Image Formation and Sensing

(How do Images Form and Are Captured?)

Computer Vision: CS 566

Computer Science

University of Wisconsin-Madison

Image Formation and Optics

Image: Projection of 3D scene onto 2D plane.
We need to understand Geometric and Radiometric relations between the scene and its image.

Topics:

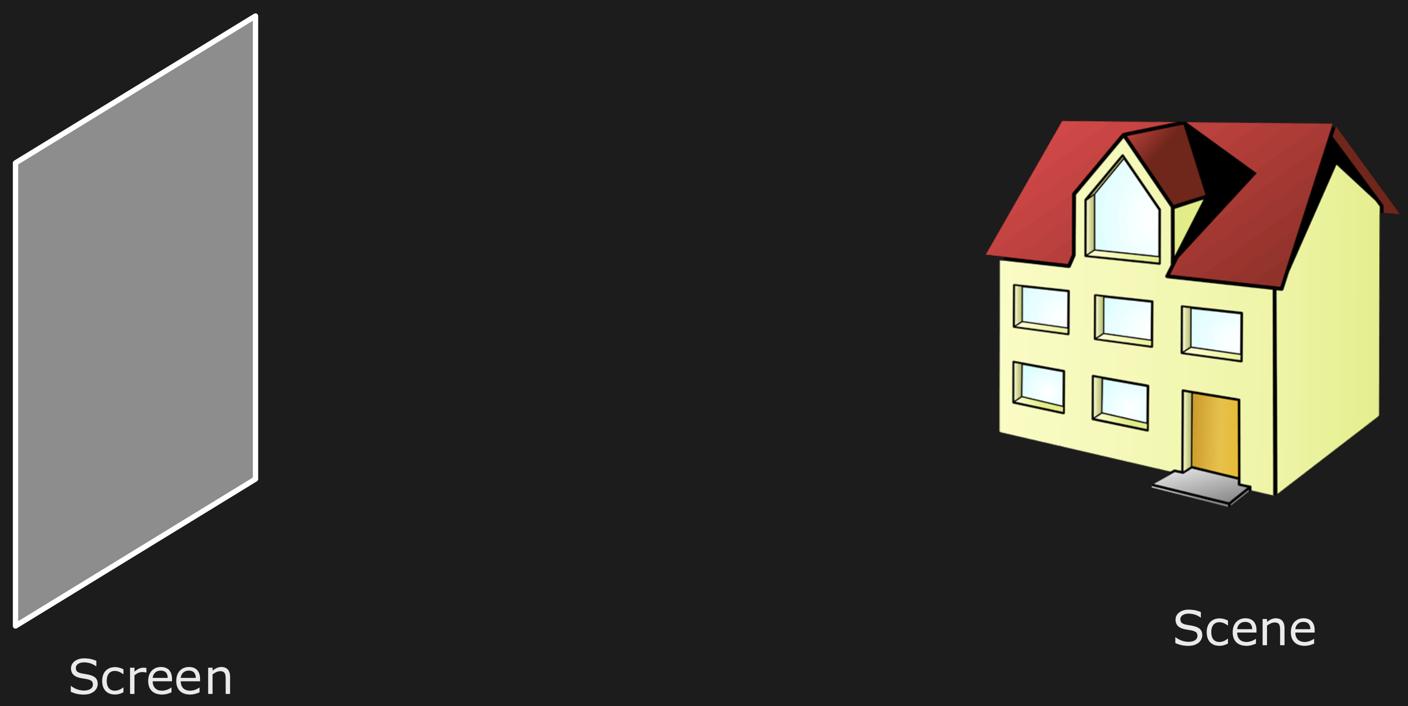
(1) Pinhole and Perspective Projection

(2) Vanishing Point

(3) Image formation using Lenses

(4) Lens Related Issues

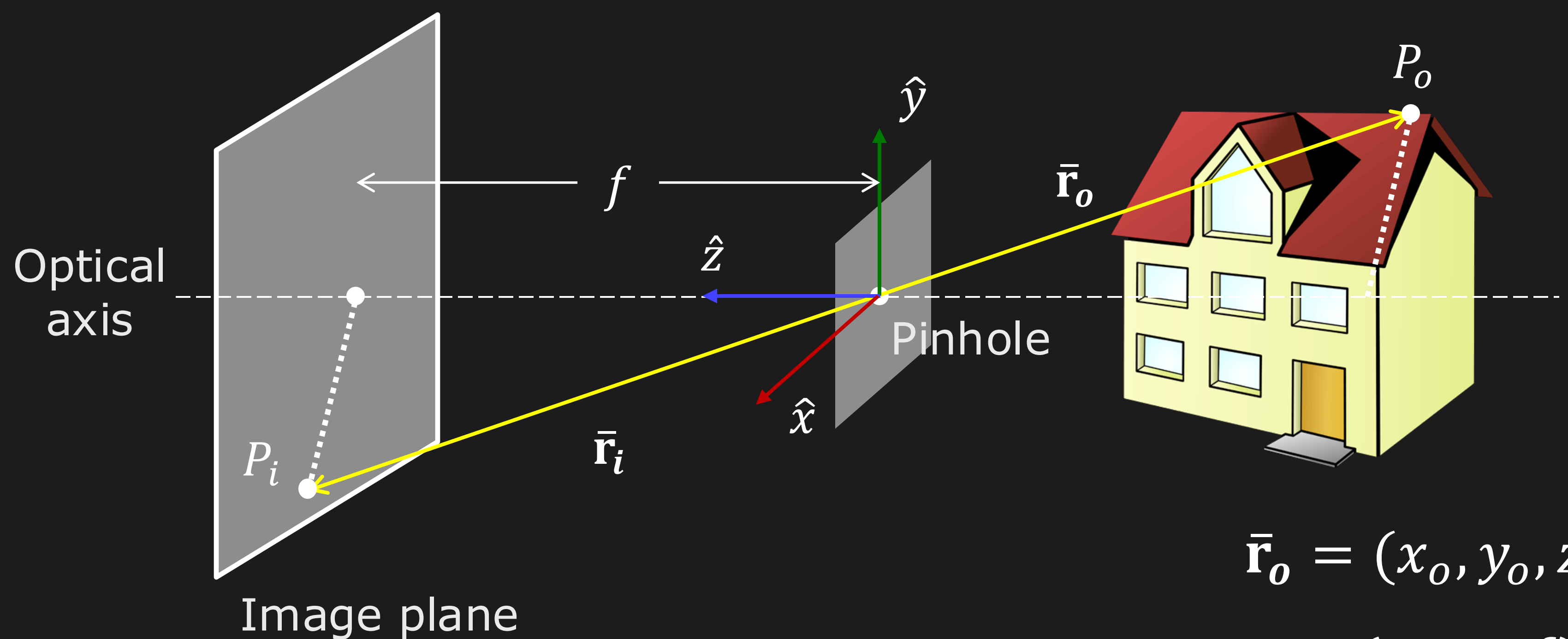
Image Formation



Is an image being formed on the screen?

Yes! But not a “clear” one.

