Asymmetric Risks: Alphas or Betas?*

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Abstract

TBA.

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1 Introduction

Throughout the last few decades, so-called factor zoo emerged–a large number of factors that are supposed to price the cross-section of stock returns. Still to this date, there is not a clear consensus on how researchers should feel about this claim. Some results suggest that a substantial portion of the factors is proxies for underlying common risks and by including them, we can average out the noise related to each of them and identify the driving force behind the formation of expected returns (Kozak et al., 2020).

Another ongoing discussion in empirical asset pricing regards the discussion of characteristics vs covariances. Risk-based explanation of expected returns claims that only exposures to common movements should constitute price determinants for the cross-section of asset returns. If a characteristic predicts future returns, it should be because this characteristic is a good proxy of systematic risk exposure. Similarly, as in the factor zoo discussion, there is still no obvious conclusion. Some results claim that we can form an arbitrage portfolio that enjoys abnormal returns without exposure to systematic risk (Kim et al., 2020; Lopez-Lira and Roussanov, 2020), while others suggest that characteristics capture all the important pricing information (Kelly et al., 2019, 2023). Moreover, the exposures to the common fluctuations should be fully described by the betas, which are based on simple covariance measures of dependence.

Using innovative techniques, much of the progress was made in recent years in both strands of the literature separately, but also simultaneously. Unfortunately, those research efforts usually focus on various accounting variables and characteristics based on simple market frictions, only, and neglect various measures of non-linear systematic dependence between stock and various factors. Those measures capture the joint behavior of stock and factor during extreme market events, which are not detected by usual regression coefficients (betas) obtained from regressing stock returns on some tradable factors. I utilize two types of asymmetric risk measures here. The first one captures systematic exposure using an asymmetric non-linear type of dependence, such as the downside beta of Ang et al. (2006). The second one is defined by utilizing an asymmetric non-linear type of aggregate risk factor, such as the tail risk beta of Kelly and Jiang (2014).

Studies usually avoid this type of market friction characteristic probably because of slightly higher difficulties related to their estimation in comparison to accounting variables. I argue that these kinds of risks are interesting because they possess a special place among characteristics. They capture the joint behavior of stock return and some aggregate measure of risk (e.g., return on the whole market), but the measured dependence goes beyond linearity and symmetry of standard covariances with tradable factors. Consequently, it is difficult to decide how much of the risk premium associated with the asymmetric risk measures is due to the non-linear nature of the dependence and how much is attributable to the overall covariance-based dependence.

I aim to fill this void by entertaining a representative set of systematic asymmetric risk measures and employing them in their multivariate context. Moreover, I evaluate the additional information they possess for asset prices when controlling for conventional characteristics previously used in similar studies.

1.1 Theoretical Motivation

The empirical research, centered around the expected utility assumption, focuses on the implementation of the equation

$$\mathbb{E}_t[m_{t+1} r_{i,t+1}] = 0, \tag{1}$$

which can be interpreted in terms of (co)variances as

$$\mathbb{E}_t[r_{i,t+1}] = \underbrace{\frac{\mathbb{C}ov_t(m_{t+1}, r_{i,t+1})}{\mathbb{V}ar_t(m_{t+1})}}_{\beta^m_{i,t}} \underbrace{\left(-\frac{\mathbb{V}ar_t(m_{t+1})}{\mathbb{E}_t[m_{t+1}]}\right)}_{\lambda_t}. \tag{2}$$

This statement implies that the priced exposure to the risk is adequately measured by the regression coefficient, $\beta^m_{i,t}$, obtained from regressing excess stock return on the stochastic discount factor, m_{t+1}. Further, if we assume linearity of the discount factor in some set of factors f, which proxy for the growth of marginal substitution, i.e., $m_{t+1} = \delta \frac{u'(c_{t+1})}{u'(c_t)} \approx a + b' f_{t+1}$, this leads to

$$\mathbb{E}_t[r_{i,t+1}] = \alpha_{i,t} + \lambda' \beta_{i,t} \tag{3}$$

$$r_{i,t+1} = \alpha_{i,t} + \beta'_{i,t} f_{t+1} + \epsilon_{i,t+1} \tag{4}$$

where $\beta_{i,t}$ are the multiple regression coefficients of $r_{i,t}$ on f_t, and λ is vector of risk prices associated with factors f. In the case of tradable factors, λ is equal to the expected value of f. This line of reasoning constitutes a base for the empirical factor literature such as the arbitrage pricing theory of Ross (1976), the three-factor model of Fama and French (1993), etc. One of the main implications of the theory is that the non-systematic part of the risk, $\alpha_{i,t}$, should be equal to zero. Statistical tests such as Gibbons et al. (1989) provide inference on goodness of fit by testing this restriction.

On the other hand, there are models that deviate from the expected utility framework and/or linearity assumption of the stochastic discount factor. Examples of the former are models that introduce some form of behavioral bias, such as the disappointment aversion utility of Gul (1991). Based on that framework, Ang et al. (2006) introduced a cross-sectional relation between expected returns and downside beta, dependence between market and stock return conditional on the market being below its mean. A pioneer of the later violation is the work of Harvey and Siddique (2000), which assumes that the stochastic discount factor is *quadratic* in the market return, which introduces conditional systematic skewness as a priced risk characteristic. Based on the recursive utility with disappointment aversion of Routledge and Zin (2010), Farago and Tédongap (2018) argue that betas with various asymmetric specifications of market return and volatility should be significantly priced in the cross-section.

Based on those arguments, risk exposure cannot be sufficiently captured by the simple betas with tradable factors. The cross-sectional relation between stock returns and risk changes to

$$\mathbb{E}_t[r_{i,t+1}] = \delta' g(r_{i,t+1}, f^*_{t+1}) + \lambda' \beta_{i,t} \tag{5}$$

$$r_{i,t+1} = \delta' g(r_{i,t+1}, f^*_{t+1}) + \beta'_{i,t} f_{t+1} + \epsilon_{i,t+1} \tag{6}$$

where g is a function of asset return and some factor–*asymmetric risk measure* (ARM) with a vector of related prices of risk δ. The uniqueness of the ARM can lie either in the choice of the dependence function g or in the choice of the factor f^*. We can see that these specifications lead to the rejection of the non-significant alpha assumption from above.

In recent years, researchers proposed many asymmetric risk measures to possess the ability to explain and predict stock returns. Those studies usually control for some pre-specified set of factors and conclude that abnormal returns cannot be explained by exposure to those factors. Because the choice of the factors will be always somewhat arbitrary, I want to entertain the question of whether there is any set of factors that can eliminate significant alphas related to the asymmetric risk measures. I investigate this question in their multivariate setting using a representative set of asymmetric risk measures. I estimate arbitrage returns that can be exploited after hedging out the common movements related to the asymmetric risks and decide whether the arbitrage returns survive.

Next, I inspect the question of whether there is a redundancy among the asymmetric risk measures. In other words, I ask the question of how many independent dimensions the asymmetric risk measures possess in regard to the expected returns and which of them are significant. Related to this, I evaluate the asymmetric risk measures in the context of commonly used characteristics that were employed in similar studies.

1.2 Main Results

The main result of the paper constitutes the finding that anomaly returns associated with asymmetric risk measures can be explained by a factor model. I let the asymmetric risk measures proxy for the exposure to common linear factors and investigate residual power for creating abnormal returns. Using a set of eleven asymmetric risk measures, six factors are needed to eliminate those anomaly returns in the case of all stocks. When working with a highly liquid dataset, the anomaly returns vanish after the inclusion of four latent factors.

Although the anomaly returns disappear after the inclusion of a sufficiently high number of latent factors, asymmetric risk measures can be combined into a trading strategy with an out-of-sample Sharpe ratio of up to 0.97 in case of all-stock analysis, and 0.8 in case of no-penny investigation. On the other hand, investing based on tangency portfolios with asymmetric risks explaining the factor loadings can yield a Sharpe ratio of around 1.15 and 1.10 for respective datasets.

The anomaly portfolio returns are generally exposed to the momentum factor. Assuming a constant relation between asymmetric risk measures and arbitrage portfolio formation, the abnormal returns are only partially diminished by accounting for this exposure. Allowing for time variation in the relation, the loss of efficiency leads to the anomaly returns being seized fully by the momentum.

When assessing the significance of abnormal returns related to the asymmetric risk measures separately, latent factors explain the related risk premiums. On the other hand, when we form pure-alpha portfolios that hedge the common exposures (given by one to six latent factors), resulting abnormal returns cannot be explained by previously proposed latent factor models.

The rest of the paper is structured as follows...

2 Asymmetric Risk Measures

In this section, I provide a first look at the asymmetric risk measures that are employed in the main analysis.

2.1 Data

In the empirical investigation, I employ a representative set of eleven asymmetric risk measures. Those are coskewness (`coskew`) of Harvey and Siddique (2000), cokurtosis (`cokurt`) of Dittmar (2002), downside beta (`beta_down`) of Ang et al. (2006), downside correlation (`down_corr`) based on Hong et al. (2006) and Jiang et al. (2018), hybrid tail covariance risk (`htcr`) of Bali et al. (2014), tail risk beta (`beta_tr`) of Kelly and Jiang (2014), exceedance coentropy measure (`coentropy`) based on Backus et al. (2018) and Jiang et al. (2018), predicted systematic coskewness (`cos_pred`) of Langlois (2020), negative semibeta (`beta_neg`) of Bollerslev et al. (2021), multivariate crash risk (`mcrash`) of Chabi-Yo et al. (2022), and downside common idiosyncratic quantile risk (CIQ) beta (`ciq_down`) of Barunik and Nevrla (2022). The choice of the variables corresponds to the fact that they capture different aspects of the return dependence in terms of non-linearity and/or asymmetry. I provide an overview how the measures are estimated in Appendix A. I estimate those measures using either daily or monthly return data from the CRSP database that starts in January 1963 and ends in December 2018.

As a control variables, I use a set of 32 characteristics from Freyberger et al. (2020). Those characteristics are intersection between data used in Freyberger et al. (2020) and Kelly et al. (2019). Those characteristics are used as controls for assessing an additional value of the ARMs with respect to asset prices. Moreover, I include only observations that posses information on all the characteristics. I use an initial window of 5 years to estimate the ARMs; because of that, the first prediction period constitutes January 1968. Besides the full dataset, I also employ dataset that strips down penny stocks, which I define as stocks with a price less than $5 or with capitalization below 10% quantile of the NYSE-traded stocks each month.

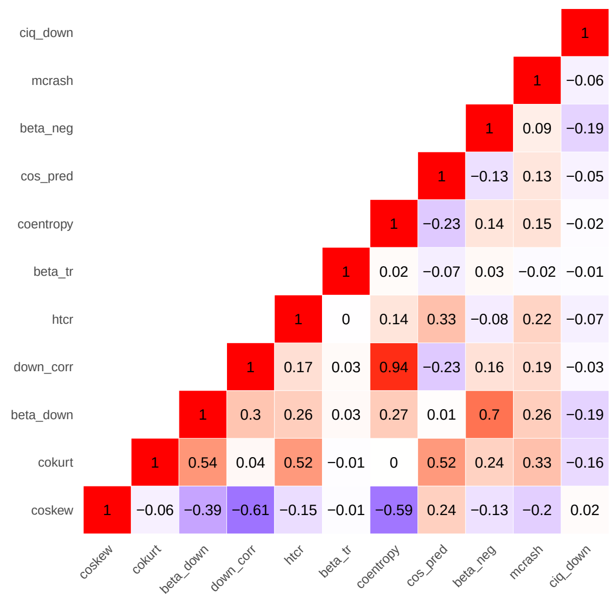
I merge the dataset of ARMs with the characteristics dataset to work with a stock universe that is fully transparent for investors and is eligible for trading based on wide variety of characteristics. The full merged dataset of all available stocks then contains 1,519,754 stock-month observations of 12,505 unique stocks. The dataset that excludes penny stocks yields 947,897 stock-month observations of 8,477 unique stocks.

2.2 Correlation Structure

First, I investigate the correlation structure of the ARM characteristics. Figure 1 contains correlations between ARMs themselves. Correlations are obtained as time-series averages of the cross-sectional correlations. We can see that the highest absolute values of correlations are between coentropy and downside correlation with value of 0.94, downside beta and negative semibeta with value of 0.70, and coskewness and downside correlation with value of -0.61.

Panel A of Table 1 summarizes how each measure is in general related to the others by reporting average absolute correlations across all measures. We observe that the downside beta possess the highest level of similarity with other measures with the average absolute

Figure 1: *Correlation structure across ARMs.* The figure captures time-series averages of cross-sectional correlations between asymmetric risk measures. Data include all available stocks and the period between January 1968 and December 2018.



correlation being equal to 0.29. On the other hand, the least correlated measure is tail risk beta with the average value of only 0.02. Results using the non-penny dataset draw similar results with values being of the same magnitude.

2.3 Fama-MacBeth Regressions

Next, I present first results on how the ARMs align with the cross-section of asset returns. To do that, I run Fama and MacBeth (1973) cross-sectional regressions and report the results in Table 2 in Panel A. I report both univariate estimates and estimates obtained by controlling for four characteristics widely employed in the literature – market beta, size, book-to-market, and momentum. Below the estimated coefficient, I include t-statistics based on the Newey-West robust standard errors.

From the univariate results, it is obvious that the ARMs vary considerably in their significance for the cross-sectional pricing implications. Looking at the all-stock results, the highest significance possess the downside CIQ beta with t-statistics of 2.69. Cokurtosis yields t-statistics of -3.15, unfortunately, the sign of the coefficient is counterintuitive. Coskewness is, on the other side, significant with an expected sign. Tail risk beta is borderline significant with a t-stat of 1.89. The rest of the variables are deemed insignificant in the presented setting. When we move to the controlled setting, most of the variables become slightly less significant with few exceptions such as tail risk beta, which becomes significant (t-stat=2.10)

Table 1: *Average correlations of ARMs.* Panel A of the table reports time-series averages of cross-sectional correlations for each ARM averaged across all other ARMs or 32 characteristics employed in Kelly et al. (2019). Panel B reports average correlations between managed portfolios. The average correlation for each ARM is obtained by averaging correlations across all other AMR portfolios or by averaging across 32 characteristic managed portfolios. The table reports results for all stocks or dataset that excludes stocks with a price less than $5 or market cap below 10% quantile of NYSE stocks. Data cover the period between January 1968 and December 2018.

Panel A: Variables	*All stocks*		*No penny stocks*	
	with ARMs	with characteristics	with ARMs	with characteristics
`coskew`	0.24	0.02	0.25	0.01
`cokurt`	0.24	0.11	0.22	0.06
`beta_down`	0.29	0.08	0.32	0.10
`down_corr`	0.27	0.02	0.28	0.03
`htcr`	0.19	0.11	0.17	0.06
`beta_tr`	0.02	0.02	0.04	0.03
`coentropy`	0.25	0.02	0.27	0.02
`cos_pred`	0.20	0.12	0.19	0.10
`beta_neg`	0.19	0.13	0.23	0.14
`mcrash`	0.16	0.05	0.17	0.03
`ciq_down`	0.08	0.04	0.07	0.05
Panel B: Managed portfolios				
`coskew`	0.32	0.16	0.36	0.14
`cokurt`	0.30	0.35	0.30	0.23
`beta_down`	0.40	0.36	0.43	0.40
`down_corr`	0.39	0.22	0.41	0.19
`htcr`	0.29	0.48	0.20	0.16
`beta_tr`	0.07	0.08	0.11	0.14
`coentropy`	0.39	0.24	0.41	0.19
`cos_pred`	0.39	0.43	0.36	0.31
`beta_neg`	0.36	0.47	0.38	0.41
`mcrash`	0.32	0.25	0.35	0.17
`ciq_down`	0.21	0.18	0.29	0.30

or downside beta, which becomes also significant, but with a negative sign.

Panel B of Table 2 reports the results using a dataset that excludes penny stocks. Generally, coefficients become more significant (or less significant, if they possess a counterintuitive sign in the all-stock sample). For example, hybrid tail covariance risk (t-stat=4.57) or downside correlation (t-stat=2.38) become highly significant. Some of the variables become even more significant when controlling for other risk measures, such as multivariate crash risk (t-stat=2.04) or tail risk beta (t-stat=3.52).

2.4 Portfolio Sorts

Next, to briefly inspect the tradability of the ARMs, I perform simple univariate portfolio sorts based on the ARMs. I focus here on a simple portfolio formation based on the following

Table 2: *Fama-MacBeth regressions.* The table reports the risk premiums of the ARMs estimated using Fama-MacBeth regressions. Below the coefficients, I include their HAC t-statistics based on Newey and West (1987) using lag auto-selection of Newey and West (1994). Panel A reports results using all stocks, Panel B excludes stocks with a price less than \$5 or market cap below 10% quantile of NYSE stocks. Data cover the period between January 1968 and December 2018.

	Panel A: All stocks						***Panel B: No penny stocks***					
	univariate	multivariate					univariate	multivariate				
	ARM	ARM	β	Size	BM	MOM	ARM	ARM	β	Size	BM	MOM
`coskew`	-0.57	-0.39	-0.13	-0.15	0.22	0.49	-0.62	-0.36	-0.26	-0.12	0.12	0.53
	(-2.17)	(-1.62)	(-0.75)	(-1.72)	(3.23)	(3.23)	(-2.21)	(-1.56)	(-1.41)	(-1.63)	(1.41)	(3.41)
`cokurt`	-0.21	-0.12	-0.10	-0.14	0.21	0.51	-0.08	0.04	-0.27	-0.18	0.13	0.53
	(-3.15)	(-1.28)	(-0.47)	(-1.95)	(3.39)	(3.51)	(-1.24)	(0.60)	(-1.39)	(-2.58)	(1.45)	(3.35)
`beta_down`	-0.12	-0.14	-0.02	-0.15	0.21	0.50	-0.07	-0.05	-0.20	-0.12	0.12	0.53
	(-1.29)	(-2.43)	(-0.14)	(-1.58)	(3.08)	(3.26)	(-0.52)	(-0.63)	(-1.24)	(-1.55)	(1.36)	(3.43)
`down_corr`	0.18	-0.03	-0.13	-0.16	0.22	0.50	0.35	0.08	-0.25	-0.12	0.13	0.52
	(1.47)	(-0.32)	(-0.76)	(-1.66)	(3.20)	(3.19)	(2.38)	(0.83)	(-1.40)	(-1.57)	(1.51)	(3.31)
`htcr`	34.30	-1.55	-0.13	-0.16	0.19	0.53	201.36	140.20	-0.24	-0.17	0.12	0.51
	(0.76)	(-0.05)	(-0.75)	(-1.91)	(3.00)	(3.84)	(4.57)	(4.28)	(-1.35)	(-2.37)	(1.40)	(3.29)
`beta_tr`	0.16	0.15	-0.13	-0.14	0.21	0.51	0.28	0.25	-0.24	-0.11	0.12	0.51
	(1.89)	(2.10)	(-0.78)	(-1.52)	(3.06)	(3.31)	(2.77)	(3.52)	(-1.37)	(-1.52)	(1.38)	(3.29)
`coentropy`	0.13	-0.08	-0.13	-0.16	0.22	0.50	0.35	0.03	-0.25	-0.12	0.13	0.53
	(0.82)	(-0.64)	(-0.75)	(-1.72)	(3.21)	(3.22)	(1.76)	(0.21)	(-1.39)	(-1.64)	(1.52)	(3.36)
`cos_pred`	-3.05	-0.20	-0.15	-0.18	0.21	0.49	-1.97	1.29	-0.29	-0.19	0.14	0.56
	(-1.78)	(-0.11)	(-0.88)	(-2.69)	(3.33)	(3.17)	(-1.16)	(0.91)	(-1.68)	(-3.00)	(1.64)	(3.52)
`beta_neg`	-0.12	0.30	-0.26	-0.14	0.20	0.51	-0.53	-0.45	-0.06	-0.13	0.11	0.54
	(-0.29)	(0.78)	(-2.12)	(-1.65)	(2.92)	(3.48)	(-1.33)	(-1.39)	(-0.42)	(-1.81)	(1.25)	(3.51)
`mcrash`	0.24	0.29	-0.14	-0.17	0.23	0.49	1.55	1.19	-0.26	-0.14	0.13	0.52
	(0.29)	(0.50)	(-0.80)	(-1.78)	(3.34)	(3.18)	(1.85)	(2.04)	(-1.45)	(-1.78)	(1.54)	(3.30)
`ciq_down`	0.09	0.05	-0.12	-0.15	0.21	0.49	0.09	0.04	-0.25	-0.12	0.13	0.52
	(2.69)	(2.05)	(-0.72)	(-1.64)	(3.17)	(3.14)	(2.24)	(1.58)	(-1.43)	(-1.62)	(1.50)	(3.31)

scheme

$$x_{t+1} = \frac{Z_t' r_{t+1}}{N_{t+1}} \tag{7}$$

where Z_t is a vector of some stock characteristic observed at time t, r_{t+1} is a vector of excess returns of the stocks in the next period, and N_{t+1} is the number of stock observations in a given month. I will refer to this type of portfolio as a *managed portfolio* with a corresponding return x_{t+1}. We can see that the return of the managed portfolio is obtained as a weighted average of stock returns where the weights are the values of the characteristics and normalized by the number of stock observations.

To obtain the weights corresponding to a given characteristic, every month, I cross-sectionally rank the values of the characteristic, divide the rank by the number of observations in the month, and subtract 0.5. This procedure transforms the characteristics into the interval $[-0.5, 0.5]$. This eliminates the effect of outliers and the resulting return can be interpreted as a zero-cost portfolio return associated with the characteristic. This return is also used for the later analysis using the instrumented principal component analysis.

Table 3: *Managed portfolio returns.* The table contains annualized out-of-sample returns of the managed portfolios sorted on various asymmetric risk measures. It reports corresponding t-statistics, Sharpe ratio (SR), and annualized 6-factor alphas and their t-statistics with respect to the four factors of Carhart (1997), CIV shocks of Herskovic et al. (2016), and BAB factor of Frazzini and Pedersen (2014). I use the HAC t-statistics of Newey and West (1987) with 6 lags. Panel A reports results using all stocks, Panel B excludes stocks with a price less than \$5 or market cap below 10% quantile of NYSE stocks. Data cover the period between January 1968 and December 2018.

