

COMPSCI639_Undergr...
PDF

I need to do this assignment: Each week you have an assigned weekly reading to which you will submit a response on Gradescope. These responses will be used to shape the week's lecture and discussion.
Submit by 12pm central time on Monday of the week due.

Possible response types:

Questions

Critiques or suggestions for extensions.

What you want to learn more about.

Thoughts on what you find most important.

Simply summarizing the reading will not receive credit. If you include a summary or your own notes, please put it below your questions / critiques / etc. In general, there is not a specific required length or number of responses to submit but I have some guidelines:

I try to read all responses before preparing class sessions for the week. With that in mind, concise and to-the-point responses are appreciated. I suggest you put your most important questions and thoughts first.

For your grade, I'm looking for evidence that the reading was completed so it will be helpful to have some mention of a variety of the read sections throughout your responses. If your response only concerns the first section of a multi-section reading, it is difficult for me to determine that you read the entire assigned reading. You don't have to reference everything in the reading but reference to a variety of parts of it are necessary for full points.

With the above in mind, depending on the length of the assignment, 3-5 response points (questions / critiques / etc) will be sufficient.

Excessive use of GenAI will result in a zero for reading

responses.

The following are comments / questions from students in my reinforcement learning class that can serve as a useful model for crafting your responses.

In the plots on page 29 which compare the (%optimal arm) of the epsilon-greedy strategies for different epsilon, the plot clearly shows that $\text{eps}=0.1$ outperforms $\text{eps}=0.01$. However this is a function of the horizon: if we took say 10^9 steps instead of 10^4 , then $\text{eps}=0.1$ will asymptote at playing the optimal arm with at most 0.9 probability, while $\text{eps}=0.01$ will eventually play the optimal arm with probability 0.99, which is obviously better, and with this many steps, the fact that $\text{eps}=0.1$ does better for the first 1000 would be insignificant. Thus it seems like a better strategy would be to be epsilon-greedy for a decaying epsilon. From a more theoretical perspective, any fixed epsilon-greedy strategy will incur linear regret, but maybe there is a sublinear regret bound for playing epsilon-greedy strategy with appropriate decay of epsilon?

I would love more insight about why baselines work. Is it easy to determine what the optimal baseline is, and does it have a simple interpretation?

I don't know anything about Thompson sampling; I'd be very interested in what these algorithms look like. I'm aware that for example UCB motivates algorithms for more general RL settings; is the same true of Thompson sampling?

In section 2.3, we can see how various values of epsilon impact how well an agent learns. As mentioned, the task (eg. variance of the reward for each arm) can determine how well each epsilon value does. Then, is there some special reason why the ten armed test bed used ten arms and drew the rewards from a standard gaussian? ie. Would we see much different behavior if we used a different k or drew rewards from a different distribution? "The reward signal is your way of communicating to the agent what you want achieved, not how you want it achieved". I can see what the authors mean by this, but

how would we enforce certain behaviors for very sparse rewards (eg. winning a chess match)? Wouldn't having smaller rewards for achieving subgoals (eg. taking opposing pieces) be helpful?

on this pdf file:

Got it — this is a **reading response**, not a summary, and it needs to sound like you *actually engaged with the Bayes Filter → Kalman Filter material*, across **multiple sections** of the notes. Below is a **clean, Gradescope-ready response** with **5 concise but substantive points**, modeled closely on the RL examples your instructor shared.

You can paste this directly. It references **state estimation, recursive Bayes filtering, limitations, smoothing, and Kalman filters**, which checks the “variety of sections” box.

Reading Response (Bayes Filter & Kalman Filter)

1. The motivation for introducing *belief distributions* instead of point estimates in Sections 3–4 was compelling, especially the argument that single observations are incomplete and noisy. However, I'm curious how practitioners decide *how expressive* the state representation should be in practice—since overly compact states risk violating the Markov assumption, while richer states quickly become intractable, as discussed in the naive belief computation section. How is this tradeoff typically managed in real robotic systems? COMPSCI639_Undergraduate Elect...
2. The recursive formulation of the Bayes filter (prediction + correction) is elegant and clearly more scalable than the direct Bayes rule expansion. That said, the prediction step explicitly increases uncertainty while the correction step reduces it—are there realistic settings where repeated noisy observations actually *increase* uncertainty overall? If so, how does this affect long-horizon belief stability?
 COMPSCI639_Undergraduate Elect...
3. The limitations listed in Section 6 raise an important concern: the Markov assumption and known transition/observation models are both quite strong. In

practice, model mismatch seems inevitable. Is there a principled way to detect when belief updates are becoming unreliable due to incorrect dynamics or sensor models, rather than just observing degraded downstream performance?

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4. I found the Bayes smoother example intuitive, especially the speech-in-noise analogy. Conceptually, it makes sense that future observations refine past beliefs, but I'm wondering how often smoothing is used online versus purely offline. Are there hybrid approaches that approximate smoothing in near-real-time for safety-critical systems? COMPSCI639_Undergraduate Elect...
5. The Kalman filter's linear-Gaussian assumptions enable exact inference and computational efficiency, but the inability to represent multimodal beliefs seems like a major limitation. In scenarios like global localization or perceptual aliasing, is the standard approach to abandon Kalman filtering entirely in favor of particle methods, or are there common extensions that partially address this while retaining efficiency? COMPSCI639_Undergraduate Elect...

If you want:

- a shorter 3-point version
- a more math-heavy version
- or something that sounds slightly more informal / more critical

just say the word and I'll tune it exactly to your instructor's style.

Sources



can you interpret the whole pdf file

Yes — I'll interpret it like a lecture explanation, not just summarize. Think of this as:

👉 What is this PDF trying to teach you conceptually?

👉 Why does it matter in robotics / ML / state estimation?

Big Picture: What This PDF Is About

The whole document is about **how robots figure out where they are and what's happening in the world when sensors are noisy.**

Core idea:

Instead of guessing the world state, maintain a **probability distribution over possible states** and update it over time using actions + sensor data.

This is called **probabilistic state estimation.**

Section-by-Section Interpretation

1 Robot Interaction Model → The World as a Probabilistic Process

Interpretation:

The robot world is modeled as:

SCSS

State → Action → New State → Observation → Repeat

Meaning:

- Robot acts → world changes probabilistically
- Robot senses → gets noisy measurement

 Copy code

Key idea:

You never know the true state directly — only noisy observations.

Why this matters:

This is literally how:

- self-driving cars
 - drones
 - SLAM systems
 - tracking systems
- all operate.
-

2 Why State Estimation Exists

Main philosophical message:

- 👉 Observations alone are not enough
- 👉 Full history is too big to use
- 👉 State is the **compressed summary of history**

They argue:

- Observations are incomplete (occlusion example)
- Using full history → exponential explosion
- State = minimal info needed to predict future

This is basically the **Markov worldview** of control + RL + robotics.

