prob = result.predict(X_validation1)
        y_pred_class = np.where(y_pred_prob > thresholds[1], 1, 0)
        sp500_data_interval = sp500_data.loc[interval[0]:interval[1]]
        investment_returns = investment_return(y_pred_class, 
sp500_data_interval)
        matrix[1][j] = round(investment_returns["dollars"].iloc[-1], 
2)
        j = j + 1

    # Long-Short Term Memory
    j = 0
    for interval in intervals:
        X_validation2_reshaped = 
X_validation2_df.loc[interval[0]:interval[1]].values.reshape(-1, 
n_steps, n_features)
        y_pred_prob = model5.predict(X_validation2_reshaped)
        y_pred_class = np.where(y_pred_prob > thresholds[4], 1, 0)
        sp500_data_interval = sp500_data.loc[interval[0]:interval[1]]
        investment_returns = investment_return(y_pred_class.reshape(-
1), sp500_data_interval)
        matrix[4][j] = investment_returns["dollars"].iloc[-1]
        j = j + 1

    # Long-Short Term Memory Improved
```

```
    j = 0
    for interval in intervals:
        X_validation3_reshaped = 
X_validation3_df.loc[interval[0]:interval[1]].values.reshape(-1, 
n_steps, n_features)
        y_pred_prob = model6.predict(X_validation3_reshaped)
        y_pred_class = np.where(y_pred_prob > thresholds[5], 1, 0)
        sp500_data_interval = sp500_data.loc[interval[0]:interval[1]]
        investment_returns = investment_return(y_pred_class.reshape(-
1), sp500_data_interval)
        matrix[5][j] = investment_returns["dollars"].iloc[-1]
        j = j + 1

    x_labels = intervals
    y_labels = ['fit_random_forest', 'fit_logistic_regression', 
'fit_knn', 'fit_svm', 'lstm', 'lstm_improved', 'SP500']
    plt.figure(figsize=(10, 10))
    matrix = np.around(matrix, decimals=2)

    # Creating heatmap using Seaborn
    sns.heatmap(matrix, cmap='coolwarm', annot=True, 
xticklabels=x_labels, yticklabels=y_labels)

    # Setting plot title and axis labels
    plt.title('Interval matrix')
    plt.xlabel('Intervals')
    plt.ylabel('Model')

    # Displaying the plot
    plt.show()

    end_time = time.time()
    elapsed_time = end_time - start_time
    print(f"The code took {elapsed_time:.5f} seconds to run.")
    return matrix
```

Hyperparameter tunning

```
def knn_hyper(X, Y, sp500_data):
    """
    Calculates and vizulizes the NORD values of K-Nearest Neighbors 
algorithm for different parameters.

    Parameters:
    X (pandas.DataFrame): DataFrame containing the values of features 
used in our algorithms.
    Y (pandas.Series): Direction column.
    sp500_data (pandas.DataFrame): A DataFrame containing the daily 
prices of the S&P 500.
```

```
    Returns:
    Matrix containing NORD values for various parameters of the K-
Nearest Neighbors algorithm.
    The function also returns the execution time.
    """
    start_time = time.time()
    matrix = [[0 for j in range(20)] for i in range(2)]
    neighbors = [1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15,
16, 17, 25, 30, 50]

    # Calculating NORD for different neighbors
    # Threshold equals 0.5
    i = 0
    for neighbor in neighbors:
        [accuracy, y_test, y_pred_class, y_pred_prob] = fit_knn(X, Y,
threshold=0.5, n_neighbors=neighbor)
        investment_returns = investment_return(y_pred_class,
sp500_data)
        matrix[0][i] = round(investment_returns['dollars'].iloc[-1],
2)
        i = i + 1

    # Calculating NORD for different neighbors
    # Threshold equals 0.4
    i = 0
    for neighbor in neighbors:
        [accuracy, y_test, y_pred_class, y_pred_prob] = fit_knn(X, Y,
threshold=0.4, n_neighbors=neighbor)
        investment_returns = investment_return(y_pred_class,
sp500_data)
        matrix[1][i] = round(investment_returns['dollars'].iloc[-1],
2)
        i = i + 1


    plt.figure(figsize=(15, 3))
    matrix = np.round(matrix, decimals=2)

    # Creating heatmap using Seaborn
    sns.heatmap(matrix, cmap='coolwarm', annot=True, yticklabels=
[0.5, 0.4], xticklabels=neighbors)

    # Setting plot title and axis labels
    plt.title('KNN Hyperparameter tunning')
    plt.xlabel('Neighbors')
    plt.ylabel('Thresholds')

    # Displaying the plot
    plt.show()
```

```
    end_time = time.time()
    elapsed_time = end_time - start_time
    print(f"The code took {elapsed_time:.5f} seconds to run.")
    return matrix

def hypertune_random_forest(X, Y, sp500_data):
    """
    Calculates and vizulizes the NORD values of the Random Forest
algorithm for different parameters.

    Parameters:
    X (pandas.DataFrame): DataFrame containing the values of features
used in our algorithms.
    Y (pandas.Series): Direction column.
    sp500_data (pandas.DataFrame): A DataFrame containing the daily
prices of the S&P 500.

    Returns:
    Matrix containing NORD values for various parameters of the RF
algorithm.
    The function also returns the execution time.
    """
    start_time = time.time()
    matrix = [[0 for j in range(7)] for i in range(2)]
    n_estimators = [3, 5, 10, 50, 100, 150, 200]
    max_debths = [1, 3, 5, 10, 20, 40, 80]

    # Calculating NORD for different estimators
    i = 0
    for n_estimator in n_estimators:
        [accuracy, y_test, y_pred_class, y_pred_prob] =
fit_random_forest(X, Y, threshold=0.4, n_estimators=n_estimator)
        investment_returns = investment_return(y_pred_class,
sp500_data)
        matrix[0][i] = round(investment_returns['dollars'].iloc[-1],
2)
        i = i + 1

    # Calculating NORD for different debths
    i = 0
    for max_debth in max_debths:
        [accuracy, y_test, y_pred_class, y_pred_prob] =
fit_random_forest(X, Y, threshold=0.4,depth=max_debth)
        investment_returns = investment_return(y_pred_class,
sp500_data)
        matrix[1][i] = round(investment_returns['dollars'].iloc[-1],
2)
        i = i + 1
```

```
    plt.figure(figsize=(15, 3))
    matrix = np.round(matrix, decimals=2)

    # Creating heatmap using Seaborn
    sns.heatmap(matrix, cmap='coolwarm', annot=True,
yticklabels=['n_estimators', 'max_depth'])

    # Setting plot title and axis labels
    plt.title('Random Forest Hyperparameter tunning')

    # Displaying the plot
    plt.show()
    print("n_estimators:", n_estimators)
    print("max_debths:", max_debths)

    end_time = time.time()
    elapsed_time = end_time - start_time
    print(f"The code took {elapsed_time:.5f} seconds to run.")
    return matrix
```

Feature selections

```
def methodology_A(fixed_features, features, dataframe, NORD_max = 0):
    """
    Applies feature selection methodology A for the given features.

    Parameters:
    fixed_features (list): List of features that will remain fixed in
each iteration.
    features (list): List of features used by Logistic regression.
    dataframe (pandas.DataFrame): A DataFrame containing the daily
prices of the S&P 500.
    NORD_max (float): The initial NORD_max used in the methodology A.

    Returns:
    A table containg NORD value for each list of features.
    """
    # Generating matrix
    matrix = [[0 for i in range(2)] for i in range(len(features))]
    best_features = []

    # Calculating the NORD for each list of features. Logistic
regression was used to generate predictions.
    for i in range(len(features)):
        if fixed_features is not None:
            current_features = fixed_features + best_features +
[features[i]]
        else:
            current_features = best_features + [features[i]]
```

```
        # Defining the training set and validation set.
        X = dataframe[current_features]
        Y = dataframe['Direction']
        X_train = X.loc[:'2017-9-25']
        X_validation = X.loc['2017-9-26':'2021-11-22']
        y_train = Y.loc[:'2017-9-25']
        y_validation = Y.loc['2017-9-26':'2021-11-22']
        X_train = sm.add_constant(X_train)
        X_validation = sm.add_constant(X_validation)
        # Trianing Logistic regression
        model = LogisticRegression(max_iter=10000)
        model.fit(X_train, y_train)
        val_pred = model.predict(X_validation)
        # Calculating the NORD value from the predicitions generated
by Logistic regression.
        NORD = investment_return(val_pred, dataframe['2017-09-
26':'2021-11-22'])["dollars"].iloc[-1]
        matrix[i][0]= current_features
        matrix[i][1]= NORD
        if NORD > NORD_max:
            NORD_max = NORD
            best_features.append(features[i])
    results_df = pd.DataFrame(matrix, columns=['Features', 'NORD'])
    results = results_df.style.set_properties(subset=['Features'],
**{'width':'300xp'})
    return results

def best_features(features, dataframe):
    """
    Calculates the NORD for each list of features.

    Parameters:
    features (List[Lists]): List of lists containing features used by
Logistic Regression.
    dataframe (pandas.DataFrame): A DataFrame containing the daily
prices of the S&P 500.
    NORD_max (float): The initial NORD_max used in the methodology A.

    Returns:
    A table containg NORD value for each list of features.
    """
    # Generating matrix
    matrix = [[0 for i in range(2)] for i in range(len(features))]
    results = []

    # Calculating the NORD for each list of features. Logistic
regression was used to generate predictions.
    for i in range(len(features)):
        # Defining the training set and validation set.
        X = dataframe[features[i]]
```

```
        Y = dataframe['Direction']
        X_train = X.loc[:'2017-9-25']
        X_validation = X.loc['2017-9-26':'2021-11-22']
        y_train = Y.loc[:'2017-9-25']
        y_validation = Y.loc['2017-9-26':'2021-11-22']
        X_train = sm.add_constant(X_train)
        X_validation = sm.add_constant(X_validation)
        # Trianing Logistic regression
        model = LogisticRegression(max_iter=10000)
        model.fit(X_train, y_train)
        val_pred = model.predict(X_validation)
        val_return = investment_return(val_pred, dataframe['2017-9-
26':'2021-11-22'])["dollars"].iloc[-1]
        matrix[i][0]= features[i]
        matrix[i][1]= val_return

    results_df = pd.DataFrame(matrix, columns=['Features', 'NORD'])
    results_2 = results_df.style.set_properties(subset=['Features'],
**{'width':'300xp'})
    return results_2
```

In the subsequent sections, we select features for our model, with each section representing a distinct feature group. The algorithms within each feature group are trained on different intervals. To enhance code clarity, a summary is provided at the end of each feature group.

For the Random Forest and K-Nearest Neighbors algorithms, we choose the parameters that result in the highest NORD value. Similarly, for each algorithm, we choose the threshold with the highest NORD value.

Prior Day Percentage Changes

```
# Calculating the daily percantage change of i days before
for i in range(1, 10):
    sp500['Day_' + str(i) + '_before'] = sp500['pct_change'].shift(i)
sp500.head(5)
```

```
                   Open         High          Low        Close     Adj
Close  \
Date

2001-01-16  1318.319946  1327.810059  1313.329956  1326.650024
1326.650024
2001-01-17  1326.650024  1346.920044  1325.410034  1329.469971
1329.469971
2001-01-18  1329.890015  1352.709961  1327.410034  1347.969971
1347.969971
2001-01-19  1347.969971  1354.550049  1336.739990  1342.540039
1342.540039
2001-01-22  1342.540039  1353.619995  1333.839966  1342.900024
1342.900024
```

```
                Volume  Close day before  pct_change  Direction
Day_1_before  \
Date

2001-01-16  1205700000               NaN         NaN          0
NaN
2001-01-17  1349100000       1326.650024    0.212561          1
NaN
2001-01-18  1445000000       1329.469971    1.391532          1
0.212561
2001-01-19  1407800000       1347.969971   -0.402823          0
1.391532
2001-01-22  1164000000       1342.540039    0.026814          1      -
0.402823

            Day_2_before  Day_3_before  Day_4_before  Day_5_before  \
Date
2001-01-16           NaN           NaN           NaN           NaN
2001-01-17           NaN           NaN           NaN           NaN
2001-01-18           NaN           NaN           NaN           NaN
2001-01-19      0.212561           NaN           NaN           NaN
2001-01-22      1.391532      0.212561           NaN           NaN

            Day_6_before  Day_7_before  Day_8_before  Day_9_before
Date
2001-01-16           NaN           NaN           NaN           NaN
2001-01-17           NaN           NaN           NaN           NaN
2001-01-18           NaN           NaN           NaN           NaN
2001-01-19           NaN           NaN           NaN           NaN
2001-01-22           NaN           NaN           NaN           NaN
```

```python
# Cutting off the test set to ensure unbiased evaluation
sp500_2 = sp500.loc['2001-01-30':"2021-11-22"]
sp500_2
```

```
                   Open         High          Low        Close    Adj
Close  \
Date

2001-01-30  1364.170044  1375.680054  1356.199951  1373.729980
1373.729980
2001-01-31  1373.729980  1383.369995  1364.660034  1366.010010
1366.010010
2001-02-01  1366.010010  1373.500000  1359.339966  1373.469971
1373.469971
2001-02-02  1373.469971  1376.380005  1348.719971  1349.469971
1349.469971
2001-02-05  1349.469971  1354.560059  1344.479980  1354.310059
1354.310059
```

```
...                   ...          ...          ...          ...
...
2021-11-16  4679.419922  4714.950195  4679.419922  4700.899902
4700.899902
2021-11-17  4701.500000  4701.500000  4684.410156  4688.669922
4688.669922
2021-11-18  4700.720215  4708.799805  4672.779785  4704.540039
4704.540039
2021-11-19  4708.439941  4717.750000  4694.220215  4697.959961
4697.959961
2021-11-22  4712.000000  4743.830078  4682.169922  4682.939941
4682.939941

                Volume  Close day before  pct_change  Direction
Day_1_before  \
Date

2001-01-30  1149800000       1364.170044    0.700788          1
0.680475
2001-01-31  1295300000       1373.729980   -0.561971          0
0.700788
2001-02-01  1118800000       1366.010010    0.546113          1     -
0.561971
2001-02-02  1048400000       1373.469971   -1.747399          0
0.546113
2001-02-05  1013000000       1349.469971    0.358666          1     -
1.747399
...                ...               ...         ...        ...
...
2021-11-16  3972640000       4682.799805    0.386523          1     -
0.001074
2021-11-17  3969070000       4700.899902   -0.260163          0
0.386523
2021-11-18  4226410000       4688.669922    0.338478          1     -
0.260163
2021-11-19  4253180000       4704.540039   -0.139867          0
0.338478
2021-11-22  4441100000       4697.959961   -0.319714          0     -
0.139867

            Day_2_before  Day_3_before  Day_4_before  Day_5_before  \
Date
2001-01-30     -0.188585     -0.497694      0.286682      1.303150
2001-01-31      0.680475     -0.188585     -0.497694      0.286682
2001-02-01      0.700788      0.680475     -0.188585     -0.497694
2001-02-02     -0.561971      0.700788      0.680475     -0.188585
2001-02-05      0.546113     -0.561971      0.700788      0.680475
...                  ...           ...           ...           ...
2021-11-16      0.722266      0.055094     -0.822582     -0.349878
2021-11-17     -0.001074      0.722266      0.055094     -0.822582
```

```
2021-11-18      0.386523     -0.001074      0.722266      0.055094
2021-11-19     -0.260163      0.386523     -0.001074      0.722266
2021-11-22      0.338478     -0.260163      0.386523     -0.001074

            Day_6_before  Day_7_before  Day_8_before  Day_9_before
Date
2001-01-30      0.026814     -0.402823      1.391532      0.212561
2001-01-31      1.303150      0.026814     -0.402823      1.391532
2001-02-01      0.286682      1.303150      0.026814     -0.402823
2001-02-02     -0.497694      0.286682      1.303150      0.026814
2001-02-05     -0.188585     -0.497694      0.286682      1.303150
...                  ...           ...           ...           ...
2021-11-16      0.088779      0.373280      0.418194      0.646128
2021-11-17     -0.349878      0.088779      0.373280      0.418194
2021-11-18     -0.822582     -0.349878      0.088779      0.373280
2021-11-19      0.055094     -0.822582     -0.349878      0.088779
2021-11-22      0.722266      0.055094     -0.822582     -0.349878

[5238 rows x 18 columns]
```

```
# Calculating NORD for the Buy and Hold strategy
# Array of predictions: In the Buy and Hold strategy, each prediction
is set to one.
# The interval ["2017-9-26":'2021-11-22'] has a length of 1048.
y_pred_class_buy_and_hold = np.ones(1048)
val_return_buy_and_hold = investment_return(y_pred_class_buy_and_hold,
sp500_2["2017-9-26":'2021-11-22'])["dollars"].iloc[-1]
val_return_buy_and_hold
```

```
1.8756819535944504
```

```
# Using methodology A for feature selection.
features_percentage_changes = ['Day_1_before', 'Day_2_before',
'Day_3_before', 'Day_4_before', 'Day_5_before',
'Day_6_before', 'Day_7_before', 'Day_8_before', 'Day_9_before']
# Since no features are selected prior to this point, the initial
value of NORD_max is NORD - BUY AND HOLD
# No fixed features are specified for repetition in each iteration.
result_percentage_changes = methodology_A(None,
features_percentage_changes, sp500_2, val_return_buy_and_hold)
result_percentage_changes
```

```
<pandas.io.formats.style.Styler at 0x16155787070>
```

Methodology A identifies features Day_1_before, Day_4_before, and Day_5_before as important. The NORD value for the model with the feature Day_1_before is 2.81, exceeding the threshold of 1.88. Therefore, Methodology A classifies the feature Day_1_before as important. Our next step involves applying Methodology B to further refine the selection of important features.

```
# Round 1 of methodology B
features_round_1_percentage_changes = ['Day_1_before', 'Day_2_before',
'Day_3_before', 'Day_4_before', 'Day_5_before',
             'Day_6_before', 'Day_7_before', 'Day_8_before',
'Day_9_before']
result_round_1_percentage_changes =
best_features(features_round_1_percentage_changes, sp500_2)
result_round_1_percentage_changes
```

```
<pandas.io.formats.style.Styler at 0x16155f90b20>
```

```
# Round 2 of methodology B
features_round_2_percentage_changes  = [
    ['Day_1_before', 'Day_2_before'],
    ['Day_1_before', 'Day_3_before'],
    ['Day_1_before', 'Day_4_before'],
    ['Day_1_before', 'Day_5_before'],
    ['Day_1_before', 'Day_6_before'],
    ['Day_1_before', 'Day_7_before'],
    ['Day_1_before', 'Day_8_before'],
    ['Day_1_before', 'Day_9_before']
]
result_round_2_percentage_changes  =
best_features(features_round_2_percentage_changes , sp500_2)
result_round_2_percentage_changes
```

```
<pandas.io.formats.style.Styler at 0x16155fbdb80>
```

```
# Round 3 of methodology B
features_round_3_percentage_changes = [
    ['Day_1_before', 'Day_5_before', 'Day_2_before'],
    ['Day_1_before', 'Day_5_before', 'Day_3_before'],
    ['Day_1_before', 'Day_5_before', 'Day_4_before'],
    ['Day_1_before', 'Day_5_before', 'Day_6_before'],
    ['Day_1_before', 'Day_5_before', 'Day_7_before'],
    ['Day_1_before', 'Day_5_before', 'Day_8_before'],
    ['Day_1_before', 'Day_5_before', 'Day_9_before'],
]
result_round_3_percentage_changes  =
best_features(features_round_3_percentage_changes, sp500_2)
result_round_3_percentage_changes
```

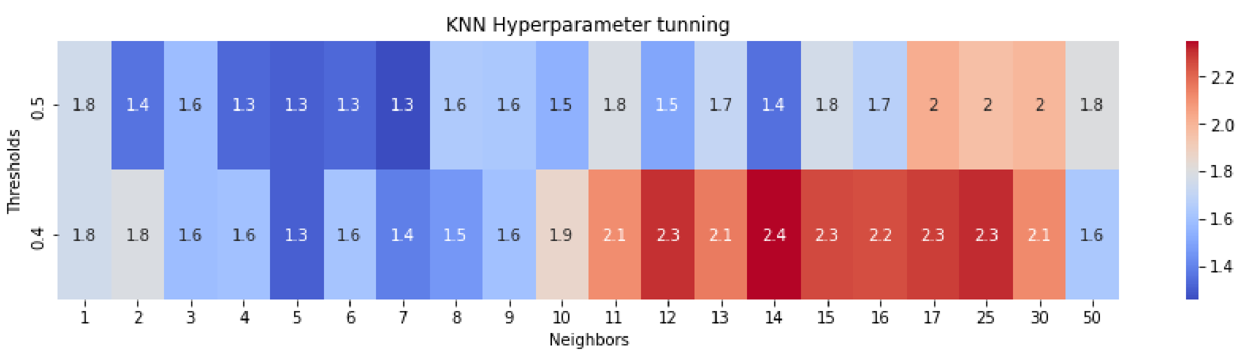
```
<pandas.io.formats.style.Styler at 0x16155fdb040>
```

```
# Rround 4 of methodology B
features_round_4_percentage_changes = [
    ['Day_1_before', 'Day_5_before', 'Day_4_before', 'Day_2_before'],
    ['Day_1_before', 'Day_5_before', 'Day_4_before', 'Day_3_before'],
    ['Day_1_before', 'Day_5_before', 'Day_4_before', 'Day_6_before'],
    ['Day_1_before', 'Day_5_before', 'Day_4_before', 'Day_7_before'],
    ['Day_1_before', 'Day_5_before', 'Day_4_before', 'Day_8_before'],
```

```
    ['Day_1_before', 'Day_5_before', 'Day_4_before', 'Day_9_before']
]
result_round_4_percentage_changes =
best_features(features_round_4_percentage_changes, sp500_2)
result_round_4_percentage_changes

<pandas.io.formats.style.Styler at 0x16155d1cfa0>
```

According to methodology B, the best combination of features consists of Day_1_before and Day_5_before. However, both methodologies skip Day_2_before and Day_3_before before including Day_4_before. To address this, we adopt an alternative approach: systematically add features one by one until we establish a cutoff point.

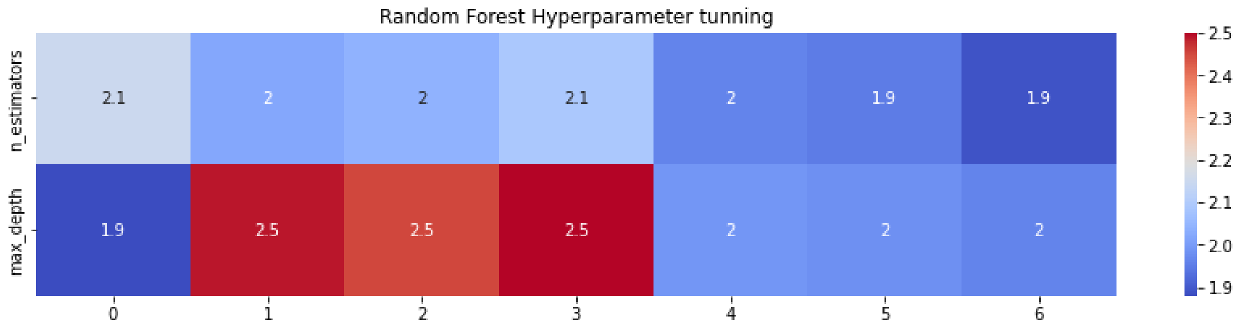
```
# Selecting Day_1_before as a feature
Features_D1B = sp500_2['Day_1_before']
Y = sp500_2['Direction']

Features_D1B = np.array(Features_D1B).reshape(-1, 1)
Features_D1B_df = pd.DataFrame(Features_D1B, columns=['feature'],
index=Y.index)

# Selecting the right parameters for KNN
matrix_knn_parameters_percentage_changes = knn_hyper(Features_D1B_df,
Y, sp500_2['2017-9-26':])
```




```
The code took 22.20946 seconds to run.

