Beyond Volatility: Common Factors in Idiosyncratic Quantile Risks*

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Abstract

This study extracts latent factors from the cross-sectional quantiles of firm-level idiosyncratic returns and demonstrates that they carry information that is missed by conventional volatility measures. Notably, exposure to the lower-tail common idiosyncratic quantile factor entails a distinctive risk premium that cannot be explained by existing volatility, downside or tail-related risk factors or characteristics. Furthermore, we demonstrate that factor structures derived from quantiles across the return distribution–which capture its asymmetric features–also possess predictive capabilities regarding aggregate market returns.

Keywords: Cross-section of asset returns, factor structure of asset returns, idiosyncratic risk, quantiles, asymmetric risk

JEL: C21; C58; G12

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1 Introduction

We investigate whether the distributional characteristics of the cross-section of idiosyncratic stock returns carry information beyond the traditional mean-variance characterization of risk that aggregates asymmetries and tails, as well as other nonlinear information. We also examine whether such risk is priced in the cross section of stock returns. This question is motivated by the importance of idiosyncratic risk for underdiversified investors (Van Nieuwerburgh and Veldkamp, 2010; Roche et al., 2013) and is linked to nontradable human capital risk (Eiling, 2013). Previous studies have summarized idiosyncratic risk primarily by reference to volatility (Herskovic et al., 2016) or focused on tail events under strong parametric assumptions or the existence of a model that generates returns (Kelly and Jiang, 2014; Chabi-Yo et al., 2022). In contrast to these approaches, we identify a finer risk characterization on the basis of conditional quantiles while remaining agnostic about the data-generating process related to the idiosyncratic returns.

Our findings reveal a new dimension of idiosyncratic risk that differs from popular volatility structures. We introduce three common idiosyncratic quantile (CIQ) factors that track movements in the lower, upper, and median quantiles of firm-specific returns. Importantly, only the lower-tail CIQ factor is robustly priced in the cross-section of stock returns. Stocks with high exposure to downside idiosyncratic shocks earn substantially higher average returns than those with lower exposure to this risk. A portfolio formed using this observation achieves an annualized return spread of approximately 8%. Notably, this lower-tail quantile risk premium is not accounted for by standard factor models or firm characteristics. In contrast, exposure to the upper-tail and central CIQ factors is not associated with higher returns, and their estimated prices of risk are small in economic terms and statistically nonsignificant. Investors therefore require compensation for exposure to extreme idiosyncratic losses but not for idiosyncratic upside potential. This asymmetric pricing is inconsistent with a purely volatility-driven narrative: if common idiosyncratic volatility were the only driving force, upside and downside risks would command similar premiums. Instead, our evidence identifies a unique lower-tail risk that market prices.

Our approach and results enhance existing measurements of risk. Unlike the common idiosyncratic volatility factor (CIV) or principal component on squared residuals (PCA-SQ) approach, which capture common variation but treat upside and downside movements symmetrically (Herskovic et al., 2016), CIQ measures asymmetric tail risks directly. Consequently, this more precise measurement leads to superior pricing performance: even when we control for the CIV or the PCA-SQ factor, the effect of the lower-tail CIQ factor remains statistically significant and large in terms of magnitude.

Furthermore, in contrast to the tail risk (TR) factor of Kelly and Jiang (2014), which assumes a specific functional form of the left-tail idiosyncratic returns, CIQ factors are estimated independently of any parametric assumption regarding idiosyncratic returns. Additionally, our approach is not limited to investigating left-tail events, as we can contrast common trends across the cross-sectional distribution of idiosyncratic returns. The effectiveness of our risk measurement approach is demonstrated by the cross-sectional pricing results, even when controlling for exposure to the TR factor. In essence, CIQ factors identify a type of systematic risk that is hidden from other volatility- or tail-based factors.

In addition to its cross-sectional pricing power, the CIQ framework provides valuable insights into the predictability of the aggregate market. Fluctuations in the downside CIQ factor contain significant time series information: when idiosyncratic lower-tail risk is high, future market-wide returns are also high. A one-standard-deviation increase in our lower-tail CIQ measure forecasts an increase in the annual excess market return of approximately 5–7%. This predictive relationship is highly significant in both the in-sample and out-of-sample periods and persists after we control for a wide range of well-known return predictors, including tail risk measures (Kelly and Jiang, 2014) or the CIV volatility factor (Herskovic et al., 2016). In contrast, the predictive power of the upper-tail CIQ factor is weaker (at most 3% in-sample) and disappears out-of-sample, thus reinforcing the claim that it is the downside idiosyncratic risk that is relevant to both cross-sectional pricing and aggregate risk premiums.

Our approach also offers a fresh perspective on factor models in empirical finance. Just as quantile regression extends classical linear regression, our quantile factor model of asset returns based on Chen et al. (2021) extends the approximate factor models used in the empirical asset pricing literature. Following the approach of the popular principal component analysis, which recovers the conditional mean, we work with more general quantile factor models. These models are sufficiently flexible to capture quantile-dependent phenomena that cannot be captured by standard tools. Unlike standard principal component analysis, quantile factor models can capture hidden factors that alter distributional properties, such as moments or quantiles. Crucially, these factors differ from the typical mean and volatility factors when we abandon the traditional location and scale shift model structure, allowing for more general and potentially unknown data-generating processes. In effect, quantile-dependent risk is treated as a constant in factor models on the basis of these assumptions.

One plausible explanation for why CIQ risk is linked to a premium is that it is driven by heterogeneous household-level risks and macroeconomic uncertainty. Idiosyncratic cash-flow shocks with heavy downside tails can lead to household consumption risk, particularly when some households are unable to insure themselves against these shocks fully. Indeed, recent evidence shows that households respond very differently to asymmetric income shocks, such that poorer and younger households are more vulnerable. If households' consumption risk inherits firms' idiosyncratic tail risk, a common lower-tail factor would enter the pricing kernel as a state variable that commands a premium. This mechanism is similar to the insight provided by Herskovic et al. (2016) with respect to volatility risk but extends that insight to encompass nonlinear, quantile-specific risks. More broadly, our results are consistent with asset pricing theories that emphasize rare disasters (Rietz, 1988; Barro, 2006) or disappointment aversion (Routledge and Zin, 2010; Farago and Tédongap, 2018), thus implying that investors are particularly averse to downside risk. By identifying a priced idiosyncratic tail factor, we provide direct empirical evidence indicating that, although diversifiable in principle, extreme idiosyncratic shocks collectively generate systematic risk that markets cannot ignore.

1.1 Relation to the Literature

Our work is related to several streams of literature. The first relates to the factor-based asset pricing models pioneered by the results of Ross (1976). This stream of literature has yielded highly successful results with respect to the parsimonious specifications (Fama and French, 1993), as well as exploration of statistically motivated latent factors (Chamberlain and Rothschild, 1983; Connor and Korajczyk, 1986). More recently, Lettau and Pelger (2020) proposed risk–premium principal component analysis, which allows for systematic time series factors that incorporate information from the first and second moments.[1]. In sharp contrast to this literature, our approach remains agnostic about the nature of the true data generating process and uses the conditional quantiles of observed returns without imposing moment conditions.

The second stream of literature to which we contribute is the study of idiosyncratic risk that comoves across assets, particularly by exploring common trends that are not captured by first-moment factors. The bulk of this research is motivated by the results of Ang et al. (2006a).[2] More recently, Herskovic et al. (2016) introduced a risk factor on the basis of the common volatility of firm-level idiosyncratic returns and demonstrated its pricing capabilities for the cross section of different asset classes.

The third stream of literature addresses the asymmetric and generally nonlinear properties of systematic risk and how they are incorporated into asset prices. Interest in this type of model was reignited by Ang et al. (2006) and their introduction of the downside beta, which

[1] Other related important results include, e.g., Kelly et al. (2019) and Kozak et al. (2020)

[2] Detzel et al. (2023) provided an updated view of their results with respect to extended periods and additional factor models.

Bollerslev et al. (2021) further decomposed into semibetas. More recently, Bollerslev et al. (2025) argued that betas are granular and associated with a risk premium that depends on the relevant part of the return distributions. A number of studies have focused specifically on the tails or extremes in the cross-section of returns (Van Oordt and Zhou, 2016; Chabi-Yo et al., 2018, 2022).

Attempts have also been made to combine these research agendas. For example, Kelly and Jiang (2014); Allen et al. (2012); Jondeau et al. (2019); Baruník and Čech (2021) explored the factor risks associated with skewness, tails and extremes. Giglio et al. (2016) estimated quantile-specific latent factors using systemic risk and financial market distress variables to predict macroeconomic activity. Recently, Massacci et al. (2025) introduced a conditional factor model that depends on the state of the economy and thus differs between good and bad times. Moreover, Amengual and Sentana (2020) reported a nonlinear dependence structure in short-term reversals and momentum, and Ando and Bai (2020) documented common factor structures that explain the upper and lower tails of the asset return distributions. In addition, Ma et al. (2021) introduced a semiparametric quantile factor panel model that exploits stock-specific characteristics. In contrast to these authors, our approach is more general since it allows not only loadings but also factors to be quantile dependent, does not require the loadings to depend on observables, and exhibits a direct relation to the approximate factor models that are ubiquitous in the finance literature.

From a theoretical perspective, many justifications for the asymmetric forms of the utility function can be provided. The information captured by quantile-dependent factors can be related to the behavior of investors with quantile preferences (de Castro and Galvao, 2019) and utility-free representation of risk. Another stream of literature explicitly focuses on modeling aversion to downside or tail risks. Barro (2006), building on the work of Rietz (1988), introduced the rare disaster model and showed that tail events can have a significant ability to explain various asset pricing puzzles. Routledge and Zin (2010) defined the generalized disappointment aversion model, which is a building block for the intertemporal equilibrium asset pricing model of Farago and Tédongap (2018) and highlights the importance of disappointment-related factors both theoretically and empirically.

Much of the research investigating common tail risk related to asset prices relies on options data. For example, Bollerslev and Todorov (2011) combined high-frequency and options data and used a nonparametric approach to conclude that a large part of the equity and variance risk premium is related to the jump tail risk. Recently, Lu and Murray (2019) introduced a beta that captures exposure to bear market risk. In contrast, our approach does not rely on option data, which are limited with respect to their time span and width of the available cross section.

The remainder of the paper is structured as follows. Section 2 proposes the quantile factor model for idiosyncratic returns, discusses its estimation and relates it to other factors. Section 3 examines the cross-sectional asset pricing implications of the exposure to these factors using portfolio sorts and cross-sectional firm-level regressions. Section 4 examines extensions of our proposed measures and tests the stability of the discovered premium. Section 5 investigates the relationship between this factor the equity premium. Section 6 concludes the study.

2 Common Idiosyncratic Quantile (CIQ) Factors

We start by introducing the general framework used for the quantile factor model. Next, we discuss its empirical implementation and extract common idiosyncratic factors from the U.S. data.

2.1 Quantile Factor Model

We assume a panel of returns of length T and width N after eliminating the common mean factors from the time series regression

$$r_{i,t} = \alpha_i + \beta_i^\top f_t + \epsilon_{i,t}, \tag{1}$$

to have a τ-dependent structure-$g_t(\tau)$-in idiosyncratic errors that we coin the common idiosyncratic quantile–CIQ(τ)-factors, which satisfies

$$Q_{\epsilon_{i,t}}\left[\tau \middle| g_t(\tau)\right] = \gamma_i^\top(\tau) g_t(\tau), \tag{2}$$

which implies

$$\epsilon_{i,t} = \gamma_i^\top(\tau) g_t(\tau) + u_{i,t}(\tau), \tag{3}$$

where $g_t(\tau)$ is an $r(\tau) \times 1$ vector of random common factors, $\gamma_i(\tau)$ is an $r(\tau) \times 1$ vector of nonrandom factor loadings with $r(\tau) \ll N$, $Q_{\epsilon_{i,t}}\left[\tau \middle| g_t(\tau)\right]$ is a conditional quantile function of $\epsilon_{i,t}$ at τ, and the quantile-dependent idiosyncratic error $u_{i,t}(\tau)$ almost certainly satisfies the quantile restriction $P[u_{i,t}(\tau) < 0 | g_t(\tau)] = \tau$ for all $\tau \in (0, 1)$.

To estimate the common factors that capture the comovement of quantile-specific features of distributions of the idiosyncratic parts of the stock returns, we use quantile factor analysis (QFA) introduced by Chen et al. (2021). In contrast to principal component analysis (PCA), QFA facilitates the capture of hidden factors that may shift more general characteristics, such as moments or quantiles of the distribution of returns other than the mean. The methodology is also suitable for large panels and requires less strict assumptions about the data generation process.

The quantile-dependent factors and their loadings can be estimated as follows:

$$\underset{(\gamma_1,\ldots,\gamma_N,g_1,\ldots,g_T)}{\operatorname{argmin}} \frac{1}{NT}\sum_{i=1}^{N}\sum_{t=1}^{T}\rho_\tau\left(\epsilon_{it}-\gamma_i^\top(\tau)g_t(\tau)\right) \tag{4}$$

where $\rho_\tau(u) = (\tau - \mathbf{1}\{u \leq 0\})u$ is the check function while imposing the following normalizations: $\frac{1}{T}\sum_{t=1}^{T} g_t(\tau)g_t(\tau)^\top = \mathbb{I}_r$, and $\frac{1}{N}\sum_{i=1}^{N}\gamma_i(\tau)\gamma_i(\tau)^\top$ is diagonal with nonincreasing diagonal elements.

As discussed in Chen et al. (2021), this estimator is related to the PCA estimator studied in Bai and Ng (2002) and Bai (2003), similar to quantile regression, which is related to classical least squares regression. Unlike the PCA estimator of Bai (2003), this estimator does not yield an analytical closed-form solution. To solve for the stationary points of the objective function, Chen et al. (2021) proposed a computational algorithm called iterative quantile regression. These authors also showed that the estimator possesses the same convergence rate as the PCA estimators for the approximate factor model. We follow their approach when estimating the quantile factors.[3]

Unlike previous studies that have attempted to recover possibly nonlinear commonalities and dependence structures, our model remains agnostic about the nature of the true data generation process and uses the conditional quantiles of the observed data to capture nonlinearities in factor models.[4] Additionally, unlike previous studies, we do not require the idiosyncratic errors to satisfy specific moment conditions. Thus, our approach is more flexible, as it estimates factors that shift relevant parts of the return distributions without restrictive assumptions by relying on density properties. The approach also differs from the existing factor literature in that it does not require the loadings to depend on observables and considers the factors as quantile-dependent objects.

Quantiles of stock returns can be related to a variety of quantities as well as distributional characteristics in specific cases. A specifically important quantity in finance that can relate to quantiles of the return distribution for a typically assumed location-scale model is volatility. As discussed by the rich body of literature initiated by Ang et al. (2006b), a genuine factor

[3]We employ the authors' MATLAB codes, which are provided on the Econometrica webpage.

[4]For example, Gorodnichenko and Ng (2017) estimated joint level and volatility factors simultaneously or some studies employ copulas to model asymmetric or tail dependence (Amengual and Sentana, 2020; Oh and Patton, 2017).

structure exists in the idiosyncratic volatility of a panel of asset returns. When PCA (or cross-sectional averages) is applied to squared residuals, once the mean factors have been removed from the returns (PCA-SQ), that structure will be recovered. We use this approach to study the relationship between quantile-specific factors and the data.

It is important to note that the full factor structure is recovered by the PCA-SQ approach only if the data-generating process is known and is well characterized by the first two moments of the distribution. However, regarding more general or even unknown data-generating processes that cannot be well characterized by the first two moments, this approach fails to characterize the risks precisely. Quantile factor models provide a more useful estimate. To illustrate this point in further detail, in Appendix A, we present a simple theoretical example that shows the restrictiveness of a typical location-scale model.

2.2 Data and Empirical Setting

To estimate the CIQ factors, we use monthly stock returns from the Center for Research in Securities Prices (CRSP) database sampled between January 1960 and December 2018. We include all stocks with codes 10 and 11 in the estimation. We adjust the returns for delisting as described in Bali et al. (2016). We follow the standard practice in the literature and exclude all "penny stocks" with prices below one dollar to avoid biases related to these stocks.[5] We also conduct the analysis by reference to all stocks, and the results do not qualitatively change.