	Panel A: All stocks					*Panel B: No penny stocks*				
	Mean	t-stat	SR	α	t-stat	Mean	t-stat	SR	α	t-stat
`coskew`	-0.30	-2.51	-0.32	-0.23	-1.52	-0.28	-2.34	-0.30	-0.10	-0.66
`cokurt`	-0.39	-2.29	-0.29	-0.10	-0.57	-0.07	-0.48	-0.06	0.23	1.73
`beta_down`	-0.27	-1.27	-0.16	0.09	0.63	-0.13	-0.53	-0.07	0.11	0.84
`down_corr`	0.15	1.80	0.22	0.09	0.84	0.24	2.64	0.33	0.04	0.39
`htcr`	0.00	0.01	0.00	-0.14	-0.66	0.37	2.86	0.42	0.32	2.63
`beta_tr`	0.32	2.28	0.32	0.31	1.44	0.35	2.56	0.36	0.18	1.12
`coentropy`	0.11	1.37	0.16	0.07	0.61	0.18	2.03	0.25	-0.01	-0.08
`cos_pred`	-0.46	-1.76	-0.26	-0.50	-1.85	-0.22	-0.97	-0.14	-0.11	-0.56
`beta_neg`	-0.13	-0.38	-0.05	0.34	1.83	-0.35	-1.18	-0.16	-0.03	-0.26
`mcrash`	0.03	0.36	0.05	0.06	0.63	0.16	1.74	0.25	0.14	1.52
`ciq_down`	0.41	2.83	0.42	0.52	3.58	0.36	2.29	0.34	0.44	3.24

The returns of the managed portfolios sorted based on ARMs are summarized in Table 3. In the case of all stocks, the highest absolute Sharpe ratio possess the downside CIQ beta with a value of 0.42. In the case of non-penny stocks, the highest Sharpe ratio attains hybrid tail covariance risk with the same value of 0.42. As hinted from the Fama-MacBeth regressions, some of the variables possess a counterintuitive negative premium, e.g., cokurtosis possess a significantly negative risk premium in the universe of all stocks. Another notable example is downside beta which attains negative risk premiums in both samples, but the associated average returns are not significantly different from zero.

Table 3 also reports annualized 6-factor alphas and their t-statistics with respect to six commonly used risk factors. As a general benchmark of risk, I employ four factors of Carhart (1997) including market, size factor, value factor, and momentum factor. To control for the effect of the common volatility, which may be a driver of many tail events, I use the CIV shocks of Herskovic et al. (2016). The BAB factor of Frazzini and Pedersen (2014) aims at controlling the effect of the well-known beta mispricing anomaly. When I control for the exposures to those six factors, the significance of some of the ARM premiums deteriorates, such as in the case of tail risk beta in both samples. On the other hand, some of the premiums do not suffer any decrease in significance if we control for the exposure to those common factors. For example, controlled risk premiums associated with the downside common idiosyncratic quantile risk deliver significant t-stats of 3.58 and 3.24.

To gain some more insight regarding the ARM abnormal returns, in Appendix B, I employ more conventional specifications of the sorts. Tables 18 and 19 summarize portfolio returns from sorting the stock into five and ten portfolios, respectively, with monthly rebalancing. Tables contain results using equal- and value-weighted schemes for both data samples. Results correspond to the implications obtained from the Fama-MacBeth regressions. In the case of all stocks, the highest risk premium carries predicted coskewness using both equal- and value-weighted returns and sorting into either quintile or decile portfolios, although with varying levels of significance.

To gain some intuition regarding the factor structure of the ARMS, in Appendix B in Figure 7, I depict the time-series correlations between managed portfolios sorted on ARMs. Moreover, Panel B of Table 1 contains averages of those correlations for each characteristic. Correlations are noticeably higher than in the case of the values of the characteristics, which can be expected. The most correlated with other ARMs is downside beta, which is closely followed by downside correlation and coentropy in both samples. From those values, there is clearly some common structure, but the question remains whether the exposures to that common structure represent priced determinants of risk.

From the presented results, it is obvious that there is a sizable variation in the magnitude and significance of the risk premiums associated with the ARMs. Those variations can be caused either by selecting the weighting scheme, universe of stocks, number of portfolios, or by their combinations. Moreover, some of the premiums can be at least partly explained by other common factors. Based on those observations, I will explore the formation of the risk premiums associated with ARMs. The following analysis will explore the question of whether those premiums can be attributed to the exposures to the common fluctuations or whether they represent anomalies (alpha vs beta discussion). I will also examine whether there is a possibility to combine the ARMs and form abnormal returns (alphas) that can be exploited in an out-of-sample setting. An interesting question also constitutes the relation of the ARM risk premiums in the context of other characteristics; I will delve into that issue, also.

3 Conditional Risk Premium

In this section, I contribute to the alpha versus beta discussion. I estimate a latent factor model that utilizes the ARMs to account for the maximal possible explanation of the factor loadings to the common factors. Then I look at the residual returns and assess whether the ARMs still possess significant risk premium not explained by those exposures to common movements. The results will indicate whether the special features of the AMRs can be explained by simple linear factors.

3.1 Model and Tests

To estimate the risk premium associated with the ARMs, I use the instrumented principal component analysis (IPCA) model of Kelly et al. (2019, 2020), which can be written as

$$\begin{aligned} r_{i,t+1} &= \alpha_{i,t} + \beta_{i,t} f_{t+1} + \epsilon_{i,t+1}, \\ \alpha_{i,t} &= z'_{i,t}\Gamma_\alpha + \nu_{\alpha,i,t},\ \beta_{i,t} = z'_{i,t}\Gamma_\beta + \nu_{\beta,i,t} \end{aligned} \tag{8}$$

where $\beta_{i,t}$ contains dynamic loadings on $(K \times 1)$ vector of latent factors f_{t+1}. The vector of factor loadings may depend on the instrument $(L \times 1)$ vector $z_{i,t}$ of observable asset characteristics (which includes a constant) through the matrix Γ_β. I use the set of 11 ARMs as the characteristics that may proxy for the exposure to the common factors. Mapping between characteristics and factor loadings serves two purposes. First, it enables the exploitation of other information than just simply return data for the estimation of latent factor loadings and thus makes the estimation more efficient. Second, it naturally makes the loadings time-varying as they are a function of the characteristics and thus makes it valuable tool for estimation of conditional risk premium. Moreover, the model admits the possibility that the characteristics align with the returns in addition to their relation to systematic risk. This feature captures $(L \times 1)$ vector of coefficients Γ_α that maps the characteristics into their anomaly intercepts.

This feature can be used to investigate how well the ARMs proxy for the exposure to the systematic risk, and to test whether they contain some important information beyond that and yield some anomaly (mispricing) returns. To do that, I test the null hypothesis that the ARMs do not proxy for the anomaly alpha. I use the fit of two specifications of the Model 8 and compare their performances. First, the *restricted* model is estimated by setting the Γ_α vector to zero. Second, the *unrestricted* model is obtained by allowing expected returns to align with the ARMs beyond their relation with the systematic risk exposure and thus Γ_α is estimated freely.

To test the hypothesis *in-sample*, I follow Kelly et al. (2019), using Model 8, I test a null hypothesis of $H_0 : \Gamma_\alpha = 0_{L\times 1}$ against an alternative hypothesis $H_1 : \Gamma_\alpha \neq 0_{t\times 1}$. Under the null hypothesis, the characteristics do not yield significant alphas after controlling for their explanatory power regarding the loadings on latent factors. The procedure follows three steps. First, the unrestricted IPCA model is estimated and the parameters and the residuals are saved. I compute a Wald-type test statistic that measures the distance between the restricted and unrestricted model, $W_\alpha = \hat{\Gamma}'_\alpha \hat{\Gamma}_\alpha$. Second, the inference regarding the test statistic is performed using residual bootstrap. In each bootstrap replication, I generate a sample of new managed portfolio returns using the estimated residuals, estimate $\hat{\Gamma}_\beta$

(both from the original unrestricted model) and the restricted model's specification (setting $\Gamma_\alpha = 0$). Then, the generated sample is used to estimate the unrestricted model and the simulated test statistic is saved. Third, the resulting inference is obtained from the simulated distribution of bootstrapped test statistics. A resulting p-value of the test is calculated as a proportion of bootstrapped test statistics that exceed the value of the test statistic from the actual data.

To assess the ARMs *out-of-sample*, I investigate the performance of two portfolios. First, I use the restricted model to form a factor tangency portfolio. Each time t, I estimate the restricted model and set weights of the factor portfolios proportional to $\Sigma_t^{-1}\mu_t$, where Σ_t and μ_t are a covariance matrix and vector of average returns of IPCA factors, respectively, both estimated using information up to time t. The portfolio weights are re-scaled to target 1% monthly volatility based on the historical estimate. The performance of this portfolio indicates how well the ARMs align with the exposures to the common factors and whether those exposures are priced.

Second, I use estimates of the unrestricted model to exploit abnormal returns related to the ARMs. Each time t, I estimate the unrestricted model and form arbitrage portfolio with weights set equal to $w_t = Z_t(Z_t'Z_t)^{-1}\Gamma_\alpha$, which yields conditional factor neutrality. The performance of the pure-alpha portfolios constitutes a natural test of the restricted vs. unrestricted model. The pure-alpha portfolio captures an opportunity for an investor to avoid systematic risk while enjoying the premium related to the ARMs. An appropriate factor model should not leave such an opportunity on the table. Moreover, the performance of the pure-alpha portfolio shows whether the ARMs can be combined to form abnormal returns beyond performances of single-variable sorts.

Those tests decide whether the ARMs line up with the expected returns because they proxy for linear exposure to the systematic risk ($\Gamma_\alpha = 0_{L\times 1}$) or whether the ARMs are related to the average returns without being (completely) related to the factor exposures ($\Gamma_\alpha \neq 0_{L\times 1}$).

3.2 Estimation and Model Fit

Following Kelly et al. (2019), estimation of the restricted model with $\Gamma_\alpha = 0$ is performed using *alternating least squares* and iterating between the first-order conditions for Γ_β and f_{t+1}

$$f_{t+1} = \left(\hat{\Gamma}_\beta' Z_t' Z_t \hat{\Gamma}_\beta\right)^{-1} \hat{\Gamma}_\beta' Z_t' r_{t+1}, \quad \forall t \tag{9}$$

and

$$\mathrm{vec}(\hat{\Gamma}_\beta') = \left(\sum_{t=1}^{T-1} Z_t' Z_t \otimes \hat{f}_{t+1} \otimes \hat{f}_{t+1}'\right)^{-1} \left(\sum_{t=1}^{T-1} \left[Z_t \otimes \hat{f}_{t+1}'\right]' r_{t+1}\right) \tag{10}$$

where r_{t+1} is the $N \times 1$ vector of stock returns and Z_t is the $N \times L$ matrix of stock characteristics. The identifying restrictions are that $\hat{\Gamma}_\beta' \hat{\Gamma}_\beta = \mathbb{I}_K$, the unconditional second moment matrix of f_t is diagonal with descending diagonal entries, and the mean of f_t is non-negative.[1] In the case of the unrestricted version of the model with $\Gamma_\alpha \neq 0$, the estimation proceeds similarly, the only difference is that we augment the vector of factors to include a constant.

I evaluate the performance of the model in terms of two metrics. The first one, *total* R^2, describes how is the model able to capture time variation of the realized returns using conditional loadings and factor realizations

$$\text{Total}\, R^2 = 1 - \frac{\sum_{i,t}\left(r_{i,t+1} - z_{i,t}'(\hat{\Gamma}_\alpha + \hat{\Gamma}_\beta \hat{f}_{t+1})\right)^2}{\sum_{i,t} r_{i,t+1}^2}. \tag{11}$$

The total R^2 aims to quantify how is the model successful at capturing the riskiness of the assets. Total R^2 is related to the estimation procedure. Similarly as in the case of principal component analysis, the estimation targets to maximize the model's explanatory power of the time variation of returns. In the case of the out-of-sample fits, the model parameters are estimated using the information up to time t, the same as the factors that are formed using the information up to time t and the out-of-sample realized factor returns are then recorded.

The second metric, *predictive* R^2, captures how is the model capable of explaining the conditional expected returns

$$\text{Predictive}\, R^2 = 1 - \frac{\sum_{i,t}\left(r_{i,t+1} - z_{i,t}'(\hat{\Gamma}_\alpha + \hat{\Gamma}_\beta \hat{\lambda})\right)^2}{\sum_{i,t} r_{i,t+1}^2} \tag{12}$$

where $\hat{\lambda}$ is a vector of factor means. In the case of out-of-sample analysis, $\hat{\lambda}$ is estimated up to time t. The predictive R^2 captures how much is the model able to describe the risk-return trade-off of the assets. We can restrict the model to $\Gamma_\alpha = 0$ and compare the performance with the unrestricted model. When we impose the restriction, the predictive R^2 tells us how much the risk compensation can be explained by the exposure to the systematic risk approximated by the ARMs. When we do not impose this restriction, the predictive R^2 summarizes how much of the variation of the expected returns can be explained through the

[1]Those restrictions do not posses any economic implications for the model.

characteristics via their relation to either systematic risk exposure or anomaly intercepts.

Moreover, the IPCA model has a natural interpretation in terms of managed portfolios. Using managed portfolio interpretation is important for estimation (e.g., for initial guess of the numerical optimization), its relation to the classical PCA estimator, and for bootstrap procedure described above. More importantly for the presented analysis, I will use both single stock and managed portfolio returns to evaluate the performance of the IPCA models. Asset pricing literature frequently prefers to use portfolios because of their lower levels of unrelated idiosyncratic risk. The corresponding metrics are defined as

$$\text{Total } R^2 = 1 - \frac{\sum_t \left(x_{t+1} - Z_t' Z_t(\hat{\Gamma}_\alpha + \hat{\Gamma}_\beta \hat{f}_{t+1})\right)' \left(x_{t+1} - Z_t' Z_t(\hat{\Gamma}_\alpha + \hat{\Gamma}_\beta \hat{f}_{t+1})\right)}{\sum_t x_{t+1}' x_{t+1}} \tag{13}$$

and

$$\text{Predictive } R^2 = 1 - \frac{\sum_t \left(x_{t+1} - Z_t' Z_t(\hat{\Gamma}_\alpha + \hat{\Gamma}_\beta \hat{\lambda})\right)' \left(x_{t+1} - Z_t' Z_t(\hat{\Gamma}_\alpha + \hat{\Gamma}_\beta \hat{\lambda})\right)}{\sum_t x_{t+1}' x_{t+1}}. \tag{14}$$

3.3 In-sample Results

For the first set of results using the IPCA framework, I perform in-sample analysis by estimating the IPCA models with varying number of latent factors over the whole dataset using the ARMs as instruments (hence ARM-IPCA). Table 4 summarizes the results of both restricted and unrestricted versions of the IPCA models. The first segment of each panel captures the results using individual stocks. The second segment describes the results using the managed portfolios. The third segment then reports the results of the test regarding the zero alpha assumption.

We see that the test rejects the null hypothesis of non-significant alphas for the first five IPCA specifications in the case of all stocks and for the first four specifications in the case of the no-penny dataset. The predictive R^2s suggest that there is little difference between the restricted and non-restricted model for the IPCA(3) models in the case of all- and no-penny-stock datasets. However, it is difficult to assess the importance of those differences as only a small increase of R^2 may lead to large investment gains. They may play an even bigger role if we look at the out-of-sample results.

Moreover, we can see that the latent factors capture the time-variation of stocks much more in the case of non-penny stocks. This confirms the notion that the less liquid stocks are much more driven by idiosyncratic shocks. The results hold similarly for the managed portfolios, although the difference is considerably lower than in the case of single stocks. We

Table 4: *In-Sample ARM-IPCA Results.* The table reports in-sample results of the IPCA models with varying numbers of latent factors and using ARMs as the instruments. The asset pricing test reports p-values of the null hypothesis that $\Gamma_\alpha = 0$. Panel A reports results using all stocks, Panel B excludes stocks with a price less than \$5 or market cap below 10% quantile of NYSE stocks. Data cover the period between January 1968 and December 2018.

		IPCA(K)							
		1	2	3	4	5	6	7	8
Panel A: All stocks									
Individual stocks									
Total R^2	$\Gamma_\alpha = 0$	15.95	17.30	17.99	18.46	18.70	18.83	18.94	19.02
	$\Gamma_\alpha \neq 0$	16.02	17.36	18.00	18.47	18.71	18.83	18.94	19.02
Predictive R^2	$\Gamma_\alpha = 0$	0.29	0.31	0.35	0.35	0.36	0.36	0.35	0.36
	$\Gamma_\alpha \neq 0$	0.37	0.37	0.36	0.36	0.36	0.36	0.36	0.36
Managed portfolios									
Total R^2	$\Gamma_\alpha = 0$	96.28	98.35	99.45	99.66	99.79	99.85	99.90	99.94
	$\Gamma_\alpha \neq 0$	96.35	98.41	99.46	99.67	99.79	99.85	99.90	99.94
Predictive R^2	$\Gamma_\alpha = 0$	1.85	1.88	1.95	1.94	1.95	1.95	1.94	1.95
	$\Gamma_\alpha \neq 0$	1.97	1.96	1.96	1.96	1.96	1.96	1.96	1.95
Asset pricing test									
W_α p-value		0.00	0.00	4.70	0.80	2.50	16.40	7.60	77.60
Panel B: No penny stocks									
Individual stocks									
Total R^2	$\Gamma_\alpha = 0$	22.22	23.97	24.85	25.28	25.69	25.90	26.09	26.22
	$\Gamma_\alpha \neq 0$	22.33	24.01	24.87	25.29	25.69	25.90	26.09	26.22
Predictive R^2	$\Gamma_\alpha = 0$	0.33	0.41	0.43	0.43	0.44	0.44	0.44	0.44
	$\Gamma_\alpha \neq 0$	0.45	0.45	0.45	0.45	0.45	0.44	0.44	0.44
Managed portfolios									
Total R^2	$\Gamma_\alpha = 0$	96.74	98.64	99.33	99.60	99.74	99.84	99.92	99.93
	$\Gamma_\alpha \neq 0$	96.82	98.62	99.35	99.61	99.74	99.84	99.92	99.93
Predictive R^2	$\Gamma_\alpha = 0$	1.54	1.63	1.63	1.64	1.65	1.65	1.65	1.65
	$\Gamma_\alpha \neq 0$	1.65	1.65	1.65	1.65	1.65	1.65	1.65	1.65
Asset pricing test									
W_α p-value		0.10	3.10	5.30	4.20	21.30	11.40	47.40	87.50

also see that the predictive R^2s for single stocks are higher among the non-penny dataset. On the other hand, if we focus on the managed portfolios, we see that those average returns are better explained using all stocks.

Generally, the results are similar to the results obtained by Kelly et al. (2019) or Kelly et al. (2023) in sense that only a few instrumented latent factors are needed to explain the asset returns. These results suggest that if we let the ARMs explain the exposures into latent factors, their residual abnormal alpha returns vanishe. The main difference between my results and results obtained by Kelly et al. (2019) is that their dataset contains 36 characteristics and needs 6 latent factors to not reject the null hypothesis of $\Gamma_\alpha = 0$. In my case, I use only 11 characteristics and need the same number of factors to not reject the hypothesis.

3.4 Out-of-sample Results

The in-sample results represent only a first indication regarding the beta vs alpha discussion. In a real-world setting, there still may be arbitrage opportunities that hedge the systematic risk while enjoying abnormal returns associated with ARMs. To assess this proposition closely, I investigate the out-of-sample performance of the pure-alpha portfolios. Table 5 summarizes those results. The models are estimated using an expanding window. First, I estimate the models with the first 60 observations of the sample and predict the next observation which is not included in the estimation process. Then, I expand the estimation period by one observation and predict the next. I repeat the procedure until the dataset is fully exhausted. I report total and predictive R^2s for both individual stocks and managed portfolios. Moreover, as discussed above, I include two more sections that were not included in the in-sample results. First, related to the restricted model, I report asummary of the tangency portfolios for each IPCA model. Second, using the unrestricted model, I report summary statistics related to the arbitrage portfolios.

Results regarding total and predictive R^2 hold similarly as in the case of the in-sample analysis. In the case of all stocks, the results show that we have to include six or seven factors to eliminate statistically significant arbitrage portfolio returns when we employ the universe of all stocks. Moreover, the results of the tangency and arbitrage portfolios enable us to understand better the small differences of the predictive R^2s for the restricted and unrestricted models. Predictive R^2 for restricted and unrestricted IPCA(5) models are 0.27 and 0.28, respectively, but the pure-alpha portfolio of the unrestricted model still delivers abnormal returns of 8.06% p.a. with significant t-statistics of 2.86.[2] But once we get to seven latent factors, those arbitrage opportunities vanish. On the other hand, using only a liquid set of stocks, there are only four factors needed to eliminate arbitrage opportunity related to the ARMs.

Results regarding the restricted vs unrestricted models are similar to the results of the bootstrap tests obtained from the in-sample analysis. They further quantify the differences between restricted and unrestricted models. We see that there is a need to include multiple latent factors to erase the significant effect of the ARM characteristics. This suggests that there is less duplicity in the information regarding the expected returns among the ARMs than one might expect. In the universe of all stocks, the proportion of the number of factors needed to eliminate arbitrage opportunity and the number of ARMs is more than half. In the case of highly liquid stocks, the effect of the abnormal returns can be captured by four factors. Based on the performances of the tangency portfolios, the results also suggest that

[2]For comparability reasons, I scale the portfolio returns to have an unconditional standard deviation of 20% p.a., which does not affect the significance of the results.

