Asymmetric Risks: Alphas or Betas?*

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November 7, 2023

Abstract

I propose a simple approach to combining systematic asymmetric risk measures and obtaining anomalous returns beyond premiums associated with each measure separately. I show that a multivariate regression setup that combines the asymmetric risk measures performs poorly. Instead, I use instrumented principal component analysis and construct portfolios that hedge exposures to the common sources of risk related to those measures. The resulting portfolios enjoy abnormal returns that any other factor model cannot fully explain. Allowing for time-variation of compensation for bearing asymmetric risks leads to a loss of efficiency, and the momentum factor spans the resulting portfolios. I also show that some asymmetric risk measures can significantly contribute to the performance of the model that assumes a linear factor structure.

Keywords: Cross-section of asset returns, factor structure, asymmetric risk, downside risk, instrumented principal component analysis
JEL: C23; G11; G12

*I appreciate helpful comments from participants at various seminars, workshops and conferences. Support from the Czech Science Foundation under the 19-28231X (EXPRO) project is gratefully acknowledged.

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1 Introduction

Non-linear systematic behavior of stock returns constitutes a fruitful research playground in the empirical asset pricing literature. Many statistical measures that capture the vital essence of those features were proposed to be significant predictors of the cross-section. All of them try to capture the natural human aversion to extreme adverse events that occur, especially during bad times. However, the definition of those extreme events and bad times generally differ across specifications. There is no theoretical answer to which specification is the right one. I propose an approach to combining these measures to form a portfolio that exploits related premiums.

I obtain the majority of the presented results by employing instrumented principal component analysis (IPCA) proposed by Kelly et al. (2019). Using the unrestricted version of their model, I can distill the non-linear features that cannot be captured by simple covariances with latent factors. Based on that information, I am able to form a portfolio that conditionally hedges the exposures to the latent factors related to the asymmetric risk measures. The resulting portfolio then yields an annualized Sharpe ratio up to 0.97.

Moreover, those abnormal returns cannot be fully explained by any other factor model, including IPCA factors estimated using the original dataset of 32 characteristics. On the other hand, the anomaly portfolio returns are generally exposed to the momentum factor. When I assume a constant relation between asymmetric risk measures and arbitrage portfolio formation, the abnormal returns are only partially diminished by accounting for this exposure. If I allow for time variation in the relation, the loss of efficiency leads to the anomaly returns being seized fully by the momentum.

I also investigate how the asymmetric risk measures align with the exposures to the common linear factors. To capture anomaly returns related to eleven such measures, a six-factor model that utilizes asymmetric risk measures to proxy for the exposures to those latent factors is needed. This observation indicates that there is little redundancy among these variables concerning asset prices. Moreover, a mean-variance efficient tangency portfolio with asymmetric risks explaining the factor loadings can yield a Sharpe ratio of around 1.15.

When I evaluate the asymmetric risk measures in a controlled environment of 32 characteristics from Kelly et al. (2019), I show that three measures significantly affect the fit of the latent factor model: downside beta, hybrid tail covariance risk, and negative semibeta. Moreover, if evaluated jointly, asymmetric risk measures yield mildly significant values of around 7%.

The presented analysis is related to a few streams of the literature. The first deals with the emergence of so-called factor zoo–many factors that are supposed to price the cross-section of stock returns. Still, to this date, there is not a clear consensus on how researchers should feel about this claim. Some results suggest that a substantial portion of the factors is proxies for underlying common risks, and by including them, we can average out the noise related to each of them and identify the driving force behind the formation of expected returns (Kozak et al., 2020).

Another ongoing discussion in empirical asset pricing regards characteristics vs. covariances. A risk-based explanation of expected returns claims that only exposures to common movements should constitute price determinants for the cross-section of asset returns. If a characteristic predicts future returns, it should be because this characteristic is a good proxy of systematic risk exposure. Similarly, as in the factor zoo discussion, there is still no obvious conclusion. Some results claim that we can form an arbitrage portfolio that enjoys abnormal returns without exposure to systematic risk (Kim et al., 2020; Lopez-Lira and Roussanov, 2020), while others suggest that exposures capture all the essential pricing information (Kelly et al., 2019, 2023). Moreover, those exposures to the common fluctuations should be fully described by the betas, which are based on a simple covariance measure of dependence.

Using innovative techniques, much of the progress was made in recent years in both strands of the literature separately but also simultaneously. Unfortunately, those research efforts usually focus on various accounting variables and characteristics based on simple market frictions, only and neglect various measures of non-linear systematic dependence between stock and various factors. Those measures capture the joint behavior of stock and factor during extreme market events, which are not detected by usual regression coefficients (betas) obtained from regressing stock returns on some tradable factors. I utilize two types of asymmetric risk measures here. The first one captures systematic exposure using an asymmetric non-linear type of dependence, such as the downside beta of Ang et al. (2006). The second one is defined by utilizing an asymmetric non-linear type of aggregate risk factor, such as the tail risk beta of Kelly and Jiang (2014).

Studies usually avoid this type of market friction characteristic probably because of slightly higher difficulties related to their estimation in comparison to accounting variables. I argue that these kinds of risks are interesting in the investigation of the factor structure of asset return because they possess a special place among characteristics. They capture the joint behavior of stock return and some aggregate measure of risk (e.g., return on the whole market), but the measured dependence goes beyond linearity and symmetry of standard covariances with tradable factors. Consequently, it is difficult to decide how much of the risk premium associated with the asymmetric risk measures is due to the non-linear nature of the dependence and how much is attributable to the overall covariance-based dependence.

I aim to fill this void by entertaining a representative set of systematic asymmetric risk measures and employing them in their multivariate context. I use the asymmetric risk measures to proxy for the exposure to the common linear factors and form a portfolio that hedges those exposures. Moreover, I evaluate the additional information they possess for asset prices when controlling for conventional characteristics previously used in similar studies.

1.1 Theoretical Motivation

The empirical research, centered around the expected utility assumption, focuses on the implementation of the equation

$$\mathbb{E}_t[m_{t+1} r_{i,t+1}] = 0, \tag{1}$$

which can be interpreted in terms of (co)variances as

$$\mathbb{E}_t[r_{i,t+1}] = \underbrace{\frac{\mathbb{C}ov_t(m_{t+1}, r_{i,t+1})}{\mathbb{V}ar_t(m_{t+1})}}_{\beta^m_{i,t}} \underbrace{\left(-\frac{\mathbb{V}ar_t(m_{t+1})}{\mathbb{E}_t[m_{t+1}]}\right)}_{\lambda_t}. \tag{2}$$

This statement implies that the priced exposure to the risk is adequately measured by the regression coefficient, $\beta^m_{i,t}$, obtained from regressing excess stock return on the stochastic discount factor, m_{t+1}. Further, if we assume linearity of the discount factor in some set of factors f, which proxy for the growth of marginal substitution, i.e., $m_{t+1} = \delta \frac{u'(c_{t+1})}{u'(c_t)} \approx a + b' f_{t+1}$, this leads to

$$\mathbb{E}_t[r_{i,t+1}] = \alpha_{i,t} + \lambda' \beta_{i,t} \tag{3}$$

$$r_{i,t+1} = \alpha_{i,t} + \beta'_{i,t} f_{t+1} + \epsilon_{i,t+1} \tag{4}$$

where $\beta_{i,t}$ are the multiple regression coefficients of $r_{i,t}$ on f_t, and λ is vector of risk prices associated with factors f. In the case of tradable factors, λ is equal to the expected value of f. This line of reasoning constitutes a base for the empirical factor literature such as the arbitrage pricing theory of Ross (1976), the three-factor model of Fama and French (1993), etc. One of the main implications of the theory is that the non-systematic part of the risk, $\alpha_{i,t}$, should be equal to zero. Statistical tests such as Gibbons et al. (1989) provide inference on goodness of fit by testing this restriction.

On the other hand, there are models that deviate from the expected utility framework and/or linearity assumption of the stochastic discount factor. Examples of the former are models that introduce some form of behavioral bias, such as the disappointment aversion utility of Gul (1991). Based on that framework, Ang et al. (2006) introduced a cross-sectional relation between expected returns and downside beta, dependence between market and stock return conditional on the market being below its mean. A pioneer of the later violation is the work of Harvey and Siddique (2000), which assumes that the stochastic discount factor is *quadratic* in the market return, which introduces conditional systematic skewness as a priced risk characteristic. More recently, based on the recursive utility with disappointment aversion of Routledge and Zin (2010), Farago and Tédongap (2018) argue that betas with various asymmetric specifications of market return and volatility should be significantly priced in the cross-section.

Based on those arguments, risk exposure cannot be sufficiently captured by the simple betas with tradable factors. The cross-sectional relation between stock returns and risk changes to

$$\mathbb{E}_t[r_{i,t+1}] = \delta' g(r_{i,t+1}, f^*_{t+1}) + \lambda' \beta_{i,t} \tag{5}$$

$$r_{i,t+1} = \delta' g(r_{i,t+1}, f^*_{t+1}) + \beta'_{i,t} f_{t+1} + \epsilon_{i,t+1} \tag{6}$$

where g is a function of asset return and some factor–*asymmetric risk measure* (ARM) with a vector of related prices of risk δ. The uniqueness of the ARM can lie either in the choice of the dependence function g or in the choice of the factor f^*. We can see that these specifications lead to the rejection of the non-significant alpha assumption from above.

In recent years, researchers proposed many asymmetric risk measures to possess the ability to explain and predict stock returns. Those studies usually control for some pre-specified set of factors and conclude that abnormal returns cannot be explained by exposure to those factors. Because the choice of the factors will always be somewhat arbitrary, I want to entertain the question of whether there is any set of factors that can eliminate significant alphas related to the asymmetric risk measures. I investigate this question in their multivariate setting using a representative set of eleven asymmetric risk measures.

I construct portfolios that hedge against exposures to one to five common factors linked to asymmetric risk measures, and demonstrate that the corresponding risk-adjusted premiums sustain their significance. Additionally, I examine the factor structure related to asymmetric risk measures. I further assess the asymmetric risk measure with regards to characteristics that act as reliable proxies for common exposures.

The rest of the paper is structured as follows. Section 2 introduces data and asymmetric risk measures that I use in the further analysis. Section 3 investigates the arbitrage returns related to the asymmetric risk measures. Section 5 describes their factor structure. And finally, 6 concludes the whole investigation.

2 Asymmetric Risk Measures

In this section, I provide a first look at the asymmetric risk measures that are employed in the main analysis. I show that they possess a sizable variation of significance of the related anomaly premiums based on the research setting, in which you estimate them. This observation supports the intention to evaluate the asymmetric risk measures jointly to extract the important component for the asset prices.

2.1 Data

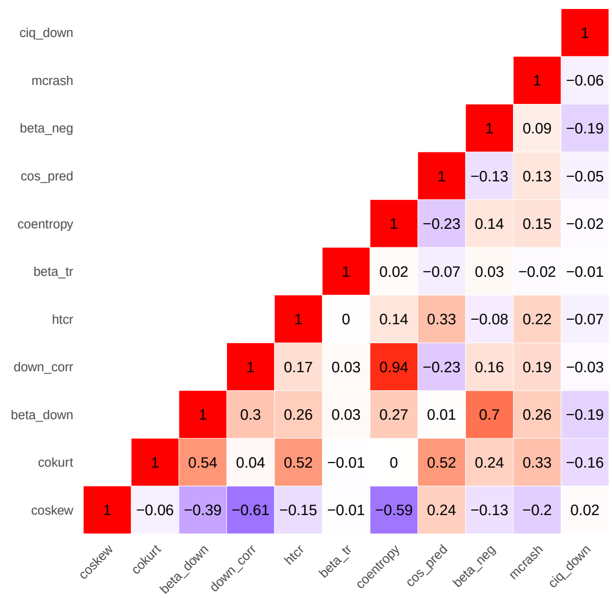
In the empirical investigation, I employ a representative set of eleven asymmetric risk measures. Those measures are coskewness (`coskew`) of Harvey and Siddique (2000), cokurtosis (`cokurt`) of Dittmar (2002), downside beta (`beta_down`) of Ang et al. (2006), downside correlation (`down_corr`) based on Hong et al. (2006) and Jiang et al. (2018), hybrid tail covariance risk (`htcr`) of Bali et al. (2014), tail risk beta (`beta_tr`) of Kelly and Jiang (2014), exceedance coentropy measure (`coentropy`) based on Backus et al. (2018) and Jiang et al. (2018), predicted systematic coskewness (`cos_pred`) of Langlois (2020), negative semibeta (`beta_neg`) of Bollerslev et al. (2021), multivariate crash risk (`mcrash`) of Chabi-Yo et al. (2022), and downside common idiosyncratic quantile risk (CIQ) beta (`ciq_down`) of Barunik and Nevrla (2022). The choice of the variables corresponds to the fact that they capture different aspects of the return dependence in terms of non-linearity and/or asymmetry. I provide an overview how the measures are estimated in Appendix A. I estimate those measures using either daily or monthly return data from the CRSP database that starts in January 1963 and ends in December 2018.

In the further analysis, I also use a set of 32 characteristics from Freyberger et al. (2020). Those characteristics are intersection between data used in Freyberger et al. (2020) and Kelly et al. (2019). Those characteristics are used to estimate baseline specification of the model of Kelly et al. (2019). Moreover, I include only observations that posses information on all the characteristics. I use an initial window of 5 years to estimate the ARMs; because of that, the first prediction period constitutes January 1968 in case of in-sample analysis. When performing out-of-sample exercises, I set the initial estimation period to be 60 month, so the out-of-sample prediction starts in January 1973.

I merge the dataset of ARMs with the characteristics dataset to work with a stock universe that is fully transparent for investors and eligible for trading based on wide variety of strategies. The full merged dataset then contains 1,519,754 stock-month observations of 12,505 unique stocks.

In the initial investigation of the ARMs, I also employ dataset that strips down penny

Figure 1: *Correlation structure across ARMs.* The figure captures time-series averages of cross-sectional correlations between asymmetric risk measures. Data include the period between January 1968 and December 2018.



stocks, which I define as stocks with a price less than $5 or with capitalization below 10% quantile of the NYSE-traded stocks each month. The dataset that excludes penny stocks yields 947,897 stock-month observations of 8,477 unique stocks.

2.2 Correlation Structure

First, to gain some intuition regarding common variation of the ARMs, I investigate their correlation structure. Figure 1 contains correlations between ARMs themselves. Correlations are obtained as time-series averages of the cross-sectional correlations. We can see that the highest absolute values of correlations are between coentropy and downside correlation with value of 0.94, downside beta and negative semibeta with value of 0.70, and coskewness and downside correlation with value of -0.61.

Panel A of Table 1 summarizes how each measure is in general related to the others by reporting average absolute correlations across all measures. We observe that the downside beta possess the highest level of similarity with other measures with the average absolute correlation being equal to 0.29. On the other hand, the least correlated measure is tail risk beta with the average value of only 0.02.

Table 1: *Average correlations of ARMs.* Panel A of the table reports time-series averages of cross-sectional correlations for each ARM averaged across all other ARMs or 32 characteristics employed in Kelly et al. (2019). Panel B reports average correlations between managed portfolios. The average correlation for each ARM is obtained by averaging correlations across all other ARM portfolios or by averaging across 32 characteristic managed portfolios. Data cover the period between January 1968 and December 2018.

	Variables		*Managed portfolios*	
	with ARMs	with others	with ARMs	with others
`coskew`	0.24	0.02	0.32	0.16
`cokurt`	0.24	0.11	0.30	0.35
`beta_down`	0.29	0.08	0.40	0.36
`down_corr`	0.27	0.02	0.39	0.22
`htcr`	0.19	0.11	0.29	0.48
`beta_tr`	0.02	0.02	0.07	0.08
`coentropy`	0.25	0.02	0.39	0.24
`cos_pred`	0.20	0.12	0.39	0.43
`beta_neg`	0.19	0.13	0.36	0.47
`mcrash`	0.16	0.05	0.32	0.25
`ciq_down`	0.08	0.04	0.21	0.18

2.3 Fama-MacBeth Regressions

Next, I present first results on how the ARMs align with the cross-section of asset returns. To do that, I run Fama and MacBeth (1973) cross-sectional regressions and report the results in Table 2 in Panel A. I report both univariate estimates and estimates obtained by controlling for four characteristics widely employed in the literature – market beta, size, book-to-market, and momentum. Below the estimated coefficient, I include t-statistics based on the Newey-West robust standard errors.

From the univariate results, it is obvious that the cross-sectional pricing implications of ARMs vary considerably in their significance. Looking at the all-stock results, the highest significance possess the downside CIQ beta with t-statistics of 2.69. Cokurtosis yields t-statistics of -3.15, unfortunately, the sign of the coefficient is counterintuitive. Coskewness is, on the other side, significant with an expected sign. Tail risk beta is borderline significant with a t-stat of 1.89. The rest of the variables are deemed insignificant in the presented setting. When we move to the controlled setting, most of the variables become slightly less significant with few exceptions such as tail risk beta, which becomes significant (t-stat=2.10) or downside beta, which becomes also significant, but with a negative sign.