In the process of factor estimation, we take several steps. First, we use a moving window of 60 months and select the stocks that have all the observations in this window. For each of these stocks, we run a time series regression to eliminate the influence of the common (linear) factors

$$r_{i,t} = \alpha_i + \beta_i^{MKT} MKT_t + \beta_i^{SMB} SMB_t + \beta_i^{HML} HML_t + e_{i,t}, \quad t = 1, \ldots, T \tag{5}$$

and save the residuals $e_{i,t}$. For the common factors, which we eliminate from the stock returns, we resort to the three factors of Fama and French (1993) (hence FF3). We use this model as the baseline model for various reasons; unless otherwise stated, the CIQ factors are estimated with respect to this linear specification. First, it connects with previously discussed results in the literature. Second, if we consider the variance ratios of realized idiosyncratic quantiles from monthly cross-sections of stock returns estimated with respect to various factor models, we observe that after saturating a model with three factors, additional factors do not substantially alter their time variation. More specifically, after we eliminate

[5] See, e.g., Amihud (2002).

the effects of these three factors, the variance ratios between the 20% (80%) idiosyncratic quantiles with respect to the three- and five-factor models are 0.90 (0.97) and 0.95 (0.81) between the five- and six-factor models, respectively.[6,7]

Then, we use the residuals[8] from the first step and, for a given τ, estimate the common idiosyncratic quantile factors, $CIQ_t(\tau)$

$$\forall\tau : e_{i,t} = \gamma_i(\tau)CIQ_t(\tau) + u_{i,t}(\tau) \tag{6}$$

where the quantile-dependent idiosyncratic error $u_{i,t}(\tau)$ satisfies the quantile restriction following the methodology discussed in the previous subsection. We use only the first–i.e., the most informative–estimated factor for our purposes. In the overwhelming majority of the cases, the algorithms proposed in Chen et al. (2021) select exactly one factor to be the correct number of factors that explain the panels of idiosyncratic returns.[9]

In line with the mainstream volatility factor literature, we focus on changes in CIQ(τ) and work with ΔCIQ(τ) factors.[10,11] Intuitively, we examine how investors price the innovations of these risks rather than the levels.[12]

We estimate the quantile-specific factors using the idiosyncratic returns instead of the raw returns for two main reasons. First, this approach relates our results to previous findings in the literature that investigated common movements in idiosyncratic volatility and tail risk.[13] Second, from a practical perspective, quantiles of a random variable are functions of both

[6]Later in the text, we also show the robustness of our pricing results to the linear specifications with five and six factors of Fama and French (2015) and Fama and French (2018), respectively.

[7]We provide the full results regarding the variance ratios of the realized quantiles in Panel A of Table 11 in Appendix C. Moreover, in Figure 3, we provide visual clues concerning how much the monthly realized idiosyncratic quantiles vary in comparison to the quantiles obtained from raw excess returns. A visual assessment reveals that substantial time dynamics persist in the quantiles even after these three factors are eliminated.

[8]Similarly, as in the case of the principal component analysis, we standardize the residuals by their standard deviation.

[9]This approach is due to the sensible choice of the linear factor model using the three-factor model of Fama and French (1993), as discussed later in the text.

[10]Unless otherwise stated, in the rest of the paper, we perform all analyses using ΔCIQ(τ) factors. We replicate the main analysis using levels of the CIQ factors and the results are qualitatively the same.

[11]Differences are calculated using estimates from a single window, i.e., the CIQ factor values are estimated in a given window, and differences are calculated using estimates from that window. Differences are therefore not calculated across different windows.

[12]The notion that the volatility-like risk measures are related to asset prices is based on the ICAPM model of Merton (1973). The idea in this context is that changes in the investment opportunity set should be related to asset prices. This approach is fairly standard and is used, for example, in Ang et al. (2006b) or Herskovic et al. (2016).

[13]For example, Ang et al. (2006b) estimated idiosyncratic volatility with respect to the three factors identified by Fama and French (1993), which are the identical to those highlighted in Kelly and Jiang (2014) in the case of their tail-risk factors.

the location and scale of its distribution. By eliminating the location effect by previously well-documented linear factors, we can focus solely on the underresearched scale effect of the cross-sectional distribution of stock returns. While the quantile factor models with sufficient number of latent factors can capture both of these effects, we want to avoid the danger of data mining by estimating more than one quantile-specific factor for each value of τ. Third, linear factor pricing models are well-researched models with plenty of evidence against which we aim to benchmark to highlight the usefulness of our methodology.

In our empirical investigation, we focus on three CIQ factors. We choose a factor for the 20th percentile, or $\tau = 0.2$, to be our *lower-tail* CIQ factor and to represent the common movements in the left part of the idiosyncratic returns' distribution. We resort to this value for multiple reasons. First, we are motivated by recent results in the finance literature that associate investors' disappointment with approximately the worst 20% of events.[14] Second, from the correlations between CIQ factors estimated for a range of τ values, we conclude that the 20% ΔCIQ factor highly correlates with the ΔCIQ factors that lie below the median factor (e.g., the correlations with the 0.1 and 0.3 ΔCIQ factors are 0.93 and 0.95, respectively).[15]. Finally, this choice represents a compromise between focusing on significantly unfavorable common events and relying on many observations for precise estimation.

Our *upper-tail* CIQ factor is a mirror image of the downside factor, as we choose the 80th percentile, or the $\tau = 0.8$ CIQ factor, to represent the common upside movements of the idiosyncratic returns. We also employ the *central* CIQ factor with $\tau = 0.5$ to capture the central tendency of the stock returns' residuals.

In Panel A of Table 1, we provide a short descriptive summary of the estimated ΔCIQ factors. We observe that the means of the factors are close to zero across all factors. The standard deviations of the factors are high compared with their means, indicating that the factors vary considerably over the period. The skewness values of the factors indicate that the distributions are nearly symmetric. On the other hand, the kurtosis of the factors is, on average, well above three, suggesting that their distributions have heavy tails. Finally, a negative autocorrelation is evident at the first lag across the factors. The remaining autocorrelations disappear after the first lag.

In addition, Panel B presents the correlations between the factors. The correlation between the lower- and upper-tail factors with a value of 0.1 shows that there is relatively

[14] Specifically, Giglio et al. (2016) employed the 20th percentile when investigating macroeconomic downside risk; Delikouras and Kostakis (2019) provided estimates of disappointing events as lying approximately one standard deviation below the conditional mean of consumption growth; Massacci et al. (2025) reported the proportion of realized disappointment states to be approximately 29% in their factor model with bear market risk; and Farago and Tédongap (2018) reported a 16.0% unconditional probability of disappointment in their asset pricing model with generalized disappointment aversion.

[15] We present full summary statistics regarding the ΔCIQ factors in Table 12.

Table 1: Summary of the ΔCIQ Factors

The table provides summary statistics of the estimated lower-tail ($\tau = 0.2$), central ($\tau = 0.5$) and upper-tail ($\tau = 0.8$) ΔCIQ factors. Factors are estimated using idiosyncratic returns with respect to the three-factor model of Fama and French (1993) and using the quantile factor analysis of Chen et al. (2021). Factors are estimated using 60-month moving window. In Panel A, we report descriptive statistics of the ΔCIQ(τ) factors, including their means, standard deviations, skewness, kurtosis and autocorrelation coefficients of order between one and three. In Panel B, we report correlations between the ΔCIQ factors themselves, as well as correlations with other related factors: differences of the PCA-SQ factor (ΔPCA-SQ), differences of the common idiosyncratic variance factor (ΔCIV) of Herskovic et al. (2016), differences of the tail risk factor (ΔTR) of Kelly and Jiang (2014), and differences of the cross-sectional bivariate idiosyncratic volatility (ΔCBIV) factor of Han and Li (2025). The data cover the period from January 1965 to December 2018. * indicates $p < 0.1$, ** indicates $p < 0.05$ and *** indicates $p < 0.01$.

	Lower-Tail	Central	Upper-Tail
Panel A: Descriptive Statistics			
Mean $\times 10^3$	-3.84	12.09	-6.70
St. Dev.	0.18	1.43	0.22
Skewness	-0.14	0.01	-0.12
Kurtosis	4.79	5.51	7.17
AR(1)	-0.44	-0.36	-0.43
AR(2)	-0.05	0.05	-0.05
AR(3)	0.06	0.01	0.06
Panel B: Correlations			
Lower-Tail	1.00	0.19***	0.10**
Central		1.00	0.43***
Upper-tail			1.00
ΔPCA-SQ	-0.42***	0.24***	0.53***
ΔCIV	-0.16***	0.09**	0.14***
ΔTR	0.07*	-0.05	-0.26***
ΔCBIV	-0.03	0.03	-0.01

small overlap between the two factors. This observation therefore suggests that the factors are unlikely to rescale the information contained in the common volatility factor. This result suggests the existence of the potential for heterogeneous pricing information across common quantiles.

To show how the newly proposed factors relate to previously discovered factors related to volatility and asymmetric risk, we also report the correlations with them. First, to investigate how the quantile-dependent factors relate to volatility, we report the correlations with the increments of the PCA-SQ factor. We estimate this factor by using the residuals from Equation 5, squaring them and estimating the first principal component using PCA. This PCA-SQ factor will not capture the full factor structure if the distribution of idiosyncratic returns has nonnormal characteristics (Chen et al., 2021).[16] Naturally, we can expect these correlations to be quite high, as every distribution's quantiles are related to various degrees of variance (if they exist). We observe that the correlations are relatively high and significant for all three factors, with the highest value of 0.53 for the upper-tail quantile factor. The most important question is whether the reduced information from the variance subsumes the

[16] As in the case of quantile factors, we focus on increments of the factor, ΔPCA-SQ, and if not stated otherwise, we work with the increments of the factor.

pricing information that is specific to a given part of the cross-section of stock returns. We explore this question later in the text. However, we can still observe that the correlations are far from perfect and that they vary across values of ΔCIQ factors.

The correlations with the increments of the CIV factor of Herskovic et al. (2016) are slightly greater for the lower-tail idiosyncratic factor, with a value of -0.16. Surprisingly, the correlations with increments of the tail risk factor (ΔTR) of Kelly and Jiang (2014) and with increments of the cross-sectional bivariate idiosyncratic volatility (ΔCBIV) of Han and Li (2025) are relatively low, with the single exception of the correlation between the upper-tail ΔCIQ and ΔTR, with a value of -0.26. This situation is probably due to the fact that the TR factor captures a persistent component of the extreme risk in the stock market, unlike our factors, which are antipersistent.

The usefulness of the CIQ factors is illustrated by the observation that the time series of the realized cross-sectional lower-tail quantiles is captured much more precisely by the lower-tail CIQ factor, as it explains 49% of the variation in the series in comparison to 32% explained by the time series of cross-sectional volatility. Moreover, each month, we perform an Anderson–Darling normality test on the cross-sectional idiosyncratic return distribution, and the null hypothesis of normality is rejected in every case. These findings further underscore the necessity of considering factors that range beyond volatility in the context of assessing risk in the market. On the other hand, the added value of the CIQ factor is reduced in the case of the upper-tail factor, as the upper-tail factor explains 46% of the variation in the upper-tail idiosyncratic quantiles, whereas volatility captures 42% of the variability.[17]

Figure 1 shows the estimated CIQ factors and the PCA-SQ factor. The shaded areas represent recessions, as recognized by the National Bureau of Economic Research (NBER).

3 Pricing the CIQ(τ) Risks in the Cross-Section

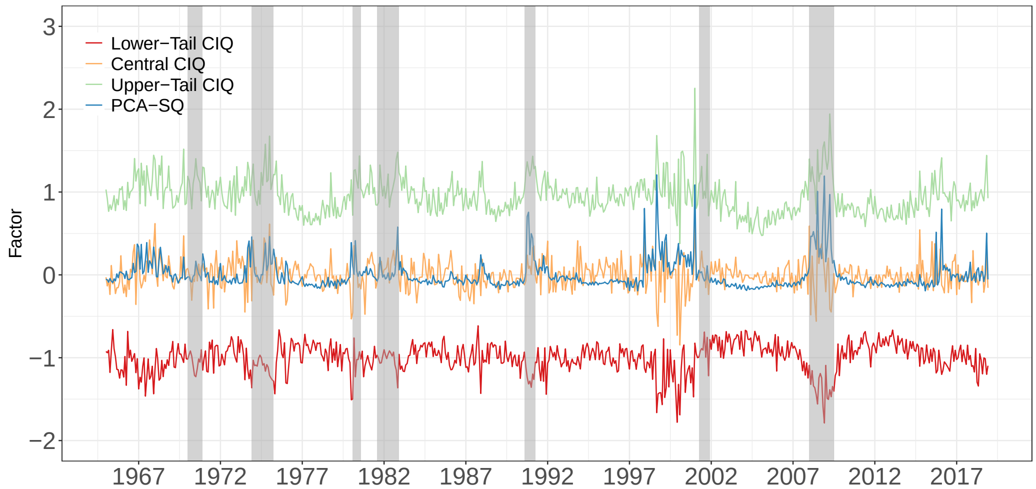
Now, we investigate how the exposures to the ΔCIQ factors are priced in the cross-section of stock returns. We document heterogeneity in the risk prices associated with the lower, upper and central parts of the joint cross-sectional distribution of the idiosyncratic returns. More specifically, the results presented here clearly indicate that only exposure to common idiosyncratic lower-tail movements is important for investors and that the corresponding premium is not captured by exposure to common volatility (captured either by the PCA-SQ or the ΔCIV factor) or any other phenomenon.

To measure "a stock's sensitivity to the lower-tail (ΔCIQ^{LT}), central (ΔCIQ^{C}) or upper-tail (ΔCIQ^{UT}) factors, we estimate its betas separately by a simple time series regression

[17] Panel B of Table 11 contains the full regression results.

Figure 1: CIQ Factors

The figure captures the lower-tail (20%), upper-tail (80%) and central (50%) CIQ factors, alongside the PCA-SQ factor. The factors are estimated on monthly idiosyncratic returns with respect to the three-factor model developed by Fama and French (1993) on the basis of either the quantile factor analysis suggested by Chen et al. (2021) (CIQ) or a principal component analysis using squared idiosyncratic returns (PCA-SQ). Factors are estimated using a rolling window of 60 months. In each window, the last estimated value is plotted. The data come from the CRSP and cover the period from January 1965 to December 2018. We exclude penny stocks with prices below $1. The shaded areas represent NBER recessions.



of its return on the factor in the form of

$$r_{i,t} = a_i + \beta_{i,j}^{CIQ} \Delta CIQ_t^j + v_{i,t}, \quad j = LT, C, UT \tag{7}$$

using the least-square estimator.[18] In the same manner as the factors, the betas are estimated using a 60-month rolling window.[19] We include stocks with at least 48 monthly observations. The betas and factors computed up to time t are used to predict returns at time $t+1$ or further–there is no overlap between the estimation and prediction periods. If not explicitly stated otherwise, we use monthly out-of-sample returns following the estimation window as our predicted variable. Subsequently, we also try to predict longer horizon returns using portfolios to assess the persistence of the ΔCIQ betas and thus indirectly investigate the transaction costs related to the trading of these factors.

[18]We measure the exposure to the ΔCIQ factors using time series regression coefficients, betas, instead of the γ_i from Equation 6 for three reasons. First, we focus on capturing the sensitivity of a stock return to *changes* in the investment opportunity set by working with the increments of the CIQ factors (the γ_i coefficient captures the relation with the levels of the factor). This approach renders our results directly comparable with those of previous studies that investigated the pricing implications of idiosyncratic returns. Second, by estimating the exposures using time series regressions, we are not limited to stocks that possess all the observations during the 60-month window, as the QFA methodology requires a balanced panel of data for estimation purposes. Third, using betas renders our results comparable to those of previous studies, e.g., Herskovic et al. (2016) or Kelly and Jiang (2014).

[19]To estimate the betas, we estimate the CIQ factors in a given window and then use these values to estimate the corresponding betas.

The data cover the usual cross-sectional asset pricing period between January 1963 and December 2018. The first returns that we predict pertain to January 1968. In total, our baseline dataset consists of 1,909,340 stock-month observations.

3.1 Univariate Portfolio Sorts

We start our analysis by investigating the performance of the portfolios formed on the basis of the ΔCIQ betas. Every month, we sort stocks on the basis of their estimated ΔCIQ betas into either five or ten portfolios; then, we record the portfolios' performance at time $t+1$ using either an equal- or value-weighted scheme based on market capitalization during the formation period. We then move one month ahead, reestimate all the betas, and create new portfolios. These univariate portfolio results are summarized in Table 2. In Panel A, we report the results for the decile sorts. We observe that for the lower-tail ΔCIQ, an increasing pattern of returns from the low-exposure portfolios to the high-exposure portfolios is observed; however, no significant return spread for the portfolio returns sorted on the central or upper-tail ΔCIQ factors is observed.

Moreover, to formally assess the presence of compensation for bearing the CIQ risks, we present the returns of the high-minus-low portfolios obtained as the difference between the returns of portfolios with the highest ΔCIQ betas and those of portfolios with the lowest ΔCIQ betas. These returns correspond to the investment strategy that buys stocks with high exposures and sells stocks with low exposures to the ΔCIQ factors. These portfolios are zero-cost portfolios and capture the risk premium associated with the specific joint movements of the idiosyncratic returns. We observe a significant positive premium for the difference portfolio only for the lower-tail ΔCIQ factor. This premium is both economically and statistically significant. In the case of the decile equal-weighted portfolio, the premium reaches 6.39% on an annual basis with a robust t statistic on the basis of Newey and West (1987) standard error correction with six lags of 3.53. The premium of the value-weighted portfolio achieves an even better performance of 8.09% p.a. with a t statistic of 3.06. The stocks that hedge the lower-tail CIQ movements possess particularly low excess returns. Investors clearly value this feature, which pushes up the price of these stocks and lowers their expected returns.

We also observe sizeable spread in the exposures across the portfolios captured by the time series average of equally weighted stock-level betas within portfolios during the formation. The low-exposure portfolio has an average β_{LT}^{CIQ} of -0.30, which is the greatest difference with respect to a neighborhood portfolio across all the portfolios (the second-lowest-exposure portfolio has an average β_{LT}^{CIQ} of -0.18, a difference of 0.12).