3 Belief Distributions (Core Concept)

Instead of:

css

"**I think I am at position x**"

We say:

 Copy code

arduino

"**I have probability distribution over positions**"

Formally:

 Copy code

cpp

bel(x_t) = P(x_t | past observations, past actions)

Why this is powerful:

 Copy code

- Represents uncertainty
- Lets robot hedge decisions
- Enables optimal control under noise

This is the same philosophy behind:

- POMDPs
 - Bayesian inference
 - Probabilistic ML models
-

4 Naive Bayes Belief Computation → Why It's Impossible

The naive equation requires summing over:

ALL possible state histories.

This grows exponentially with time.

Interpretation:

This section exists to justify why we need **recursive filtering**.

5 Bayes Filter (THE MAIN IDEA)

Core algorithm:

Two steps every timestep:

Prediction

"What do I expect to happen given my action?"

Adds uncertainty.

Correction

"Given new sensor data, how should I adjust belief?"

Reduces uncertainty.

Pseudocode basically says:

csharp

```
Predict using motion model  
Then multiply by observation likelihood  
Then normalize
```

This is the same structure as:

- Hidden Markov Models filtering
- POMDP belief updates
- Recursive Bayesian estimation

 Copy code

6 Why Bayes Filter Is Mostly Theoretical

Three huge problems:

✗ Too expensive computationally

Need to sum/integrate over all states.

✗ Markov assumption unrealistic

Real world often depends on long history.

✗ Need perfect models

Transition + sensor models rarely known exactly.

This is why real systems use approximations.

7 Bayes Smoother → Using the Future to Fix the Past

Interpretation:

Filtering = online belief

Smoothing = offline improved belief

Example:

You misheard a word → later context clarifies it.

In robotics:

- Offline trajectory estimation
- Post-processing SLAM maps

Key insight:

Future data improves past state estimates.

8 Continuous State → Enter Kalman Filter

Problem:

Bayes filter with continuous states requires integrals → impossible in general.

Solution:

Assume special case:

- ✓ Linear dynamics
- ✓ Gaussian noise

Then everything becomes linear algebra.

9 Kalman Filter = Efficient Bayes Filter (Special Case)

Key modeling assumption:

State update:

cpp

$$\mathbf{x}_t = \mathbf{A} \mathbf{x}_{t-1} + \mathbf{B} \mathbf{u}_t + \text{noise}$$

Observation:

 Copy code

cpp

$$\mathbf{z}_t = \mathbf{H} \mathbf{x}_t + \text{noise}$$

Interpretation:

 Copy code

Everything is linear + Gaussian noise → math stays Gaussian.

10 Kalman Filter Steps (Very Important Conceptually)

Prediction

Propagate mean + covariance forward.

Update

Blend prediction with measurement using Kalman Gain.

Kalman Gain = trust measurement vs prediction.

If sensor noisy → trust prediction more.

If sensor clean → trust measurement more.

This is extremely elegant.

1 1 Strengths vs Weaknesses

✓ Strengths

- Optimal (if assumptions true)
- Fast (polynomial time)
- Tracks uncertainty

✗ Weaknesses

- Only Gaussian distributions
- Cannot represent multiple hypotheses
- Fails for strong nonlinear systems

Example failure:

Robot unsure between two locations → Kalman cannot represent that.

Meta Interpretation: What This PDF Is REALLY Teaching

It's teaching a hierarchy of state estimation methods:

```
Full Bayes Inference (Impossible)
↓
Bayes Filter (General but expensive)
↓
Kalman Filter (Fast but restrictive)
↓
Later → Particle Filters (Flexible but approximate)
```

This is the standard progression in robotics + tracking theory.

If You're ML / RL Oriented (Important for You)

Connections to your background:

RL

 Copy code

Belief state \approx POMDP belief representation.

CV / Tracking

Kalman filter = classic object tracking baseline.

Control / Robotics

Used everywhere:

- IMU fusion
 - GPS fusion
 - SLAM
 - Sensor fusion
-

One-Sentence Core Takeaway

👉 The PDF teaches how to **recursively estimate hidden world state under uncertainty**, starting from general Bayesian reasoning and narrowing to practical algorithms like the Kalman filter.

If you want, I can also:

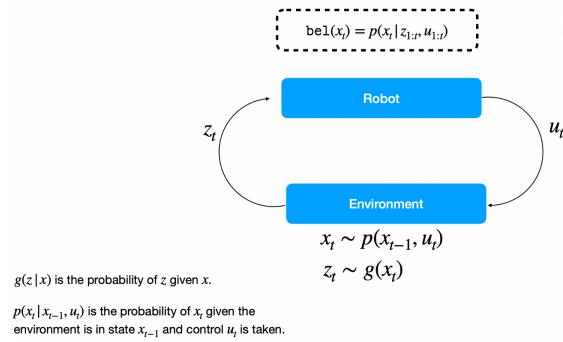
- Turn this into **reading-response-style interpretation**
- Give **ELI5 version**
- Give **math intuition version**
- Connect it directly to **SLAM / CV / tracking / RL POMDPs**

Just tell me 

Sources



Probabilistic Interaction Model



Nice — this slide is actually the **foundation of the entire PDF**. I'll interpret it like your professor would want you to understand it conceptually.

🧠 What This Slide Is Really Saying

This slide defines how robots interact with the world under uncertainty.

It's basically saying:

The robot does not know the world state.

It only gets noisy sensor observations.

So it must maintain a probability belief about the state.

⟳ The Loop: Robot ↔ Environment

The diagram shows a **closed loop system**:

Robot → Environment

Robot sends:

SCSS

u_t (control action)

Examples:

- move forward
- turn wheels
- apply motor torque
- change steering angle

 Copy code

Environment → Robot

Environment sends:

SCSS

z_t (sensor observation)

Examples:

 Copy code

- camera image
- lidar scan
- GPS reading
- IMU measurement

📦 Hidden Variable: The True State

Hidden inside environment:

ini

```
x_t = true world state
```

Examples:

 Copy code

- true robot position
- true velocity
- true map state
- true object location

Robot NEVER sees this directly.

🎯 The Most Important Equation on the Slide

Belief Definition

cpp

$$\text{bel}(x_t) = P(x_t \mid z_{1:t}, u_{1:t})$$

Translation:

 Copy code

- 👉 "Given everything I've seen and done so far..."
- 👉 What is the probability I am in state x_t ?"

This is literally the **Bayesian posterior over state**.

📈 Environment Dynamics (Motion Model)

State Evolution

cpp

$$x_t \sim p(x_t \mid x_{\{t-1\}}, u_t)$$

 Copy code

Meaning:

Next state depends on:

- previous state
- action taken
- randomness (noise)

Example:

If you tell a robot to move 1 meter:

- maybe it moves 0.98m
 - maybe 1.02m
 - maybe slips
-

Sensor Model

Observation Generation

cpp

$$z_t \sim g(z_t | x_t)$$

Meaning:

Sensor reading depends on true state + noise.