Panel B of Table 2 reports the results using a dataset that excludes penny stocks. Generally, coefficients become more significant (or less significant, if they possess a counterintuitive sign in the all-stock sample). For example, hybrid tail covariance risk (t-stat=4.57) or downside correlation (t-stat=2.38) become highly significant. Some of the variables become even more significant when controlling for other risk measures, such as multivariate crash risk (t-stat=2.04) or tail risk beta (t-stat=3.52).

Table 2: *Fama-MacBeth regressions.* The table reports the risk premiums of the ARMs estimated using Fama-MacBeth regressions. Below the coefficients, I include their HAC t-statistics based on Newey and West (1987) using lag auto-selection of Newey and West (1994). Panel A reports results using all stocks, Panel B excludes stocks with a price less than \$5 or market cap below 10% quantile of NYSE stocks. Data cover the period between January 1968 and December 2018.

	Panel A: All stocks						***Panel B: No penny stocks***					
	univariate	multivariate					univariate	multivariate				
	ARM	ARM	β	Size	BM	MOM	ARM	ARM	β	Size	BM	MOM
`coskew`	-0.57	-0.39	-0.13	-0.15	0.22	0.49	-0.62	-0.36	-0.26	-0.12	0.12	0.53
	(-2.17)	(-1.62)	(-0.75)	(-1.72)	(3.23)	(3.23)	(-2.21)	(-1.56)	(-1.41)	(-1.63)	(1.41)	(3.41)
`cokurt`	-0.21	-0.12	-0.10	-0.14	0.21	0.51	-0.08	0.04	-0.27	-0.18	0.13	0.53
	(-3.15)	(-1.28)	(-0.47)	(-1.95)	(3.39)	(3.51)	(-1.24)	(0.60)	(-1.39)	(-2.58)	(1.45)	(3.35)
`beta_down`	-0.12	-0.14	-0.02	-0.15	0.21	0.50	-0.07	-0.05	-0.20	-0.12	0.12	0.53
	(-1.29)	(-2.43)	(-0.14)	(-1.58)	(3.08)	(3.26)	(-0.52)	(-0.63)	(-1.24)	(-1.55)	(1.36)	(3.43)
`down_corr`	0.18	-0.03	-0.13	-0.16	0.22	0.50	0.35	0.08	-0.25	-0.12	0.13	0.52
	(1.47)	(-0.32)	(-0.76)	(-1.66)	(3.20)	(3.19)	(2.38)	(0.83)	(-1.40)	(-1.57)	(1.51)	(3.31)
`htcr`	34.30	-1.55	-0.13	-0.16	0.19	0.53	201.36	140.20	-0.24	-0.17	0.12	0.51
	(0.76)	(-0.05)	(-0.75)	(-1.91)	(3.00)	(3.84)	(4.57)	(4.28)	(-1.35)	(-2.37)	(1.40)	(3.29)
`beta_tr`	0.16	0.15	-0.13	-0.14	0.21	0.51	0.28	0.25	-0.24	-0.11	0.12	0.51
	(1.89)	(2.10)	(-0.78)	(-1.52)	(3.06)	(3.31)	(2.77)	(3.52)	(-1.37)	(-1.52)	(1.38)	(3.29)
`coentropy`	0.13	-0.08	-0.13	-0.16	0.22	0.50	0.35	0.03	-0.25	-0.12	0.13	0.53
	(0.82)	(-0.64)	(-0.75)	(-1.72)	(3.21)	(3.22)	(1.76)	(0.21)	(-1.39)	(-1.64)	(1.52)	(3.36)
`cos_pred`	-3.05	-0.20	-0.15	-0.18	0.21	0.49	-1.97	1.29	-0.29	-0.19	0.14	0.56
	(-1.78)	(-0.11)	(-0.88)	(-2.69)	(3.33)	(3.17)	(-1.16)	(0.91)	(-1.68)	(-3.00)	(1.64)	(3.52)
`beta_neg`	-0.12	0.30	-0.26	-0.14	0.20	0.51	-0.53	-0.45	-0.06	-0.13	0.11	0.54
	(-0.29)	(0.78)	(-2.12)	(-1.65)	(2.92)	(3.48)	(-1.33)	(-1.39)	(-0.42)	(-1.81)	(1.25)	(3.51)
`mcrash`	0.24	0.29	-0.14	-0.17	0.23	0.49	1.55	1.19	-0.26	-0.14	0.13	0.52
	(0.29)	(0.50)	(-0.80)	(-1.78)	(3.34)	(3.18)	(1.85)	(2.04)	(-1.45)	(-1.78)	(1.54)	(3.30)
`ciq_down`	0.09	0.05	-0.12	-0.15	0.21	0.49	0.09	0.04	-0.25	-0.12	0.13	0.52
	(2.69)	(2.05)	(-0.72)	(-1.64)	(3.17)	(3.14)	(2.24)	(1.58)	(-1.43)	(-1.62)	(1.50)	(3.31)

2.4 Portfolio Sorts

Next, to briefly inspect the tradability of the ARMs, I perform simple univariate portfolio sorts based on the ARMs. I focus here on a simple portfolio formation based on the following scheme

$$x_{t+1} = \frac{Z_t' r_{t+1}}{N_{t+1}} \tag{7}$$

where Z_t is a vector of some stock characteristic observed at time t, r_{t+1} is a vector of excess returns of the stocks in the next period, and N_{t+1} is the number of stock observations in a given month. I will refer to this type of portfolio as a *managed portfolio* with a corresponding return x_{t+1}. We can see that the return of the managed portfolio is obtained as a weighted average of stock returns where the weights are the values of the characteristics and normalized by the number of stock observations.

To obtain the weights corresponding to a given characteristic, every month, I cross-sectionally rank the values of the characteristic, divide the rank by the number of observations in the month, and subtract 0.5. This procedure transforms the characteristics into the interval $[-0.5, 0.5]$. This eliminates the effect of outliers and the resulting return can be interpreted as a zero-cost portfolio return associated with the characteristic. This return is also used for the later analysis using the instrumented principal component analysis.

The returns of the managed portfolios sorted based on ARMs are summarized in Table 3. In the case of all stocks, the highest absolute Sharpe ratio possess the downside CIQ beta with a value of 0.42. In the case of non-penny stocks, the highest Sharpe ratio attains hybrid tail covariance risk with the same value of 0.42. As hinted from the Fama-MacBeth regressions, some of the variables possess a counterintuitive negative premium, e.g., cokurtosis possess a significantly negative risk premium in the universe of all stocks. Another notable example is downside beta which attains negative risk premiums in both samples, but the associated average returns are not significantly different from zero.

Table 3 also reports annualized 6-factor alphas and their t-statistics with respect to six commonly used risk factors. As a general benchmark of risk, I employ four factors of Carhart (1997) including market, size, value, and momentum factor. To control for the effect of the common volatility, which may be a driver of many tail events, I use the common idiosyncratic volatility (CIV) shocks of Herskovic et al. (2016). The betting-against-beta (BAB) factor of Frazzini and Pedersen (2014) aims at controlling the effect of the well-known beta mispricing anomaly. When I control for the exposures to those six factors, the significance of some of the ARM premiums deteriorates, such as in the case of tail risk beta in both samples. On the other hand, some of the premiums do not suffer any decrease in significance if we control for the exposure to those common factors. For example, controlled risk premiums associated with the downside common idiosyncratic quantile risk deliver significant t-stats of 3.58 and 3.24.

In Appendix B, I employ more conventional specifications of the sorts. Tables 20 and 21 summarize portfolio returns from sorting the stock into five and ten portfolios, respectively, with monthly rebalancing. Tables contain results using equal- and value-weighted schemes for both data samples. In the case of all stocks, the highest risk premium carries predicted coskewness using both equal- and value-weighted returns and sorting into either quintile or decile portfolios, although with varying levels of significance.

In Appendix B in Figure 6, I depict the time-series correlations between managed portfolios sorted on ARMs. Moreover, Table 1 contains also averages of those correlations for each characteristic. Correlations are noticeably higher than in the case of the values of the characteristics, which can be expected. The most correlated with other ARMs is downside beta, which is closely followed by downside correlation, coentropy, and predicted coskewness. From those values, there is clearly some common structure, but the question remains whether the exposures to that common structure represent priced determinants of risk.

Table 3: *Managed portfolio returns.* The table contains annualized out-of-sample returns of the managed portfolios sorted on various asymmetric risk measures. It reports corresponding t-statistics, Sharpe ratio (SR), and annualized 6-factor alphas and their t-statistics with respect to the four factors of Carhart (1997), CIV shocks of Herskovic et al. (2016), and BAB factor of Frazzini and Pedersen (2014). I use the HAC t-statistics of Newey and West (1987) with 6 lags. Panel A reports results using all stocks, Panel B excludes stocks with a price less than \$5 or market cap below 10% quantile of NYSE stocks. Data cover the period between January 1968 and December 2018.

	Panel A: All stocks					*Panel B: No penny stocks*				
	Mean	t-stat	SR	α	t-stat	Mean	t-stat	SR	α	t-stat
`coskew`	-0.30	-2.51	-0.32	-0.23	-1.52	-0.28	-2.34	-0.30	-0.10	-0.66
`cokurt`	-0.39	-2.29	-0.29	-0.10	-0.57	-0.07	-0.48	-0.06	0.23	1.73
`beta_down`	-0.27	-1.27	-0.16	0.09	0.63	-0.13	-0.53	-0.07	0.11	0.84
`down_corr`	0.15	1.80	0.22	0.09	0.84	0.24	2.64	0.33	0.04	0.39
`htcr`	0.00	0.01	0.00	-0.14	-0.66	0.37	2.86	0.42	0.32	2.63
`beta_tr`	0.32	2.28	0.32	0.31	1.44	0.35	2.56	0.36	0.18	1.12
`coentropy`	0.11	1.37	0.16	0.07	0.61	0.18	2.03	0.25	-0.01	-0.08
`cos_pred`	-0.46	-1.76	-0.26	-0.50	-1.85	-0.22	-0.97	-0.14	-0.11	-0.56
`beta_neg`	-0.13	-0.38	-0.05	0.34	1.83	-0.35	-1.18	-0.16	-0.03	-0.26
`mcrash`	0.03	0.36	0.05	0.06	0.63	0.16	1.74	0.25	0.14	1.52
`ciq_down`	0.41	2.83	0.42	0.52	3.58	0.36	2.29	0.34	0.44	3.24

2.5 Naive Portfolio Formation

To set the stage, I combine the information across the ARMs using multivariate regression. I form the portfolios based on the multivariate Fama-MacBeth regressions in the spirit of Lewellen (2015). Using the set of 11 ARMs, I estimate expanding and moving regression where on the left-hand side are stock returns at time $t+1$ and on the right-hand side are the ARMs at time t. I use an out-of-sample setting with a 60-month initial or moving period. I estimate the model up to time T and use the model to predict the return at time $T+1$. I use the prediction of the out-of-sample return to construct the portfolio and observe its realized return. Then, I expand the estimation window and repeat the procedure until the sample is exhausted. I use either the managed portfolio approach or the difference between high and low portfolios based on quintile or decile sorts. Difference portfolios are weighted using an equal- or value-weighted scheme. I call those portfolios *regression portfolios.*

Table 4 summarizes the results. The first thing that we observe is the fact that the returns that those portfolios yield are only borderline significant at best. This observation is further supported by insignificant t-statistics with respect to the six-factor model that was previously used in case of single-sorted portfolios. Moreover, the returns are leptokurtic with slightly negative skewness in most of the cases. Last three columns employ rescaled returns so that the unconditional yearly volatility is 20%. Based on that, I report maximum drawdown and worst- and best-month returns.

From those results, it is evident that the simple portfolio formation cannot efficiently combine the information from the ARMs to yield abnormal returns beyond premiums as-

Table 4: *Regression portfolio returns.* The table contains out-of-sample results for the regression portfolios estimated using Fama-MacBeth regressions and various weighting schemes. It reports annualized mean, corresponding HAC t-statistics of Newey and West (1987) with 6 lags, Sharpe ratio (SR), alpha and its t-statistic with respect to the four factors of Carhart (1997), CIV shocks of Herskovic et al. (2016), and BAB factor of Frazzini and Pedersen (2014), skewness, kurtosis, the maximum drawdown, and best- and worst-month returns. Values are in percentages. I use expanding (moving) window estimation with a 60-month initial (moving) period. Data cover the period between January 1968 and December 2018.

	Mean	t-stat	SR	α	t-stat	Skewness	Kurtosis	Maximum drawdown	Worst month	Best month
Panel A: Expanding window										
Managed	0.44	2.02	0.31	0.33	1.35	-0.59	12.87	66.28	-39.35	44.30
Quintile EW	4.18	1.93	0.30	3.20	1.31	-0.61	12.35	66.88	-38.81	44.01
Decile EW	4.65	1.77	0.27	3.41	1.09	-0.81	11.37	65.68	-38.56	41.61
Quintile VW	2.66	1.12	0.17	1.02	0.41	0.16	7.45	68.59	-28.74	41.83
Decile VW	4.54	1.65	0.25	1.91	0.64	0.20	6.37	58.47	-25.81	42.59
Panel B: Moving window										
Managed	0.51	1.88	0.28	0.16	0.59	-0.38	9.35	66.11	-31.16	42.61
Quintile EW	4.87	1.78	0.27	1.31	0.48	-0.48	9.01	66.57	-33.13	41.36
Decile EW	4.70	1.41	0.22	-0.12	-0.03	-0.60	8.28	66.20	-32.49	39.58
Quintile VW	4.63	1.62	0.23	1.78	0.65	0.49	10.84	74.02	-24.74	49.35
Decile VW	5.61	1.61	0.24	1.89	0.55	0.09	6.99	77.69	-26.95	43.38

sociated with single sorts. In addition, the regression portfolios are highly exposed to the common factors and thus do not yield any significant risk-adjusted premium. High correlations between some of the ARMs may cause high estimation errors, which may be attenuated even more in the out-of-sample setting with shorter estimation periods. The fact that the moving-window estimation approach yields lower significance of the results further supports this claim.

From the presented results, it is obvious that there is a sizable variation in the magnitude and significance of the risk premiums associated with the ARMs. Those variations can be caused either by selecting the weighting scheme, universe of stocks, number of portfolios, research design, or by their combinations. Moreover, some of the premiums can be at least partly explained by other common factors. On top of that, a simple approach based on multivariate regression does not perform well in regards to combining the ARMs into portfolios. Based on those observations, I will explore the question of whether there is a possibility to efficiently form a portfolio that combines pricing information from all the ARMs without being exposed to common fluctuations.

3 Combining Asymmetric Risk Measures

In this section, I present an approach to portfolio construction that enjoys the abnormal returns associated with the ARMs without being exposed to common sources of risk. I estimate a latent factor model that utilizes the ARMs to account for the maximal possible explanation of the factor loadings to the common factors. Then, I form a portfolio that hedges those exposures and show that it still possess a significant risk premium not explained by any other factor model.

3.1 Model and Estimation

To estimate the risk premium associated with the ARMs, I use the instrumented principal component analysis (IPCA) model of Kelly et al. (2019, 2020), which can be written as

$$\begin{aligned} r_{i,t+1} &= \alpha_{i,t} + \beta_{i,t} f_{t+1} + \epsilon_{i,t+1}, \\ \alpha_{i,t} &= z'_{i,t}\Gamma_\alpha + \nu_{\alpha,i,t},\ \beta_{i,t} = z'_{i,t}\Gamma_\beta + \nu_{\beta,i,t} \end{aligned} \tag{8}$$

where $r_{i,t+1}$ is an excess return, $\beta_{i,t}$ contains dynamic loadings on $(K \times 1)$ vector of latent factors f_{t+1}. The vector of factor loadings may depend on the instrument $(L \times 1)$ vector $z_{i,t}$ of observable asset characteristics (which includes a constant) through the matrix Γ_β. I use the set of 11 ARMs as the characteristics that may proxy for the exposure to the common factors (hence ARM-IPCA). Mapping between characteristics and factor loadings serves two purposes. First, it enables the exploitation of other information than just simply return data for the estimation of latent factor loadings and thus makes the estimation more efficient. Second, it naturally makes the loadings time-varying as they are a function of the characteristics and thus makes it valuable tool for estimation of conditional risk premium. Moreover, the model admits the possibility that the characteristics align with the returns in addition to their relation to systematic risk. This feature captures $(L \times 1)$ vector of coefficients Γ_α that maps the characteristics into their anomaly intercepts.

This feature can be used to investigate how well the ARMs proxy for the exposure to the systematic risk, and to test whether they contain some important information beyond that and yield some anomaly (mispricing) returns. To do that, I can investigate features of the Γ_α estimate and test the null hypothesis that the ARMs do not proxy for the anomaly alpha. Throughout the text, I use fit of two specifications of the Model 8. First, the *restricted* model is estimated by setting the Γ_α vector to zero. Second, the *unrestricted* model is obtained by allowing expected returns to align with the ARMs beyond their relation with the systematic risk exposure and thus Γ_α is estimated freely.