Table 2: Portfolios Sorted on Exposures to the ΔCIQ Factors

The table reports the annualized out-of-sample excess returns of portfolios sorted on the exposure to the lower-tail ($\tau = 0.2$), central ($\tau = 0.5$) and upper-tail ($\tau = 0.8$) ΔCIQ factors. We also report the returns of zero-cost portfolios obtained by buying the high-exposure portfolio and selling the low-exposure portfolio (High - Low). The corresponding t-statistics (in parentheses) are computed using the robust standard errors suggested by Newey and West (1987) with six lags. Stocks are either sorted into ten portfolios in Panel A or into five portfolios in Panel B, with portfolio returns obtained by either equally weighting stock returns (EW) or value weighting by their market capitalization (VW). The portfolios are formed each month based on the sensitivity to the ΔCIQ factors estimated using time-series regression over the previous 60 months. We also report time-series averages of equally-weighted stock-level ΔCIQ betas within portfolios during the formation. The return sample covers period between January 1968 and December 2018. Each month, we use all the CRSP stocks for which at least 48 monthly observations are available over the last 60 months and exclude penny stocks with prices below $1.

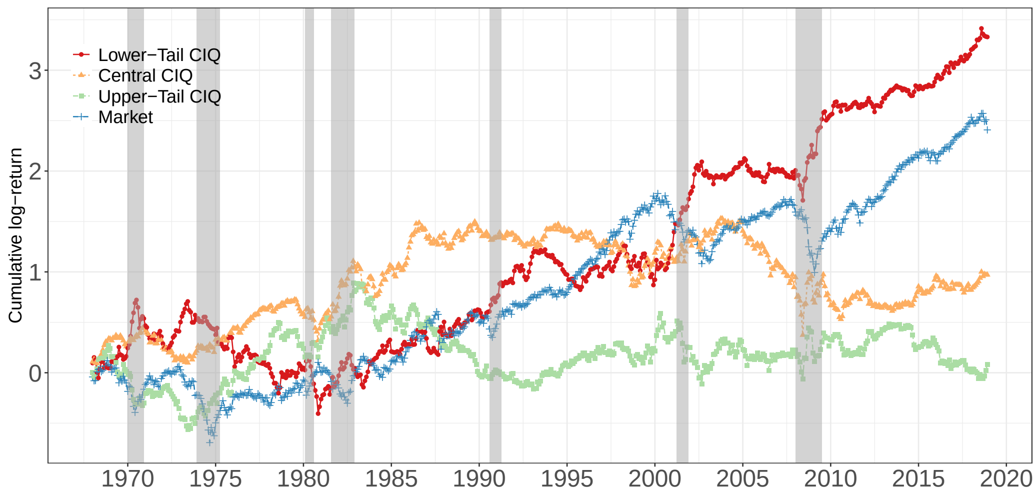
	Lower-Tail			Central			Upper-Tail		
	β^{CIQ}_{LT}	EW	VW	β^{CIQ}_{C}	EW	VW	β^{CIQ}_{UT}	EW	VW
Panel A: Decile Sorts									
Low β^{CIQ}	-0.30	3.89	1.91	-0.03	6.71	3.83	-0.12	8.19	4.75
2	-0.18	7.36	5.79	-0.01	9.02	4.98	-0.04	9.21	6.99
3	-0.14	8.88	6.24	-0.01	9.89	5.21	-0.01	9.73	6.95
4	-0.10	9.33	5.52	-0.01	8.81	5.47	0.01	8.92	6.00
5	-0.08	8.92	6.27	-0.00	9.62	6.73	0.03	9.08	7.03
6	-0.05	10.20	7.51	0.00	9.48	7.81	0.05	9.18	6.76
7	-0.03	10.07	7.16	0.00	9.51	7.36	0.07	9.37	6.18
8	-0.00	9.90	7.54	0.01	9.72	6.77	0.10	8.52	5.90
9	0.04	10.52	8.91	0.01	9.20	5.09	0.13	9.32	5.33
High β^{CIQ}	0.13	10.28	10.00	0.03	7.39	7.23	0.22	7.81	6.41
High - Low		6.39	8.09		0.69	3.40		-0.38	1.66
t-statistic		(3.53)	(3.06)		(0.39)	(1.55)		(-0.24)	(0.68)
Panel B: Quintile Sorts									
Low β^{CIQ}	-0.24	5.62	4.33	-0.02	7.86	4.43	-0.08	8.70	6.01
2	-0.12	9.10	5.85	-0.01	9.35	5.31	0.00	9.32	6.46
3	-0.07	9.56	6.84	-0.00	9.55	7.20	0.04	9.13	6.67
4	-0.01	9.99	7.22	0.01	9.62	7.17	0.09	8.95	6.01
High β^{CIQ}	0.08	10.40	9.06	0.02	8.29	5.66	0.18	8.57	6.00
High - Low		4.78	4.73		0.43	1.23		-0.14	-0.01
t-statistic		(3.12)	(2.34)		(0.32)	(0.72)		(-0.10)	(-0.01)

To show that the premium is not driven by a particular choice of the sorting scheme, in Panel B, we report the results from sorting the stocks into quintiles. Although the premium is smaller than in the case of decile sorts, it remains both economically and statistically significant at 4.78% p.a. with a t statistic of 3.12 in the case of an equally-weighted portfolio and 4.73% p.a. with a t statistic of 2.34 in the case of a value-weighted portfolio. This slightly lower significance in the case of the value-weighted portfolio may be partially caused by the fact that the value-weighted portfolios possess a higher concentration, which leads to more volatile returns.

On the other hand, if we consider the returns of the zero-cost portfolios associated with either central or upper-tail ΔCIQ factors, these portfolios yield premiums indistinguishable from zero. This observation holds across weighting and sorting schemes. Investors clearly value exposures to the downside and upside idiosyncratic events asymmetrically, and they exhibit a clear preference to hedge against the former but not the latter. The fact that

Figure 2: Performance of the ΔCIQ Portfolios

The figure depicts performance of a strategy that buy stocks with high exposure to the ΔCIQ factors and sell stocks with low exposure. It plots cumulative log-return obtained from sorting the stocks into decile portfolios with value-weighting the stocks. The return sample covers period between January 1968 and December 2018. Each month, we use all the CRSP stocks with at least 48 monthly observations over the last 60 months and exclude penny stocks with prices below $1. The shaded areas represent NBER recessions. We include the performance of the excess market return as a comparison.



only the exposures to the lower-tail common movements yield a premium suggests that the ΔCIQ risks are not driven by the effect of the common volatility. If volatility was the main driver of the factors, we would observe symmetrical compensation for the exposures to both the downside and upside factors, which we do not observe here. To illustrate this point in further detail, in Appendix B, we provide evidence indicating that, in an economy that specifically rewards holding assets with exposure to a common volatility factor, the upside and downside factors are priced symmetrically. On the basis of this result, we argue that the typical location-scale model is not consistent with the premiums that we observe here.

To visually inspect the performance of the portfolios sorted on the basis of the exposure to the ΔCIQ factors, we present in Figure 2 the cumulative log-returns of the value-weighted high-minus-low portfolios. Consistent with the numerical portfolio results, only the portfolio based on the lower-tail ΔCIQ factor provides strong performance during the sample period that is no worse than the performance of the aggregate market as measured by the value-weighted return of all CRSP firms.

The newly discovered lower-tail ΔCIQ premium raises the question of whether it simply mirrors previously reported risks associated with a particular factor model. Accordingly, we regress the returns of the high-minus-low portfolios on various sets of factors and report estimated intercepts–alphas–from this exercise. We summarize the results in Table 3. We report the annualized alphas for both the equal- and value-weighted portfolios that are sorted either in quintiles or deciles. We start the investigation by regressing the returns on

Table 3: Alphas of the Zero-Cost Lower-Tail ΔCIQ Portfolios

The table reports annualized abnormal returns of zero-cost portfolios obtained from buying high-exposure and selling low-exposure stocks with respect to the lower-tail ΔCIQ factor. Each month, stocks are sorted on the exposure estimated from the last 60 months into decile or quintile portfolios. Returns within a portfolio are either equal- or value-weighted based on the market capitalization at the time of the portfolio formation. Zero-cost portfolio return is calculated as a difference between high-beta and low-beta portfolio. The portfolio is then held for one month and subsequently rebalanced. We report estimated intercepts (alphas) from regressing the returns on various sets of asset pricing factors: market (CAPM), three factors of Fama and French (1993) (FF3), five factors of Fama and French (2015) (FF5) and its extension with the momentum factor of Fama and French (2018) (FF6), five factors of Hou et al. (2020), and four factors of Stambaugh and Yuan (2016) (M4). Moreover, we formulate an ad-hoc model based on four factors of Carhart (1997), CIV shocks of Herskovic et al. (2016), and BAB factor of Frazzini and Pedersen (2014) (FF alt.) and augment it with either short-term reversal factor (Rev.) or liquidity factor of Pastor and Stambaugh (2003) (Liquid.). The corresponding t-statistics (in parentheses) are computed using the robust standard errors suggested by Newey and West (1987) with six lags. The return sample covers period between January 1968 and December 2018. Each month, we use all the CRSP stocks for which at least 48 monthly observations are available over the last 60 months and exclude penny stocks with prices below $1.

	Decile		Quintile	
	EW	VW	EW	VW
CAPM	8.79	10.24	6.79	6.65
	(5.51)	(4.15)	(5.03)	(3.54)
FF3	7.07	7.58	5.42	4.23
	(4.75)	(3.42)	(4.34)	(2.66)
FF5	6.65	8.23	5.71	4.75
	(4.04)	(3.38)	(4.29)	(2.76)
FF6	7.22	9.23	6.28	5.90
	(4.01)	(3.65)	(4.37)	(3.40)
FF alt.	6.96	9.22	5.83	5.95
	(3.59)	(3.38)	(3.71)	(3.26)
FF alt. + Rev.	7.02	9.61	5.88	5.60
	(3.23)	(3.23)	(3.30)	(2.78)
FF alt. + Liquid.	6.88	9.31	5.77	5.96
	(3.49)	(3.33)	(3.61)	(3.21)
Q5	4.98	7.45	4.37	4.57
	(2.59)	(2.67)	(2.75)	(2.15)
M4	5.94	7.43	5.47	5.16
	(3.17)	(2.77)	(3.79)	(2.73)

the market factor (CAPM). We observe that the market factor is not successful in explaining the abnormal returns across the specifications. We repeat the regressions on the basis of the three-factor model of Fama and French (1993) (FF3), the five-factor model of Fama and French (2015) (FF5), and its extension incorporating the momentum factor of Fama and French (2018) (FF6). None of these models performs particularly well in explaining these abnormal returns, since neither the levels nor the significances deteriorate.

Next, we form an extension of the four-factor model of Carhart (1997) by augmenting it with the ΔCIV factor of Herskovic et al. (2016) and the betting-against-beta (BAB) factor of Frazzini and Pedersen (2014). We then add either the short-term reversal (Rev.) or liquidity factor of Pastor and Stambaugh (2003) (Liquid.). Similarly, we observe that the premiums associated with these risks do not subsume the abnormal returns of the lower-tail ΔCIQ portfolio.

Finally, we also regress the portfolio returns on the five-factor model of Hou et al. (2020) (Q5) and the four factors of Stambaugh and Yuan (2016), which include two mispricing factors (M4). The results also suggest that these models do not span the abnormal premium associated with the lower-tail ΔCIQ risk.

3.2 Bivariate Portfolio Sorts

Although none of the factor models considered here can explain the ΔCIQ premium, it may be the case that some stock-level characteristics do. For this reason, we investigate how stock-level lower-tail exposures relate to other stock-specific characteristics and exposures by performing dependent bivariate sorts. Every month, we first sort the stocks into ten (five) portfolios on the basis of the value of their control variable. Then, within each of the control-variable-sorted portfolios, we sort the stocks into another ten (five) portfolios on the basis of their lower-tail ΔCIQ betas. Finally, we form each ΔCIQ portfolio by collapsing all the corresponding control-variable-sorted portfolios into one ΔCIQ portfolio. This procedure yields single-sorted portfolios that vary in terms of their exposure to the lower-tail ΔCIQ factor but exhibit approximately equal values of the control variable.

The obtained results are summarized in Table 4. We focus here on some of the most researched stock characteristics and most related risk measures.[20] We start the investigation by using the market beta, firm size, book-to-price ratio and momentum. We observe that the zero-cost portfolio returns remain both economically and statistically significant when either decile or quintile sorts are used, with an annual premium of at least 5.01% (t statistic of 2.84) in the case of decile sorts and 4.14 (t statistic of 3.2) in the case of quintile sorts. Moreover, we report the six-factor alphas with respect to the model of Fama and French (2018) for the portfolios and observe that the premiums are not subsumed by the model.

Next, we focus on risk measures related to idiosyncratic risk and examine nonsystematic measures of idiosyncratic volatility and idiosyncratic skewness. We observe that even these measures do not erase the economic and statistical significance of the lower-tail ΔCIQ risk premium. The premium remains approximately 5% for decile sorts and 4% for quintile sorts, with t statistics above 3 in every case.

We then investigate the cases in which we control for nonlinear systematic risk measures. We start with coskewness and cokurtosis and observe that very significant risk premiums remain after we control for these measures. Then, we consider three measures that aim to capture exposure to systematic common volatility. First, we investigate exposure to the ΔPCA-SQ factor. If the performance of the lower-tail ΔCIQ portfolio is due to the effect of the common idiosyncratic volatility, the exposure to the ΔPCA-SQ factor should erase

[20]We provide information on how we obtain the control variables in Appendix D. We tried to adhere to the originally proposed specifications to the greatest extent possible. Stock characteristics are obtained following the specifications of Langlois (2020).

Table 4: Dependent Bivariate Portfolio Sorts

The table reports annualized out-of-sample excess returns of portfolios double-sorted on the exposure to the lower-tail ΔCIQ factor and a control variable. In Panel A, the portfolios are constructed by first sorting the stocks into deciles based on a control variable and then within each portfolio we sort stocks into deciles based on the exposure to the lower-tail ΔCIQ factor. Final portfolio returns are calculated by averaging the returns across the control deciles for every decile of the β^{CIQ}_{LT}. This procedure yields spread in the exposure to the lower-tail ΔCIQ factor, while holding control variable approximately constant across portfolios. In Panel B, we repeat the procedure by sorting stocks into quintiles. The portfolios are formed every month and returns within portfolios are equally weighted. We also report returns of the high minus low portfolios, annualized alphas with respect to the six-factor model of Fama and French (2018) and their t-statistics using the robust standard errors suggested by Newey and West (1987) with six lags. The return sample covers period between January 1968 and December 2018. Each month, we use all the CRSP stocks for which at least 48 monthly observations are available over the last 60 months and exclude penny stocks with prices below $1.

	β^{CAPM}	Size	B/P	MOM	IVOL	ISKEW	CSK	CKT	β^{down}	$\beta^{PCA\text{-}SQ}$	β^{CIV}	β^{tail}	β^{VIX}
Panel A: Decile Sorts													
Low	4.54	4.43	5.58	4.82	6.34	4.20	4.24	5.01	5.47	5.12	4.40	5.04	6.06
2	7.55	6.76	7.13	7.54	7.10	7.12	7.53	6.99	7.53	8.51	7.62	6.80	8.71
3	8.03	9.08	9.55	8.40	8.07	8.56	8.36	8.63	8.69	8.00	9.07	8.69	9.56
4	9.09	9.11	9.27	9.08	9.02	9.03	9.25	9.47	8.86	9.07	9.00	9.11	10.32
5	9.16	9.29	8.86	8.96	8.77	9.95	9.46	8.93	9.08	9.41	9.48	9.25	9.98
6	9.76	10.33	9.55	10.13	9.03	10.33	9.85	10.02	9.07	9.69	10.03	10.30	10.69
7	9.77	10.10	9.30	10.04	10.24	9.37	9.79	9.74	9.71	9.63	9.88	10.13	10.97
8	10.37	10.20	9.75	9.40	9.44	10.35	9.60	10.02	9.78	9.56	10.19	9.21	11.25
9	10.78	10.68	10.47	10.35	10.43	10.52	10.85	10.55	10.64	9.81	10.57	10.73	12.60
High	10.89	9.45	10.79	10.51	11.02	10.00	10.43	9.98	10.52	10.58	9.06	10.00	12.70
High - Low	6.35	5.01	5.22	5.70	4.68	5.80	6.20	4.97	5.05	5.47	4.66	4.95	6.63
t-statistic	(5.09)	(2.84)	(3.30)	(3.67)	(3.33)	(3.31)	(3.57)	(3.04)	(3.50)	(3.71)	(2.71)	(3.03)	(3.06)
α^{FF6}	6.61	5.52	7.37	7.06	5.35	6.42	7.09	6.03	6.76	5.65	4.64	6.28	6.19
t-statistic	(4.76)	(3.59)	(4.35)	(5.06)	(4.24)	(3.83)	(4.15)	(3.86)	(4.18)	(3.81)	(2.77)	(4.02)	(2.84)
Panel B: Quintile Sorts													
Low	5.95	5.44	6.37	6.05	6.75	5.70	5.75	5.95	6.30	6.69	5.99	5.88	7.34
2	8.53	9.27	9.22	8.96	8.56	8.78	8.88	8.96	8.86	8.35	8.97	8.75	9.82
3	9.52	9.67	9.26	9.42	8.92	9.81	9.78	9.56	9.23	9.81	9.64	9.73	10.59
4	10.15	10.14	9.75	9.67	9.79	10.06	9.68	9.88	9.72	9.62	10.18	10.06	10.99
5	10.79	10.16	10.51	10.51	10.70	10.33	10.57	10.32	10.54	10.21	9.83	10.19	12.66
High - Low	4.85	4.72	4.14	4.47	3.95	4.62	4.82	4.37	4.24	3.53	3.83	4.31	5.32
t-statistic	(4.31)	(3.27)	(3.02)	(3.44)	(3.23)	(3.11)	(3.31)	(3.09)	(3.45)	(3.10)	(2.71)	(3.10)	(3.16)
α^{FF6}	5.39	5.68	6.09	6.07	4.77	5.88	6.33	5.71	5.86	3.84	4.18	5.92	5.68
t-statistic	(4.50)	(4.51)	(4.35)	(5.19)	(4.35)	(4.30)	(4.49)	(4.28)	(4.49)	(3.40)	(3.36)	(4.62)	(3.24)

its significance. However, we do not observe that and still observe a very sizeable premium, which is not even explained by the six-factor model, with alphas equal to 5.65% p.a. (t statistic of 3.85) and 3.84% p.a. (t statistic of 3.40) with respect to the quantile and decile sorts, respectively.