matrix_knn_parameters_percentage_changes

array([[1.75, 1.36, 1.58, 1.33, 1.31, 1.33, 1.26, 1.64, 1.63, 1.54,
1.78,
        1.5 , 1.71, 1.35, 1.76, 1.67, 2.02, 1.96, 1.99, 1.8 ],
       [1.75, 1.79, 1.58, 1.6 , 1.31, 1.61, 1.39, 1.46, 1.6 , 1.86,
2.11,
        2.3 , 2.15, 2.35, 2.26, 2.25, 2.28, 2.31, 2.12, 1.63]])
```

The parameter combination of 14 neighbors with a threshold of 0.4 results in the highest NORD.

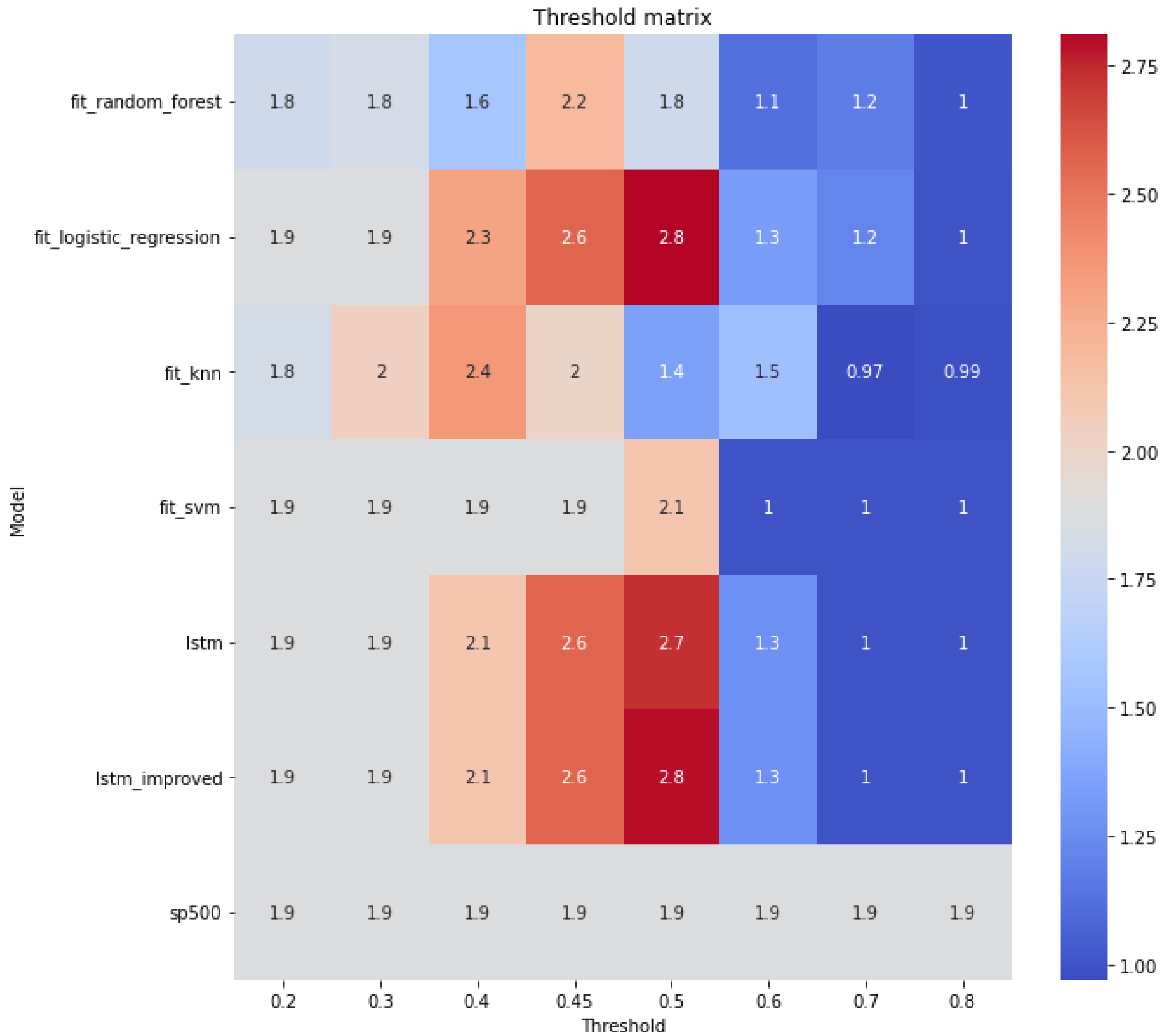
```
# Selecting the right parameters for RF
matrix_rf_parameters_percentage_changes =
hypertune_random_forest(Features_D1B_df, Y, sp500_2['2017-9-26':])
```




```
n_estimators: [3, 5, 10, 50, 100, 150, 200]
max_debths: [1, 3, 5, 10, 20, 40, 80]
The code took 13.03370 seconds to run.

matrix_rf_parameters_percentage_changes

array([[2.15, 2.02, 2.04, 2.09, 1.96, 1.95, 1.89],
       [1.88, 2.49, 2.45, 2.5 , 1.99, 1.98, 1.96]])
```

A max depth of 10 and n_estimators of 3 result in the highest NORD.

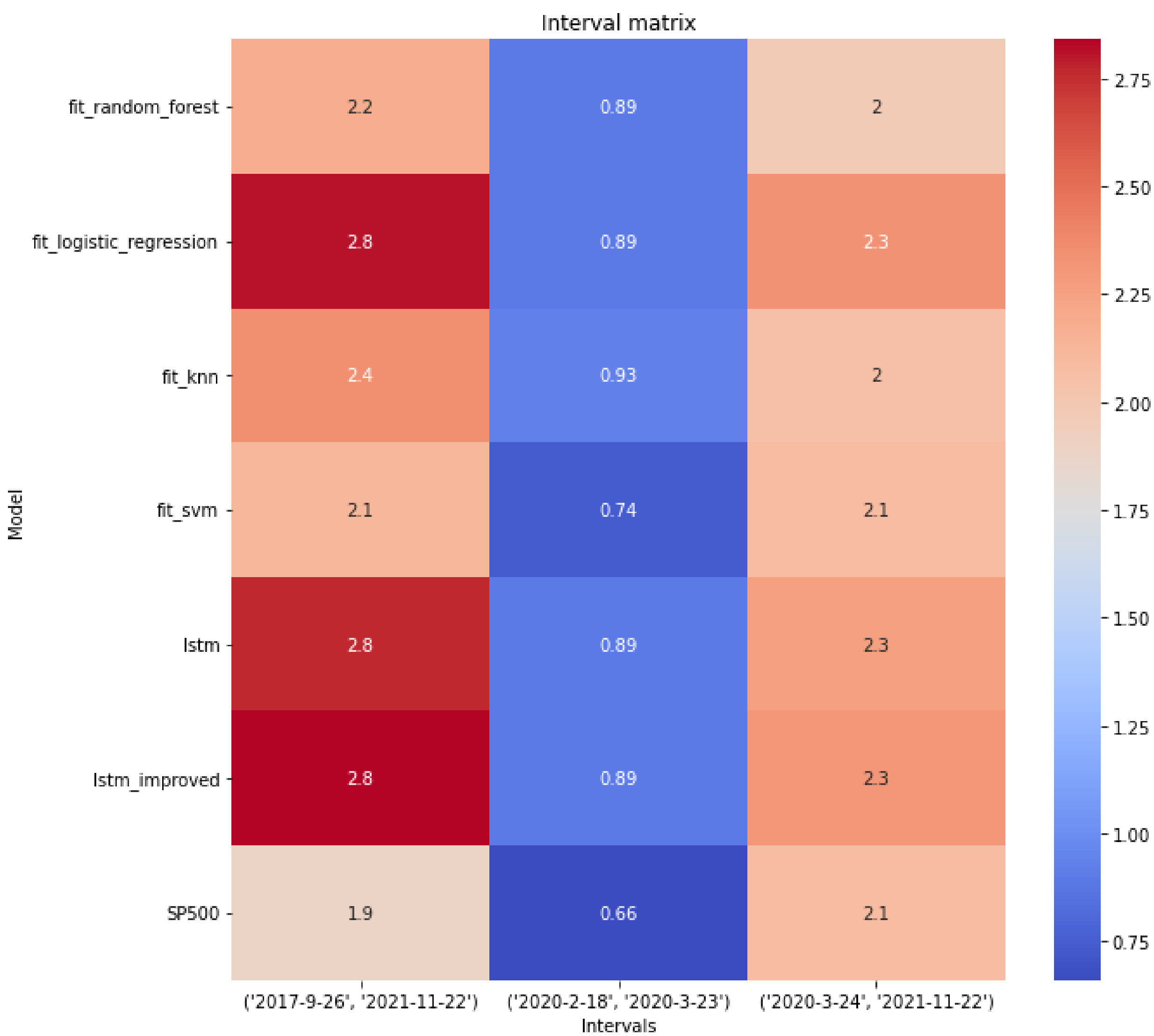
```
matrix_D1B_threshold = treshold_matrix(Features_D1B_df, Y,
sp500_2["2017-9-26":], 1, n_steps=1, n_estimators=3, max_depth=10,
neighbors=14)

Optimization terminated successfully.
         Current function value: 0.689174
         Iterations 4
33/33 [==============================] - 1s 1ms/step
33/33 [==============================] - 0s 1ms/step
```



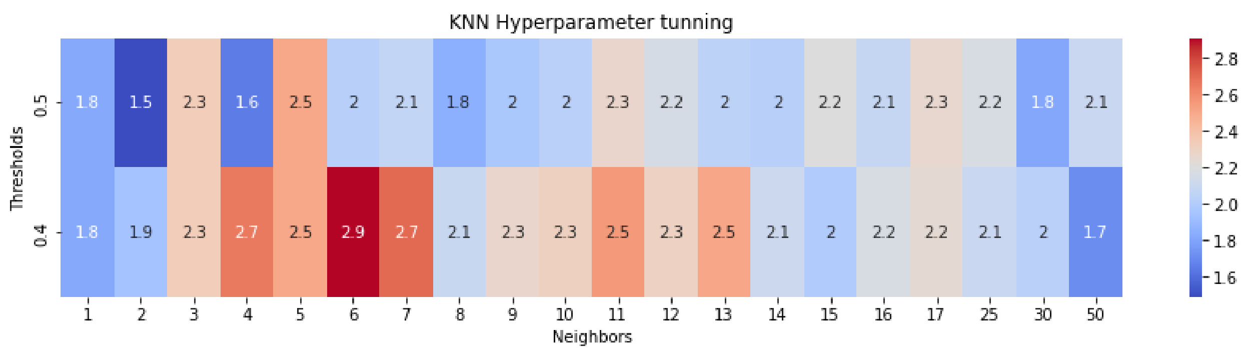

```
The code took 51.94077 seconds to run.

matrix_D1B_threshold

array([[1.79, 1.83, 1.57, 2.18, 1.79, 1.12, 1.18, 1.01],
       [1.88, 1.88, 2.3 , 2.56, 2.81, 1.33, 1.22, 1.  ],
       [1.82, 2.04, 2.35, 2.  , 1.35, 1.53, 0.97, 0.99],
       [1.88, 1.88, 1.88, 1.88, 2.11, 1.  , 1.  , 1.  ],
       [1.88, 1.88, 2.11, 2.56, 2.73, 1.27, 1.  , 1.  ],
       [1.88, 1.88, 2.11, 2.56, 2.79, 1.27, 1.  , 1.  ],
       [1.88, 1.88, 1.88, 1.88, 1.88, 1.88, 1.88, 1.88]])
```

The thresholds 0.45, 0.5, 0.4, 0.5, 0.5, 0.5, corresponding to the algorithms Random Forest, Logistic Regression, K-Nearest Neighbors, Support Vector Classifier, Long-Short Term Memory, and Long-Short Term Memory Improved, respectively, yield the highest NORD.

```
thresholds_D1B = [0.45, 0.5, 0.4, 0.5, 0.5, 0.5]

matrix_D1B_interval = interval_matrix(Features_D1B_df, Y, interval1,
interval2, interval3, sp500_2["2017-9-26":], 1, thresholds_D1B,
neighbors=14, n_estimators=3, max_depth=10)

Optimization terminated successfully.
         Current function value: 0.689174
         Iterations 4
33/33 [==============================] - 0s 1ms/step
1/1 [==============================] - 0s 18ms/step
14/14 [==============================] - 0s 1ms/step
33/33 [==============================] - 2s 1ms/step
1/1 [==============================] - 0s 17ms/step
14/14 [==============================] - 0s 1ms/step
```



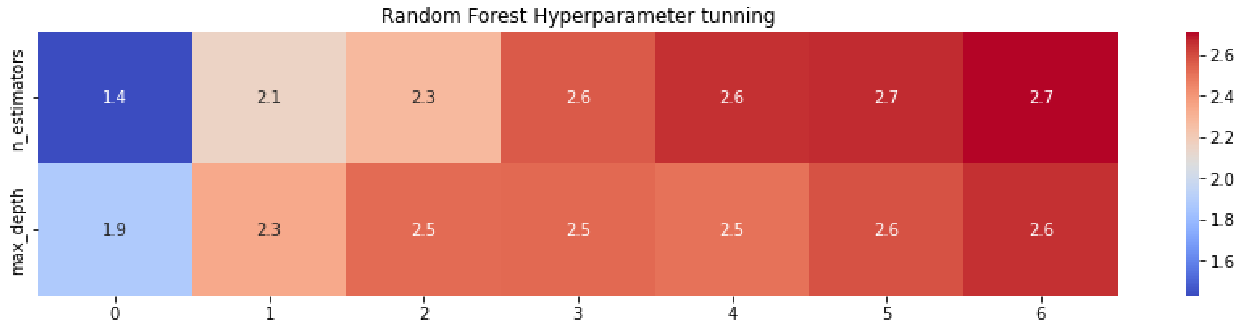
```
The code took 101.90484 seconds to run.

matrix_D1B_interval

array([[2.18, 0.89, 1.97],
       [2.81, 0.89, 2.33],
       [2.35, 0.93, 2.05],
       [2.12, 0.74, 2.09],
       [2.77, 0.89, 2.26],
       [2.84, 0.89, 2.31],
       [1.88, 0.66, 2.09]])

# Selecting Day_1_before, Day_2_before as our features
Features_D1B_D2B = sp500_2[['Day_1_before', 'Day_2_before']]

# Selecting the right parameters for KNN
matrix_knn_parameters_percentage_changes_D1B_D2B =
knn_hyper(Features_D1B_D2B, Y, sp500_2['2017-9-26':])
```




```
The code took 21.56551 seconds to run.

matrix_knn_parameters_percentage_changes_D1B_D2B

array([[1.82, 1.49, 2.33, 1.65, 2.5 , 2.02, 2.07, 1.84, 1.97, 2.03,
2.27,
        2.16, 2.04, 2.02, 2.2 , 2.08, 2.26, 2.17, 1.81, 2.1 ],
       [1.82, 1.94, 2.33, 2.66, 2.5 , 2.91, 2.71, 2.09, 2.27, 2.31,
2.55,
        2.3 , 2.51, 2.11, 1.99, 2.17, 2.24, 2.1 , 2.03, 1.71]])
```

The parameter combination of 6 neighbors with a threshold of 0.4 results in the highest NORD.

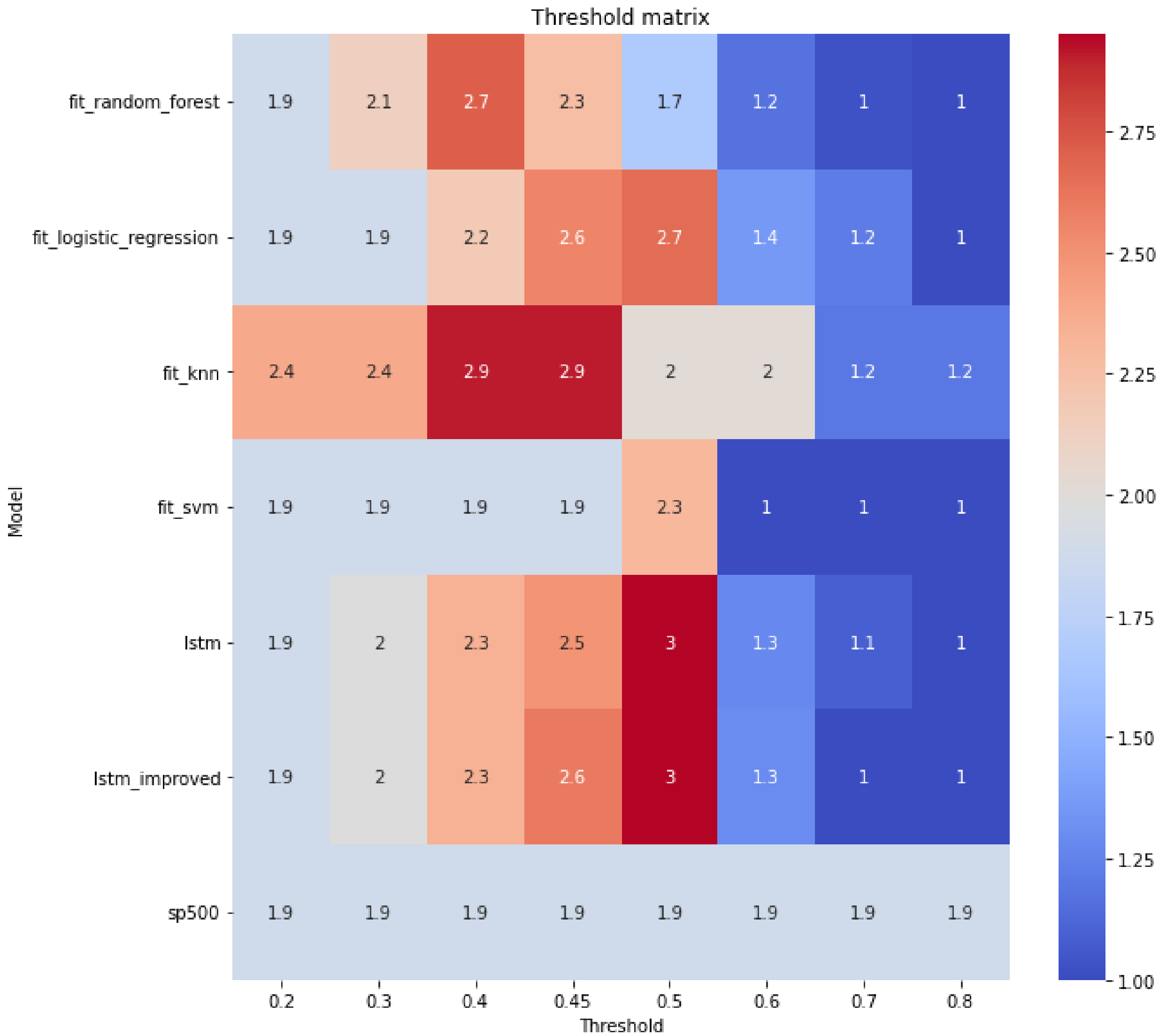
```
# Selecting the right parameters for RF
matrix_rf_parameters_percentage_changes_D1B_D2B =
hypertune_random_forest(Features_D1B_D2B, Y, sp500_2['2017-9-26':])
```




```
n_estimators: [3, 5, 10, 50, 100, 150, 200]
max_debths: [1, 3, 5, 10, 20, 40, 80]
The code took 11.81645 seconds to run.

matrix_rf_parameters_percentage_changes_D1B_D2B

array([[1.43, 2.15, 2.28, 2.56, 2.65, 2.66, 2.71],
       [1.88, 2.34, 2.52, 2.53, 2.5 , 2.58, 2.65]])
```

A max depth of 80 and n_estimators of 200 result in the highest NORD.

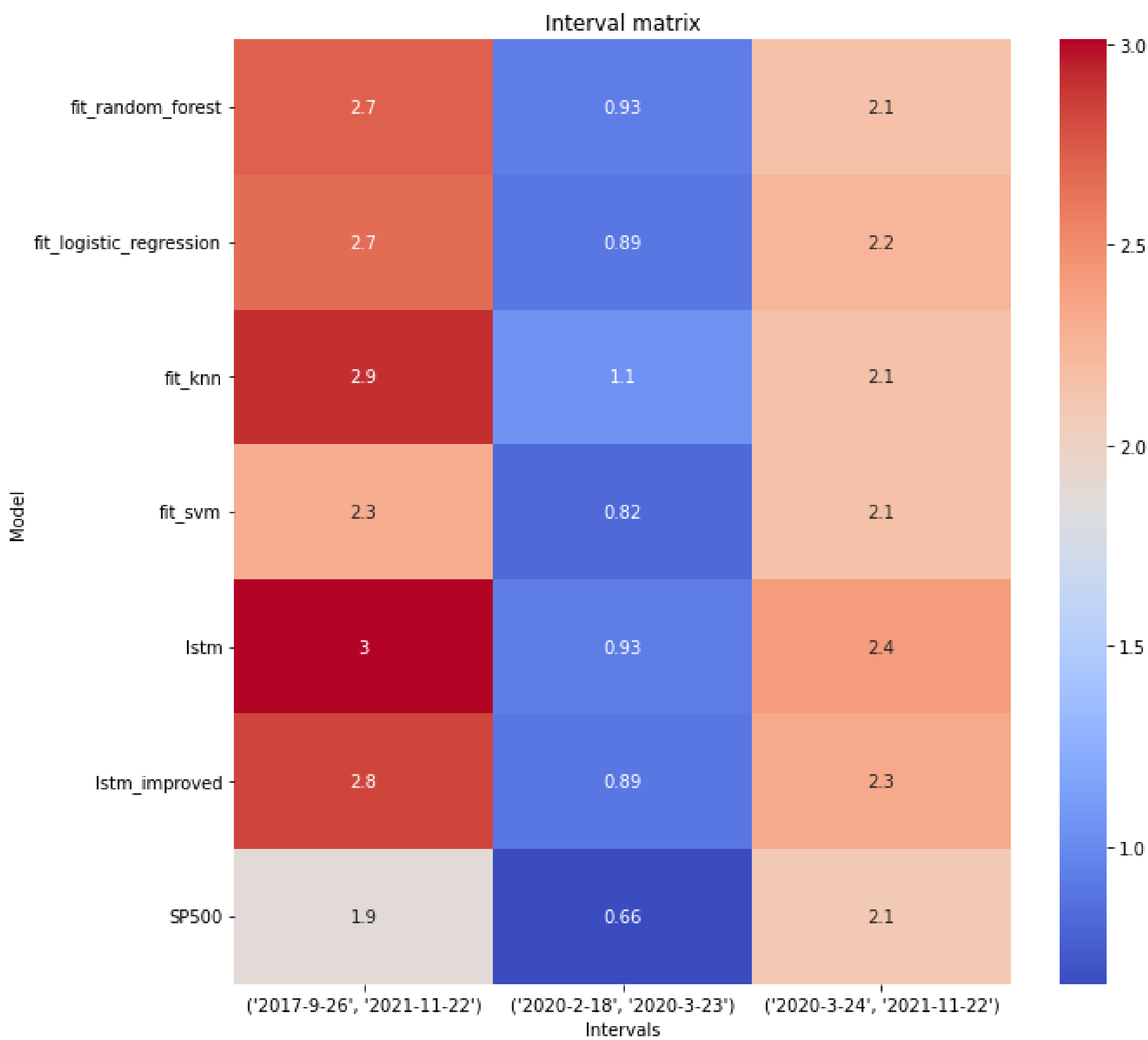
```
matrix_D1B_D2B_threshold = treshold_matrix(Features_D1B_D2B, Y,
sp500_2["2017-9-26":], 2, n_steps=1, n_estimators=200, max_depth=80,
neighbors=6)

Optimization terminated successfully.
         Current function value: 0.689169
         Iterations 4
33/33 [==============================] - 0s 973us/step
33/33 [==============================] - 1s 1ms/step
```



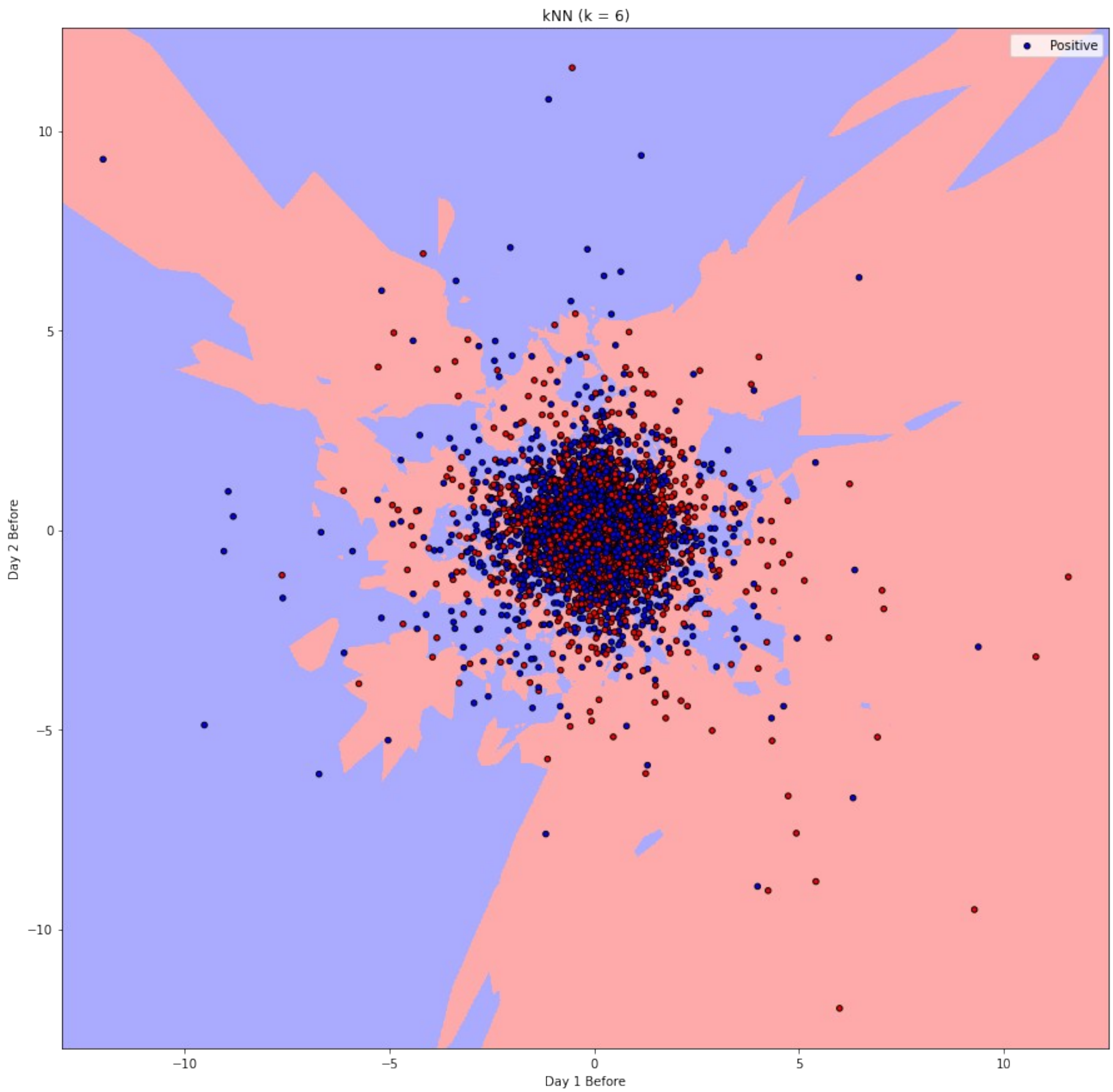