To construct the hedging portfolio, I use estimates of the unrestricted model to exploit abnormal returns related to the ARMs. Each time t, I estimate the unrestricted model and form arbitrage portfolio with weights set equal to

$$w_t = Z_t(Z'_t Z_t)^{-1}\Gamma_\alpha, \tag{9}$$

which yields conditional factor neutrality.[1] I denote this portfolio as *pure-alpha* portfolio. The performance of the pure-alpha portfolio constitutes a natural test of abnormal returns associated with ARMs beyond exposure to common factors. The pure-alpha portfolio captures an opportunity for an investor to avoid systematic risk while enjoying the premium related to the ARMs. A successful factor model should not leave such an opportunity on the table. Moreover, the performance of the pure-alpha portfolio shows whether the ARMs can be combined to form abnormal returns beyond performances of single-variable sorts.

Following Kelly et al. (2019), estimation of the restricted model with $\Gamma_\alpha = 0$ is performed using *alternating least squares* and iterating between the first-order conditions for Γ_β and f_{t+1}

$$f_{t+1} = \left(\hat{\Gamma}'_\beta Z'_t Z_t \hat{\Gamma}_\beta\right)^{-1} \hat{\Gamma}'_\beta Z'_t r_{t+1}, \quad \forall t \tag{10}$$

and

$$\mathrm{vec}(\hat{\Gamma}'_\beta) = \left(\sum_{t=1}^{T-1} Z'_t Z_t \otimes \hat{f}_{t+1} \otimes \hat{f}'_{t+1}\right)^{-1} \left(\sum_{t=1}^{T-1} \left[Z_t \otimes \hat{f}'_{t+1}\right]' r_{t+1}\right) \tag{11}$$

where r_{t+1} is the $N \times 1$ vector of stock returns and Z_t is the $N \times L$ matrix of stock characteristics. The identifying restrictions are that $\hat{\Gamma}'_\beta \hat{\Gamma}_\beta = \mathbb{I}_K$, the unconditional second moment matrix of f_t is diagonal with descending diagonal entries, and the mean of f_t is non-negative.[2] In the case of the unrestricted version of the model with $\Gamma_\alpha \neq 0$, the estimation proceeds similarly, the only difference is that we augment the vector of factors to include a constant.

The proposed approach is particularly suitable for combining asymmetric risk measures for various reasons. First, by using ARMs to approximate the exposures to common factors, I can extract the risk premium associated solely with the non-linear features related to the measures. Moreover, the algorithm minimizes the risk that other risk factors will span the resulting abnormal returns. This is especially true for the market factor. From the previous literature, see, e.g., Hou et al. (2018), it is a well-documented fact that the exposure to the market factor is negatively priced across stock returns, even though it represents a counterintuitive observation. It is reasonable to expect that the linear relation with the overall market will dilute some asymmetric risk measures. As the market return usually explains the most time-series variation of stock returns, IPCA considers this fact, and the puzzle does not contaminate the pure-alpha portfolios.

[1]For comparability reasons, I scale the portfolio returns to have an unconditional standard deviation of 20% p.a., which does not affect the significance of the results.

[2]Those restrictions do not posses any economic implications for the model.

Second, this procedure mitigates a possible problem of multicollinearity among the ARMs, as well. If some variables similarly proxy for the exposure to the common factors, IPCA controls those associations when assigning the weights for the pure-alpha portfolio.

3.2 Pure-Alpha Portfolios

To combine the ARMs while hedging exposure to linear factors, I form the pure-alpha portfolios and investigate their out-of-sample performance. The models are estimated using an expanding window. First, I estimate the IPCA models with the first 60 observations of the sample and use the estimates $\hat{\Gamma}_\alpha$ to form the pure-alpha portfolios and record the out-of-sample return in the next period. Then, I expand the estimation period by one observation and predict the next. I repeat the procedure until the dataset is fully exhausted. The first out-of-sample prediction period corresponds to January 1973.

Table 5 summarizes basic features of the pure-alpha portfolios for the ARM-IPCA model with range of one to eight common latent factors. Results show that portfolios estimated using between one and five factors yield highly significant returns with HAC t-statistic of Newey and West (1987) and 6 lags of up to 6.27 corresponding to the ARM-IPCA(2) specification. Sharpe ratio achieves up to 0.97. I report also skewness and kurtosis of the pure-alpha portfolios. Those values do not indicate any extreme behavior of the portfolios as the distributions are close to symmetric and without signs of extreme heavy tails. In comparison to the results obtained using the regression portfolios based on Fama-MacBeth regression, returns of the pure-alpha portfolios exhibit features much closer to the normal distribution without signs of an extreme behavior. Moreover, I include the maximum drawdowns that every portfolio yielded as well as their best and worst months. In Appendix C in Table 22, I include summary results for the pure-alpha portfolios estimated separately in two disjoint sub-intervals and show that the implications hold similarly over those periods.

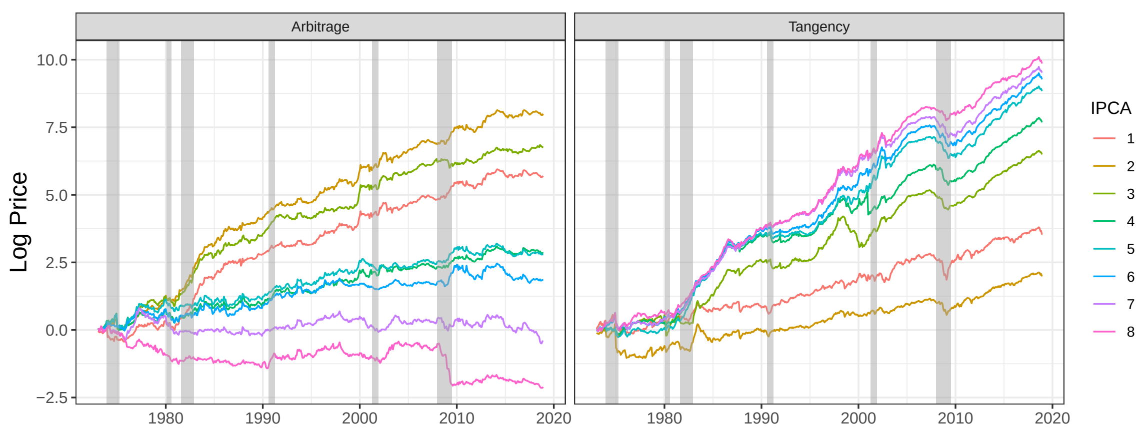
To further assess the performance of the pure-alpha portfolios, left panel of Figure 2 captures the cumulative log return of those portfolios. We see that the pure-alpha portfolios based on up to five latent factors grow constantly over the whole period without a noticeable sign of slowing down. Those results suggest that it is possible to strip the ARMs down from their exposures to the common linear factors and combine them into a highly profitable strategy. This strategy is up to more than twice as big as the best strategy based on a single-variable sort. Moreover, the features of the pure-alpha portfolios suggest that the resulting returns do not exhibit extreme behavior that may be expected due to the nature of the ARMs.

Next, I investigate whether the arbitrage returns associated with the pure-alpha portfolios

Table 5: *Pure-alpha portfolio returns.* The table contains out-of-sample results for the pure-alpha portfolios estimated using ARM-IPCA model with various numbers of latent factors. It reports annualized mean, corresponding HAC t-statistics of Newey and West (1987) with 6 lags, Sharpe ratio (SR), skewness, kurtosis, the maximum drawdown, and best- and worst-month returns. Values are in percentages. I use expanding window estimation with 60-month initial period. Data cover the period between January 1968 and December 2018.

K factors	Mean	t-stat	SR	Skewness	Kurtosis	Maximum drawdown	Worst month	Best month
1	14.36	4.73	0.72	0.09	3.59	41.40	-31.14	25.53
2	19.36	6.27	0.97	0.15	2.87	31.17	-25.47	27.23
3	16.78	5.35	0.84	-0.00	6.59	43.45	-39.64	25.67
4	8.20	3.04	0.41	-0.12	5.41	45.88	-40.07	24.14
5	8.06	2.86	0.40	0.34	3.69	38.36	-32.42	27.70
6	5.97	2.05	0.30	0.79	3.20	51.45	-17.98	27.29
7	1.07	0.34	0.05	0.48	2.25	73.12	-20.19	26.80
8	-2.53	-0.81	-0.13	-0.40	2.76	89.97	-31.47	21.61

Figure 2: *Performance of the ARM-IPCA portfolios.* The figure shows out-of-sample performance results of the pure-alpha and tangency portfolios estimated using IPCA models with the ARMs as instruments. Models are estimated with an expanding window and a 60-month initial period. Tangency portfolios are based on the restricted IPCA model, and pure-alpha portfolios on the unrestricted model. Data cover the period between January 1973 and December 2018.



are not driven by exposures to other known factors. I regress returns of the pure-alpha portfolios on various sets of factors that were proven to be successful in capturing risk premium. I report the annualized alphas and their HAC t-statistics of Newey and West (1987) with 6 lags. Table 6 reports risk-adjusted returns when controlling for the exposures to the 3- and 5-factor models of Fama and French (1993) and Fama and French (2015), while also using specification of Carhart (1997) and combining it with CIV shocks of Herskovic et al. (2016), and BAB factor of Frazzini and Pedersen (2014). We can see that the returns of the pure-alpha portfolios are not subsumed by those other specifications. Although, it is obvious that momentum factor and betting-against-beta factor capture a non-trivial part of the returns of the pure-alpha portfolios.

Table 6: *Fama-French risk-adjusted returns of the pure-alpha portfolios.* The table reports annualized alphas and their HAC t-statistics of Newey and West (1987) with 6 lags obtained by regressing the pure-alpha portfolio returns on various factor models and their combinations: Fama and French (1993), Carhart (1997), Fama and French (2015), CIV shocks of Herskovic et al. (2016), and BAB factor of Frazzini and Pedersen (2014). Data cover the period between January 1973 and December 2018.

K factors	CAPM	FF3	FF3+MOM	FF3+MOM +CIV	FF3+MOM +CIV+BAB	FF5	FF5+MOM	FF5+MOM +CIV	FF5+MOM +CIV+BAB
1	14.31	13.58	9.12	9.16	6.27	12.38	8.77	8.79	6.87
	(4.75)	(4.54)	(2.71)	(2.74)	(1.85)	(3.69)	(2.53)	(2.54)	(2.00)
2	19.65	18.70	13.28	13.31	10.63	17.39	13.00	13.02	11.18
	(6.54)	(6.19)	(3.95)	(3.98)	(3.15)	(5.02)	(3.73)	(3.73)	(3.23)
3	17.04	16.88	11.49	11.50	10.03	15.97	11.59	11.60	10.34
	(5.68)	(5.41)	(4.23)	(4.22)	(3.46)	(4.65)	(4.06)	(4.03)	(3.51)
4	8.44	6.68	5.87	5.90	5.22	6.67	6.02	6.03	5.37
	(3.27)	(2.55)	(2.27)	(2.28)	(1.88)	(2.67)	(2.32)	(2.32)	(1.91)
5	7.89	6.57	5.59	5.60	5.61	7.41	6.54	6.55	6.14
	(2.94)	(2.32)	(1.94)	(1.94)	(1.94)	(2.76)	(2.33)	(2.33)	(2.13)
6	6.07	4.14	4.51	4.52	4.57	5.90	6.07	6.07	5.61
	(2.13)	(1.47)	(1.48)	(1.48)	(1.52)	(2.13)	(2.01)	(2.01)	(1.88)

Table 7: *Exposures of the ARM-IPCA pure-alpha portfolios.* The table reports estimated coefficients and their t-statistics from regressing returns of the pure-alpha ARM-IPCA(K) portfolios on five factors of Fama and French (2015), augmented by momentum factor of Carhart (1997), CIV shocks of Herskovic et al. (2016), and BAB factor of Frazzini and Pedersen (2014). Data cover the period between January 1973 and December 2018.

K	α	Mkt	SMB	HML	RMW	CMA	MOM	CIV	BAB
1	6.87	0.05	0.06	0.06	-0.21	-0.06	0.33	-0.02	0.48
	(2.00)	(0.69)	(0.38)	(0.39)	(-0.97)	(-0.30)	(2.73)	(-0.47)	(3.63)
2	11.18	0.02	0.08	0.10	-0.27	0.02	0.42	-0.02	0.46
	(3.23)	(0.26)	(0.53)	(0.57)	(-1.39)	(0.12)	(3.35)	(-0.62)	(3.88)
3	10.34	0.04	-0.03	-0.05	-0.42	0.27	0.44	0.01	0.31
	(3.51)	(0.57)	(-0.21)	(-0.24)	(-2.19)	(1.12)	(4.50)	(0.33)	(2.76)
4	5.37	0.04	-0.21	0.27	-0.27	0.16	0.06	-0.04	0.17
	(1.91)	(0.59)	(-1.12)	(1.13)	(-1.61)	(0.60)	(0.67)	(-1.14)	(1.37)
5	6.14	0.09	-0.23	0.26	-0.40	0.11	0.09	-0.01	0.10
	(2.13)	(1.24)	(-1.38)	(1.16)	(-2.75)	(0.48)	(0.88)	(-0.37)	(0.98)
6	5.61	-0.01	-0.05	0.35	-0.46	-0.07	-0.04	0.00	0.11
	(1.88)	(-0.17)	(-0.55)	(2.50)	(-3.20)	(-0.37)	(-0.45)	(-0.10)	(1.09)

Table 7 summarizes the exposures of the pure-alpha arbitrage portfolios to eight factors based on five-factor model of Fama and French (2015), augmented by momentum factor of Carhart (1997), CIV shocks of Herskovic et al. (2016), and BAB factor of Frazzini and Pedersen (2014). The pure-alpha portfolios of the ARM-IPCA models possess a significant exposures to the momentum and betting-against-beta factors. Although these exposures diminishes the abnormal returns, the remaining risk premium remains still significant.

Next, I control for the exposure to the q-factor models of Hou et al. (2014) and Hou et al. (2020) augmented by momentum factor, CIV shocks, and BAB factor. Results summarizes Table 8. The abnormal returns of the pure-alpha portfolios cannot be erased by those combinations, either. Especially strong remain the abnormal returns for portfolios constructed from two- or three-factor specifications of the ARM-IPCA.

Table 8: *q-model risk-adjusted returns of the pure-alpha portfolios.* The table reports annualized alphas and their HAC t-statistics of Newey and West (1987) with 6 lags obtained by regressing the pure-alpha portfolio returns on factor models of Hou et al. (2014) and Hou et al. (2020), and augmented by CIV shocks of Herskovic et al. (2016), and BAB factor of Frazzini and Pedersen (2014). Data cover the period between January 1973 and December 2018.

K factors	Q4	Q5	Q5+MOM	Q5+MOM +CIV	Q5+MOM +CIV+BAB
1	8.03	7.40	7.81	7.64	6.29
	(2.23)	(2.15)	(2.42)	(2.37)	(1.95)
2	12.22	11.05	11.58	11.41	10.16
	(3.27)	(3.13)	(3.60)	(3.51)	(3.17)
3	11.39	8.69	9.29	9.24	8.54
	(2.98)	(2.57)	(3.06)	(3.00)	(2.74)
4	5.98	6.00	6.04	5.91	5.37
	(2.01)	(1.93)	(1.96)	(1.89)	(1.69)
5	6.11	6.50	6.63	6.59	6.39
	(1.97)	(2.03)	(2.11)	(2.08)	(2.04)
6	6.33	6.81	6.83	6.81	6.40
	(2.16)	(2.16)	(2.18)	(2.18)	(2.08)

Table 9: *IPCA risk-adjusted returns of the pure-alpha portfolios.* The table reports annualized alphas and their HAC t-statistics of Newey and West (1987) with 6 lags obtained by regressing the pure-alpha portfolio returns on out-of-sample IPCA factors with one to six latent factors and 32 characteristics from Kelly et al. (2019) as instruments. Data cover the period between January 1973 and December 2018.

K factors	IPCA1	IPCA2	IPCA3	IPCA4	IPCA5	IPCA6
1	14.12	14.60	12.95	8.60	10.62	12.07
	(4.77)	(4.99)	(2.71)	(2.01)	(2.34)	(2.69)
2	18.85	19.50	18.83	12.15	13.65	18.33
	(6.30)	(6.89)	(3.57)	(2.54)	(2.74)	(3.62)
3	16.40	16.95	21.09	16.29	16.42	18.87
	(5.26)	(6.07)	(4.12)	(3.21)	(3.03)	(3.25)
4	7.12	7.47	3.29	3.04	7.94	10.61
	(2.62)	(2.94)	(0.88)	(0.81)	(2.03)	(2.16)
5	7.25	7.40	4.44	3.82	7.96	11.83
	(2.58)	(2.76)	(1.24)	(1.05)	(2.12)	(2.58)
6	5.47	4.52	1.90	3.12	2.12	4.50
	(1.92)	(1.65)	(0.65)	(0.98)	(0.64)	(1.22)

Finally, I put the anomaly returns of the pure-alpha portfolios against their closest competitor. I investigate whether the out-of-sample IPCA factors estimated using the original set of 32 characteristics can explain abnormal returns related to the pure-alpha portfolios estimated using the set of 11 ARMs. The results of this analysis are contained in Table 9. We observe that returns of the pure-alpha portfolios cannot be explained by the original IPCA factors.