 Copy code

Example:

If true position = (10, 5):

GPS might return:

- (10.1, 4.9)
 - (9.7, 5.2)
 - etc.
-

Why This Model Exists (Deep Reason)

Without this framework:

Robot would have to:

- Trust sensors completely (bad)
- Ignore sensors (worse)

Instead:

- Fuse prediction + measurement probabilistically

This is exactly what:

- Kalman Filter
 - Particle Filter
 - HMM filtering
 - POMDP belief update
- all do.

How This Connects to the Rest of the PDF

This slide = Problem Definition

Then PDF builds:

Step 1

General Bayes inference → too expensive

Step 2

Recursive Bayes Filter → practical concept

Step 3

Kalman Filter → efficient special case

If I Explain It Like Robotics Intuition

Every timestep robot asks:

1 If my model is right, where should I be now?

(Prediction)

2 Given sensor reading, how wrong was I?

(Correction)

Repeat forever.



If I Explain It In ML Language (For You)

This is basically:

Hidden state model + sequential Bayesian inference.

Equivalent views:

| Field | Equivalent Concept |
|-------------------|---------------------------|
| RL | Belief state in POMDP |
| CV tracking | State estimator |
| Signal processing | Recursive Bayesian filter |
| Time series | State space model |

★ The One Sentence Meaning

👉 This slide formalizes the world as a **hidden state probabilistic dynamical system with noisy observations**, and defines the robot's job as recursively estimating the posterior over states.

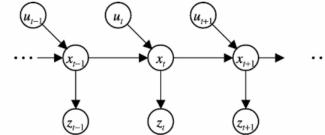
If you want, I can:

- Explain how this becomes Bayes Filter step-by-step
- Explain how this becomes Kalman Filter mathematically
- Show numerical example (like 1D robot position tracking)
- Or help turn this into reading response language

What would help most?



Dependency Graph



- Directed graphical model: nodes are variables and edges represent direct dependencies.
- Graph enables easy checking of conditional independence.
- Two nodes, A and B are conditionally independent given C if the node for C blocks all directed paths from A to B.*

* Represents a simplification of conditional independence check.

Josiah Hanna, University of Wisconsin – Madison

Good — this slide is **the math structure behind the probabilistic interaction model**.

If the last slide told you *what happens*, this slide tells you *how we reason about dependencies mathematically*.

🧠 Big Idea of This Slide

This is a **probabilistic graphical model** (specifically a Dynamic Bayesian Network / HMM-style chain).

It encodes:

- 👉 What variables directly influence what
- 👉 What can be ignored when computing probabilities

🔗 What Each Node Means

State Nodes (Middle Row)

cpp

$x_{\{t-1\}} \rightarrow x_{\{t\}} \rightarrow x_{\{t+1\}}$

These are **hidden states** (true world state).

 Copy code

Example:

- robot position
 - velocity
 - map location
 - object pose
-

Control Nodes (Top Row)

cpp

$$\begin{aligned} u_{\{t-1\}} &\rightarrow x_{\{t-1\}} \\ u_t &\rightarrow x_t \\ u_{\{t+1\}} &\rightarrow x_{\{t+1\}} \end{aligned}$$

Meaning:

State at time t depends on:

- previous state
- action taken

 Copy code

This encodes:

cpp

$$p(x_t | x_{\{t-1\}}, u_t)$$

 Copy code

Observation Nodes (Bottom Row)

cpp

$$x_t \rightarrow z_t$$

Meaning:

Observation depends ONLY on current state.

 Copy code

Encodes:

cpp

$p(z_t \mid x_t)$

 Copy code

⭐ The Most Important Concept: Conditional Independence

This slide is secretly teaching:

You don't need full history to reason about probability.

Key Property From Graph

From the graph:

pgsql

x_t depends only on x_{t-1} , u_t
NOT on x_{t-2} , z_{t-5} , etc

This is the **Markov Property**.

Mathematically:

 Copy code

cpp

$p(x_t \mid x_0:t-1, u_1:t)$
= $p(x_t \mid x_{t-1}, u_t)$

🔬 Why This Is Powerful (Computation)

 Copy code

Without graph:

Need to consider ALL past history.

With graph:

Only need:

- previous belief
- current action
- current observation

This is why recursive filtering works.

➡ Why This Enables Bayes Filter

Because dependencies are local:

You can do:

SCSS

Belief(t-1)

↓

Prediction

↓

Correction

↓

Belief(t)

Instead of:

CSS

All past states + all past observations + all actions

 Copy code
[copy link]

📊 Why They Mention “Blocking Paths”

They are hinting at **d-separation** intuition (but simplified).

Rule on slide:

A and B are independent given C if C blocks paths.

Example here:

If you know current state x_t :

Then:

Past observations $z_{\{t-1\}}$

and

Future observations $z_{\{t+1\}}$

Become independent.

Because everything must go through x_t .

Intuition Example

If you know:

Robot is exactly at position (10,5)

Then:

Old sensor readings give no extra info.

Future sensor readings depend only on current state.

How This Connects to Filters

This graph structure allows:

Filtering

Estimate current state using past data.

Prediction

Estimate future state.

Smoothing

Estimate past state using future data.

🧠 ML Connections (Important for You)

This graph is basically:

Hidden Markov Model structure

State chain + emissions.

State Space Model

Linear Gaussian → Kalman Filter.

POMDP Belief Graph

Belief = distribution over hidden state.

⭐ One Sentence Meaning

👉 This graph encodes the Markov structure that makes recursive Bayesian state estimation computationally feasible.

🧭 Why Your Professor Put This Slide

They want you to understand:

Without conditional independence → impossible inference

With graphical structure → efficient recursive inference

If you want, next I can explain:

- How to derive Bayes filter equation directly from this graph
- How this becomes Kalman filter math
- Or how to read independence directly from graph (exam style)

Which would you want next?



Conditional independence

Two random variables **A** and **B** are **conditionally independent given a third variable C** if, once the value of **C** is known, knowledge about **A** provides **no additional information** about **B**, and vice versa.

$$P(A,B|C)=P(A|C)\cdot P(B|C) \sim P(A|B,C)=P(A|C) \text{ when } P(B,C)>0.$$

$A \leftarrow C \rightarrow B$ independence Trivially satisfied fork

$A \rightarrow C \rightarrow B$ $P(B|A,C)=P(B|C)$ conditionally independent

Here, **C** blocks the (only) directed path from **A** to **B**.

Nice — this is a **core probability concept** and also one of the most confusing ones the first time you see it. I'll decode it step-by-step and tie it back to your robotics slides.

🧠 Plain English Meaning

The definition says:

A and B are conditionally independent given C if, once you know C, learning A tells you nothing new about B.

Think of it as:

css

Before knowing C → **A** and **B** might be related

After knowing C → **A** and **B** become unrelated

📐 Mathematical Meaning

 Copy code

Two equivalent ways to say conditional independence:

Factorization Form

mathematica

$$P(A,B | C) = P(A | C) \cdot P(B | C)$$

Meaning:

Given C, the joint probability splits into separate pieces.