```
The code took 49.85473 seconds to run.

matrix_D1B_D2B_threshold

array([[1.88, 2.13, 2.71, 2.26, 1.68, 1.17, 1.03, 1.  ],
       [1.88, 1.88, 2.18, 2.56, 2.66, 1.36, 1.22, 1.  ],
       [2.38, 2.38, 2.91, 2.91, 2.02, 2.02, 1.2 , 1.2 ],
       [1.88, 1.88, 1.88, 1.88, 2.3 , 1.  , 1.  , 1.  ],
       [1.88, 1.98, 2.34, 2.49, 2.95, 1.28, 1.09, 1.  ],
       [1.88, 1.98, 2.34, 2.6 , 2.95, 1.29, 1.  , 1.  ],
       [1.88, 1.88, 1.88, 1.88, 1.88, 1.88, 1.88, 1.88]])
```

The thresholds 0.4, 0.5, 0.45, 0.5, 0.5, 0.5, corresponding to the algorithms Random Forest, Logistic Regression, K-Nearest Neighbors, Support Vector Classifier, Long-Short Term Memory, and Long-Short Term Memory Improved, respectively, yield the highest NORD.

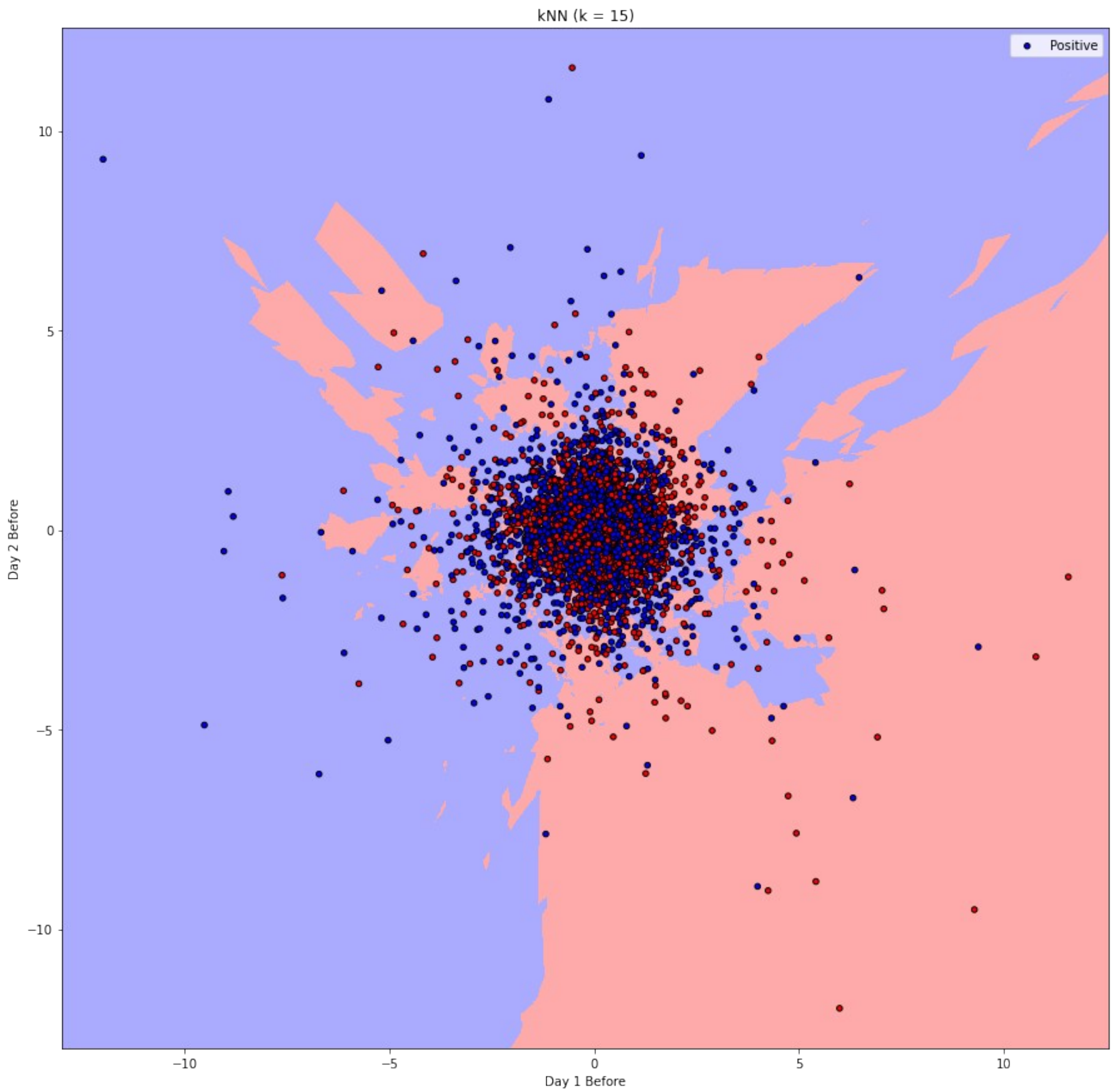
```
thresholds_D1B_D2B = [0.4, 0.5, 0.45, 0.5, 0.5, 0.5]

matrix_D1B_D2B_interval = interval_matrix(Features_D1B_D2B, Y,
interval1, interval2, interval3, sp500_2["2017-9-26":], 2,
thresholds_D1B_D2B, neighbors=6, n_estimators=200, max_depth=80)

Optimization terminated successfully.
         Current function value: 0.689169
         Iterations 4
33/33 [==============================] - 0s 974us/step
1/1 [==============================] - 0s 16ms/step
14/14 [==============================] - 0s 965us/step
33/33 [==============================] - 2s 2ms/step
1/1 [==============================] - 0s 17ms/step
14/14 [==============================] - 0s 2ms/step
```



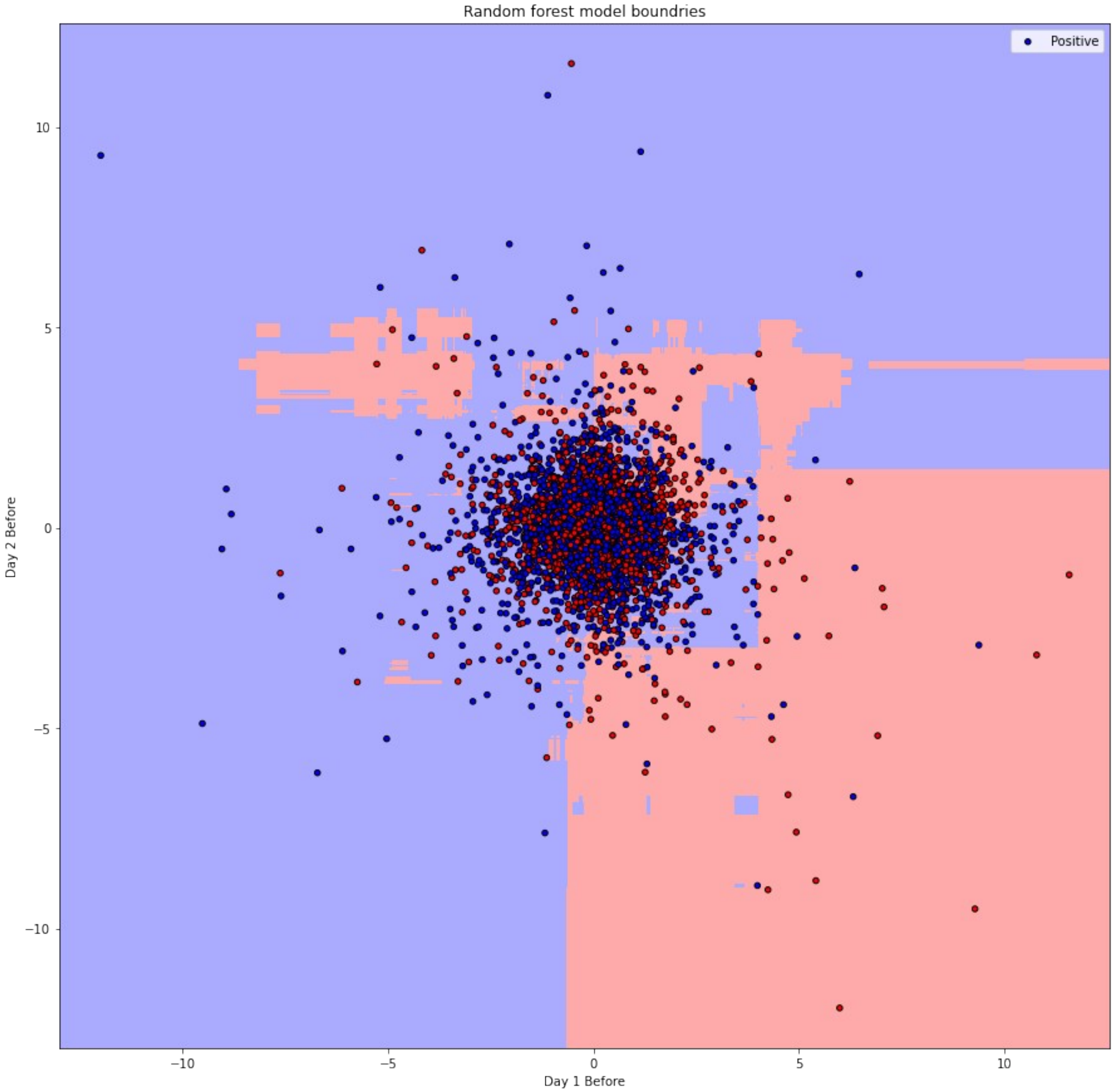
```
The code took 94.36026 seconds to run.

matrix_D1B_D2B_interval

array([[2.71, 0.93, 2.15],
       [2.66, 0.89, 2.24],
       [2.91, 1.05, 2.15],
       [2.3 , 0.82, 2.15],
       [3.01, 0.93, 2.41],
       [2.83, 0.89, 2.33],
       [1.88, 0.66, 2.09]])

# Calculating the difference in NORD between interval matrices to
identify which algorithms on
# specific intervals have shown improvement.
print(matrix_D1B_D2B_interval - matrix_D1B_interval)

[[ 0.53  0.04  0.18]
 [-0.15  0.   -0.09]
 [ 0.56  0.12  0.1 ]
 [ 0.18  0.08  0.06]
 [ 0.24  0.04  0.15]
 [-0.01  0.    0.02]
 [ 0.    0.    0.  ]]

# Visualizing the K-Nearest-Neighbors algorithm
plot_knn(Features_D1B_D2B, Y, n_neighbors=6)

C:\Users\sembm\anaconda3\lib\site-packages\sklearn\base.py:450:
UserWarning: X does not have valid feature names, but
KNeighborsClassifier was fitted with feature names
  warnings.warn(
```



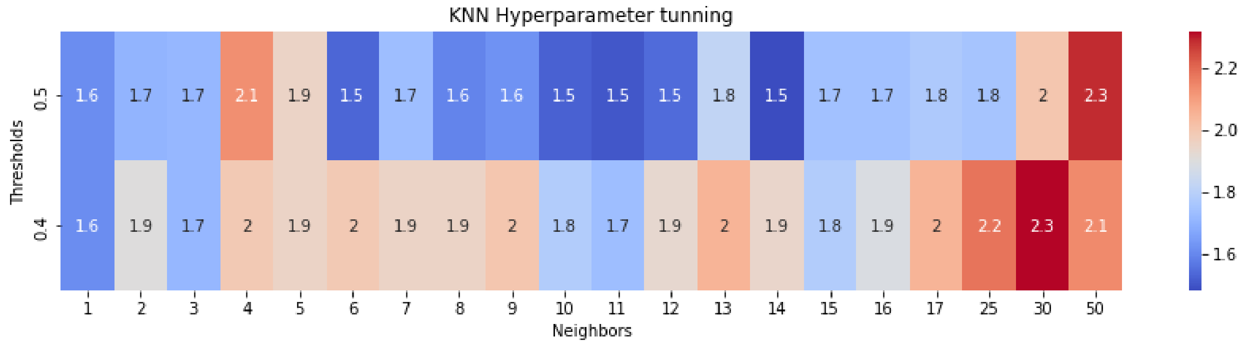

```
The code took 75.40009 seconds to run.

plt.savefig('plot_knn_neighbors_6.svg', format='svg')

<Figure size 432x288 with 0 Axes>

# Visualizing the K-Nearest-Neighbors algorithm with different number
of neighbors
plot_knn(Features_D1B_D2B, Y, n_neighbors=15)

C:\Users\sembm\anaconda3\lib\site-packages\sklearn\base.py:450:
UserWarning: X does not have valid feature names, but
KNeighborsClassifier was fitted with feature names
  warnings.warn(
```




```
The code took 99.15543 seconds to run.

plt.savefig('plot_knn_neighbors_15.svg', format='svg')

<Figure size 432x288 with 0 Axes>

# Visualizing the Random Forest algorithm
plot_random_forest(Features_D1B_D2B, Y)

C:\Users\sembm\anaconda3\lib\site-packages\sklearn\base.py:450:
UserWarning: X does not have valid feature names, but
RandomForestClassifier was fitted with feature names
  warnings.warn(
```



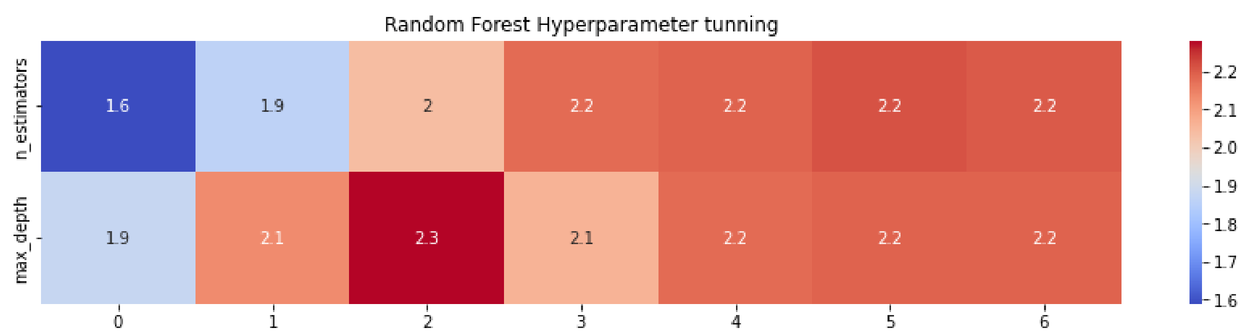

```
The code took 29.51022 seconds to run.

plt.savefig('plot_rf.svg', format='svg')

<Figure size 432x288 with 0 Axes>

# Selecting Day_1_before, Day_2_before, Day_3_before as our features
Features_D1B_D2B_D3B = sp500_2[['Day_1_before', 'Day_2_before',
'Day_3_before']]

# Selecting the right parameters for KNN
matrix_knn_parameters_percentage_changes_D1B_D2B_D3B =
knn_hyper(Features_D1B_D2B_D3B, Y, sp500_2['2017-9-26':])
```




```
The code took 40.70950 seconds to run.

matrix_knn_parameters_percentage_changes_D1B_D2B_D3B

array([[1.61, 1.7 , 1.71, 2.13, 1.95, 1.53, 1.73, 1.59, 1.62, 1.52,
1.5 ,
        1.54, 1.82, 1.48, 1.74, 1.74, 1.77, 1.75, 2.  , 2.29],
       [1.61, 1.9 , 1.71, 1.99, 1.95, 1.99, 1.95, 1.95, 2.  , 1.78,
1.73,
        1.93, 2.05, 1.94, 1.78, 1.89, 2.05, 2.18, 2.32, 2.14]])
```

The parameter combination of 30 neighbors with a threshold of 0.4 results in the highest NORD.

```
# Selecting the right parameters for RF
matrix_rf_parameters_percentage_changes_D1B_D2B_D3B =
hypertune_random_forest(Features_D1B_D2B_D3B, Y, sp500_2['2017-9-
26':])
```

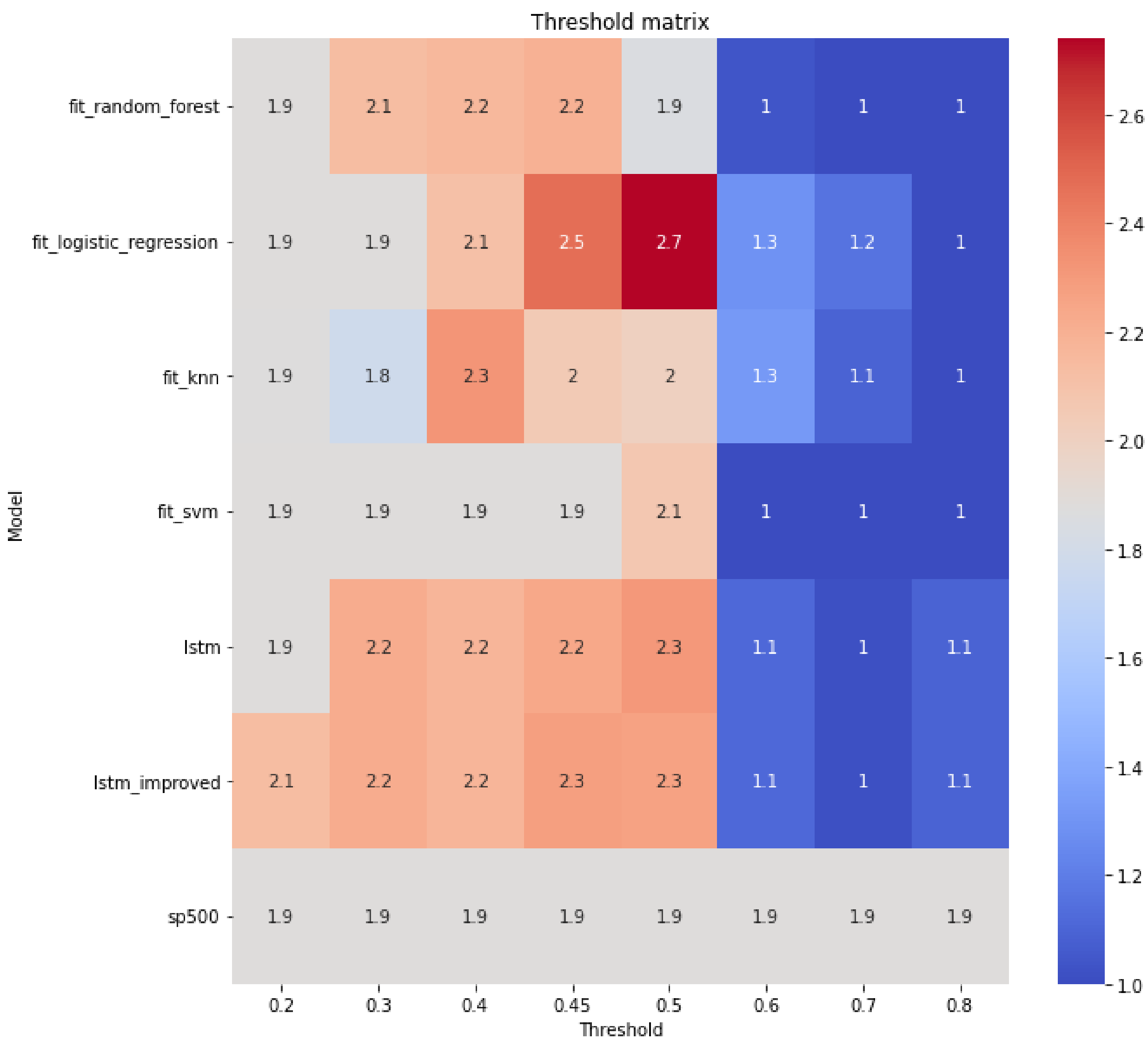



```
n_estimators: [3, 5, 10, 50, 100, 150, 200]
max_debths: [1, 3, 5, 10, 20, 40, 80]
The code took 23.74523 seconds to run.

matrix_rf_parameters_percentage_changes_D1B_D2B_D3B

array([[1.59, 1.86, 2.04, 2.18, 2.19, 2.21, 2.2 ],
       [1.88, 2.13, 2.28, 2.06, 2.18, 2.19, 2.19]])
```

A max depth of 10 and n_estimators of 200 result in the highest NORD.

```
matrix_D1B_D2B_D3B_threshold = treshold_matrix(Features_D1B_D2B_D3B,
Y, sp500_2["2017-9-26":], 3, n_steps=1, n_estimators=200,
max_depth=10, neighbors=30)
```

```
Optimization terminated successfully.
         Current function value: 0.689107
         Iterations 4
33/33 [==============================] - 1s 2ms/step
33/33 [==============================] - 1s 2ms/step
```




```
The code took 99.34682 seconds to run.
```

```
matrix_D1B_D2B_D3B_threshold
```

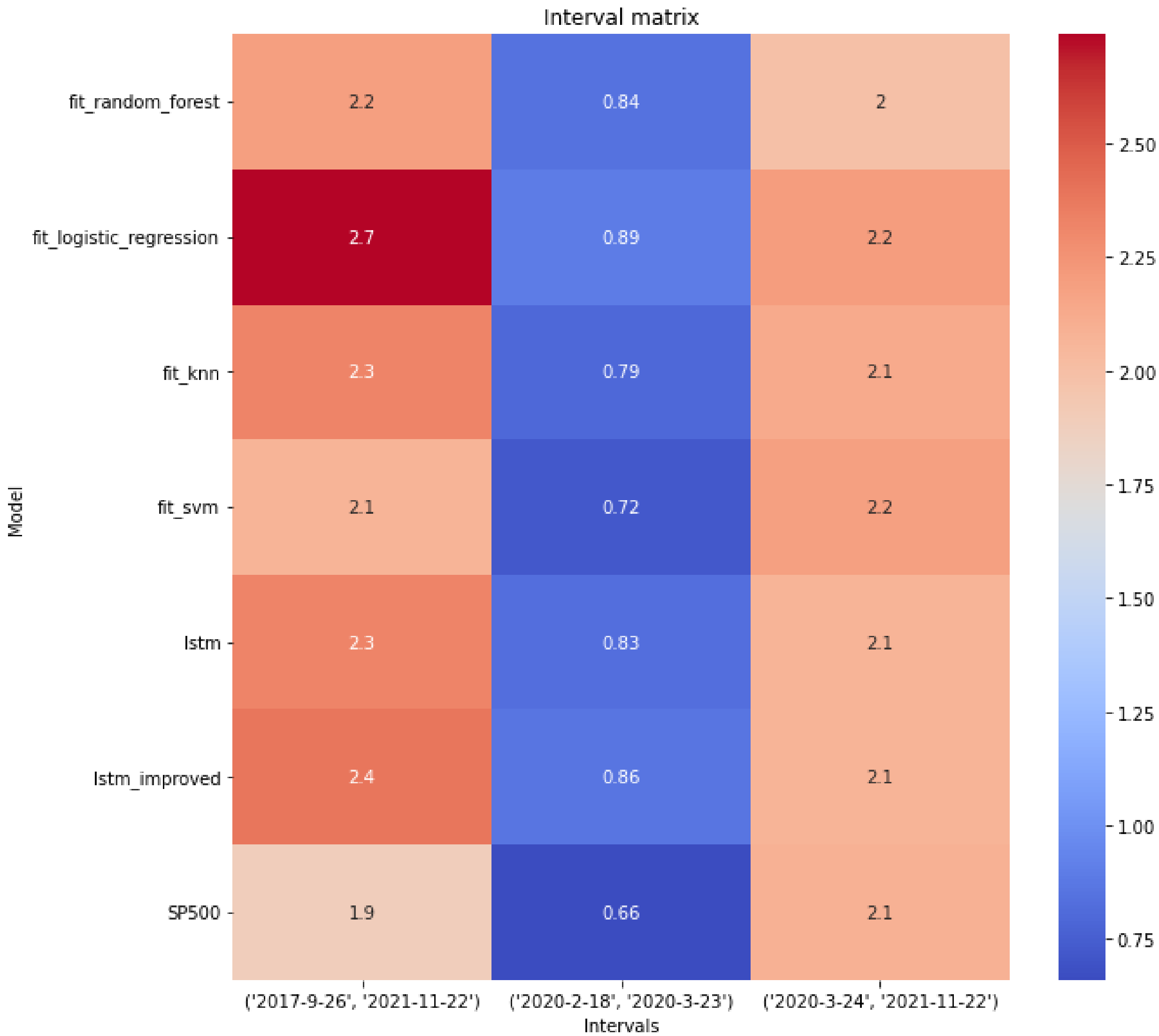
```
array([[1.88, 2.13, 2.16, 2.19, 1.85, 1.04, 1.  , 1.  ],
       [1.88, 1.88, 2.11, 2.47, 2.74, 1.29, 1.16, 1.  ],
```

```
       [1.87, 1.78, 2.32, 2.05, 2.  , 1.32, 1.09, 1.  ],
       [1.88, 1.88, 1.88, 1.88, 2.07, 1.  , 1.  , 1.  ],
       [1.88, 2.22, 2.17, 2.24, 2.31, 1.11, 1.02, 1.09],
       [2.13, 2.22, 2.17, 2.28, 2.27, 1.11, 1.02, 1.09],
       [1.88, 1.88, 1.88, 1.88, 1.88, 1.88, 1.88, 1.88]])
```

The thresholds 0.45, 0.5, 0.4, 0.5, 0.45, 0.45, corresponding to the algorithms Random Forest, Logistic Regression, K-Nearest Neighbors, Support Vector Classifier, Long-Short Term Memory, and Long-Short Term Memory Improved, respectively, yield the highest NORD.