The fact that the pure-alpha portfolios are exposed to the momentum risk relates to recent results in the literature. There has been much work done that investigates the relationship between momentum returns and tail risk. More specifically, some studies investigate co-called momentum crashes and propose methods to avoid them. Barroso and Santa-Clara (2015) propose a volatility-managed approach to solving the problem of extreme drawdowns and excess kurtosis related to the momentum strategy. Daniel and Moskowitz (2016) propose an alternative approach that maximizes the Sharpe ratio based on predicting both risk and return of the momentum strategy.

Min and Kim (2016) investigate the momentum strategy in relation to the economic states. They find that the strategy performs poorly when the marginal utility of wealth is the highest captured by the expectation of the market risk premium. They conclude that the momentum premium is substantially related to the downside risk. Atilgan et al. (2020) report the presence of left-tail momentum that is characterized by the continuation of extreme left-tail events of stocks that experienced such events in the past. Unlike their results, my pure-alpha portfolios that invest in stocks with high systematic left-tail stocks report economically intuitive positive returns and positive exposure to the momentum factor.

Although the pure-alpha portfolios are significantly exposed to the momentum factor, they do not possess such extreme behavior. During the investigated period, momentum possesses a negative skewness of -1.35. On the other hand, the lowest value of skewness that a pure-alpha portfolio yields is -0.12, obtained from the IPCA model with four latent factors. On top of that, unlike the distribution of the momentum returns that exhibit highly leptokurtic features with a value of kurtosis equal to 10.92, the pure alpha portfolio attains a value of 6.59 at the highest.

Kelly et al. (2021) investigate momentum in relation to the IPCA model. They conclude that the momentum premium is explainable due to the fact that the momentum characteristic is a proxy for the exposure to the common factors. Even though the original set of IPCA factors can erase the abnormal returns of the momentum factor, my pure-alpha portfolios cannot be explainable by this set of factors.

3.3 Variable Importance

This section investigates which ARMs contribute the most to the performance of the pure-portfolio. Table 10 reports estimates of the Γ_α vector from the out-of-sample procedure in the last prediction period. Because the ARM variable are standardized, their magnitudes are comparable. We can observe that coefficients of some variables change considerably across the range of common latent factors that are controlled for when forming the portfolios. This fact is caused by the process of using more ARMs as a proxies for exposures to another common factors and potentially losing some predictive ability for anomaly returns of the pure-alpha portfolio.

Moreover, in Figure 3, I capture the estimates of Γ_α from the expanding window estimation of the ARM-IPCA(2) model. We can see that the coefficients are relatively stable

Table 10: *Estimated coefficients of* Γ_α *vector.* The table summarizes estimated coefficients of Γ_α vector of the ARM-IPCA model. This vector is used for the construction of the pure-alpha portfolios. Reported are coefficients estimated using the last prediction window before exhausting the full dataset. Coefficients are multiplied by 1,000 for better readability. Data cover the period between January 1973 and December 2018.

	K factors					
	1	2	3	4	5	6
`coskew`	-5.50	-4.11	0.08	-0.78	-0.54	-1.57
`cokurt`	2.61	1.72	2.70	1.88	2.03	2.55
`beta_down`	-9.10	-7.71	-1.82	-2.76	-3.32	-3.00
`down_corr`	2.62	1.84	0.18	1.04	0.42	-0.13
`htcr`	2.92	3.10	5.87	4.89	4.56	2.73
`beta_tr`	2.43	1.71	0.71	2.71	-1.19	0.81
`coentropy`	-4.18	-3.12	-1.72	-2.40	-1.66	-0.71
`cos_pred`	-4.23	-4.85	-4.92	-1.45	-0.36	-0.40
`beta_neg`	-2.53	-2.32	0.11	0.92	1.14	1.11
`mcrash`	0.95	0.95	1.52	1.34	1.77	1.14
`ciq_down`	3.82	3.30	2.98	3.52	1.21	-1.32

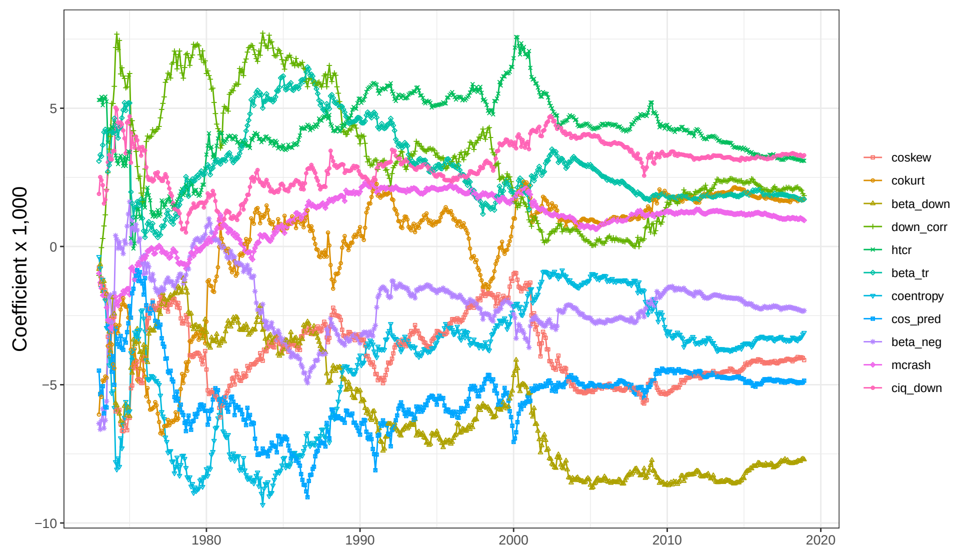
across time and the variables possess the same sign during the most of the period.

Next, I assess the variable importance for the out-of-sample results based on setting the effect of a variable on the formation of the pure-alpha portfolio to zero. For a given number of latent factors, I estimate the unrestricted IPCA model using all ARMs. Then, when forming the arbitrage portfolio, I set element of Γ_α corresponding to the investigated ARM to zero and record the out-of-sample return next period. I exhaust the full dataset and compute the realized out-of-sample Sharpe ratio. I repeat this procedure for each ARM and range between one and six factors.[3] Table 11 reports this effect.

We can see that there are three variables with highly negative omission impact across all six specifications of the pure-alpha portfolios: downside correlation, coentropy, and downside CIQ beta. Those variables significantly improve the performance of the pure-alpha portfolio beyond their effect on exposures to the common factors. Hybrid tail covariance risk contributes positively to the first three specifications of the pure-alpha portfolios, which possess the highest Sharpe ratios among the specifications.

[3] I avoid the analysis based on entirely leaving a variable out from the whole estimation procedure of an unrestricted model because, in this case, the resulting effect on the Sharpe ratio combine two forces. First, there is less information that can be used for the formation of the arbitrage portfolio. This should generally lead to a decrease in the out-of-sample Sharpe ratio. Second, leaving one variable out restricts the information that can be used for the exploitation of the common factor structure of the returns. Consequently, this effect saves more potential pricing information for the construction of the arbitrage portfolio, which should generally lead to an increase in the Sharpe ratio.

Figure 3: Γ_α *estimates from the out-of-sample estimation.* The figure shows estimates of the Γ_α vector from the unrestricted ARM-IPCA(2) model using expanding window estimation and a 60-month initial period. Data cover the period between January 1973 and December 2018.



4 Time-Varying Risk Premium

The IPCA framework may not fully capture the arbitrage opportunities if the compensation for bearing risk associated with the ARMs is not stable across time periods. To investigate and potentially exploit the time-varying nature of the risk premium associated with the ARMs, I employ the projected principal component analysis (PPCA) framework proposed by Fan et al. (2016) and extended by Kim et al. (2020). In comparison to the IPCA framework, PPCA enables changes of cross-sectional relations between alphas/betas and characteristics. This variation may be potentially important if the relation between ARMs and risk/mispricing changes over time due to various reasons, such as being arbitraged away or beta-ARM relation changes.[4]

The PPCA framework first assigns maximal explanatory power of the characteristics to the systematic risk exposures before relating the characteristics to their alphas. The resulting arbitrage portfolio thus aims to hedge sources of systematic risk related to the characteristics while enjoying the residual (true anomaly) returns associated with the ARMs. Moreover, it enables the arbitrage portfolios to reflect the time variation in compensation for the ARMs by being consistently estimated over short samples. This feature comes at cost of less efficiency

[4]An example of the former constitutes the results of Mclean and Pontiff (2016), which state that the relation changes by investors' usage of academic publications to learn about mispricing and forming their investment decisions based on that. An example of the latter represents Cho (2020) who argues that financial intermediaries through their arbitrage process and common exposure to funding liquidity shocks and arbitrageur wealth portfolio shocks turn alphas into betas.

Table 11: *Variable Importance of the ARMs for the pure-alpha portfolios.* The table reports decreases of the out-of-sample Sharpe ratios of the pure-alpha portfolios from leave-one-out procedure. For each ARM, I report difference (in % points) between Sharpe ratio obtained without the ARM and Sharpe ratio obtained from the model with all ARMs. Data cover the period between January 1968 and December 2018.

	IPCA1	IPCA2	IPCA3	IPCA4	IPCA5	IPCA6
Sharpe ratio	0.72	0.97	0.84	0.41	0.40	0.30
	Decrease of Sharpe ratio in %					
coskew	15.17	9.19	7.21	26.12	14.21	34.82
cokurt	11.24	-1.56	-14.72	4.65	-6.52	26.40
beta_down	5.96	0.48	-10.53	-40.93	-47.86	-71.74
down_corr	-15.21	-4.98	-13.34	-26.57	-29.53	-57.84
htcr	-5.44	-6.91	-22.21	3.71	15.90	21.43
beta_tr	0.09	2.27	3.97	-3.61	-66.96	-142.62
coentropy	-39.70	-32.00	-25.15	-2.26	-16.14	-41.76
cos_pred	1.35	-21.29	-7.45	25.48	11.38	32.18
beta_neg	-1.99	0.64	7.24	-12.51	-61.93	-51.72
mcrash	1.27	-1.63	-1.61	-0.07	0.03	-10.14
ciq_down	-31.39	-25.61	-15.85	-22.89	-15.85	-74.26

if the relationship between characteristics and model parameters are constant because we use less data to estimate the model and form the arbitrage portfolio.

4.1 Model and Estimation

Similarly, as in the case of the IPCA model, I assume that the excess return of stock follows the structure

$$r_{i,t} = \alpha_i + \beta_i f_t + \epsilon_{i,t} \tag{12}$$

where the main difference in comparison to IPCA is that now I assume that the return-generating process for individual stocks (characterized by α_i and β_i) is stable over short time periods (12 months in the empirical investigation) $t = 1, \ldots, T$. In a matrix format for N assets over T periods, this can be rewritten as

$$\boldsymbol{R} = \boldsymbol{\alpha} \boldsymbol{1}_T' + \boldsymbol{B}\boldsymbol{F}' + \boldsymbol{E} \tag{13}$$

where $\boldsymbol{R}$ is the $(N \times T)$ matrix of returns, $\boldsymbol{\alpha}$ is the $(N \times 1)$ mispricing vector, $\boldsymbol{B}$ is the $(N \times K)$ matrix with i-th row corresponding to factor exposure β_i', $\boldsymbol{F}$ is $(T \times K)$ matrix of latent factors with t-th row being $f_t' = [f_{1,T}, \ldots, f_{K,t}]$. This specification allows the systematic exposure matrix $\boldsymbol{B}$ and vector of mispricing being nonparametric functions of the asset-specific characteristics. I stack each of the L characteristics into the $(N \times L)$ matrix $\boldsymbol{Z}$ and impose the following structure

$$\boldsymbol{\alpha} = \boldsymbol{G}_{\alpha}(\boldsymbol{Z}) + \Gamma_{\alpha} \tag{14}$$

$$\boldsymbol{B} = \boldsymbol{G}_{\beta}(\boldsymbol{Z}) + \Gamma_{\beta} \tag{15}$$

where the mis-pricing function is defined as $\boldsymbol{G}_{\alpha}(\boldsymbol{Z}) : \mathbb{R}^{N \times L} \to \mathbb{R}^{N}$, and the factor loading function is $\boldsymbol{G}_{\beta}(\boldsymbol{Z}) : \mathbb{R}^{N \times L} \to \mathbb{R}^{N \times K}$, and the $(N \times 1)$ vector Γ_{α} and the $(N \times K)$ matrix Γ_{β} are cross-sectionally orthogonal to the characteristics $\boldsymbol{Z}$. To estimate this model, I follow projected principal component analysis (PPCA) proposed by Fan et al. (2016) and generalized by Kim et al. (2020) to allow for presence of the mis-pricing contained in $\boldsymbol{\alpha}$.

The formation of the arbitrage portfolio proceeds in three steps. First, I demean the returns and apply PCA to obtain an estimate of $\boldsymbol{G}_{\beta}(\boldsymbol{Z})$. Second, I cross-sectionally regress the average returns on the characteristics space which is orthogonal to the estimate $\boldsymbol{G}_{\beta}(\boldsymbol{Z})$ from the first step to obtain the estimate of $\boldsymbol{G}_{\alpha}(\boldsymbol{Z})$. Third, I use the estimate of $\boldsymbol{G}_{\alpha}(\boldsymbol{Z})$ to form the portfolio, which is held for the next period. I denote this portfolio as *arbitrage portfolio*.

The main advantage of this methodology over the IPCA framework is that it is suited for the estimation over short time periods and thus enables to exploit the dynamics of the compensation for the ARMs. The model is estimated on a rolling-window basis, setting T equal to the short horizon. This allows for a change in cross-sectional relation between ARMs and returns either in terms of systematic risk or/and mispricing. Moreover, the model does not require to have all relevant characteristics for risk and mispricing, as the missing information may be contained in Γ_{α} and Γ_{β}. The aim of this model is to exploit mispricing captured by $\boldsymbol{\alpha}$ while hedging the systematic risk characterized by the ARMs and captured by $\boldsymbol{B}$.

This greater flexibility comes at a cost, however. The methodology does not exploit the time-variation of the characteristics during the estimation window. It employs only the values of characteristics at the first period of the estimation and assumes that these values proxy sufficiently for characteristics in the subsequent periods during the window. If the true relationship between characteristics and the model is constant, this will lead to a loss of estimation efficiency.

Following the original empirical PPCA implementation, I cross-sectionally demean the characteristics so that the resulting arbitrage portfolio costs zero dollars. Moreover, I target the in-sample volatility of the portfolio at 20% per year. I report the results for a range between one and ten latent factors. All the results are purely out-of-sample as the model is fitted using 12 months of data, the arbitrage portfolio is formed at the end of this period,

Table 12: *Summary of the arbitrage portfolio returns.* The table contains out-of-sample results for the arbitrage portfolios estimated using extended PPCA framework of Kim et al. (2020) using a rolling window estimation of 12 months and various numbers of latent factors. It reports annualized mean, corresponding HAC t-statistics of Newey and West (1987) with 6 lags, Sharpe ratio (SR), skewness, kurtosis, the maximum drawdown, and best- and worst-month returns. Data cover the period between January 1968 and December 2018.

K factors	Mean	t-stat	SR	Skewness	Kurtosis	Maximum drawdown	Worst month	Best month
1	3.29	1.30	0.20	0.79	8.13	60.81	-28.64	32.97
2	8.63	3.07	0.47	0.10	5.03	60.14	-31.22	30.56
3	7.73	2.99	0.49	0.40	6.92	55.43	-29.57	30.11
4	7.76	2.83	0.43	-0.00	4.61	56.98	-30.22	28.39
5	7.66	2.92	0.45	0.15	5.89	61.76	-28.80	30.67
6	7.84	2.96	0.46	-0.08	6.19	50.07	-31.86	27.03
7	7.71	3.01	0.46	-0.49	8.05	46.79	-34.48	26.16
8	8.90	3.35	0.53	-0.30	8.81	47.13	-33.95	30.00
9	6.32	2.35	0.39	-0.06	7.05	60.23	-29.87	28.11
10	5.68	2.26	0.35	-0.52	18.54	52.76	-44.09	32.44

and then the return in the next month is recorded.