Our simulation results in Appendix B, which are based on the stock returns obtained from a location-scale model with a symmetrical common volatility factor, show that exposure to the ΔPCA-SQ factor would yield a significant premium in that case. Therefore, if the returns are generated by the location-scale model, the quantile risk is captured by this volatility risk.

We then control for the exposure to the ΔCIV factor of Herskovic et al. (2016). We observe that out of all the controls, this exposure drives the lower-tail ΔCIQ premium the lowest, but even in this case, the premium remains solid at 4.66% p.a. (t statistic of 2.71) and 3.83% p.a. (t statistic of 2.71) for the decile and quintile sorts, respectively. Moreover, the remaining premium is not explainable by the six-factor model. The last volatility-related systemic measure that we explore is the VIX beta of Ang et al. (2006b). Once again, we observe that the premium is not subsumed by this exposure. Finally, we include the tail-risk beta of Kelly and Jiang (2014) and draw the same conclusion.

3.3 Fama-MacBeth Regressions

In the next step, we perform two-stage Fama and MacBeth (1973) predictive regressions. In contrast to portfolio sorting, this type of asset pricing test conveniently allows for simultaneous estimation of many risk premiums associated with various stock-level characteristics. This means that we can estimate the risk premium associated with the ΔCIQ_j factors $j \in \{LT, C, UT\}$ while controlling for other risk measures that have previously been proposed in the literature. Specifically, for each time $t = 1, \ldots, T-1$ using all of the stocks $i = 1, \ldots, N$ available at time t and $t+1$,[21] we cross-sectionally regress all the returns at time $t+1$ on the betas estimated using only the information available up to time t. This procedure yields estimates of prices of risk $\lambda^{CIQ}_{t+1,j}$ while controlling for the most widely used measures of risk. More specifically, we use variations of the following cross-sectional regressions:

$$r_{i,t+1} = \alpha_{t+1} + \sum_j \beta^{CIQ}_{i,t,j}\lambda^{CIQ}_{t+1,j} + Z'_{i,t}\lambda^{Z}_{t+1} + e_{i,t+1} \tag{8}$$

where $Z'_{i,t}$ is a vector of control variables and λ^{Z}_{t+1} is a vector of corresponding prices of risk. Using $T-1$ cross-sectional estimates of the prices of risk, we compute the average price of risk associated with λ^{CIQ}_j as

$$\widehat{\lambda}^{CIQ}_j = \frac{1}{T-1}\sum_{t=2}^{T}\widehat{\lambda}^{CIQ}_{t,j}, \quad j = LT, C, UT \tag{9}$$

and report them alongside their t statistics on the basis of the Newey and West (1987) robust standard errors with six lags. Moreover, unlike in the case of portfolio sorts, this setup enables us to jointly evaluate the risk premiums of the ΔCIQ factors by including them simultaneously in the regressions.

We summarize the first set of results in Panel A of Table 5, where we report the estimation outcomes of the regressions with general risk measures. First, by considering the settings featuring all three ΔCIQ exposures, we observe that only the lower-tail exposure significantly predicts future returns, with a coefficient of 1.2 (t statistic of 2.44). Adding idiosyncratic volatility, total and idiosyncratic skewness and the market beta to the regression does not alter the results, and the coefficient for lower-tail exposure is 1.31 (t statistic of 3.81) in the

[21] A stock is identified as available if it possesses at least 48 monthly return observations during the last 60-month window up to time t and an observation at time $t+1$.

Table 5: Fama–MacBeth Regressions with General Characteristics

The table shows estimated prices of risk and their t-statistics from Fama-MacBeth predictive regressions. Each month, we cross-sectionally regress next-month stock returns on current-month estimate of the exposures to the ΔCIQ factors while controlling for various stock and firm characteristics. In Panel A, we control for idiosyncratic volatility (IVOL) and skewness (ISKEW), total skewness (SKEW) and market beta (β^{CAPM}). In Panel B, we focus on lower-tail ΔCIQ exposure and various firm-specific characteristics. The resulting coefficients are calculated as averages of the monthly estimated coefficients and corresponding t-statistics are based on the robust standard errors suggested by Newey and West (1987) with six lags. The return sample covers period between January 1968 and December 2018. Each month, we use all the CRSP stocks with at least 48 monthly observations over the last 60 months, and exclude penny stocks with prices below $1. Note the coefficients are multiplied by 100 to ensure the clarity of the presentation.

Panel A: Risk Characteristics

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
β_{LT}^{CIQ}	1.18			1.20	1.13	1.20	1.21	1.31	1.31
	(2.79)			(2.44)	(2.58)	(2.48)	(2.49)	(3.48)	(3.81)
β_{C}^{CIQ}		1.29		-0.05	-0.91	-0.21	-0.20	-0.14	-1.18
		(0.45)		(-0.02)	(-0.37)	(-0.07)	(-0.07)	(-0.05)	(-0.46)
β_{UT}^{CIQ}			-0.17	-0.44	-0.34	-0.39	-0.40	-0.29	-0.23
			(-0.37)	(-0.98)	(-0.85)	(-0.89)	(-0.92)	(-0.77)	(-0.65)
IVOL					-18.79				-19.42
					(-4.53)				(-4.97)
SKEW						0.00			-0.08
						(0.08)			(-1.08)
ISKEW							0.01		0.11
							(0.48)		(1.42)
β^{CAPM}								0.11	0.16
								(1.01)	(1.50)
Intercept	0.79	0.76	0.77	0.83	1.17	0.81	0.81	0.78	1.09
	(3.31)	(2.98)	(3.20)	(3.66)	(6.65)	(3.65)	(3.63)	(3.97)	(6.59)
R^2_{adj}	0.67	0.41	0.42	1.52	2.67	1.67	1.66	4.10	5.18
$\bar{n}$	3120	3120	3120	3120	3114	3114	3114	3114	3114
T	612	612	612	612	612	612	612	612	612

Panel B: Firm Characteristics

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
β_{LT}^{CIQ}	1.20	1.17	1.08	1.04	1.17	1.17	1.04	0.92
	(2.88)	(2.96)	(2.90)	(2.52)	(2.71)	(2.77)	(2.54)	(2.57)
Size	-0.02							-0.02
	(-1.91)							(-1.80)
Book-to-price		0.11						0.10
		(1.75)						(1.66)
Net payout yield			1.34					0.84
			(1.63)					(1.08)
Turnover				-0.08				-0.10
				(-1.41)				(-2.08)
Illiquidity					1.44			1.88
					(1.14)			(1.09)
Profit						0.45		0.46
						(3.64)		(3.58)
Investment							-0.44	-0.38
							(-6.81)	(-7.02)
Intercept	0.80	0.67	0.77	0.81	0.77	0.65	0.85	0.59
	(3.29)	(3.10)	(3.15)	(3.46)	(3.35)	(2.75)	(3.62)	(2.81)
R^2_{adj}	1.01	1.39	1.18	1.54	1.42	1.21	1.08	3.17
$\bar{n}$	3120	2896	2898	3120	2925	2883	2883	2721
T	612	612	612	612	612	612	612	612

full setting. On the other hand, exposures to the central or upper-tail ΔCIQ factors remain unpriced.

We also investigate whether firm-specific characteristics based on accounting and trading information can capture the lower-tail premium.[22]. In Panel B, we predict the returns using the exposure to the lower-tail ΔCIQ factor and size, book-to-price ratio, net payout yield, turnover, illiquidity, profit and investment. All these specifications do not affect the significance and magnitude of the effect of β_{LT}^{CIQ} on the expected returns, and we observe that the exposure to the common idiosyncratic lower-tail events is robustly compensated in the cross-section of stock returns. For example, in the setting featuring all the characteristics, the estimated price of risk is equal to 0.92 (t statistic of 2.57).

Finally, we focus on nonlinear risk measures that may be correlated with exposure to common idiosyncratic lower-tail movements. We report the results from the regressions in Table 6. We start by considering coskewness and cokurtosis, following the specifications of Harvey and Siddique (2000) and Dittmar (2002), respectively, and control for these measures simultaneously. We believe that these measures do not drive out the significance of the lower-tail ΔCIQ betas and that both factors are nonsignificant in this setting. Next, we consider the effect of a stock's lagged return and its momentum (cumulative return over the last twelve months while excluding the most recent return). Both control variables are highly significant, but the lower-tail ΔCIQ exposure remains significant, with a slightly diminished coefficient of 0.83 (t statistic of 2.18).

Extending the bivariate portfolio results, we consider the simultaneous effect of the exposures to three systematic volatility measures- the ΔPCA-SQ factor, the ΔCIV factor of Herskovic et al. (2016) and the ΔVIX factor–on the lower-tail premium. We can conclude that the lower-tail factor extracts different pricing information, as the volatility exposures do not erase either its magnitude or significance. One must investigate the common distribution in further depth if one wants to identify priced information regarding the common distributional movements.

Similarly, we jointly consider exposure to the tail-risk factor of Kelly and Jiang (2014) and the downside beta of Ang et al. (2006). The results show that these two measures do not drive out the effect of the lower-tail ΔCIQ betas, which remains significant, similar to the univariate specification.

Finally, we consider the following three measures separately: the hybrid tail covariance risk (HTCR) proposed by Bali et al. (2014), the multivariate crash risk (MCRASH) of Chabi-Yo et al. (2022) and the predicted systematic skewness (PSS) of Langlois (2020). Although the HTCR and MCRASH are highly significant predictors of expected returns, these factors do not alter the effect of CIQ(τ) risks.

[22] We construct the variables in the same vein as in Langlois (2020).

Table 6: Fama–MacBeth Regressions with Asymmetric Risk Measures

The table shows estimated prices of risk and their t-statistics from Fama-MacBeth predictive regressions. Each month, we cross-sectionally regress next-month stock returns on current-month estimates of exposure to the lower-tail ΔCIQ factor and other risk measure that captures firm's risk exposure. We control for co-skewness (CSK), co-kurtosis (CKT), previous-month return (STR), cumulative return over $t-11$ to $t-1$ months (MOM), exposure to the ΔPCA-SQ factor (β^{PCA-SQ}), exposure to the ΔCIV factor (β^{CIV}), exposure to the ΔVIX (β^{VIX}), tail-risk beta (β^{tail}), downside-risk beta (β^{down}), hybrid tail covariance risk (HTCR), multivariate crash risk (MCRASH) and predicted co-skewness (PSS). The resulting coefficients are calculated as averages of the monthly estimated coefficients and corresponding t-statistics are based on the robust standard errors suggested by Newey and West (1987) with six lags. The return sample covers period between January 1968 and December 2018. Each month, we use all the CRSP stocks with at least 48 monthly observations over the last 60 months, and exclude penny stocks with prices below \$1. Note the coefficients are multiplied by 100 to ensure the clarity of the presentation.

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
β_{LT}^{CIQ}	1.18	1.11	0.83	1.05	0.86	1.25	1.32	1.18
	(2.79)	(2.72)	(2.18)	(2.52)	(2.42)	(3.01)	(2.84)	(2.98)
CSK		-0.15						
		(-0.53)						
CKT		-0.11						
		(-1.42)						
STR			-4.08					
			(-9.81)					
MOM			0.72					
			(4.30)					
$\beta^{PCA\text{-}SQ}$				2.73				
				(0.09)				
β^{CIV}				-0.61				
				(-1.68)				
β^{VIX}				-4.12				
				(-1.00)				
β^{down}					-0.11			
					(-1.17)			
β^{tail}					0.09			
					(1.16)			
HTCR						1.18		
						(2.97)		
MCRASH							2.28	
							(2.61)	
PSS								-2.32
								(-1.36)
Intercept	0.79	0.78	0.68	0.77	0.84	0.88	0.52	1.96
	(3.31)	(2.93)	(2.91)	(2.93)	(4.20)	(3.96)	(2.16)	(1.97)
R^2_{adj}	0.67	1.86	2.67	1.59	2.47	1.59	1.48	2.14
$\bar{n}$	3120	3118	3114	3402	3116	3119	1990	2717
T	612	612	612	346	612	612	612	612

3.4 CIQ Risks

Using portfolio sorts and firm-level cross-sectional regressions, we have shown that exposure to lower-tail idiosyncratic common events is significantly priced in the cross-section of stock returns and that none of the discussed risks drive the significance of these results. On the other hand, the exposure to the idiosyncratic upside potential captured by the upper-tail ΔCIQ factor does not have significant pricing implications for the cross-section of stock returns. This asymmetry further supports the hypothesis that common volatility is not the reason behind the significant pricing consequences of our lower-tail quantile factor.

These results suggest that the stochastic discount factor increases in the lower-tail ΔCIQ risk, as the risk-averse investor's marginal utility is high in the states of high ΔCIQ risk. On the basis of that hypothesis, assets that perform poorly in states with high lower-tail ΔCIQ risk will require a higher risk premium for holding by investors. As investors in many cases hold underdiversified portfolios, they are particularly averse to events in which bad stocks perform badly, as this may have catastrophic consequences for their wealth. On the other hand, assets that perform well during these states serve as a hedging tool and will be traded with higher prices and thus lower expected returns. Moreover, the results show that investors do not particularly value assets that do particularly well when there is high potential for high-performing stocks.

4 Robustness Checks

In this section, we investigate how the lower-tail ΔCIQ premium holds across specifications. First, we explore the alteration of the data that we use and the specification of the model that lies behind the abnormal returns. Then, we introduce several new approaches for estimating the exposure to the ΔCIQ factors.

4.1 Stability Checks

In this section, we investigate the stability of the premium associated with the lower-tail ΔCIQ factor across various alternative data and model specifications. First, in Panel A of Table 7, we further address the concern that the results are driven by volatility. To alleviate the risk that the driving force behind the premium is the firm-level time-varying idiosyncratic volatility, we standardize the idiosyncratic returns from Equation 5 by their estimate of time-varying volatility using the simple exponentially weighted moving average (EWMA) model and use the standardized returns to estimate the CIQ factors.[23] We present the results of both the equal- and value-weighted decile portfolios. We observe that the premium remains highly significant, with annual returns of 5.74% (t statistic of 3.18) and 7.27% (t statistic of 2.78) in the cases of the equal- and value-weighted portfolios, respectively. Furthermore, the premium is not subsumed by the six-factor model of Fama and French (2018).

Next, to precisely measure the sensitivity of stocks to the lower-tail ΔCIQ factor while controlling for the sensitivity to the changes in common idiosyncratic volatility, we estimate the lower-tail ΔCIQ betas from Equation 7 using multiple regression, in which we also include the ΔPCA-SQ factor as a control. The results show that this alternative does not affect the

[23]The EWMA volatility model for random variable e_t is defined as $\sigma^2_{t+1} = \lambda\sigma^2_t + (1-\lambda)e^2_t$, where we opt for the standard value of $\lambda = 0.94$.

Table 7: Lower-Tail ΔCIQ Premium across Different Specifications

The table contains risk premiums associated with exposure to the lower-tail ΔCIQ factor across various specifications. Premiums are obtained as differences between high- and low-exposure decile portfolio returns. We also report alphas with respect to FF6 model of Fama and French (2018). In Panel A, firstly, we report results based on the lower-tail ΔCIQ factor that is estimated by reference to idiosyncratic returns that are standardized by their time-varying volatility using the EWMA model. Second, we report results using ΔCIQ betas that are estimated from multiple regression when controlling for the exposure to the ΔPCA-SQ factor. In Panel B, we provide results for different values of τ that define the quantile level of the lower-tail ΔCIQ factor. In Panel C, we report results from models that use FF5 and FF6 model to compute the idiosyncratic returns. In Panel D, we report results using data that do not exclude stocks based on their price, as well as for dataset that includes stocks that are traded with price above \$5. In Panel E, we progressively exclude stocks with market capitalization below certain quantile level of NYSE traded companies. In Panel F, we report separately the premiums across two disjoint time periods. Panel G captures returns that are obtained from average multi-period stock returns followed after the formation of the portfolios. The returns are either computed using all twelve months after the formation period or exclude the month directly following the formation period. The reported t-statistics are computed using Newey and West (1987) robust standard errors with six lags. Returns are either equal- or value-weighted by their market capitalization at the formation period. The return sample covers period between January 1968 and December 2018. Each month, we use all the CRSP stocks with at least 48 monthly observations over the last 60 months, and exclude penny stocks with prices below \$1 if not stated otherwise.