```
thresholds_D1B_D2B_D3B = [0.45, 0.5, 0.4, 0.5, 0.45, 0.45]
matrix_D1B_D2B_D3B_interval = interval_matrix(Features_D1B_D2B_D3B, Y,
interval1, interval2, interval3, sp500_2["2017-9-26":], 3,
thresholds_D1B_D2B_D3B, neighbors=30, n_estimators=200, max_depth=10)
```

```
Optimization terminated successfully.
         Current function value: 0.689107
         Iterations 4
33/33 [==============================] - 1s 2ms/step
1/1 [==============================] - 0s 23ms/step
14/14 [==============================] - 0s 2ms/step
33/33 [==============================] - 3s 4ms/step
1/1 [==============================] - 0s 41ms/step
14/14 [==============================] - 0s 6ms/step
```

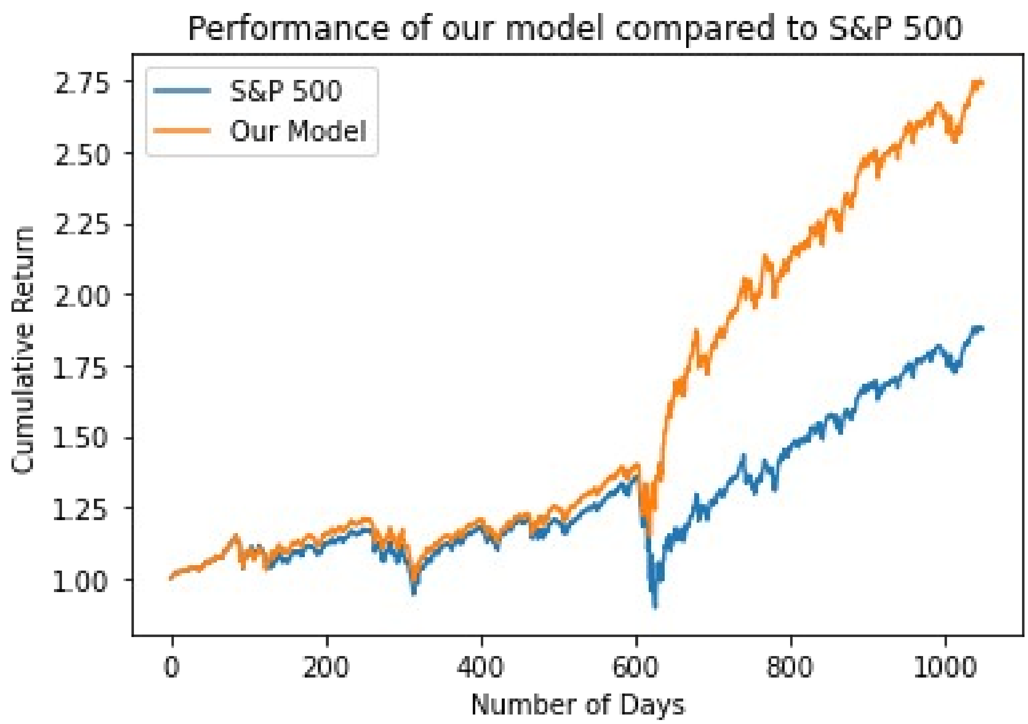



```
The code took 202.59032 seconds to run.

matrix_D1B_D2B_D3B_interval

array([[2.19, 0.84, 1.99],
       [2.74, 0.89, 2.2 ],
       [2.32, 0.79, 2.13],
       [2.07, 0.72, 2.18],
       [2.32, 0.83, 2.07],
       [2.39, 0.86, 2.07],
       [1.88, 0.66, 2.09]])

# Calculating the difference in NORD between interval matrices to
identify which algorithms on
# specific intervals have shown improvement.
print(matrix_D1B_D2B_D3B_interval - matrix_D1B_D2B_interval)
```

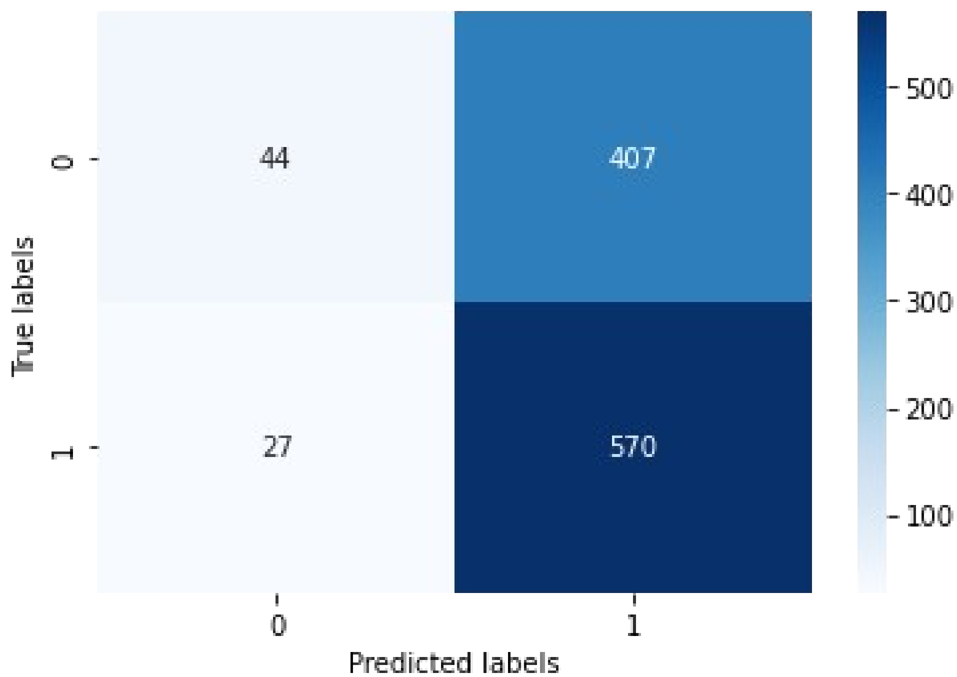
```
[[-0.52 -0.09 -0.16]
 [ 0.08  0.   -0.04]
 [-0.59 -0.26 -0.02]
 [-0.23 -0.1   0.03]
 [-0.69 -0.1  -0.34]
 [-0.44 -0.03 -0.26]
 [ 0.    0.    0.  ]]
```

```
# Generating predictions by using Logistic regression
accuracy_D1B_D2B_D3B, y_test_D1B_D2B_D3B, y_pred_class_D1B_D2B_D3B,
y_pred_prob_D1B_D2B_D3B =
fit_logistic_regression(Features_D1B_D2B_D3B, Y, threshold = 0.5)
val_return_D1B_D2B_D3B = investment_return(y_pred_class_D1B_D2B_D3B,
sp500["2017-9-26":'2021-11-22'])["dollars"].iloc[-1]
# Plotting the results into a graph
plot_investment_performance(y_pred_class_D1B_D2B_D3B, sp500["2017-9-
26":'2021-11-22'])
```

```
Optimization terminated successfully.
         Current function value: 0.689107
         Iterations 4
```



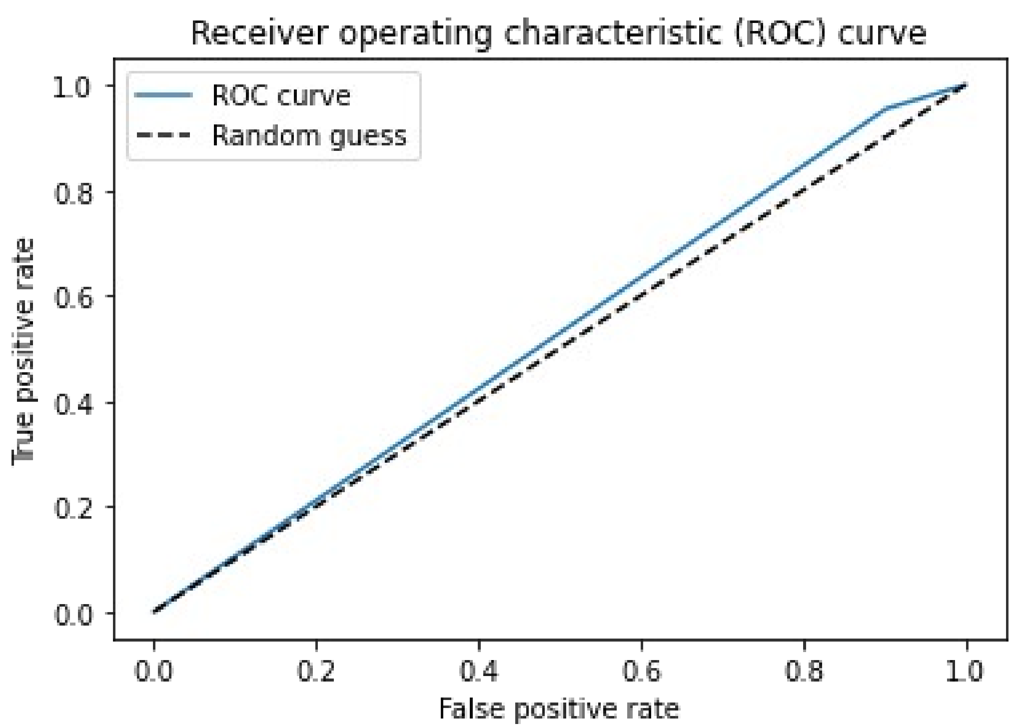

```
# Generating confusion matrix
c_matrix(y_test_D1B_D2B_D3B, y_pred_class_D1B_D2B_D3B)
```




```
plt.savefig('confusion_matrix.svg', format='svg')
```

<Figure size 432x288 with 0 Axes>

```
# Generating ROC curve
roc(y_test_D1B_D2B_D3B, y_pred_class_D1B_D2B_D3B)
```



```
plt.savefig('roc_curve.svg', format='svg')
```

```
<Figure size 432x288 with 0 Axes>
```

Summary:

Algorithms are trained on the interval from 30.1.2001, until 25.9.2017.

The NORD value on the validation set of a model using Logistic Regression as the algorithm, with features Day_1_before, Day_2_before, and Day_3_before, and trained on the interval from 30.1.2001, until 25.9.2017, is 2.73.

The NORD - BUY AND HOLD on the validation set is equal to 1.88.

Methodology A identifies Day_1_before, Day_4_before, and Day_5_before as important features.

Methodology B identifies the same features as important as Methodology A.

Upon applying common sense, we decide to include additional features to our model: Day_1_before, Day_2_before, and Day_3_before.

Our feature vector changes from an empty set to X=[Day_1_before, Day_2_before, Day_3_before].

Open, Close, High, Low

```
# Calculating Opening price a day ago, Highest price a day ago, Lowest
price a day ago
sp500['Open_day_before'] = sp500['Open'].shift(1)
sp500['High_day_before'] = sp500['Open'].shift(1)
sp500['Low_day_before'] = sp500['Open'].shift(1)

sp500['Open_pct_change'] = (sp500['Open'] - sp500['Open_day_before'])
/ sp500['Open_day_before'] * 100
sp500['High_pct_change'] = (sp500['High'] - sp500['High_day_before'])
/ sp500['High_day_before'] * 100
sp500['Low_pct_change'] = (sp500['Low'] - sp500['Low_day_before'])  /
sp500['Low_day_before'] * 100

sp500['Open_pct_change_Day_before'] =
sp500['Open_pct_change'].shift(1)
sp500['High_pct_change_Day_before'] =
sp500['High_pct_change'].shift(1)
sp500['Low_pct_change_Day_before'] = sp500['Low_pct_change'].shift(1)

sp500_2 = sp500.loc['2001-01-30':"2021-11-22"]

sp500_2
```

```
                       Open          High            Low          Close     Adj
Close  \
Date
```

```
2001-01-30  1364.170044  1375.680054  1356.199951  1373.729980
1373.729980
2001-01-31  1373.729980  1383.369995  1364.660034  1366.010010
1366.010010
2001-02-01  1366.010010  1373.500000  1359.339966  1373.469971
1373.469971
2001-02-02  1373.469971  1376.380005  1348.719971  1349.469971
1349.469971
2001-02-05  1349.469971  1354.560059  1344.479980  1354.310059
1354.310059
...                 ...          ...          ...          ...
...
2021-11-16  4679.419922  4714.950195  4679.419922  4700.899902
4700.899902
2021-11-17  4701.500000  4701.500000  4684.410156  4688.669922
4688.669922
2021-11-18  4700.720215  4708.799805  4672.779785  4704.540039
4704.540039
2021-11-19  4708.439941  4717.750000  4694.220215  4697.959961
4697.959961
2021-11-22  4712.000000  4743.830078  4682.169922  4682.939941
4682.939941

                Volume  Close day before  pct_change  Direction
Day_1_before  \
Date

2001-01-30  1149800000       1364.170044    0.700788          1
0.680475
2001-01-31  1295300000       1373.729980   -0.561971          0
0.700788
2001-02-01  1118800000       1366.010010    0.546113          1     -
0.561971
2001-02-02  1048400000       1373.469971   -1.747399          0
0.546113
2001-02-05  1013000000       1349.469971    0.358666          1     -
1.747399
...                ...               ...         ...        ...
...
2021-11-16  3972640000       4682.799805    0.386523          1     -
0.001074
2021-11-17  3969070000       4700.899902   -0.260163          0
0.386523
2021-11-18  4226410000       4688.669922    0.338478          1     -
0.260163
2021-11-19  4253180000       4704.540039   -0.139867          0
0.338478
2021-11-22  4441100000       4697.959961   -0.319714          0     -
0.139867
```

```
            ...  Day_9_before  Open_day_before  High_day_before  \
Date        ...
2001-01-30  ...      0.212561      1354.920044      1354.920044
2001-01-31  ...      1.391532      1364.170044      1364.170044
2001-02-01  ...     -0.402823      1373.729980      1373.729980
2001-02-02  ...      0.026814      1366.010010      1366.010010
2001-02-05  ...      1.303150      1373.469971      1373.469971
...         ...           ...              ...              ...
2021-11-16  ...      0.646128      4689.299805      4689.299805
2021-11-17  ...      0.418194      4679.419922      4679.419922
2021-11-18  ...      0.373280      4701.500000      4701.500000
2021-11-19  ...      0.088779      4700.720215      4700.720215
2021-11-22  ...     -0.349878      4708.439941      4708.439941

            Low_day_before  Open_pct_change  High_pct_change
Low_pct_change  \
Date

2001-01-30     1354.920044         0.682697         1.532194
0.094464
2001-01-31     1364.170044         0.700788         1.407446
0.035919
2001-02-01     1373.729980        -0.561971        -0.016741        -
1.047514
2001-02-02     1366.010010         0.546113         0.759145        -
1.265733
2001-02-05     1373.469971        -1.747399        -1.376798        -
2.110712
...                    ...              ...              ...
...
2021-11-16     4689.299805        -0.210690         0.546998        -
0.210690
2021-11-17     4679.419922         0.471855         0.471855
0.106642
2021-11-18     4701.500000        -0.016586         0.155265        -
0.610873
2021-11-19     4700.720215         0.164224         0.362280        -
0.138277
2021-11-22     4708.439941         0.075610         0.751632        -
0.557935

            Open_pct_change_Day_before  High_pct_change_Day_before  \
Date
2001-01-30                   -0.190788                    0.591526
2001-01-31                    0.682697                    1.532194
2001-02-01                    0.700788                    1.407446
2001-02-02                   -0.561971                   -0.016741
2001-02-05                    0.546113                    0.759145
...                                ...                         ...
```

```
2021-11-16                           0.731639                            0.906069
2021-11-17                          -0.210690                            0.546998
2021-11-18                           0.471855                            0.471855
2021-11-19                          -0.016586                            0.155265
2021-11-22                           0.164224                            0.362280

            Low_pct_change_Day_before
Date
2001-01-30                  -0.526701
2001-01-31                   0.094464
2001-02-01                   0.035919
2001-02-02                  -1.047514
2001-02-05                  -1.265733
...                               ...
2021-11-16                   0.378490
2021-11-17                  -0.210690
2021-11-18                   0.106642
2021-11-19                  -0.610873
2021-11-22                  -0.138277

[5238 rows x 27 columns]

# Applying methodology A
# Using methodology A for feature selection.
new_features_O_H_L = ['Open_pct_change_Day_before',
'High_pct_change_Day_before', 'Low_pct_change_Day_before']
# We have already decided to include features Day_1_before,
Day_2_before, and Day_3_before
# in our model; therefore, they will be fixed and present in each
iteration.
fixed_features = ['Day_1_before', 'Day_2_before', 'Day_3_before']
# The NORD value for Logistic Regression with features Day_1_before,
Day_2_before, and Day_3_before
# on the validation set is 2.73. Therefore, NORD_max = 2.73.
result_O_H_L = methodology_A(fixed_features, new_features_O_H_L,
sp500_2, 2.73)
result_O_H_L

<pandas.io.formats.style.Styler at 0x16100a628e0>
```

Methodology does not identify any of the new features as important because each option yields a lower NORD than the NORD of the feature combination Day_1_before, Day_2_before, Day_3_before.

```
# Round 1 of methodlogy B
features_round_1_O_H_L = [
    ['Day_1_before', 'Day_2_before', 'Day_3_before'],
    ['Day_1_before', 'Day_2_before', 'Day_3_before',
'Open_pct_change_Day_before'],
    ['Day_1_before', 'Day_2_before', 'Day_3_before',
```

```
'High_pct_change_Day_before'],
    ['Day_1_before', 'Day_2_before', 'Day_3_before',
'Low_pct_change_Day_before']
]
result_round_1_O_H_L = best_features(features_round_1_O_H_L, sp500_2)
result_round_1_O_H_L
```

```
<pandas.io.formats.style.Styler at 0x1610636d0d0>
```

Methodology B does not select any of the new features as important.

Summary:

Algorithms are trained on the interval from 30.1.2001, until 25.9.2017.

The NORD value on the validation set of a model using Logistic Regression as the algorithm, with features Day_1_before, Day_2_before, and Day_3_before, and trained on the interval from 30.1.2001, until 25.9.2017, is 2.73.

The NORD - BUY AND HOLD on the validation set is equal to 1.88.

Methodologies A and B do not identify any of the new features as important.

Upon applying common sense, we decide to not include any new features to our model.

Our feature vector does not change and remains the same: X=[Day_1_before, Day_2_before, Day_3_before].

Opening Price of the Targeted Day

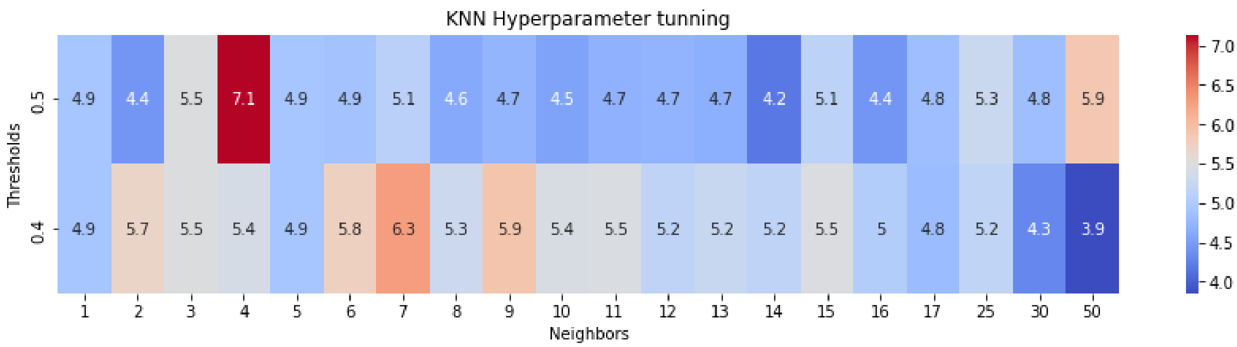
```
# Calculating the Percentage Change between the Open price and Close
price
sp500['Change_Open_Close'] = (sp500['Open'] - sp500['Close day
before']) / sp500['Close day before']*100
sp500_2 = sp500.loc['2001-01-30':"2021-11-22"]
sp500_2
```

```
                   Open         High          Low        Close     Adj
Close  \
Date

2001-01-30  1364.170044  1375.680054  1356.199951  1373.729980
1373.729980
2001-01-31  1373.729980  1383.369995  1364.660034  1366.010010
1366.010010
2001-02-01  1366.010010  1373.500000  1359.339966  1373.469971
1373.469971
2001-02-02  1373.469971  1376.380005  1348.719971  1349.469971
1349.469971
2001-02-05  1349.469971  1354.560059  1344.479980  1354.310059
1354.310059
```

```
...                    ...          ...          ...          ...
...
2021-11-16  4679.419922  4714.950195  4679.419922  4700.899902
4700.899902
2021-11-17  4701.500000  4701.500000  4684.410156  4688.669922
4688.669922
2021-11-18  4700.720215  4708.799805  4672.779785  4704.540039
4704.540039
2021-11-19  4708.439941  4717.750000  4694.220215  4697.959961
4697.959961
2021-11-22  4712.000000  4743.830078  4682.169922  4682.939941
4682.939941

                Volume  Close day before  pct_change  Direction
Day_1_before  \
Date

2001-01-30  1149800000       1364.170044    0.700788          1
0.680475
2001-01-31  1295300000       1373.729980   -0.561971          0
0.700788
2001-02-01  1118800000       1366.010010    0.546113          1     -
0.561971
2001-02-02  1048400000       1373.469971   -1.747399          0
0.546113
2001-02-05  1013000000       1349.469971    0.358666          1     -
1.747399
...                ...               ...         ...        ...
...
2021-11-16  3972640000       4682.799805    0.386523          1     -
0.001074
2021-11-17  3969070000       4700.899902   -0.260163          0
0.386523
2021-11-18  4226410000       4688.669922    0.338478          1     -
0.260163
2021-11-19  4253180000       4704.540039   -0.139867          0
0.338478
2021-11-22  4441100000       4697.959961   -0.319714          0     -
0.139867

            ...  Open_day_before  High_day_before  Low_day_before  \
Date        ...
2001-01-30  ...      1354.920044      1354.920044     1354.920044
2001-01-31  ...      1364.170044      1364.170044     1364.170044
2001-02-01  ...      1373.729980      1373.729980     1373.729980
2001-02-02  ...      1366.010010      1366.010010     1366.010010
2001-02-05  ...      1373.469971      1373.469971     1373.469971
...         ...              ...              ...             ...
2021-11-16  ...      4689.299805      4689.299805     4689.299805
2021-11-17  ...      4679.419922      4679.419922     4679.419922
```

```
2021-11-18  ...      4701.500000      4701.500000     4701.500000
2021-11-19  ...      4700.720215      4700.720215     4700.720215
2021-11-22  ...      4708.439941      4708.439941     4708.439941

            Open_pct_change  High_pct_change  Low_pct_change  \
Date
2001-01-30         0.682697         1.532194        0.094464
2001-01-31         0.700788         1.407446        0.035919
2001-02-01        -0.561971        -0.016741       -1.047514
2001-02-02         0.546113         0.759145       -1.265733
2001-02-05        -1.747399        -1.376798       -2.110712
...                     ...              ...             ...
2021-11-16        -0.210690         0.546998       -0.210690
2021-11-17         0.471855         0.471855        0.106642
2021-11-18        -0.016586         0.155265       -0.610873
2021-11-19         0.164224         0.362280       -0.138277
2021-11-22         0.075610         0.751632       -0.557935

            Open_pct_change_Day_before  High_pct_change_Day_before  \
Date
2001-01-30                   -0.190788                    0.591526
2001-01-31                    0.682697                    1.532194
2001-02-01                    0.700788                    1.407446
2001-02-02                   -0.561971                   -0.016741
2001-02-05                    0.546113                    0.759145
...                                ...                         ...
2021-11-16                    0.731639                    0.906069
2021-11-17                   -0.210690                    0.546998
2021-11-18                    0.471855                    0.471855
2021-11-19                   -0.016586                    0.155265
2021-11-22                    0.164224                    0.362280

            Low_pct_change_Day_before  Change_Open_Close
Date
2001-01-30                  -0.526701           0.000000
2001-01-31                   0.094464           0.000000
2001-02-01                   0.035919           0.000000
2001-02-02                  -1.047514           0.000000
2001-02-05                  -1.265733           0.000000
...                               ...                ...
2021-11-16                   0.378490          -0.072177
2021-11-17                  -0.210690           0.012766
2021-11-18                   0.106642           0.257009
2021-11-19                  -0.610873           0.082897
2021-11-22                  -0.138277           0.298854

[5238 rows x 28 columns]
```

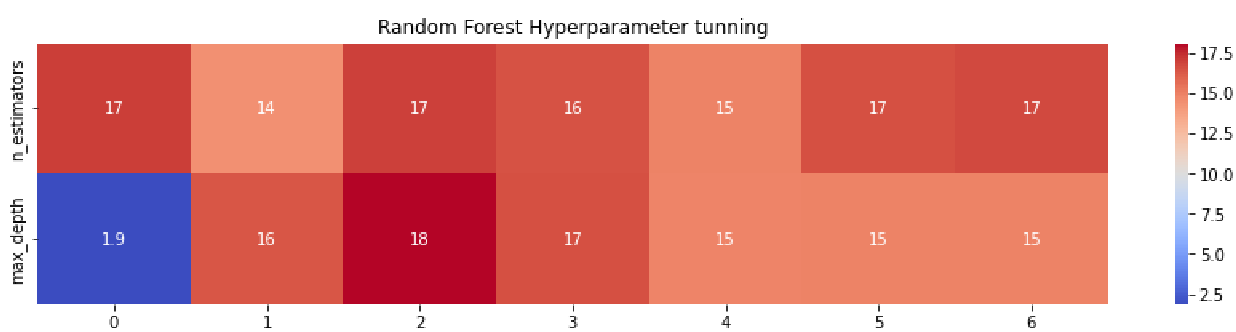
```
# Selecting features
Features_D1B_D2B_D3B_COC = sp500_2[['Day_1_before', 'Day_2_before',
'Day_3_before', 'Change_Open_Close']]

# Selecting the right parameters for KNN
matrix_knn_parameters_COC = knn_hyper(Features_D1B_D2B_D3B_COC, Y,
sp500_2['2017-9-26':])
```




```
The code took 41.40019 seconds to run.

matrix_knn_parameters_COC

array([[4.92, 4.44, 5.48, 7.14, 4.92, 4.88, 5.1 , 4.61, 4.73, 4.55,
4.69,
        4.72, 4.68, 4.19, 5.13, 4.43, 4.83, 5.27, 4.83, 5.87],
       [4.92, 5.68, 5.48, 5.4 , 4.92, 5.75, 6.29, 5.28, 5.9 , 5.42,
5.45,
        5.16, 5.24, 5.15, 5.46, 5.04, 4.81, 5.17, 4.31, 3.85]])
```

The parameter combination of 4 neighbors with a threshold of 0.5 results in the highest NORD.