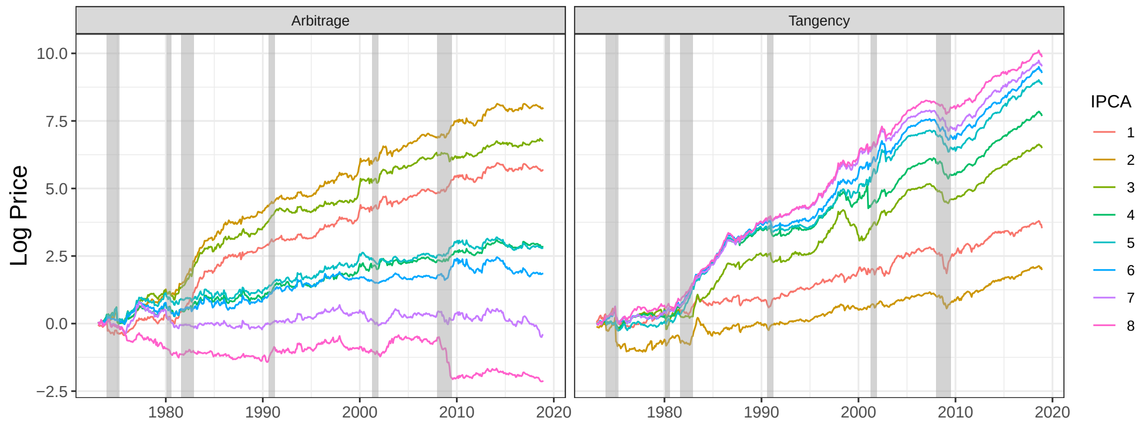
Table 5: *Out-of-Sample ARM-IPCA Results.* The table reports out-of-sample results of the IPCA models with varying numbers of latent factors and using ARMs as the instruments. Models are estimated with an expanding window and a 60-month initial period. Tangency portfolios are based on the restricted IPCA model, and pure-alpha portfolios on the unrestricted model. Panel A reports results using all stocks, Panel B excludes stocks with a price less than $5 or market cap below 10% quantile of NYSE stocks. Data cover the period between January 1973 and December 2018.

		IPCA(K)							
		1	2	3	4	5	6	7	8
Panel A: All stocks									
Individual stocks									
Total R^2	$\Gamma_\alpha = 0$	15.49	16.81	17.47	17.99	18.25	18.38	18.49	18.57
	$\Gamma_\alpha \neq 0$	15.47	16.80	17.37	17.98	18.24	18.36	18.48	18.57
Predictive R^2	$\Gamma_\alpha = 0$	0.23	0.23	0.26	0.26	0.27	0.28	0.28	0.28
	$\Gamma_\alpha \neq 0$	0.28	0.28	0.28	0.28	0.28	0.28	0.28	0.28
Managed portfolios									
Total R^2	$\Gamma_\alpha = 0$	96.30	98.35	99.28	99.63	99.77	99.83	99.89	99.93
	$\Gamma_\alpha \neq 0$	95.91	98.04	99.08	99.56	99.74	99.81	99.88	99.92
Predictive R^2	$\Gamma_\alpha = 0$	1.55	1.56	1.64	1.67	1.69	1.69	1.69	1.69
	$\Gamma_\alpha \neq 0$	1.69	1.69	1.69	1.69	1.69	1.69	1.69	1.70
Tangency portfolios									
Mean		9.74	6.64	16.36	19.11	21.37	22.37	22.93	23.68
t-stat		3.10	2.27	4.45	6.00	6.06	6.66	7.10	7.43
Sharpe		0.49	0.33	0.82	0.96	1.07	1.12	1.15	1.18
Pure-alpha portfolios									
Mean		14.36	19.36	16.78	8.20	8.06	5.97	1.07	-2.53
t-stat		4.73	6.27	5.35	3.04	2.86	2.05	0.34	-0.81
Sharpe		0.72	0.97	0.84	0.41	0.40	0.30	0.05	-0.13
Panel B: No penny stocks									
Individual stocks									
Total R^2	$\Gamma_\alpha = 0$	21.75	23.48	24.37	24.82	25.26	25.47	25.67	25.80
	$\Gamma_\alpha \neq 0$	21.53	23.37	24.30	24.79	25.24	25.46	25.66	25.79
Predictive R^2	$\Gamma_\alpha = 0$	0.24	0.30	0.30	0.33	0.33	0.33	0.33	0.33
	$\Gamma_\alpha \neq 0$	0.32	0.34	0.33	0.33	0.33	0.33	0.33	0.33
Managed portfolios									
Total R^2	$\Gamma_\alpha = 0$	96.56	98.53	99.28	99.52	99.74	99.81	99.90	99.92
	$\Gamma_\alpha \neq 0$	95.25	97.95	98.96	99.48	99.69	99.77	99.87	99.90
Predictive R^2	$\Gamma_\alpha = 0$	1.16	1.27	1.29	1.35	1.36	1.36	1.36	1.36
	$\Gamma_\alpha \neq 0$	1.35	1.37	1.36	1.36	1.36	1.36	1.36	1.36
Tangency portfolios									
Mean		7.87	12.98	14.44	22.43	21.55	22.37	22.28	21.30
t-stat		2.60	3.85	4.57	6.92	6.83	7.00	6.99	6.41
Sharpe		0.39	0.65	0.72	1.12	1.08	1.12	1.11	1.07
Pure-alpha portfolios									
Mean		13.91	15.95	11.65	2.61	0.68	1.26	2.25	1.63
t-stat		4.32	4.58	3.49	0.96	0.25	0.49	0.86	0.55
Sharpe		0.70	0.80	0.58	0.13	0.03	0.06	0.11	0.08

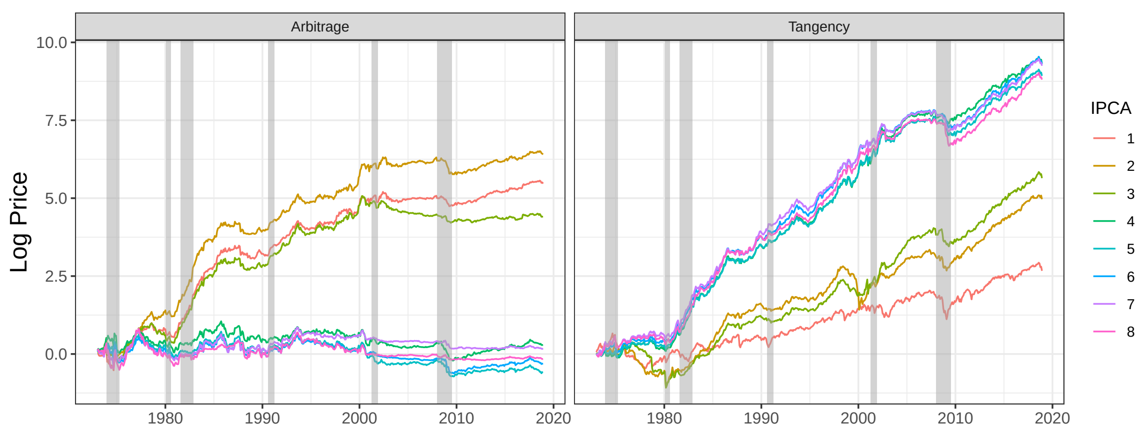
ARMs successfully proxy for the exposures to the common factors.

To further assess the performance of both tangency and pure-alpha portfolios, Figure 2 captures the cumulative log return of those portfolios. We see that in the case tangency portfolios, they grow over the whole period without a noticeable sign of slowing down. The arbitrage portfolios, on the other hand, grow constantly only in the case of all stocks for up to five factors. Arbitrage portfolios which are formed using only non-penny stocks show a significant drop in performance around the burst of the dot-com bubble in the early 2000s,

Figure 2: *Performance of the ARM-IPCA portfolios.* The figure shows out-of-sample performance results of the arbitrage and pure-alpha portfolios estimated using IPCA models with the ARMs as instruments. Models are estimated with an expanding window and a 60-month initial period. Tangency portfolios are based on the restricted IPCA model, and pure-alpha portfolios on the unrestricted model. Panel A reports results using all stocks, Panel B excludes stocks with a price less than \$5 or market cap below 10% quantile of NYSE stocks. Data cover the period between January 1973 and December 2018.



(a) All stocks



(b) No penny stocks

from which they did not fully recover.

Table 6 captures alphas of ARM-managed portfolios with relation to out-of-sample ARM-IPCA models with a range between one and six latent factors. Those alphas are obtained from time-series regressions of the returns of the managed portfolios on the ARM-IPCA factors. We can see that the higher-order IPCA specifications are quite successful in describing the abnormal returns of the managed portfolios with few exceptions. Most notably, predicted coskewness, even in the case of IPCA(6) model, yields a significant abnormal return with a sign that corresponds with the economic intuition. On the other hand, abnormal

Table 6: *ARM-IPCA alphas of the ARM-managed portfolios.* The table reports alphas and their t-statistics of the returns of the ARM-managed portfolios with respect to out-of-sample IPCA factors with one to six latent factors and the ARMs as instruments. Panel A reports results using all stocks, Panel B excludes stocks with a price less than \$5 or market cap below 10% quantile of NYSE stocks. Data cover the period between January 1968 and December 2018.

	Panel A: All stocks						*Panel B: No penny stocks*					
	IPCA1	IPCA2	IPCA3	IPCA4	IPCA5	IPCA6	IPCA1	IPCA2	IPCA3	IPCA4	IPCA5	IPCA6
`coskew`	-0.20	-0.24	-0.32	-0.28	-0.13	-0.20	-0.22	-0.46	-0.26	-0.15	-0.08	-0.16
	(-1.49)	(-1.80)	(-2.28)	(-1.96)	(-0.92)	(-1.51)	(-1.58)	(-2.99)	(-1.82)	(-1.14)	(-0.66)	(-1.34)
`cokurt`	-0.50	-0.62	-0.26	-0.35	-0.44	-0.64	-0.12	-0.06	0.36	0.18	0.12	0.06
	(-2.49)	(-3.22)	(-1.26)	(-2.50)	(-2.25)	(-3.58)	(-0.72)	(-0.39)	(2.60)	(1.43)	(0.89)	(0.49)
`beta_down`	-0.78	-0.67	0.23	0.17	-0.02	-0.05	-0.50	0.24	0.43	0.30	0.19	0.06
	(-3.19)	(-3.00)	(1.09)	(0.97)	(-0.08)	(-0.26)	(-2.12)	(0.96)	(1.56)	(1.14)	(0.71)	(0.28)
`down_corr`	0.05	0.09	0.20	0.14	-0.01	0.04	0.19	0.40	0.23	0.11	0.06	0.09
	(0.54)	(0.96)	(2.21)	(1.46)	(-0.06)	(0.46)	(1.81)	(3.69)	(2.33)	(1.22)	(0.70)	(1.01)
`htcr`	0.03	-0.14	-0.20	-0.48	-0.42	-0.65	0.31	0.17	0.45	0.29	0.25	0.19
	(0.16)	(-0.71)	(-0.84)	(-2.02)	(-1.76)	(-2.97)	(2.25)	(1.15)	(3.00)	(2.02)	(2.29)	(1.41)
`beta_tr`	0.23	0.25	0.24	0.46	-0.19	0.06	0.34	0.15	0.38	0.25	0.25	0.06
	(1.51)	(1.61)	(1.39)	(2.32)	(-1.12)	(0.34)	(2.19)	(0.84)	(2.67)	(1.78)	(1.50)	(0.50)
`coentropy`	0.02	0.05	0.15	0.12	-0.02	0.03	0.12	0.34	0.16	0.06	0.01	0.05
	(0.16)	(0.58)	(1.69)	(1.16)	(-0.24)	(0.39)	(1.22)	(3.06)	(1.68)	(0.66)	(0.16)	(0.62)
`cos_pred`	-0.32	-0.57	-0.84	-0.78	-0.51	-0.78	-0.16	-0.91	-0.32	-0.23	-0.17	-0.15
	(-1.31)	(-2.90)	(-2.94)	(-3.08)	(-2.54)	(-3.89)	(-0.72)	(-3.46)	(-1.31)	(-1.35)	(-0.98)	(-0.92)
`beta_neg`	-0.79	-0.51	0.73	0.77	0.58	0.64	-0.78	0.18	0.31	0.24	0.13	-0.04
	(-2.18)	(-1.51)	(2.08)	(2.28)	(1.68)	(1.82)	(-2.72)	(0.56)	(0.94)	(0.77)	(0.40)	(-0.15)
`mcrash`	-0.02	-0.05	0.06	0.01	-0.01	-0.09	0.08	0.17	0.28	0.20	0.16	0.14
	(-0.14)	(-0.49)	(0.62)	(0.16)	(-0.13)	(-0.98)	(0.86)	(1.91)	(3.16)	(2.20)	(1.93)	(1.62)
`ciq_down`	0.47	0.48	0.32	0.36	0.19	0.20	0.42	0.12	0.13	0.13	0.28	0.09
	(3.24)	(3.21)	(2.04)	(2.32)	(1.53)	(1.68)	(2.55)	(0.70)	(0.74)	(0.89)	(2.02)	(0.78)

returns of the managed portfolio sorted based on cokurtosis or hybrid tail covariance risk yield negative average returns that do not agree with the expectations. In this sense, the non-penny dataset yields more intuitive results. But in order to erase the significant returns, six latent factors are also needed.

Table 7 summarizes the exposures of the ARM-managed portfolios to the ARM-IPCA factors. For the all-stocks dataset, I report exposures to factors from the IPCA(6) specification and in the case of no-penny dataset, I provide exposures to factors from the IPCA(4) model. We see that the ARM portfolios load differently on the latent factors. These results further confirm the notion that the returns of the ARMs are not driven by one or two common factors.

Table 8 summarizes the exposures of the pure-alpha arbitrage portfolios to the six commonly used factors. The pure-alpha portfolios of the ARM-IPCA models with a lower number of factors possess significant exposure to the momentum factor. This exposure vanishes for the specifications with a higher number of factors in the case of all stocks, on the other hand, the momentum factor (along with the BAB and the market factor) erases the significance of the pure-alpha portfolio returns in the case of non-penny stocks. This observation suggests that the abnormal returns of the ARMs are at least partially related to the momentum

Table 7: *Exposures of the ARM-managed portfolios to the ARM-IPCA factors.* The table reports estimated coefficients and their t-statistics from regressing the returns of ARM-managed portfolios on the out-of-sample ARM-IPCA factors. Panel A reports results using all stocks, Panel B excludes stocks with a price less than \$5 or market cap below 10% quantile of NYSE stocks. Data cover the period between January 1973 and December 2018.

	Panel A: All stocks						
	α	IPC1	IPC2	IPC3	IPC4	IPC5	IPC6
`coskew`	-0.20	-0.79	-1.22	-1.10	0.03	0.26	2.98
	(-1.51)	(-2.92)	(-2.59)	(-2.57)	(0.04)	(0.34)	(3.02)
`cokurt`	-0.64	-0.08	-3.03	4.97	4.48	-1.82	2.63
	(-3.58)	(-0.24)	(-6.59)	(7.43)	(7.14)	(-1.94)	(2.67)
`beta_down`	-0.05	2.67	-1.83	3.34	1.74	-5.08	-0.51
	(-0.26)	(5.82)	(-1.93)	(4.02)	(1.55)	(-5.71)	(-0.47)
`down_corr`	0.04	0.75	1.12	0.85	0.56	-0.20	-1.78
	(0.46)	(3.65)	(3.20)	(3.07)	(1.19)	(-0.36)	(-3.03)
`htcr`	-0.65	-1.11	-1.62	4.97	3.74	2.18	4.05
	(-2.97)	(-2.41)	(-3.14)	(6.36)	(2.66)	(1.90)	(2.32)
`beta_tr`	0.06	0.50	-0.66	0.67	3.88	0.63	-1.69
	(0.34)	(1.67)	(-0.82)	(1.30)	(4.63)	(0.75)	(-1.22)
`coentropy`	0.03	0.67	1.07	0.61	0.47	-0.44	-2.00
	(0.39)	(3.31)	(3.04)	(2.20)	(0.97)	(-0.79)	(-3.12)
`cos_pred`	-0.78	-2.45	-5.99	2.61	1.52	2.82	3.01
	(-3.89)	(-5.10)	(-7.92)	(4.40)	(1.23)	(2.22)	(2.09)
`beta_neg`	0.64	4.14	-0.93	-0.06	-0.93	-7.43	-1.68
	(1.82)	(5.11)	(-0.56)	(-0.05)	(-0.42)	(-4.73)	(-0.82)
`mcrash`	-0.09	0.15	-0.84	2.37	1.02	-0.31	1.05
	(-0.98)	(0.98)	(-4.38)	(6.14)	(2.84)	(-0.62)	(2.05)
`ciq_down`	0.20	0.08	0.46	-3.79	-0.55	4.84	2.61
	(1.68)	(0.42)	(1.73)	(-12.67)	(-1.07)	(9.98)	(3.48)
	Panel B: No penny stocks						
`coskew`	-0.15	0.09	-2.52	0.41	0.14		
	(-1.14)	(0.27)	(-4.33)	(0.66)	(0.13)		
`cokurt`	0.18	-0.03	-0.40	1.11	-6.76		
	(1.43)	(-0.09)	(-0.77)	(1.23)	(-5.80)		
`beta_down`	0.30	-0.49	1.89	-2.32	-7.09		
	(1.14)	(-0.64)	(1.28)	(-1.68)	(-4.46)		
`down_corr`	0.11	-0.03	2.59	-0.28	-0.38		
	(1.22)	(-0.11)	(6.82)	(-0.64)	(-0.57)		
`htcr`	0.29	0.60	0.09	2.57	-4.00		
	(2.02)	(3.05)	(0.22)	(4.13)	(-4.61)		
`beta_tr`	0.25	-0.48	-0.06	3.21	-3.65		
	(1.78)	(-1.39)	(-0.06)	(4.23)	(-5.40)		
`coentropy`	0.06	-0.08	2.34	-0.38	-0.16		
	(0.66)	(-0.31)	(5.94)	(-0.84)	(-0.22)		
`cos_pred`	-0.23	-0.91	-5.99	3.90	-2.09		
	(-1.35)	(-2.08)	(-8.23)	(3.77)	(-1.96)		
`beta_neg`	0.24	-0.91	1.92	-3.92	-7.38		
	(0.77)	(-0.97)	(0.96)	(-2.29)	(-4.42)		
`mcrash`	0.20	0.09	0.42	0.52	-2.65		
	(2.20)	(0.45)	(1.54)	(1.18)	(-3.89)		
`ciq_down`	0.13	1.24	-0.72	2.55	1.48		
	(0.89)	(3.64)	(-1.62)	(3.99)	(1.30)		

factor, which in case of the liquid dataset, plays a significant role in erasing the abnormal returns.

Table 8: *Exposures of the ARM-IPCA pure-alpha portfolios.* The table reports estimated coefficients and their t-statistics from regressing returns of the pure-alpha ARM-IPCA(K) portfolios on four factors of Carhart (1997), CIV shocks of Herskovic et al. (2016), and BAB factor of Frazzini and Pedersen (2014). Panel A reports results using all stocks, Panel B excludes stocks with a price less than $5 or market cap below 10% quantile of NYSE stocks. Data cover the period between January 1973 and December 2018.

K	α	Mkt	SMB	HML	CIV	BAB	MOM
	Panel A: All stocks						
1	6.27	0.07	0.13	0.06	-0.02	0.42	0.33
	(1.85)	(1.01)	(0.81)	(0.47)	(-0.57)	(3.49)	(2.69)
2	10.63	0.03	0.17	0.15	-0.02	0.39	0.43
	(3.15)	(0.48)	(1.08)	(1.06)	(-0.72)	(3.32)	(3.28)
3	10.03	0.04	0.11	0.09	0.00	0.22	0.46
	(3.46)	(0.55)	(0.64)	(0.52)	(0.19)	(1.80)	(4.29)
4	5.22	0.04	-0.11	0.33	-0.04	0.10	0.07
	(1.88)	(0.57)	(-0.72)	(1.82)	(-1.24)	(0.93)	(0.78)
5	5.61	0.10	-0.10	0.30	-0.02	0.00	0.10
	(1.94)	(1.49)	(-0.70)	(1.66)	(-0.52)	(0.00)	(0.99)
6	4.57	0.02	0.08	0.35	-0.01	-0.01	-0.03
	(1.52)	(0.41)	(0.86)	(2.79)	(-0.37)	(-0.08)	(-0.33)
7	-0.02	0.07	0.14	0.44	-0.04	-0.12	-0.01
	(-0.01)	(1.10)	(1.36)	(3.05)	(-1.14)	(-1.30)	(-0.10)
8	-2.60	-0.08	0.14	0.30	0.02	-0.19	0.14
	(-0.68)	(-1.05)	(1.50)	(1.81)	(0.77)	(-1.66)	(1.12)
	Panel B: No penny stocks						
1	2.79	0.36	0.06	0.21	0.02	0.39	0.47
	(1.00)	(4.83)	(0.41)	(1.20)	(0.56)	(3.25)	(5.10)
2	4.96	0.24	0.15	0.17	0.02	0.36	0.59
	(1.74)	(3.49)	(1.23)	(1.07)	(0.71)	(3.16)	(6.77)
3	4.13	0.29	-0.05	0.29	0.01	0.13	0.41
	(1.37)	(4.03)	(-0.36)	(1.66)	(0.25)	(1.07)	(3.92)
4	-1.98	0.17	-0.30	0.10	-0.04	0.22	0.18
	(-0.79)	(1.89)	(-1.82)	(0.41)	(-1.28)	(1.76)	(1.93)
5	-2.83	0.22	-0.22	0.12	-0.01	0.10	0.12
	(-1.16)	(2.26)	(-1.25)	(0.45)	(-0.31)	(0.76)	(1.37)
6	-2.76	0.19	-0.22	0.12	-0.01	0.14	0.15
	(-1.12)	(1.86)	(-1.25)	(0.43)	(-0.23)	(1.02)	(1.66)
7	-1.63	0.20	-0.18	0.17	-0.01	0.13	0.11
	(-0.65)	(1.70)	(-1.32)	(0.72)	(-0.34)	(0.84)	(1.19)
8	-1.72	0.13	-0.17	0.26	0.01	0.11	0.08
	(-0.59)	(1.53)	(-1.29)	(1.49)	(0.60)	(1.02)	(0.98)

3.5 Sub-sample Analysis

As a simple robustness check, I perform the out-of-sample analysis over two sub-intervals. Table 20 in Appendix C summarizes the out-of-sample results of the ARM-IPCA models using all stocks estimated separately in two disjoint time periods. The first period covers range between January 1968 and December 1993, the second period spans time between January 1994 and December 2018. Results regarding the tangency and arbitrage portfolios are both in agreement with the results obtained over the whole period. There is although difference in predictive R^2, which is noticeably lower in the first period than in the second one or even over the whole period. Generally, the results are stable over disjoint periods, as the number of latent factors needed to eliminate the arbitrage opportunities is around six.

Table 9: *Summary statistics of the ARM-IPCA factors.* The table reports summary statistics of the instrumented principal components from the IPCA(6) and IPCA(4) models using all-stock and no-penny datasets, respectively. The factors are standardized to have an unconditional standard deviation of 20% p.a.