Information Form

mathematica

$$P(A \mid B, C) = P(A \mid C)$$

Meaning:

Knowing B does not change probability of A once C is known.

Why This Matters in Graphical Models

Graph structure tells you:

- 👉 What information flows
- 👉 What gets blocked

The slide is showing **path blocking** intuition.

Case 1: Common Cause

Structure

css

$$A \leftarrow C \rightarrow B$$

Example:

nginx

`Weather` → Traffic
`Weather` → Umbrella usage

Before knowing weather:

Traffic and umbrellas look correlated.

After knowing weather:

They become independent.

⭐ Case 2: Chain Structure

Structure

css

A → C → B

Example:

nginx

`Engine health → Engine temperature → Warning light`

If you know temperature:

Engine health tells you nothing more about warning light.

Because all influence flows through C.

📦 “C Blocks Path” Meaning

Information flows through paths.

If you condition on the middle variable → path stops carrying info.

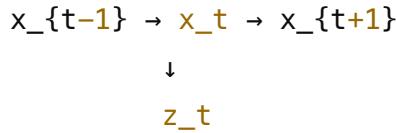
This is what your slide means by:

C blocks directed path from A to B.

🔬 Why This Is Critical for Bayes Filters

In your dependency graph:

cpp



Key results:

If you know x_t :

Past and future become independent.

 Copy code

Meaning:

kotlin

Past **data** \perp Future **data** | Current state

This is EXACTLY why filtering works.

 Copy code

🧠 Deep Intuition (Super Important)

Conditional independence is about:

- 👉 What you can safely ignore
- 👉 What you don't need to compute
- 👉 Why recursive inference is possible

Without it:

Inference cost explodes.

📊 Real Robotics Example

Suppose:

- A = past sensor readings
- B = future sensor readings

- C = current state estimate

Once you know current state:

Past sensors give no extra info about future sensors.

⭐ One Sentence Rule

Condition on the "middle cause" → endpoints become independent.

⚠ Important Caveat (Advanced But Useful)

There is a third pattern (not shown here) where conditioning **creates** dependence:

Collider:

css

A → C ← B

But your slide is focusing on the first two cases only.

 Copy code

⌚ Why Your Professor Put This Slide

They want you to understand:

- 👉 Graph structure → probability factorization
 - 👉 Probability factorization → efficient filtering
 - 👉 Efficient filtering → Bayes / Kalman filters work
-

If I Compress Everything

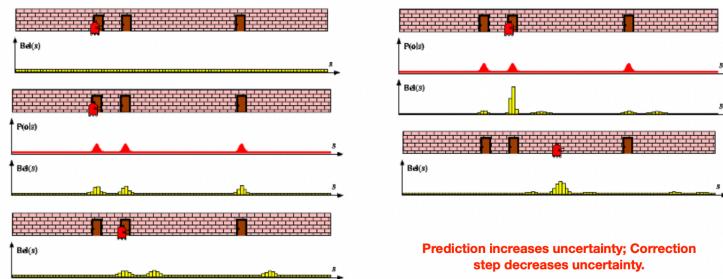
Conditional independence means:

Once you know the right variable, other variables stop giving extra information.

If you want, I can next explain:

- The **collider case** (super important later for ML + causal graphs)
- How to read independence directly from your dependency graph slide
- How this derives Bayes filter math

Just tell me 



This is a **great intuition slide** — it's showing visually what the Bayes filter is doing over time.

I'll walk through it like a story.

Big Message of This Slide

- 👉 Prediction step = spreads belief (more uncertainty)
- 👉 Correction step = sharpens belief (less uncertainty)

That red sentence at the bottom is literally the takeaway.

What You're Looking At

Each panel shows:

Top: hallway with doors = environment

Middle: red = observation likelihood $P(o|s)$

Bottom: yellow = belief over state $Bel(s)$

Where:

- s = robot position along hallway
 - Doors = landmarks sensor can detect
-

🟡 Yellow = Belief $Bel(s)$

This is:

arduino

Bel(s) = probability robot is at position s

If flat → robot is totally unsure

 Copy code

If spike → robot confident about location

If multiple spikes → ambiguous location

🔴 Red = Observation Model $P(o|s)$

This is:

arduino

Likelihood of seeing **this** sensor reading **if** robot were at position s

If sensor sees "door":

 Copy code

Then red spikes appear at all door positions.

Because:

If robot were at ANY door → that observation is likely.

📘 Left Side: Multiple Hypotheses (Ambiguity)

Step 1 — Uniform Belief

Robot starts totally unsure.

Flat yellow line.

Step 2 — Gets Observation

Sees something like "door".

Red likelihood peaks at door positions.

Step 3 — Correction Step

Belief becomes spikes at all door locations.

Robot knows:

"I am at a door"

But not WHICH door.

This is **multi-modal belief**.

Right Side: Movement + Observation

Now robot moves.

Prediction Step (Motion)

Robot moves forward → belief spreads.

Why?

Motion has noise:

- wheel slip
- motor error
- terrain variation

So each spike becomes wider.

👉 Prediction increases uncertainty.

Correction Step (New Observation)

Robot sees new landmark pattern.

Multiply prediction belief \times observation likelihood.

Only locations consistent with BOTH survive.

Belief collapses toward true location.

👉 Correction decreases uncertainty.



Why Prediction Always Increases Uncertainty

Prediction = convolution with motion noise.

Mathematically:

cpp

$$\text{Bel_pred} = \int P(x_t | x_{t-1}, u_t) \text{Bel_prev} dx$$

This spreads probability mass.

Copy code



Why Correction Decreases Uncertainty

Correction = multiply by likelihood.

cpp

$$\text{Bel_new} \propto P(z_t | x_t) \text{Bel_pred}$$

Multiplying:

- Suppresses wrong states

Copy code

- Amplifies consistent states
-

Intuition Analogy

Prediction = walking in fog

You get more uncertain.

Correction = seeing a landmark

You get more certain.

Why This Is Powerful

This explains why robots can localize using:

- noisy motion
- noisy sensors

Because uncertainty is constantly:

Expanded → Then reduced → Then expanded → Then reduced

Connection to Kalman Filter

Exact same idea but:

- Instead of spikes → Gaussian curve
- Instead of hallway → continuous state

Prediction:

Mean moves + covariance grows.

Correction:

Mean shifts + covariance shrinks.

One Sentence Summary

 Motion uncertainty spreads belief, sensor observations focus belief.

If you want, I can next explain:

- Why Bayes filters can represent **multiple spikes** but Kalman can't
- How this relates to **particle filters** (this slide is secretly particle-filter intuition)
- Or turn this into **reading response language**

What would help most?



explain each step again in detail

Sure — I'll go step-by-step through what the slide is showing, **exactly in the order the pictures go**, and what each plot means.