```
# Selecting the right parameters for RF
matrix_rf_parameters_COC =
hypertune_random_forest(Features_D1B_D2B_D3B_COC, Y, sp500_2['2017-9-
26':])
```



```
n_estimators: [3, 5, 10, 50, 100, 150, 200]
max_debths: [1, 3, 5, 10, 20, 40, 80]
The code took 24.46156 seconds to run.
```

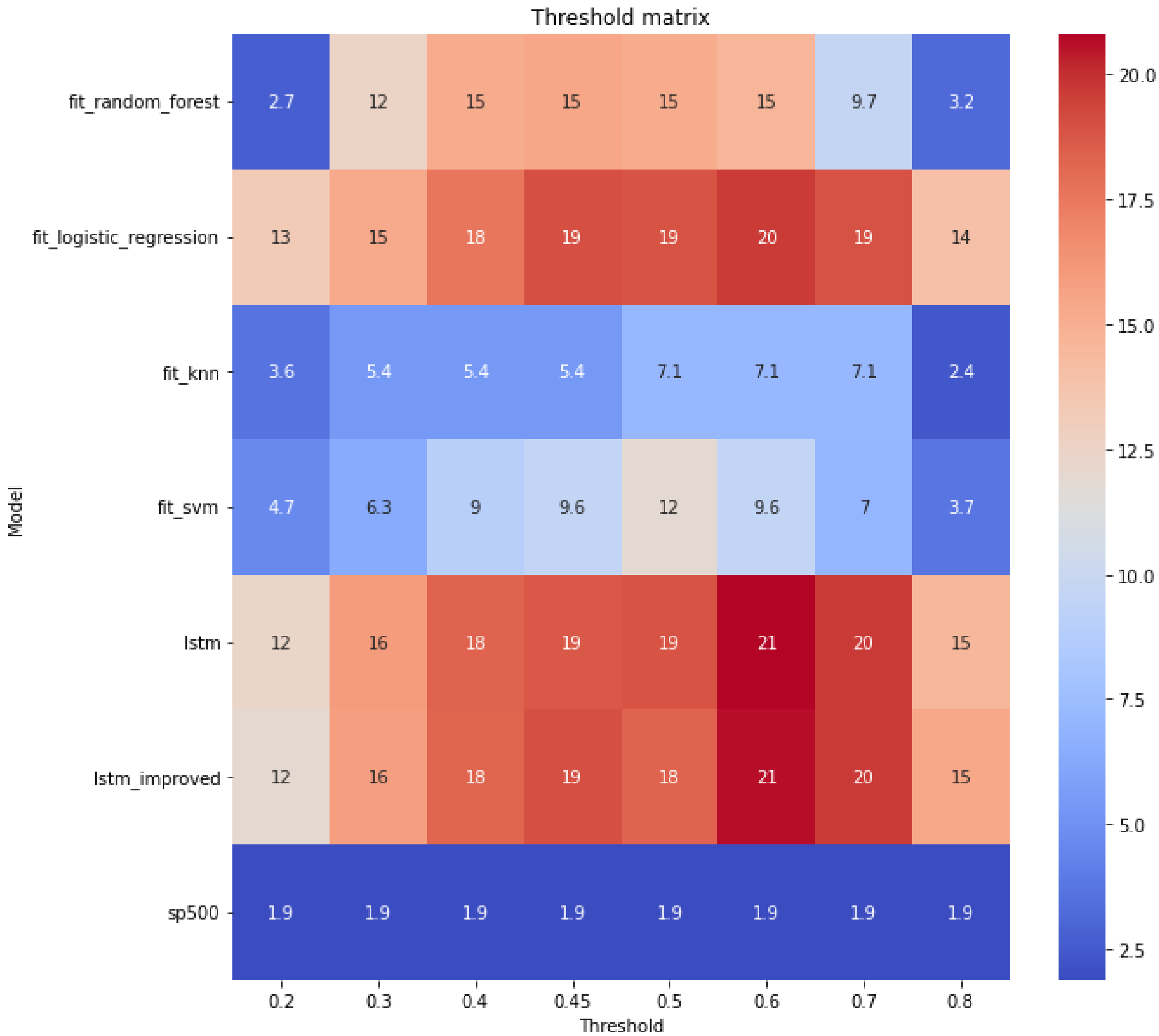
```
matrix_rf_parameters_COC
```

```
array([[17.05, 14.41, 16.99, 16.5 , 14.9 , 16.52, 16.71],
       [ 1.88, 16.33, 18.09, 16.57, 14.84, 14.9 , 14.9 ]])
```

A max depth of 5 and n_estimators of 3 result in the highest NORD.

```
matrix_D1B_D2B_D3B_COC_threshold =
treshold_matrix(Features_D1B_D2B_D3B_COC, Y, sp500_2["2017-9-26":], 4,
n_steps=1, n_estimators=3, max_depth=5, neighbors=4)
```

```
Optimization terminated successfully.
         Current function value: 0.646107
         Iterations 6
33/33 [==============================] - 1s 1ms/step
33/33 [==============================] - 1s 2ms/step
```

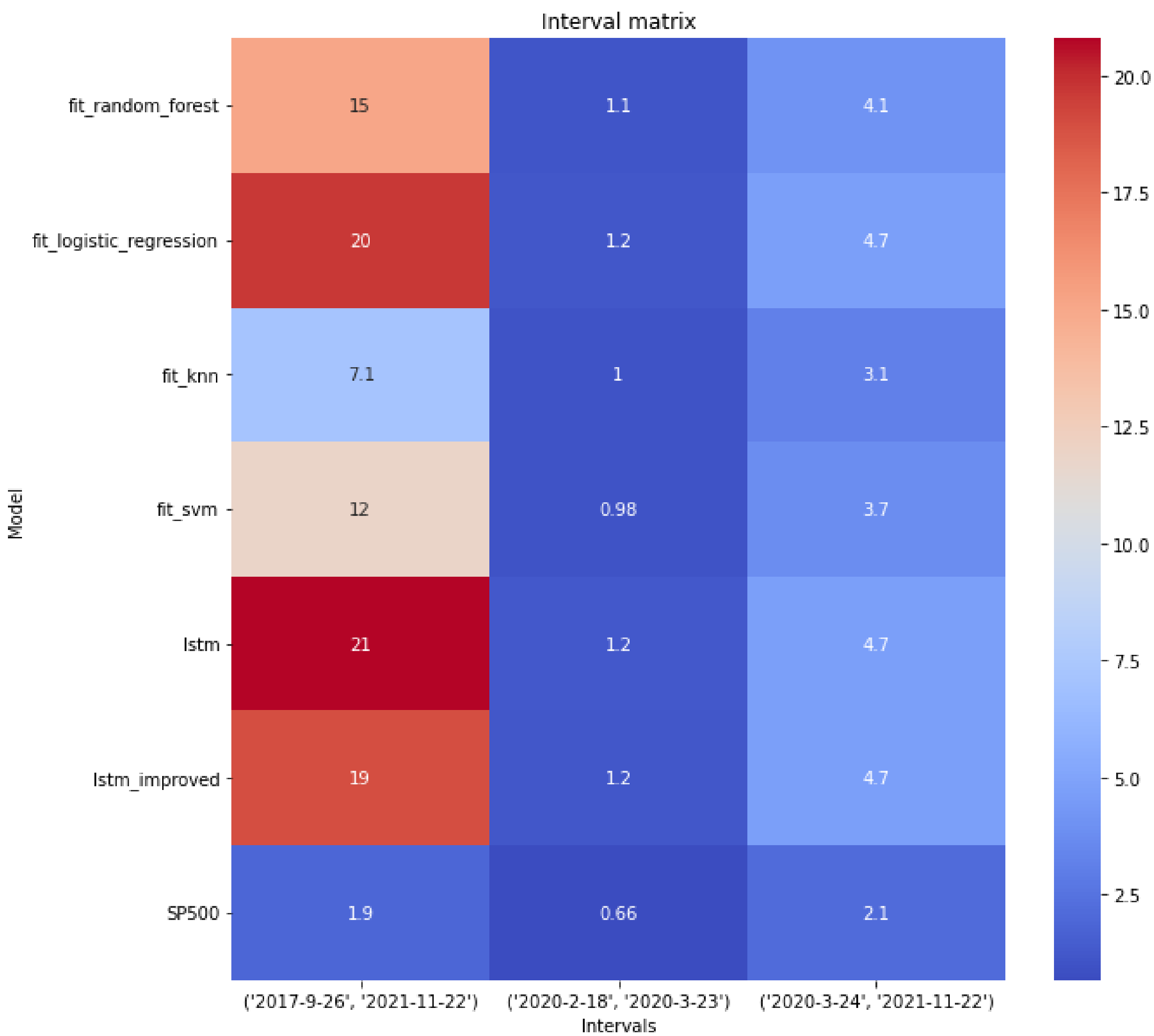



```
The code took 95.44896 seconds to run.

matrix_D1B_D2B_D3B_COC_threshold

array([[ 2.69, 12.5 , 15.13, 15.45, 15.12, 14.66,  9.68,  3.16],
       [13.26, 15.33, 17.59, 19.  , 18.82, 19.65, 18.9 , 13.98],
       [ 3.61,  5.4 ,  5.4 ,  5.4 ,  7.14,  7.14,  7.14,  2.37],
       [ 4.68,  6.33,  8.96,  9.63, 11.86,  9.59,  6.99,  3.67],
       [12.32, 15.96, 18.31, 18.7 , 18.83, 20.8 , 19.68, 14.58],
       [12.  , 15.79, 18.23, 19.  , 18.3 , 20.61, 19.67, 15.35],
       [ 1.88,  1.88,  1.88,  1.88,  1.88,  1.88,  1.88,  1.88]])
```

The thresholds 0.5, 0.6, 0.6, 0.5, 0.6, 0.6, corresponding to the algorithms Random Forest, Logistic Regression, K-Nearest Neighbors, Support Vector Classifier, Long-Short Term Memory, and Long-Short Term Memory Improved, respectively, yield the highest NORD.

```
thresholds_D1B_D2B_D3B_COC = [0.5, 0.6, 0.6, 0.5, 0.6, 0.6]

matrix_D1B_D2B_D3B_COC_interval =
interval_matrix(Features_D1B_D2B_D3B_COC, Y, interval1, interval2,
interval3, sp500_2["2017-9-26":], 4, thresholds_D1B_D2B_D3B_COC,
neighbors=4, n_estimators=3, max_depth=5)
```

```
Optimization terminated successfully.
         Current function value: 0.646107
         Iterations 6
33/33 [==============================] - 1s 2ms/step
1/1 [==============================] - 0s 25ms/step
14/14 [==============================] - 0s 2ms/step
33/33 [==============================] - 3s 4ms/step
1/1 [==============================] - 0s 40ms/step
14/14 [==============================] - 0s 4ms/step
```



```
The code took 195.62787 seconds to run.
```

```
matrix_D1B_D2B_D3B_COC_interval
```

```
array([[15.12,  1.08,  4.09],
       [19.65,  1.2 ,  4.74],
       [ 7.14,  1.04,  3.14],
       [11.86,  0.98,  3.73],
       [20.8 ,  1.23,  4.71],
       [18.93,  1.2 ,  4.71],
       [ 1.88,  0.66,  2.09]])
```

```
# Dropping unnecessary columns
sp500 = sp500.drop(columns=['Change_Open_Close'])
```

Summary:

Algorithms are trained on the interval from 30.1.2001, until 25.9.2017.

The NORD value on the validation set of a model using Logistic Regression as the algorithm, with features Day_1_before, Day_2_before, and Day_3_before, and trained on the interval from 30.1.2001, until 25.9.2017, is 2.73.

The NORD - BUY AND HOLD on the validation set is equal to 1.88.

The feature Change_Open_Close significantly improved the NORD across all algorithms. However, there is a logical error. Incorporating this feature into our model would allow our model to 'see' the future. Therefore, we will not include this feature in our model.

Our feature vector does not change and remains the same: X=[Day_1_before, Day_2_before, Day_3_before].

Volume

```
# Calculating percentage change in volume one day before
sp500['Volume'] = sp500['Volume'] / 1000000
sp500['Volume day before'] = sp500['Volume'].shift(1)
sp500['Change_volume'] = ((sp500['Volume'] - sp500['Volume day
before']) / sp500['Volume day before']*100).shift(1)
sp500_2 = sp500.loc['2001-01-30':"2021-11-22"]
sp500_2
```

```
                   Open         High          Low        Close     Adj
Close  \
Date

2001-01-30  1364.170044  1375.680054  1356.199951  1373.729980
1373.729980
2001-01-31  1373.729980  1383.369995  1364.660034  1366.010010
1366.010010
2001-02-01  1366.010010  1373.500000  1359.339966  1373.469971
```

```
1373.469971
2001-02-02  1373.469971  1376.380005  1348.719971  1349.469971
1349.469971
2001-02-05  1349.469971  1354.560059  1344.479980  1354.310059
1354.310059
...                 ...          ...          ...          ...
...
2021-11-16  4679.419922  4714.950195  4679.419922  4700.899902
4700.899902
2021-11-17  4701.500000  4701.500000  4684.410156  4688.669922
4688.669922
2021-11-18  4700.720215  4708.799805  4672.779785  4704.540039
4704.540039
2021-11-19  4708.439941  4717.750000  4694.220215  4697.959961
4697.959961
2021-11-22  4712.000000  4743.830078  4682.169922  4682.939941
4682.939941

             Volume  Close day before  pct_change  Direction
Day_1_before  \
Date

2001-01-30  1149.80       1364.170044    0.700788          1
0.680475
2001-01-31  1295.30       1373.729980   -0.561971          0
0.700788
2001-02-01  1118.80       1366.010010    0.546113          1     -
0.561971
2001-02-02  1048.40       1373.469971   -1.747399          0
0.546113
2001-02-05  1013.00       1349.469971    0.358666          1     -
1.747399
...             ...               ...         ...        ...
...
2021-11-16  3972.64       4682.799805    0.386523          1     -
0.001074
2021-11-17  3969.07       4700.899902   -0.260163          0
0.386523
2021-11-18  4226.41       4688.669922    0.338478          1     -
0.260163
2021-11-19  4253.18       4704.540039   -0.139867          0
0.338478
2021-11-22  4441.10       4697.959961   -0.319714          0     -
0.139867

            ...  High_day_before  Low_day_before  Open_pct_change  \
Date        ...
2001-01-30  ...      1354.920044     1354.920044         0.682697
2001-01-31  ...      1364.170044     1364.170044         0.700788
2001-02-01  ...      1373.729980     1373.729980        -0.561971
```

```
2001-02-02  ...      1366.010010     1366.010010         0.546113
2001-02-05  ...      1373.469971     1373.469971        -1.747399
...         ...              ...             ...              ...
2021-11-16  ...      4689.299805     4689.299805        -0.210690
2021-11-17  ...      4679.419922     4679.419922         0.471855
2021-11-18  ...      4701.500000     4701.500000        -0.016586
2021-11-19  ...      4700.720215     4700.720215         0.164224
2021-11-22  ...      4708.439941     4708.439941         0.075610

            High_pct_change  Low_pct_change
Open_pct_change_Day_before  \
Date

2001-01-30         1.532194        0.094464                    -
0.190788
2001-01-31         1.407446        0.035919
0.682697
2001-02-01        -0.016741       -1.047514
0.700788
2001-02-02         0.759145       -1.265733                    -
0.561971
2001-02-05        -1.376798       -2.110712
0.546113
...                     ...             ...                         ..
.
2021-11-16         0.546998       -0.210690
0.731639
2021-11-17         0.471855        0.106642                    -
0.210690
2021-11-18         0.155265       -0.610873
0.471855
2021-11-19         0.362280       -0.138277                    -
0.016586
2021-11-22         0.751632       -0.557935
0.164224

            High_pct_change_Day_before  Low_pct_change_Day_before  \
Date
2001-01-30                    0.591526                  -0.526701
2001-01-31                    1.532194                   0.094464
2001-02-01                    1.407446                   0.035919
2001-02-02                   -0.016741                  -1.047514
2001-02-05                    0.759145                  -1.265733
...                                ...                        ...
2021-11-16                    0.906069                   0.378490
2021-11-17                    0.546998                  -0.210690
2021-11-18                    0.471855                   0.106642
2021-11-19                    0.155265                  -0.610873
2021-11-22                    0.362280                  -0.138277
```

```
              Volume day before  Change_volume
Date
2001-01-30             1053.10      -4.089253
2001-01-31             1149.80       9.182414
2001-02-01             1295.30      12.654375
2001-02-02             1118.80     -13.626187
2001-02-05             1048.40      -6.292456
...                        ...            ...
2021-11-16             3488.41      -6.441828
2021-11-17             3972.64      13.881109
2021-11-18             3969.07      -0.089865
2021-11-19             4226.41       6.483635
2021-11-22             4253.18       0.633398

[5238 rows x 29 columns]
```

```
# Methodology A
features_V_CV = ['Volume day before', 'Change_volume']
fixed_features = ['Day_1_before', 'Day_2_before', 'Day_3_before']
result_V_CV = methodology_A(fixed_features, features_V_CV, sp500_2,
2.73)
result_V_CV

<pandas.io.formats.style.Styler at 0x16131c283d0>
```

Methodology does not identify any of the new features as important because each option yields a lower NORD than the NORD of the feature combination Day_1_before, Day_2_before, Day_3_before. The NORD value of the feature combination Day_1_before, Day_2_before, Day_3_before is 2.73.

```
# Round 1 of methodology B
Features_round_1_V_CV = [
    ['Day_1_before', 'Day_2_before', 'Day_3_before'],
    ['Day_1_before', 'Day_2_before', 'Day_3_before', 'Volume'],
    ['Day_1_before', 'Day_2_before', 'Day_3_before', 'Change_volume'],
]
result_round_1_V_CV = best_features(Features_round_1_V_CV, sp500_2)
result_round_1_V_CV

<pandas.io.formats.style.Styler at 0x16131d2ab20>
```

Methodologies A does not deem any of the new features important.

VIX

```
# Downloading data of VIX index
yahoo_financials = YahooFinancials('^VIX')
hist_market_data = yahoo_financials.get_historical_price_data('2001-
01-18', '2024-1-24', 'daily')
prices_data = hist_market_data['^VIX']['prices']
```

```
df_VIX = pd.DataFrame(prices_data)
df_VIX['formatted_date'] = pd.to_datetime(df_VIX['formatted_date'])
df_VIX.set_index('formatted_date', inplace=True)
df_VIX

                      date       high        low       open      close  \
formatted_date

2001-01-18       979804800  24.530001  23.180000  24.480000  23.370001

2001-01-19       979891200  24.080000  23.000000  23.620001  23.240000

2001-01-22       980150400  24.309999  23.240000  24.270000  23.250000

2001-01-23       980236800  23.150000  21.469999  23.000000  21.570000

2001-01-24       980323200  22.410000  21.870001  21.980000  22.030001

...                    ...        ...        ...        ...        ...

2024-01-17      1705478400  15.400000  14.380000  14.590000  14.790000

2024-01-18      1705564800  14.890000  13.890000  14.850000  14.130000

2024-01-19      1705651200  14.580000  13.280000  13.800000  13.300000

2024-01-22      1705910400  13.840000  13.170000  13.770000  13.190000

2024-01-23      1705996800  13.290000  12.530000  13.200000  12.550000


                volume   adjclose
formatted_date
2001-01-18         0.0  23.370001
2001-01-19         0.0  23.240000
2001-01-22         0.0  23.250000
2001-01-23         0.0  21.570000
2001-01-24         0.0  22.030001
...                ...        ...
2024-01-17         0.0  14.790000
2024-01-18         0.0  14.130000
2024-01-19         0.0  13.300000
2024-01-22         0.0  13.190000
2024-01-23         0.0  12.550000

[6004 rows x 7 columns]

# Mergind dataframe df_VIX and sp500 into single dataframe
merged_df_VIX = sp500.merge(df_VIX, how='inner', left_index=True,
right_index=True, suffixes=('_SP500', '_VIX'))
```

```python
merged_df_VIX['VIX_Close_day_before'] =
merged_df_VIX['close'].shift(1)
merged_df_VIX['VIX_change'] = (((merged_df_VIX['close'] -
merged_df_VIX['VIX_Close_day_before']) /
merged_df_VIX['VIX_Close_day_before'])*100).shift(1)
merged_df_VIX
```

```
                   Open         High          Low        Close    Adj
Close  \
2001-01-18  1329.890015  1352.709961  1327.410034  1347.969971
1347.969971
2001-01-19  1347.969971  1354.550049  1336.739990  1342.540039
1342.540039
2001-01-22  1342.540039  1353.619995  1333.839966  1342.900024
1342.900024
2001-01-23  1342.900024  1362.900024  1339.630005  1360.400024
1360.400024
2001-01-24  1360.400024  1369.750000  1357.280029  1364.300049
1364.300049
...                 ...          ...          ...          ...
...
2024-01-17  4739.129883  4744.229980  4714.819824  4739.209961
4739.209961
2024-01-18  4760.100098  4785.790039  4740.569824  4780.939941
4780.939941
2024-01-19  4796.279785  4842.069824  4785.870117  4839.810059
4839.810059
2024-01-22  4853.419922  4868.410156  4844.049805  4850.430176
4850.430176
2024-01-23  4856.799805  4866.479980  4844.370117  4864.600098
4864.600098

             Volume  Close day before  pct_change  Direction
Day_1_before  \
2001-01-18  1445.00       1329.469971    1.391532          1
0.212561
2001-01-19  1407.80       1347.969971   -0.402823          0
1.391532
2001-01-22  1164.00       1342.540039    0.026814          1     -
0.402823
2001-01-23  1232.60       1342.900024    1.303150          1
0.026814
2001-01-24  1309.00       1360.400024    0.286682          1
1.303150
...             ...               ...         ...        ...
...
2024-01-17  3928.60       4765.979980   -0.561690          0     -
0.373134
2024-01-18  4019.00       4739.209961    0.880526          1     -
0.561690
```

```
2024-01-19  4287.20       4780.939941    1.231350          1
0.880526
2024-01-22  4297.61       4839.810059    0.219433          1
1.231350
2024-01-23  3912.80       4850.430176    0.292137          1
0.219433

            ...  Change_volume        date       high        low
open  \
2001-01-18  ...      11.893506   979804800  24.530001  23.180000
24.480000
2001-01-19  ...       7.108443   979891200  24.080000  23.000000
23.620001
2001-01-22  ...      -2.574394   980150400  24.309999  23.240000
24.270000
2001-01-23  ...     -17.317801   980236800  23.150000  21.469999
23.000000
2001-01-24  ...       5.893471   980323200  22.410000  21.870001
21.980000
...         ...            ...         ...        ...        ...
...
2024-01-17  ...      22.206956  1705478400  15.400000  14.380000
14.590000
2024-01-18  ...      -7.791248  1705564800  14.890000  13.890000
14.850000
2024-01-19  ...       2.301074  1705651200  14.580000  13.280000
13.800000
2024-01-22  ...       6.673302  1705910400  13.840000  13.170000
13.770000
2024-01-23  ...       0.242816  1705996800  13.290000  12.530000
13.200000

                close  volume   adjclose  VIX_Close_day_before
VIX_change
2001-01-18  23.370001     0.0  23.370001                   NaN
NaN
2001-01-19  23.240000     0.0  23.240000             23.370001
NaN
2001-01-22  23.250000     0.0  23.250000             23.240000   -
0.556273
2001-01-23  21.570000     0.0  21.570000             23.250000
0.043030
2001-01-24  22.030001     0.0  22.030001             21.570000   -
7.225808
...               ...     ...        ...                   ...
...
2024-01-17  14.790000     0.0  14.790000             13.840000
8.976381
2024-01-18  14.130000     0.0  14.130000             14.790000
```

```
6.864160
2024-01-19  13.300000     0.0  13.300000             14.130000   -
4.462474
2024-01-22  13.190000     0.0  13.190000             13.300000   -
5.874026
2024-01-23  12.550000     0.0  12.550000             13.190000   -
0.827072

[5789 rows x 38 columns]

merged_df_2_VIX = merged_df_VIX['2001-01-30':"2021-11-23"]
# Applying methodology A and B
features_round_1_VIX = [
    ['Day_1_before', 'Day_2_before', 'Day_3_before'],
    ['Day_1_before', 'Day_2_before', 'Day_3_before', 'VIX_change']
]
result_round_1_VIX = best_features(features_round_1_VIX,
merged_df_2_VIX)
result_round_1_VIX

<pandas.io.formats.style.Styler at 0x1610636d400>
```

Summary:

Algorithms are trained on the interval from 30.1.2001, until 25.9.2017.

The NORD value on the validation set of a model using Logistic Regression as the algorithm, with features Day_1_before, Day_2_before, and Day_3_before, and trained on the interval from 30.1.2001, until 25.9.2017, is 2.73.

The NORD - BUY AND HOLD on the validation set is equal to 1.88.

Methodologies A and B do not identify the VIX_change as important.

Upon applying common sense, we decide to not include VIX_change to our model.

Our feature vector does not change and remains the same: X=[Day_1_before, Day_2_before, Day_3_before].

Currency exchange rates

```
# Defining the currencies we are interested in
currencies = ['EURUSD=X', 'GBPUSD=X', 'JPYUSD=X', 'CHFUSD=X',
'CNYUSD=X', 'ZARUSD=X', 'BRLUSD=X']
currencies_df = yf.download(currencies, start='2001-01-18', end='2024-
1-24')['Close']
# Renaming columns for clarity
currencies_df.columns = ['EUR', 'GBP', 'JPY', 'CHF', 'CNY', 'ZAR',
'BRL']
# Changing the index format to 'YYYY-MM-DD'
currencies_df.index = currencies_df.index.strftime('%Y-%m-%d')
```

```
# Converting index to datetime
currencies_df.index = pd.to_datetime(currencies_df.index)
currencies_df

[*********************100%***********************]  7 of 7 completed

                 EUR       GBP       JPY       CHF       CNY       ZAR  \
Date

2001-01-18       NaN       NaN       NaN       NaN       NaN  0.008482

2001-01-19       NaN       NaN       NaN       NaN       NaN  0.008538

2001-01-22       NaN       NaN       NaN       NaN       NaN  0.008587

2001-01-23       NaN       NaN       NaN       NaN       NaN  0.008576

2001-01-24       NaN       NaN       NaN       NaN       NaN  0.008482

...              ...       ...       ...       ...       ...       ...