Perspective Imaging with Pinhole



$$\bar{\mathbf{r}}_o = (x_o, y_o, z_o)$$

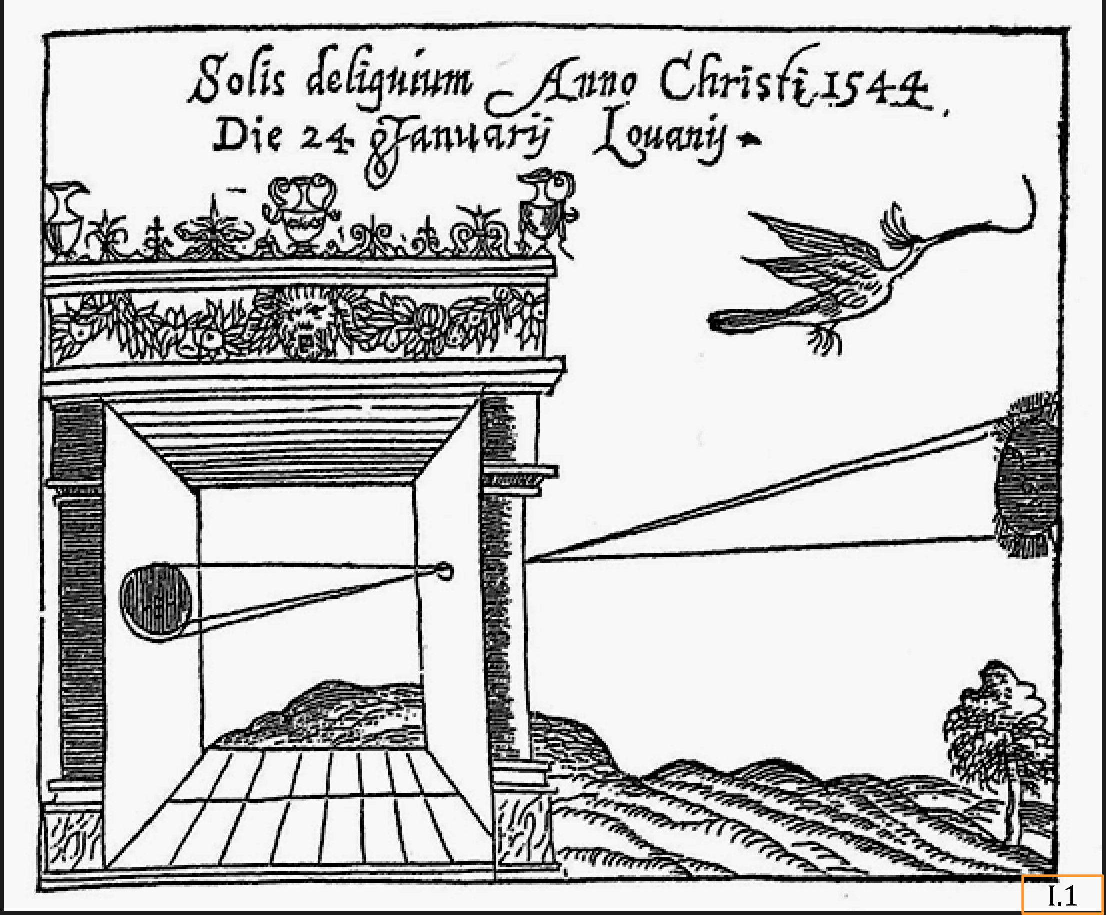
$$\bar{\mathbf{r}}_i = (x_i, y_i, f)$$

Using similar triangles:

$$\boxed{\frac{\bar{\mathbf{r}}_i}{f} = \frac{\bar{\mathbf{r}}_o}{z_o}} \rightarrow \boxed{\frac{x_i}{f} = \frac{x_o}{z_o}\,, \quad \frac{y_i}{f} = \frac{y_o}{z_o}}$$

f: Effective Focal Length

Camera Obscura



I.1

"Dark Chamber"

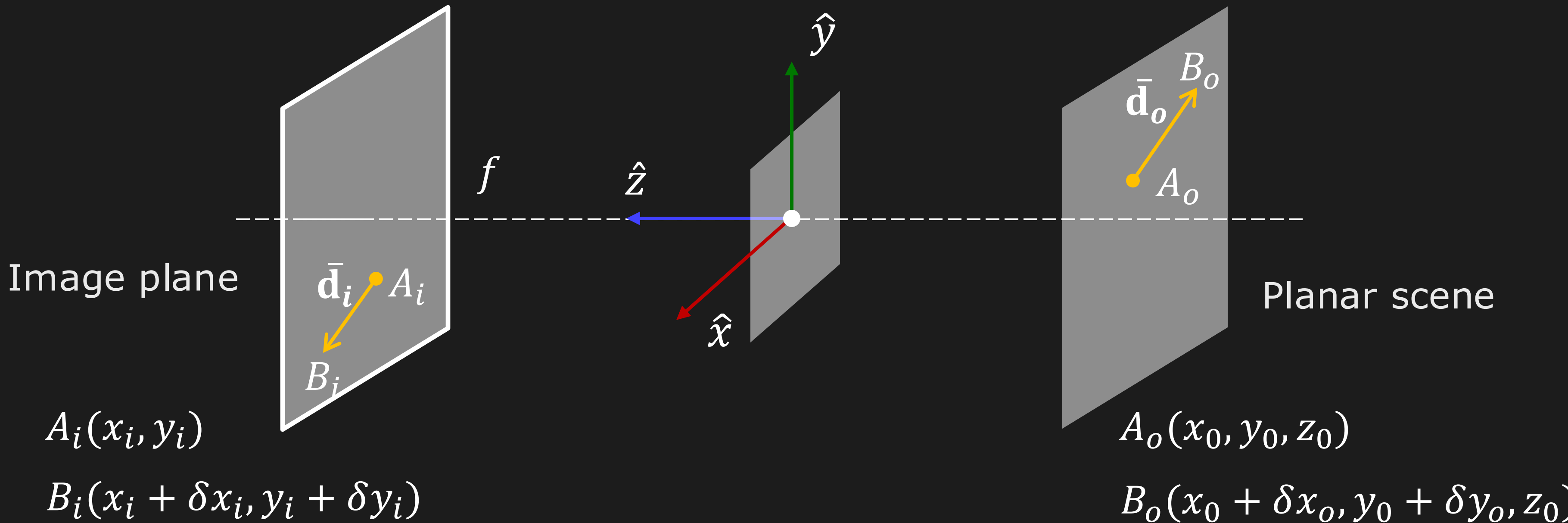
[Frisius 1545]

Pinhole Eye of *Nautilus pompilius*

I.2

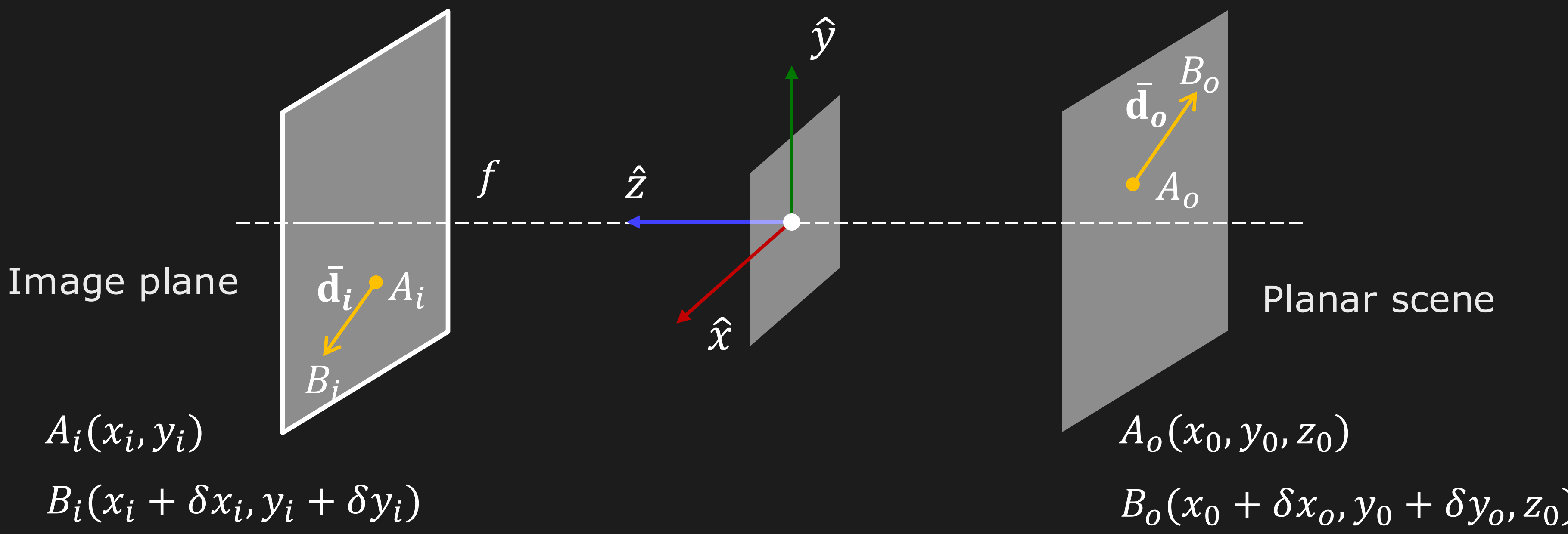
I.3

Image Magnification



Magnification: $$|m| = \frac{\|\bar{\mathbf{d}}_i\|}{\|\bar{\mathbf{d}}_o\|} = \sqrt{{\delta x_i}^2 + {\delta y_i}^2} \Big/ \sqrt{{\delta x_o}^2 + {\delta y_o}^2}$$

Image Magnification



From Perspective Projection:

$$\frac{x_i}{f} = \frac{x_o}{z_o} \quad \text{and} \quad \frac{y_i}{f} = \frac{y_o}{z_o} \quad \text{-------------} \quad \text{(A)}$$

$$\frac{x_i + \delta x_i}{f} = \frac{x_o + \delta x_o}{z_o} \quad \text{and} \quad \frac{y_i + \delta y_i}{f} = \frac{y_o + \delta y_o}{z_o} \quad \text{-------} \quad \text{(B)}$$

Image Magnification

From (A) and (B) we get:

$$\frac{\delta x_i}{f} = \frac{\delta x_o}{z_o} \quad \text{and} \quad \frac{\delta y_i}{f} = \frac{\delta y_o}{z_o}$$