What the symbols/plots mean (quick reminder)

- s (x-axis): position along the hallway.
- **Yellow bars** = $Bel(s)$: your current belief (probability) you're at each position.
- **Red curve** = $P(o | s)$: how likely your *current sensor observation o* would be if you were at position s .
- **Doors** are landmarks: many different positions can produce similar sensor readings (that's what creates ambiguity).

LEFT COLUMN (starting uncertain → first correction → then prediction after motion)

Step L1 — Initial belief (top-left)

What you see: Yellow is basically flat.

Meaning:

The robot has **no idea** where it is initially.

So it assigns roughly equal probability to every position:

$$Bel_0(s) \approx \text{uniform}$$

Step L2 — Sensor likelihood for the current observation (middle-left, red)

What you see: Red has **several peaks** near door locations.

Meaning:

The robot observed something like "I see a door" (or a landmark pattern).

If the robot were standing at any door, that observation would be likely.

So the sensor model says:

$$P(o | s) \text{ is high at door positions, low elsewhere}$$

That's why you get multiple peaks: **several places could explain the observation.**

Step L3 — Correction step (middle-left, yellow below red)

What you see: Yellow becomes **peaks at doors**.

Meaning:

This is Bayes update / measurement update:

$$Bel(s) \propto P(o | s) \cdot Bel_{\text{prior}}(s)$$

Since the prior was uniform, multiplying by the likelihood basically gives you:

- high probability at door positions
- low probability elsewhere

Interpretation:

Robot now knows: "**I'm probably at a door.**"

But it still doesn't know *which* door → belief is **multi-modal** (multiple peaks).

Step L4 — Prediction step after motion (bottom-left)

What you see: Robot has moved forward; yellow peaks are still there but **more spread** and slightly shifted.

Meaning:

Robot took an action (e.g., "move right"). Motion is noisy, so each possible location spreads out.

Prediction equation:

$$Bel^-(s_t) = \sum_{s_{t-1}} P(s_t | s_{t-1}, u_t) Bel(s_{t-1})$$

What this does visually:

- **shifts** the belief in the direction of motion
- **widens** each peak (uncertainty grows)

Interpretation:

Robot thinks: "I moved forward, but I'm not sure exactly how far."

RIGHT COLUMN (new sensor update → belief collapses → then next prediction)

Step R1 — New sensor likelihood $P(o | s)$ (top-right, red)

What you see: Red again has peaks, but now the pattern matches a *different* place(s).

Meaning:

Robot took another observation after moving.

The sensor likelihood again says: "These positions are consistent with what I'm seeing now."

Sometimes the new observation is **more informative** (unique door spacing), sometimes not.

Step R2 — Correction step (middle-right, yellow below red)

What you see: Yellow becomes dominated by **one big spike** (or much fewer peaks).

Meaning:

Now you multiply:

$$Bel(s) \propto P(o | s) \cdot Bel^-(s)$$

Key idea:

- The prediction $Bel^-(s)$ already limited where you *could* be.
- The likelihood $P(o | s)$ says where the observation makes sense.
- Their overlap is usually much smaller.

So most hypotheses die off, one survives strongly.

Interpretation:

Robot goes from "could be at several doors" to "it's almost certainly this one."

That's why **correction decreases uncertainty**.

Step R3 — Next prediction (bottom-right, yellow only)

What you see: Yellow shifts again and widens again.

Meaning:

Robot moves again → we apply the motion model again:

- shift belief along hallway
- widen due to motion noise

Same logic as Step L4.

Why the slide's message is true **Prediction increases uncertainty**

Because you're applying a noisy motion model → it spreads probability.

 Correction decreases uncertainty

Because you multiply by the observation likelihood → it "selects" only consistent states and renormalizes.

A tiny mental model to keep it straight

- **Prediction:** "Where could I be after moving?" → spread out
- **Correction:** "Given what I see, where *must* I be?" → sharpen

If you want, I can also map each step to the exact Bayes filter pseudocode lines:

Prediction: $Bel^-(x_t) = \sum p(x_t | u_t, x_{t-1}) Bel(x_{t-1})$

Correction: $Bel(x_t) = \eta p(z_t | x_t) Bel^-(x_t)$

Just say "map to code" and I'll do it.



Bayes Filter Derivation

$$\begin{aligned}
 & \bullet \text{bel}(x_t) = p(x_t | z_{1:t}, u_{1:t}) \\
 & = \eta p(z_t | x_t, z_{1:t-1}, u_{1:t}) p(x_t | z_{1:t-1}, u_{1:t}) && \text{Bayes Rule} \\
 & = \eta p(z_t | x_t) p(x_t | z_{1:t-1}, u_{1:t}) && \text{Markov Assumption} \\
 & = \eta p(z_t | x_t) \int p(x_t | z_{1:t-1}, u_{1:t}, x_{t-1}) p(x_{t-1} | z_{1:t-1}, u_{1:t}) dx_{t-1} && \text{Law of total probability} \\
 & = \eta p(z_t | x_t) \int p(x_t | x_{t-1}, u_t) p(x_{t-1} | z_{1:t-1}, u_{1:t}) dx_{t-1} && \text{Markov Assumption} \\
 & = \eta p(z_t | x_t) \int p(x_t | x_{t-1}, u_t) p(x_{t-1} | z_{1:t-1}, u_{1:t-1}) dx_{t-1} && \text{Markov Assumption}
 \end{aligned}$$

what is ni

Great question — the **η (eta)** here is super important and shows up everywhere in Bayesian filtering.

🧠 Short Answer

👉 η (eta) is a normalization constant.

Its job is to make sure the belief distribution sums (or integrates) to 1.

Why We Need η

After Bayes update, we compute something like:

$$Bel(x_t) \propto p(z_t|x_t) \cdot Bel^-(x_t)$$

But this product is usually “unnormalized” — meaning:

- Probabilities might not sum to 1
- It’s just proportional to the true distribution

η fixes that.

What η Actually Is

Mathematically:

$$\eta = \frac{1}{\int p(z_t|x_t) Bel^-(x_t) dx_t}$$

Or in discrete state case:

$$\eta = \frac{1}{\sum_{x_t} p(z_t|x_t) Bel^-(x_t)}$$

So $\eta = 1 / \text{total probability mass}$

Intuition Example

Suppose after multiplying prediction \times likelihood you get:

| State | Value |
|-------|-------|
| A | 0.2 |
| B | 0.3 |
| C | 0.5 |

Sum = 1 → $\eta = 1$ (already normalized)

But if you get:

| State | Value |
|-------|-------|
| A | 4 |
| B | 6 |
| C | 10 |

Sum = 20

Then:

$$\eta = \frac{1}{20}$$

Multiply everything by 1/20 → valid probability distribution.

🧠 Conceptual Meaning

η is basically saying:

"Scale everything so total probability = 1."

It does NOT change relative probabilities.

It just rescales them.

🔗 Where η Shows Up in Bayes Rule

General Bayes rule:

$$P(A|B) = \frac{P(B|A)P(A)}{P(B)}$$

Here:

$$\eta = \frac{1}{P(z_t)}$$

So η is just hiding the denominator:

nginx

Evidence / marginal likelihood

 Copy code

⭐ Why People Write η Instead of Full Expression

Because computing denominator explicitly is expensive.