2024-01-17  0.203050  1.161036  0.140519  1.087879  1.263855  0.006791

2024-01-18  0.202675  1.156976  0.140558  1.088578  1.267684  0.006751

2024-01-19  0.203000  1.151968  0.140548  1.087914  1.270987  0.006751

2024-01-22  0.203888  1.151437  0.140704  1.089230  1.269986  0.006751

2024-01-23  0.200469  1.150589  0.139030  1.088021  1.270696  0.006750


                 BRL
Date
2001-01-18       NaN
2001-01-19       NaN
2001-01-22       NaN
2001-01-23       NaN
2001-01-24       NaN
...              ...
2024-01-17  0.052745
2024-01-18  0.052413
2024-01-19  0.052851
2024-01-22  0.052607
2024-01-23  0.052102

[6000 rows x 7 columns]

# Identifying the earliest date with non-NaN values for each currency.
first_non_nan_dates = currencies_df.apply(lambda col:
```

```
col.dropna().index.min())
first_non_nan_dates

EUR   2003-12-01
GBP   2003-09-17
JPY   2001-06-25
CHF   2003-12-01
CNY   2003-12-01
ZAR   2001-01-18
BRL   2003-12-01
dtype: datetime64[ns]

# Eliminating NaN values from the dataset.
currencies_df_2 = currencies_df['2003-12-01':]
currencies_df_2

                 EUR       GBP       JPY       CHF       CNY       ZAR
\
Date

2003-12-01  0.342114  0.770357  0.120960  1.196501  1.718597  0.009160

2003-12-02  0.341180  0.776880  0.120960  1.208897  1.730313  0.009197

2003-12-03  0.341180  0.778089  0.120961  1.212298  1.728101  0.009234

2003-12-04  0.339789  0.776096  0.120963  1.208094  1.720697  0.009236

2003-12-05  0.340832  0.785916  0.120964  1.218695  1.733102  0.009292

...              ...       ...       ...       ...       ...       ...

2024-01-17  0.203050  1.161036  0.140519  1.087879  1.263855  0.006791

2024-01-18  0.202675  1.156976  0.140558  1.088578  1.267684  0.006751

2024-01-19  0.203000  1.151968  0.140548  1.087914  1.270987  0.006751

2024-01-22  0.203888  1.151437  0.140704  1.089230  1.269986  0.006751

2024-01-23  0.200469  1.150589  0.139030  1.088021  1.270696  0.006750


                 BRL
Date
2003-12-01  0.157672
2003-12-02  0.158854
2003-12-03  0.161488
2003-12-04  0.157851
2003-12-05  0.160769
...              ...
```

```
2024-01-17  0.052745
2024-01-18  0.052413
2024-01-19  0.052851
2024-01-22  0.052607
2024-01-23  0.052102

[5253 rows x 7 columns]
```

```
# We want to merge currencies_df and sp500. Therefore we want the same
dates for each.
sp500_3 = merged_df_VIX['2003-12-01':]

# Merging currencies_df and sp500_3 into one dataframe
merged_df_currencies = sp500_3.merge(currencies_df, how='inner',
left_index=True, right_index=True, suffixes=('_SP500', '_currencies'))

# Calculating Percentage change of each currency one day before
currencies = ['EUR', 'GBP', 'JPY', 'CHF', 'CNY', 'ZAR', 'BRL']
for currency in currencies:
    merged_df_currencies[currency + str('Day before')] =
merged_df_currencies[currency].shift(1)
    merged_df_currencies[currency + str('change')] =
(((merged_df_currencies[currency] - merged_df_currencies[currency +
str('Day before')]) / merged_df_currencies[currency + str('Day
before')])*100).shift(1)
# Eliminating NaNs
merged_df_currencies = merged_df_currencies['2003-12-03':]
merged_df_currencies
```

```
                   Open         High          Low        Close     Adj
Close  \
2003-12-03  1066.619995  1074.300049  1064.630005  1064.729980
1064.729980
2003-12-04  1064.729980  1070.369995  1063.150024  1069.719971
1069.719971
2003-12-05  1069.719971  1069.719971  1060.089966  1061.500000
1061.500000
2003-12-08  1061.500000  1069.589966  1060.930054  1069.300049
1069.300049
2003-12-09  1069.300049  1071.939941  1059.160034  1060.180054
1060.180054
...                 ...          ...          ...          ...
...
2024-01-17  4739.129883  4744.229980  4714.819824  4739.209961
4739.209961
2024-01-18  4760.100098  4785.790039  4740.569824  4780.939941
4780.939941
2024-01-19  4796.279785  4842.069824  4785.870117  4839.810059
4839.810059
2024-01-22  4853.419922  4868.410156  4844.049805  4850.430176
```

```
4850.430176
2024-01-23  4856.799805  4866.479980  4844.370117  4864.600098
4864.600098

             Volume  Close day before  pct_change  Direction
Day_1_before  \
2003-12-03  1441.70       1066.619995   -0.177197          0     -
0.327066
2003-12-04  1463.10       1064.729980    0.468663          1     -
0.177197
2003-12-05  1265.90       1069.719971   -0.768423          0
0.468663
2003-12-08  1218.90       1061.500000    0.734814          1     -
0.768423
2003-12-09  1465.50       1069.300049   -0.852894          0
0.734814
...             ...               ...         ...        ...
...
2024-01-17  3928.60       4765.979980   -0.561690          0     -
0.373134
2024-01-18  4019.00       4739.209961    0.880526          1     -
0.561690
2024-01-19  4287.20       4780.939941    1.231350          1
0.880526
2024-01-22  4297.61       4839.810059    0.219433          1
1.231350
2024-01-23  3912.80       4850.430176    0.292137          1
0.219433

            ...  JPYDay before  JPYchange  CHFDay before  CHFchange  \
2003-12-03  ...       0.120960   0.000000       1.208897   1.036029
2003-12-04  ...       0.120961   0.001226       1.212298   0.281255
2003-12-05  ...       0.120963   0.001207       1.208094  -0.346723
2003-12-08  ...       0.120964   0.001213       1.218695   0.877452
2003-12-09  ...       0.120961  -0.002421       1.222001   0.271296
...         ...            ...        ...            ...        ...
2024-01-17  ...       0.140905   0.085946       1.094571  -0.327277
2024-01-18  ...       0.140519  -0.274007       1.087879  -0.611375
2024-01-19  ...       0.140558   0.028112       1.088578   0.064214
2024-01-22  ...       0.140548  -0.007029       1.087914  -0.060920
2024-01-23  ...       0.140704   0.111153       1.089230   0.120906

            CNYDay before  CNYchange  ZARDay before  ZARchange  BRLDay
before  \
2003-12-03       1.730313   0.681747       0.009197   0.404667
0.158854
2003-12-04       1.728101  -0.127889       0.009234   0.397041
0.161488
2003-12-05       1.720697  -0.428446       0.009236   0.027717
0.157851
```

```
2003-12-08       1.733102   0.720966       0.009292   0.603967
0.160769
2003-12-09       1.734214   0.064168       0.009310   0.195517
0.155732
...                   ...        ...            ...        ...
...
2024-01-17       1.271617  -0.481949       0.006859  -0.513058
0.053622
2024-01-18       1.263855  -0.610438       0.006791  -0.986098
0.052745
2024-01-19       1.267684   0.302981       0.006751  -0.588721
0.052413
2024-01-22       1.270987   0.260548       0.006751  -0.006752
0.052851
2024-01-23       1.269986  -0.078739       0.006751   0.004725
0.052607

            BRLchange
2003-12-03   0.749786
2003-12-04   1.658488
2003-12-05  -2.252536
2003-12-08   1.848850
2003-12-09  -3.133321
...               ...
2024-01-17  -0.021449
2024-01-18  -1.635047
2024-01-19  -0.629537
2024-01-22   0.835571
2024-01-23  -0.461885

[5064 rows x 59 columns]
```

```
merged_df_currencies[['EUR', 'EURDay before', 'EURchange']]
```

```
                 EUR  EURDay before  EURchange
2003-12-03  0.341180       0.341180  -0.272940
2003-12-04  0.339789       0.341180   0.000000
2003-12-05  0.340832       0.339789  -0.407744
2003-12-08  0.340136       0.340832   0.306751
2003-12-09  0.340948       0.340136  -0.204085
...              ...            ...        ...
2024-01-17  0.203050       0.205668   0.154257
2024-01-18  0.202675       0.203050  -1.273127
2024-01-19  0.203000       0.202675  -0.184436
2024-01-22  0.203888       0.203000   0.160374
2024-01-23  0.200469       0.203888   0.437499

[5064 rows x 3 columns]
```

```python
# Filling up NaNs
merged_df_currencies = merged_df_currencies.fillna(method='ffill')
merged_df_currencies = merged_df_currencies.fillna(method='bfill')
merged_df_currencies_2 = merged_df_currencies['2003-12-03':'2021-11-
22']
print(merged_df_currencies_2.isna().sum())
```

```
Open                          0
High                          0
Low                           0
Close                         0
Adj Close                     0
Volume                        0
Close day before              0
pct_change                    0
Direction                     0
Day_1_before                  0
Day_2_before                  0
Day_3_before                  0
Day_4_before                  0
Day_5_before                  0
Day_6_before                  0
Day_7_before                  0
Day_8_before                  0
Day_9_before                  0
Open_day_before               0
High_day_before               0
Low_day_before                0
Open_pct_change               0
High_pct_change               0
Low_pct_change                0
Open_pct_change_Day_before    0
High_pct_change_Day_before    0
Low_pct_change_Day_before     0
Volume day before             0
Change_volume                 0
date                          0
high                          0
low                           0
open                          0
close                         0
volume                        0
adjclose                      0
VIX_Close_day_before          0
VIX_change                    0
EUR                           0
GBP                           0
JPY                           0
CHF                           0
CNY                           0
```

```
ZAR                               0
BRL                               0
EURDay before                     0
EURchange                         0
GBPDay before                     0
GBPchange                         0
JPYDay before                     0
JPYchange                         0
CHFDay before                     0
CHFchange                         0
CNYDay before                     0
CNYchange                         0
ZARDay before                     0
ZARchange                         0
BRLDay before                     0
BRLchange                         0
dtype: int64
```

The NORD_max varies and is not equal to 2.73 due to changes in the training data. The training set interval switches from (30.2001, 22.11.2022) to (3.12.2003, 22.11.2022). As the model is trained on different data, it generates different predictions.

```
# Calculating NORD_max
nord_max_currencies = [
    ['Day_1_before', 'Day_2_before', 'Day_3_before']
]
result_nord_max = best_features(nord_max_currencies,
merged_df_currencies_2)
result_nord_max

<pandas.io.formats.style.Styler at 0x16135e0c8b0>

# Using methodology A for feature selection
features_currencies = ['EURchange', 'GBPchange', 'JPYchange',
'CHFchange', 'CNYchange',
                       'ZARchange', 'BRLchange']
fixed_features = ['Day_1_before', 'Day_2_before', 'Day_3_before']
result_currencies = methodology_A(fixed_features, features_currencies,
merged_df_currencies_2, 2.63)
result_currencies

<pandas.io.formats.style.Styler at 0x16135dfd4c0>
```

Methodology A identifies features EURchange, GBPchange, JPYchange, CNYchange as important.

```
# Round 1 of methodology B
features_round_1_currencies = [
    ['Day_1_before', 'Day_2_before', 'Day_3_before'],
```

```
    ['Day_1_before', 'Day_2_before', 'Day_3_before', 'EURchange'],
    ['Day_1_before', 'Day_2_before', 'Day_3_before', 'GBPchange'],
    ['Day_1_before', 'Day_2_before', 'Day_3_before', 'JPYchange'],
    ['Day_1_before', 'Day_2_before', 'Day_3_before', 'CHFchange'],
    ['Day_1_before', 'Day_2_before', 'Day_3_before', 'CNYchange'],
    ['Day_1_before', 'Day_2_before', 'Day_3_before', 'ZARchange'],
    ['Day_1_before', 'Day_2_before', 'Day_3_before', 'BRLchange']
]
result_round_1_currencies = best_features(features_round_1_currencies,
merged_df_currencies_2)
result_round_1_currencies
```

```
<pandas.io.formats.style.Styler at 0x161323655e0>
```

```
# Round 2 of methodology B
features_round_2_currencies = [
    ['Day_1_before', 'Day_2_before', 'Day_3_before', 'JPYchange',
'EURchange'],
    ['Day_1_before', 'Day_2_before', 'Day_3_before', 'JPYchange',
'GBPchange'],
    ['Day_1_before', 'Day_2_before', 'Day_3_before', 'JPYchange',
'CHFchange'],
    ['Day_1_before', 'Day_2_before', 'Day_3_before', 'JPYchange',
'CNYchange'],
    ['Day_1_before', 'Day_2_before', 'Day_3_before', 'JPYchange',
'ZARchange'],
    ['Day_1_before', 'Day_2_before', 'Day_3_before', 'JPYchange',
'BRLchange']
]
result_round_2_currencies = best_features(features_round_2_currencies,
merged_df_currencies_2)
result_round_2_currencies
```

```
<pandas.io.formats.style.Styler at 0x16135e0c6a0>
```

```
# Round 3 of methodology B
features_round_3_currencies = [
    ['Day_1_before', 'Day_2_before', 'Day_3_before', 'JPYchange',
'CNYchange', 'EURchange'],
    ['Day_1_before', 'Day_2_before', 'Day_3_before', 'JPYchange',
'CNYchange', 'GBPchange'],
    ['Day_1_before', 'Day_2_before', 'Day_3_before', 'JPYchange',
'CNYchange', 'CHFchange'],
    ['Day_1_before', 'Day_2_before', 'Day_3_before', 'JPYchange',
'CNYchange', 'ZARchange'],
    ['Day_1_before', 'Day_2_before', 'Day_3_before', 'JPYchange',
'CNYchange', 'BRLchange']
]
result_round_3_currencies = best_features(features_round_3_currencies,
```

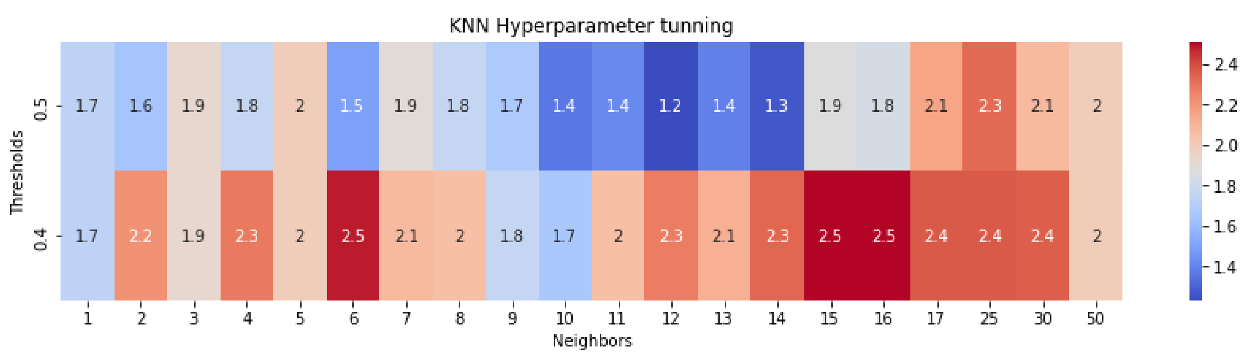
```
merged_df_currencies_2)
result_round_3_currencies

<pandas.io.formats.style.Styler at 0x16132364280>
```

Methodology B identifies features JPYchange, CNYchange as important.

```
# Using methodology A for feature selection with different order
features_currencies = ['GBPchange', 'EURchange', 'JPYchange',
'GBPchange', 'CHFchange', 'CNYchange',
                       'ZARchange', 'BRLchange']
fixed_features = ['Day_1_before', 'Day_2_before', 'Day_3_before']
result_currencies = methodology_A(fixed_features, features_currencies,
merged_df_currencies_2, 2.63)
result_currencies

<pandas.io.formats.style.Styler at 0x1613426f490>
```

If we alter the order and switch EURchange and GBPchange, the methodology does not identify the feature EURchange as important.

```
Features_currencies = merged_df_currencies_2[['Day_1_before',
'Day_2_before', 'Day_3_before', 'JPYchange', 'CNYchange',
'GBPchange']]
Y_currencies =  merged_df_currencies_2['Direction']

matrix_knn_parameters_currencies = knn_hyper(Features_currencies,
Y_currencies, merged_df_currencies_2['2017-9-26':])
```

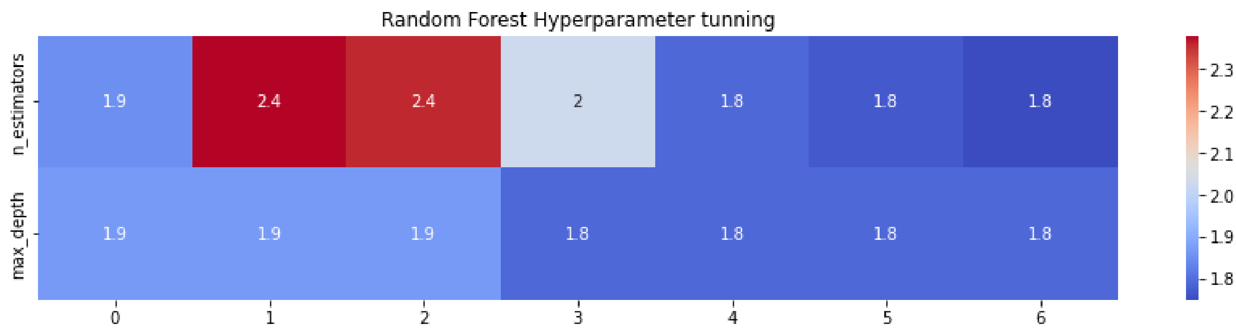



```
The code took 42.59273 seconds to run.

matrix_knn_parameters_currencies

array([[1.73, 1.64, 1.92, 1.76, 1.98, 1.49, 1.89, 1.76, 1.67, 1.36,
1.42,
        1.23, 1.39, 1.26, 1.87, 1.83, 2.15, 2.32, 2.07, 1.98],
       [1.73, 2.21, 1.92, 2.28, 1.98, 2.48, 2.07, 2.05, 1.76, 1.66,
```

```
2.05,
        2.27, 2.11, 2.33, 2.51, 2.51, 2.36, 2.36, 2.35, 1.99]])
```

The parameter combination of 15 neighbors with a threshold of 0.4 results in the highest NORD.

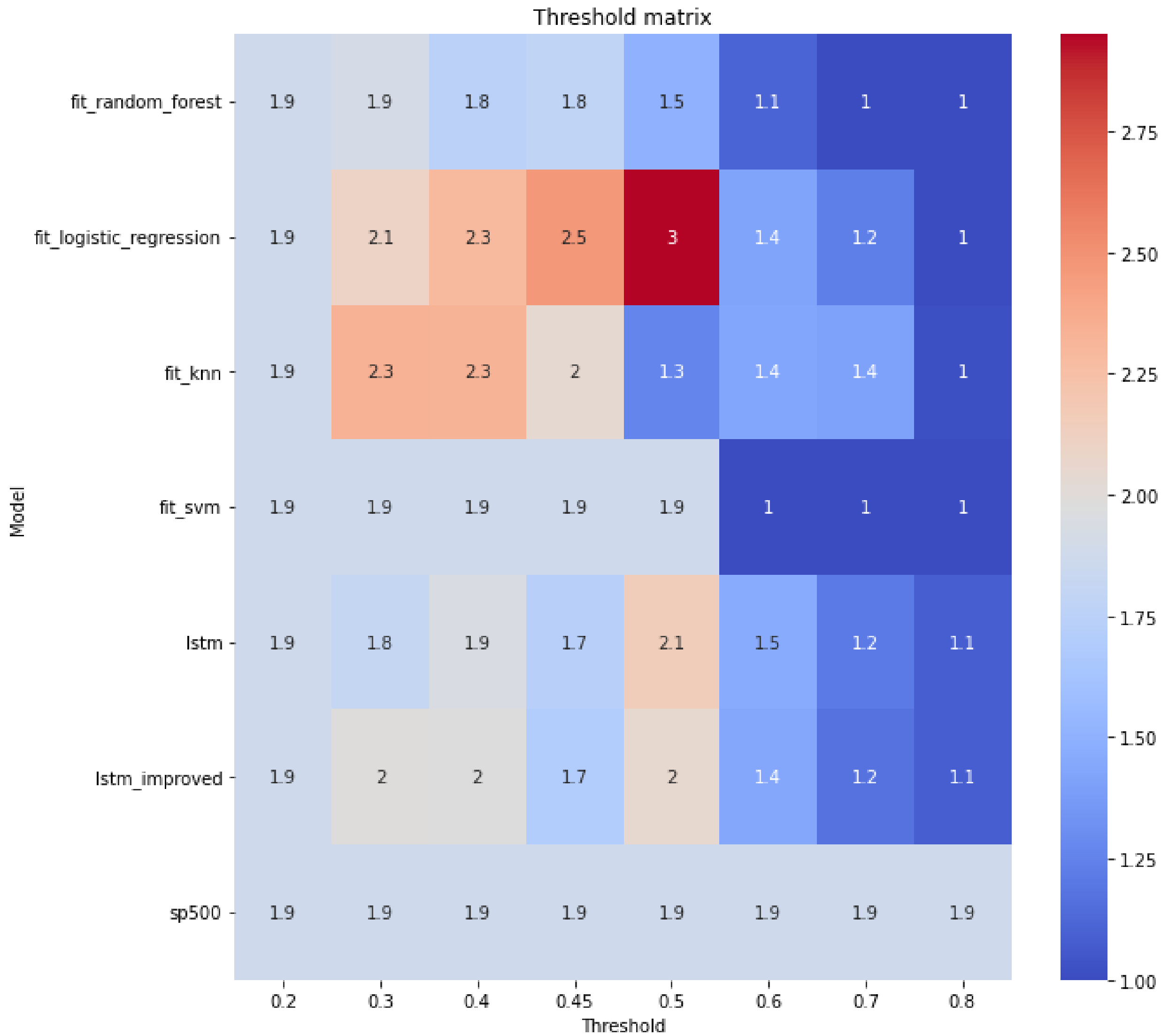
```
matrix_rf_parameters_currencies =
hypertune_random_forest(Features_currencies, Y_currencies,
merged_df_currencies_2['2017-9-26':])
```




```
n_estimators: [3, 5, 10, 50, 100, 150, 200]
max_debths: [1, 3, 5, 10, 20, 40, 80]
The code took 24.09786 seconds to run.

matrix_rf_parameters_currencies

array([[1.85, 2.38, 2.36, 2.03, 1.79, 1.77, 1.75],
       [1.87, 1.87, 1.87, 1.79, 1.79, 1.79, 1.79]])
```

A max depth of 5 and n_estimators of 5 result in the highest NORD.