Magnification:

$$|m| = \frac{\|\bar{\mathbf{d}}_i\|}{\|\bar{\mathbf{d}}_o\|} = \sqrt{\delta x_i{}^2 + \delta y_i{}^2} \Big/ \sqrt{\delta x_o{}^2 + \delta y_o{}^2} = \left|\frac{f}{z_o}\right|$$

$$\boxed{m = \frac{f}{z_o}}$$

m is negative when image is inverted

Image Magnification


I.4

I.5

$$\boxed{m = \frac{f}{z_o}}$$

Image size inversely proportional to depth

Image Magnification

Notes:

- m can be assumed to be constant if the range of scene depth Δz is much smaller than the average scene depth $\tilde{z}$

- $$\frac{Area_i}{Area_o} = m^2$$

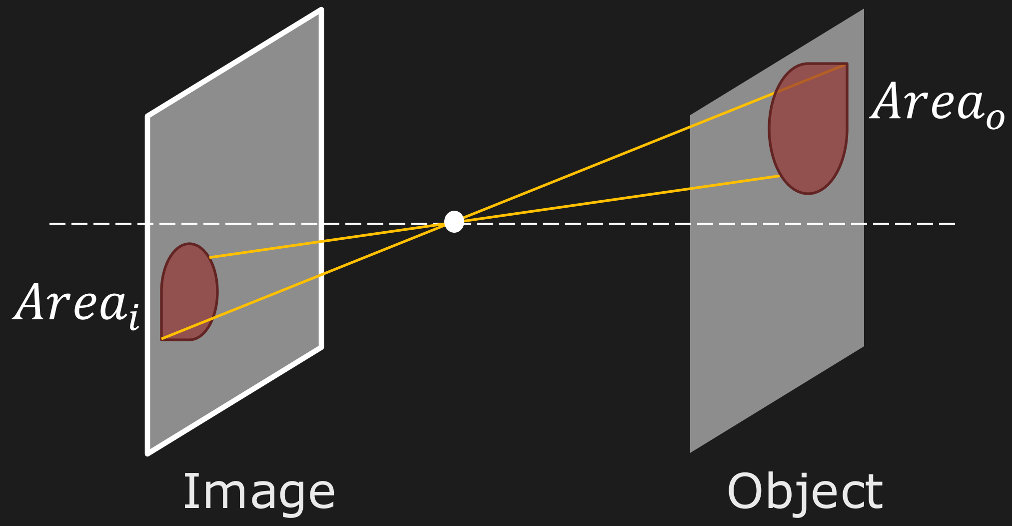


Image Magnification

I.4

I.5