	In-sample			Out-of-sample		
Factor	Mean	Std. Dev.	Sharpe	Mean	Std. Dev.	Sharpe
Panel A: All stocks						
1	4.10	33.72	0.12	2.02	34.24	0.06
2	6.94	18.35	0.38	3.69	18.04	0.20
3	3.84	13.80	0.28	5.76	13.53	0.43
4	0.37	9.72	0.04	5.70	10.91	0.52
5	8.92	8.58	1.04	8.35	9.32	0.90
6	3.56	7.06	0.50	-0.06	8.22	-0.01
Panel B: No penny stocks						
1	1.12	29.99	0.04	-4.72	28.87	-0.16
2	4.82	17.38	0.28	6.79	16.15	0.42
3	10.60	10.40	1.02	8.65	11.06	0.78
4	2.35	7.90	0.30	4.09	8.81	0.46

Table 21 in Appendix C summarizes the out-of-sample analysis over split samples using non-penny stocks. As we can see, the results that we obtained over the whole sample hold very similarly in the case of split samples. Although in the second period, we need slightly more factors to erase the significance of the arbitrage portfolios, we are able to retrieve considerably higher Sharpe ratios in the first period – up to 1.15 against 0.68 in the second period.

3.6 Factors and Characteristic Importance

This section further delves into the features of the latent factors estimated using the IPCA procedure. Panel A of Table 9 summarizes the latent factors from the ARM-IPCA(6) model using the all-stock dataset. We see that the higher Sharpe ratios, both in-sample and out-of-sample, possess mostly higher-order (three and higher) factors. The first instrumented principal component, which explains the most time variation of the returns, leaves the predictive power to the other factors. This is a similar result as obtained by Lettau and Pelger (2020), which also reports high Sharpe ratios for higher-order factors.

Panel B of Table 9 reports a summary of the ARM-IPCA(4) factors estimated using the non-penny dataset. Results are similar as in the case of all stocks–the highest Sharpe ratio with the highest mean possess the third factor, both in-sample and out-of-sample.

Figure 10 from Appendix C shows loadings of the ARMs on the latent factors from the restricted IPCA(6) model with all stocks and no penny stocks, respectively. The first two factors are clearly related to the negative semibeta and predicted coskewness, respectively. The fifth factor, which possesses the highest Sharpe ratio both in and out-of-sample, noticeably loads on tail risk beta and downside common idiosyncratic quantile risk betas. Figure

11 from Appendix C presents the results for the non-penny dataset.

To formally assess the importance of each variable for the performance of the restricted IPCA model, I perform a bootstrap test proposed in Kelly et al. (2019). For given IPCA model with K latent factors, let the l^{th} row in the matrix $\Gamma_\beta = [\gamma_{\beta,1}, \ldots, \gamma_{\beta,L}]$ maps the l^{th} characteristic to the loadings on the K latent factors. The null hypothesis assumes that the l^{th} row is equal to zero, i.e., this characteristic does not proxy for the dynamics of the factor loadings. To test the hypothesis, I estimate the alternative model that admits the possibility of the contribution of the l^{th} characteristic, and form a Wald-type characteristic of the form $W_{\beta,l} = \hat{\gamma}'_{\beta,l}\hat{\gamma}_{\beta,l}$. I save the estimated model parameters, factors, and managed portfolio residuals. Then, I simulate a new bootstrap sample under the null hypothesis of $\gamma_{\beta,l}$ being equal to zero by resampling the returns of the characteristic-managed portfolios using the wild bootstrap procedure and the estimated parameters. Using the new sample, I estimate the alternative model and form test statistic $\tilde{W}^b_{\beta,l}$. The resulting p-value of the test is calculated as the proportion of $\tilde{W}^b_{\beta,l}$ that exceeds $W_{\beta,l}$.

Table 10 reports simulated p-values for each variable and each specification of the IPCA model for both samples. In the case of all stocks, we know that around 6 latent factors is needed to eliminate the arbitrage opportunity, so I focus on the IPCA(6) specification. In this case, seven variables are highly significant and drive the explanatory power of the model – downside beta, hybrid tail covariance risk, predicted coskewness, negative semibeta, MCRASH, and downside CIQ beta. Using the no-penny dataset and looking at the IPCA(4) specification, four variables significantly proxy for the exposure to the common factors: downside beta, hybrid tail covariance risk, predicted coskewness, and negative semibeta.

I assess the variable importance for the out-of-sample results based on the leave-one-out variable decrease of the Sharpe ratio. In Table 11, I report the effects of excluding given ARM on the performance of the pure-alpha portfolio. For each variable, I report the decrease in the Sharpe ratio in percentage points compared to the model that includes all the variables. The result of leaving a variable out of the estimation of an unrestricted model results in two effects. First, there is less information that can be used for the formation of the arbitrage portfolio. This should generally lead to a decrease in the out-of-sample Sharpe ratio. Second, leaving one variable out restricts the information that can be used for the exploitation of the common factor structure of the returns. Consequently, this effect saves more potential pricing information for the construction of the arbitrage portfolio, which should generally lead to an increase in the Sharpe ratio.

Next, I aim to disentangle the two opposing effects. I estimate the unrestricted model using the full set of characteristics and then set the effect of a given variable on the formation of pure-alpha portfolio to zero. This approach should isolate the first effect. This effect is

Table 10: *Variable Importance of the ARMs.* The table reports p-values of the bootstrap tests that given ARM does not significantly contribute to the in-sample ARM-IPCA model's fit. Panel A reports results using all stocks, Panel B excludes stocks with a price less than \$5 or market cap below 10% quantile of NYSE stocks. Data cover the period between January 1968 and December 2018.

	IPCA1	IPCA2	IPCA3	IPCA4	IPCA5	IPCA6	IPCA7	IPCA8
Panel A: All stocks								
coskew	22.60	16.90	9.10	2.30	3.30	59.90	46.90	0.00
cokurt	17.70	18.30	9.80	4.80	7.10	55.60	1.90	0.50
beta_down	9.90	4.90	0.20	0.30	0.10	0.80	0.00	0.00
down_corr	0.00	3.00	18.40	7.30	9.20	13.80	33.30	64.90
htcr	0.00	4.20	0.10	0.80	0.40	0.00	0.30	0.00
beta_tr	97.80	8.60	18.80	21.70	0.00	0.00	0.00	0.00
coentropy	2.50	2.90	25.70	17.10	18.10	18.20	40.40	51.40
cos_pred	0.10	26.60	46.30	0.00	0.00	0.00	0.00	0.00
beta_neg	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
mcrash	49.40	6.40	2.80	3.60	2.90	1.40	4.70	8.90
ciq_down	75.40	8.90	13.30	4.00	0.00	0.00	0.00	0.00
Panel B: No penny stocks								
coskew	7.40	5.20	6.20	19.30	11.50	0.60	0.00	0.00
cokurt	42.80	8.80	11.90	20.20	45.30	1.10	0.50	0.00
beta_down	83.80	12.70	20.80	2.00	2.20	3.00	3.30	7.30
down_corr	0.00	15.50	13.10	29.70	5.00	27.50	16.50	15.60
htcr	0.00	11.30	23.20	2.70	2.70	4.90	3.00	0.00
beta_tr	13.50	1.90	3.20	43.80	0.00	0.00	0.00	0.00
coentropy	7.50	48.80	29.20	49.00	6.10	13.60	7.80	9.60
cos_pred	0.10	32.60	0.00	0.00	0.00	0.00	0.00	0.00
beta_neg	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
mcrash	5.40	39.60	55.20	12.00	3.10	9.30	2.70	3.20
ciq_down	10.90	12.90	11.30	79.60	0.00	0.00	0.00	0.00

more interesting for the investigation of the performance of the arbitrage portfolio.

TBA

4 ARMs and other Characteristics

In this section, I investigate how the ARMs relate to other characteristics that have been proven to be significant proxies for factor exposures. To do that, I use data from Freyberger et al. (2020) and Kim et al. (2020) and select 32 variables that were employed in Kelly et al. (2019). Those variables are: market beta (`beta`), assets-to-market (`a2me`), total assets (`at`), sales-to-assets (`ato`), book-to-market (`beme`), cash-to-short-term-investment (`c`), capital turnover (`cto`), ratio of change in property, plants and equipment to the change in total assets (`dpi2a`), earnings-to-price (`e2p`), cash flow-to-book (`freecf`), idiosyncratic volatility with respect to the FF3 model (`idiovol`), investment (`invest`), market capitalization (`lme`), turnover (`lturnover`), net operating assets (`noa`), operating accruals (`oa`), operating leverage (`ol`), price-to-cost margin (`pcm`), profit margin (`pm`), gross profitability (`prof`), Tobin's Q (`q`), price relative to its 52-week high (`rel_to_high_price`), return on net operating assets

Table 11: *Variable Importance of the ARMs for the pure-alpha portfolios.* The table reports decreases of the out-of-sample Sharpe ratios of the pure-alpha portfolios from leave-one-out procedure. For each ARM, I report difference (in % points) between Sharpe ratio obtained without the ARM and Sharpe ratio obtained from the model with all ARMs. Data cover the period between January 1968 and December 2018.

	IPCA1	IPCA2	IPCA3	IPCA4	IPCA5	IPCA6
`coskew`	2.07	0.57	-1.49	-3.08	-10.51	-19.18
`cokurt`	13.99	7.30	-5.71	13.06	14.50	52.11
`beta_down`	0.92	-3.01	-11.18	-35.41	-40.66	-36.29
`down_corr`	-7.89	-0.58	-3.89	-10.40	-6.48	-34.38
`htcr`	-0.63	-3.98	-18.47	7.34	11.89	13.07
`beta_tr`	-3.69	-3.82	-0.30	-2.83	-44.95	-101.92
`coentropy`	-7.89	-1.02	-6.42	-11.44	-7.69	-35.14
`cos_pred`	1.65	-16.31	-12.54	27.58	16.76	27.15
`beta_neg`	-2.00	0.82	7.36	-11.69	-61.16	-53.24
`mcrash`	2.04	-0.83	-1.16	-1.97	-4.43	-19.32
`ciq_down`	-0.69	-1.35	0.81	-5.92	-2.52	-51.49

(`rna`), return on assets (`roa`), return on equity (`roe`), momentum (`cum_return_12_2`), intermediate momentum (`cum_return_12_7`), short-term reversal (`cum_return_1_0`), long-term reversal (`cum_return_36_13`), sales-to-price (`s2p`), bid-ask spread (`spread_mean`), and unexplained volume (`suv`).[3]

Figure 3 contains correlations between ARMs and characteristics used in Kelly et al. (2019). The highest correlation that we observe is between market beta and negative semibeta with an average value of 0.75, and market beta and downside beta with a value of 0.58. Both these correlations are expected to be quite high as their definitions are closely related. Negative semibeta is also closely related to idiosyncratic volatility with an average correlation of 0.49. The second column of Table 1 summarizes the average absolute correlations between each ARM and all other characteristics. We observe that the average values are noticeably lower than in the case of correlations with other ARMs. The lowest correlated ARM is coentropy with value 0.0w and the highest value of 0.13 negative semibeta in the case of all stocks.

Panel B of Table 1 reports average correlations between returns of the ARM-managed portfolios. Naturally, we observe higher correlations than in the case of the raw variables. Correlations with characteristics are generally lower if we work with the liquid subsample. Generally, the lowest average correlations possess tail risk beta.

[3]Due to availability in the updated sample, I have omitted four variables relative to the original IPCA specification from Kelly et al. (2019). Those variables are: capital intensity (`d2a`) fixed costs-to-sales (`fc2y`) leverage (`lev`), the ratio of sales and price (`s2p`). None of the variables was shown to be significant in the baseline IPCA(5) specification.

Figure 3: *Correlations between ARMs and other characteristics.* The figure captures time-series averages of cross-sectional correlations between asymmetric risk measures and characteristics used in Kelly et al. (2019). Data include all available stocks and the period between January 1968 and December 2018.

	coskew	cokurt	beta_down	down_corr	htcr	beta_tr	coentropy	cos_pred	beta_neg	mcrash	ciq_down
suv	0	−0.02	−0.01	0	−0.02	0	0	−0.03	0	−0.01	0
spread_mean	0.08	−0.33	−0.06	−0.04	−0.42	0	−0.03	−0.24	0.26	−0.09	0.02
s2p	0.01	−0.17	−0.04	0	−0.15	0.02	0	−0.21	0.06	−0.06	0.06
roe	0	0.09	0.01	−0.01	0.09	0	−0.01	0.11	−0.07	0.04	−0.02
roa	−0.01	0.15	−0.03	−0.03	0.17	−0.04	−0.02	0.22	−0.19	0.06	−0.02
rna	0	0.01	0	0	0.01	−0.01	0	0.02	−0.01	0	−0.01
rel_to_high_price	−0.03	0.11	−0.17	0.05	0.35	0.02	0.02	0.02	−0.46	0.08	0.06
q	0.01	0.14	0.15	−0.01	0.04	−0.02	0	0.1	0.14	0.04	−0.08
prof	0	−0.02	0.02	0	−0.04	0.01	0	−0.05	0.05	0	−0.01
pm	0.01	0.06	−0.03	−0.02	0.08	−0.01	−0.02	0.11	−0.1	0.02	0.02
pcm	0.01	0.05	−0.01	−0.01	0.05	−0.02	−0.01	0.08	−0.04	0.02	−0.01
ol	−0.01	−0.11	−0.01	0	−0.11	0.01	0.01	−0.2	0.05	−0.05	0.03
oa	0	0.01	0	0	0	0	0	0	0	0	−0.01
noa	0	0.04	0.05	−0.01	0.01	−0.03	−0.01	0.03	0.05	0.01	−0.03
lturnover	−0.03	0.15	0.31	0.05	0	0.01	0.05	−0.04	0.34	0.04	−0.13
lme	0.01	0.26	0.02	−0.02	0.15	−0.02	−0.02	0.27	−0.07	0.1	0
investment	−0.01	0.08	0.09	0	0.03	0	0.01	0.06	0.08	0.03	−0.06
idio_vol	0.05	−0.3	0.1	−0.01	−0.48	0.02	0	−0.36	0.49	−0.15	−0.03
free_cf	−0.01	0.04	−0.04	−0.01	0.06	−0.02	−0.01	0.06	−0.11	0.02	0.01
e2p	−0.03	0.11	−0.05	0	0.19	0	0	0.15	−0.23	0.06	0.01
dpi2a	0	0.06	0.07	0	0.03	−0.03	0	0.06	0.06	0.02	−0.05
cum_return_36_13	−0.03	0.16	0.12	0.01	0.11	0.04	0.01	0.1	0.03	0.08	−0.07
cum_return_1_0	0	−0.01	−0.01	0.03	0.05	0.03	0.02	−0.17	−0.05	0	0
cum_return_12_7	−0.06	0.08	0.08	0.1	0.12	0.05	0.08	−0.12	−0.07	0.08	−0.01
cum_return_12_2	−0.07	0.06	0.06	0.13	0.18	0.07	0.1	−0.21	−0.11	0.08	−0.01
cto	−0.01	−0.05	0.02	0	−0.06	0.01	0	−0.12	0.04	−0.03	0
c	0	−0.05	0.08	0.02	−0.06	0.03	0.03	−0.13	0.14	−0.02	−0.05
beta	0	0.34	0.58	0.03	0.01	0.04	0.02	0.08	0.75	0.08	−0.28
beme	0.03	−0.24	−0.14	−0.01	−0.16	0.03	−0.01	−0.23	−0.03	−0.1	0.11
ato	−0.01	−0.01	0.01	0	−0.01	−0.01	0	−0.03	0.01	0	0
at	0	0.21	0.02	−0.02	0.12	0	−0.02	0.21	−0.04	0.07	0.02
a2me	0.01	−0.1	−0.08	−0.01	−0.07	0.05	−0.01	−0.07	−0.03	−0.04	0.06

4.1 Alphas of the Managed Portfolios

Table 23 reports correlations between out-of-sample latent factors estimated using the original dataset of 32 variables and latent factors estimated using 11 ARMs. Generally speaking, there is only a little commonality between those two sets of factors. Only the first IPCs from the all-stock dataset are noticeably correlated with value of a 0.43. Surprisingly, in the case of no-penny dataset, the first IPCs are even negatively correlated with value of -0.38.

Next, I investigate how those other characteristics and corresponding latent factors align with the ARM returns. I look at the alphas of the ARM-managed portfolios when regressed on the IPCA factors which are obtained from the original set of 32 characteristics. The results of this exercise capture Table 12. We can see that the returns of the ARM portfolios become insignificant as we move to the higher-order models by looking at the magnitude and t-statistics of the average portfolio returns. In the case of all stocks, the most successful model

Table 12: *IPCA alphas of the ARM-managed portfolios.* The table reports alphas and their t-statistics of the returns of the ARM-managed portfolios with respect to out-of-sample IPCA factors with one to six latent factors and 32 characteristics from Kelly et al. (2019) as instruments. Panel A reports results using all stocks, Panel B excludes stocks with a price less than \$5 or market cap below 10% quantile of NYSE stocks. Data cover the period between January 1968 and December 2018.

	PanelA: All stocks						*Panel B: No penny stocks*					
	IPCA1	IPCA2	IPCA3	IPCA4	IPCA5	IPCA6	IPCA1	IPCA2	IPCA3	IPCA4	IPCA5	IPCA6
coskew	-0.20	-0.21	-0.40	0.05	0.18	0.19	-0.21	-0.30	-0.19	-0.08	-0.15	-0.17
	(-1.48)	(-1.50)	(-1.98)	(0.23)	(0.72)	(0.76)	(-1.50)	(-2.12)	(-0.94)	(-0.54)	(-0.96)	(-1.03)
cokurt	-0.50	-0.16	-0.18	-0.57	0.57	0.14	-0.14	-0.04	0.22	0.28	0.38	-0.12
	(-2.49)	(-0.81)	(-1.14)	(-2.91)	(2.11)	(0.69)	(-0.80)	(-0.24)	(1.32)	(1.98)	(2.33)	(-0.65)
beta_down	-0.79	-0.68	-0.02	-0.92	-0.07	-0.20	-0.56	-0.31	0.23	0.24	0.33	-0.38
	(-3.20)	(-3.14)	(-0.10)	(-2.98)	(-0.21)	(-0.59)	(-2.35)	(-1.70)	(0.85)	(0.96)	(1.19)	(-1.61)
down_corr	0.05	0.08	0.36	-0.10	-0.28	-0.19	0.18	0.27	0.11	0.01	0.10	0.05
	(0.53)	(0.81)	(2.84)	(-0.69)	(-1.61)	(-1.09)	(1.70)	(2.60)	(0.70)	(0.10)	(0.83)	(0.44)
htcr	0.04	0.41	-0.10	-0.36	0.61	0.36	0.32	0.33	0.11	0.12	0.22	0.02
	(0.18)	(2.51)	(-0.67)	(-1.62)	(1.85)	(1.36)	(2.28)	(2.20)	(0.68)	(0.81)	(1.36)	(0.11)
beta_tr	0.22	0.33	0.21	-0.09	0.01	-0.04	0.34	0.28	-0.06	0.24	-0.01	-0.08
	(1.48)	(2.31)	(0.72)	(-0.50)	(0.03)	(-0.14)	(2.14)	(1.80)	(-0.31)	(1.47)	(-0.07)	(-0.46)
coentropy	0.01	0.03	0.31	-0.08	-0.32	-0.25	0.12	0.21	0.09	0.00	0.03	-0.01
	(0.15)	(0.32)	(2.57)	(-0.53)	(-1.96)	(-1.53)	(1.14)	(2.07)	(0.60)	(0.02)	(0.30)	(-0.04)
cos_pred	-0.32	-0.01	-1.54	-0.68	0.34	-0.15	-0.13	-0.30	-0.46	-0.15	0.06	-0.15
	(-1.30)	(-0.05)	(-4.57)	(-1.51)	(0.66)	(-0.39)	(-0.62)	(-1.47)	(-1.24)	(-0.77)	(0.25)	(-0.58)
beta_neg	-0.81	-0.96	0.20	-0.77	-0.27	-0.19	-0.85	-0.56	0.23	0.15	0.21	-0.56
	(-2.19)	(-4.11)	(0.71)	(-1.62)	(-0.50)	(-0.42)	(-2.98)	(-2.56)	(0.73)	(0.51)	(0.60)	(-1.99)
mcrash	-0.02	0.11	0.08	-0.22	0.20	0.10	0.08	0.14	0.09	0.10	0.20	-0.01
	(-0.16)	(0.98)	(0.82)	(-2.16)	(1.61)	(0.81)	(0.78)	(1.47)	(0.85)	(1.15)	(2.19)	(-0.14)
ciq_down	0.48	0.40	0.19	0.28	0.34	0.42	0.44	0.30	0.27	0.05	-0.13	0.24
	(3.27)	(3.00)	(0.74)	(1.33)	(1.60)	(2.11)	(2.72)	(1.94)	(1.41)	(0.35)	(-0.70)	(1.32)

is IPCA(6), which yields only one significant alpha, which corresponds to the downside CIQ beta of Barunik and Nevrla (2022). The rest of the variables yield insignificant premiums. This suggests that the other characteristics are quite successful in capturing the risk related to the ARMs.

The model is also successful when we look at the no-penny dataset. In this case, the IPCA(3) model is able to eliminate all the significant returns related to the managed portfolios. This confirms the observation from the ARM-IPCA models that the ARM premiums using a more liquid dataset can be explained by a smaller number of latent factors.

4.2 IPCA Alphas of the Pure-Alpha Portfolios

In this subsection, I investigate whether the IPCA factors estimated using the original set of 32 characteristics can explain abnormal returns related to the pure-alpha portfolios estimated using the set of 11 ARMs. The results of this analysis are contained in Table 13. We observe that when combined, returns of the pure-alpha portfolios cannot be explained by the IPCA factors estimated using the original IPCA factors. That result is in contrast to the results of managed portfolios single-sorted on the ARMs.

Table 13: *IPCA alphas of the ARM-IPCA pure-alpha portfolios.* The table reports alphas and their t-statistics of the returns of the ARM-IPCA pure-alpha portfolios with respect to out-of-sample IPCA factors with one to six latent factors and 32 characteristics from Kelly et al. (2019) as instruments. Panel A reports results using all stocks, Panel B excludes stocks with a price less than \$5 or market cap below 10% quantile of NYSE stocks. Data cover the period between January 1968 and December 2018.

