Matrix Operation Cheat Sheet

Definitions

Let $\mathbb{K}$ be a field of real $\mathbb{R}$ or complex numbers $\mathbb{C}$. Then we can define matrix A a *table of numbers*

$$A = (a_{ij}) = \begin{bmatrix} a_{11} & \dots & a_{1m} \\ \vdots & \ddots & \vdots \\ a_{n1} & \dots & a_{nm} \end{bmatrix} \in \mathbb{K}^{n \times m}$$

Inverse matrix

A matrix $A \in \mathbb{C}^{n \times n}$ is called invertible if exists an inverse matrix $B \in \mathbb{C}^{n \times n}$ such that:

$$AB = BA = I_n$$

`numpy.linalg.inv()` / `INV()`

Hermitian matrix

For matrix $(a_{ij}) = A \in \mathbb{C}^{n \times m}$ a hermitian is A^*

$$A^* = \overline{A}^\top = (\overline{a}_{ji})$$

Often denoted as conjugate transpose.

`A.H or A.getH()` / `A'`

Unitary matrix

A matrix $U \in \mathbb{C}^{n \times m}$ is unitary if its hermitian is its inverse.

$$U^* U = U U^* = I$$

Moore–Penrose inverse

For $A \in \mathbb{C}^{n \times m}$ a pseudo-inverse of A is defined as a matrix $A^\dagger \in \mathbb{C}^{m \times n}$ satisfying:

1. $AA^\dagger A = A$
2. $A^\dagger A A^\dagger = A^\dagger$
3. $(AA^\dagger)^* = AA^\dagger$
4. $(A^\dagger A)^* = A^\dagger A$

When A has linearly independent columns:

$$A^\dagger = (AA^*)^{-1} A^* => A^\dagger A = I$$

When A has linearly independent rows:

$$A^\dagger = A^* (AA^*)^{-1} => AA^\dagger = I$$

`numpy.linalg.pinv()` / `PINV()`

Matrix decomposition

Singular value decomposition (SVD)