$$\boxed{m = \frac{f}{z_o}}$$

Image size inversely proportional to depth

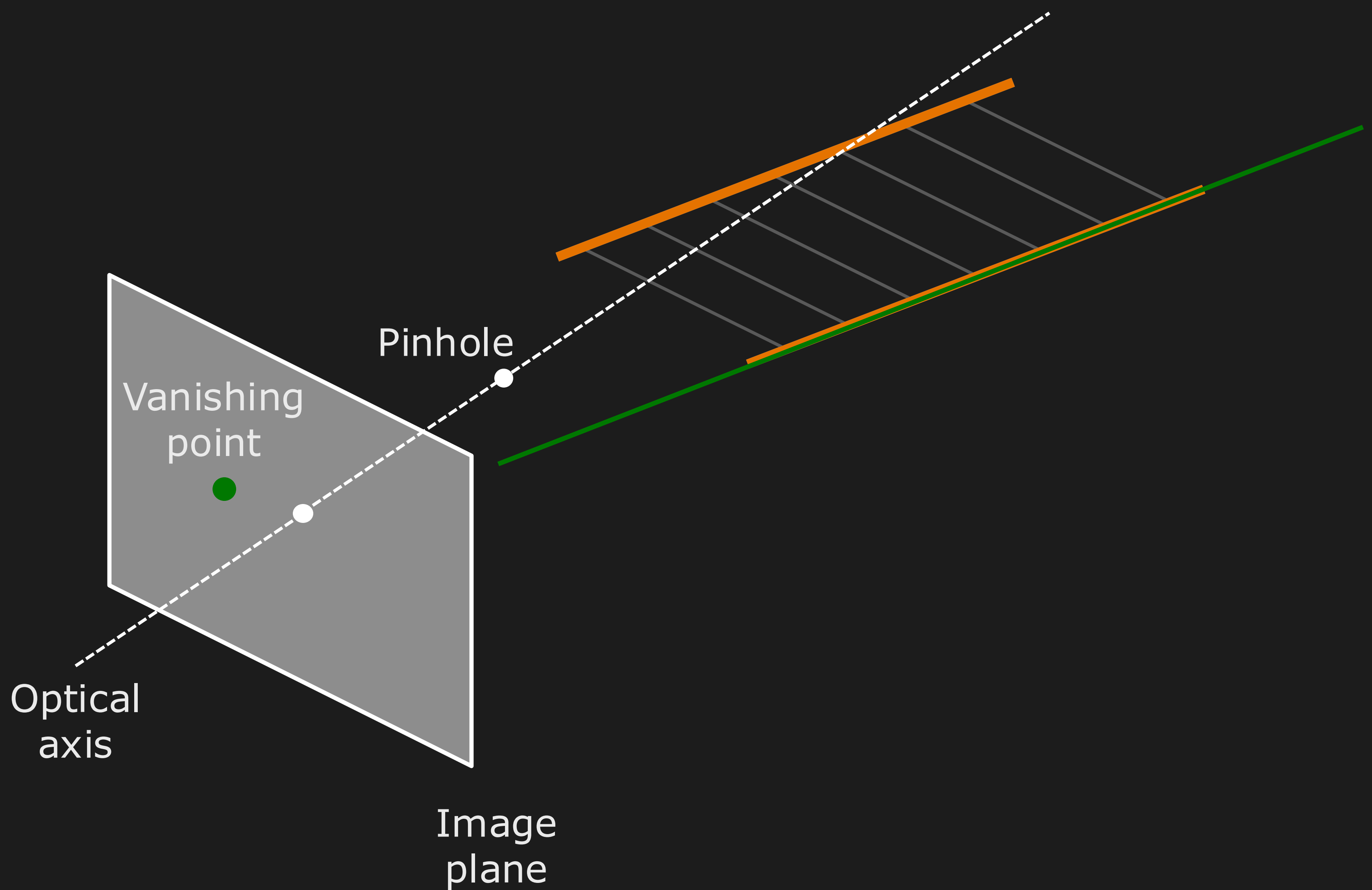
Vanishing Point

Parallel straight lines converge at a single image point

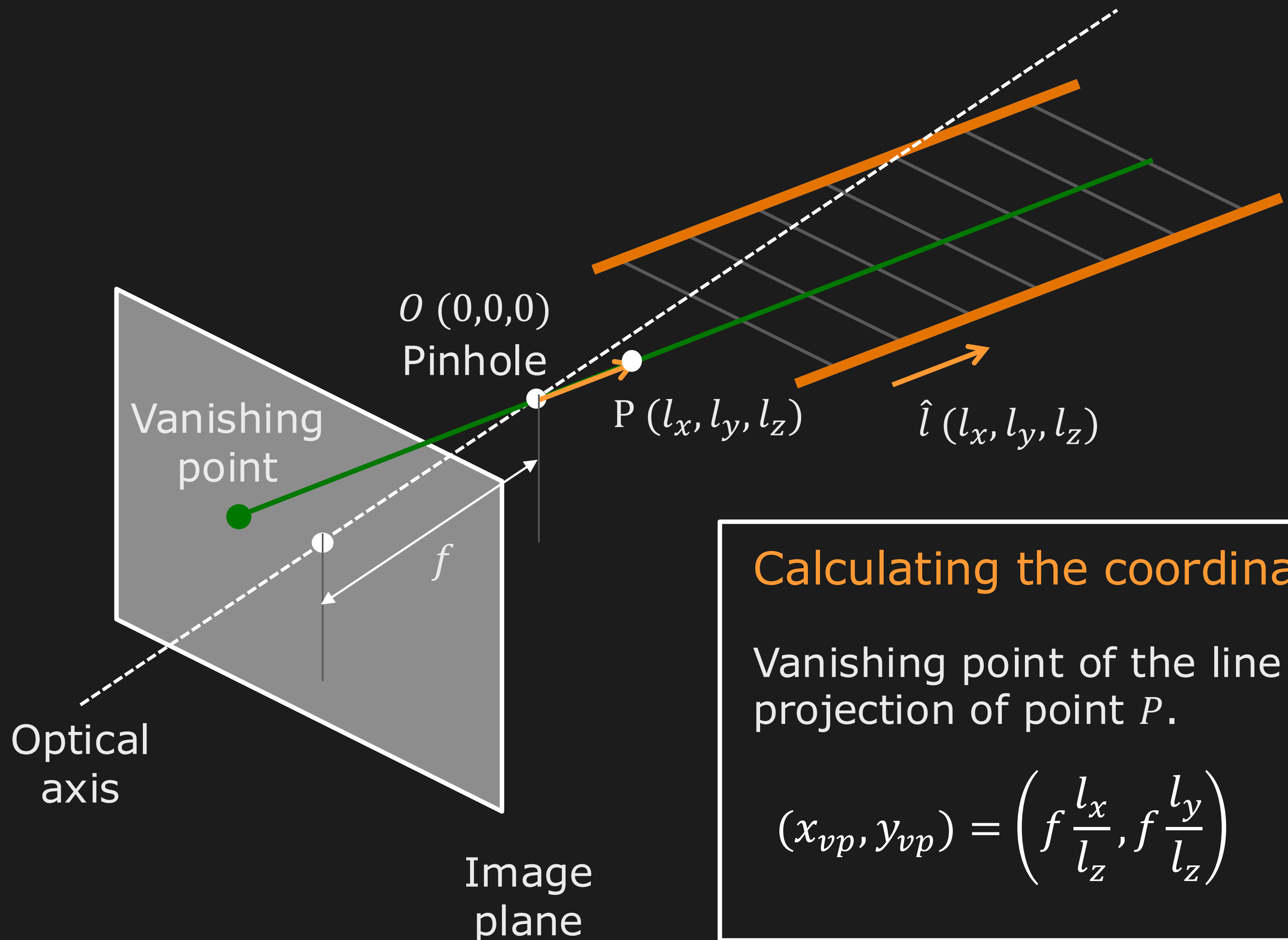
Vanishing Point

Location of Vanishing Point depends on the orientation of parallel straight lines.

Finding the Vanishing Point



Finding Vanishing Point

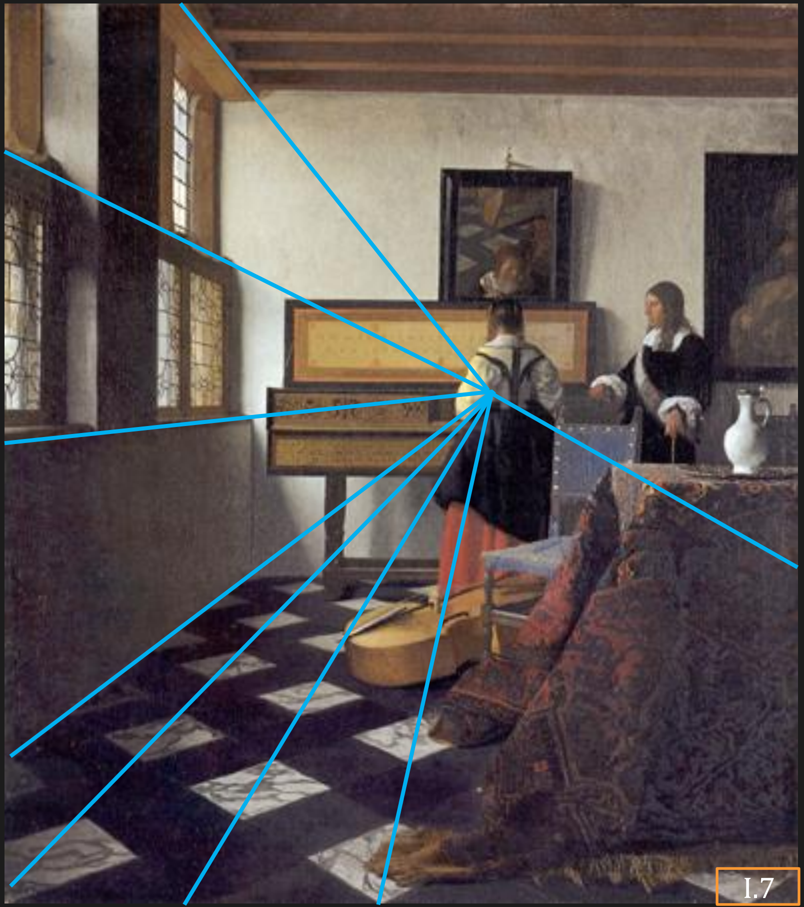


Calculating the coordinates:

Vanishing point of the line is the projection of point P.

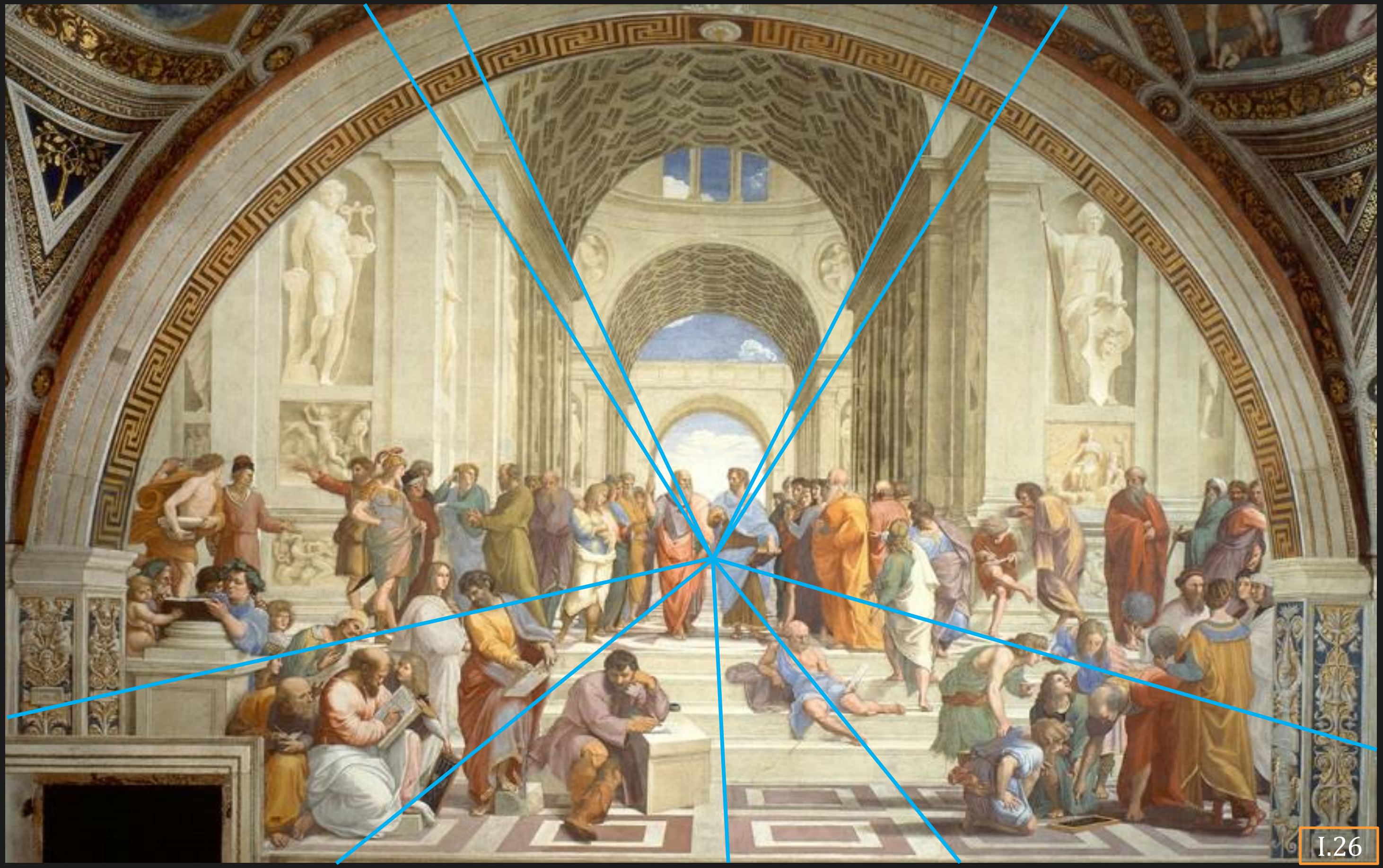
$$(x_{vp}, y_{vp}) = \left(f\,\frac{l_x}{l_z}, f\,\frac{l_y}{l_z} \right)$$