	PanelA: All stocks						*Panel B: No penny stocks*					
	IPCA1	IPCA2	IPCA3	IPCA4	IPCA5	IPCA6	IPCA1	IPCA2	IPCA3	IPCA4	IPCA5	IPCA6
ARM-IPCA1	14.12	14.60	12.95	8.60	10.62	12.07	13.54	13.51	6.21	1.76	8.10	9.84
	(4.77)	(4.99)	(2.71)	(2.01)	(2.34)	(2.69)	(4.04)	(3.73)	(1.52)	(0.63)	(2.08)	(2.45)
ARM-IPCA2	18.85	19.50	18.83	12.15	13.65	18.33	15.72	15.87	7.81	4.99	8.52	11.01
	(6.30)	(6.89)	(3.57)	(2.54)	(2.74)	(3.62)	(4.52)	(4.33)	(1.68)	(1.55)	(2.07)	(2.54)
ARM-IPCA3	16.40	16.95	21.09	16.29	16.42	18.87	11.05	10.39	5.51	2.72	6.69	7.02
	(5.26)	(6.07)	(4.12)	(3.21)	(3.03)	(3.25)	(3.24)	(2.91)	(1.47)	(0.89)	(1.83)	(1.91)
ARM-IPCA4	7.12	7.47	3.29	3.04	7.94	10.61	2.68	1.49	-3.46	-1.37	2.89	1.15
	(2.62)	(2.94)	(0.88)	(0.81)	(2.03)	(2.16)	(0.92)	(0.49)	(-1.20)	(-0.44)	(0.81)	(0.36)
ARM-IPCA5	7.25	7.40	4.44	3.82	7.96	11.83	0.59	-0.25	-4.20	-2.87	3.17	-0.34
	(2.58)	(2.76)	(1.24)	(1.05)	(2.12)	(2.58)	(0.20)	(-0.08)	(-1.53)	(-0.94)	(0.87)	(-0.11)
ARM-IPCA6	5.47	4.52	1.90	3.12	2.12	4.50	1.47	0.64	-3.33	-2.65	3.17	-0.05
	(1.92)	(1.65)	(0.65)	(0.98)	(0.64)	(1.22)	(0.52)	(0.21)	(-1.22)	(-0.87)	(0.85)	(-0.02)

Discuss the results in more detail HERE.

4.3 Model with All Characteristics

Next, I investigate how the ARMs can improve the original specification of the restricted IPCA model. To do so, I estimate the IPCA model using both the original set of 32 variables of Kelly et al. (2019) and 11 additional ARM variables, hence All-IPCA. Table 22 from Appendix C reports the in-sample IPCA results. We observe that, similarly as in the case ARM-IPCA, around six factors are needed to obtain an appropriate model that provides an adequate description of the behavior of stock returns.

To assess how ARMs contribute to the fit of the model, I test whether ARMs as a whole possess coefficients significantly different from zero. This is a generalization of the test discussed earlier which tests the importance of each variable separately. The testing procedure follows the same logic based on wild bootstrap. One difference is the definition of the Wald-type test statistic. In this case, we test whether a subset of J characteristics contributes significantly to the performance, so the statistic is $W_{\beta,l_1,\dots,l_J} = \hat{\gamma}'_{\beta,l_1}\hat{\gamma}_{\beta,l_1} + \ldots + \hat{\gamma}'_{\beta,l_J}\hat{\gamma}_{\beta,l_J}$. The other difference is that the restricted model sets contribution to all J tested characteristics to zero. The logic behind the rest of the test is the same.

As both datasets display a good fit with approximately six factors, I focus on the IPCA(6) model. The resulting test possess a mildly significant p-value of 6.8% in the case of all stocks. In the case of the liquid dataset, the test yields an insignificant p-value of 17.9%. Those results suggest that the ARMs are able to contribute to the explanation of stock returns in the case of the full dataset, but they do not provide significant additional information

Table 14: *All-IPCA alphas of the ARM-managed portfolios.* The table reports alphas and their t-statistics of returns of the ARM-managed portfolios with respect to out-of-sample IPCA factors with one to six latent factors and 32 characteristics from Kelly et al. (2019) and 11 ARMs as instruments. Panel A reports results using all stocks, Panel B excludes stocks with a price less than \$5 or market cap below 10% quantile of NYSE stocks. Data cover the period between January 1968 and December 2018.

	Panel A: All stocks						*Panel B: No penny stocks*					
	IPCA1	IPCA2	IPCA3	IPCA4	IPCA5	IPCA6	IPCA1	IPCA2	IPCA3	IPCA4	IPCA5	IPCA6
coskew	-0.20	-0.20	-0.39	0.23	0.23	-0.06	-0.21	-0.38	-0.07	0.01	-0.27	0.02
	(-1.47)	(-1.44)	(-1.84)	(1.06)	(1.03)	(-0.22)	(-1.51)	(-2.67)	(-0.32)	(0.08)	(-1.61)	(0.14)
cokurt	-0.50	-0.16	-0.04	-0.06	0.42	-0.36	-0.14	0.04	0.17	0.30	-0.01	0.02
	(-2.49)	(-0.82)	(-0.23)	(-0.23)	(1.97)	(-1.71)	(-0.79)	(0.25)	(1.07)	(1.76)	(-0.03)	(0.15)
beta_down	-0.79	-0.68	0.11	-0.61	-0.03	-0.23	-0.55	0.00	0.17	0.21	-0.04	-0.28
	(-3.21)	(-3.15)	(0.55)	(-1.82)	(-0.09)	(-0.75)	(-2.31)	(0.02)	(0.47)	(0.87)	(-0.15)	(-1.27)
down_corr	0.05	0.08	0.35	-0.21	-0.16	-0.02	0.18	0.32	0.04	-0.02	0.14	0.06
	(0.52)	(0.73)	(2.62)	(-1.39)	(-1.09)	(-0.11)	(1.71)	(2.91)	(0.26)	(-0.18)	(1.16)	(0.53)
htcr	0.04	0.42	0.00	0.13	0.52	-0.29	0.32	0.31	0.00	0.05	0.16	0.20
	(0.18)	(2.63)	(0.03)	(0.45)	(1.68)	(-1.12)	(2.28)	(2.09)	(-0.02)	(0.33)	(0.99)	(1.28)
beta_tr	0.23	0.34	0.08	0.00	0.29	-0.17	0.34	0.21	-0.21	0.13	-0.20	-0.08
	(1.49)	(2.36)	(0.28)	(0.00)	(1.02)	(-0.83)	(2.15)	(1.38)	(-1.07)	(0.75)	(-1.22)	(-0.47)
coentropy	0.01	0.02	0.29	-0.22	-0.20	-0.04	0.12	0.26	0.04	-0.03	0.10	0.00
	(0.15)	(0.24)	(2.28)	(-1.48)	(-1.38)	(-0.30)	(1.14)	(2.48)	(0.23)	(-0.23)	(0.85)	(0.04)
cos_pred	-0.32	0.00	-1.50	-0.17	0.07	-0.96	-0.14	-0.48	-0.50	-0.22	0.00	-0.12
	(-1.30)	(0.00)	(-4.05)	(-0.42)	(0.19)	(-2.18)	(-0.63)	(-1.80)	(-1.13)	(-0.96)	(0.01)	(-0.56)
beta_neg	-0.81	-0.96	0.30	-0.65	-0.30	-0.01	-0.84	-0.20	0.23	0.22	-0.27	-0.45
	(-2.20)	(-4.28)	(1.20)	(-1.41)	(-0.64)	(-0.03)	(-2.94)	(-0.64)	(0.51)	(0.79)	(-0.91)	(-1.68)
mcrash	-0.02	0.11	0.16	-0.06	0.09	-0.08	0.08	0.20	0.06	0.12	0.19	0.18
	(-0.16)	(1.03)	(1.67)	(-0.51)	(0.76)	(-0.67)	(0.79)	(1.99)	(0.54)	(0.98)	(1.61)	(1.85)
ciq_down	0.48	0.41	0.14	0.19	0.03	0.29	0.44	0.20	0.20	-0.01	-0.08	0.30
	(3.28)	(3.09)	(0.60)	(0.88)	(0.19)	(1.47)	(2.71)	(1.21)	(0.95)	(-0.05)	(-0.45)	(1.87)

regarding factor exposures when focusing on the liquid dataset.

Table 14 summarizes the alphas of the ARM portfolios with respect to the number of latent factors of the All-IPCA model. In this case, the IPCA model proves to be very successful in describing the results of the ARMs. IPCA(4) is able to erase all the significant returns of the ARM portfolios in the case of all stocks; in the case of no-penny stocks, IPCA(3) yields insignificant returns of the managed portfolios.

5 Time-Varying Risk Premium

The IPCA framework may not fully capture the arbitrage opportunities if the compensation for bearing risk associated with the ARMs is not stable across time periods. To investigate and potentially exploit the time-varying nature of the risk premium associated with the ARMs, I employ the projected principal component analysis (PPCA) framework proposed by Fan et al. (2016) and extended by Kim et al. (2020). In comparison to the IPCA framework, PPCA enables changes of cross-sectional relations between alphas/betas and characteristics. This variation may be potentially important if the relation between ARMs and risk/mispricing changes over time due to various reasons, such as being arbitraged away or beta-ARM relation changes.[4]

The PPCA framework first assigns maximal explanatory power of the characteristics to the systematic risk exposures before relating the characteristics to their alphas. The resulting arbitrage portfolio thus aims to hedge sources of systematic risk related to the characteristics while enjoying the residual (true anomaly) returns associated with the ARMs. Moreover, it enables the arbitrage portfolios to reflect the time variation in compensation for the ARMs by being consistently estimated over short samples.

5.1 Model and Estimation

Similarly, as in the case of the IPCA model, I assume that the excess return of stock follows the structure

$$r_{i,t} = \alpha_i + \beta_i f_t + \epsilon_{i,t} \tag{15}$$

where the main difference in comparison to IPCA is that now I assume that the return-generating process for individual stocks (characterized by α_i and β_i) is stable over short time periods (12 months in the empirical investigation) $t = 1, \ldots, T$. In a matrix format for N assets over T periods, this can be rewritten as

$$\boldsymbol{R} = \boldsymbol{\alpha} \boldsymbol{1}'_T + \boldsymbol{B}\boldsymbol{F}' + \boldsymbol{E} \tag{16}$$

where $\boldsymbol{R}$ is the $(N \times T)$ matrix of returns, $\boldsymbol{\alpha}$ is the $(N \times 1)$ mispricing vector, $\boldsymbol{B}$ is the $(N \times K)$ matrix with i-th row corresponding to factor exposure β'_i, $\boldsymbol{F}$ is $(T \times K)$ matrix of latent factors with t-th row being $f'_t = [f_{1,T}, \ldots, f_{K,t}]$. This specification allows the systematic exposure matrix $\boldsymbol{B}$ and vector of mispricing being nonparametric functions of the asset-specific characteristics. I stack each of the L characteristics into the $(N \times L)$ matrix $\boldsymbol{Z}$ and impose the following structure

$$\boldsymbol{\alpha} = \boldsymbol{G}_\alpha(\boldsymbol{Z}) + \Gamma_\alpha \tag{17}$$

$$\boldsymbol{B} = \boldsymbol{G}_\beta(\boldsymbol{Z}) + \Gamma_\beta \tag{18}$$

[4]An example of the former constitutes the results of Mclean and Pontiff (2016), which state that the relation changes by investors' usage of academic publications to learn about mispricing and forming their investment decisions based on that. An example of the latter represents Cho (2020) who argues that financial intermediaries through their arbitrage process and common exposure to funding liquidity shocks and arbitrageur wealth portfolio shocks turn alphas into betas.

where the mis-pricing function is defined as $\boldsymbol{G}_\alpha(\boldsymbol{Z}) : \mathbb{R}^{N\times L} \to \mathbb{R}^N$, and the factor loading function is $\boldsymbol{G}_\beta(\boldsymbol{Z}) : \mathbb{R}^{N\times L} \to \mathbb{R}^{N\times K}$, and the $(N \times 1)$ vector Γ_α and the $(N \times K)$ matrix Γ_β are cross-sectionally orthogonal to the characteristics $\boldsymbol{Z}$. To estimate this model, I follow projected principal component analysis proposed by Fan et al. (2016) and generalized by Kim et al. (2020) to allow for presence of the mis-pricing contained in $\boldsymbol{\alpha}$.

The formation of the arbitrage portfolio proceeds in three steps. First, I demean the returns and apply PCA to obtain an estimate of $\boldsymbol{G}_\beta(\boldsymbol{Z})$. Second, I cross-sectionally regress the average returns on the characteristics space which is orthogonal to the estimate $\boldsymbol{G}_\beta(\boldsymbol{Z})$ from the first step to obtain the estimate of $\boldsymbol{G}_\alpha(\boldsymbol{Z})$. Third, I use the estimate of $\boldsymbol{G}_\alpha(\boldsymbol{Z})$ to form the portfolio, which is held for the next period.

The main advantage of this methodology over the IPCA framework is that it is suited for the estimation over short time periods and thus enables to exploit the dynamics of the compensation for the ARMs. The model is estimated on a rolling-window basis, setting T equal to the short horizon. This allows for a change in cross-sectional relation between ARMs and returns either in terms of systematic risk or/and mispricing. Moreover, the model does not require to have all relevant characteristics for risk and mispricing, as the missing information may be contained in Γ_α and Γ_β. The aim of this model is to exploit mispricing captured by $\boldsymbol{\alpha}$ while hedging the systematic risk characterized by the ARMs and captured by $\boldsymbol{B}$.

In the empirical implementation, I cross-sectionally demean the characteristics so that the resulting arbitrage portfolio costs zero dollars. Moreover, I target the in-sample volatility of the portfolio at 20% per year. I report the results for a range between one and ten latent factors. All the results are purely out-of-sample as the model is fitted using 12 months of data, the arbitrage portfolio is formed at the end of this period, and then the return in the next month is recorded.

5.2 Arbitrage Returns

Table 15 summarizes the performances of the arbitrage portfolios that exploit the ARMs using a range between one and ten latent factors. Because I work with cross-sectionally demeaned values of characteristics, the resulting portfolio is based on zero investment. Panel A reports the results of the arbitrage portfolios that employ all stocks. We can see that when we use between two and ten factors in the model, we can obtain significant anomaly returns above the exposure to the common risks. The annual risk premium that we can obtain constitutes around 7.5% per year with a Sharpe ratio of around 0.45 and highly significant t-statistics of the mean return around three. Important results constitute alphas from re-

Table 15: *Arbitrage portfolios using ARMs.* The table reports abnormal mean returns, their t-statistics, Sharpe ratios, and alphas and their t-statistcs with respect to four factors of Carhart (1997), CIV shocks of Herskovic et al. (2016), and BAB factor of Frazzini and Pedersen (2014). The formation of the arbitrage portfolios is based on the extended PPCA framework of Kim et al. (2020) using a rolling window estimation of 12 months. Arbitrage returns are purely out-of-sample. Panel A reports results using all stocks, Panel B excludes stocks with a price less than \$5 or market cap below 10% quantile of NYSE stocks. Data cover the period between January 1968 and December 2018.

	Panel A: All stocks						*Panel B: No penny stocks*					
N factors	Mean	Std. Dev.	Sharpe	t-stat	α	t-stat	Mean	Std. Dev.	Sharpe	t-stat	α	t-stat
1	3.29	16.60	0.20	1.30	3.44	1.06	3.53	16.28	0.22	1.30	2.82	0.94
2	8.63	18.23	0.47	3.07	4.74	1.57	7.02	17.78	0.39	2.33	3.91	1.19
3	7.73	15.91	0.49	2.99	4.21	1.53	6.90	16.40	0.42	2.41	3.55	1.19
4	7.76	17.94	0.43	2.83	3.42	1.20	6.96	17.59	0.40	2.37	3.75	1.19
5	7.66	16.84	0.45	2.92	3.72	1.29	6.24	16.99	0.37	2.16	2.65	0.85
6	7.84	17.16	0.46	2.96	3.84	1.33	6.54	17.09	0.38	2.27	3.16	1.01
7	7.71	16.77	0.46	3.01	3.83	1.33	6.03	16.25	0.37	2.23	2.61	0.87
8	8.90	16.68	0.53	3.35	5.19	1.72	4.11	16.47	0.25	1.47	0.32	0.10
9	6.32	16.32	0.39	2.35	2.58	0.94	4.13	16.70	0.25	1.56	-0.44	-0.15
10	5.68	16.23	0.35	2.26	1.28	0.46	3.93	15.66	0.25	1.61	-0.47	-0.18

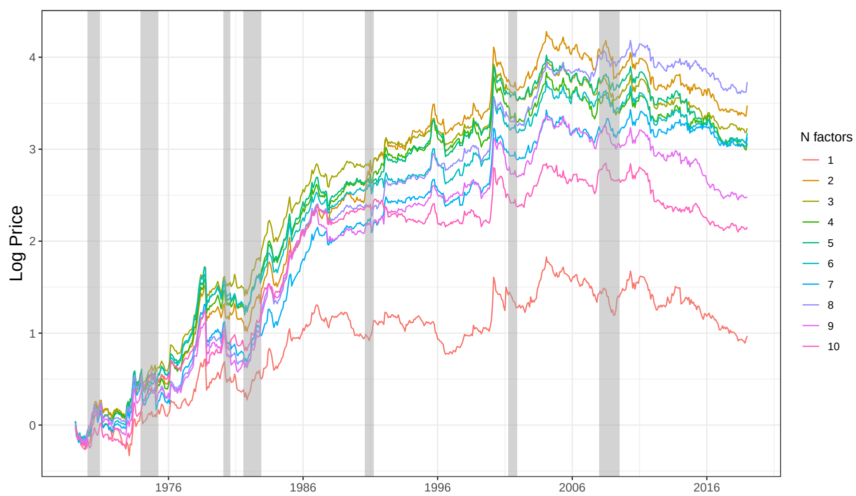
gressing the arbitrage returns on four factors of Carhart (1997), CIV shocks of Herskovic et al. (2016), and BAB factor of Frazzini and Pedersen (2014). We observe that the 6-factor alpha significantly reduces the premium associated with the ARMs. More specifically, the annual mean decreases to around 4% per annum and the corresponding t-statistic falls below two in all models.

Panel B reports similar results based on the panel of non-penny stocks. Arbitrage returns are slightly less significant, but generally draw the same conclusions. We can obtain around 6.5% anomaly return with a 0.4 Sharpe ratio, but the t-statistics of the 6-factor alpha falls at around 1. In Figure 4, I plot the cumulative logarithmic return of the arbitrage portfolios. We observe that the performance significantly deteriorates after the year 2000.

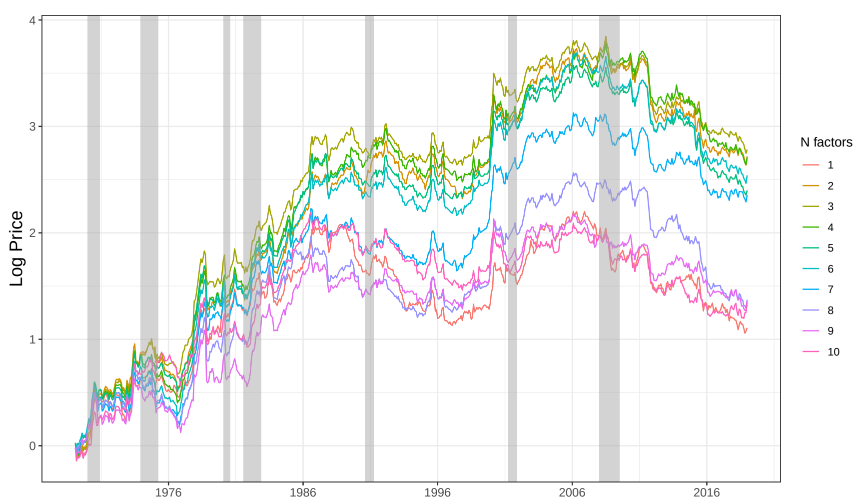
To further investigate which factors explain the arbitrage returns, in Table 16, I report estimated time-series coefficients for each number of latent factors. It is obvious that the arbitrage portfolio returns have an especially high and significant exposure to the momentum factor with t-statistic being above five for most of the specifications. Moreover, in the case of all stocks, HML factor significantly captures the time-variation of the arbitrage portfolio, as well. In thecase of dataset without penny stocks, arbitrage returns load significantly on the SMB factor.

We observe that the arbitrage returns obtained using PPCA do not yield abnormal returns beyond exposures to the common factors. Especially strong is the exposure to the momentum factor. Those observations suggest that the time variation of the prices of risk of the ARMs does not outweigh the loss of efficiency due to the short-window estimation. In comparison, pure-alpha portfolio returns obtained from the IPCA procedure using all stocks

Figure 4: *Cumulative return of the arbitrage portfolios.* The figure depicts the cumulative logarithm price of the arbitrage portfolios based on the PPCA framework of Kim et al. (2020) with the number of latent factors between one and ten. Arbitrage returns are purely out-of-sample. Panel A reports results using all stocks, Panel B excludes stocks with a price less than \$5 or market cap below 10% quantile of NYSE stocks. Data cover the period between January 1968 and December 2018.



(a) All stocks



(b) No penny stocks

Table 16: *Exposures of the arbitrage portfolios.* The table reports estimated coefficients and their t-statistics from regressing returns of the arbitrage portfolios on four factors of Carhart (1997), CIV shocks of Herskovic et al. (2016), and BAB factor of Frazzini and Pedersen (2014). The formation of the arbitrage portfolios is based on the extended PPCA framework of Kim et al. (2020) using a rolling window estimation of 12 months. Arbitrage returns are purely out-of-sample. Panel A reports results using all stocks, Panel B excludes stocks with a price less than \$5 or market cap below 10% quantile of NYSE stocks. Data cover the period between January 1968 and December 2018.