4.2 Arbitrage Returns

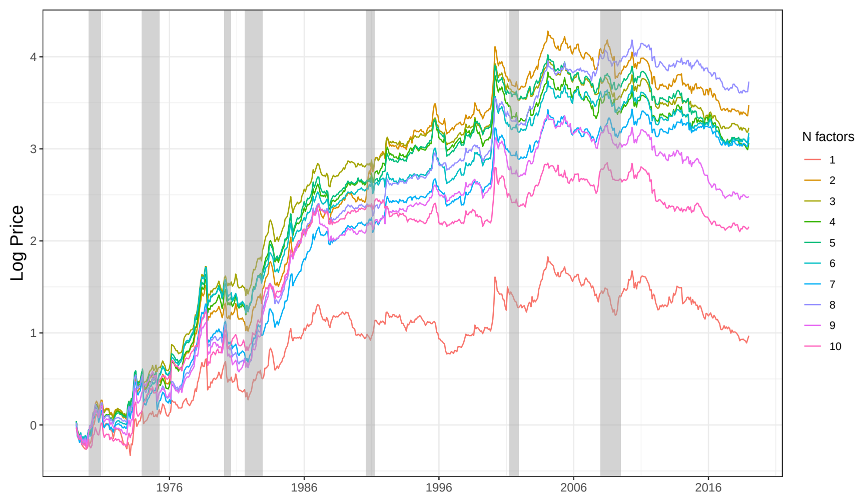
Table 12 summarizes the performances of the arbitrage portfolios that exploit the ARMs. We can see that when we use between two and ten factors in the model, we can obtain significant abnormal returns above the exposure to the common risks. The annual risk premium that we can obtain constitutes around 7.5% per year with a Sharpe ratio of around 0.45 and highly significant t-statistics of the mean return of value of around three. Regarding the distributional features of the returns, we see that they are close to symmetrically distributed. On the other hand, the estimated values of the kurtosis suggest that the returns are more heavy-tailed than the returns of the pure-alpha portfolios estimated using the ARM-IPCA model. This fact also affects the maximum drawdowns of the portfolios, which are also higher in case of the arbitrage portfolios than in the case of pure-alpha portfolios.

Figure 4 plots the cumulative returns of the arbitrage portfolios. We see that the portfolios grow constantly up until around the financial crisis. Around that time, returns sizably deteriorate and do not recover since then.

Table 13 summarizes risk-adjusted returns of the arbitrage portfolios with respect to various factor models based on 3-factor model of Fama and French (1993). While 3- and 5-factor models of Fama and French (1993) and Fama and French (2015) are not able to explain the associated anomaly returns, results that include momentum factor erase their significance.

To further investigate the relationship between arbitrage returns and other factors, I

Figure 4: *Cumulative return of the arbitrage portfolios.* The figure depicts the cumulative logarithm price of the arbitrage portfolios based on the PPCA framework of Kim et al. (2020) with the number of latent factors between one and ten. Arbitrage returns are purely out-of-sample. Data cover the period between January 1968 and December 2018.



report in Table 14 exposures to six-factor model based on four factors of Carhart (1997) augmented by CIV shocks of Herskovic et al. (2016), and BAB factor of Frazzini and Pedersen (2014). We observe that the six-factor alpha significantly shrinks to a value of around 4% per annum and the corresponding t-statistic falls below two in all models. Similarly as in the case of pure-alpha portfolios, the arbitrage portfolio possess a significant exposure to the momentum factor. In this case, however, the momentum strategy, along with other factors, explains the whole significant part of the arbitrage returns. Well-documented momentum crashes may partially explain the leptokurtic features of the portfolio, similarly they may be related to the high drawdowns that the portfolios experienced.

We observe that the arbitrage returns associated with the ARMs do not benefit from allowing for time-variation of the prices of risk. This is evident from the fact that the arbitrage portfolios do not yield abnormal returns beyond exposures to the common factors, especially when accounting for the relation to the momentum factor. Those observations suggest that the time variation of the prices of risk of the ARMs does not outweigh the loss of efficiency due to the short-window estimation. This was already hinted by the regression portfolios that performed better when they were estimated using the expanding window in comparison to moving window. Similarly, alphas of those portfolios were much less affected when they were estimated using the expanding window.

In comparison, pure-alpha portfolio returns obtained from the IPCA procedure using up to five factors yield a significant premium after controlling for those six common factors. Moreover, the Sharpe ratios that attain the pure-alpha portfolios are considerably higher

Table 13: *Fama-French risk-adjusted returns of the arbitrage portfolios.* The table reports annualized alphas and their HAC t-statistics of Newey and West (1987) with 6 lags obtained by regressing the arbitrage portfolio returns on various factor models and their combinations: Fama and French (1993), Carhart (1997), Fama and French (2015), CIV shocks of Herskovic et al. (2016), and BAB factor of Frazzini and Pedersen (2014). Data cover the period between January 1973 and December 2018.

K factors	CAPM	FF3	FF3+MOM	FF3+MOM +CIV	FF3+MOM +CIV+BAB	FF5	FF5+MOM	FF5+MOM +CIV	FF5+MOM +CIV+BAB
1	2.38	1.85	1.36	1.36	3.44	2.81	2.40	2.40	3.38
	(0.90)	(0.70)	(0.46)	(0.46)	(1.06)	(0.91)	(0.71)	(0.71)	(0.99)
2	8.31	7.80	3.02	3.02	4.74	7.53	3.71	3.71	4.51
	(2.78)	(2.51)	(1.07)	(1.07)	(1.57)	(2.20)	(1.22)	(1.22)	(1.47)
3	7.04	6.55	2.73	2.73	4.21	6.64	3.57	3.57	4.20
	(2.61)	(2.43)	(1.09)	(1.09)	(1.53)	(2.18)	(1.27)	(1.27)	(1.47)
4	7.31	7.00	2.12	2.12	3.42	6.88	2.95	2.94	3.47
	(2.57)	(2.43)	(0.82)	(0.82)	(1.20)	(2.10)	(1.03)	(1.03)	(1.18)
5	7.04	6.71	2.49	2.48	3.72	6.68	3.28	3.28	3.77
	(2.57)	(2.42)	(0.95)	(0.96)	(1.29)	(2.11)	(1.14)	(1.15)	(1.27)
6	7.37	7.04	2.73	2.73	3.84	7.27	3.77	3.77	4.14
	(2.66)	(2.51)	(1.05)	(1.05)	(1.33)	(2.29)	(1.33)	(1.33)	(1.41)
7	7.24	7.11	2.72	2.72	3.83	7.30	3.73	3.73	4.09
	(2.68)	(2.59)	(1.05)	(1.05)	(1.33)	(2.31)	(1.30)	(1.30)	(1.38)
8	8.44	8.65	4.38	4.38	5.19	8.44	5.02	5.02	5.23
	(3.06)	(3.09)	(1.60)	(1.60)	(1.72)	(2.62)	(1.66)	(1.66)	(1.68)
9	6.02	5.78	1.92	1.92	2.58	5.40	2.33	2.33	2.50
	(2.17)	(2.09)	(0.74)	(0.74)	(0.94)	(1.74)	(0.84)	(0.84)	(0.90)
10	5.05	4.69	0.58	0.58	1.28	4.51	1.21	1.20	1.43
	(1.93)	(1.75)	(0.23)	(0.23)	(0.46)	(1.40)	(0.43)	(0.43)	(0.50)

Table 14: *Exposures of the arbitrage portfolios.* The table reports estimated coefficients and their t-statistics from regressing returns of the arbitrage portfolios on four factors of Carhart (1997), CIV shocks of Herskovic et al. (2016), and BAB factor of Frazzini and Pedersen (2014). The formation of the arbitrage portfolios is based on the extended PPCA framework of Kim et al. (2020) using a rolling window estimation of 12 months. Arbitrage returns are purely out-of-sample. Data cover the period between January 1968 and December 2018.

N factors	α	Mkt	SMB	HML	CIV	BAB	MOM
1	3.44	0.11	0.34	0.27	-0.05	-0.31	0.14
	(1.06)	(1.58)	(2.31)	(2.73)	(-1.72)	(-2.88)	(1.32)
2	4.74	0.11	0.30	0.40	-0.02	-0.26	0.53
	(1.57)	(1.47)	(2.19)	(3.23)	(-0.97)	(-2.68)	(5.84)
3	4.21	0.15	0.30	0.34	-0.03	-0.22	0.43
	(1.53)	(2.44)	(2.27)	(3.37)	(-1.50)	(-2.38)	(5.03)
4	3.42	0.13	0.26	0.33	-0.03	-0.19	0.53
	(1.20)	(1.94)	(1.88)	(2.81)	(-1.07)	(-2.07)	(5.95)
5	3.72	0.14	0.27	0.30	-0.03	-0.19	0.46
	(1.29)	(2.35)	(1.96)	(2.99)	(-1.19)	(-1.96)	(5.12)
6	3.84	0.12	0.28	0.30	-0.02	-0.17	0.46
	(1.33)	(1.86)	(2.12)	(2.92)	(-0.71)	(-1.82)	(5.49)
7	3.83	0.13	0.19	0.26	-0.02	-0.17	0.47
	(1.33)	(2.01)	(1.48)	(2.58)	(-0.72)	(-1.95)	(5.40)
8	5.19	0.11	0.21	0.16	-0.02	-0.12	0.45
	(1.72)	(1.59)	(1.60)	(1.61)	(-0.87)	(-1.27)	(4.79)
9	2.58	0.08	0.26	0.23	-0.01	-0.10	0.40
	(0.94)	(1.25)	(2.17)	(2.30)	(-0.32)	(-1.10)	(4.57)
10	1.28	0.13	0.35	0.27	-0.02	-0.11	0.43
	(0.46)	(2.07)	(2.82)	(3.46)	(-0.84)	(-1.20)	(5.05)

than in the case of the arbitrage portfolios based on the PPCA. All these results suggest that the relationship between ARMs and anomalous returns is quite stable in time.

5 ARM Latent Factors

Although we are able to form arbitrage returns related to the ARMs, I investigate how the ARMs can also be used as an approximation for the exposures to the common factors. In this section, I dissect the IPCA model fit using mostly restricted specification of the ARM-IPCA model. I investigate which variables proxy for the exposures to the common factors and how they relate to the original IPCA model results using the set of 32 variables.

5.1 Model Fit and Tests

I evaluate the performance of the IPCA models in terms of two metrics. The first one, *total* R^2, describes how is the model able to capture time variation of the realized returns using conditional loadings and factor realizations

$$\text{Total}\, R^2 = 1 - \frac{\sum_{i,t}\left(r_{i,t+1} - z'_{i,t}(\hat{\Gamma}_\alpha + \hat{\Gamma}_\beta \hat{f}_{t+1})\right)^2}{\sum_{i,t} r^2_{i,t+1}}. \tag{16}$$

The total R^2 aims to quantify how is the model successful at capturing the riskiness of the assets. Total R^2 is related to the estimation procedure. Similarly as in the case of principal component analysis, the estimation targets to maximize the model's explanatory power of the time variation of returns. In the case of the out-of-sample fits, the model parameters are estimated using the information up to time t, the same as the factors that are formed using the information up to time t and the out-of-sample realized factor returns are then recorded.

The second metric, *predictive* R^2, captures how is the model capable of explaining the conditional expected returns

$$\text{Predictive}\, R^2 = 1 - \frac{\sum_{i,t}\left(r_{i,t+1} - z'_{i,t}(\hat{\Gamma}_\alpha + \hat{\Gamma}_\beta \hat{\lambda})\right)^2}{\sum_{i,t} r^2_{i,t+1}} \tag{17}$$

where $\hat{\lambda}$ is a vector of factor means. In the case of out-of-sample analysis, $\hat{\lambda}$ is estimated up to time t. The predictive R^2 captures how much is the model able to describe the risk-return trade-off of the assets. We can restrict the model to $\Gamma_\alpha = 0$ and compare the performance with the unrestricted model. When we impose the restriction, the predictive R^2 tells us how much the risk compensation can be explained by the systematic risk with the exposures approximated by the ARMs. When we do not impose this restriction, the predictive R^2 summarizes how much of the variation of the expected returns can be explained through the characteristics via their relation to either systematic risk exposure or anomaly intercepts.

Moreover, the IPCA model has a natural interpretation in terms of managed portfolios. Using managed portfolio interpretation is important for estimation (e.g., for initial guess of the numerical optimization), its relation to the classical PCA estimator, and for various bootstrap testing procedures. More importantly for the presented analysis, I will use both single stock and managed portfolio returns to evaluate the performance of the IPCA models. Asset pricing literature frequently prefers to use portfolios because of their lower levels of unrelated idiosyncratic risk. The corresponding metrics are defined as

$$\text{Total } R^2 = 1 - \frac{\sum_t \left(x_{t+1} - Z_t' Z_t(\hat{\Gamma}_\alpha + \hat{\Gamma}_\beta \hat{f}_{t+1})\right)' \left(x_{t+1} - Z_t' Z_t(\hat{\Gamma}_\alpha + \hat{\Gamma}_\beta \hat{f}_{t+1})\right)}{\sum_t x_{t+1}' x_{t+1}} \tag{18}$$

and

$$\text{Predictive } R^2 = 1 - \frac{\sum_t \left(x_{t+1} - Z_t' Z_t(\hat{\Gamma}_\alpha + \hat{\Gamma}_\beta \hat{\lambda})\right)' \left(x_{t+1} - Z_t' Z_t(\hat{\Gamma}_\alpha + \hat{\Gamma}_\beta \hat{\lambda})\right)}{\sum_t x_{t+1}' x_{t+1}}. \tag{19}$$

To formally decide between restricted or unrestricted model specification for given number of latent factors *in-sample*, I follow Kelly et al. (2019), using Model 8, I test a null hypothesis of $H_0 : \Gamma_\alpha = 0_{L\times 1}$ against an alternative hypothesis $H_1 : \Gamma_\alpha \neq 0_{t\times 1}$. Under the null hypothesis, the characteristics do not yield significant alphas after controlling for their explanatory power regarding the loadings on latent factors. The procedure follows three steps.

First, the unrestricted IPCA model is estimated and the parameters and the residuals are saved. I compute a Wald-type test statistic that measures the distance between the restricted and unrestricted model, $W_\alpha = \hat{\Gamma}_\alpha' \hat{\Gamma}_\alpha$. Second, the inference regarding the test statistic is performed using residual bootstrap. In each bootstrap replication, I generate a sample of new managed portfolio returns using the estimated residuals, estimate $\hat{\Gamma}_\beta$ (both from the original unrestricted model) and the restricted model's specification (setting $\Gamma_\alpha = 0$). Then, the generated sample is used to estimate the unrestricted model and the simulated test statistic is saved. Third, the resulting inference is obtained from the simulated distribution of bootstrapped test statistics. A resulting p-value of the test is calculated as a proportion of bootstrapped test statistics that exceed the value of the test statistic from the actual data.

To assess the ARMs *out-of-sample*, I investigate the performances of two portfolios. Beside the pure-alpha portfolios estimated from the unrestricted model, I use the restricted model to form a factor tangency portfolio. Each time t, I estimate the restricted model and set weights of the factor portfolios proportional to $\Sigma_t^{-1}\mu_t$, where Σ_t and μ_t are a covariance matrix and vector of average returns of IPCA factors, respectively, both estimated using

Table 15: *ARM-IPCA results.* The table reports in-sample and out-of-sample results of the IPCA models with varying numbers of latent factors and using ARMs as the instruments. The asset pricing test reports p-values of the null hypothesis that $\Gamma_\alpha = 0$. Data cover the period between January 1968 and December 2018.

		IPCA(K)							
		1	2	3	4	5	6	7	8
Panel A: In-sample results									
Individual stocks									
Total R^2	$\Gamma_\alpha = 0$	15.95	17.30	17.99	18.46	18.70	18.83	18.94	19.02
	$\Gamma_\alpha \neq 0$	16.02	17.36	18.00	18.47	18.71	18.83	18.94	19.02
Predictive R^2	$\Gamma_\alpha = 0$	0.29	0.31	0.35	0.35	0.36	0.36	0.35	0.36
	$\Gamma_\alpha \neq 0$	0.37	0.37	0.36	0.36	0.36	0.36	0.36	0.36
Managed portfolios									
Total R^2	$\Gamma_\alpha = 0$	96.28	98.35	99.45	99.66	99.79	99.85	99.90	99.94
	$\Gamma_\alpha \neq 0$	96.35	98.41	99.46	99.67	99.79	99.85	99.90	99.94
Predictive R^2	$\Gamma_\alpha = 0$	1.85	1.88	1.95	1.94	1.95	1.95	1.94	1.95
	$\Gamma_\alpha \neq 0$	1.97	1.96	1.96	1.96	1.96	1.96	1.96	1.95
Asset pricing test									
W_α p-value		0.00	0.00	4.70	0.80	2.50	16.40	7.60	77.60
Panel B: Out-of-sample results									
Individual stocks									
Total R^2	$\Gamma_\alpha = 0$	15.49	16.81	17.47	17.99	18.25	18.38	18.49	18.57
	$\Gamma_\alpha \neq 0$	15.47	16.80	17.37	17.98	18.24	18.36	18.48	18.57
Predictive R^2	$\Gamma_\alpha = 0$	0.23	0.23	0.26	0.26	0.27	0.28	0.28	0.28
	$\Gamma_\alpha \neq 0$	0.28	0.28	0.28	0.28	0.28	0.28	0.28	0.28
Managed portfolios									
Total R^2	$\Gamma_\alpha = 0$	96.30	98.35	99.28	99.63	99.77	99.83	99.89	99.93
	$\Gamma_\alpha \neq 0$	95.91	98.04	99.08	99.56	99.74	99.81	99.88	99.92
Predictive R^2	$\Gamma_\alpha = 0$	1.55	1.56	1.64	1.67	1.69	1.69	1.69	1.69
	$\Gamma_\alpha \neq 0$	1.69	1.69	1.69	1.69	1.69	1.69	1.69	1.70
Tangency portfolios									
Mean		9.74	6.64	16.36	19.11	21.37	22.37	22.93	23.68
t-stat		3.10	2.27	4.45	6.00	6.06	6.66	7.10	7.43
Sharpe		0.49	0.33	0.82	0.96	1.07	1.12	1.15	1.18

information up to time t. The portfolio weights are re-scaled to target 1% monthly volatility based on the historical estimate. The performance of this portfolio indicates how well the ARMs align with the exposures to the common factors and whether those exposures are priced.