Use of Vanishing Point in Art



The Music Lesson, Johannes Vermeer, c. 1662-1664

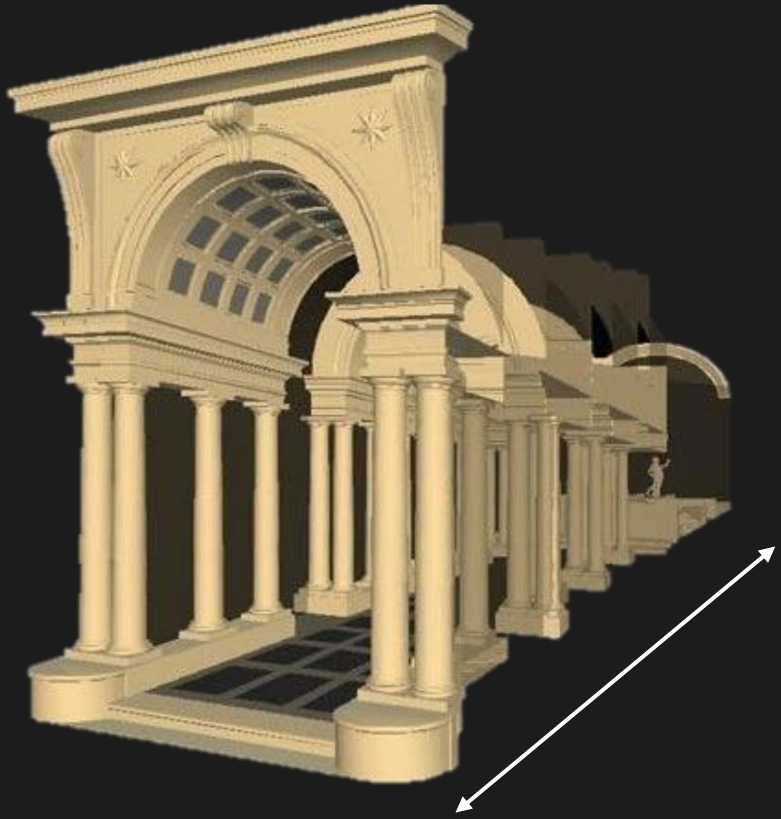
Use of Vanishing Point in Art



The Scholar of Athens, Raphael, c. 1509-1510

False Perspective

Depth appears to be ~155 feet

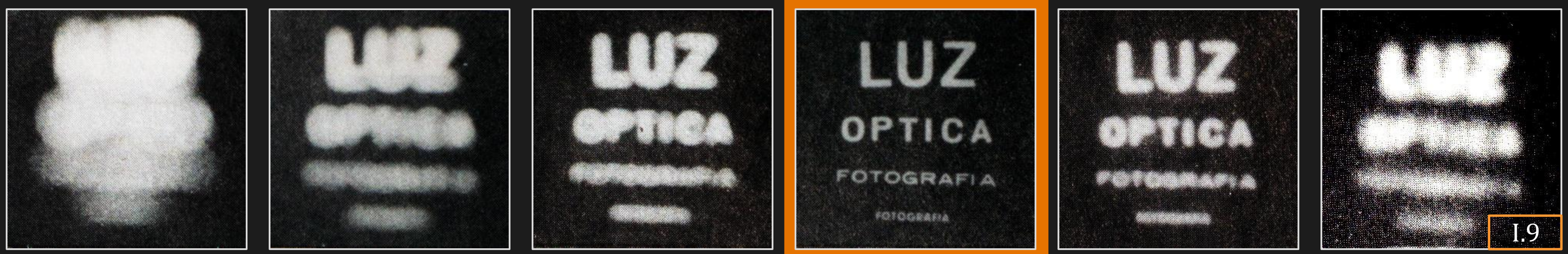


Depth is actually ~30 feet

Galleria Spada, Francesco Borromini, 1652

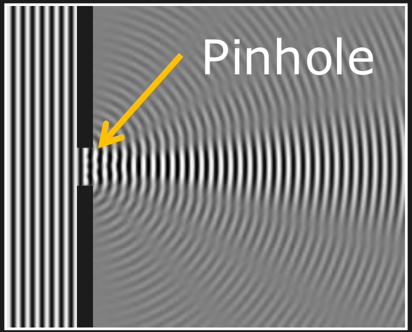
ART

What is the Ideal Pinhole Size?



2 mm | 1 mm | 0.6 mm | 0.35 mm | 0.15 mm | 0.07 mm

The pinhole must be tiny,
but if it’s too tiny it will cause diffraction.



Diffraction

Ideal pinhole diameter: $\boxed{d \approx 2\sqrt{f\lambda}}$

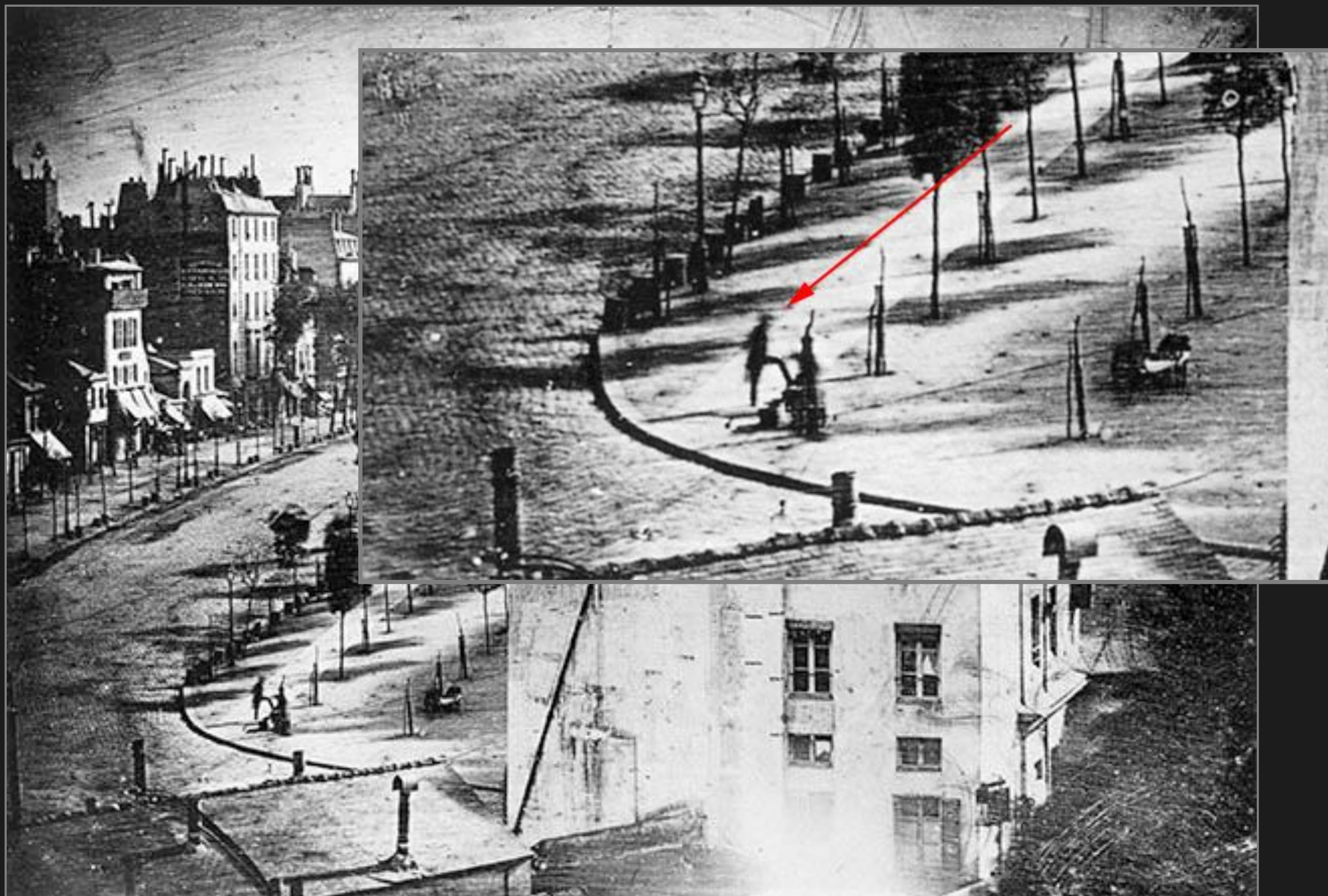
f: effective focal length
λ: wavelength

What about Exposure Time?

Pinholes pass less light and hence require long exposures to capture bright images.