	Panel A: All stocks						
N factors	α	Mkt	SMB	HML	CIV	BAB	MOM
1	3.44	0.11	0.34	0.27	-0.05	-0.31	0.14
	(1.06)	(1.58)	(2.31)	(2.73)	(-1.72)	(-2.88)	(1.32)
2	4.74	0.11	0.30	0.40	-0.02	-0.26	0.53
	(1.57)	(1.47)	(2.19)	(3.23)	(-0.97)	(-2.68)	(5.84)
3	4.21	0.15	0.30	0.34	-0.03	-0.22	0.43
	(1.53)	(2.44)	(2.27)	(3.37)	(-1.50)	(-2.38)	(5.03)
4	3.42	0.13	0.26	0.33	-0.03	-0.19	0.53
	(1.20)	(1.94)	(1.88)	(2.81)	(-1.07)	(-2.07)	(5.95)
5	3.72	0.14	0.27	0.30	-0.03	-0.19	0.46
	(1.29)	(2.35)	(1.96)	(2.99)	(-1.19)	(-1.96)	(5.12)
6	3.84	0.12	0.28	0.30	-0.02	-0.17	0.46
	(1.33)	(1.86)	(2.12)	(2.92)	(-0.71)	(-1.82)	(5.49)
7	3.83	0.13	0.19	0.26	-0.02	-0.17	0.47
	(1.33)	(2.01)	(1.48)	(2.58)	(-0.72)	(-1.95)	(5.40)
8	5.19	0.11	0.21	0.16	-0.02	-0.12	0.45
	(1.72)	(1.59)	(1.60)	(1.61)	(-0.87)	(-1.27)	(4.79)
9	2.58	0.08	0.26	0.23	-0.01	-0.10	0.40
	(0.94)	(1.25)	(2.17)	(2.30)	(-0.32)	(-1.10)	(4.57)
10	1.28	0.13	0.35	0.27	-0.02	-0.11	0.43
	(0.46)	(2.07)	(2.82)	(3.46)	(-0.84)	(-1.20)	(5.05)
	Panel B: No penny stocks						
1	2.82	0.19	0.31	0.23	-0.02	-0.22	0.09
	(0.94)	(2.66)	(3.59)	(2.27)	(-0.73)	(-2.30)	(1.08)
2	3.91	0.10	0.33	0.28	-0.04	-0.26	0.48
	(1.19)	(1.48)	(3.02)	(2.50)	(-1.68)	(-2.53)	(4.91)
3	3.55	0.17	0.37	0.33	-0.05	-0.25	0.42
	(1.19)	(2.66)	(3.66)	(3.12)	(-2.24)	(-2.46)	(4.65)
4	3.75	0.09	0.32	0.23	-0.03	-0.23	0.49
	(1.19)	(1.40)	(2.98)	(1.87)	(-1.51)	(-2.21)	(5.07)
5	2.65	0.13	0.27	0.29	-0.04	-0.20	0.45
	(0.85)	(2.06)	(2.79)	(2.54)	(-2.06)	(-1.95)	(5.03)
6	3.16	0.14	0.30	0.29	-0.04	-0.23	0.45
	(1.01)	(2.16)	(3.10)	(2.55)	(-1.77)	(-2.34)	(4.90)
7	2.61	0.17	0.24	0.29	-0.02	-0.21	0.41
	(0.87)	(2.72)	(2.71)	(2.59)	(-1.11)	(-2.05)	(4.41)
8	0.32	0.17	0.29	0.23	-0.02	-0.15	0.42
	(0.10)	(2.51)	(3.15)	(2.05)	(-0.81)	(-1.50)	(4.36)
9	-0.44	0.17	0.30	0.19	-0.01	-0.12	0.48
	(-0.15)	(2.60)	(2.63)	(1.80)	(-0.30)	(-1.29)	(5.10)
10	-0.47	0.15	0.30	0.27	-0.03	-0.06	0.36
	(-0.18)	(2.39)	(3.49)	(2.93)	(-1.60)	(-0.77)	(5.36)

and up to five factors yield a significant premium after controlling for those six common factors. Moreover, the Sharpe ratios that attain the pure-alpha portfolios are considerably higher than in the case of the arbitrage portfolios based on the PPCA.

Table 17: *Variable importance of the arbitrage portfolios.* The table reports time-series averages of the variable importances of each variable separately for long and short legs of the arbitrage portfolio, and also their differences. Arbitrage portfolios are based on extended PPCA of Kim et al. (2020) with five latent factors. β_{G_b} is the averaged normalized projection coefficient of $\hat{\boldsymbol{G}}_\beta(\boldsymbol{Z})$ on the characteristic vector; β_{G_α} is the averaged normalized projection coefficient of $\hat{\boldsymbol{G}}_\alpha(\boldsymbol{Z})$ on the characteristic vector.

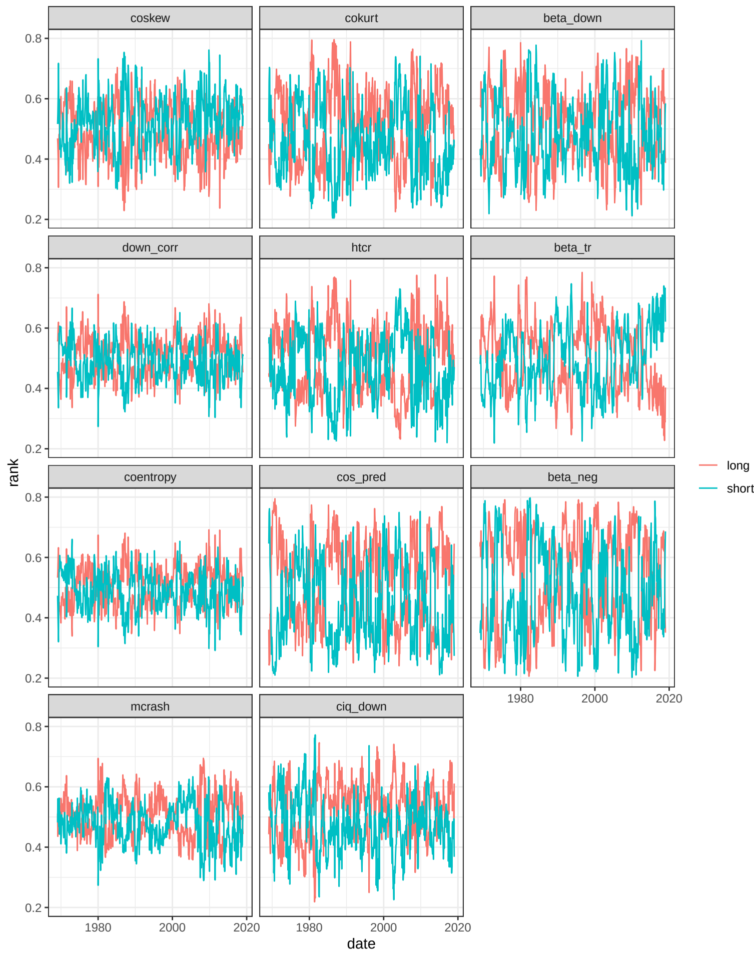
	Panel A: All stocks					*Panel B: No penny stocks*				
	Long	Short	L-S	β_{G_b}	β_{G_α}	Long	Short	L-S	β_{G_b}	β_{G_α}
`coskew`	0.49	0.52	-0.04	0.55	0.34	0.49	0.52	-0.03	0.56	0.35
`cokurt`	0.51	0.46	0.06	0.65	0.40	0.51	0.47	0.04	0.63	0.40
`beta_down`	0.52	0.48	0.04	0.53	0.42	0.56	0.46	0.10	0.61	0.42
`down_corr`	0.51	0.49	0.02	0.40	0.43	0.51	0.48	0.03	0.41	0.43
`htcr`	0.50	0.47	0.03	0.42	0.28	0.49	0.49	0.00	0.40	0.26
`beta_tr`	0.50	0.50	0.00	0.34	0.38	0.51	0.49	0.02	0.33	0.35
`coentropy`	0.51	0.49	0.02	0.38	0.43	0.51	0.49	0.03	0.37	0.40
`cos_pred`	0.53	0.44	0.10	0.88	0.58	0.52	0.47	0.06	0.81	0.50
`beta_neg`	0.55	0.49	0.06	0.66	0.68	0.58	0.45	0.13	0.77	0.69
`mcrash`	0.51	0.48	0.03	0.13	0.14	0.51	0.48	0.03	0.16	0.15
`ciq_down`	0.53	0.47	0.05	0.23	0.29	0.52	0.47	0.05	0.25	0.35

5.3 ARM's Importance

Next, I look how the arbitrage portfolio returns align with the ARMs. I will focus on the case with five common factors and report the corresponding results. Table 17 reports the average normalized ranks for each of the characteristics separately for the long and short leg of the arbitrage portfolio, respectively. Each month and for every ARM, I compute $\tilde{c}_{i,t} = \frac{\text{rank}(c_{i,t})}{N_t+1}$, then each variable is averaged over long and short leg of the portfolio, respectively, with weights being equal to the portfolio weights. Then, I report the time-series means of those averages. In the next column, I report the difference between long and short legs. Positive values correspond to the fact that the arbitrage portfolio is on average long in firms with a high value of a given ARM and vice versa. The magnitude of the difference corresponds to the significance of a given ARM for arbitrage portfolio performance. The portfolio is on average long in stocks with high levels of predicted coskewness, negative semibeta, cokurtosis, and downside CIQ beta.

To further investigate the relation between ARMs and systematic or anomaly risk, I follow the procedure from Kim et al. (2020) and project the estimated factor loadings, $\hat{\boldsymbol{G}}_\beta(\boldsymbol{Z})$, or the mispricing function, $\hat{\boldsymbol{G}}_\alpha(\boldsymbol{Z})$, onto the character vector each time period. I normalize the ARMs cross-sectionally so that the highest coefficient always receives a value of one. Values of β_{G_b} in Table 17 correspond to the average value of the systematic risk coefficient, β_{G_α} captures the average mispricing coefficient. We see that the predicted coskewness and negative semibeta are highly related to the systematic risk exposure, which agrees with the observations from the analysis using the IPCA model, in which case the predicted coskewness and negative semibeta heavily load on the first and second instrumented component,

Figure 5: *Variable importances for short and long legs of the arbitrage portfolio.* The figure depicts the time-evolution of variable importances for the short and long leg of the arbitrage portfolio based on the PPCA of Kim et al. (2020) with five latent factors. I use all available stocks between the period of January 1968 and December 2018.

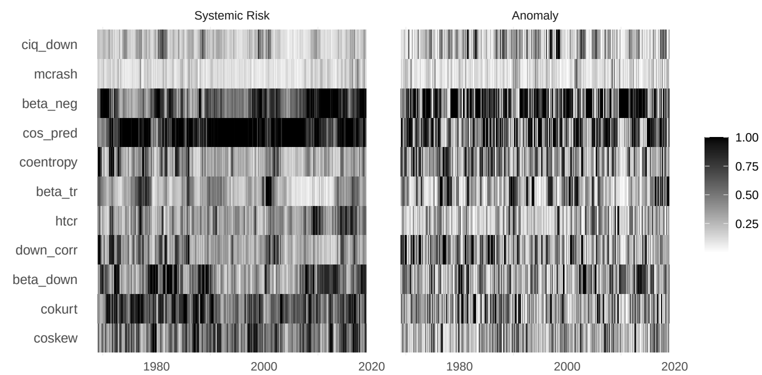


respectively. Negative semibeta, for example, is significantly related to mispricing.

Figure 5 depicts the dynamics of the ranks of the ARMs of short and long legs of the arbitrage portfolio. We observe that the relation between arbitrage portfolio weights and ARMs is quite volatile. Moreover, no ARM clearly dominates others in terms of their importance for the arbitrage portfolio.

Figure 6 captures how the relative importances of the ARMs vary in relation to systematic and anomaly risk, respectively. We can see that the relationship between ARMs and systematic risk is quite stable with the highest importance having the predicted coskewness, cokurtosis, and also negative semibeta. In terms of anomaly risk, both predicted coskewness and negative semibeta posses a significant effect, but the relative importance is more spread across all the ARMs. All the presented results are robust to the inclusion of the non-penny stocks only. I report the results for the dataset that excludes penny stocks in Appendix D.

Figure 6: *Relative importances of the ARMs for the systematic and anomaly risk.* The figure depicts the time-evolution of relative importances of the ARMs for systematic and anomaly risk of the PPCA model of Kim et al. (2020) with five latent factors. I use all available stocks between the period of January 1968 and December 2018.



6 Conclusion

TBA

References

Ang, A., J. Chen, and Y. Xing (2006, 03). Downside Risk. *The Review of Financial Studies 19*(4), 1191–1239.

Backus, D., N. Boyarchenko, and M. Chernov (2018). Term structures of asset prices and returns. *Journal of Financial Economics 129*(1), 1–23.

Bali, T. G., N. Cakici, and R. F. Whitelaw (2014, 09). Hybrid Tail Risk and Expected Stock Returns: When Does the Tail Wag the Dog? *The Review of Asset Pricing Studies 4*(2), 206–246.

Barunik, J. and M. Nevrla (2022). Common idiosyncratic quantile risk. *arXiv preprint arXiv:2208.14267*.

Bollerslev, T. (1986). Generalized autoregressive conditional heteroskedasticity. *Journal of Econometrics 31*(3), 307–327.

Bollerslev, T., A. J. Patton, and R. Quaedvlieg (2021). Realized semibetas: Disentangling "good" and "bad" downside risks. *Journal of Financial Economics*.

Carhart, M. M. (1997). On persistence in mutual fund performance. *The Journal of Finance 52*(1), 57–82.

Chabi-Yo, F., M. Huggenberger, and F. Weigert (2022). Multivariate crash risk. *Journal of Financial Economics 145*(1), 129–153.

Chen, L., J. J. Dolado, and J. Gonzalo (2021). Quantile factor models. *Econometrica 89*(2), 875–910.

Cho, T. (2020). Turning alphas into betas: Arbitrage and endogenous risk. *Journal of Financial Economics 137*(2), 550–570.

Dittmar, R. F. (2002). Nonlinear pricing kernels, kurtosis preference, and evidence from the cross section of equity returns. *The Journal of Finance 57*(1), 369–403.

Fama, E. F. and K. R. French (1993). Common risk factors in the returns on stocks and bonds. *Journal of Financial Economics 33*(1), 3–56.

Fama, E. F. and K. R. French (2015). A five-factor asset pricing model. *Journal of Financial Economics 116*(1), 1–22.

Fama, E. F. and J. D. MacBeth (1973). Risk, return, and equilibrium: Empirical tests. *Journal of Political Economy 81*(3), 607–636.

Fan, J., Y. Liao, and W. Wang (2016). Projected principal component analysis in factor models. *The Annals of Statistics 44*(1), 219–254.

Farago, A. and R. Tédongap (2018). Downside risks and the cross-section of asset returns.

Journal of Financial Economics 129(1), 69–86.

Frazzini, A. and L. H. Pedersen (2014). Betting against beta. *Journal of Financial Economics 111*(1), 1–25.

Freyberger, J., A. Neuhierl, and M. Weber (2020, 04). Dissecting Characteristics Nonparametrically. *The Review of Financial Studies 33*(5), 2326–2377.

Gibbons, M. R., S. A. Ross, and J. Shanken (1989). A test of the efficiency of a given portfolio. *Econometrica 57*(5), 1121–1152.

Gul, F. (1991). A theory of disappointment aversion. *Econometrica 59*(3), 667–686.

Harvey, C. R. and A. Siddique (2000). Conditional skewness in asset pricing tests. *The Journal of Finance 55*(3), 1263–1295.

Herskovic, B., B. Kelly, H. Lustig, and S. Van Nieuwerburgh (2016). The common factor in idiosyncratic volatility: Quantitative asset pricing implications. *Journal of Financial Economics 119*(2), 249–283.

Hong, Y., J. Tu, and G. Zhou (2006, 09). Asymmetries in Stock Returns: Statistical Tests and Economic Evaluation. *The Review of Financial Studies 20*(5), 1547–1581.

Jiang, L., K. Wu, and G. Zhou (2018). Asymmetry in stock comovements: An entropy approach. *Journal of Financial and Quantitative Analysis 53*(4), 1479–1507.

Kelly, B. and H. Jiang (2014, 06). Tail Risk and Asset Prices. *The Review of Financial Studies 27*(10), 2841–2871.

Kelly, B., D. Palhares, and S. Pruitt (2023). Modeling corporate bond returns. *The Journal of Finance 78*(4), 1967–2008.

Kelly, B. T., S. Pruitt, and Y. Su (2019). Characteristics are covariances: A unified model of risk and return. *Journal of Financial Economics 134*(3), 501–524.

Kelly, B. T., S. Pruitt, and Y. Su (2020). Instrumented principal component analysis. *Available at SSRN 2983919*.

Kim, S., R. A. Korajczyk, and A. Neuhierl (2020, 09). Arbitrage Portfolios. *The Review of Financial Studies 34*(6), 2813–2856.

Kozak, S., S. Nagel, and S. Santosh (2020). Shrinking the cross-section. *Journal of Financial Economics 135*(2), 271–292.

Langlois, H. (2020). Measuring skewness premia. *Journal of Financial Economics 135*(2), 399–424.

Lettau, M. and M. Pelger (2020). Factors that fit the time series and cross-section of stock returns. *The Review of Financial Studies 33*(5), 2274–2325.

Lopez-Lira, A. and N. L. Roussanov (2020). Do common factors really explain the cross-section of stock returns? *Jacobs Levy Equity Management Center for Quantitative Financial Research Paper*.

Mclean, R. D. and J. Pontiff (2016). Does academic research destroy stock return predictability? *The Journal of Finance 71*(1), 5–32.

Newey, W. K. and K. D. West (1987). A simple, positive semi-definite, heteroskedasticity and autocorrelation consistent covariance matrix. *Econometrica 55*(3), 703–708.

Newey, W. K. and K. D. West (1994). Automatic lag selection in covariance matrix estimation. *The Review of Economic Studies 61*(4), 631–653.

Ross, S. A. (1976). The arbitrage theory of capital asset pricing. *Journal of Economic Theory 13*(3), 341–360.

Routledge, B. R. and S. E. Zin (2010). Generalized disappointment aversion and asset prices. *The Journal of Finance 65*(4), 1303–1332.

A Appendix A – Definitions of the ARMs

This Appendix provides a simple exposition to the estimation process of each of the asymmetric risk measures employed in the main text. For further details regarding the nuances of the related computations, consult the original papers.

I use two sources of data to compute the asymmetric risk measures. First, I use either daily or monthly data of stock returns from the CRSP database. Second, I use value-weighted return of the CRSP stocks from the Kenneth French's online library to approximate the overall market return.

Variables are estimated using moving window of various lengths following the procedures proposed in their original papers. In case of measures estimated from the daily stock returns, I use mostly moving window of one year. I require at least 200 daily observations during the window to be included. I estimate measures based on monthly return data using window of at least 60 months and demand at least 36 monthly observations.

The measures are estimated following their definition proposed in the literature. In some cases, I slightly change the requirements regarding the minimal history of stocks to be included in the analysis. This modification aims at the precision of the estimates as well as the widest possible dataset.

Throughout the section, I use $r_{i,t}$ and $r^e_{i,t}$ to denote a raw and excess return of an asset i at time t, respectively. The raw and excess market return is denoted by f_t and f^e_t. Corresponding variables with bar denote their time-series averages computed in a given window.

A.1 Coskewness

Coskewness (`coskew`) of Harvey and Siddique (2000) is estimated using daily excess returns and is defined as

$$CSK_i = \frac{\frac{1}{T}\sum_{t=1}^{T}(r^e_{i,t} - \bar{r}^e_i)(f^e_t - \bar{f}^e)^2}{\sqrt{\frac{1}{T}\sum_{t=1}^{T}(r^e_{i,t} - \bar{r}^e_i)^2}\frac{1}{T}\sum_{t=1}^{T}(f^e_t - \bar{f}^e)^2}. \tag{19}$$

Estimation window is set to 1 year, at least 200 daily observations are required.

A.2 Cokurtosis

Cokurtosis (`cokurt`) of Dittmar (2002) is estimated using daily data and is defined as

$$CKT_i = \frac{\frac{1}{T}\sum_{t=1}^{T}(r_{i,t}^e - \bar{r}_i^e)(f_t^e - \bar{f}^e)^3}{\sqrt{\frac{1}{T}\sum_{t=1}^{T}(r_{i,t}^e - \bar{r}_i^e)^2}\frac{1}{T}\left(\sum_{t=1}^{T}(f_t^e - \bar{f}^e)^2\right)^{3/2}} \tag{20}$$

Estimation window is set to 1 year, at least 200 daily observations are required.

A.3 Downside Beta

Downside (`beta_down`) beta of Ang et al. (2006) is estimated using daily data and is defined as

$$\beta_i^{DR} = \frac{\sum\limits_{f_t^e < \bar{f}^e}(r_{i,t}^e - \bar{r}_i^e)(f_t^e - \bar{f}^e)}{\sum\limits_{f_t^e < \bar{f}^e}(f_t^e - \bar{f}^e)^2}. \tag{21}$$

Estimation window is set to 1 year, at least 200 daily observations are required.

A.4 Downside Correlation

Downside correlation (`down_corr`) based on Hong et al. (2006) and Jiang et al. (2018) is estimated using daily data and is defined as

$$\mathbb{C}or_i^- = \mathbb{C}or(r_i, f | r_i < 0, f < 0) - \mathbb{C}or(r_i, f | r_i > 0, f > 0) \tag{22}$$

using empirical counterpart of the correlation. Minimum of 200 observations in the 1-year window is demanded.

A.5 Hybrid Tail Covariance Risk

Hybrid tail covariance risk (`htcr`) of Bali et al. (2014) is estimated using daily data using 6-month window with at least 80 daily observations as

$$HTCR_i = \sum_{r_{i,t} < h_i}(r_{i,t} - h_i)(f_t - h_f) \tag{23}$$

where h_i and h_f are the 10% empirical quantiles of stock and market return, respectively.

A.6 Tail Risk Beta

Tail risk beta (`beta_tr`) of Kelly and Jiang (2014) is estimated using monthly return data using 120-month window with requirement of at least 36 monthly observations. Beta is computed by means of least-square estimator from the predictive regression of the form

$$r_{i,t+1} = \mu_i + \beta_i^{TR}\lambda_t + \epsilon_{t+1,i} \tag{24}$$

where the tail risk factor is obtained as

$$\lambda_t = \frac{1}{K_t}\sum_{k=1}^{K_t} \ln\frac{e_{k,t}}{u_t} \tag{25}$$

where $e_{k,t}$ is the kth daily idiosyncratic return that falls below an extreme value threshold u_t during month t, and K_t is the total number of such exceedences within month t. Idiosyncratic return is computed relative to 3-factor model of Fama and French (1993), and the threshold value is taken to be 5% quantile from the monthly cross-section of daily returns.