Instead:

1. Compute numerator for all states
2. Normalize at end

Much faster in practice.

⌚ In Your Slide Specifically

In:

$$bel(x_t) = \eta p(z_t|x_t) \int p(x_t|x_{t-1}, u_t) bel(x_{t-1}) dx_{t-1}$$

η ensures final belief is valid probability distribution.

⭐ One Sentence Memory Trick

👉 η = "make probabilities sum to 1" constant.

If you want, I can also explain:

- Why η disappears in Kalman filter derivation
- How η relates to log-likelihood in ML

- Or how normalization works in particle filters

Just tell me 



Bayes Smoother

- Forward pass: Bayes filter over the data. Get $\text{bel}(x_t)$ for $t \in \{1, \dots, T\}$.

- Backward (smoothing) pass:

- $\text{bel}'(x_T) \leftarrow \text{bel}(x_T)$

The belief at the final time-step is just the belief from the filter.

- Work backwards in time from T to 1 using the update:

$$\text{bel}'(x_t) \leftarrow \text{bel}(x_t) \cdot \sum_{x_{t+1}} \frac{p(x_{t+1} | x_t, u_{t+1}) \cdot \text{bel}'(x_{t+1})}{\text{bel}(x_{t+1})}$$

Josiah Hahn, University of Wisconsin - Madison

Belief from filtering

Nice — this slide is about **smoothing**, which is like “*filtering but with hindsight*.”

I'll go slowly and explain every line in plain language and math intuition.

🧠 Big Idea: Filtering vs Smoothing

Filtering (what you already learned)

Estimate current state using **past data only**:

$$\text{bel}(x_t) = P(x_t | z_{1:t}, u_{1:t})$$

Smoothing (new idea here)

Estimate past state using **all data (past + future)**:

$$\text{bel}'(x_t) = P(x_t | z_{1:T}, u_{1:T})$$

Key difference:

- Filtering = online estimate

- Smoothing = offline best estimate
-

⭐ Why Smoothing Exists (Intuition)

Example:

You hear:

"The ___ will rise"

You're unsure if it was *price* or *prize*.

Later you hear:

"...due to inflation"

Now you know it was *price*.

Future info improved past estimate.



Step-by-Step Through Slide

➊ Forward Pass (Filtering)

Slide says:

Run Bayes filter forward → get $\text{bel}(x_t)$ for all t

Meaning:

For $t = 1 \rightarrow T$:

mathematica

Predict → Correct → Store belief

You now have:

Copy code

SCSS

bel(x_1), **bel**(x_2), ..., **bel**(x_T)

But these only use data up to time t.

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➊ Initialization of Backward Pass

Slide:

$$\text{bel}'(x_T) = \text{bel}(x_T)$$

Why?

Because:

At final time T:

There is no future data beyond T.

So filtered belief = smoothed belief.

🔁 Backward Pass (The Real Smoothing Step)

Slide formula:

$$\text{bel}'(x_t) = \text{bel}(x_t) \cdot \sum_{x_{t+1}} \frac{p(x_{t+1}|x_t, u_{t+1}) \text{bel}'(x_{t+1})}{\text{bel}(x_{t+1})}$$

Let's decode.

🧩 Part 1 – $\text{bel}(x_t)$

This is:

Filtered belief at time t.

Think:

"What we believed at time t using past data only."

🧩 Part 2 – The Summation Term

This is the future correction factor.

It answers:

Given future info, how likely was each state x_t ?

Part 3 — Transition Model

$$p(x_{t+1} | x_t, u_{t+1})$$

This says:

If you were at x_t , how likely to go to next state x_{t+1} ?

Motion model.

Part 4 — $\text{bel}'(x_{t+1})$

This is:

Smoothed belief at next time step.

Meaning:

What we now believe about next state using all data.

Part 5 — Division by $\text{bel}(x_{t+1})$

This part is subtle.

It prevents double-counting information from filtering step.

Intuition:

We already used some info when computing $\text{bel}(x_{t+1})$.

So we correct for that here.

Big Picture of Backward Step

You are adjusting old belief using:

- Transition model (how states evolve)
 - Future smoothed belief (what we now know happened later)
-



Why This Improves Estimates

Filtering only used:

Past → Present

Smoothing uses:

Past → Present → Future → Back to Present

So smoothing is always equal or better.



Visual Intuition

Filtering:

ini

t=1 → 2 → 3 → 4 → 5

Smoothing:

Copy code

cpp

Forward pass

Then

Backward correction using future knowledge



Important Special Case

Copy code

If motion + sensors are perfect:

Filtering ≈ Smoothing

If sensors noisy:

Smoothing helps a lot.

🔗 Connection to Kalman Smoother

This is general version.

Kalman smoother = same idea but:

- Gaussian math
 - Matrix equations
 - Much faster
-

⭐ One Sentence Summary

👉 Smoothing takes filtered beliefs and adjusts them using future data to produce best possible estimate of every past state.

If you want, next I can explain:

- Why smoothing can be written as forward × backward messages (HMM view)
- Why smoothing is used in SLAM and trajectory estimation
- Or walk through a tiny numeric example

Just tell me 👍



what is large T compared to t

Good — this is a **notation / time-index meaning** question, and it's important for smoothing.

🧠 Short Answer

👉 **T = final time step (end of dataset)**

👉 **t = some earlier time step**

So:

r

T ≥ t

And usually when people say “**large T compared to t**” they mean:

 Copy code

- 👉 You are early in the sequence
 - 👉 There is a lot of future data available after time t
-

📊 Example Timeline

Suppose you collected data for 100 steps:

css

| | | | | | | | | |
|-------|---|---|---|---|-----|----|-----|-----|
| Time: | 1 | 2 | 3 | 4 | ... | 50 | ... | 100 |
| | ↑ | | | | | | | |
| | | t | | | | | | |
| | | | ↑ | | | | | |
| | | | | T | | | | |

Here:

ini

t = 50

T = 100

You still have 50 future observations after t.

 Copy code

⭐ Why This Matters for Smoothing

Remember smoothing:

$$\text{bel}'(x_t) = P(x_t | z_{1:T})$$

If T is much larger than t:

- ➡ You have lots of future info to improve estimate at time t.

If t is close to T:

- ➡ Very little future info → smoothing ≈ filtering.
-

Intuition Example

Imagine tracking a robot:

If t = 10, T = 100

You can use:

- observations 11 → 100
to refine belief at time 10.

Huge improvement possible.

If t = 99, T = 100

Only 1 future observation → small improvement.

Why Papers Sometimes Say “Large T”

They usually mean:

- 👉 Long trajectory
 - 👉 Lots of data after time t
 - 👉 Smoothing becomes very powerful
-

In Math / Algorithms

Backward pass runs:

r

$T \rightarrow T-1 \rightarrow T-2 \rightarrow \dots \rightarrow 1$

So if T is large:

Backward pass has more influence on early times.