5.2 IPCA Estimation Results

Panel A of Table 15 summarizes the in-sample results of both restricted and unrestricted versions of the IPCA models with varying number of latent factors. The models are estimated over the whole sample. The first segment of each panel captures the results using individual stocks. The second segment describes the results using the managed portfolios. The third segment then reports the results of the test regarding the zero alpha assumption.

We see that the test rejects the null hypothesis of non-significant alphas for the first five IPCA specifications. The predictive R^2s suggest that there is little difference between the restricted and non-restricted model for the IPCA(3) models. However, it is difficult to assess the importance of those differences as only a small increase of R^2 may lead to large investment gains. They may play an even bigger role if we look at the out-of-sample results.

Generally, the results are similar to the results obtained by Kelly et al. (2019) or Kelly et al. (2023) in sense that only a few instrumented latent factors are needed to explain the asset returns. These results suggest that if we let the ARMs explain the exposures into latent factors, their residual abnormal alpha returns vanishes. The main difference between my results and results obtained by Kelly et al. (2019) is that their dataset contains 36 characteristics and needs 6 latent factors to not reject the null hypothesis of $\Gamma_\alpha = 0$. In my case, I use only 11 characteristics and need the same number of factors to not reject the hypothesis.

The out-of-sample estimation proceeds the same as in the case of formation of the arbitrage portfolios. The models are estimated using an expanding window with 60-month initial period. Results regarding total and predictive R^2 hold similarly as in the case of the in-sample analysis. The results of the pure-alpha portfolios from Section 3 show that we have to include around six factors to eliminate statistically significant arbitrage returns. Those observations enable us to understand better the small differences of the predictive R^2s for the restricted and unrestricted models. Predictive R^2 for the restricted and unrestricted IPCA(5) models are 0.27 and 0.28, respectively, but the pure-alpha portfolio of the unrestricted model still delivers abnormal returns of 8.06% p.a. with significant t-statistics of 2.86. But once we get to seven latent factors, those arbitrage opportunities vanish.

These out-of-sample results are similar to the results of the bootstrap tests obtained from the in-sample analysis. We see that there is a need to include multiple latent factors to erase the significant effect of the ARM characteristics. This suggests that there is less duplicity in the information regarding the expected returns among the ARMs than one might expect. Proportion of the number of factors needed to eliminate arbitrage opportunity and the number of ARMs is more than a half.

Based on the performances of the tangency portfolios, the results also suggest that ARMs successfully proxy for the exposures to the common factors. Tangency portfolio yields up to around 1.15 Sharpe ratio. Right panel of Figure 2 captures the cumulative log return of those portfolios. We see that the tangency portfolios grow over the whole period without a noticeable sign of slowing down.

As a simple robustness check, I perform the out-of-sample analysis over two sub-intervals. Table 22 in Appendix C summarizes the out-of-sample results of the ARM-IPCA models using all stocks estimated separately in two disjoint time periods. The first period covers range between January 1968 and December 1993, the second period spans time between January 1994 and December 2018. Results regarding the tangency and arbitrage portfolios

Table 16: *Summary statistics of the ARM-IPCA factors.* The table reports summary statistics of the instrumented principal components from the IPCA(6) model. The factors are standardized to have an unconditional standard deviation of 20% p.a.

	In-sample			Out-of-sample		
Factor	Mean	Std. Dev.	Sharpe	Mean	Std. Dev.	Sharpe
1	4.10	33.72	0.12	2.02	34.24	0.06
2	6.94	18.35	0.38	3.69	18.04	0.20
3	3.84	13.80	0.28	5.76	13.53	0.43
4	0.37	9.72	0.04	5.70	10.91	0.52
5	8.92	8.58	1.04	8.35	9.32	0.90
6	3.56	7.06	0.50	-0.06	8.22	-0.01

are both in agreement with the results obtained over the whole period. Generally, the results are stable over disjoint periods, as the number of latent factors needed to eliminate the arbitrage opportunities is around six.

5.3 Factors and Characteristic Importance

This section delves further into the features of the latent factors of the ARM-IPCA model. Table 16 summarizes the latent factors from the ARM-IPCA(6) model. We see that the higher Sharpe ratios, both in-sample and out-of-sample, possess mostly higher-order (three and higher) factors. The first instrumented principal component, which explains the most time variation of the returns, leaves the predictive power to the other factors. This is a similar result as obtained by Lettau and Pelger (2020), which also reports high Sharpe ratios for higher-order factors.

Figure 8 from Appendix C shows loadings of the ARMs on the latent factors from the restricted IPCA(6). The first two factors are clearly related to the negative semibeta and predicted coskewness, respectively. The fifth factor, which possesses the highest Sharpe ratio both in- and out-of-sample, noticeably loads on tail risk beta and downside CIQ betas.

To formally assess the importance of each variable for the performance of the restricted IPCA model, I perform a bootstrap test proposed in Kelly et al. (2019). For given IPCA model with K latent factors, let the l^{th} row in the matrix $\Gamma_\beta = [\gamma_{\beta,1}, \ldots, \gamma_{\beta,L}]$ maps the l^{th} characteristic to the loadings on the K latent factors. The null hypothesis assumes that the l^{th} row is equal to zero, i.e., this characteristic does not proxy for the dynamics of the factor loadings. To test the hypothesis, I estimate the alternative model that admits the possibility of the contribution of the l^{th} characteristic, and form a Wald-type characteristic of the form $W_{\beta,l} = \hat{\gamma}'_{\beta,l}\hat{\gamma}_{\beta,l}$. I save the estimated model parameters, factors, and managed portfolio residuals. Then, I simulate a new bootstrap sample under the null hypothesis of $\gamma_{\beta,l}$ being equal to zero by resampling the returns of the characteristic-managed portfolios

Table 17: *Variable importance of the ARMs.* The table reports p-values (in %) of the bootstrap tests that given ARM does not significantly contribute to the in-sample ARM-IPCA model's fit. Data cover the period between January 1968 and December 2018.

	IPCA1	IPCA2	IPCA3	IPCA4	IPCA5	IPCA6	IPCA7	IPCA8
`coskew`	22.60	16.90	9.10	2.30	3.30	59.90	46.90	0.00
`cokurt`	17.70	18.30	9.80	4.80	7.10	55.60	1.90	0.50
`beta_down`	9.90	4.90	0.20	0.30	0.10	0.80	0.00	0.00
`down_corr`	0.00	3.00	18.40	7.30	9.20	13.80	33.30	64.90
`htcr`	0.00	4.20	0.10	0.80	0.40	0.00	0.30	0.00
`beta_tr`	97.80	8.60	18.80	21.70	0.00	0.00	0.00	0.00
`coentropy`	2.50	2.90	25.70	17.10	18.10	18.20	40.40	51.40
`cos_pred`	0.10	26.60	46.30	0.00	0.00	0.00	0.00	0.00
`beta_neg`	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
`mcrash`	49.40	6.40	2.80	3.60	2.90	1.40	4.70	8.90
`ciq_down`	75.40	8.90	13.30	4.00	0.00	0.00	0.00	0.00

using the wild bootstrap procedure and the estimated parameters. Using the new sample, I estimate the alternative model and form test statistic $\tilde{W}^b_{\beta,l}$. The resulting p-value of the test is calculated as the proportion of $\tilde{W}^b_{\beta,l}$ that exceeds $W_{\beta,l}$.

Table 17 reports simulated p-values for each variable and each specification of the IPCA model for both samples. We know that around 6 latent factors is needed to eliminate the arbitrage opportunity, so I focus on the IPCA(6) specification here. In this case, seven variables are highly significant and drive the explanatory power of the model – downside beta, hybrid tail covariance risk, predicted coskewness, negative semibeta, MCRASH, and downside CIQ beta.

5.4 ARMs and other Characteristics

In this section, I investigate how the ARMs relate to other characteristics that have been proven to be significant proxies for factor exposures. To do that, I use data from Freyberger et al. (2020) and Kim et al. (2020) and select 32 variables that were employed in Kelly et al. (2019). Those variables are: market beta (`beta`), assets-to-market (`a2me`), total assets (`at`), sales-to-assets (`ato`), book-to-market (`beme`), cash-to-short-term-investment (`c`), capital turnover (`cto`), ratio of change in property, plants and equipment to the change in total assets (`dpi2a`), earnings-to-price (`e2p`), cash flow-to-book (`freecf`), idiosyncratic volatility with respect to the FF3 model (`idiovol`), investment (`invest`), market capitalization (`lme`), turnover (`lturnover`), net operating assets (`noa`), operating accruals (`oa`), operating leverage (`ol`), price-to-cost margin (`pcm`), profit margin (`pm`), gross profitability (`prof`), Tobin's Q (`q`), price relative to its 52-week high (`rel_to_high_price`), return on net operating assets (`rna`), return on assets (`roa`), return on equity (`roe`), momentum (`cum_return_12_2`), intermediate momentum (`cum_return_12_7`), short-term reversal (`cum_return_1_0`), long-term

Table 18: *Correlations between original IPCA and ARM-IPCA factors.* The table reports correlations between IPCA latent factors estimated using set of original 32 variables and IPCA latent factors estimated using 11 ARMs.

	ARM-IPC1	ARM-IPC2	ARM-IPC3	ARM-IPC4	ARM-IPC5	ARM-IPC6
IPC1	0.43	-0.38	-0.38	0.05	-0.12	-0.11
IPC2	0.30	0.15	-0.13	-0.13	0.04	0.05
IPC3	-0.08	0.32	-0.03	0.02	-0.12	-0.10
IPC4	-0.28	-0.32	0.16	0.03	0.00	-0.11
IPC5	-0.23	0.22	0.26	-0.05	0.41	0.08
IPC6	-0.03	0.07	0.18	0.01	0.21	0.10

reversal (`cum_return_36_13`), sales-to-price (`s2p`), bid-ask spread (`spread_mean`), and unexplained volume (`suv`).[5]

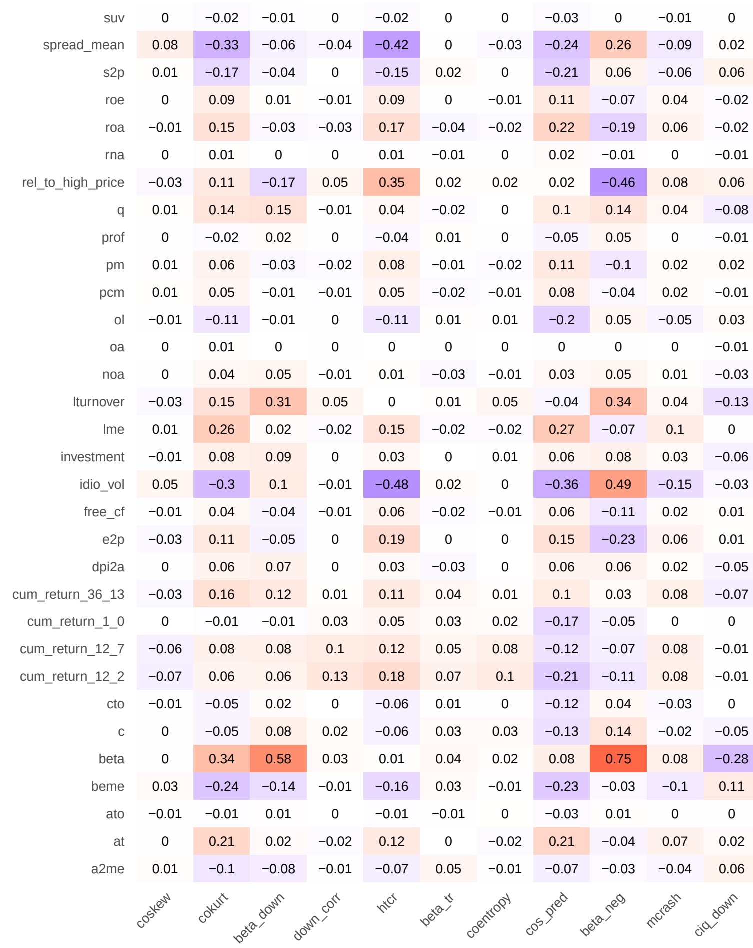
Figure 5 contains correlations between ARMs and characteristics used in Kelly et al. (2019). The highest correlation that we observe is between market beta and negative semibeta with an average value of 0.75, and market beta and downside beta with a value of 0.58. Both these correlations are expected to be quite high as their definitions are closely related. Negative semibeta is also highly correlated with idiosyncratic volatility with an average correlation of 0.49. Table 1 summarizes the average absolute correlations between each ARM and all other characteristics. We observe that the average values are noticeably lower than in the case of correlations with other ARMs. The lowest correlated ARMs are coskewness, downside correlation, tail risk beta and coentropy with value around 0.02. The highest average correlation possess negative semibeta with value of 0.13.

Right panel of Table 1 reports average correlations between returns of the ARM-managed portfolios and managed portfolios sorted on other characteristics. Naturally, we observe higher correlations than in the case of the raw variables. The highest correlations possess hybrid tail covariance risk and negative semibeta, the lowest average correlations possess tail risk beta.

Table 18 reports correlations between out-of-sample latent factors estimated using the original dataset of 32 variables and latent factors estimated using 11 ARMs. Generally speaking, there is only a little commonality between those two sets of factors. Only the first IPCs from the all-stock dataset are noticeably correlated with value of a 0.43. This observation suggests that the ARMs possess a specific common factor structure without a clear link to the structure obtained from the original dataset.

[5] Due to availability in the updated sample, I have omitted four variables relative to the original IPCA specification from Kelly et al. (2019). Those variables are: capital intensity (`d2a`) fixed costs-to-sales (`fc2y`) leverage (`lev`), the ratio of sales and price (`s2p`). None of the variables was shown to be significant in the baseline IPCA(5) specification.

Figure 5: *Correlations between ARMs and other characteristics.* The figure captures time-series averages of cross-sectional correlations between asymmetric risk measures and characteristics used in Kelly et al. (2019). Data include all available stocks and the period between January 1968 and December 2018.



5.5 Model with All Characteristics

Next, I investigate whether the ARMs possess additional information for the factor exposures over the variables that were previously employed. To do so, I estimate the restricted and unrestricted IPCA models that utilize both the original set of 32 variables of Kelly et al. (2019) and 11 additional ARM variables, hence All-IPCA. Table 23 from Appendix C reports the in-sample IPCA results. Based on the p-values of test that $\Gamma_\alpha = 0$, similarly as in the case ARM-IPCA, around six factors are needed to obtain an appropriate model that provides adequate description of the behavior of stock returns.

Table 19 reports p-values of the variable importance tests for each ARM. I focus on specifications with five and six latent factors due to their best fit. We can see that three ARM variables significantly contribute to the model performance: downside beta, hybrid tail covariance risk, and negative semibeta. These non-linear systematic measures of risk can

Table 19: *Variable importance results from the All-IPCA models.* The table reports p-values (in %) of the significance tests regarding the importance of the ARMs in relation to the restricted All-IPCA model fit. It also contains results regarding joint importance of the ARMs for the model fit. All-IPCA model is estimated using set of original 32 variables from Kelly et al. (2019) and 11 ARMs.

	coskew	cokurt	beta_down	down_corr	htcr	beta_tr	coentropy	cos_pred	beta_neg	mcrash	ciq_down	Joint test
All-IPCA(5)	6.8	28.7	0.6	28.6	1.8	8	22.5	16.9	2.2	58.6	26.1	6.7
All-IPCA(6)	24.2	37.3	2.5	23.9	2.4	11.7	26.2	8.2	1.2	94.9	17	6.8

significantly improve the description of the stock exposures to the common linear factors.

To assess how the ARMs contribute to the fit of the model as whole, I test whether ARMs jointly possess coefficients significantly different from zero. This is a generalization of the test discussed earlier which inspects the importance of each variable separately. The testing procedure follows the same logic based on wild bootstrap. One difference is the definition of the Wald-type test statistic. In this case, we test whether a subset of J characteristics contributes significantly to the performance, so the statistic is $W_{\beta,l_1,\ldots,l_J} = \hat{\gamma}'_{\beta,l_1}\hat{\gamma}_{\beta,l_1} + \ldots + \hat{\gamma}'_{\beta,l_J}\hat{\gamma}_{\beta,l_J}$. In the resampling procedure, restricted model then sets contribution to all J tested characteristics to zero. The logic behind the rest of the test is the same.

The resulting tests for the All-IPCA models with five and six latent factors possess mildly significant p-values of 6.7% and 6.8%, respectively. This result suggests that the ARMs can contribute to the explanation of the stock returns based on a common factor structure.