A.7 Exceedance Coentropy

Exceedance coentropy (`coentropy`) measure based on Backus et al. (2018) and Jiang et al. (2018) using daily data and 1-year estimation window with at least 200 observations is based on

$$C^{+}(0, r_i, f) = \frac{L(r_i f) - L(r_i) - L(f)}{L(r_i) + L(f)}\Big|(r_i > 0, y > 0) \tag{26}$$

$$C^{-}(0, r_i, f) = \frac{L(r_i f) - L(r_i) - L(f)}{L(r_i) + L(f)}\Big|(r_i < 0, y < 0) \tag{27}$$

where $L(x) = \ln\mathbb{E}(x) - \mathbb{E}(\ln x)$. The measure is then defined as

$$Coentropy = C^{-}(0, r_i, f) - C^{+}(0, r_i, f). \tag{28}$$

A.8 Predicted Systematic Coskewness

Predicted systematic coskewness (`cos_pred`) of Langlois (2020) is based on

$$Cos_{i,t} = \mathbb{C}ov_{t-1}\big(r_{i,t}, f_t^2\big), \tag{29}$$

then, each month I run the panel regression using all available stocks and history of data

$$F\big(Cos_{i,k-12\to k-1}\big) = \kappa + F\big(Y_{i,k-24\to k-13}\big)\theta + F\big(X_{i,k-13}\big)\phi + \epsilon_{i,k-12\to k-1} \tag{30}$$

where $Cos_{i,k-12\to k-1}$ is the coskewness from Equation 29 computed using daily returns from month $k-12$ to month $k-1$, $Y_{i,k-24\to k-13}$ are risk measures (volatility, market beta, etc.) estimated using daily data from month $k-24$ to month $k-13$, and $X_{i,k-13}$ are characteristics (size, book-to-price, etc.) observed at the end of month $k-13$. The function $F(x_{i,t}) = \frac{Rank(x_{i,t})}{N_t+1}$ transforms the original variable into its normalized rank in the cross-section of variable x_t, which posses N_t observations.

The predicted systematic coskewness for each stock is then obtained using the estimated coefficients of $\hat{\kappa}, \hat{\theta}, \hat{\phi}$ as

$$F\big(\widehat{Cos_{i,t\to t+11}}\big) = \hat{\kappa} + F\big(Y_{i,t-12\to t-1}\big)\hat{\theta} + F\big(X_{i,t-1}\big)\hat{\phi}. \tag{31}$$

The choice of risk measures and characteristics employed in the prediction of systematic skewness follows closely Langlois (2020).

A.9 Semibeta

Negative semibeta (`beta_neg`) of Bollerslev et al. (2021) is estimated using daily data with 1-year moving window as

$$\beta_i^N = \frac{\sum\limits_{r_{i,t}<0, f_t<0} r_{i,t} f_{i,t}}{\sum\limits_t f_t^2} \tag{32}$$

with the requirement of at least 200 daily observations.

A.10 Multivariate Crash Risk

Multivariate crash risk (`mcrash`) of Chabi-Yo et al. (2022) is estimated using daily data with 1-year window and minimum of 200 observations in the following steps. First, for each stock separately, using stock and N factor returns, I estimate $N+1$ GARCH(1,1) models of Bollerslev (1986) to obtain a series of conditional distribution functions $F_{i,t}(h) = \mathbb{P}_{t-1}[r_{i,t} \leq h]$ and use it to compute probability integral transforms as $\hat{u}_{i,t} = F_{i,t}(r_{i,t})$. Second, I estimate

MCRASH as

$$\text{MCRASH}_{i,t} = \frac{\sum_t \mathbb{I}(\{\hat{u}_{1,t} \leq p\}) \cdot \mathbb{I}(\cup_{j=2}^{N+1}\{\hat{u}_{j,t} \leq p\})}{\sum_t \mathbb{I}(\cup_{j=2}^{N+1}\{\hat{u}_{j,t} \leq p\})} \tag{33}$$

where $\mathbb{I}$ denotes the indicator function and p is set to 0.05. I follow the baseline specification of Chabi-Yo et al. (2022) and use the five factors of Fama and French (2015), momentum factor of Carhart (1997) and betting-against-beta factor of Frazzini and Pedersen (2014).

A.11 Downside CIQ Beta

Downside common idiosyncratic quantile risk beta (`ciq_down`) of Barunik and Nevrla (2022) is estimated using monthly data with 60-month window and requirement of at least 48 observations as

$$\beta_i^{down} = \sum_{\tau \in \tau_{down}} F\big(\beta_i(\tau)\big) \tag{34}$$

which gives the average cross-sectional rank of the common idiosyncratic quantile (CIQ) betas for downside τ CIQ factors. CIQ betas are estimated from time-series regression of stock returns on the increments of CIQ factors. The CIQ factors are estimated using residuals from Fama and French (1993) factors and following the quantile factor model of Chen et al. (2021).

B Appendix B – ARM Portfolio Returns

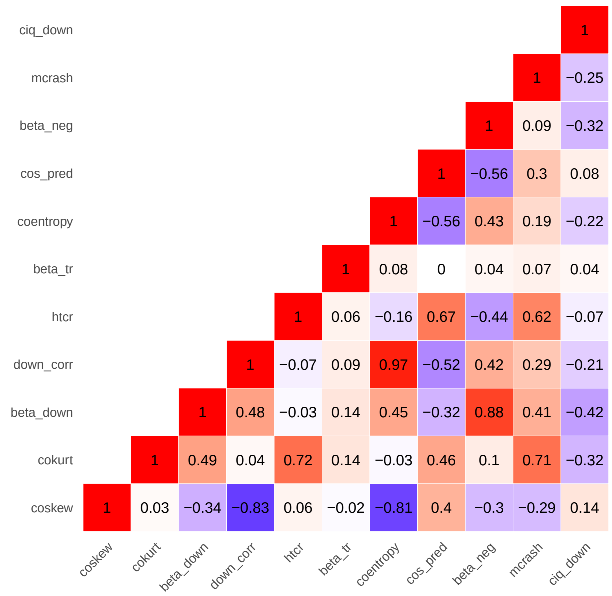
Table 18: *Quintile portfolio sorts.* The table contains annualized out-of-sample returns of five monthly rebalanced portfolios sorted on various asymmetric risk measures. It also reports returns of the high minus low (H - L) portfolios, their t-statistics, and annualized 6-factor alphas and their t-statistics with respect to the four factors of Carhart (1997), CIV shocks of Herskovic et al. (2016), and BAB factor of Frazzini and Pedersen (2014). I use HAC t-statistics of Newey and West (1987) with 6 lags. Panel A reports results using all stocks, Panel B excludes stocks with a price less than \$5 or market cap below 10% quantile of NYSE stocks. Data cover the period between January 1968 and December 2018.

Variable	Low	2	3	4	High	H-L	t-stat	α	t-stat
Panel A: All stocks									
Equal-weighted									
`coskew`	11.08	10.61	9.68	8.84	8.43	-2.65	-2.24	-1.69	-1.12
`cokurt`	11.69	10.44	9.84	8.90	7.78	-3.91	-2.40	-1.10	-0.63
`beta_down`	11.08	9.88	9.76	9.91	8.01	-3.07	-1.44	0.64	0.47
`down_corr`	8.73	9.51	10.14	9.96	10.31	1.57	1.98	0.99	0.97
`htcr`	9.99	9.00	9.98	9.93	9.75	-0.24	-0.12	-1.63	-0.74
`beta_tr`	8.20	9.07	9.44	10.48	11.47	3.27	2.36	3.10	1.47
`coentropy`	9.11	9.35	9.77	10.01	10.40	1.29	1.61	0.87	0.83
`cos_pred`	12.43	10.60	9.16	8.51	7.95	-4.48	-1.78	-4.69	-1.79
`beta_neg`	9.67	10.40	10.38	10.02	8.17	-1.50	-0.43	3.30	1.84
`mcrash`	10.03	9.84	9.47	10.00	9.91	-0.13	-0.13	0.18	0.19
`ciq_down`	7.16	9.51	10.22	10.22	11.54	4.38	2.98	5.51	3.67
Value-weighted									
`coskew`	6.93	7.57	7.66	6.18	4.47	-2.46	-1.60	1.39	0.71
`cokurt`	5.30	7.26	6.89	6.62	5.74	0.45	0.24	4.34	2.57
`beta_down`	5.92	7.05	6.69	6.34	5.18	-0.74	-0.27	1.51	0.70
`down_corr`	5.70	5.10	6.97	7.41	7.91	2.21	1.84	-1.35	-0.93
`htcr`	5.79	5.66	6.37	6.57	5.92	0.13	0.06	1.10	0.65
`beta_tr`	4.34	5.98	7.11	7.72	8.88	4.54	2.59	5.85	2.58
`coentropy`	4.73	6.05	6.62	7.25	7.71	2.98	2.18	-0.64	-0.42
`cos_pred`	11.66	8.56	8.10	6.43	5.57	-6.09	-2.31	-3.42	-1.44
`beta_neg`	7.06	6.56	6.60	5.94	2.85	-4.21	-1.17	-0.65	-0.31
`mcrash`	4.99	7.04	6.64	6.02	6.42	1.43	1.05	-0.07	-0.04
`ciq_down`	5.18	5.68	7.07	7.07	8.08	2.90	1.52	4.27	2.49
Pabel B: No penny stocks									
Equal-weighted									
`coskew`	9.30	9.06	8.83	7.98	6.59	-2.71	-2.24	-0.84	-0.56
`cokurt`	8.07	9.08	8.53	8.49	7.58	-0.48	-0.34	2.36	1.76
`beta_down`	8.26	8.64	9.09	9.14	6.62	-1.63	-0.66	0.96	0.69
`down_corr`	6.83	7.97	8.97	8.73	9.25	2.42	2.76	0.48	0.51
`htcr`	5.65	8.12	8.90	9.89	9.19	3.54	2.82	3.24	2.72
`beta_tr`	6.10	8.38	8.26	9.26	9.75	3.64	2.62	1.70	1.07
`coentropy`	7.12	8.07	8.94	8.77	8.85	1.73	1.98	0.05	0.05
`cos_pred`	9.68	8.58	8.16	7.75	7.59	-2.09	-0.94	-1.03	-0.56
`beta_neg`	8.82	9.38	9.39	9.03	5.15	-3.67	-1.21	-0.21	-0.16
`mcrash`	7.47	7.75	8.64	8.62	9.13	1.66	1.71	1.49	1.54
`ciq_down`	5.31	8.85	9.09	8.91	9.59	4.28	2.64	5.17	3.71
Value-weighted									
`coskew`	6.68	6.99	7.42	7.14	4.23	-2.44	-1.64	1.25	0.70
`cokurt`	5.93	6.73	5.97	7.16	5.53	-0.40	-0.25	3.51	2.31
`beta_down`	6.01	7.09	7.02	5.57	5.31	-0.71	-0.27	1.30	0.69
`down_corr`	5.49	5.31	6.69	7.47	7.72	2.23	1.92	-1.37	-0.96
`htcr`	4.92	6.42	6.82	6.11	6.00	1.09	0.71	1.89	1.20
`beta_tr`	4.86	6.24	6.48	7.48	8.21	3.36	2.10	3.61	1.77
`coentropy`	4.99	5.99	6.34	7.40	7.43	2.44	1.93	-1.13	-0.80
`cos_pred`	9.76	7.75	7.03	5.47	5.86	-3.90	-1.65	-0.66	-0.31
`beta_neg`	6.52	6.54	6.69	5.42	3.60	-2.92	-0.92	0.37	0.20
`mcrash`	5.98	6.20	6.32	5.81	6.34	0.35	0.28	-1.00	-0.63
`ciq_down`	4.74	5.66	6.45	6.96	7.67	2.93	1.54	3.58	2.34

Table 19: *Decile portfolio sorts.* The table contains annualized out-of-sample returns of ten monthly rebalanced portfolios sorted on various asymmetric risk measures. It also reports returns of the high minus low (H - L) portfolios, their t-statistics, and annualized 6-factor alphas and their t-statistics with respect to the four factors of Carhart (1997), CIV shocks of Herskovic et al. (2016), and BAB factor of Frazzini and Pedersen (2014). I use HAC t-statistics of Newey and West (1987) with 6 lags. Panel A reports results using all stocks, Panel B excludes stocks with a price less than \$5 or market cap below 10% quantile of NYSE stocks. Data cover period the between January 1968 and December 2018.

Variable	Low	2	3	4	5	6	7	8	9	High	H-L	t-stat	α	t-stat
Panel A: All stocks														
Equal-weighted														
`coskew`	11.46	10.70	10.58	10.64	10.08	9.29	9.21	8.47	9.40	7.46	-4.00	-2.86	-2.62	-1.44
`cokurt`	12.34	11.04	10.69	10.19	9.60	10.08	8.97	8.82	8.33	7.23	-5.11	-2.65	-2.14	-1.06
`beta_down`	11.82	10.35	9.85	9.91	10.08	9.44	9.78	10.04	8.79	7.24	-4.58	-1.79	0.17	0.11
`down_corr`	9.26	8.21	9.49	9.52	9.46	10.81	9.65	10.27	10.45	10.16	0.90	0.91	0.25	0.20
`htcr`	10.90	9.07	9.25	8.76	9.96	9.99	9.51	10.35	10.10	9.40	-1.51	-0.61	-3.06	-1.13
`beta_tr`	8.40	7.99	9.03	9.10	9.69	9.18	10.17	10.79	11.40	11.53	3.13	1.71	3.25	1.19
`coentropy`	9.56	8.66	9.40	9.29	10.00	9.54	10.46	9.56	10.92	9.89	0.33	0.35	-0.24	-0.19
`cos_pred`	13.10	11.75	11.17	10.04	8.95	9.37	8.22	8.80	8.31	7.59	-5.52	-1.77	-5.71	-1.80
`beta_neg`	9.18	10.15	10.27	10.53	10.03	10.74	10.54	9.51	8.90	7.44	-1.74	-0.41	4.30	1.84
`mcrash`	9.86	8.71	10.48	9.85	8.22	9.38	11.47	9.77	9.06	10.55	0.68	0.53	0.61	0.50
`ciq_down`	6.33	7.99	9.03	10.00	9.90	10.55	10.03	10.40	11.51	11.57	5.24	2.97	5.71	3.14
Value-weighted														
`coskew`	8.06	6.67	7.66	7.66	8.21	6.97	6.31	6.20	5.75	2.78	-5.28	-2.92	-0.93	-0.39
`cokurt`	6.94	4.52	8.07	7.03	6.33	7.36	6.32	6.80	7.04	5.35	-1.59	-0.72	3.44	1.65
`beta_down`	6.75	6.37	7.02	7.23	6.96	6.36	6.20	6.85	5.83	4.42	-2.33	-0.64	0.23	0.08
`down_corr`	5.01	6.02	5.42	4.86	6.05	8.34	7.16	7.83	8.74	6.73	1.72	1.13	-2.67	-1.80
`htcr`	5.94	5.47	6.20	5.35	6.39	6.39	6.18	6.89	7.20	5.08	-0.86	-0.31	0.35	0.15
`beta_tr`	5.01	3.94	5.50	6.56	7.10	7.32	8.03	7.55	8.66	8.75	3.75	1.63	4.43	1.54
`coentropy`	4.60	4.95	5.86	6.29	6.07	7.04	7.13	7.66	8.44	6.83	2.23	1.41	-2.33	-1.40
`cos_pred`	12.52	11.04	9.85	7.50	8.67	7.85	7.15	6.14	5.24	5.81	-6.70	-2.05	-4.53	-1.44
`beta_neg`	7.03	7.59	7.04	6.34	6.46	6.92	6.01	6.02	4.67	-0.62	-7.65	-1.77	-4.11	-1.58
`mcrash`	5.00	4.62	9.77	5.85	6.65	6.32	5.74	7.07	6.22	7.02	2.02	0.97	-0.28	-0.12
`ciq_down`	3.05	6.65	5.36	5.85	6.04	7.90	6.50	7.68	7.84	9.75	6.70	2.55	7.60	2.94
Panel B: No penny stocks														
Equal-weighted														
`coskew`	9.51	9.09	9.25	8.87	8.78	8.87	8.25	7.71	7.38	5.80	-3.72	-2.52	-1.32	-0.70
`cokurt`	8.07	8.07	8.65	9.51	8.71	8.34	8.66	8.33	8.27	6.90	-1.16	-0.67	2.11	1.32
`beta_down`	7.97	8.55	8.26	9.01	9.35	8.82	8.82	9.47	7.78	5.46	-2.50	-0.80	0.73	0.42
`down_corr`	6.64	7.02	7.38	8.55	8.15	9.78	8.64	8.83	9.29	9.22	2.57	2.28	0.27	0.23
`htcr`	4.74	6.57	7.98	8.25	9.12	8.68	9.10	10.68	9.46	8.93	4.18	2.73	3.98	2.65
`beta_tr`	4.73	7.48	8.05	8.72	8.32	8.19	9.22	9.30	9.39	10.10	5.37	3.04	3.52	1.71
`coentropy`	7.10	7.14	7.68	8.46	8.49	9.38	8.90	8.64	8.75	8.95	1.85	1.64	-0.16	-0.14
`cos_pred`	10.47	8.88	8.48	8.68	8.16	8.15	8.18	7.32	8.42	6.76	-3.71	-1.36	-2.61	-1.14
`beta_neg`	9.01	8.62	9.44	9.32	9.10	9.67	9.69	8.37	7.78	2.52	-6.50	-1.74	-2.14	-1.31
`mcrash`	7.35	7.06	9.28	8.12	7.68	12.10	7.38	12.21	8.37	9.30	1.95	1.62	1.60	1.36
`ciq_down`	4.24	6.38	8.86	8.85	8.84	9.34	9.42	8.40	9.40	9.77	5.53	2.77	5.91	3.27
Value-weighted														
`coskew`	6.85	6.99	6.99	7.34	6.59	8.00	7.29	7.00	4.36	4.36	-2.49	-1.37	2.47	1.15
`cokurt`	6.35	5.56	6.88	6.71	6.46	5.62	7.49	7.03	6.12	5.25	-1.10	-0.56	2.87	1.64
`beta_down`	5.41	6.88	6.38	7.54	6.66	7.40	6.24	5.52	6.13	4.29	-1.12	-0.32	1.51	0.62
`down_corr`	5.94	5.20	5.29	5.21	6.01	7.48	7.93	6.92	8.42	6.88	0.94	0.66	-3.22	-2.16
`htcr`	4.14	5.49	6.15	6.72	6.29	7.05	6.28	6.36	7.12	4.98	0.84	0.39	2.11	1.14
`beta_tr`	4.02	5.23	5.32	7.17	6.69	6.50	7.13	7.77	8.37	8.95	4.94	2.39	5.21	1.99
`coentropy`	5.43	4.89	5.65	6.26	5.59	7.03	7.81	7.47	8.10	6.69	1.25	0.79	-3.11	-1.84
`cos_pred`	11.30	9.00	6.74	8.29	7.43	6.62	6.30	4.86	5.93	5.88	-5.43	-1.92	-2.24	-0.79
`beta_neg`	7.40	6.11	6.86	6.47	6.97	6.52	6.35	4.97	4.34	1.93	-5.46	-1.31	-2.17	-0.90
`mcrash`	4.94	7.09	6.40	7.63	6.04	9.35	3.51	9.83	6.12	6.48	1.54	0.91	-0.50	-0.25
`ciq_down`	3.20	5.75	5.85	5.79	6.88	6.57	7.16	7.07	7.33	8.70	5.50	2.37	5.09	2.44

Figure 7: *Correlation structure across ARM-managed portfolios.* The figure captures the averages of time-series correlations between managed portfolios sorted on various asymmetric risk measures. Data include all available stocks and the period between January 1968 and December 2018.



C Appendix: IPCA Estimation Results

This Appendix provides some estimation results of the ARM-IPCA models.

Table 20: *Out-of-sample ARM-IPCA results using all stocks and split samples.* The table reports out-of-sample results of the IPCA models with varying numbers of latent factors and using ARMs as the instruments. Models are estimated with an expanding window and a 60-month initial period. Tangency portfolios are based on the restricted IPCA model, pure-alpha portfolios on the unrestricted model. I include all available stocks. The first period covers the interval between January 1968 and December 1993, the second period January 1994 and December 2018.

		IPCA(K)							
		1	2	3	4	5	6	7	8
Panel A: Period 1/1968-12/1993									
Individual stocks									
Total R^2	$\Gamma_\alpha = 0$	19.09	20.43	20.87	21.30	21.51	21.61	21.72	21.81
	$\Gamma_\alpha \neq 0$	18.99	20.42	20.87	21.29	21.51	21.60	21.71	21.80
Predictive R^2	$\Gamma_\alpha = 0$	0.11	0.10	0.11	0.15	0.16	0.16	0.16	0.16
	$\Gamma_\alpha \neq 0$	0.16	0.17	0.16	0.17	0.17	0.17	0.17	0.16
Managed portfolios									
Total R^2	$\Gamma_\alpha = 0$	97.49	98.99	99.43	99.74	99.82	99.86	99.91	99.94
	$\Gamma_\alpha \neq 0$	96.87	98.76	99.39	99.68	99.79	99.84	99.90	99.93
Predictive R^2	$\Gamma_\alpha = 0$	0.59	0.57	0.60	0.68	0.69	0.70	0.70	0.69
	$\Gamma_\alpha \neq 0$	0.68	0.69	0.69	0.70	0.70	0.70	0.70	0.70
Tangency portfolios									
Mean		8.34	3.62	13.45	21.59	21.28	21.43	24.19	23.68
t-stat		1.85	0.83	2.92	4.15	3.90	4.37	5.31	5.07
Sharpe		0.42	0.18	0.67	1.08	1.06	1.07	1.21	1.18
Pure-alpha portfolios									
Mean		16.89	25.17	20.85	8.98	9.99	9.12	3.63	-1.54
t-stat		3.87	5.83	4.72	2.31	2.54	2.26	0.78	-0.38
Sharpe		0.84	1.26	1.04	0.45	0.50	0.46	0.18	-0.08
Panel B: Period 1/1994-12/2018									
Individual stocks									
Total R^2	$\Gamma_\alpha = 0$	14.71	16.06	16.81	17.49	17.73	17.86	17.98	18.08
	$\Gamma_\alpha \neq 0$	14.72	15.94	16.62	17.48	17.72	17.86	17.98	18.08
Predictive R^2	$\Gamma_\alpha = 0$	0.19	0.19	0.19	0.25	0.24	0.25	0.25	0.25
	$\Gamma_\alpha \neq 0$	0.25	0.25	0.24	0.24	0.24	0.24	0.24	0.24
Managed portfolios									
Total R^2	$\Gamma_\alpha = 0$	94.47	97.52	98.79	99.54	99.69	99.79	99.86	99.90
	$\Gamma_\alpha \neq 0$	94.24	96.29	97.27	99.51	99.66	99.76	99.85	99.89
Predictive R^2	$\Gamma_\alpha = 0$	2.25	2.21	2.22	2.24	2.24	2.25	2.24	2.24
	$\Gamma_\alpha \neq 0$	2.26	2.25	2.24	2.24	2.23	2.24	2.24	2.24
Tangency portfolios									
Mean		9.72	9.65	13.06	21.65	22.42	23.47	21.82	23.28
t-stat		2.00	2.17	2.27	3.64	3.70	3.69	3.45	3.79
Sharpe		0.49	0.48	0.65	1.08	1.12	1.17	1.09	1.16
Pure-alpha portfolios									
Mean		13.38	15.61	17.70	11.71	10.72	9.30	2.56	0.40
t-stat		2.67	3.01	3.74	2.49	2.23	1.93	0.59	0.09
Sharpe		0.67	0.78	0.88	0.59	0.54	0.47	0.13	0.02

Table 21: *Out-of-sample ARM-IPCA results using non-penny stocks and split samples.* The table reports out-of-sample results of the IPCA models with varying numbers of latent factors and using ARMs as the instruments. Models are estimated with an expanding window and a 60-month initial period. Tangency portfolios are based on the restricted IPCA model, pure-alpha portfolios on the unrestricted model. I excludes stocks with a price less than \$5 or market cap below 10% quantile of NYSE stocks. The first period covers the interval between January 1968 and December 1993, the second period January 1994 and December 2018.