 Copy code

⭐ Key Memory Trick

Filtering:

nginx

Past → Present

Smoothing:

 Copy code

css

Past → Present → Future → Back to Present

The more "future" you have → the more smoothing helps.

 Copy code

⌚ One Sentence Summary

👉 "Large T compared to t " means you are early in the sequence and have lots of future observations to improve the estimate at time t .

If you want, I can also explain:

- Why smoothing error decreases as T grows
- How this relates to HMM forward-backward algorithm
- Or how this shows up in trajectory estimation / SLAM

Just tell me 

     ...

Example of a Linear Gaussian System

- Consider a robot moving in a 2D plane.
 - State is $x = [x, \dot{x}, y, \dot{y}]$, i.e., position and velocity
 - Action is $u = [\ddot{x}, \ddot{y}]$, i.e., acceleration
 - Observation is noisy position: $z = [\tilde{x}, \tilde{y}]$.
- $$\begin{aligned} x_t &= A_{1,1}x_{t-1} + A_{1,2}\dot{x}_{t-1} + B_{1,1}\ddot{x}_t + w_t(0) & \dot{x}_t &= A_{2,2}\dot{x}_{t-1}B_{2,1}\ddot{x}_t + w_t(1) \\ \tilde{x}_t &= H_{1,1}x_t + v_t(0) \end{aligned}$$
- Similar transition and observation definitions for the y-coordinate.

what is x tilda

Great — this is a **notation question**, and the tilde here is easy to miss.

Short Answer

👉 \tilde{x} (x-tilde) = noisy measurement of x

So:

arduino

```
x_t = true position
tilde_x_t = measured position (with sensor noise)
```

In Your Slide Specifically

 Copy code

They define observation as:

ini

$z = [\tilde{x}, \tilde{y}]$

So:

 Copy code

- True state contains real position: x, y
 - Sensor gives noisy version: \tilde{x}, \tilde{y}
-



Why They Use Tilde

Tilde usually means:

- 👉 "Observed version"
- 👉 "Noisy estimate"
- 👉 "Measurement of"

Not universal, but very common in controls + estimation.



Look at This Equation From Slide

cpp

$$\tilde{x}_t = H_{\{1,1\}} x_t + v_t(0)$$

Breakdown:

Copy code

x_t

True position

$H_{1,1}$

Observation matrix element

(Usually just 1 if directly measuring position)

v_t

Measurement noise

Usually Gaussian noise:

SCSS

$v_t \sim N(0, R)$
Copy code

Intuition Example

True robot position:

ini

 $x_t = 10.0$ meters

Sensor reading:

Copy code

python

 $\tilde{x}_t = 9.8 \text{ or } 10.3 \text{ or } 10.1$

Because sensors are noisy.

Copy code

Connection to Kalman Filter

Kalman uses:

cpp

 $z_t = H x_t + v_t$

Here they are just writing components:

Copy code

cpp

 $\tilde{x}_t = \text{noisy measurement of } x$
 $\tilde{y}_t = \text{noisy measurement of } y$

Important Conceptual Difference

Copy code

| Symbol | Meaning |
|---------------|-------------------------|
| x_t | True hidden state |
| \tilde{x}_t | What sensor reports |
| z_t | Full observation vector |

🧭 Why This Matters

Kalman filter's job is basically:

- 👉 Combine prediction of true state
- 👉 With noisy measurements (tilde values)
- 👉 To estimate best true state

⭐ One Sentence Memory Trick

- 👉 Tilde = "measured version with noise."

If you want, I can also explain:

- Why observation matrix H exists (when measurement isn't direct)
- Why sometimes we don't use tilde and just use z
- How this maps into Kalman update equations

Just tell me 



Kalman Filter

- The Kalman filter is a Bayes filter that represents $\text{bel}(x_t)$ with a Gaussian distribution, $\mathcal{N}(\mu_t, \Sigma_t)$.
- The initial belief is Gaussian: $\text{bel}(x_0) = \mathcal{N}(x_0; \mu_0, \Sigma_0)$.
- Under our assumptions, the posterior remains a Gaussian distribution using the updates from the Bayes filter:

$$p(x_t | z_{1:t}, u_{1:t}) = \mathcal{N}(x_t; \mu_t, \Sigma_t)$$

- Intuition for correctness: plug Gaussian beliefs and linear Gaussian system state transitions and observations into Bayes filter updates.

Nice — this slide is the “why Kalman filter works” slide.

Let’s unpack it line by line and translate it into intuition + math meaning.

Big Idea of This Slide

- 👉 Kalman Filter = Bayes Filter
 - 👉 BUT assumes everything is **Gaussian + linear**
 - 👉 That makes the math closed-form and fast.
-

Line 1

“Kalman filter represents belief with Gaussian distribution”

$$\text{bel}(x_t) = \mathcal{N}(\mu_t, \Sigma_t)$$

Meaning

Instead of storing full probability distribution:

We store only:

- Mean (best guess state)
 - Covariance (uncertainty)
-

Intuition

Instead of storing:

arduino

Probability at every position

We store:

 Copy code

SCSS

Center of distribution + spread

Huge memory + compute savings.

 Copy code

Line 2

Initial belief is Gaussian

$$bel(x_0) = \mathcal{N}(x_0; \mu_0, \Sigma_0)$$

Meaning:

We assume starting uncertainty is Gaussian.

Example:

Robot starts near origin with some uncertainty.

Line 3

Posterior remains Gaussian

$$p(x_t | z_{1:t}, u_{1:t}) = \mathcal{N}(x_t; \mu_t, \Sigma_t)$$

This is the magic property.

Why This Is Amazing

Normally:

Bayes updates make distributions weird shapes.

But if:

- Motion is linear
- Observation is linear
- Noise is Gaussian

Then:

Gaussian \times Gaussian \rightarrow Gaussian

So distribution shape never changes.



This Is Called Closure Property

Gaussian distributions are closed under:

- Linear transforms
 - Gaussian noise addition
 - Bayesian updates
-

Line 4

Why This Works

Slide says:

Plug Gaussian beliefs + linear Gaussian system into Bayes filter.

Meaning:

Take full Bayes filter math:

Prediction + Correction

Then assume:

cpp

$$\begin{aligned} \mathbf{x}_t &= A \mathbf{x}_{t-1} + B \mathbf{u}_t + \text{noise} \\ \mathbf{z}_t &= H \mathbf{x}_t + \text{noise} \end{aligned}$$

 Copy code

Everything stays Gaussian.

⭐ Why This Makes Kalman Efficient

Instead of computing integrals over distributions:

We just update:

cpp

$$\begin{aligned} \boldsymbol{\mu}_t \\ \boldsymbol{\Sigma}_t \end{aligned}$$

Using matrix equations.