	Equal-Weighted				Value-Weighted			
	Premium	t-stat	α^{FF6}	t-stat	Premium	t-stat	α^{FF6}	t-stat
Panel A: Volatility Control								
TV volatility std.	5.74	3.18	6.55	3.65	7.27	2.78	8.58	3.37
PCA-SQ control	5.78	3.47	4.87	2.96	7.43	2.95	6.19	2.57
Panel B: Lower-tail τ								
$\tau = 0.10$	4.85	2.68	5.06	2.83	5.41	2.17	5.00	2.16
$\tau = 0.15$	5.62	3.03	5.97	3.29	6.41	2.41	6.61	2.68
$\tau = 0.30$	5.71	3.33	6.70	4.25	7.00	2.56	8.35	3.20
$\tau = 0.40$	4.95	3.20	5.60	3.76	5.62	2.31	7.75	3.22
Panel C: Linear Specification								
FF5	5.88	3.47	7.19	4.20	6.90	2.82	8.33	3.40
FF6	4.21	2.54	5.98	3.29	4.33	1.78	5.83	2.18
Panel D: Stock Price								
All stocks	6.61	3.05	6.85	3.63	6.47	2.60	5.76	2.60
Price > \$5	6.01	3.35	6.31	4.32	7.04	2.91	6.51	2.98
Panel E: Firm Size								
Market cap > $q_{NYSE}(10\%)$	6.68	3.09	6.91	3.71	6.58	2.64	5.84	2.64
Market cap > $q_{NYSE}(20\%)$	6.32	2.83	6.76	3.50	6.16	2.54	5.56	2.77
Market cap > $q_{NYSE}(50\%)$	5.09	2.26	5.12	2.77	5.69	2.41	5.13	2.60
Panel F: Time Split								
01/1968 - 12/1993	4.83	1.88	4.23	1.97	6.19	1.60	7.68	2.17
01/1994 - 12/2018	8.01	3.17	7.15	2.69	10.06	2.84	8.09	2.47
Panel G: Multi-Period Returns								
Next 3 months	4.80	3.03	5.71	3.15	6.66	2.84	7.64	3.08
Next 6 months	4.41	3.09	5.40	2.88	6.44	3.10	7.74	3.06
Next 12 months	4.00	3.56	3.13	2.06	5.75	3.62	4.04	1.91
$t+2$ to $t+12$	3.91	3.38	3.52	2.71	5.52	3.37	5.01	2.54

premium associated with lower-tail ΔCIQ risks, as the premiums remain at 5.78% ($t = 3.47$) and 7.43% ($t = 2.95$) in the cases of the equal- and value-weighted portfolios, respectively.

In Panel B, we vary the value of τ, which defines the quantile level of our lower-tail ΔCIQ factor. We investigate additional values of 0.1, 0.15, 0.3 and 0.4. We observe a stable premium that is very close to the baseline specification with $\tau = 0.2$.

In Panel C, we report the portfolio results using idiosyncratic returns with respect to the FF5 and FF6 models. While the FF5 specification does not quantitatively alter the results from our baseline FF3 specification, the results using the FF6 model slightly diminish the premium. As the only difference between the FF5 and FF6 models is the inclusion of the momentum factor in the linear specification, we can conclude that the momentum exposure is partially related to the common lower-tail events. The observation that the momentum returns are associated with extreme negative events, so-called momentum crashes, has been well documented; see, for example, Daniel and Moskowitz (2016) or Barroso and Santa-Clara (2015). This observation sheds further light on the drivers of the momentum risk premium capturing the risk of lower-tail events in the cross-section of stock returns. Moreover, the alphas associated with the lower-tail ΔCIQ risk remain high and significant, with annualized values of 5.98% ($t = 3.29$) and 5.83% ($t = 2.18$) for the equal- and value-weighted portfolios, respectively.

Next, in Panel D, we vary the stocks that we consider when forming the portfolios. First, as we restrict our universe to stocks with prices above $1 in our baseline specification, we report here the results using all the available stocks. We observe that the premiums estimated using the equal and value sorts remain high and significant. This situation holds if we restrict our universe to stocks with prices above the $5 threshold.

In Panel E, we restrict our investment universe to stocks with market capitalization above the p percentile of the distribution of stocks traded on the New York Stock Exchange at a given time. We report the results for the stocks with market capitalization greater than the 10%, 20% and 50% percentiles. We observe a stable premium that, even if we focus on the stocks with capitalization above the NYSE median, yields annual equal- or value weighted returns of 5.09% ($t = 2.26$) and 5.69% ($t = 2.41$).

In Panel F, we report separate results for two disjointed periods. The first period covers the time between January 1968 and December 1993, and the second period covers the time from January 1994 to December 2018. We observe that although the premium is relatively high—4.83% for the equal-weighted portfolio and 6.19% for the value-weighted portfolio– they are only borderline significant, with t statistics of 1.88 and 1.60, respectively. Figure 2 shows that this situation is caused mostly by the period between approximately 1974 and 1980, in which the value-weighted strategy experienced its largest drawdown. On the other hand, since that time, the strategy has experienced relatively steady growth, with an equal-weighted premium of 8.01% (t statistic of 3.17) and a value-weighted premium of 10.06% (t statistic of 2.84).

Finally, in Panel G, we consider the performance of the lower-tail ΔCIQ-sorted portfolios as captured by the following multiperiod returns. This exercise helps us understand two points. First, we examine whether the investment strategy based on the lower-tail ΔCIQ factor is feasible in terms of turnover by examining the returns of portfolios that are rebalanced with a lower than monthly frequency. Second, we can infer whether the premium associated with the exposure to the lower-tail ΔCIQ factor is a compensation for risk and not just a reversal effect. If risk is the driving force underlying the abnormal returns, then multiperiod returns should remain economically and statistically significant. We proceed as follows. Each month, we form the portfolios as in the previous case. Instead of saving the next one-month return of the sorted portfolios, we record the three-, six- and twelve-month returns that follow after the formation period. We observe returns that are consistent with the results obtained using the one-month returns; for example, the high-minus-low portfolio rebalanced every six months yields a 4.41% ($t = 3.09$) equal-weighted premium and a 6.44% ($t = 3.10$) value-weighted premium on an annual basis. These results suggest that an investor does not have to suffer the high turnover costs associated with the strategy to exploit the associated risk premium.

To mitigate the effect of return reversals, we also extend this analysis by working with one-year returns but excluding returns immediately following the formation period. We report the results in the last row of Panel G. The resulting returns are almost indistinguishable from the returns over the full one-year period, with an equal-weighted return of 3.91% ($t = 3.38$) and a value-weighted return of 5.52% ($t = 3.37$).

4.2 Beyond ΔCIQ Betas

In this subsection, we extend the ΔCIQ betas to further demonstrate the importance of considering downside- and upside-specific risks and their heterogeneous implications. In particular, this approach has two objectives. First, we specifically capture additional information beyond the median dependence from the downside and upside parts of the distribution and define *the relative* ΔCIQ betas as follows:

$$\beta_{i,j}^{CIQ,rel} := \beta_{i,j}^{CIQ} - \beta_{i,C}^{CIQ}, \quad j = LT, UT \tag{10}$$

The results of the portfolio sorts on the basis of relative betas are summarized in Table 8. These results are similar to the ΔCIQ results presented above. The high-minus-low portfolio sorted on the lower-tail relative betas yields an annual excess return of 6.89% ($t = 3.69$) with a six-factor $\alpha = 7.65$ ($t = 4.06$) for the equal-weighted portfolio. In the case of the value-weighted portfolio, we obtain an annual return of 7.98% ($t = 3.04$) and $\alpha = 8.75$ ($t = 3.56$). Similarly, as in the previous section, we also present the results for the quintile sorts and

Table 8: Portfolios Sorted on Alternative Specifications of the ΔCIQ Exposures

The table reports the annualized out-of-sample excess returns of portfolios sorted on exposure to the lower-tail and upper-tail ΔCIQ(τ) factors. Relative betas are computed by subtracting central ΔCIQ exposure from either lower- and upper-tail exposures. Combination betas are obtained as average rank of either lower- and upper-tai ΔCIQ exposures. Relative combination exposures are obtained by first computing relative exposures and then by averaging across ranks of either lower- or upper-tail Δ exposures. We also report the returns of zero-cost portfolios obtained by buying a high-exposure portfolio and selling a low-exposure portfolio (High - Low). The corresponding t-statistics (in parentheses) are computed using the robust standard errors suggested by Newey and West (1987) with six lags. Stocks are either sorted into ten portfolios in Panel A or into five portfolios in Panel B, with portfolio returns obtained by either equally weighting stock returns or value weighting by their market capitalization. The portfolios are formed each month based on the sensitivity to the ΔCIQ(τ) factors estimated using time-series regression over the previous 60 months. The return sample covers period between January 1968 and December 2018. Each month, we use all the CRSP stocks for which at least 48 monthly observations are available over the last 60 months and exclude penny stocks with prices below $1.

	Relative				Combination				Rel. Comb.			
	Lower-Tail		Upper-Tail		Lower-Tail		Upper-Tail		Lower-Tail		Upper-Tail	
	EW	VW	EW	VW	EW	VW	EW	VW	EW	VW	EW	VW
Panel A: Decile Sorts												
Low	3.89	2.57	8.22	3.85	4.51	2.41	8.48	4.64	4.23	2.46	8.60	5.84
2	7.44	5.35	9.83	7.65	6.71	6.42	8.76	6.37	7.29	6.07	9.32	7.14
3	8.99	6.37	9.40	6.19	8.89	5.77	9.59	5.79	8.55	5.96	9.04	7.16
4	8.85	6.35	8.62	6.84	9.37	6.02	8.91	6.67	8.98	6.33	8.83	5.74
5	9.02	6.18	8.83	7.38	9.05	5.80	9.36	6.34	9.67	6.43	8.97	6.55
6	9.96	7.15	9.50	5.99	10.51	8.24	9.14	7.38	9.77	7.20	9.45	6.01
7	10.26	7.33	9.15	6.37	9.59	7.09	9.17	6.23	10.12	8.08	9.15	6.78
8	10.02	7.13	8.98	5.73	10.12	6.93	8.97	6.09	9.93	6.43	8.69	6.66
9	10.15	8.96	8.82	5.23	10.48	8.67	8.58	5.57	10.42	8.31	8.76	5.18
High	10.78	10.55	7.98	6.27	10.12	9.90	8.38	7.01	10.38	10.39	8.54	6.59
High - Low	6.89	7.98	-0.23	2.41	5.60	7.49	-0.10	2.37	6.15	7.93	-0.05	0.75
t-statistic	(3.69)	(3.04)	(-0.15)	(0.94)	(3.16)	(2.85)	(-0.06)	(1.06)	(3.25)	(2.93)	(-0.04)	(0.33)
α^{FF6}	7.65	8.75	-2.61	2.14	6.60	8.31	-2.65	1.87	6.99	8.49	-2.49	0.27
t-statistic	(4.06)	(3.56)	(-1.53)	(0.84)	(3.99)	(3.46)	(-1.39)	(0.76)	(3.85)	(3.48)	(-1.50)	(0.11)
Panel B: Quintile Sorts												
Low	5.66	4.26	9.02	6.22	5.61	5.01	8.62	5.61	5.76	4.80	8.96	6.68
2	8.92	6.36	9.01	6.53	9.13	5.84	9.25	6.28	8.76	6.17	8.93	6.38
3	9.49	6.65	9.16	6.56	9.78	6.98	9.25	6.79	9.72	6.82	9.21	6.17
4	10.14	7.14	9.07	6.06	9.86	7.03	9.07	6.12	10.02	7.25	8.92	6.66
High	10.46	9.27	8.40	5.63	10.29	8.81	8.48	6.09	10.40	8.74	8.65	5.52
High - Low	4.80	5.01	-0.62	-0.59	4.68	3.80	-0.14	0.49	4.64	3.94	-0.31	-1.16
t-statistic	(3.10)	(2.39)	(-0.47)	(-0.33)	(3.13)	(1.97)	(-0.10)	(0.28)	(3.02)	(1.95)	(-0.25)	(-0.69)
α^{FF6}	6.17	5.73	-2.61	-1.05	6.14	5.02	-1.95	0.02	6.00	4.87	-2.11	-1.29
t-statistic	(4.31)	(3.33)	(-1.82)	(-0.56)	(4.42)	(3.00)	(-1.28)	(0.01)	(4.20)	(2.94)	(-1.54)	(-0.73)

report highly significant abnormal returns and alphas. Moreover, the results suggest that the incremental upside exposure does not result in any premium with zero-cost portfolio returns being statistically indistinguishable from zero.

Second, to show robustness with respect to the quantile level, τ, which we choose to compute the lower- and upper-tail exposures and to provide a way to aggregate the information from the downside and upside parts, we define two compressed measures. To summarize the dependence across the entire lower or upper part of the factor structure, we define the *combination* lower-tail and upper-tail ΔCIQ betas as follows:

$$\begin{aligned}\beta_{i,LT}^{CIQ,comb} &:= \sum_{\tau \in \tau_{LT}} F\big(\beta_{i,\tau}^{CIQ}\big)\\ \beta_{i,UT}^{CIQ,comb} &:= \sum_{\tau \in \tau_{UT}} F\big(\beta_{i,\tau}^{CIQ}\big)\end{aligned} \tag{11}$$

where $F\big(\beta_{i,\tau}^{CIQ}\big) = \frac{Rank(\beta_{i,\tau}^{CIQ})}{N_t+1}$ and $\beta_{i,\tau}^{CIQ}$ is estimated using the ΔCIQ factor for a given quantile level τ, where we use a set of τs of 0.1, 0.15, 0.2, 0.3 and 0.4 for the lower tail and a set of 0.6, 0.7, 0.8, 0.85 and 0.9 for the upper tail. We obtain the combination lower- and upper-tail ΔCIQ betas as the average cross-sectional ranks of the ΔCIQ betas for the lower- and upper-tail τs, respectively. The results of the portfolio sorts on the basis of those betas are also summarized in Table 8. We observe that the long-short portfolios sorted on the basis of the combination lower-tail ΔCIQ betas provide significant excess annual returns of 5.60% (t = 3.16) and 7.49% (t = 2.85) using decile sorting and equal- and value-weighted schemes, respectively. On the other hand, an investment strategy based on the combination upper-tail betas does not yield significant abnormal returns when either weighting approach is used.

Finally, to summarize the relative betas through the whole downside or upside parts of the joint structure, we introduce the *relative combination* betas as follows:

$$\begin{aligned}\beta_{i,LT}^{CIQ,rel-comb} &:= \sum_{\tau \in \tau_{LT}} F\big(\beta_{i,\tau}^{CIQ,rel}\big),\\ \beta_{i,UT}^{CIQ,rel-comb} &:= \sum_{\tau \in \tau_{UT}} F\big(\beta_{i,\tau}^{CIQ,rel}\big),\end{aligned} \tag{12}$$

which are obtained as a mean cross-sectional rank of the relative betas associated with the exposure to the lower- or upper-tail ΔCIQ(τ) factors, respectively. We compute the relative betas with respect to the same quantiles as in the case of the combination betas. The associated returns are also summarized in Table 8. Similarly, as in the case of the relative betas, the lower-tail relative combination betas provide an investment strategy with significant abnormal returns of 6.15% (t = 3.25) and 7.93% (t = 2.93) on an annual basis using the equal- or value-weighted returns, respectively. The returns of the portfolios based on the relative combination upper-tail betas remain nonsignificant.

The results presented here confirm the previous hypothesis that lower-tail idiosyncratic comovements present a priced risk factor in the economy. On the other hand, the upside idiosyncratic factor does not constitute a risk to which the average investor assigns a premium.

5 Economic Drivers

In this section, we explore the origins of the premium associated with the ΔCIQ risk in further detail and consider how the ΔCIQ risk relates to the time series of the equity premium.[24]

5.1 Predictability of the Equity Premium

In this section, we show that the lower-tail ΔCIQ factor robustly predicts the equity premium in the economy, providing further evidence for the hypothesis that investors want to hedge against such risk. We report here the ability of the lower-tail ΔCIQ factor to forecast subsequent short-term market returns, which we approximate by the value-weighted returns of all CRSP firms. We demonstrate that this predictability is not due to correlation with any related factors and holds in an out-of-sample setting.

First, we report the results of in-sample univariate predictive regressions of the following form:

$$r_{m,t+1} = \gamma_0 + \gamma_1 \times \Delta CIQ_t^j + \epsilon_{t+1}, \quad j = LT, C, UT \tag{13}$$

where $r_{m,t}$ is the monthly market excess return. We report the results in Table 9.[25] We report the estimated scaled coefficients that capture the effect of a one standard deviation increase in the independent variable on the subsequent annualized market return. The corresponding t statistics are computed using Newey and West (1987) robust standard errors using six lags. We document the significant predictive power using the lower-tail ΔCIQ factor, with an increase (decrease) of one standard deviation in the factor predicting a subsequent decrease (increase) of 5.00 percent ($t = -2.14$) in annualized market returns.[26] There is also some predictive power for the upper-tail factor, but the effect is smaller, with only a 3.81 percent ($t = 1.74$) increase in the annualized market return accompanied by an in-sample (IS) R^2 value of only 0.51% in comparison to 0.88% for the lower-tail factor. From the perspective of an investor, in times of high risk–captured by large negative increments of the lower-tail CIQ factor–they require a premium for investing. Thus, these risky periods correlate with the high marginal utility states of investors.

[24]Moreover, in Appendix E.1, we report the heterogeneous relationships between the ΔCIQ risk and household and firm cash flow risk. In addition, in Appendix F, we investigate the cross-sectional origins of the exposures to the presented risks and reveal which firms are more susceptible to the common idiosyncratic quantile movements.

[25]For each month, we estimate the CIQ factors over the past 60 months, calculate the differences and use the latest value to predict the out-of-sample market returns.