		IPCA(K)							
		1	2	3	4	5	6	7	8
Panel A: Period 1/1968-12/1993									
Individual stocks									
Total R^2	$\Gamma_\alpha = 0$	27.00	27.92	28.68	29.32	29.64	29.82	29.95	30.08
	$\Gamma_\alpha \neq 0$	26.61	27.76	28.55	29.29	29.60	29.78	29.92	30.05
Predictive R^2	$\Gamma_\alpha = 0$	0.14	0.20	0.18	0.27	0.27	0.26	0.27	0.26
	$\Gamma_\alpha \neq 0$	0.26	0.28	0.26	0.27	0.27	0.27	0.27	0.27
Managed portfolios									
Total R^2	$\Gamma_\alpha = 0$	97.98	98.79	99.42	99.72	99.82	99.86	99.90	99.93
	$\Gamma_\alpha \neq 0$	96.24	98.17	98.85	99.67	99.76	99.79	99.85	99.89
Predictive R^2	$\Gamma_\alpha = 0$	0.37	0.43	0.46	0.57	0.59	0.58	0.59	0.58
	$\Gamma_\alpha \neq 0$	0.57	0.61	0.59	0.59	0.59	0.59	0.59	0.59
Tangency portfolios									
Mean		6.10	10.74	9.38	24.98	25.07	24.69	25.37	23.98
t-stat		1.32	2.35	1.90	4.93	5.32	5.43	5.67	5.06
Sharpe		0.31	0.54	0.47	1.25	1.25	1.23	1.27	1.20
Pure-alpha portfolios									
Mean		18.98	23.04	18.26	5.41	4.51	4.32	5.13	4.71
t-stat		4.08	4.69	3.97	1.43	1.22	1.20	1.36	1.06
Sharpe		0.95	1.15	0.91	0.27	0.23	0.22	0.26	0.24
Panel B: Period 1/1994-12/2018									
Individual stocks									
Total R^2	$\Gamma_\alpha = 0$	19.68	22.10	23.13	23.58	23.96	24.22	24.40	24.56
	$\Gamma_\alpha \neq 0$	19.60	21.88	23.09	23.58	23.95	24.20	24.39	24.55
Predictive R^2	$\Gamma_\alpha = 0$	0.10	0.16	0.20	0.19	0.17	0.18	0.18	0.17
	$\Gamma_\alpha \neq 0$	0.18	0.19	0.19	0.18	0.17	0.17	0.17	0.18
Managed portfolios									
Total R^2	$\Gamma_\alpha = 0$	94.05	98.03	99.09	99.42	99.63	99.79	99.87	99.91
	$\Gamma_\alpha \neq 0$	93.53	96.82	99.05	99.34	99.61	99.76	99.84	99.89
Predictive R^2	$\Gamma_\alpha = 0$	1.43	1.45	1.42	1.41	1.40	1.41	1.41	1.41
	$\Gamma_\alpha \neq 0$	1.41	1.42	1.42	1.41	1.40	1.40	1.40	1.41
Tangency portfolios									
Mean		7.81	11.65	18.38	16.39	17.64	18.36	16.73	15.01
t-stat		1.74	2.65	4.03	3.48	3.53	3.59	3.08	2.59
Sharpe		0.39	0.58	0.92	0.82	0.88	0.92	0.84	0.75
Pure-alpha portfolios									
Mean		10.53	13.67	8.14	10.80	9.19	10.25	7.00	2.97
t-stat		2.40	2.84	1.77	2.19	1.70	1.81	1.27	0.59
Sharpe		0.53	0.68	0.41	0.54	0.46	0.51	0.35	0.15

Figure 8: *Factor loadings of the ARM-IPCA(6) using all stocks.* The figure reports columns of the estimated Γ_β IPCA matrix with six latent factors and ARMs as instruments. Results are based on the in-sample analysis using all stocks.

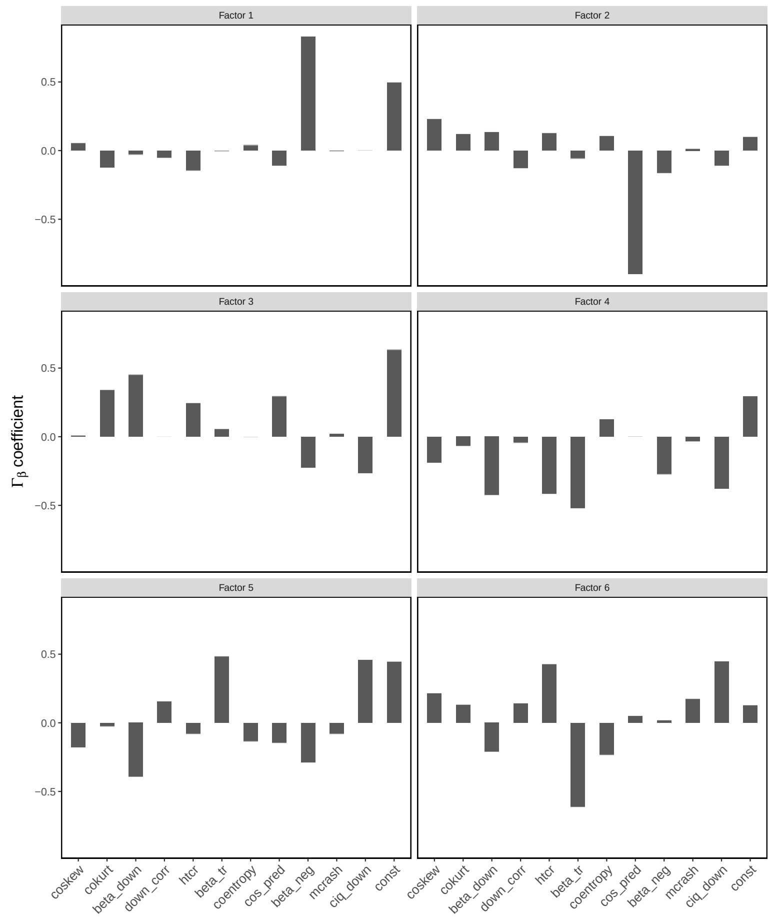


Figure 9: *Factor loadings of the ARM-IPCA(6) excluding penny stocks.* The figure reports columns of the estimated Γ_β IPCA matrix with six latent factors and ARMs as instruments. Results are based on the in-sample analysis using dataset that excludes stocks with a price less than \$5 or market cap below 10% quantile of NYSE stocks.

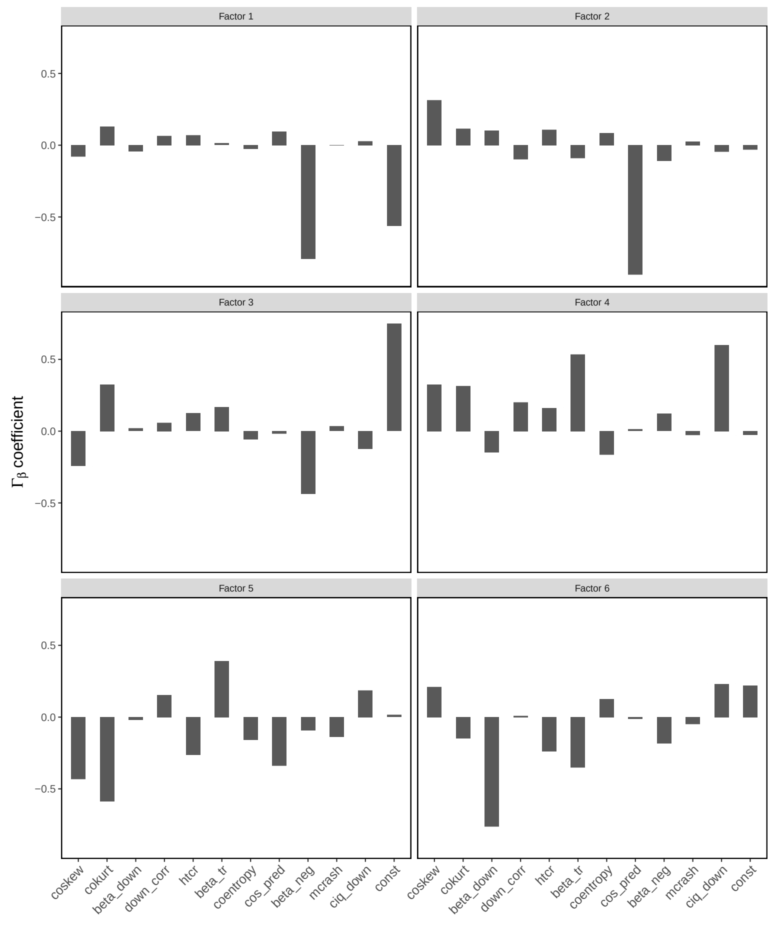


Figure 10: *Alphas of the ARM-IPCA models using all stocks.* The figure reports estimated Γ_α IPCA vectors with number of latent factors between one and six and ARMs as instruments. Results are based on the in-sample analysis using all stocks.

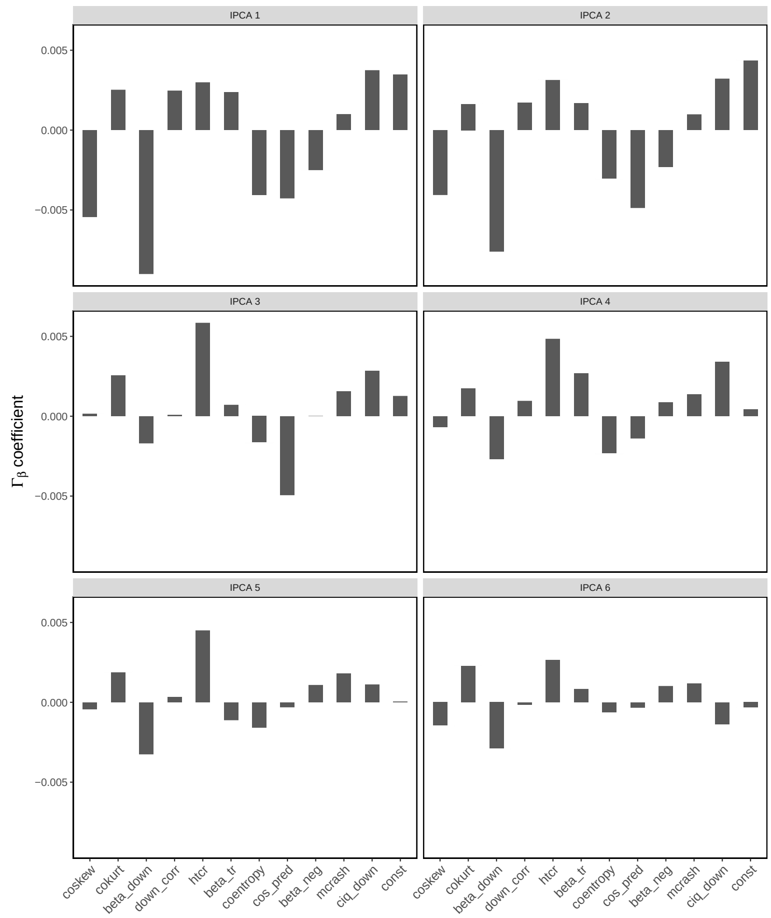


Figure 11: *Alphas of the ARM-IPCA models excluding penny stocks.* The figure reports estimated Γ_α IPCA vectors with number of latent factors between one and six and ARMs as instruments. Results are based on the in-sample analysis using dataset that excludes stocks with a price less than $5 or market cap below 10% quantile of NYSE stocks.

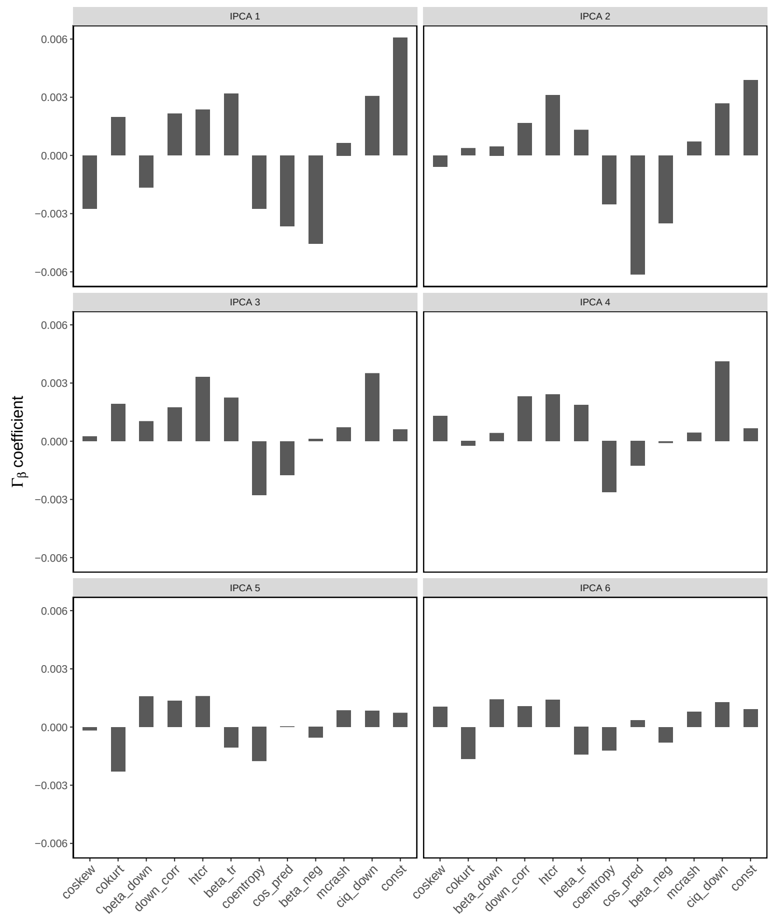


Table 22: *IPCA Results – All variables.*

		IPCA(K)							
		1	2	3	4	5	6	7	8
		Panel A: All stocks							
Individual stocks									
Total R^2	$\Gamma_\alpha = 0$	16.54	18.28	19.46	20.13	20.66	21.01	21.28	21.48
	$\Gamma_\alpha \neq 0$	16.94	18.65	19.77	20.42	20.79	21.05	21.32	21.51
Predictive R^2	$\Gamma_\alpha = 0$	0.35	0.35	0.41	0.42	0.65	0.67	0.66	0.67
	$\Gamma_\alpha \neq 0$	0.73	0.72	0.71	0.71	0.70	0.70	0.69	0.69
Managed portfolios									
Total R^2	$\Gamma_\alpha = 0$	89.35	94.89	96.74	97.95	98.29	98.77	99.07	99.22
	$\Gamma_\alpha \neq 0$	89.90	95.29	96.89	98.08	98.57	98.79	99.10	99.24
Predictive R^2	$\Gamma_\alpha = 0$	1.61	1.63	1.77	1.82	2.02	2.03	2.02	2.04
	$\Gamma_\alpha \neq 0$	2.21	2.15	2.13	2.12	2.10	2.08	2.07	2.07
Asset pricing test									
W_α p-value		0.10	0.00	0.00	0.00	3.90	71.80	27.70	61.90
		Panel B: No penny stocks							
Individual stocks									
Total R^2	$\Gamma_\alpha = 0$	22.67	25.08	26.33	27.32	27.91	28.42	28.83	29.20
	$\Gamma_\alpha \neq 0$	23.02	25.34	26.57	27.54	28.07	28.56	28.94	29.25
Predictive R^2	$\Gamma_\alpha = 0$	0.35	0.42	0.46	0.47	0.58	0.58	0.61	0.61
	$\Gamma_\alpha \neq 0$	0.68	0.67	0.66	0.66	0.66	0.66	0.65	0.65
Managed portfolios									
Total R^2	$\Gamma_\alpha = 0$	90.21	94.43	96.02	97.44	97.91	98.57	98.81	99.08
	$\Gamma_\alpha \neq 0$	90.63	94.52	96.13	97.52	98.30	98.64	98.96	99.10
Predictive R^2	$\Gamma_\alpha = 0$	1.28	1.45	1.53	1.55	1.57	1.60	1.59	1.57
	$\Gamma_\alpha \neq 0$	1.70	1.67	1.65	1.65	1.65	1.64	1.63	1.62
Asset pricing test									
W_α p-value		0.00	0.00	0.10	0.00	0.30	1.60	0.80	36.50

Table 23: *Correlations between original IPCA and ARM-IPCA factors.* The table reports correlations between IPCA latent factors estimated using set of original 32 variables and IPCA latent factors estimated using 11 ARMs.

	Panel A: All stocks						*Panel B: Ne penny stocks*					
	ARM-IPC1	ARM-IPC2	ARM-IPC3	ARM-IPC4	ARM-IPC5	ARM-IPC6	ARM-IPC1	ARM-IPC2	ARM-IPC3	ARM-IPC4	ARM-IPC5	ARM-IPC6
IPC1	0.43	-0.38	-0.38	0.05	-0.12	-0.11	-0.38	0.04	-0.09	0.01	-0.06	0.06
IPC2	0.30	0.15	-0.13	-0.13	0.04	0.05	-0.11	-0.57	-0.08	0.25	-0.01	0.11
IPC3	-0.08	0.32	-0.03	0.02	-0.12	-0.10	0.16	0.21	-0.27	0.12	0.08	0.11
IPC4	-0.28	-0.32	0.16	0.03	0.00	-0.11	-0.05	-0.02	0.39	-0.01	0.04	-0.06
IPC5	-0.23	0.22	0.26	-0.05	0.41	0.08	-0.03	0.10	0.04	0.04	-0.21	0.04
IPC6	-0.03	0.07	0.18	0.01	0.21	0.10	0.01	0.06	0.03	0.03	0.10	-0.06

D Appendix: Arbitrage Portfolios without Penny Stocks

Figure 12: *Variable importances for short and long legs of the arbitrage portfolio, no penny stocks.* The figure depicts time-evolution of variable importances for the short and long leg of the arbitrage portfolio based on PPCA of Kim et al. (2020) with five latent factors. I exclude stocks with a price less than \$5 or market cap below 10% quantile of NYSE stocks. Period covers interval between January 1968 and December 2018.

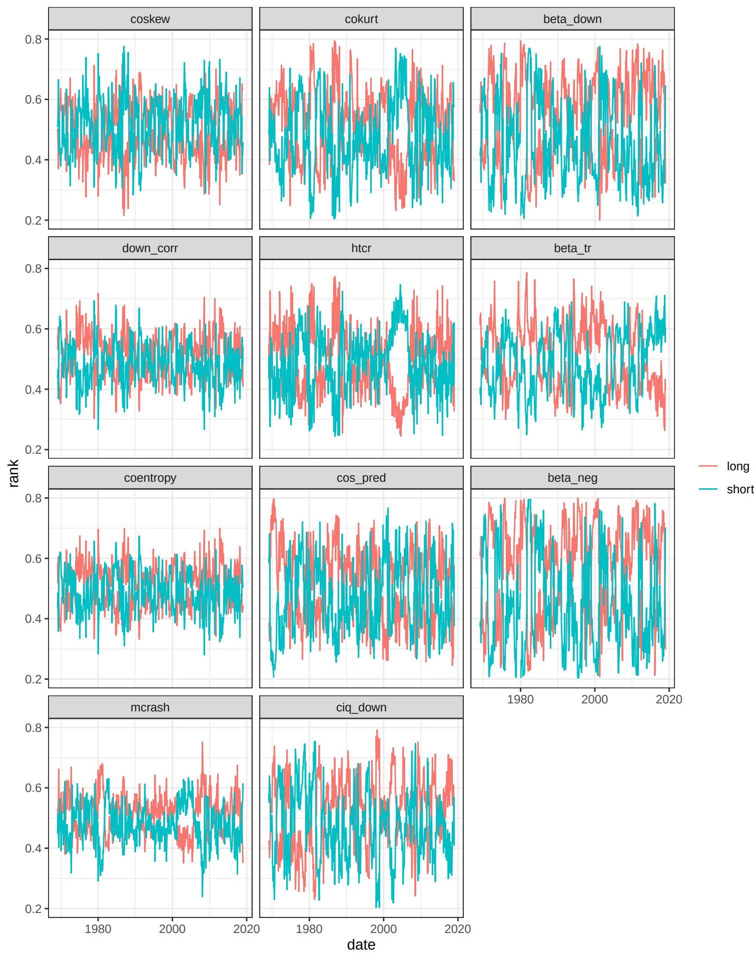


Figure 13: *Relative importances of the ARMs for the systematic and anomaly risk, no penny stocks.* The figure depicts time-evolution of relative importances of the ARMs for systematic and anomaly risk of the PPCA model of Kim et al. (2020) with five latent factors. I exclude stocks with a price less than $5 or market cap below 10% quantile of NYSE stocks. Period covers interval between January 1968 and December 2018.