6 Conclusion

I investigate asymmetric risk measures that capture non-linear systematic behavior of stock returns. I present an approach to combining them into portfolios that enjoy abnormal returns without being subsumed by exposures to other common sources of risk. I also investigate how asymmetric risk measures relate to the joint factor structure while controlling for previously researched characteristics. I show that they possess significant information that explains the behavior of stock returns.

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A Appendix A – Definitions of the ARMs

This Appendix provides a brief exposition of the estimation process of each of the asymmetric risk measures employed in the main text. For further details regarding the nuances of the related computations, consult the original papers.

I use two sources of data to compute the asymmetric risk measures. First, I use either daily or monthly data of stock returns from the CRSP database. Second, I use the value-weighted return of the CRSP stocks from Kenneth French's online library to approximate the overall market return.

Variables are estimated using moving windows of various lengths following the procedures proposed in their original papers. In the case of measures estimated from the daily stock returns, I use mostly a moving window of one year. I require at least 200 daily observations during the window to be included. I estimate measures based on monthly return data using a window of at least 60 months and demand at least 36 monthly observations.

The measures are estimated following the definition proposed in the literature. In some cases, I slightly change the requirements regarding the minimal history of stocks to be included in the analysis. This modification aims at the precision of the estimates as well as the broadest possible dataset.

Throughout the section, I use $r_{i,t}$ and $r^e_{i,t}$ to denote a raw and excess return of an asset i at time t, respectively. The raw and excess market return is denoted by f_t and f^e_t. Corresponding variables with a bar denote their time-series averages computed in a given window.

A.1 Coskewness

Coskewness (`coskew`) of Harvey and Siddique (2000) is estimated using daily excess returns and is defined as

$$CSK_i = \frac{\frac{1}{T}\sum_{t=1}^{T}(r^e_{i,t} - \bar{r}^e_i)(f^e_t - \bar{f}^e)^2}{\sqrt{\frac{1}{T}\sum_{t=1}^{T}(r^e_{i,t} - \bar{r}^e_i)^2}\frac{1}{T}\sum_{t=1}^{T}(f^e_t - \bar{f}^e)^2}. \tag{20}$$

Estimation window is set to 1 year, at least 200 daily observations are required.

A.2 Cokurtosis

Cokurtosis (`cokurt`) of Dittmar (2002) is estimated using daily data and is defined as

$$CKT_i = \frac{\frac{1}{T}\sum_{t=1}^{T}(r_{i,t}^e - \bar{r}_i^e)(f_t^e - \bar{f}^e)^3}{\sqrt{\frac{1}{T}\sum_{t=1}^{T}(r_{i,t}^e - \bar{r}_i^e)^2}\frac{1}{T}\big(\sum_{t=1}^{T}(f_t^e - \bar{f}^e)^2\big)^{3/2}} \tag{21}$$

Estimation window is set to 1 year, at least 200 daily observations are required.

A.3 Downside Beta

Downside (`beta_down`) beta of Ang et al. (2006) is estimated using daily data and is defined as

$$\beta_i^{DR} = \frac{\sum\limits_{f_t^e < \bar{f}^e} (r_{i,t}^e - \bar{r}_i^e)(f_t^e - \bar{f}^e)}{\sum\limits_{f_t^e < \bar{f}^e} (f_t^e - \bar{f}^e)^2}. \tag{22}$$

Estimation window is set to 1 year, at least 200 daily observations are required.

A.4 Downside Correlation

Downside correlation (`down_corr`) based on Hong et al. (2006) and Jiang et al. (2018) is estimated using daily data and is defined as

$$\mathbb{C}or_i^- = \mathbb{C}or(r_i, f | r_i < 0, f < 0) - \mathbb{C}or(r_i, f | r_i > 0, f > 0) \tag{23}$$

using empirical counterpart of the correlation. Minimum of 200 observations in the 1-year window is demanded.

A.5 Hybrid Tail Covariance Risk

Hybrid tail covariance risk (`htcr`) of Bali et al. (2014) is estimated using daily data using 6-month window with at least 80 daily observations as

$$HTCR_i = \sum_{r_{i,t} < h_i} (r_{i,t} - h_i)(f_t - h_f) \tag{24}$$

where h_i and h_f are the 10% empirical quantiles of stock and market return, respectively.

A.6 Tail Risk Beta

Tail risk beta (`beta_tr`) of Kelly and Jiang (2014) is estimated using monthly return data using 120-month window with requirement of at least 36 monthly observations. Beta is computed by means of least-square estimator from the predictive regression of the form

$$r_{i,t+1} = \mu_i + \beta_i^{TR}\lambda_t + \epsilon_{t+1,i} \tag{25}$$

where the tail risk factor is obtained as

$$\lambda_t = \frac{1}{K_t}\sum_{k=1}^{K_t}\ln\frac{e_{k,t}}{u_t} \tag{26}$$

where $e_{k,t}$ is the kth daily idiosyncratic return that falls below an extreme value threshold u_t during month t, and K_t is the total number of such exceedences within month t. Idiosyncratic return is computed relative to 3-factor model of Fama and French (1993), and the threshold value is taken to be 5% quantile from the monthly cross-section of daily returns.

A.7 Exceedance Coentropy

Exceedance coentropy (`coentropy`) measure based on Backus et al. (2018) and Jiang et al. (2018) using daily data and 1-year estimation window with at least 200 observations is based on

$$C^{+}(0, r_i, f) = \frac{L(r_i f) - L(r_i) - L(f)}{L(r_i) + L(f)}\Big|(r_i > 0, y > 0) \tag{27}$$

$$C^{-}(0, r_i, f) = \frac{L(r_i f) - L(r_i) - L(f)}{L(r_i) + L(f)}\Big|(r_i < 0, y < 0) \tag{28}$$

where $L(x) = \ln\mathbb{E}(x) - \mathbb{E}(\ln x)$. The measure is then defined as

$$Coentropy = C^{-}(0, r_i, f) - C^{+}(0, r_i, f). \tag{29}$$

A.8 Predicted Systematic Coskewness

Predicted systematic coskewness (`cos_pred`) of Langlois (2020) is based on

$$Cos_{i,t} = \mathbb{C}ov_{t-1}\big(r_{i,t}, f_t^2\big), \tag{30}$$

then, each month I run the panel regression using all available stocks and history of data

$$F\big(Cos_{i,k-12\to k-1}\big) = \kappa + F\big(Y_{i,k-24\to k-13}\big)\theta + F\big(X_{i,k-13}\big)\phi + \epsilon_{i,k-12\to k-1} \tag{31}$$

where $Cos_{i,k-12\to k-1}$ is the coskewness from Equation 30 computed using daily returns from month $k-12$ to month $k-1$, $Y_{i,k-24\to k-13}$ are risk measures (volatility, market beta, etc.) estimated using daily data from month $k-24$ to month $k-13$, and $X_{i,k-13}$ are characteristics (size, book-to-price, etc.) observed at the end of month $k-13$. The function $F(x_{i,t}) = \frac{Rank(x_{i,t})}{N_t+1}$ transforms the original variable into its normalized rank in the cross-section of variable x_t, which posses N_t observations.

The predicted systematic coskewness for each stock is then obtained using the estimated coefficients of $\hat{\kappa}, \hat{\theta}, \hat{\phi}$ as

$$F\big(\widehat{Cos_{i,t\to t+11}}\big) = \hat{\kappa} + F\big(Y_{i,t-12\to t-1}\big)\hat{\theta} + F\big(X_{i,t-1}\big)\hat{\phi}. \tag{32}$$

The choice of risk measures and characteristics employed in the prediction of systematic skewness follows closely Langlois (2020).

A.9 Semibeta

Negative semibeta (`beta_neg`) of Bollerslev et al. (2021) is estimated using daily data with 1-year moving window as

$$\beta_i^N = \frac{\sum\limits_{r_{i,t}<0,f_t<0} r_{i,t}f_{i,t}}{\sum\limits_t f_t^2} \tag{33}$$

with the requirement of at least 200 daily observations.

A.10 Multivariate Crash Risk

Multivariate crash risk (`mcrash`) of Chabi-Yo et al. (2022) is estimated using daily data with 1-year window and minimum of 200 observations in the following steps. First, for each stock separately, using stock and N factor returns, I estimate $N+1$ GARCH(1,1) models of Bollerslev (1986) to obtain a series of conditional distribution functions $F_{i,t}(h) = \mathbb{P}_{t-1}[r_{i,t} \leq h]$ and use it to compute probability integral transforms as $\hat{u}_{i,t} = F_{i,t}(r_{i,t})$. Second, I estimate

MCRASH as

$$\text{MCRASH}_{i,t} = \frac{\sum_t \mathbb{I}(\{\hat{u}_{1,t} \leq p\}) \cdot \mathbb{I}(\cup_{j=2}^{N+1}\{\hat{u}_{j,t} \leq p\})}{\sum_t \mathbb{I}(\cup_{j=2}^{N+1}\{\hat{u}_{j,t} \leq p\})} \tag{34}$$

where $\mathbb{I}$ denotes the indicator function and p is set to 0.05. I follow the baseline specification of Chabi-Yo et al. (2022) and use the five factors of Fama and French (2015), momentum factor of Carhart (1997) and betting-against-beta factor of Frazzini and Pedersen (2014).

A.11 Downside CIQ Beta

Downside common idiosyncratic quantile risk beta (`ciq_down`) of Barunik and Nevrla (2022) is estimated using monthly data with 60-month window and requirement of at least 48 observations as

$$\beta_i^{down} = \sum_{\tau \in \tau_{down}} F\big(\beta_i(\tau)\big) \tag{35}$$

which gives the average cross-sectional rank of the common idiosyncratic quantile (CIQ) betas for downside τ CIQ factors. CIQ betas are estimated from time-series regression of stock returns on the increments of CIQ factors. The CIQ factors are estimated using residuals from Fama and French (1993) factors and following the quantile factor model of Chen et al. (2021).

B Appendix B – ARM Portfolio Returns

Table 20: *Quintile portfolio sorts.* The table contains annualized out-of-sample returns of five monthly rebalanced portfolios sorted on various asymmetric risk measures. It also reports returns of the high minus low (H - L) portfolios, HAC t-statistics of Newey and West (1987) with 6 lags, and annualized 6-factor alphas and their t-statistics with respect to the four factors of Carhart (1997), CIV shocks of Herskovic et al. (2016), and BAB factor of Frazzini and Pedersen (2014). Panel A reports results using all stocks, Panel B excludes stocks with a price less than $5 or market cap below 10% quantile of NYSE stocks. Data cover the period between January 1968 and December 2018.

Variable	Low	2	3	4	High	H-L	t-stat	α	t-stat
Panel A: All stocks									
Equal-weighted									
`coskew`	11.08	10.61	9.68	8.84	8.43	-2.65	-2.24	-1.69	-1.12
`cokurt`	11.69	10.44	9.84	8.90	7.78	-3.91	-2.40	-1.10	-0.63
`beta_down`	11.08	9.88	9.76	9.91	8.01	-3.07	-1.44	0.64	0.47
`down_corr`	8.73	9.51	10.14	9.96	10.31	1.57	1.98	0.99	0.97
`htcr`	9.99	9.00	9.98	9.93	9.75	-0.24	-0.12	-1.63	-0.74
`beta_tr`	8.20	9.07	9.44	10.48	11.47	3.27	2.36	3.10	1.47
`coentropy`	9.11	9.35	9.77	10.01	10.40	1.29	1.61	0.87	0.83
`cos_pred`	12.43	10.60	9.16	8.51	7.95	-4.48	-1.78	-4.69	-1.79
`beta_neg`	9.67	10.40	10.38	10.02	8.17	-1.50	-0.43	3.30	1.84
`mcrash`	10.03	9.84	9.47	10.00	9.91	-0.13	-0.13	0.18	0.19
`ciq_down`	7.16	9.51	10.22	10.22	11.54	4.38	2.98	5.51	3.67
Value-weighted									
`coskew`	6.93	7.57	7.66	6.18	4.47	-2.46	-1.60	1.39	0.71
`cokurt`	5.30	7.26	6.89	6.62	5.74	0.45	0.24	4.34	2.57
`beta_down`	5.92	7.05	6.69	6.34	5.18	-0.74	-0.27	1.51	0.70
`down_corr`	5.70	5.10	6.97	7.41	7.91	2.21	1.84	-1.35	-0.93
`htcr`	5.79	5.66	6.37	6.57	5.92	0.13	0.06	1.10	0.65
`beta_tr`	4.34	5.98	7.11	7.72	8.88	4.54	2.59	5.85	2.58
`coentropy`	4.73	6.05	6.62	7.25	7.71	2.98	2.18	-0.64	-0.42
`cos_pred`	11.66	8.56	8.10	6.43	5.57	-6.09	-2.31	-3.42	-1.44
`beta_neg`	7.06	6.56	6.60	5.94	2.85	-4.21	-1.17	-0.65	-0.31
`mcrash`	4.99	7.04	6.64	6.02	6.42	1.43	1.05	-0.07	-0.04
`ciq_down`	5.18	5.68	7.07	7.07	8.08	2.90	1.52	4.27	2.49
Pabel B: No penny stocks									
Equal-weighted									
`coskew`	9.30	9.06	8.83	7.98	6.59	-2.71	-2.24	-0.84	-0.56
`cokurt`	8.07	9.08	8.53	8.49	7.58	-0.48	-0.34	2.36	1.76
`beta_down`	8.26	8.64	9.09	9.14	6.62	-1.63	-0.66	0.96	0.69
`down_corr`	6.83	7.97	8.97	8.73	9.25	2.42	2.76	0.48	0.51
`htcr`	5.65	8.12	8.90	9.89	9.19	3.54	2.82	3.24	2.72
`beta_tr`	6.10	8.38	8.26	9.26	9.75	3.64	2.62	1.70	1.07
`coentropy`	7.12	8.07	8.94	8.77	8.85	1.73	1.98	0.05	0.05
`cos_pred`	9.68	8.58	8.16	7.75	7.59	-2.09	-0.94	-1.03	-0.56
`beta_neg`	8.82	9.38	9.39	9.03	5.15	-3.67	-1.21	-0.21	-0.16
`mcrash`	7.47	7.75	8.64	8.62	9.13	1.66	1.71	1.49	1.54
`ciq_down`	5.31	8.85	9.09	8.91	9.59	4.28	2.64	5.17	3.71
Value-weighted									
`coskew`	6.68	6.99	7.42	7.14	4.23	-2.44	-1.64	1.25	0.70
`cokurt`	5.93	6.73	5.97	7.16	5.53	-0.40	-0.25	3.51	2.31
`beta_down`	6.01	7.09	7.02	5.57	5.31	-0.71	-0.27	1.30	0.69
`down_corr`	5.49	5.31	6.69	7.47	7.72	2.23	1.92	-1.37	-0.96
`htcr`	4.92	6.42	6.82	6.11	6.00	1.09	0.71	1.89	1.20
`beta_tr`	4.86	6.24	6.48	7.48	8.21	3.36	2.10	3.61	1.77
`coentropy`	4.99	5.99	6.34	7.40	7.43	2.44	1.93	-1.13	-0.80
`cos_pred`	9.76	7.75	7.03	5.47	5.86	-3.90	-1.65	-0.66	-0.31
`beta_neg`	6.52	6.54	6.69	5.42	3.60	-2.92	-0.92	0.37	0.20
`mcrash`	5.98	6.20	6.32	5.81	6.34	0.35	0.28	-1.00	-0.63
`ciq_down`	4.74	5.66	6.45	6.96	7.67	2.93	1.54	3.58	2.34

Table 21: *Decile portfolio sorts.* The table contains annualized out-of-sample returns of ten monthly rebalanced portfolios sorted on various asymmetric risk measures. It also reports returns of the high minus low (H - L) portfolios, HAC t-statistics of Newey and West (1987) with 6 lags, and annualized 6-factor alphas and their t-statistics with respect to the four factors of Carhart (1997), CIV shocks of Herskovic et al. (2016), and BAB factor of Frazzini and Pedersen (2014). Panel A reports results using all stocks, Panel B excludes stocks with a price less than \$5 or market cap below 10% quantile of NYSE stocks. Data cover the period between January 1968 and December 2018.