$f = 73\, mm,\ d = 0.2\, mm,$
Exposure, $T = 12\, s$

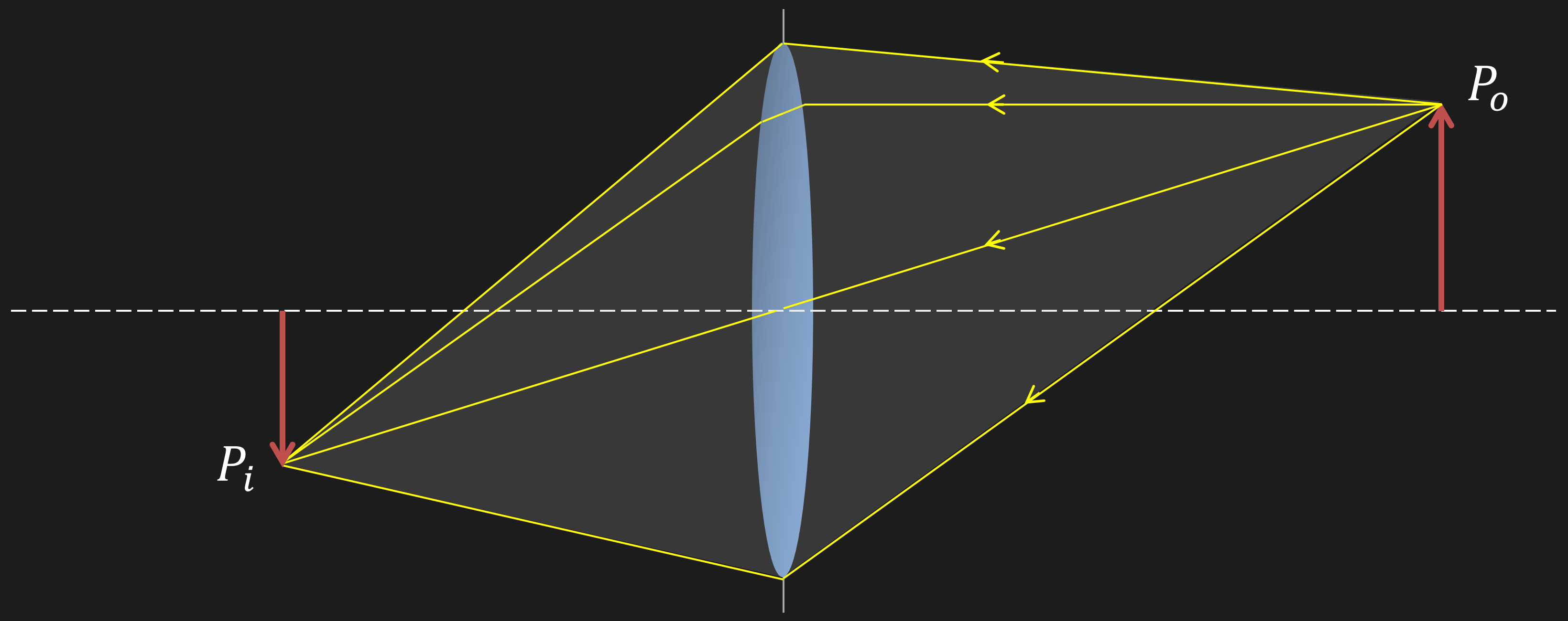
First Photograph of a Human



Louis Daguerre, “Le boulevard du Temple”, 1838, from [petapixel]

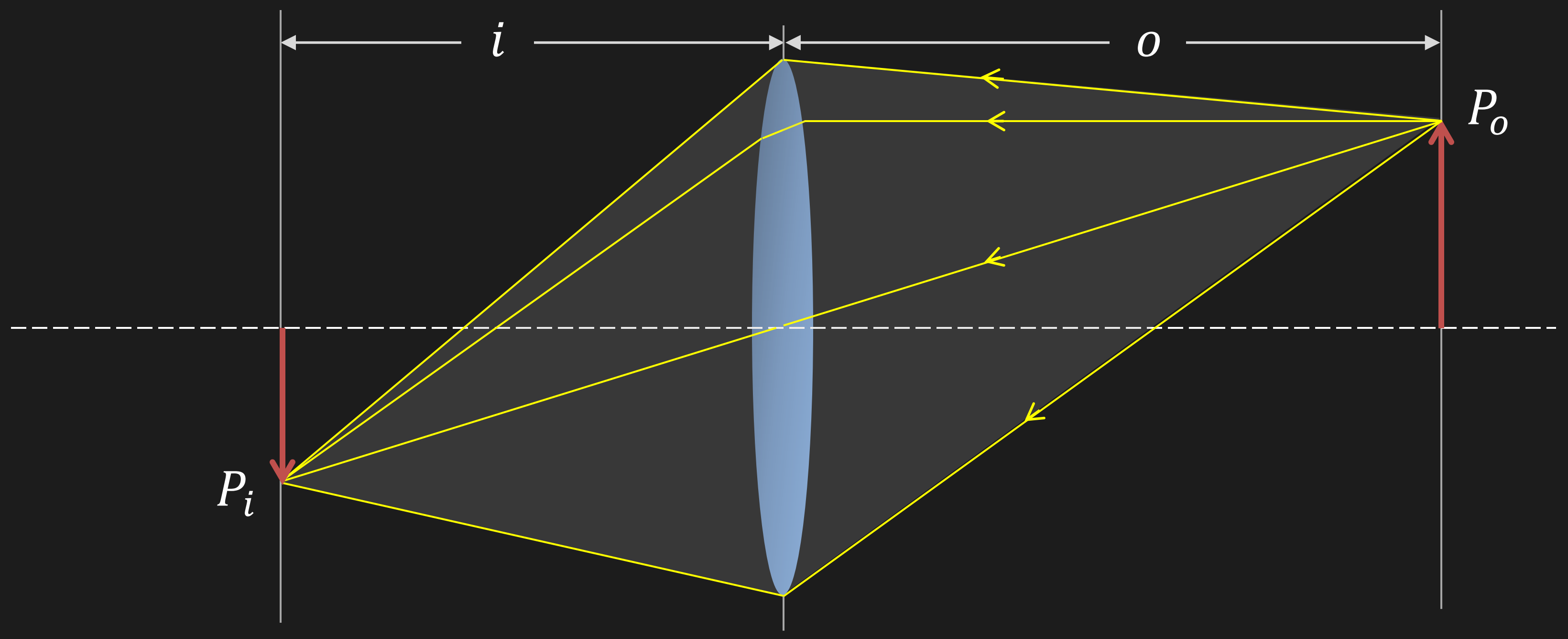
Lenses

Same projection as pinhole, but gather more light!



Focal length (f) determines the lens' bending power

Gaussian Lens (Thin Lens) Law



f: focal length

i: image distance

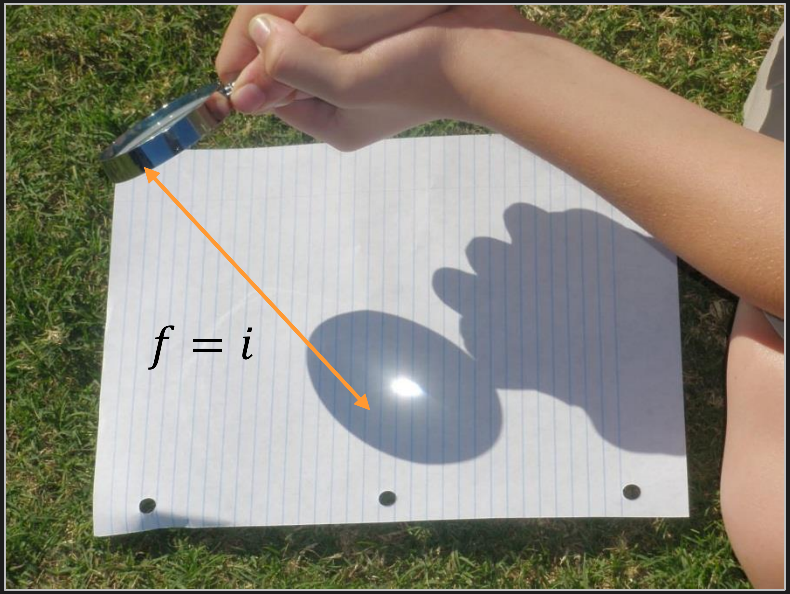
o: object distance

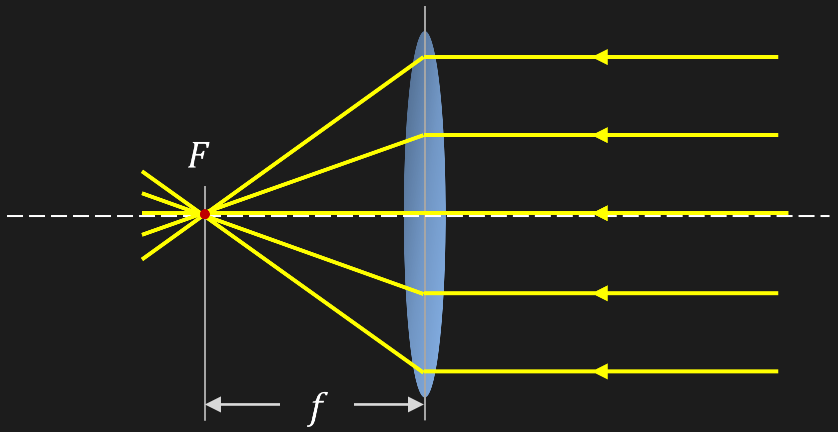
$$\boxed{\frac{1}{i} + \frac{1}{o} = \frac{1}{f}}$$

Example: If $f = 50mm$ & $o = 300mm$, then image distance $i = 60mm$

How to Find the Focal Length?