[26]Note that the lower tail factors are, on average, negative. An increase (decrease) in these factors corresponds to a decrease (increase) in risk, which leads to a decrease (increase) in the required risk premium.

Table 9: Market Return Predictability by Reference to ΔCIQ Factors

The table reports results from various specifications of predictive regressions of the value-weighted return of all CRSP firms on the ΔCIQ factors and control variables. We employ increments of the PCA-SQ factor ($\Delta PCA\text{-}SQ$), innovations of the CIV factor of Herskovic et al. (2016) (ΔCIV), tail risk factor of Kelly and Jiang (2014) (TR), lagged market return (MKT_{t-1}), cross-sectional bivariate idiosyncratic volatility of Han and Li (2025) (CBIV), and short-term reversal factor. Coefficients are scaled to capture the effect of one-standard-deviation increase in the factor on the annualized market return in percentage points. The corresponding t-statistics reported in parentheses are computed using the Newey and West (1987) robust standard errors using six lags. We report both in-sample (IS) and out-of-sample (OOS) R^2s. We also truncate the predictions at zero following Campbell and Thompson (2007) (CT) and report corresponding IS and OOS R^2s. The data cover the period from January 1965 to December 2018.

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
ΔCIQ^{LT}	-5.00			-5.37	-6.42	-5.77	-5.38	-5.39	-5.39	-5.87	-5.46	-7.14
	(-2.14)			(-2.21)	(-2.23)	(-2.24)	(-2.23)	(-2.23)	(-2.23)	(-2.22)	(-2.22)	(-2.23)
ΔCIQ^{C}		0.39		-0.51	-0.28	-0.36	-0.57	-0.40	-0.46	-0.38	-0.48	-0.03
		(0.18)		(-0.20)	(-0.11)	(-0.14)	(-0.23)	(-0.16)	(-0.18)	(-0.15)	(-0.19)	(-0.01)
ΔCIQ^{UT}			3.81	4.58	5.72	4.86	4.87	4.58	4.70	4.80	4.44	6.19
			(1.74)	(2.04)	(1.97)	(2.13)	(2.18)	(2.04)	(2.10)	(2.09)	(2.01)	(2.10)
$\Delta PCA\text{-}SQ$					-2.16							-2.39
					(-0.72)							(-0.77)
ΔCIV						-2.15						-2.00
						(-0.62)						(-0.59)
TR							4.96		2.10			2.48
							(2.42)		(0.57)			(0.66)
CBIV								5.12	3.31			2.98
								(2.47)	(0.86)			(0.76)
MKT_{t-1}										1.24		0.31
										(0.59)		(0.14)
STR											0.87	0.75
											(0.32)	(0.28)
R^2 IS	0.88	0.01	0.51	1.55	1.63	1.70	2.41	2.47	2.51	1.59	1.57	2.78
R^2 IS CT	0.73	0.01	0.42	1.08	1.23	1.13	2.00	2.07	2.14	1.04	1.10	2.24
R^2 OOS	0.68	-0.29	0.11	0.70	-0.08	-0.53	1.42	1.58	1.15	0.42	0.14	-1.93
R^2 OOS CT	1.08	-0.14	0.45	0.70	0.21	-0.35	1.38	1.53	1.37	0.44	0.38	-1.01

Alongside the in-sample R^2 values, we also report the out-of-sample (OOS) R^2 values from the expanding window scheme. We use data up to time t to estimate a prediction model and then forecast a return at time $t+1$ (the first window contains 120 monthly periods to obtain sufficiently reasonable estimates). Then, the window is extended by one observation, the prediction model is re-estimated, and a new forecast is obtained. We repeat this procedure until the whole sample is exhausted. The corresponding R^2 value is computed by comparing the conditional forecast and historical mean computed using the available data up to time t, i.e., $1 - \sum_t (r_{m,t+1} - \widehat{r}_{m,t+1|t})^2 / \sum_t (r_{m,t+1} - \bar{r}_{m,t})^2$, where $\widehat{r}_{m,t+1|t}$ is the out-of-sample forecast of the $t+1$ return using data up to time t and $\bar{r}_{m,t}$ is the historical mean of the market return computed up to date t. Unlike the case of the IS R^2 value, the OOS R^2 value can attain negative values if the conditional forecasts perform worse than the historical mean forecast. A positive value of 0.68% of the OOS R^2 value for the lower-tail ΔCIQ factor provides strong evidence to support the benefits of the factor for predicting the market return in the real-world setting. On the other hand, the OOS predictability deteriorates noticeably for the upper-tail quantile factor, with an OOS R^2 value of only 0.11%.

To assess the economic usefulness for investors, we further follow suggestions from Campbell and Thompson (2007) (hence CT). These authors proposed truncating the predictions from the estimated model at 0, as the investor would not have used a model to predict a negative premium. This nonlinear modification of the model should introduce caution into the models. On the basis of this modification, we report both the IS and OOS R^2 values. Naturally, using this transformation, the IS R^2 value does not improve for any of the models; however, the performance increases for the OOS analysis. The OOS R^2 value in the case of the lower-tail ΔCIQ factor increases to 1.08% and to 0.45% in the case of the upper-tail factor.

Furthermore, we report the regression results from the multivariate regressions of the following form:

$$r_{m,t+1} = \gamma_0 + \sum_j \gamma_{1,j} \times \Delta CIQ_t^j + \gamma_2 \times f_t + \epsilon_{t+1} \tag{14}$$

where f_t is a vector of control factors. We start by controlling for the effect of the ΔPCA-SQ factor. We observe that neither the significance nor the magnitude of the predictive power of the lower-tail ΔCIQ factor is diminished. Although the IS R^2 value increases in comparison to the univariate case with the lower-tail ΔCIQ factor only, the OOS performance deteriorates substantially to a negative value of -0.08. This finding suggests that the common volatility element is not the driving force of the predictive performance of the lower-tail idiosyncratic factor.

Next, we investigate the ΔCIV factor of Herskovic et al. (2016). The results regarding the lower-tail ΔCIQ factor remain the same, and ΔCIV proves not to predict future market returns. In the case of the TR factor of Kelly and Jiang (2014), the lower-tail ΔCIQ factor mirrors the results from the univariate regressions in terms of the coefficient and its significance. Furthermore, the TR factor proves to be a significant predictor of the equity premium, as it substantially improves the fit of the OOS regressions. Next, we include the cross-sectional bivariate idiosyncratic volatility of Han and Li (2025) (CBIV). We observe that it also constitutes a significant predictor of the short-term market return and increases the OOS R^2 value by approximately one percentage point. On the other hand, the lower-tail idiosyncratic factor maintains both its significance and magnitude. To determine whether TR and CBIV can jointly drive the predictive power of the lower-tail ΔCIQ factor down, we include them both in one regression. We observe that the lower-tail ΔCIQ factor retains its predictive power even in this setting.

We also use lagged market returns as a control. This predictor may capture the reversal or continuation effect of the market returns. We observe that the inclusion of this control does not alter the results regarding the CIQ factors and that using the lagged returns do not significantly improve the prediction fit. This claim also holds for the final control that we consider, i.e., the returns of the short-term reversal factor. Finally, we include all the previously mentioned variables in one regression and observe that the conclusion regarding the idiosyncratic factors holds even in this setting.[27]

6 Conclusion

We use a quantile-based factor model to extract latent factors from the cross-sectional distribution of idiosyncratic stock returns. The results reveal that the downside (left-tail) factor carries a robust risk premium that is distinct from known volatility or downside-related risk factors. Stocks with greater exposure to extreme idiosyncratic declines generate significantly higher average returns than those with less exposure, even when standard risk factors are taken into account. The documented return spread between firms with high and low exposure reflects "investors' strong aversion to common idiosyncratic tail losses. In contrast, we find no evidence indicating that exposure to upper-tail or median quantile idiosyncratic factors is rewarded by the market. Furthermore, we demonstrate that the lower-tail CIQ factor has substantial time series predictive power, with spikes in idiosyncratic tail risk reliably forecasting elevated future excess market returns. Together, these empirical results provide a consistent picture: market prices downside idiosyncratic quantile risk but not upside idiosyncratic swings.

Our quantile-factor approach offers significant advantages over existing methods regarding its ability to capture tail risks. Unlike idiosyncratic volatility factors, such as the common idiosyncratic volatility factor proposed by Herskovic et al. (2016) or the PCA-SQ volatility factor, which aggregates upside and downside movements, CIQ factors disentangle the distribution's tails and thereby uncover asymmetric risk that was previously masked. The lower-tail CIQ factor more effectively explains the cross-section of returns than do traditional volatility or downside risk proxies, yielding higher pricing significance and alphas that are not explained by those benchmarks. In short, the CIQ framework provides a clearer view of the idiosyncratic risk, revealing a priced downside component that is missed by volatility-based factors or parametric tail metrics. Furthermore, our method is agnostic with respect to idiosyncratic return dynamics and does not depend on options markets or higher-moment estimators. This characteristic makes it a flexible and robust tool for identifying latent risk factors.

The presence of a priced idiosyncratic tail factor has broad implications for asset pricing

[27]In unreported results, we also control for a set of eleven macroeconomic variables investigated by Welch and Goyal (2007), and the results remain identical. These results are available upon request.

theory. This finding challenges the classical claim that idiosyncratic risk is irrelevant to expected returns because it is fully diversifiable. Instead, we observe that extreme idiosyncratic shocks share a common structure with significant consequences for asset prices. This situation is likely due to the fact that these shocks coincide with economy-wide downturns or significant shifts in aggregate marginal utility. This insight links microlevel risks with macrolevel outcomes. Consistent with heterogeneous-agent models, undiversified tail risks at the firm and household levels can be combined to create a systematic risk factor. The fact that only downside idiosyncratic risk is priced is consistent with theories of rare disasters and loss aversion, in which investors demand additional compensation for exposure to severe losses but not forgone gains. Our findings suggest that asset pricing models should be refined to incorporate distributional risk factors beyond the traditional mean-variance paradigm. Such models may provide better explanations of anomalies such as the idiosyncratic volatility puzzle and improve our understanding of risk premiums in equity markets by accounting for common downside risks in idiosyncratic returns. In conclusion, recognizing and modeling common idiosyncratic quantile risks–particularly on the downside–is essential to the task of achieving a more complete and nuanced understanding of how risks are priced in financial markets.

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A Beyond Volatility Factors

To illustrate the discussion and provide the link between volatility and quantiles in such restrictive models, let's consider the data generating process to be a typical location-scale model with two unrelated factors in the first and second moments. Idiosyncratic returns $\epsilon_{i,t}$ of such model will be zero mean i.i.d. process independent of both factors with cumulative distribution function $F_{\epsilon_{i,t}}$. Further let $Q_{\epsilon_{i,t}}(\tau) = F^{-1}_{\epsilon_{i,t}}(\tau) = \inf\{s : F_{\epsilon_{i,t}}(s) \leq \tau\}$ be a quantile function of $\epsilon_{i,t}$ and assume the median is zero. Then the following model that is typical for finance

$$r_{i,t} = \beta_i f_{1,t} + (\sigma_{i,t}^{\top} f_{2,t})\epsilon_{i,t}, \tag{15}$$

where $\sigma_{i,t}$ is time-varying volatility of an ith stock and $\sigma_{i,t} f_{2,t} > 0$ can be assumed to generate returns. When $f_{1,t}$ and $f_{2,t}$ do not share common elements, then

$$Q_{r_{i,t}}\Big[\tau\Big| f_t(\tau)\Big] = \beta_i f_{1,t} + \sigma_{i,t}^{\top} f_{2,t} Q_{\epsilon_{i,t}}(\tau) \tag{16}$$

for $\tau \neq 0.5$ and $Q_{r_{i,t}}\Big[\tau\Big| f_t(\tau)\Big] = \beta_i f_{1,t}$ for $\tau = 0.5$. Note that here loadings on the factor are the only quantile-dependent objects and structure in the mean and volatility describes well the structure in quantiles. While this is already restrictive example that operates with the assumption on first two moments, even in such case standard PCA will not provide consistent estimates if the distribution of $\epsilon_{i,t}$ is heavy-tailed (Chen et al., 2021).

But what if the data follows more complicated models than the one implied by location-shift models? Consider adding asymmetric dependence such as

$$r_{i,t} = \beta_i f_{1,t} + f_{2,t}\epsilon_{i,t} + f_{3,t}\epsilon_{i,t}^3, \tag{17}$$

where $\epsilon_{i,t}$ is standard normal random variable with cumulative distribution function $\Phi(.)$. The quantiles of the returns will then follow

$$Q_{r_{i,t}}\Big[\tau\Big| f_t(\tau)\Big] = \beta_i f_{1,t} + \Phi^{-1}(\tau)\big[f_{2,t} + f_{3,t}\Phi^{-1}(\tau)^2\big], \tag{18}$$

for $\tau \neq 0.5$ and we can clearly see that second factor in $f(\tau) = [f_{1,t}, f_{2,t} + f_{3,t}\Phi^{-1}(\tau)^2]^{\top}$ is quantile dependent.

B Simulation Study

We present a simulation exercise to illustrate how the ΔCIQ premiums would look like if the driving force behind them were simply common volatility. We simulate stock returns from the following model

$$r_{i,t} = \alpha_i + \beta_i r_{m,t} + \gamma_i (V_t - \bar{V}) - \gamma_i \lambda^V + e_{i,t} \tag{19}$$

where V_t is the common variance factor, and the variance of the idiosyncratic error follows the factor structure proposed by Ding et al. (2025)

$$\begin{aligned} e_{i,t} &= \sqrt{V_{i,t}} z_{i,t}, \\ V_{i,t} &= V_t \exp(\mu_i + \sigma_i u_{i,t}) = V_t \tilde{V}_{i,t}, \\ z_{i,t},\, u_{i,t} &\sim i.i.d.\, N(0,1). \end{aligned} \tag{20}$$

Time-series variation of the returns drive two common factors – market factor, $r_{m,t}$, and variance factor V_t. The expected return of a stock is then equal to

$$\mathbb{E}[r_i] = \alpha_i + \beta_i \mathbb{E}[r_m] + \gamma_i \lambda^V. \tag{21}$$

We assume that the market factor follows a simple GARCH(1,1) process of Bollerslev (1986), which we fit on the market return from the empirical analysis. We assume that the log of the variance factor follows a modified HAR model of Corsi (2009)

$$\begin{aligned} \log V_{t+1} &= \theta_0 + \theta_m x_t^m + \theta_y x_t^y + v_{t+1} \\ v_{t+1} &\sim i.i.d.\, N(0, \sigma_v^2) \end{aligned} \tag{22}$$

where x_t^m and x_t^y are the previous month's log-variance and average log-variance over the last 12-month period, respectively. The common variance process is approximated by the cross-sectional average of the squared residuals from the time series regression of stock returns on the market factor. We fit the model from equation 22 on this time series. When simulating this time series, we initialise the process by randomly selecting 12 consequent observations of the common variance process estimated from the data and using those observations for iterating forward.

We calibrate the simulation setting to match the CRSP data sample we employ in the empirical investigation. We estimate stock-level market beta, β_i, using time-series regression of stock return on the market return. Exposure to the common variance, γ_i, is estimated by regressing the stock return on the estimate of the common variance process. Price of risk associated with the variance exposure, λ^V is chosen to be equal to 3×10^{-3}.[28] We estimate stock-level parameters of the idiosyncratic error variance–μ_i, σ_i–as the sample mean and standard deviation of $\log \tilde{V}_{i,t}$. To approximate the $\tilde{V}_{i,t}$, we use squared residuals from the time-series regression of the stock return on the market return. Then, to simulate these parameters, we approximate their distribution by normal distribution, with the mean equal to the estimates' cross-sectional average and the variance equal to the cross-sectional variance of the estimates.

We simulate a panel of 2,500 stocks with 120 observations. We repeat this simulation 1,000 times. Each time, we simulate stock returns by randomly choosing parameters for the stock-level process from the normal distribution with mean and variance corresponding to their sample counterparts. We remove the common time variation in stock returns by first forming the common linear factor

$$f_t = \frac{1}{N} \sum_{i=1}^{N} r_{i,t}, \quad t = 1, \ldots, T \tag{23}$$

and then regressing the returns on this factor

$$r_{i,t} = \alpha_i + \hat{\beta}_i f_t + \hat{e}_{i,t}, \tag{24}$$

which yields the residuals $\hat{e}_{i,t}$. Those residuals are then used to form the common volatility and quantile factors. We construct the volatility factor as the first principal component of those squared residuals. ΔCIQ(τ) factors are estimated as discussed in Section 2. Exposures to those factors are then estimated using univariate time-series regressions of stock returns on the increments of the volatility or quantile factors, respectively.

Similarly, as in the empirical investigation, we sort stocks into decile portfolios based on their estimated exposure to the factors to infer the associated risk premiums. We proxy the premiums by computing high-minus-low returns of the portfolios. Table 10 reports the average premiums for the three ΔCIQ factors that we investigate in the empirical analysis. We observe that the premium is positive for the lower-tail quantile factor, negative for the upper-tail factor and close to zero for the central factor. The magnitude of the premiums are comparable across lower and upper tail factors and in absolute value approximately equal to 9.4%. The premium associated with the exposure to the ΔPCA-SQ factor is -6.09%. We also compute associated t-statistics as a ratio between average premium and

[28] This value corresponds to approximately 6% annual high minus low premium obtained from ten portfolios portfolios sorted on the exposure to the common variance. The choice of this value is not essential for the results that we present here.