Variable	Low	2	3	4	5	6	7	8	9	High	H-L	t-stat	α	t-stat
Panel A: All stocks														
Equal-weighted														
`coskew`	11.46	10.70	10.58	10.64	10.08	9.29	9.21	8.47	9.40	7.46	-4.00	-2.86	-2.62	-1.44
`cokurt`	12.34	11.04	10.69	10.19	9.60	10.08	8.97	8.82	8.33	7.23	-5.11	-2.65	-2.14	-1.06
`beta_down`	11.82	10.35	9.85	9.91	10.08	9.44	9.78	10.04	8.79	7.24	-4.58	-1.79	0.17	0.11
`down_corr`	9.26	8.21	9.49	9.52	9.46	10.81	9.65	10.27	10.45	10.16	0.90	0.91	0.25	0.20
`htcr`	10.90	9.07	9.25	8.76	9.96	9.99	9.51	10.35	10.10	9.40	-1.51	-0.61	-3.06	-1.13
`beta_tr`	8.40	7.99	9.03	9.10	9.69	9.18	10.17	10.79	11.40	11.53	3.13	1.71	3.25	1.19
`coentropy`	9.56	8.66	9.40	9.29	10.00	9.54	10.46	9.56	10.92	9.89	0.33	0.35	-0.24	-0.19
`cos_pred`	13.10	11.75	11.17	10.04	8.95	9.37	8.22	8.80	8.31	7.59	-5.52	-1.77	-5.71	-1.80
`beta_neg`	9.18	10.15	10.27	10.53	10.03	10.74	10.54	9.51	8.90	7.44	-1.74	-0.41	4.30	1.84
`mcrash`	9.86	8.71	10.48	9.85	8.22	9.38	11.47	9.77	9.06	10.55	0.68	0.53	0.61	0.50
`ciq_down`	6.33	7.99	9.03	10.00	9.90	10.55	10.03	10.40	11.51	11.57	5.24	2.97	5.71	3.14
Value-weighted														
`coskew`	8.06	6.67	7.66	7.66	8.21	6.97	6.31	6.20	5.75	2.78	-5.28	-2.92	-0.93	-0.39
`cokurt`	6.94	4.52	8.07	7.03	6.33	7.36	6.32	6.80	7.04	5.35	-1.59	-0.72	3.44	1.65
`beta_down`	6.75	6.37	7.02	7.23	6.96	6.36	6.20	6.85	5.83	4.42	-2.33	-0.64	0.23	0.08
`down_corr`	5.01	6.02	5.42	4.86	6.05	8.34	7.16	7.83	8.74	6.73	1.72	1.13	-2.67	-1.80
`htcr`	5.94	5.47	6.20	5.35	6.39	6.39	6.18	6.89	7.20	5.08	-0.86	-0.31	0.35	0.15
`beta_tr`	5.01	3.94	5.50	6.56	7.10	7.32	8.03	7.55	8.66	8.75	3.75	1.63	4.43	1.54
`coentropy`	4.60	4.95	5.86	6.29	6.07	7.04	7.13	7.66	8.44	6.83	2.23	1.41	-2.33	-1.40
`cos_pred`	12.52	11.04	9.85	7.50	8.67	7.85	7.15	6.14	5.24	5.81	-6.70	-2.05	-4.53	-1.44
`beta_neg`	7.03	7.59	7.04	6.34	6.46	6.92	6.01	6.02	4.67	-0.62	-7.65	-1.77	-4.11	-1.58
`mcrash`	5.00	4.62	9.77	5.85	6.65	6.32	5.74	7.07	6.22	7.02	2.02	0.97	-0.28	-0.12
`ciq_down`	3.05	6.65	5.36	5.85	6.04	7.90	6.50	7.68	7.84	9.75	6.70	2.55	7.60	2.94
Panel B: No penny stocks														
Equal-weighted														
`coskew`	9.51	9.09	9.25	8.87	8.78	8.87	8.25	7.71	7.38	5.80	-3.72	-2.52	-1.32	-0.70
`cokurt`	8.07	8.07	8.65	9.51	8.71	8.34	8.66	8.33	8.27	6.90	-1.16	-0.67	2.11	1.32
`beta_down`	7.97	8.55	8.26	9.01	9.35	8.82	8.82	9.47	7.78	5.46	-2.50	-0.80	0.73	0.42
`down_corr`	6.64	7.02	7.38	8.55	8.15	9.78	8.64	8.83	9.29	9.22	2.57	2.28	0.27	0.23
`htcr`	4.74	6.57	7.98	8.25	9.12	8.68	9.10	10.68	9.46	8.93	4.18	2.73	3.98	2.65
`beta_tr`	4.73	7.48	8.05	8.72	8.32	8.19	9.22	9.30	9.39	10.10	5.37	3.04	3.52	1.71
`coentropy`	7.10	7.14	7.68	8.46	8.49	9.38	8.90	8.64	8.75	8.95	1.85	1.64	-0.16	-0.14
`cos_pred`	10.47	8.88	8.48	8.68	8.16	8.15	8.18	7.32	8.42	6.76	-3.71	-1.36	-2.61	-1.14
`beta_neg`	9.01	8.62	9.44	9.32	9.10	9.67	9.69	8.37	7.78	2.52	-6.50	-1.74	-2.14	-1.31
`mcrash`	7.35	7.06	9.28	8.12	7.68	12.10	7.38	12.21	8.37	9.30	1.95	1.62	1.60	1.36
`ciq_down`	4.24	6.38	8.86	8.85	8.84	9.34	9.42	8.40	9.40	9.77	5.53	2.77	5.91	3.27
Value-weighted														
`coskew`	6.85	6.99	6.99	7.34	6.59	8.00	7.29	7.00	4.36	4.36	-2.49	-1.37	2.47	1.15
`cokurt`	6.35	5.56	6.88	6.71	6.46	5.62	7.49	7.03	6.12	5.25	-1.10	-0.56	2.87	1.64
`beta_down`	5.41	6.88	6.38	7.54	6.66	7.40	6.24	5.52	6.13	4.29	-1.12	-0.32	1.51	0.62
`down_corr`	5.94	5.20	5.29	5.21	6.01	7.48	7.93	6.92	8.42	6.88	0.94	0.66	-3.22	-2.16
`htcr`	4.14	5.49	6.15	6.72	6.29	7.05	6.28	6.36	7.12	4.98	0.84	0.39	2.11	1.14
`beta_tr`	4.02	5.23	5.32	7.17	6.69	6.50	7.13	7.77	8.37	8.95	4.94	2.39	5.21	1.99
`coentropy`	5.43	4.89	5.65	6.26	5.59	7.03	7.81	7.47	8.10	6.69	1.25	0.79	-3.11	-1.84
`cos_pred`	11.30	9.00	6.74	8.29	7.43	6.62	6.30	4.86	5.93	5.88	-5.43	-1.92	-2.24	-0.79
`beta_neg`	7.40	6.11	6.86	6.47	6.97	6.52	6.35	4.97	4.34	1.93	-5.46	-1.31	-2.17	-0.90
`mcrash`	4.94	7.09	6.40	7.63	6.04	9.35	3.51	9.83	6.12	6.48	1.54	0.91	-0.50	-0.25
`ciq_down`	3.20	5.75	5.85	5.79	6.88	6.57	7.16	7.07	7.33	8.70	5.50	2.37	5.09	2.44

C Appendix: IPCA Estimation Results

This Appendix provides some estimation results of the ARM-IPCA models.

Table 22: *Out-of-sample ARM-IPCA results using all stocks and split samples.* The table reports out-of-sample results of the IPCA models with varying numbers of latent factors and using ARMs as the instruments. Models are estimated with an expanding window and a 60-month initial period. Tangency portfolios are based on the restricted IPCA model, pure-alpha portfolios on the unrestricted model. I include all available stocks. The first period covers the interval between January 1968 and December 1993, the second period January 1994 and December 2018.

		IPCA(K)							
		1	2	3	4	5	6	7	8
Panel A: Period 1/1968-12/1993									
Individual stocks									
Total R^2	$\Gamma_\alpha = 0$	19.09	20.43	20.87	21.30	21.51	21.61	21.72	21.81
	$\Gamma_\alpha \neq 0$	18.99	20.42	20.87	21.29	21.51	21.60	21.71	21.80
Predictive R^2	$\Gamma_\alpha = 0$	0.11	0.10	0.11	0.15	0.16	0.16	0.16	0.16
	$\Gamma_\alpha \neq 0$	0.16	0.17	0.16	0.17	0.17	0.17	0.17	0.16
Managed portfolios									
Total R^2	$\Gamma_\alpha = 0$	97.49	98.99	99.43	99.74	99.82	99.86	99.91	99.94
	$\Gamma_\alpha \neq 0$	96.87	98.76	99.39	99.68	99.79	99.84	99.90	99.93
Predictive R^2	$\Gamma_\alpha = 0$	0.59	0.57	0.60	0.68	0.69	0.70	0.70	0.69
	$\Gamma_\alpha \neq 0$	0.68	0.69	0.69	0.70	0.70	0.70	0.70	0.70
Tangency portfolios									
Mean		8.34	3.62	13.45	21.59	21.28	21.43	24.19	23.68
t-stat		1.85	0.83	2.92	4.15	3.90	4.37	5.31	5.07
Sharpe		0.42	0.18	0.67	1.08	1.06	1.07	1.21	1.18
Pure-alpha portfolios									
Mean		16.89	25.17	20.85	8.98	9.99	9.12	3.63	-1.54
t-stat		3.87	5.83	4.72	2.31	2.54	2.26	0.78	-0.38
Sharpe		0.84	1.26	1.04	0.45	0.50	0.46	0.18	-0.08
Panel B: Period 1/1994-12/2018									
Individual stocks									
Total R^2	$\Gamma_\alpha = 0$	14.71	16.06	16.81	17.49	17.73	17.86	17.98	18.08
	$\Gamma_\alpha \neq 0$	14.72	15.94	16.62	17.48	17.72	17.86	17.98	18.08
Predictive R^2	$\Gamma_\alpha = 0$	0.19	0.19	0.19	0.25	0.24	0.25	0.25	0.25
	$\Gamma_\alpha \neq 0$	0.25	0.25	0.24	0.24	0.24	0.24	0.24	0.24
Managed portfolios									
Total R^2	$\Gamma_\alpha = 0$	94.47	97.52	98.79	99.54	99.69	99.79	99.86	99.90
	$\Gamma_\alpha \neq 0$	94.24	96.29	97.27	99.51	99.66	99.76	99.85	99.89
Predictive R^2	$\Gamma_\alpha = 0$	2.25	2.21	2.22	2.24	2.24	2.25	2.24	2.24
	$\Gamma_\alpha \neq 0$	2.26	2.25	2.24	2.24	2.23	2.24	2.24	2.24
Tangency portfolios									
Mean		9.72	9.65	13.06	21.65	22.42	23.47	21.82	23.28
t-stat		2.00	2.17	2.27	3.64	3.70	3.69	3.45	3.79
Sharpe		0.49	0.48	0.65	1.08	1.12	1.17	1.09	1.16
Pure-alpha portfolios									
Mean		13.38	15.61	17.70	11.71	10.72	9.30	2.56	0.40
t-stat		2.67	3.01	3.74	2.49	2.23	1.93	0.59	0.09
Sharpe		0.67	0.78	0.88	0.59	0.54	0.47	0.13	0.02

Figure 7: *Factor loadings of the ARM-IPCA(6) model.* The figure reports columns of the estimated Γ_β IPCA matrix with six latent factors and ARMs as instruments. Results are based on the in-sample analysis. Data cover the period between January 1968 and December 2018.

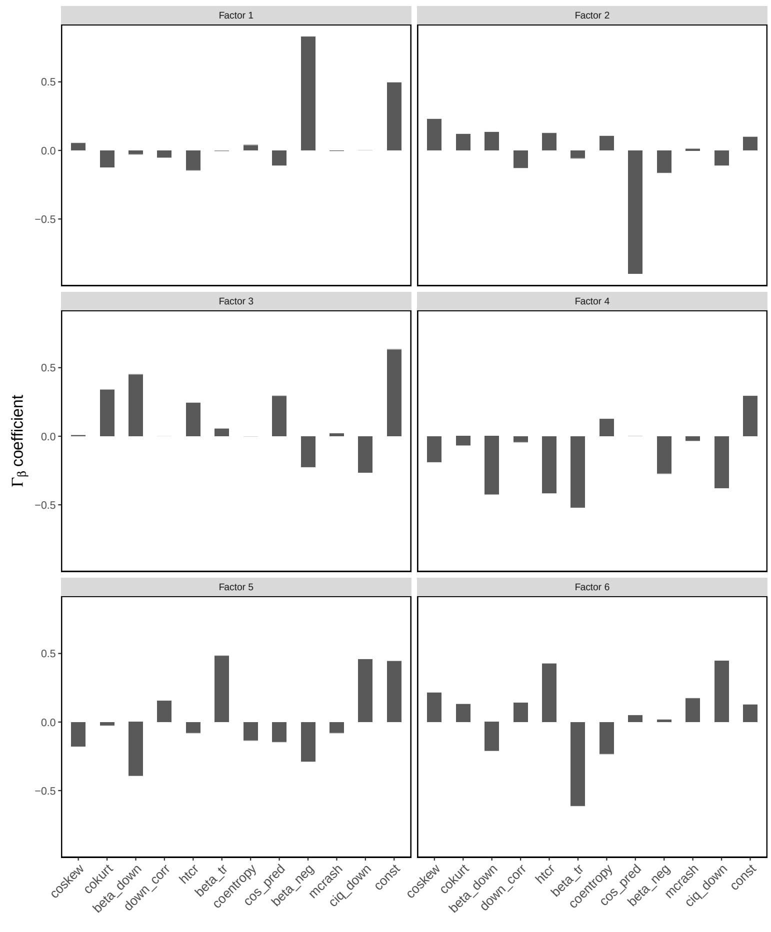

Figure 8: *Alphas of the ARM-IPCA models .* The figure reports estimated Γ_α vectors for IPCA models with number of latent factors between one and six and ARMs as instruments. Results are based on the in-sample analysis. Data cover the period between January 1968 and December 2018.

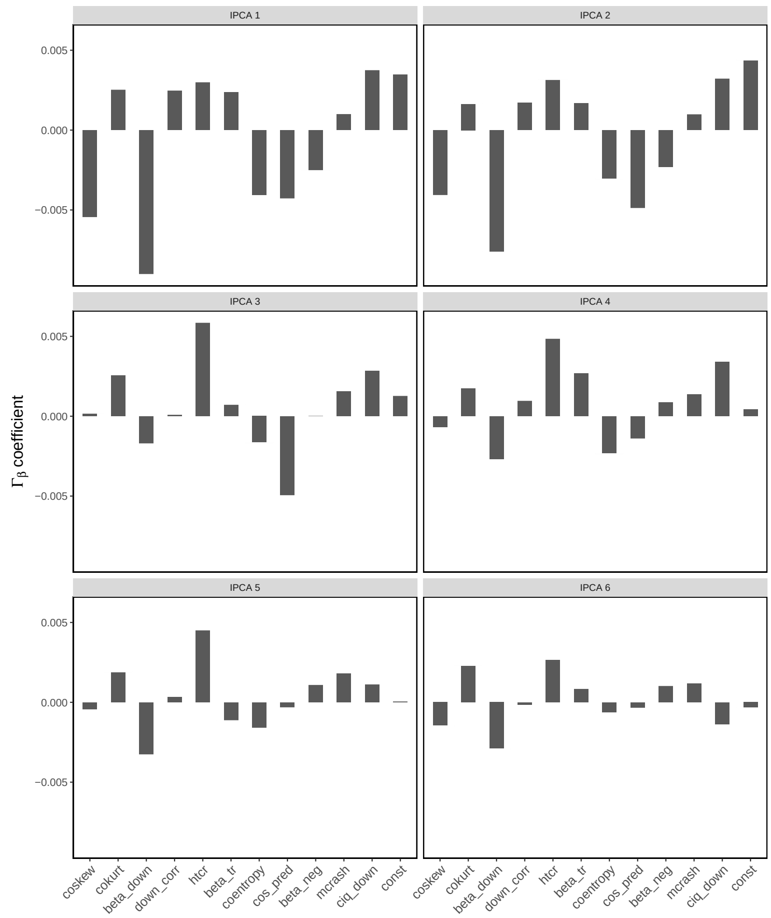

Table 23: *All-IPCA results.* The table reports in-sample estimation results of the IPCA models with varying numbers of latent factors and using 11 ARMs and 32 characteristics from Kelly et al. (2019) as the instruments. The asset pricing test reports p-values of the null hypothesis that $\Gamma_\alpha = 0$. Data cover the period between January 1968 and December 2018.

		IPCA(K)							
		1	2	3	4	5	6	7	8
		Panel A: All stocks							
Individual stocks									
Total R^2	$\Gamma_\alpha = 0$	16.54	18.28	19.46	20.13	20.66	21.01	21.28	21.48
	$\Gamma_\alpha \neq 0$	16.94	18.65	19.77	20.42	20.79	21.05	21.32	21.51
Predictive R^2	$\Gamma_\alpha = 0$	0.35	0.35	0.41	0.42	0.65	0.67	0.66	0.67
	$\Gamma_\alpha \neq 0$	0.73	0.72	0.71	0.71	0.70	0.70	0.69	0.69
Managed portfolios									
Total R^2	$\Gamma_\alpha = 0$	89.35	94.89	96.74	97.95	98.29	98.77	99.07	99.22
	$\Gamma_\alpha \neq 0$	89.90	95.29	96.89	98.08	98.57	98.79	99.10	99.24
Predictive R^2	$\Gamma_\alpha = 0$	1.61	1.63	1.77	1.82	2.02	2.03	2.02	2.04
	$\Gamma_\alpha \neq 0$	2.21	2.15	2.13	2.12	2.10	2.08	2.07	2.07
Asset pricing test									
W_α p-value		0.10	0.00	0.00	0.00	3.90	71.80	27.70	61.90

Figure 6: *Correlation structure across ARM-managed portfolios.* The figure captures the time-series correlations between managed portfolios sorted on various asymmetric risk measures. Data cover the period between January 1968 and December 2018.