```
matrix_currencies_threshold = treshold_matrix(Features_currencies,
Y_currencies, merged_df_currencies_2['2017-9-26':], 6, n_steps=1,
n_estimators=5, max_depth=5, neighbors=14)

Optimization terminated successfully.
         Current function value: 0.686413
         Iterations 5
33/33 [==============================] - 1s 2ms/step
33/33 [==============================] - 1s 2ms/step
```

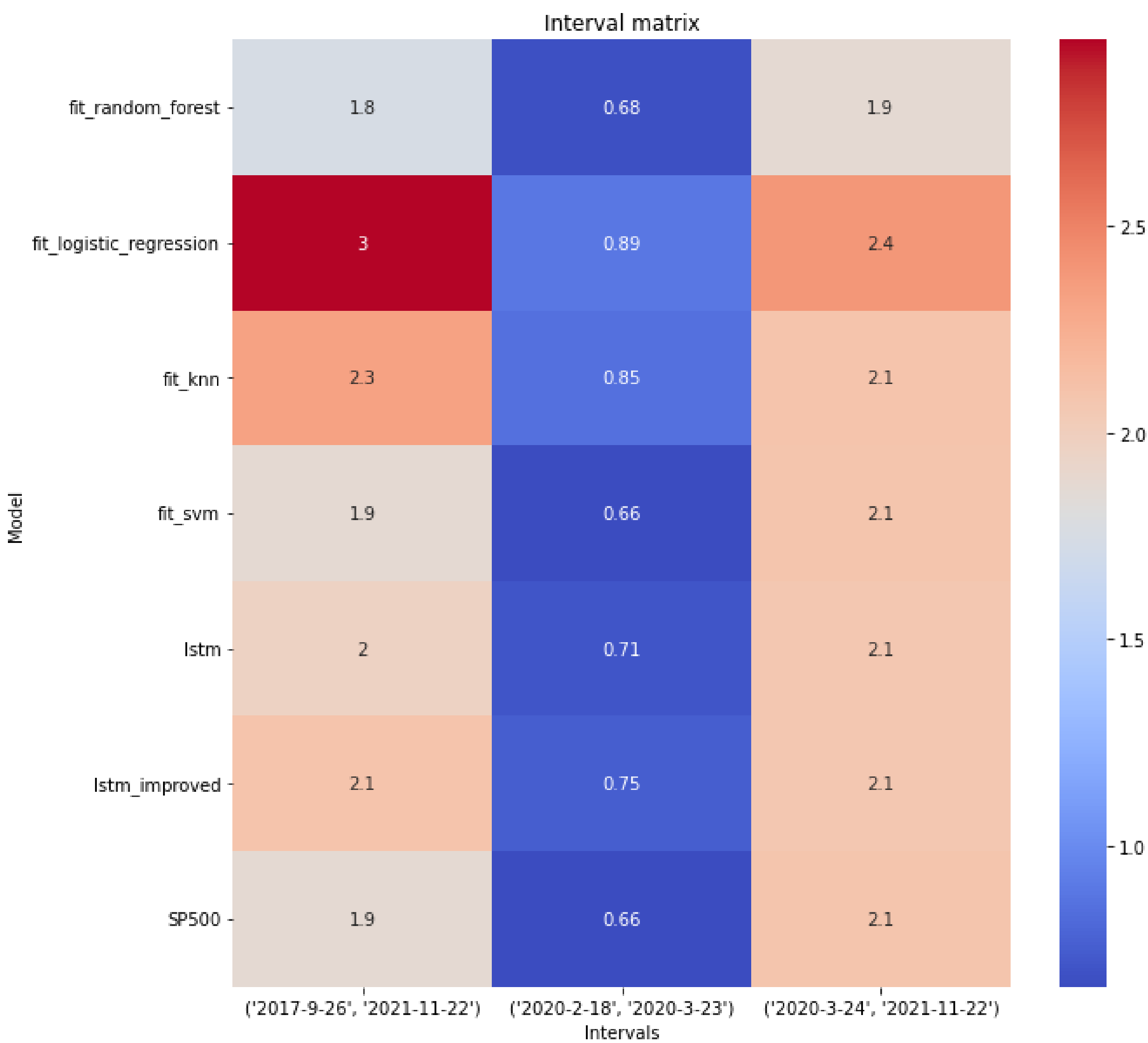



```
The code took 88.47010 seconds to run.

matrix_currencies_threshold

array([[1.87, 1.91, 1.75, 1.78, 1.5 , 1.11, 1.  , 1.  ],
       [1.87, 2.1 , 2.28, 2.47, 2.95, 1.42, 1.23, 1.  ],
       [1.87, 2.34, 2.33, 2.04, 1.26, 1.43, 1.41, 1.02],
       [1.87, 1.87, 1.87, 1.87, 1.87, 1.  , 1.  , 1.  ],
       [1.87, 1.81, 1.94, 1.73, 2.15, 1.46, 1.21, 1.08],
       [1.87, 1.99, 1.98, 1.7 , 2.05, 1.44, 1.17, 1.08],
       [1.87, 1.87, 1.87, 1.87, 1.87, 1.87, 1.87, 1.87]])
```

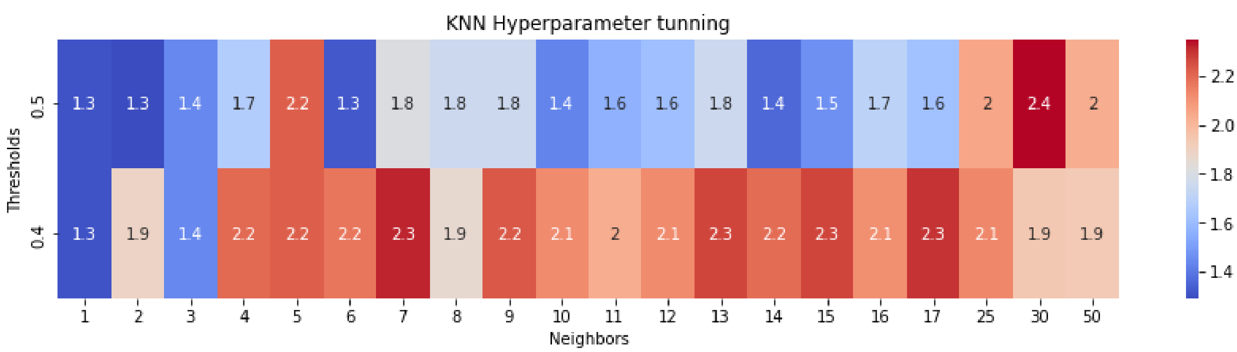
The thresholds 0.4, 0.5, 0.4, 0.5, 0.4, 0.4, corresponding to the algorithms Random Forest, Logistic Regression, K-Nearest Neighbors, Support Vector Classifier, Long-Short Term Memory, and Long-Short Term Memory Improved, respectively, yield the highest NORD.

```
thresholds_currencies = [0.4, 0.5, 0.4, 0.5, 0.4, 0.4]

matrix_currencies_interval = interval_matrix(Features_currencies,
Y_currencies, interval1, interval2, interval3,
merged_df_currencies_2['2017-9-25':], 6, thresholds_currencies,
neighbors=14, n_estimators=5, max_depth=5)
```

```
Optimization terminated successfully.
         Current function value: 0.686413
         Iterations 5
33/33 [==============================] - 1s 1ms/step
1/1 [==============================] - 0s 29ms/step
14/14 [==============================] - 0s 2ms/step
33/33 [==============================] - 3s 4ms/step
1/1 [==============================] - 0s 39ms/step
14/14 [==============================] - 0s 4ms/step
```



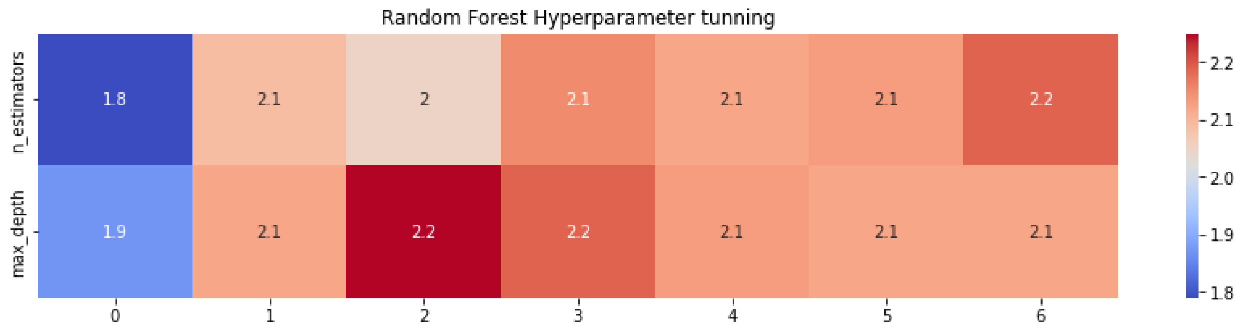
```
The code took 165.60821 seconds to run.

matrix_currencies_interval

array([[1.75, 0.68, 1.87],
       [2.95, 0.89, 2.39],
       [2.33, 0.85, 2.1 ],
       [1.87, 0.66, 2.09],
       [1.96, 0.71, 2.07],
       [2.1 , 0.75, 2.07],
       [1.87, 0.66, 2.09]])

# Finding the interval matrix on this interval
Features_currencies_D1B_D2b_D3B =
merged_df_currencies_2[['Day_1_before', 'Day_2_before',
'Day_3_before']]

matrix_knn_parameters_currencies_D1B_D2B_D3B =
knn_hyper(Features_currencies_D1B_D2b_D3B, Y_currencies,
merged_df_currencies_2['2017-9-26':])
```




```
The code took 42.44851 seconds to run.

matrix_knn_parameters_currencies_D1B_D2B_D3B

array([[1.31, 1.29, 1.44, 1.67, 2.22, 1.32, 1.81, 1.75, 1.75, 1.43,
1.56,
        1.61, 1.75, 1.36, 1.46, 1.7 , 1.62, 2.05, 2.35, 2.03],
       [1.31, 1.89, 1.44, 2.2 , 2.22, 2.17, 2.32, 1.86, 2.23, 2.12,
2.03,
        2.12, 2.27, 2.19, 2.27, 2.11, 2.3 , 2.13, 1.94, 1.93]])
```

The parameter combination of 30 neighbors with a threshold of 0.5 results in the highest NORD.

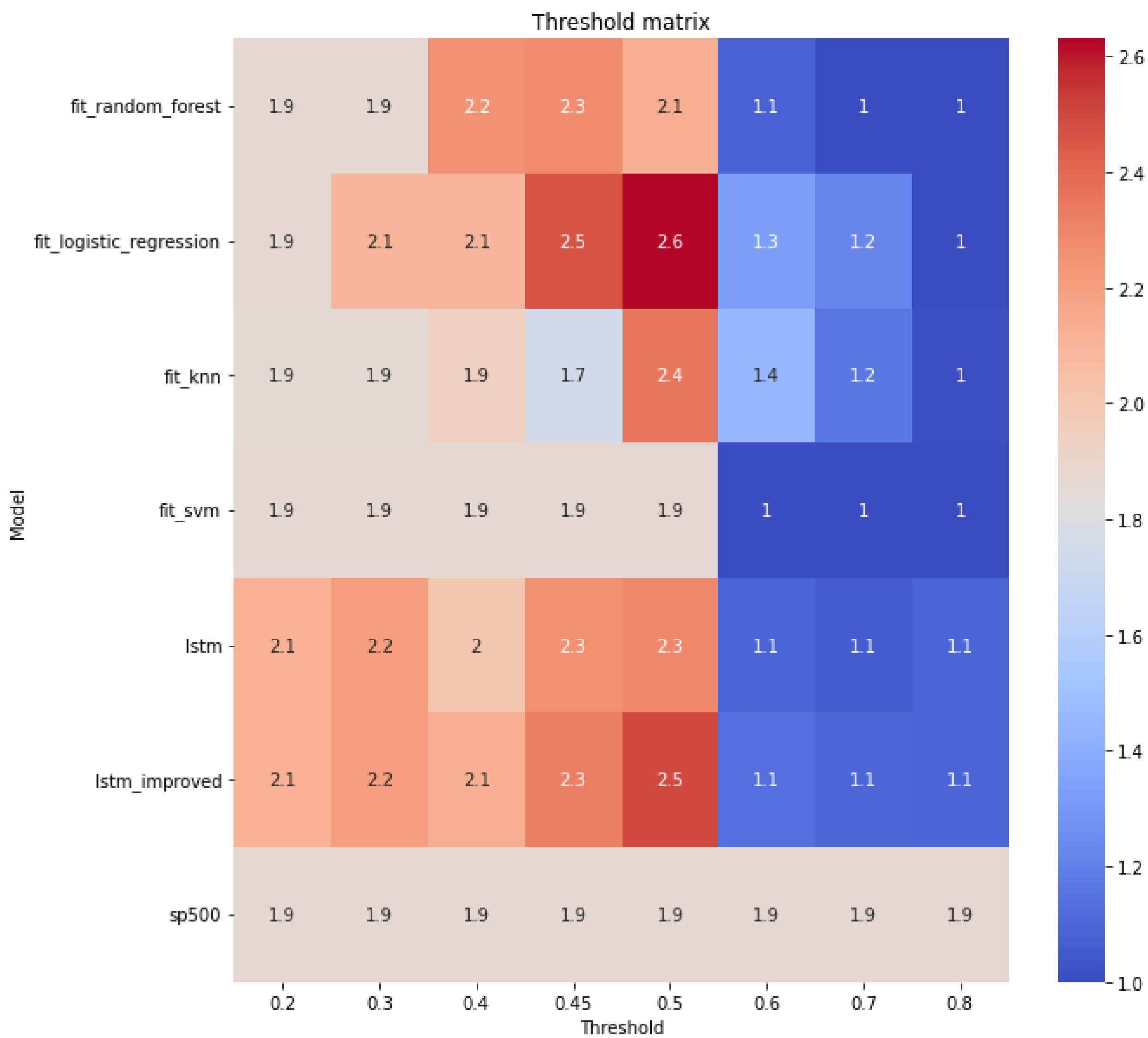
```
matrix_rf_parameters_currencies =
hypertune_random_forest(Features_currencies_D1B_D2b_D3B, Y_currencies,
merged_df_currencies_2['2017-9-26':])
```




```
n_estimators: [3, 5, 10, 50, 100, 150, 200]
max_debths: [1, 3, 5, 10, 20, 40, 80]
The code took 20.83182 seconds to run.

matrix_rf_parameters_currencies

array([[1.79, 2.09, 2.05, 2.15, 2.12, 2.13, 2.19],
       [1.87, 2.12, 2.25, 2.19, 2.13, 2.12, 2.12]])
```

A max depth of 5 and n_estimators of 200 result in the highest NORD.

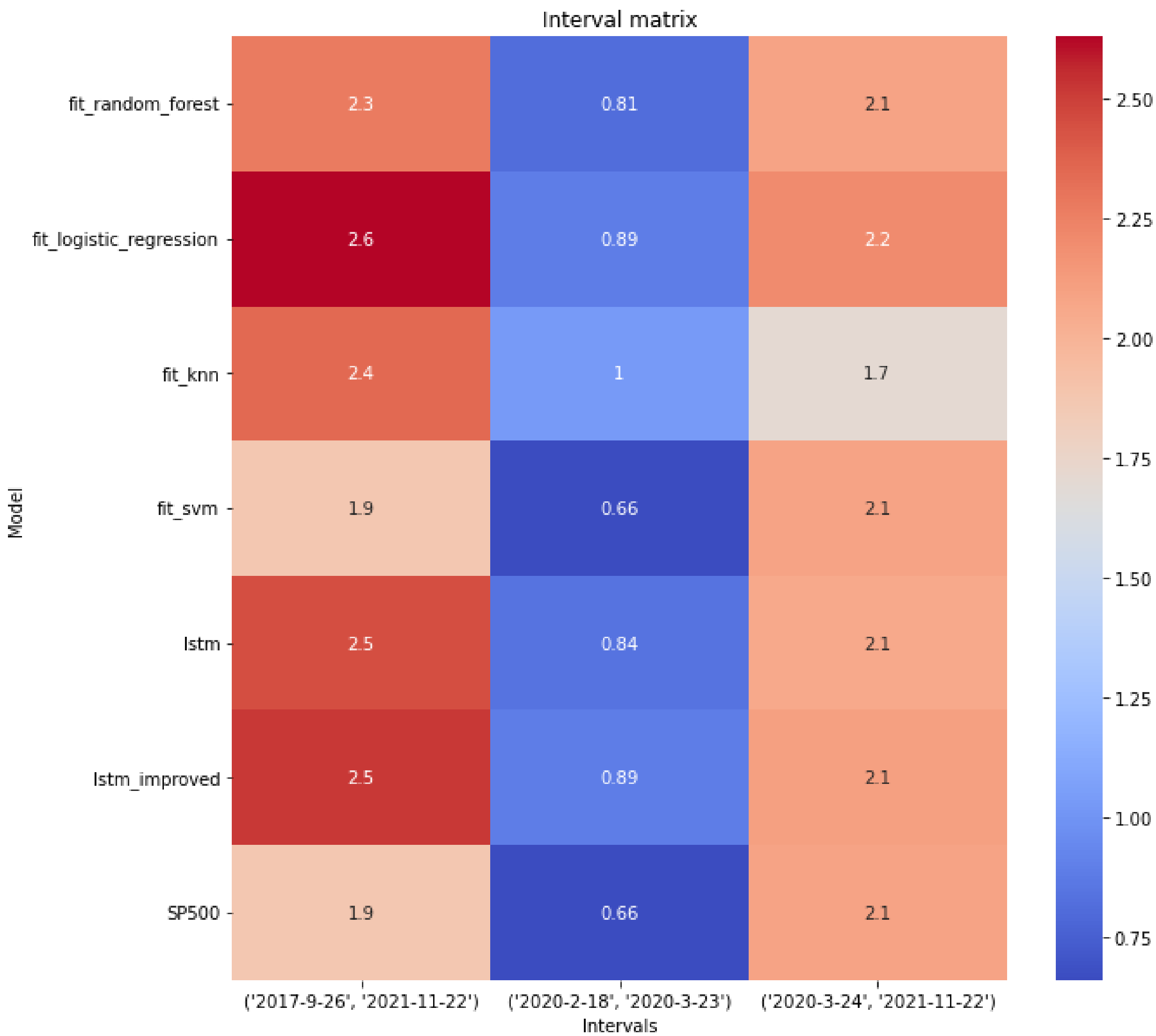
```
matrix_currencies_threshold_D1B_D2B_D3B =
treshold_matrix(Features_currencies_D1B_D2b_D3B, Y_currencies,
merged_df_currencies_2['2017-9-26':], 3, n_steps=1, n_estimators=200,
max_depth=5, neighbors=30)

Optimization terminated successfully.
         Current function value: 0.687045
         Iterations 4
33/33 [==============================] - 1s 2ms/step
33/33 [==============================] - 1s 2ms/step
```




```
The code took 90.33162 seconds to run.

matrix_currencies_threshold_D1B_D2B_D3B

array([[1.87, 1.87, 2.25, 2.27, 2.13, 1.08, 1.  , 1.  ],
       [1.87, 2.1 , 2.1 , 2.46, 2.63, 1.33, 1.22, 1.  ],
       [1.86, 1.86, 1.94, 1.74, 2.35, 1.45, 1.16, 1.01],
       [1.87, 1.87, 1.87, 1.87, 1.87, 1.  , 1.  , 1.  ],
       [2.12, 2.2 , 2.01, 2.26, 2.29, 1.09, 1.06, 1.09],
       [2.12, 2.2 , 2.13, 2.32, 2.49, 1.13, 1.1 , 1.09],
       [1.87, 1.87, 1.87, 1.87, 1.87, 1.87, 1.87, 1.87]])

thresholds_currencies_D1B_D2B_D3B = [0.45, 0.5, 0.5, 0.5, 0.5, 0.5]

matrix_currencies_interval_D1B_D2B_D3B =
interval_matrix(Features_currencies_D1B_D2b_D3B, Y_currencies,
```

```
interval1, interval2, interval3, merged_df_currencies_2['2017-9-25':],
3, thresholds_currencies_D1B_D2B_D3B, neighbors=30, n_estimators=200,
max_depth=5)
```

```
Optimization terminated successfully.
         Current function value: 0.687045
         Iterations 4
33/33 [==============================] - 1s 2ms/step
1/1 [==============================] - 0s 23ms/step
14/14 [==============================] - 0s 2ms/step
33/33 [==============================] - 3s 4ms/step
1/1 [==============================] - 0s 39ms/step
14/14 [==============================] - 0s 4ms/step
```




```
The code took 168.03514 seconds to run.
```

```
matrix_currencies_interval_D1B_D2B_D3B
```

```
array([[2.27, 0.81, 2.09],
       [2.63, 0.89, 2.2 ],
       [2.35, 1.03, 1.7 ],
       [1.87, 0.66, 2.09],
       [2.45, 0.84, 2.06],
       [2.52, 0.89, 2.1 ],
       [1.87, 0.66, 2.09]])
```

```
# Calculating the difference in NORD between interval matrices to
identify which algorithms on
# specific intervals have shown improvement.
print(matrix_currencies_interval_D1B_D2B_D3B -
matrix_currencies_interval)
```

```
[[ 0.52  0.13  0.22]
 [-0.32  0.   -0.19]
 [ 0.02  0.18 -0.4 ]
 [ 0.    0.    0.  ]
 [ 0.49  0.13 -0.01]
 [ 0.42  0.14  0.03]
 [ 0.    0.    0.  ]]
```

Summary:

Algorithms are trained on the interval from 3.12.2003, until 25.9.2017.

The NORD value on the validation set of a model using Logistic Regression as the algorithm, with features Day_1_before, Day_2_before, and Day_3_before, and trained on the interval from 30.1.2001, until 25.9.2017, is 2.63.

The NORD - BUY AND HOLD on the validation set is equal to 1.88.

Methodology A identifies features EURchange, GBPchange, JPYchange, CNYchange as important.

Methodology B identifies features JPYchange, CNYchange as important.

Upon applying common sense, we decide to include features JPYchange, CNYchange, GBPchange to our model.

Our feature vector changes to: X=[Day_1_before, Day_2_before, Day_3_before, JPYchange, CNYchange, GBPchange].

Indices

```
# Downloading data from other indices
merged_df_indices = merged_df_currencies.copy()

indices = [
    '^IXIC',    # NASDAQ Composite
```

```python
    '^DJI',    # Dow Jones Industrial Average
    '^N225',   # Nikkei 225
    '^HSI',    # Hang Seng Index
    '^FTSE',   # FTSE 100
    '^GDAXI',  # DAX
    '^FCHI',   # CAC 40
]

for i in range(len(indices)):
    data = yf.download(indices[i], start='2003-12-3', end='2024-1-24')
['Close']
    data_df = data.to_frame('Close' + indices[i])
    data_df.index = data_df.index.strftime('%Y-%m-%d')
    merged_df_indices.index =
pd.to_datetime(merged_df_indices.index).tz_localize(None)
    data_df.index = pd.to_datetime(data_df.index).tz_localize(None)
    merged_df_indices = merged_df_indices.groupby(level=0).first()
    data_df = data_df.groupby(level=0).first()
    merged_df_indices = pd.merge(merged_df_indices, data_df,
left_index=True, right_index=True, how='outer')
```

```
[*********************100%***********************]  1 of 1 completed
[*********************100%***********************]  1 of 1 completed
[*********************100%***********************]  1 of 1 completed
[*********************100%***********************]  1 of 1 completed
[*********************100%***********************]  1 of 1 completed
[*********************100%***********************]  1 of 1 completed
[*********************100%***********************]  1 of 1 completed
```

```python
merged_df_indices
```

```
                   Open         High          Low        Close    Adj
Close  \
2003-12-03  1066.619995  1074.300049  1064.630005  1064.729980
1064.729980
2003-12-04  1064.729980  1070.369995  1063.150024  1069.719971
1069.719971
2003-12-05  1069.719971  1069.719971  1060.089966  1061.500000
1061.500000
2003-12-08  1061.500000  1069.589966  1060.930054  1069.300049
1069.300049
2003-12-09  1069.300049  1071.939941  1059.160034  1060.180054
1060.180054
...                 ...          ...          ...          ...
...
2024-01-17  4739.129883  4744.229980  4714.819824  4739.209961
4739.209961
2024-01-18  4760.100098  4785.790039  4740.569824  4780.939941
4780.939941
2024-01-19  4796.279785  4842.069824  4785.870117  4839.810059
```

```
4839.810059
2024-01-22  4853.419922  4868.410156  4844.049805  4850.430176
4850.430176
2024-01-23  4856.799805  4866.479980  4844.370117  4864.600098
4864.600098

             Volume  Close day before  pct_change  Direction
Day_1_before  \
2003-12-03  1441.70       1066.619995   -0.177197        0.0     -
0.327066
2003-12-04  1463.10       1064.729980    0.468663        1.0     -
0.177197
2003-12-05  1265.90       1069.719971   -0.768423        0.0
0.468663
2003-12-08  1218.90       1061.500000    0.734814        1.0     -
0.768423
2003-12-09  1465.50       1069.300049   -0.852894        0.0
0.734814
...             ...               ...         ...        ...
...
2024-01-17  3928.60       4765.979980   -0.561690        0.0     -
0.373134
2024-01-18  4019.00       4739.209961    0.880526        1.0     -
0.561690
2024-01-19  4287.20       4780.939941    1.231350        1.0
0.880526
2024-01-22  4297.61       4839.810059    0.219433        1.0
1.231350
2024-01-23  3912.80       4850.430176    0.292137        1.0
0.219433