Table 10: Simulated Risk Premiums

The table contains risk average premiums computed from high-minus-low returns of decile portfolios sorted on exposure to the CIQ(τ) risks. We simulate the returns using common variance factor model proposed by Ding et al. (2025). We simulate panel of 2,500 stocks with 120 monthly observations. We perform the simulation 1,000 times. t-statistics are obtained by dividing the average premium by its standard deviation. We also report proportion of rejections of non-significance of ΔCIQ betas from multivariate cross-sectional regressions of average returns on those betas and market betas.

τ	Premium	t-stat	Rejections
ΔCIQ_{LT}	9.37	2.44	0.96
ΔCIQ_{C}	0.26	0.03	0.96
ΔCIQ_{UT}	-9.47	-2.56	0.96

its standard deviation across all the simulations. Premiums for the lower and upper tail factors are significant, unlike the value for the central factor, with t-statistics of around 2.6 in absolute value. The t-value associated with the ΔPCA-SQ factor is -2.33, so the premium estimated using this approach is also significant. Next, we present the proportion of rejections of non-significance of the ΔCIQ betas at a 5% significance level from multivariate cross-sectional regressions of average returns on those betas and market betas. We can see that the proportions are virtually identical for both lower- and upper-tail betas of around 96%. The ratio for the ΔPCA-SQ betas is 90%.

As we can see from the results, if there was a simple common volatility element present in the returns, which is compensated in the cross-section, the ΔCIQ risk premium would be symmetrical for lower and upper tail quantile factors. Moreover, the exposure to the ΔPCA-SQ factor would be priced in this case, too. Overall, the evidence from the simulation exercise suggests that the ΔCIQ risk premiums we observe in the data are not attributable to the common volatility compensation.

C Cross-Sectional Quantiles

Table 11: Quantiles of the Cross-Sectional Stock Returns

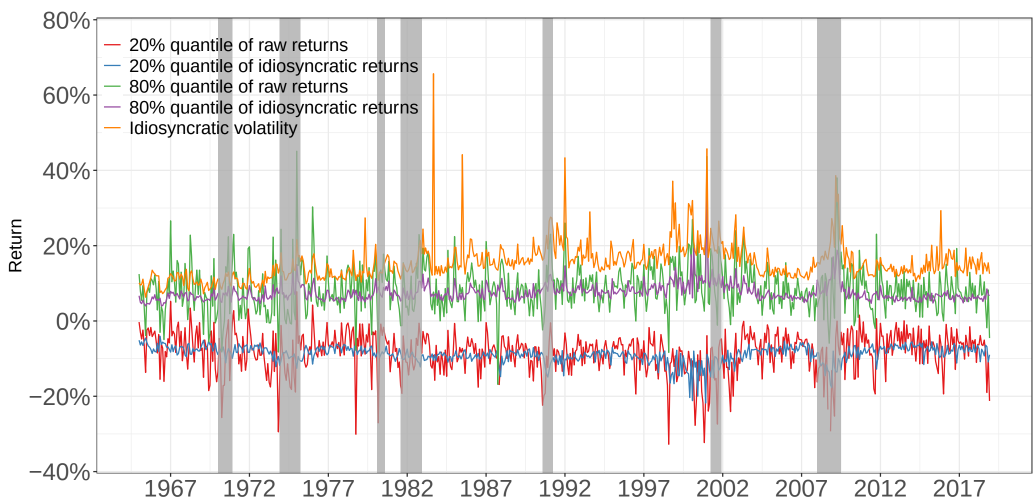
The table provides a summary of cross-sectional dispersion of excess stock returns. Each month, we compute cross-sectional quantile of raw or idiosyncratic excess stock returns with respect to market factor (CAPM), three factors of Fama and French (1993), five factors of Fama and French (2015) (FF5) or six factors of Fama and French (2018) (FF6). We report results for 20% (lower-tail) and 80% (upper-tail) quantiles. In Panel A, we report variance ratios between time-series of these quantile series. Rows correspond to the quantile series in the nominator, while columns corresponds to the quantile series in the denominator. In Panel B, we report regression results from regressing FF3 idiosyncratic quantiles on intercept, idiosyncratic volatility and CIQ (either lower- or upper-tail) factor. The data come from the CRSP database and cover the period from January 1963 to December 2018.

Panel A: Variance ratios	Lower-Tail				Upper-Tail			
	CAPM	FF3	FF5	FF6	CAPM	FF3	FF5	FF6
Raw	0.31				0.33			
CAPM		0.47				0.39		
FF3			0.90				0.97	
FF5				0.95				0.81

Panel B: Regressions	Lower-Tail			Upper-Tail		
	(1)	(2)	(3)	(4)	(5)	(6)
Intercept	-0.09	-0.05	0.00	0.08	0.03	0.01
	(-101.89)	(-22.68)	(-1.06)	(83.60)	(14.32)	(3.17)
Volatility		-0.24			0.29	
		(-17.39)			(21.60)	
CIQ factor			0.08			0.07
			(24.92)			(23.46)
adj. R^2	0.00	0.32	0.49	0.00	0.42	0.46

Figure 3: Cross-Sectional Dispersion of Stock Returns

The figure shows various measures of cross-sectional stock returns dispersion through time. For each month, we plot 20% and 80% cross-sectional quantiles of either excess or idiosyncratic excess returns. The idiosyncratic returns are computed with respect to the three-factor model of Fama and French (1993). The data come from the CRSP database and cover the period from January 1963 to December 2018. The shaded areas represent NBER recessions.



D Risk Measures Definitions

This Appendix provides a brief exposition of the estimation process of each of the control risk measures employed in the main text. For further details regarding the nuances of the related computations, consult the original papers.

We use two sources of data to compute these measures. First, we use either daily or monthly data of stock returns from the CRSP database. Second, we utilise the value-weighted return of the CRSP stocks from Kenneth French's online library to approximate the overall market return.

Variables are estimated using moving windows of various lengths following the procedures proposed in their original papers. In the case of measures estimated from the daily stock returns, we use mostly a moving window of one year. We require at least 200 daily observations during the window to be included. If we estimate a measure based on monthly return data we use a window of at least 60 months and demand at least 36 monthly observations.

The measures are estimated following the definition proposed in the literature. In some cases, we slightly change the requirements regarding the minimal history of stocks to be included in the analysis. This modification aims at the precision of the estimates as well as the broadest possible dataset.

Throughout this section, we use $r_{i,t}$ ($r^e_{i,t}$) to denote a raw (excess) return of an asset i at time t. The raw (excess) market return is denoted by f_t (f^e_t). Corresponding variables with a bar denote their time-series averages computed in a given window.

D.1 Market Beta

Market beta is estimated using daily data over the previous year for stocks that possess at least 200 observations as

$$\beta_i^{CAPM} = \frac{\sum_t (r^e_{i,t} - \bar{r}^e_i)(f^e_t - \bar{f}^e)}{\sum_t (f^e_t - \bar{f}^e)^2}. \tag{25}$$

D.2 Idiosyncratic Volatility

Following Ang et al. (2006a), idiosyncratic volatility is estimated using daily data over the previous month relative to the model of Fama and French (1993)

$$r^e_{i,t} = \alpha_i + \beta_i^{MKT} f^e_t + \beta_i^{SMB} SMB_t + \beta_i^{HML} HML_t + e_{i,t} \tag{26}$$

and taking standard deviation of the estimated residuals, $IVOL_i = \sqrt{var(e_i)}$.

D.3 Total and Idiosyncratic Skewness

Following Langlois (2020), we estimate total skewness as mean of the cubed standardised daily returns $r^e_{i,t}$, and idiosyncratic skewness from the model

$$r^e_{i,t} = \alpha_i + \beta_{1,i} f^e_t + \beta_{2,i} (f^e_t)^2 + e_{i,t} \tag{27}$$

and taking mean of the cubed standardised residuals $e_{i,t}$. We estimate these quantities using one year of data and requiring at least 200 observations.

D.4 Co-skewness

Co-skewness of Harvey and Siddique (2000) is estimated using daily excess returns and is defined as

$$CSK_i = \frac{\frac{1}{T}\sum_{t=1}^{T}(r^e_{i,t} - \bar{r}^e_i)(f^e_t - \bar{f}^e)^2}{\sqrt{\frac{1}{T}\sum_{t=1}^{T}(r^e_{i,t} - \bar{r}^e_i)^2}\,\frac{1}{T}\sum_{t=1}^{T}(f^e_t - \bar{f}^e)^2}. \tag{28}$$

Estimation window is set to one year, at least 200 daily observations are required.

D.5 Co-kurtosis

Co-kurtosis of Dittmar (2002) is estimated using daily data and is defined as

$$CKT_i = \frac{\frac{1}{T}\sum_{t=1}^{T}(r^e_{i,t} - \bar{r}^e_i)(f^e_t - \bar{f}^e)^3}{\sqrt{\frac{1}{T}\sum_{t=1}^{T}(r^e_{i,t} - \bar{r}^e_i)^2}\,\frac{1}{T}\left(\sum_{t=1}^{T}(f^e_t - \bar{f}^e)^2\right)^{3/2}}. \tag{29}$$

Estimation window is set to 1 year, at least 200 daily observations are required.

D.6 PCA-SQ Betas

Both the factor and the exposures are estimated using 60-month moving window of monthly data similarly as in the case of the ΔCIQ factors. Exposure is estimated from time series regression of regressing excess stock returns on a constant and ΔPCA-SQ factor. We require at least 48 observations during the estimation period.

D.7 CIV Beta

CIV beta is estimated by regressing monthly excess stock returns on a constant and increments of the CIV factor of Herskovic et al. (2016). We use 60-month rolling window and require at least 48 observations.

D.8 VIX Beta

VIX beta is estimated following Ang et al. (2006) using daily data over the previous month by regressing stock returns on a constant, market factor and increments of the CBOE volatility index as

$$r^e_{i,t} = \alpha_i + \beta_i^{MKT} f^e_t + \beta_i^{VIX} \Delta VIX_t + e_{i,t}. \tag{30}$$

We require at least 17 observations during the estimation month.

D.9 Downside Beta

Downside beta of Ang et al. (2006) is estimated using daily data and is defined as

$$\beta_i^{down} = \frac{\sum\limits_{f^e_t < \bar{f}^e} (r^e_{i,t} - \bar{r}^e_i)(f^e_t - \bar{f}^e)}{\sum\limits_{f^e_t < \bar{f}^e} (f^e_t - \bar{f}^e)^2}. \tag{31}$$

Estimation window is set to one year, at least 200 daily observations are required.

D.10 Tail Risk Beta

Tail risk beta of Kelly and Jiang (2014) is estimated using monthly return data using 120-month window with requirement of at least 36 monthly observations. Following the original setting, we require stocks to have price higher than \$5. Beta is computed by means of least-square estimator from the predictive regression of the form

$$r_{i,t+1} = \mu_i + \beta_i^{tail} \lambda_t + \epsilon_{t+1,i} \tag{32}$$

where the tail risk factor is obtained as

$$\lambda_t = \frac{1}{K_t} \sum_{k=1}^{K_t} \ln \frac{e_{k,t}}{u_t} \tag{33}$$

where $e_{k,t}$ is the kth daily idiosyncratic return that falls below an extreme value threshold u_t during month t, and K_t is the total number of such exceedences within month t. Idiosyncratic return is computed relative to the three-factor model of Fama and French (1993), and the threshold value is taken to be 5% quantile from the monthly cross-section of daily returns.

D.11 Hybrid Tail Covariance Risk

Hybrid tail covariance risk of Bali et al. (2014) is estimated using daily data using 6-month window with at least 80 daily observations as

$$HTCR_i = \sum_{r_{i,t} < h_i} (r_{i,t} - h_i)(f_t - h_f) \tag{34}$$

where h_i and h_f are the 10% empirical quantiles of stock and market return, respectively.

D.12 Multivariate Crash Risk

Multivariate crash risk of Chabi-Yo et al. (2022) is estimated using daily data with 1-year window and minimum of 200 nonzero observations in the following steps. First, for each stock separately, using stock and N factor returns, we estimate $N+1$ GARCH(1,1) models of Bollerslev (1986) to obtain a series of conditional distribution functions $F_{i,t}(h) = \mathbb{P}_{t-1}[r_{i,t} \leq h]$ and use it to compute probability integral transforms as $\hat{u}_{i,t} = F_{i,t}(r_{i,t})$. Second, we estimate MCRASH as

$$\text{MCRASH}_{i,t} = \frac{\sum_t \mathbb{I}(\{\hat{u}_{1,t} \leq p\}) \cdot \mathbb{I}(\cup_{j=2}^{N+1}\{\hat{u}_{j,t} \leq p\})}{\sum_t \mathbb{I}(\cup_{j=2}^{N+1}\{\hat{u}_{j,t} \leq p\})} \tag{35}$$

where $\mathbb{I}$ denotes the indicator function and p is set to 0.05. We follow the baseline specification of Chabi-Yo et al. (2022) and use the five factors of Fama and French (2015), momentum factor of Carhart (1997) and betting-against-beta factor of Frazzini and Pedersen (2014).

D.13 Predicted Systematic Co-skewness

Predicted systematic co-skewness of Langlois (2020) is based on

$$Cos_{i,t} = \mathbb{C}ov_{t-1}\big(r_{i,t}, f_t^2\big), \tag{36}$$

then, each month we run the panel regression using all available stocks and history of data

$$F\big(Cos_{i,k-12\to k-1}\big) = \kappa + F\big(Y_{i,k-24\to k-13}\big)\theta + F\big(X_{i,k-13}\big)\phi + \epsilon_{i,k-12\to k-1} \tag{37}$$

where $Cos_{i,k-12\to k-1}$ is the co-skewness from Equation 36 computed using daily returns from month $k-12$ to month $k-1$, $Y_{i,k-24\to k-13}$ are risk measures (volatility, market beta, etc.) estimated using daily data from month $k-24$ to month $k-13$, and $X_{i,k-13}$ are characteristics (size, book-to-price, etc.) observed at the end of month $k-13$. The function $F(x_{i,t}) = \frac{Rank(x_{i,t})}{N_t+1}$ transforms the original variable into its normalised rank in the cross-section of variable x_t, which posses N_t observations.

The predicted systematic co-skewness for each stock is then obtained using the estimated coefficients of $\hat{\kappa}, \hat{\theta}, \hat{\phi}$ as

$$F\big(\widehat{Cos_{i,t\to t+11}}\big) = \hat{\kappa} + F\big(Y_{i,t-12\to t-1}\big)\hat{\theta} + F\big(X_{i,t-1}\big)\hat{\phi}. \tag{38}$$

The choice of risk measures and characteristics employed in the prediction of systematic skewness follows closely Langlois (2020).

E Summary of the CIQ Factors

Table 12: Summary of the ΔCIQ(τ) Factors

The table provides summary of the estimated ΔCIQ(τ) factors. In Panel A, we report descriptive statistics of the ΔCIQ(τ) factors including their means, standard deviations, skewness, kurtosis and autocorrelation coefficients of order between one and three. In Panel B, we report correlations between ΔCIQ(τ) factors. The data cover the period from January 1965 to December 2018.

	0.1	0.15	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.85	0.9
Panel A: Descriptive statistics											
Mean $\times 10^3$	-5.76	-4.40	-3.84	-4.04	-2.73	12.09	-38.26	-8.64	-6.70	-4.06	-4.87
St. Dev.	0.15	0.16	0.18	0.25	0.47	1.43	1.33	0.35	0.22	0.19	0.17
Skewness	-0.16	-0.08	-0.14	0.02	0.09	0.01	-0.02	-0.03	-0.12	-0.19	0.04
Kurtosis	4.38	4.43	4.79	5.75	5.86	5.51	5.56	7.02	7.17	7.32	6.72
AR(1)	-0.41	-0.40	-0.44	-0.46	-0.48	-0.36	-0.34	-0.48	-0.43	-0.41	-0.38
AR(2)	-0.10	-0.08	-0.05	0.01	0.04	0.05	-0.05	0.03	-0.05	-0.07	-0.11
AR(3)	0.15	0.11	0.06	0.01	-0.03	0.01	0.00	0.02	0.06	0.07	0.07
Panel B: Correlations											
0.1	1.00	0.96	0.93	0.84	0.66	0.09	0.15	0.15	-0.09	-0.20	-0.29
0.15		1.00	0.97	0.90	0.75	0.15	0.23	0.24	0.00	-0.11	-0.22
0.2			1.00	0.95	0.81	0.19	0.28	0.34	0.10	-0.02	-0.13
0.3				1.00	0.93	0.28	0.37	0.50	0.28	0.16	0.04
0.4					1.00	0.43	0.50	0.68	0.49	0.38	0.26
0.5						1.00	0.72	0.49	0.43	0.39	0.34
0.6							1.00	0.59	0.50	0.46	0.38
0.7								1.00	0.93	0.86	0.76
0.8									1.00	0.97	0.91
0.85										1.00	0.95
0.9											1.00

E.1 CIQ Shocks and the Real Economy

A large body of literature shows that firm-level shocks can be linked to individual households via channels such as wages, employment and wealth due to incomplete insurance (Harris and Holmstrom, 1982; Brown and Medoff, 1989; Berk et al., 2010; Heathcote et al., 2014; Herskovic et al., 2016). This suggests that common quantile factors (ΔCIQ) in firm returns may proxy for household risk. We test this hypothesis by examining the correlations between the innovations in the CIQ and the cross-sectional outcomes of firm employment growth, earnings growth (by place of work), house price growth and firm sales growth. For each outcome, we compute dispersion (interquartile range, etc.) and key percentiles (20th, 40th, 60th and 80th) to capture heterogeneous results.