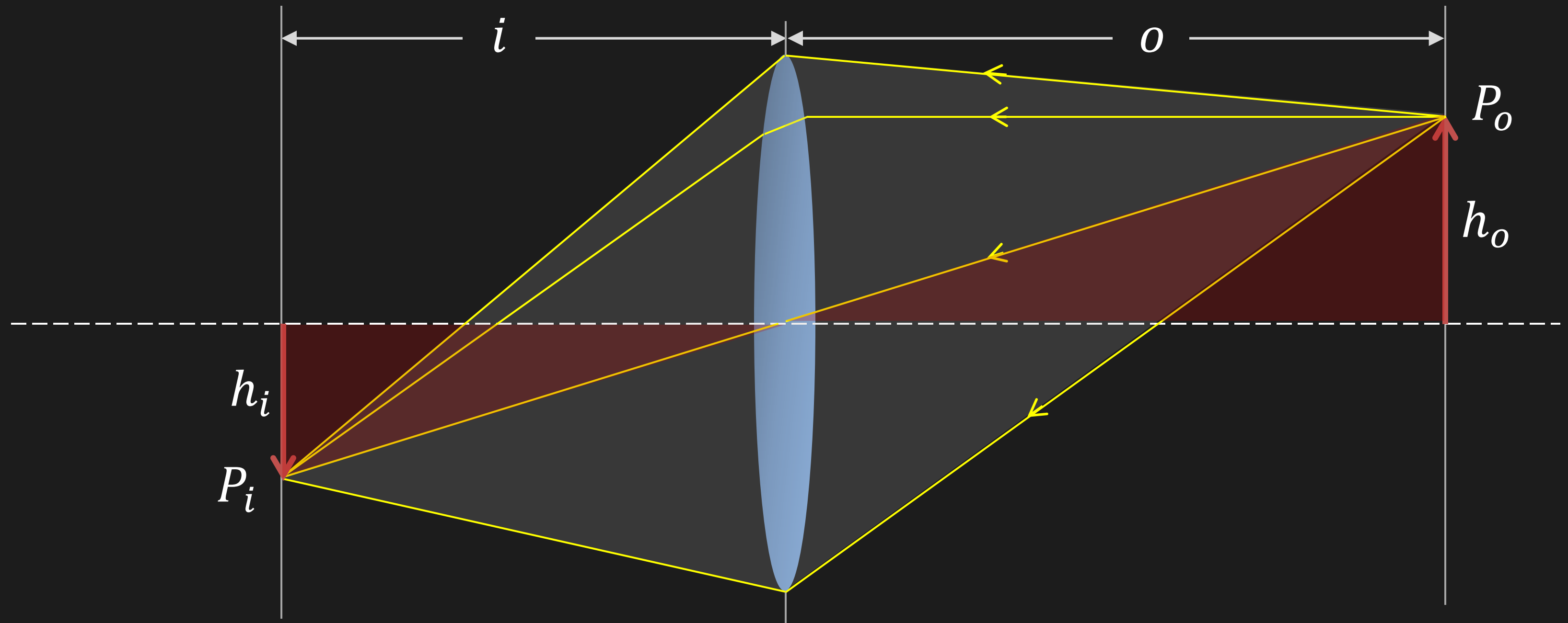
$$\frac{1}{i} + \frac{1}{o} = \frac{1}{f} \quad \Rightarrow \quad \text{If } o = \infty\text{, then } f = i$$





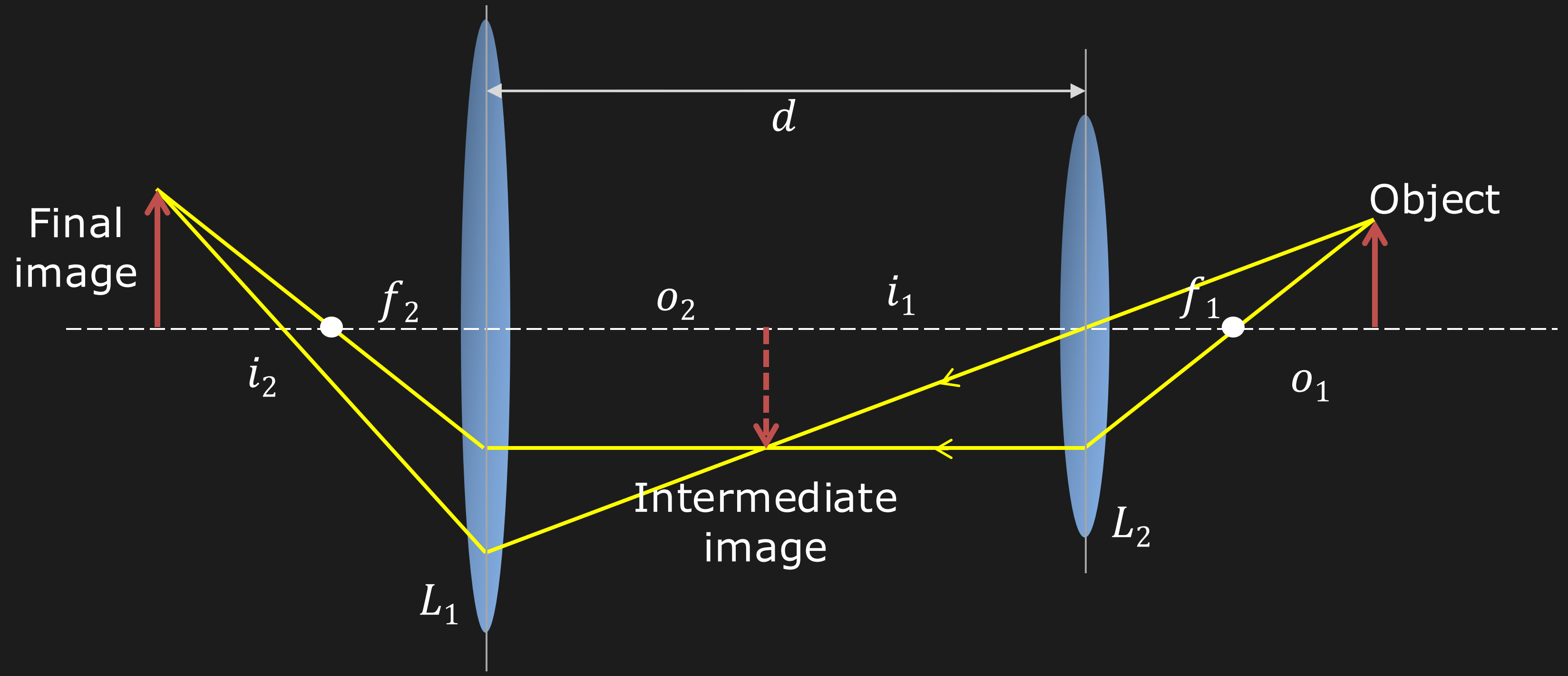
Focal length: Distance at which incoming rays that are parallel to the optical axis converge.

Image Magnification



Magnification: $\boxed{m = \frac{h_i}{h_o} = \frac{i}{o}}$

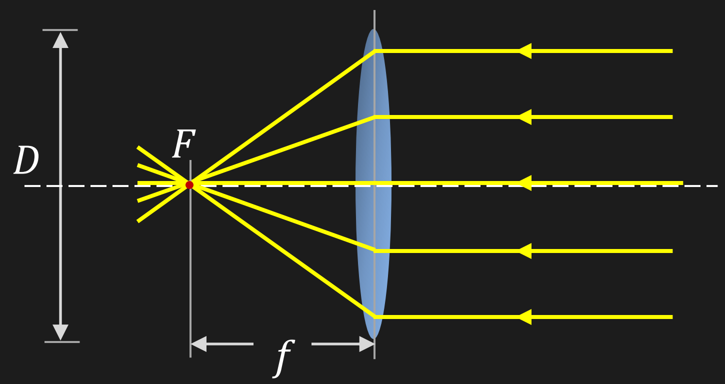
Two Lens System

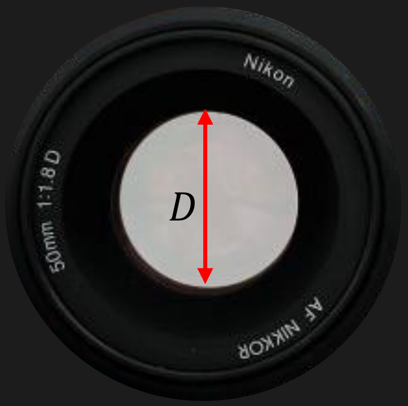


Magnification: $m = \dfrac{i_2}{o_2} \cdot \dfrac{i_1}{o_1}$

Aperture of Lens

Light receiving area of lens, indicated by lens diameter.