            ...  ZARchange  BRLDay before  BRLchange    Close^IXIC  \
2003-12-03  ...   0.404667       0.158854   0.749786   1960.250000
2003-12-04  ...   0.397041       0.161488   1.658488   1968.800049
2003-12-05  ...   0.027717       0.157851  -2.252536   1937.819946
2003-12-08  ...   0.603967       0.160769   1.848850   1948.849976
2003-12-09  ...   0.195517       0.155732  -3.133321   1908.319946
...         ...        ...            ...        ...           ...
2024-01-17  ...  -0.513058       0.053622  -0.021449  14855.620117
2024-01-18  ...  -0.986098       0.052745  -1.635047  15055.650391
2024-01-19  ...  -0.588721       0.052413  -0.629537  15310.969727
2024-01-22  ...  -0.006752       0.052851   0.835571  15360.290039
2024-01-23  ...   0.004725       0.052607  -0.461885  15425.940430

               Close^DJI    Close^N225     Close^HSI   Close^FTSE  \
2003-12-03   9873.419922  10326.389648  12361.179688  4392.000000
2003-12-04   9930.820312  10429.990234  12342.650391  4378.200195
2003-12-05   9862.679688  10373.459961  12314.730469  4367.000000
2003-12-08   9965.269531  10045.339844  12177.440430  4359.799805
2003-12-09   9923.419922  10124.280273  12393.639648  4379.600098
```

```
...                     ...           ...           ...          ...
2024-01-17  37266.671875  35477.750000  15276.900391  7446.299805
2024-01-18  37468.609375  35466.171875  15391.790039  7459.100098
2024-01-19  37863.800781  35963.269531  15308.690430  7461.899902
2024-01-22  38001.808594  36546.949219  14961.179688  7487.700195
2024-01-23  37905.449219  36517.570312  15353.980469  7485.700195

             Close^GDAXI   Close^FCHI
2003-12-03   3875.659912  3501.929932
2003-12-04   3874.780029  3496.550049
2003-12-05   3841.729980  3457.139893
2003-12-08   3806.540039  3434.909912
2003-12-09   3846.179932  3456.120117
...                  ...          ...
2024-01-17  16431.689453  7318.689941
2024-01-18  16567.349609  7401.350098
2024-01-19  16555.130859  7371.640137
2024-01-22  16683.359375  7413.250000
2024-01-23  16627.089844  7388.040039

[5241 rows x 66 columns]
```

```
# Calculating percentage change in index one day ago
indeces = ['Close^IXIC', 'Close^DJI', 'Close^N225', 'Close^HSI',
'Close^FTSE',
       'Close^GDAXI', 'Close^FCHI']
for index in indeces:
    merged_df_indices[index + str('Day before')] =
merged_df_indices[index].shift(1)
    merged_df_indices[index + str('change')] =
(((merged_df_indices[index] - merged_df_indices[index + str('Day
before')]) / merged_df_indices[index + str('Day
before')])*100).shift(1)

merged_df_indices
```

```
                   Open         High          Low        Close    Adj
Close  \
2003-12-03  1066.619995  1074.300049  1064.630005  1064.729980
1064.729980
2003-12-04  1064.729980  1070.369995  1063.150024  1069.719971
1069.719971
2003-12-05  1069.719971  1069.719971  1060.089966  1061.500000
1061.500000
2003-12-08  1061.500000  1069.589966  1060.930054  1069.300049
1069.300049
2003-12-09  1069.300049  1071.939941  1059.160034  1060.180054
1060.180054
...                 ...          ...          ...          ...
...
```

```
2024-01-17  4739.129883  4744.229980  4714.819824  4739.209961
4739.209961
2024-01-18  4760.100098  4785.790039  4740.569824  4780.939941
4780.939941
2024-01-19  4796.279785  4842.069824  4785.870117  4839.810059
4839.810059
2024-01-22  4853.419922  4868.410156  4844.049805  4850.430176
4850.430176
2024-01-23  4856.799805  4866.479980  4844.370117  4864.600098
4864.600098

             Volume  Close day before  pct_change  Direction
Day_1_before  \
2003-12-03  1441.70       1066.619995   -0.177197        0.0     -
0.327066
2003-12-04  1463.10       1064.729980    0.468663        1.0     -
0.177197
2003-12-05  1265.90       1069.719971   -0.768423        0.0
0.468663
2003-12-08  1218.90       1061.500000    0.734814        1.0     -
0.768423
2003-12-09  1465.50       1069.300049   -0.852894        0.0
0.734814
...             ...               ...         ...        ...
...
2024-01-17  3928.60       4765.979980   -0.561690        0.0     -
0.373134
2024-01-18  4019.00       4739.209961    0.880526        1.0     -
0.561690
2024-01-19  4287.20       4780.939941    1.231350        1.0
0.880526
2024-01-22  4297.61       4839.810059    0.219433        1.0
1.231350
2024-01-23  3912.80       4850.430176    0.292137        1.0
0.219433

            ...  Close^N225Day before  Close^N225change  Close^HSIDay
before  \
2003-12-03  ...                   NaN               NaN
NaN
2003-12-04  ...          10326.389648               NaN
12361.179688
2003-12-05  ...          10429.990234          1.003260
12342.650391
2003-12-08  ...          10373.459961         -0.541997
12314.730469
2003-12-09  ...          10045.339844         -3.163073
12177.440430
...         ...                   ...               ...
```

```
...
2024-01-17  ...          35619.179688         -0.787174
15865.919922
2024-01-18  ...          35477.750000         -0.397060
15276.900391
2024-01-19  ...          35466.171875         -0.032635
15391.790039
2024-01-22  ...          35963.269531          1.401611
15308.690430
2024-01-23  ...          36546.949219          1.622988
14961.179688

            Close^HSIchange  Close^FTSEDay before  Close^FTSEchange  \
2003-12-03              NaN                   NaN               NaN
2003-12-04              NaN           4392.000000               NaN
2003-12-05        -0.149899           4378.200195         -0.314203
2003-12-08        -0.226207           4367.000000         -0.255817
2003-12-09        -1.114844           4359.799805         -0.164877
...                     ...                   ...               ...
2024-01-17        -2.160847           7558.299805         -0.481904
2024-01-18        -3.712483           7446.299805         -1.481815
2024-01-19         0.752048           7459.100098          0.171901
2024-01-22        -0.539896           7461.899902          0.037535
2024-01-23        -2.270023           7487.700195          0.345760

            Close^GDAXIDay before  Close^GDAXIchange  Close^FCHIDay
before  \
2003-12-03                    NaN                NaN
NaN
2003-12-04            3875.659912                NaN
3501.929932
2003-12-05            3874.780029          -0.022703
3496.550049
2003-12-08            3841.729980          -0.852953
3457.139893
2003-12-09            3806.540039          -0.915992
3434.909912
...                           ...                ...
...
2024-01-17           16571.679688          -0.304057
7398.000000
2024-01-18           16431.689453          -0.844756
7318.689941
2024-01-19           16567.349609           0.825601
7401.350098
2024-01-22           16555.130859          -0.073752
7371.640137
2024-01-23           16683.359375           0.774555
7413.250000
```

```
            Close^FCHIchange
2003-12-03               NaN
2003-12-04               NaN
2003-12-05         -0.153626
2003-12-08         -1.127115
2003-12-09         -0.643017
...                      ...
2024-01-17         -0.184576
2024-01-18         -1.072047
2024-01-19          1.129439
2024-01-22         -0.401413
2024-01-23          0.564459

[5241 rows x 80 columns]
```

```
# Filling up the NaNs
merged_df_indices = merged_df_indices.fillna(method='ffill')
merged_df_indices = merged_df_indices.fillna(method='bfill')
merged_df_indices_2 = merged_df_indices['2003-12-3':'2021-11-22']
merged_df_indices_2.isnull().sum()
```

```
Open                        0
High                        0
Low                         0
Close                       0
Adj Close                   0
                           ..
Close^FTSEchange            0
Close^GDAXIDay before       0
Close^GDAXIchange           0
Close^FCHIDay before        0
Close^FCHIchange            0
Length: 80, dtype: int64
```

```
# Initial NORD_max
features_NORD_max_indices = [['Day_1_before', 'Day_2_before',
'Day_3_before',  'JPYchange', 'CNYchange', 'GBPchange']]
result_NORD_max_indices = best_features(features_NORD_max_indices,
merged_df_indices_2)
result_NORD_max_indices
```

```
<pandas.io.formats.style.Styler at 0x16135e0f550>
```

```
# Using methodology A for feature selection.
features_indices = ['Close^IXICchange', 'Close^DJIchange',
'Close^HSIchange', 'Close^FTSEchange',
                    'Close^GDAXIchange', 'Close^FCHIchange']
fixed_features = ['Day_1_before', 'Day_2_before', 'Day_3_before',
'JPYchange', 'CNYchange', 'GBPchange']
result_indices = methodology_A(fixed_features, features_indices,
```

```
merged_df_indices_2, 2.954338)
result_indices
```

```
<pandas.io.formats.style.Styler at 0x1615bf00340>
```

Methodologies A and B do not deem any of the new features important.

Summary:

Algorithms are trained on the interval from 3.12.2003, until 25.9.2017.

The NORD value on the validation set of a model using Logistic Regression as the algorithm, with features Day_1_before, Day_2_before, and Day_3_before, JPYchange, CNYchange, GBPchange and trained on the interval from 30.1.2001, until 25.9.2017, is 2.95

The NORD - BUY AND HOLD on the validation set is equal to 1.88.

Methodologies A and B do not deem any of the new features important.

Upon applying common sense, we decide to not include any of the new features to our model.

Our feature vector does not change and remains the same: X=[Day_1_before, Day_2_before, Day_3_before, JPYchange, CNYchange, GBPchange].

Conclusion

```
merged_df_indices
```

```
                   Open         High          Low        Close     Adj
Close  \
2003-12-03  1066.619995  1074.300049  1064.630005  1064.729980
1064.729980
2003-12-04  1064.729980  1070.369995  1063.150024  1069.719971
1069.719971
2003-12-05  1069.719971  1069.719971  1060.089966  1061.500000
1061.500000
2003-12-08  1061.500000  1069.589966  1060.930054  1069.300049
1069.300049
2003-12-09  1069.300049  1071.939941  1059.160034  1060.180054
1060.180054
...                 ...          ...          ...          ...
...
2024-01-17  4739.129883  4744.229980  4714.819824  4739.209961
4739.209961
2024-01-18  4760.100098  4785.790039  4740.569824  4780.939941
4780.939941
2024-01-19  4796.279785  4842.069824  4785.870117  4839.810059
4839.810059
2024-01-22  4853.419922  4868.410156  4844.049805  4850.430176
4850.430176
2024-01-23  4856.799805  4866.479980  4844.370117  4864.600098
```

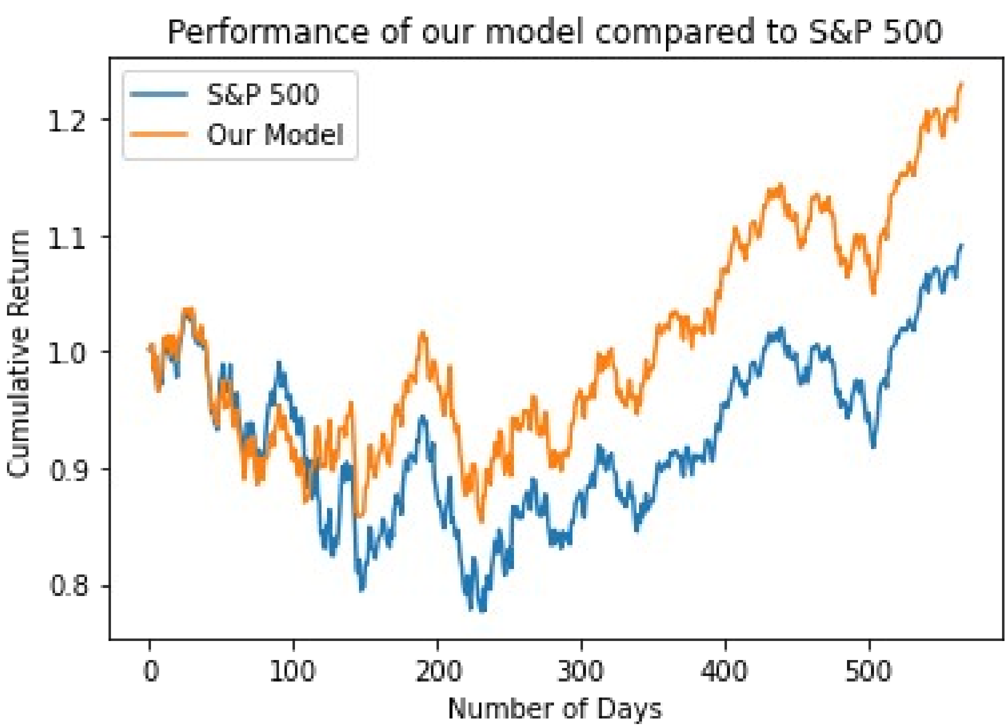
```
4864.600098

             Volume  Close day before  pct_change  Direction  
Day_1_before  \
2003-12-03  1441.70       1066.619995   -0.177197        0.0     -
0.327066
2003-12-04  1463.10       1064.729980    0.468663        1.0     -
0.177197
2003-12-05  1265.90       1069.719971   -0.768423        0.0      
0.468663
2003-12-08  1218.90       1061.500000    0.734814        1.0     -
0.768423
2003-12-09  1465.50       1069.300049   -0.852894        0.0      
0.734814
...             ...               ...         ...        ...      
...
2024-01-17  3928.60       4765.979980   -0.561690        0.0     -
0.373134
2024-01-18  4019.00       4739.209961    0.880526        1.0     -
0.561690
2024-01-19  4287.20       4780.939941    1.231350        1.0      
0.880526
2024-01-22  4297.61       4839.810059    0.219433        1.0      
1.231350
2024-01-23  3912.80       4850.430176    0.292137        1.0      
0.219433

            ...  Close^N225Day before  Close^N225change  Close^HSIDay 
before  \
2003-12-03  ...          10326.389648          1.003260           
12361.179688
2003-12-04  ...          10326.389648          1.003260           
12361.179688
2003-12-05  ...          10429.990234          1.003260           
12342.650391
2003-12-08  ...          10373.459961         -0.541997           
12314.730469
2003-12-09  ...          10045.339844         -3.163073           
12177.440430
...         ...                   ...               ...           
...
2024-01-17  ...          35619.179688         -0.787174           
15865.919922
2024-01-18  ...          35477.750000         -0.397060           
15276.900391
2024-01-19  ...          35466.171875         -0.032635           
15391.790039
2024-01-22  ...          35963.269531          1.401611           
15308.690430
```

```
2024-01-23  ...          36546.949219          1.622988
14961.179688

            Close^HSIchange  Close^FTSEDay before  Close^FTSEchange  \
2003-12-03        -0.149899           4392.000000         -0.314203
2003-12-04        -0.149899           4392.000000         -0.314203
2003-12-05        -0.149899           4378.200195         -0.314203
2003-12-08        -0.226207           4367.000000         -0.255817
2003-12-09        -1.114844           4359.799805         -0.164877
...                     ...                   ...               ...
2024-01-17        -2.160847           7558.299805         -0.481904
2024-01-18        -3.712483           7446.299805         -1.481815
2024-01-19         0.752048           7459.100098          0.171901
2024-01-22        -0.539896           7461.899902          0.037535
2024-01-23        -2.270023           7487.700195          0.345760

            Close^GDAXIDay before  Close^GDAXIchange  Close^FCHIDay
before  \
2003-12-03            3875.659912          -0.022703
3501.929932
2003-12-04            3875.659912          -0.022703
3501.929932
2003-12-05            3874.780029          -0.022703
3496.550049
2003-12-08            3841.729980          -0.852953
3457.139893
2003-12-09            3806.540039          -0.915992
3434.909912
...                           ...                ...
...
2024-01-17           16571.679688          -0.304057
7398.000000
2024-01-18           16431.689453          -0.844756
7318.689941
2024-01-19           16567.349609           0.825601
7401.350098
2024-01-22           16555.130859          -0.073752
7371.640137
2024-01-23           16683.359375           0.774555
7413.250000

            Close^FCHIchange
2003-12-03         -0.153626
2003-12-04         -0.153626
2003-12-05         -0.153626
2003-12-08         -1.127115
2003-12-09         -0.643017
...                      ...
2024-01-17         -0.184576
2024-01-18         -1.072047
```

```
2024-01-19         1.129439
2024-01-22        -0.401413
2024-01-23         0.564459

[5241 rows x 80 columns]
```

```
# Authors model
Features_authors_model = merged_df_indices[['Day_1_before',
'Day_2_before', 'Day_3_before','JPYchange', 'CNYchange', 'GBPchange']]
Y = merged_df_indices[['Direction']]
accuracy_authors, y_test_authors, y_pred_class_authors,
y_pred_prob_authors =
fit_logistic_regression_test_data(Features_authors_model, Y, threshold
= 0.5)
val_authors_model = investment_return( y_pred_class_authors.reshape(-
1), merged_df_indices['2021-11-23':])["dollars"].iloc[-1]
plot_investment_performance( y_pred_class_authors.reshape(-1),
merged_df_indices['2021-11-23':'2024-01-23'])
plt.savefig('investment_performance_plot_authors_model.svg',
format='svg')
```

```
Optimization terminated successfully.
         Current function value: 0.684770
         Iterations 5
```




```
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```
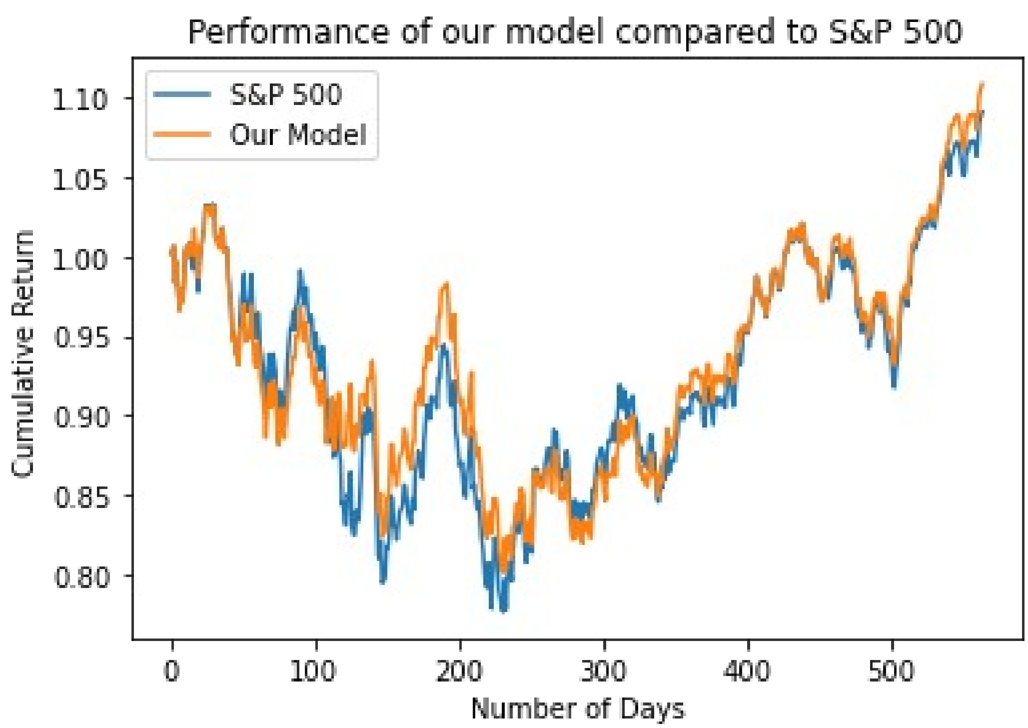

```
val_authors_model
```

```
1.2297666681535147
```

```
# Methodology B
Features_methodology_b = merged_df_indices[['Day_1_before',
'Day_4_before', 'Day_5_before','JPYchange', 'CNYchange']]
Y = merged_df_indices[['Direction']]
accuracy_methodology_b, y_test_methodology_b,
y_pred_class_methodology_b, y_pred_prob_methodology_b =
fit_logistic_regression_test_data(Features_methodology_b, Y, threshold
= 0.5)
val_methodology_b =
investment_return( y_pred_class_methodology_b.reshape(-1),
merged_df_indices['2021-11-23':])["dollars"].iloc[-1]
plot_investment_performance( y_pred_class_methodology_b.reshape(-1),
merged_df_indices['2021-11-23':'2024-01-23'])
plt.savefig('investment_performance_plot_methodology_b.svg',
format='svg')
```

```
Optimization terminated successfully.
         Current function value: 0.684559
         Iterations 5
```




```
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```
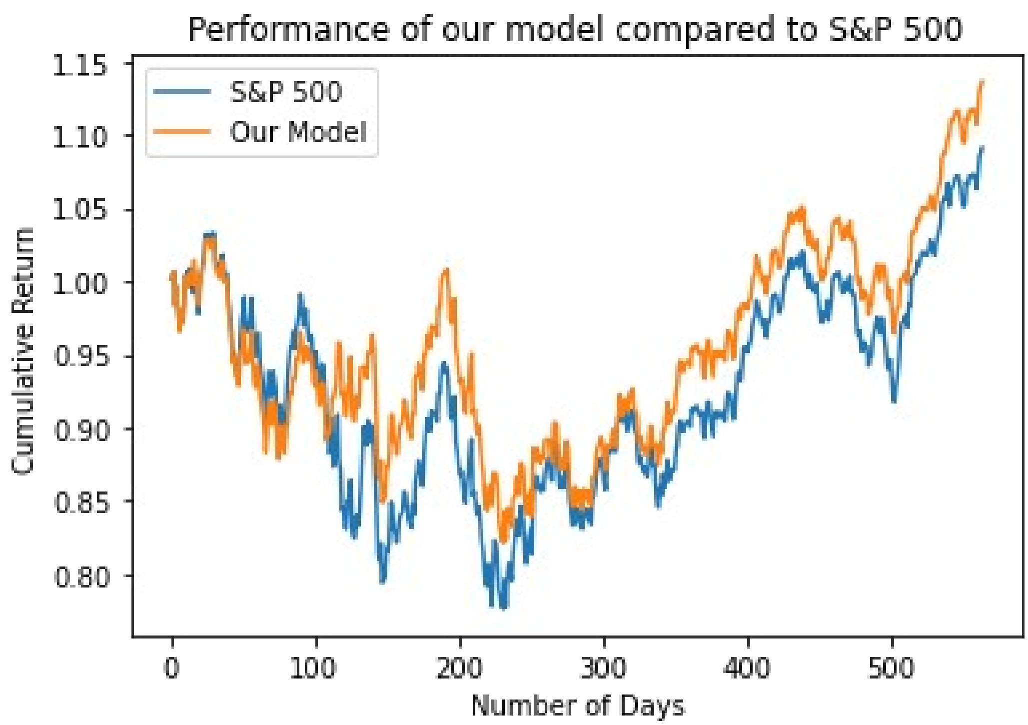

```
val_methodology_b
```

```
1.1083267819658968
```

```python
# Methodology A
Features_methodology_a = merged_df_indices[['Day_1_before',
'Day_4_before', 'Day_5_before','EURchange','JPYchange', 'CNYchange',
'GBPchange']]
Y = merged_df_indices[['Direction']]
accuracy_methodology_a, y_test_methodology_a,
y_pred_class_methodology_a, y_pred_prob_methodology_a =
fit_logistic_regression_test_data(Features_methodology_a, Y, threshold
= 0.5)
val_methodology_a =
investment_return( y_pred_class_methodology_a.reshape(-1),
merged_df_indices['2021-11-23':])["dollars"].iloc[-1]
plot_investment_performance( y_pred_class_methodology_a.reshape(-1),
merged_df_indices['2021-11-23':'2024-01-23'])
plt.savefig('investment_performance_plot_methodology_a.svg',
format='svg')
```

```
Optimization terminated successfully.
         Current function value: 0.684396
         Iterations 5
```

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```

```python
val_methodology_a
```

```
1.1362131458803497
```

```python
# All Features
Features_all_features = merged_df_indices[['Volume', 'Day_1_before',
'Day_2_before', 'Day_3_before',
'Day_4_before', 'Day_5_before', 'Day_6_before', 'Day_7_before',
'Day_8_before', 'Day_9_before',
'Open_pct_change_Day_before',
'High_pct_change_Day_before','Low_pct_change_Day_before',
'Change_volume',
'VIX_change', 'EURchange', 'GBPchange', 'JPYchange', 'CHFchange',
'CNYchange', 'BRLchange', 'ZARchange',
'Close^IXICchange','Close^DJIchange','Close^N225change',
'Close^HSIchange','Close^FTSEchange',
'Close^GDAXIchange', 'Close^FCHIchange']]
Y = merged_df_indices[['Direction']]
accuracy_all_features, y_test_all_features, y_pred_class_all_features,
y_pred_prob_all_features =
fit_logistic_regression_test_data(Features_all_features, Y, threshold
= 0.5)
val_all_features =
investment_return( y_pred_class_all_features.reshape(-1),
merged_df_indices['2021-11-23':])["dollars"].iloc[-1]
plot_investment_performance( y_pred_class_all_features.reshape(-1),
merged_df_indices['2021-11-23':'2024-01-23'])
plt.savefig('investment_performance_plot_all_features.svg',
format='svg')
```

```
Optimization terminated successfully.
         Current function value: 0.677693
         Iterations 5
```


```
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val_all_features

1.131649851439757
```