Table 13 shows the correlations between the changes in CIQ and the changes in these distributional measures. The strongest link is shown by household earnings growth: a lower-tail ΔCIQ shock is strongly associated with declines in earnings growth, whereas an upper-tail shock coincides with higher earnings growth. Sales growth (Panel E) exhibits greater asymmetry: an upper-tail shock improves the performance of the weakest firms, thereby compressing the distribution, whereas a lower-tail shock hurts these firms, thereby widening the distribution. In contrast, a volatility shock (CIV) increases dispersion and depresses all sales growth percentiles. Employment growth (Panel A) and house prices (Panels C–D) exhibit weaker yet consistent patterns. A downside CIQ shock increases employment growth dispersion and exacerbates job losses at the lower end of the scale, whereas an upside shock mitigates the worst outcomes. House price growth only responds to downside CIQ shocks, indicating that housing wealth risk is mainly driven by common downturns. Overall, these results confirm that ΔCIQ risk is associated with fluctuations in household earnings, wealth and sales. The effects differ across quantiles, indicating heterogeneous effects that would be missed by focusing only on volatility (CIV). This highlights the heterogeneous transmission of risk from firms to households and implies that asset pricing models should incorporate quantile-dependent risk (e.g. via quantile-based preferences, as discussed in de Castro and Galvao (2019)) rather than relying solely on variance.

Table 13: Employment, earnings, and wealth (transposed)

This table presents the correlations between ΔCIQ risks and changes in cross-sectional measures of employment (Panel A), earnings (Panel B), house-price (Panels C and D) growth, and sales growth (Panel E). It reports dispersion measures (interquartile range, max–min, standard deviation), selected percentiles (20%, 40%, 60%, 80%), and the cross-sectional average.

	75% − 25%	max − min	**std**	20%	40%	60%	80%	**Avg.**
Panel A: Annual employment growth								
Lower-tail CIQ	0.46 **	0.33	0.27	-0.41 *	-0.61 ***	0.13	0.25	-0.21
Central CIQ	0.18	0.30	0.34	0.13	0.64 ***	0.35	0.49 **	0.38 *
Upper-tail CIQ	-0.38 *	-0.01	-0.18	0.56 ***	0.64 ***	0.25	0.05	0.49 **
CIV	-0.10	-0.01	-0.09	-0.27	-0.25	-0.52 ***	-0.47 **	-0.51 ***
Panel B: Annual earnings growth								
Lower-tail CIQ	-0.19	-0.42 *	-0.37	-0.38 *	-0.46 **	-0.50 ***	-0.56 ***	-0.50 ***
Central CIQ	-0.11	0.28	-0.04	0.41 *	0.45 **	0.46 **	0.40 *	0.41 *
Upper-tail CIQ	0.12	0.49 **	0.20	0.45 **	0.57 ***	0.62 ***	0.59 ***	0.49 **
CIV	0.02	-0.0	-0.15	-0.48 **	-0.54 ***	-0.54 ***	-0.47 **	-0.51 ***
Panel C: Monthly house-price growth (nine areas, seasonally adjusted)								
Lower-tail CIQ	-0.03	0.06	0.04	0.13 ***	0.06	0.08	0.13 ***	0.11 **
Central CIQ	-0.02	0.09	0.07	0.05	0.05	0.00	0.04	0.05
Upper-tail CIQ	-0.02	0.01	0.01	-0.01	0.00	-0.02	-0.00	0.00
CIV	0.03	-0.04	-0.02	-0.12 **	-0.12 **	-0.06	-0.12 **	-0.13 ***
Panel D: Quarterly house-price growth (100 largest MSAs, seasonally adjusted)								
Lower-tail CIQ	0.04	0.15	0.13	0.16 *	0.21 **	0.12	0.19 **	0.18 *
Central CIQ	-0.06	0.10	0.14	0.06	0.05	0.01	0.01	0.03
Upper-tail CIQ	-0.04	-0.07	-0.07	0.00	0.05	-0.03	0.01	0.05
CIV	-0.08	0.12	-0.01	-0.11	-0.10	-0.08	-0.14	-0.12
Panel E: Sales growth								
Lower-tail CIQ	-0.26 **	0.02	-0.15	0.13	0.06	-0.08	-0.12	0.05
Central CIQ	-0.02	0.05	0.04	-0.00	-0.02	-0.04	-0.01	-0.04
Upper-tail CIQ	-0.24 **	-0.10	-0.26 **	-0.03	-0.11	-0.25 **	-0.36 ***	-0.20 *
CIV	0.28 ***	-0.23 **	-0.03	-0.43 ***	-0.45 ***	-0.39 ***	-0.31 ***	-0.41 ***

F ΔCIQ Betas and Firm Characteristics

In this section, we aim to understand which firms are more susceptible to common idiosyncratic quantile movements. We examine the cross-sectional relationships between exposures to the ΔCIQ factors and firm characteristics. In doing so, we will be better equipped to understand the risk and potential of the highly exposed firms.

We use the set of characteristics from Freyberger et al. (2020) and Kim et al. (2020).[29] We use 60 of their characteristics, which we group into six categories: *Past returns*, *Investment*, *Profitability*, *Intangibles*, *Value* and *Frictions*. We list the characteristics in Table 14. Each month, we normalise the characteristics to the interval (-1,+1) based on their cross-sectional ranking in that month. The coefficients can then be compared on the basis of their magnitude to identify the variables that are most related to the CIQ risks. We also standardise the ΔCIQ betas for two reasons. First, we can compare the magnitude of the effects of the characteristics across betas for lower- and upper-tail factors. Second, it is natural to look at the ranks rather than the levels of the ΔCIQ betas, since the ranks are what matters the most in cross-sectional pricing.

We then apply the Fama-MacBeth approach and each month regress the cross-section of the ΔCIQ betas on the set of characteristics that could be used to predict the next month's return. The estimated coefficients and t-statistics are then obtained from the time series of the estimated coefficients and by applying the correction of Newey and West (1994).

We summarise the results in Table 15, from which we can accurately identify the most informative characteristics. This table ranks the characteristics from the most important to the least based on the absolute value of the coefficient estimated for the lower-tail exposure. A number of observations can be made. First, we can see that both lower- and upper-tail exposures are best explained by the market beta (`beta`). lower-tail exposures are negatively related to the market beta, which suggests that stocks exposed to the lower-tail common idiosyncratic events have relatively low market risk. On the other hand, stocks that highly covaries with the upside common movements have higher market beta. This suggests that market beta is positively related to the common idiosyncratic upside potential and negatively related to the common idiosyncratic downside risk.

Second, the relationship between the characteristics and ΔCIQ betas is asymmetric across lower and upper tail–some of the characteristics are more important for the lower-tail and some for upper-tail ΔCIQ exposures. For example, the category of *Frictions* is highly related to the lower-tail CIQ, which, except in the case of the `beta` and `spread_mean`, is not as pronounced for the upper-tail ΔCIQ betas. On the other hand, unlike in the case of lower-

[29] We are grateful to Andreas Neuhierl for sharing the data with us.

Table 14: Firm Characteristics

The table provides a list of firm characteristics of Freyberger et al. (2020) and Kim et al. (2020). We employ them to explain the firm-level exposures to the ΔCIQ factors.

Frictions		
(1)	at	Total assets
(2)	beta	Beta
(3)	beta_daily	CAPM beta using daily returns
(4)	dto	De-trended Turnover - market Turnover
(5)	idio_vol	Idio vol of Fama-French 3 factor model
(6)	lme	Price times shares outstanding
(7)	lme_adj	Size - mean size in Fama-French 48 industry
(8)	lturnover	Last month's volume to shares outstanding
(9)	rel_to_high_price	Price to 52 week high price
(10)	ret_max	Maximum daily return
(11)	spread_mean	Average daily bid-ask spread
(12)	std_turn	Standard deviation of daily turnover
(13)	std_volume	Standard deviation of daily volume
(14)	suv	Standard unexplained volume
(15)	total_vol	Standard deviation of daily returns
Intangibles		
(16)	aoa	Absolute value of operating accruals
(17)	ol	Costs of goods sold + SG&A to total assets
(18)	tan	Tangibility
(19)	oa	Operating accruals
Investment		
(20)	investment	% change in AT
(21)	d_ceq	% change in BE
(22)	dpi2a	Change in PP&E and inventory over lagged AT
(23)	ivc	Change in inventory over average AT
(24)	noa	Net-operating assets over lagged AT
Past Returns		
(25)	cum_return_1_0	Return 1 month before prediction
(26)	cum_return_6_2	Return from 6 to 2 months before prediction
(27)	cum_return_12_7	Return from 12 to 7 months before prediction
(28)	cum_return_12_2	Return from 12 to 2 months before prediction
(29)	cum_return_36_13	Return from 36 to 13 months before prediction
Profitability		
(30)	ato	Sales to lagged net operating assets
(31)	cto	Sales to lagged total assets
(32)	d_dgm_dsales	% change in gross margin and sales
(33)	eps	Earnings per share
(34)	ipm	Pretax income over sales
(35)	pcm	Sales minus costs of goods sold to sales
(36)	pm	OI after depreciation over sales
(37)	pm_adj	Profit margin - mean PM in Fama-French 48 industry
(38)	prof	Gross profitability over BE
(39)	roa	Return on assets
(40)	roc	Size + long-term debt - total assets to cash
(41)	roe	Income before extraordinary items to lagged BE
(42)	roic	Return on invested capital
(43)	s2c	Sales to cash
(44)	sat	Sales to total assets
(45)	at_adj	SAT - mean SAT in Fama-French 48 industry
Value		
(46)	a2me	Total assets to Size
(47)	beme	Book to market ratio
(48)	beme_adj	BEME - mean BEME in Fama-French 48 industry
(49)	c	Cash to AT
(50)	c2d	Cash flow to total liabilities
(51)	d_so	Log change in split-adjusted shares outstanding
(52)	debt2p	Total debt to Size
(53)	e2p	Income before extraordinary items to Size
(54)	free_cf	Free cash flow to BE
(55)	ldp	Trailing 12-months dividends to price
(56)	nop	Net payouts to Size
(57)	o2p	Operating payouts to market cap
(58)	q	Tobin's Q
(59)	s2p	Sales to price
(60)	sales_g	Sales growth

tail betas, upper-tail betas are related to the sum of cost of goods sold and selling, general and administrative expenses over total assets (`ol`). In addition, various measures of value are more strongly related to the upside movements than to the downside movements.

Third, we observe that from the top ten most important characteristics the lower-tail exposures are negatively related to size (`lme`), last month's volume relative to shares out-

Table 15: Explanatory Firm Characteristics

The table reports regression results of Fama-French cross-sectional regressions of ΔCIQ exposures on various firm characteristics. Each month, we regress ΔCIQ exposures on the set of all characteristics. Final coefficients are computed as time-series averages of monthly estimated values, corresponding t-statistics are based on robust standard errors of Newey and West (1994). Both ΔCIQ exposures and characteristics are each month cross-sectionally ranked and standardised into interval (-1,1). The data on firm characteristics comes from Freyberger et al. (2020) and Kim et al. (2020). The data cover the period between January 1968 and December 2018.

		Lower-Tail exposure			Upper-Tail exposure		
Characteristic	Category	rank	Coefficient	t-stat	Coefficient	t-stat	rank
beta	Frictions	1	-24.82***	-7.82	20.24***	10.22	1
std_volume	Frictions	2	8.93***	6.76	-2.23	-1.47	16
sat	Profitability	3	6.80*	1.75	-7.50**	-2.33	4
lme	Frictions	4	-5.44***	-3.06	-2.06	-0.90	18
beme	Value	5	4.41**	2.06	10.00***	6.09	3
lturnover	Frictions	6	-4.27***	-4.30	-1.04	-1.03	27
spread_mean	Frictions	7	3.77***	4.71	-5.04***	-6.87	7
a2me	Value	8	3.68**	2.16	4.14**	2.40	8
std_turn	Frictions	9	-3.65***	-3.54	2.40**	2.27	15
e2p	Value	10	-3.04***	-3.19	3.20***	4.18	11
eps	Profitability	11	2.94***	2.66	-4.00***	-4.01	9
q	Value	12	2.87	1.61	5.95***	4.12	5
roe	Profitability	13	-2.83***	-2.78	2.58**	1.97	14
cto	Profitability	14	-2.78**	-2.13	0.38	0.25	46
pm_adj	Profitability	15	-2.56***	-4.87	-1.10*	-1.78	26
total_vol	Frictions	16	2.42**	2.14	-1.78	-1.40	20
ipm	Profitability	17	2.35**	2.40	-0.19	-0.19	54
ldp	Value	18	2.06*	1.84	-1.01	-1.56	28
prof	Profitability	19	1.92**	2.22	0.84	0.68	33
cum_return_36_13	Past returns	20	-1.89*	-1.78	-0.00	-0.00	60
s2p	Value	21	-1.79	-1.49	-2.90	-1.59	13
tan	Intangibles	22	-1.75***	-4.66	-0.84	-1.49	32
noa	Invesment	23	-1.72***	-2.75	-0.51	-0.87	41
ol	Intangibles	24	-1.34	-0.39	12.46***	4.15	2
rel_to_high_price	Frictions	25	1.26***	3.53	2.14***	4.94	17
o2p	Value	26	1.18**	2.20	-0.37	-0.68	47
free_cf	Value	27	1.14**	2.16	1.16*	1.78	24
cum_return_12_7	Past returns	28	-1.12**	-2.03	0.48	0.88	42
pcm	Profitability	29	-1.09*	-1.82	-1.52**	-2.25	22
idio_vol	Frictions	30	-1.05	-0.89	0.16	0.13	55
at_adj	Profitability	31	-0.86**	-2.08	0.04	0.07	57
cum_return_6_2	Past returns	32	-0.86**	-2.13	0.62	1.24	37
pm	Profitability	33	-0.84	-0.76	5.44***	4.10	6
nop	Value	34	0.82	0.79	-1.01	-1.63	30
roic	Profitability	35	0.79	0.88	-0.90	-1.13	31
debt2p	Value	36	0.78	1.09	-0.23	-0.25	53
aoa	Intangibles	37	-0.76**	-2.41	-0.41	-1.24	44
c	Value	38	-0.69	-0.57	-0.82	-0.50	34
d_ceq	Invesment	39	-0.67	-1.45	-0.58	-1.03	38
at	Frictions	40	-0.61	-0.32	-3.71**	-2.22	10
ato	Profitability	41	0.55	0.87	0.07	0.11	56
dpi2a	Invesment	42	0.52	0.77	-0.44	-0.72	43
roa	Profitability	43	0.52	0.44	-0.24	-0.19	51
ret_max	Frictions	44	-0.44**	-1.97	-0.68***	-2.62	35
oa	Intangibles	45	-0.40	-1.12	0.40	1.15	45
cum_return_1_0	Past returns	46	-0.38	-1.43	-0.28	-0.95	49
beta_daily	Frictions	47	-0.38*	-1.71	0.54*	1.96	39
investment	Invesment	48	0.32	0.61	0.27	0.56	50
lme_adj	Frictions	49	0.29	0.49	1.71***	3.25	21
sales_g	Value	50	-0.21	-0.38	-0.54	-0.88	40
ivc	Invesment	51	0.17	0.37	-0.02	-0.05	58
d_dgm_dsales	Profitability	52	0.16	0.44	0.00	0.01	59
suv	Frictions	53	-0.16	-1.32	0.33**	2.39	48
cum_return_12_2	Past returns	54	0.13	0.26	-1.01*	-1.83	29
beme_adj	Value	55	0.12	0.12	-3.09**	-2.47	12
dto	Frictions	56	-0.12	-1.19	-0.23*	-1.90	52
roc	Profitability	57	0.07	0.13	-1.10	-1.50	25
d_so	Value	58	0.02	0.05	0.63*	1.77	36
s2c	Profitability	59	0.01	0.01	-1.87	-1.35	19
c2d	Value	60	-0.01	-0.01	-1.23	-1.62	23

standing (lturnover), standard deviation of daily turnover (std_turn) and income before extraordinary items to size (e2p). On the other hand, the standard deviation of daily volume (std_volume), the sales to total assets (sat), book to market ratio (beme), the average daily bid-ask spread (spread_mean) and total assets to size (a2me) are significantly positively related to the lower-tail common movements. These observations suggest that stocks that are less liquid and smaller are more exposed to the downside idiosyncratic movements. Thus, in turbulent times with high risk, stocks that are less tradable will be more affected because they cannot be sold as quickly.

There is also a clear pattern suggesting that high-value companies are positively related to the lower-tail ΔCIQ factor. We see that companies with high book-to-market and total assets-to-size ratios are also more exposed. Similarly, sales-to-total assets is positively related to the lower-tail idiosyncratic exposure.

We then take a closer look at the firm characteristics that explain the upside exposures. We see that these characteristics are not entirely the same as in the case of the lower tail and that they are more spread across categories than in the case of the lower-tail risk. The main characteristics are spread across characteristics from the *Value*, *Profitability* and *Friction* categories, with some being from the *Intangibles* as well. We see that, similar to the lower-tail factor, high-value companies are more exposed to the upper-tail factor than low-value companies. On the other hand, more liquid stocks and stocks with less volatile trading volumes are relatively more covaried with upside factor than the opposite firms.