 Copy code

📊 Real Meaning in Practice

Kalman Filter tracks:

- Position
- Velocity
- Temperature
- Any linear dynamic system

Using:

- Mean estimate
- Uncertainty ellipse

🧠 Why Mean + Covariance Are Enough

Gaussian fully described by:

mathematica

Mean → location

Covariance → spread + correlation

Nothing else needed.

 Copy code

🔗 Connection to Bayes Filter

Bayes Filter:

Stores full distribution.

Kalman Filter:

Stores compressed Gaussian summary.

⭐ One Sentence Summary

👉 Kalman filter works because Gaussian distributions stay Gaussian under linear motion + noisy measurement updates.

⌚ If I Make It Even Simpler

Kalman filter is:

python

Bayes filter

+

Assume everything is Gaussian

+

Assume physics is linear

=

Fast exact solution

If you want, next I can explain:

- Why Kalman fails for multimodal distributions (very important)

- How particle filters fix this
- Or how μ and Σ update equations come from Bayes math

Copy code

Just tell me



Kalman Filter as a Bayesian Filter | The Kalman Gain | Evolution of Kalman Filter

bel(x_t) = $\mathcal{F}(x_0, \mu_0, \Sigma_0)$

$$K_t = \Sigma_t H^T (H\Sigma_t H^T + R)^{-1}$$

ie Kalman gain at time-step t .

In case with $H = 1$ to build intuition:

$$K_t = -\frac{\sigma_t}{\sigma_t^2 + R}$$

which tells you how much weight must the predictor gets.

$\mu_t = \mu$

$\Sigma_t = \Sigma$

plus the measurement is less reliable and the error of the prediction itself.

Advantages / Disadvantages | Practice

Practice

ng above the x-axis and has state given by its position x_t and velocity v_t . Assume the robot's initial belief is $\mu_0 = 0$ and $\sigma_0 = 1$.

$x_t + w_t$, where $w_t \sim \mathcal{N}(0, 1)$

where $v_t \sim \mathcal{N}(0, 0.2)$

a robot's belief about its location after it takes action a_t

Belief

$$\mu_t = \mu_0 + K_t (y_t - H\mu_0)$$

$$= (0.000) + 1 = 1$$

$$\mu_t = \mu_0 + K_t (y_t - H\mu_0)$$

$$\Sigma_t = (I - K_t H)^2 \Sigma_0$$

Summary

linear Gaussian model.

basic Kalman filter as an instantiation of Gaussian assumption.

of how updates change the belief.

Nice — this set of slides is basically the **full Kalman filter story from theory → intuition → example → limitations**.

I'll explain it as one coherent flow, not slide-by-slide, so it sticks conceptually.

1 Kalman Filter = Bayes Filter With Strong Assumptions

From your slides:

Kalman filter assumes:

- Linear dynamics

cpp

$$x_t = A x_{t-1} + B u_t + w_t$$

- Linear observation

Copy code

cpp

$$z_t = H x_t + v_t$$

Gaussian noise

[Copy code](#)

SCSS

$$w_t \sim N(0, Q)$$

$$v_t \sim N(0, R)$$

👉 Because of this → belief always stays Gaussian.

So instead of storing full distribution:

[Copy code](#)

You store only:

cpp

μ_t (mean state estimate)

Σ_t (uncertainty covariance)



2

Kalman = Prediction Step + Correction Step

[Copy code](#)

Same as Bayes filter.

● Prediction (Motion Update)

Mean:

$$\bar{\mu}_t = A\mu_{t-1} + Bu_t$$

Uncertainty:

$$\bar{\Sigma}_t = A\Sigma_{t-1}A^T + Q$$

Intuition

- Move belief forward using physics
- Add motion noise → uncertainty grows

● Correction (Measurement Update)

Mean:

$$\mu_t = \bar{\mu}_t + K_t(z_t - H\bar{\mu}_t)$$

Covariance:

$$\Sigma_t = (I - K_t H) \bar{\Sigma}_t$$

Intuition

vbnet

```
New estimate =
Prediction +
Gain × Measurement error
```

Measurement error = innovation =:

cpp

```
z_t - predicted measurement
```

 Copy code

🧠 3 The Kalman Gain = Trust Weight

From slide:

$$K_t = \bar{\Sigma}_t H^T (H \bar{\Sigma}_t H^T + R)^{-1}$$

In 1D case (slide shows)

$$K = \frac{\sigma_{pred}^2}{\sigma_{pred}^2 + R}$$

Interpretation

If prediction uncertainty big → trust measurement more

If measurement noise big → trust prediction more

Extreme Cases

If measurement perfect:

css

$R \rightarrow 0$

$K \rightarrow 1$

→ Fully trust measurement

If measurement terrible:

 Copy code

css

$R \rightarrow \infty$

$K \rightarrow 0$

→ Ignore measurement

 Copy code

4 The Visualization Slide (Red, Green, Blue Curves)

Red = prediction belief

Green = measurement likelihood

Blue = final belief

Blue sits between red and green depending on gain.

5 The Practice Example (Let's Re-Explain Intuition)

Given:

$\mu_0 = 0$
 $\sigma_0^2 = 1$

Motion:

cpp

```
x_t = x_{t-1} + u_t + noise
noise variance = 1
```

Measurement:

cpp

```
z_t = x_t + noise
measurement variance = 2
```

Prediction

 Copy code

Mean:

$$\bar{\mu}_1 = \mu_0 + u_1 = 0 + 1 = 1$$

Variance:

 Copy code

$$\bar{\Sigma}_1 = 1 + 1 = 2$$

 Motion made uncertainty bigger.

 Copy code

Gain

ini

$$K = 2 / (2 + 2) = 1/2$$

 Copy code

👉 Trust prediction and measurement equally.

Correction

Mean:

$$\mu_1 = 1 + 0.5(2 - 1) = 1.5$$

Variance:

 Copy code

$$\Sigma_1 = (1 - 0.5) \times 2 = 1$$

👉 Measurement reduced uncertainty.

 Copy code

6 Advantages / Disadvantages

Pros

- Very fast
 - Exact optimal solution (if assumptions hold)
 - Works in continuous space
-

Cons

- Assumes Gaussian noise
- Assumes linear dynamics
- Real robots often nonlinear

(That's why EKF / UKF / Particle Filters exist.)



7

Deep Intuition (Most Important)

Kalman filter is doing this every timestep:

powershell

 Copy code

Predict **where** you think you are

↓

Measure where sensors say you are

↓

Blend them **using** uncertainty weights

⭐ The Single Most Important Equation Conceptually

java

 Copy code

```
New estimate =  
Prediction +  
Weight × (Measurement - Prediction)
```

Everything else is just math to compute that weight.

🧭 If This Were One Sentence

👉 Kalman filter is Bayes filtering where uncertainty is tracked using Gaussian mean and covariance, and prediction vs measurement is balanced using the Kalman gain.

If you want, I can go one level deeper and explain:

- WHY covariance update has form $(I - KH)\Sigma$
- WHY gain formula minimizes mean squared error
- HOW this connects to linear regression / least squares

Just tell me