Aperture can be reduced/increased to control image brightness

f-number (f-stop, f-ratio) of Lens

Convenient to represent aperture as a fraction of focal length

$\boxed{\text{Aperture: } D = f/N}$ $\boxed{\text{F-Number: } N = f/D}$

where N is called the F-Number of lens.

Ex: A $50mm$ focal length, $f/1.8$ lens implies:

$N = 1.8\ (D = 27.8mm)$ when aperture is fully open

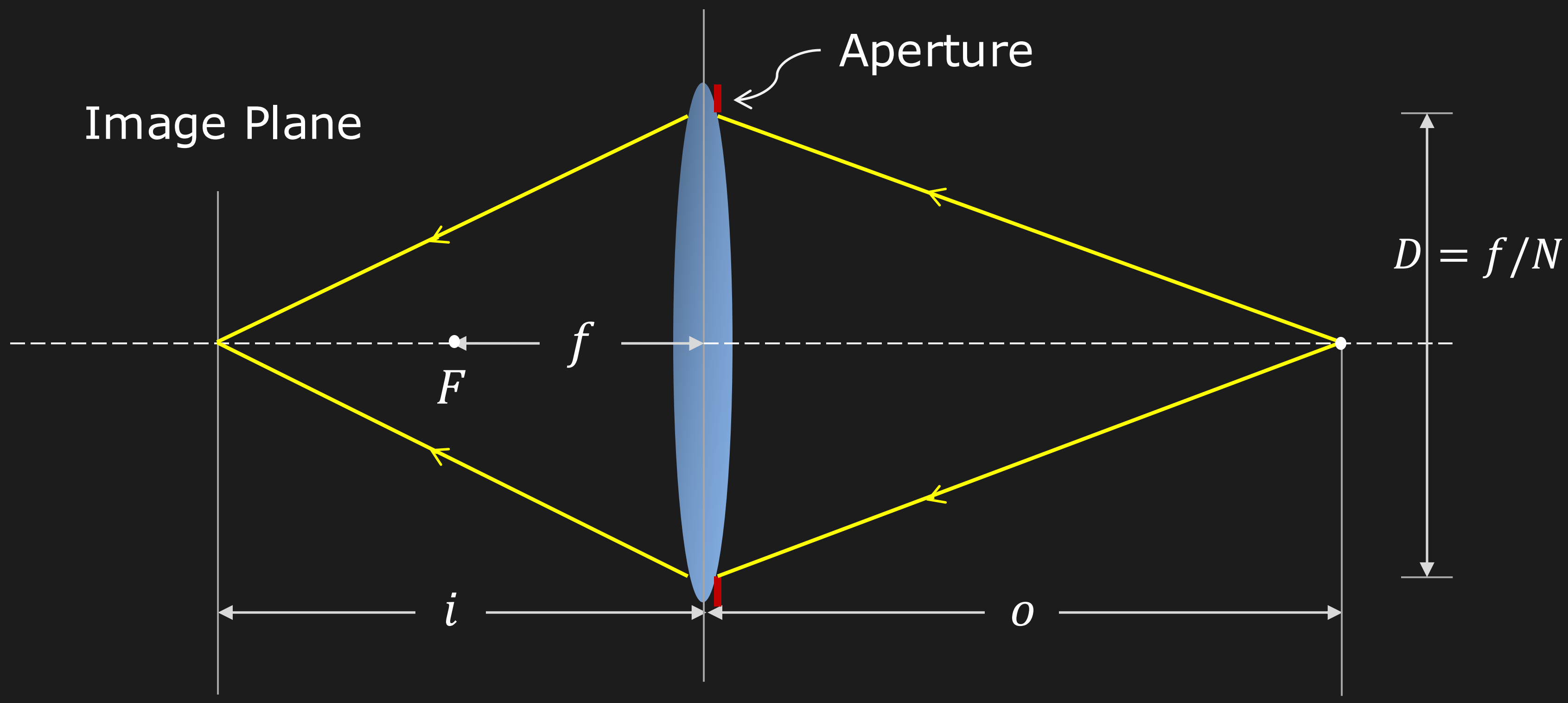
$N = 1.8$
$D = 27.78mm$

$N = 4$
$D = 12.5mm$

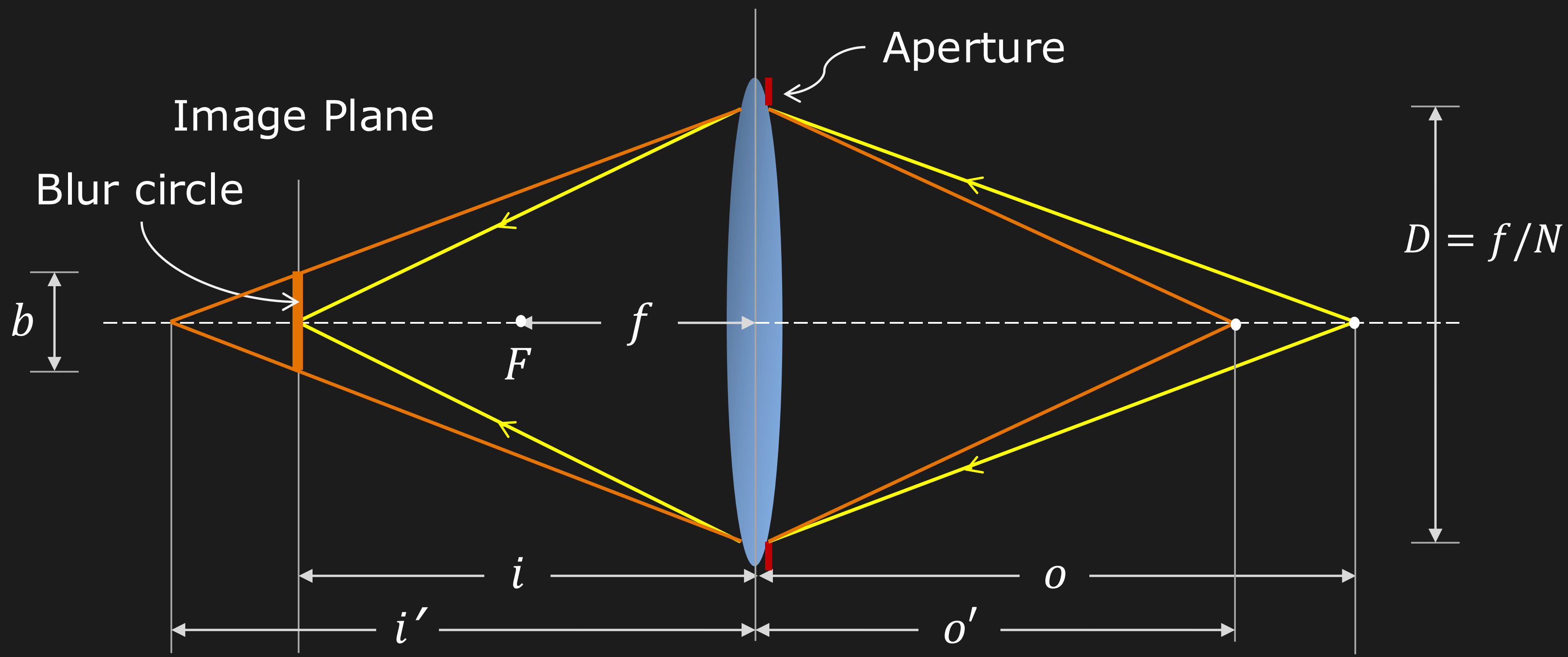
$N = 8$
$D = 6.25mm$

$N = 11$
$D = 4.45mm$

Lens Defocus



Lens Defocus



From similar triangles:

$$\frac{b}{D} = \frac{|i' - i|}{i'}$$

Blur circle diameter:

$$\boxed{b = \frac{D}{i'}|i' - i|} \quad \boxed{b \propto D \propto \frac{1}{N}}$$

Blur Circle (Defocus)

Focused Point

$$\frac{1}{i}+\frac{1}{o}=\frac{1}{f}$$

$$i=\frac{of}{o-f}$$

Defocused Point

$$\frac{1}{i'}+\frac{1}{o'}=\frac{1}{f}$$

(Gaussian Lens Law)

$$i'=\frac{o'f}{o'-f}$$

$$i'-i=\frac{f}{(o'-f)}\cdot\frac{f}{(o-f)}\cdot(o-o')$$

$$b=Df\left|\frac{(o-o')}{o'(o-f)}\right|$$

$$\boxed{b=\frac{f^2}{N}\left|\frac{(o-o')}{o'(o-f)}\right|}$$

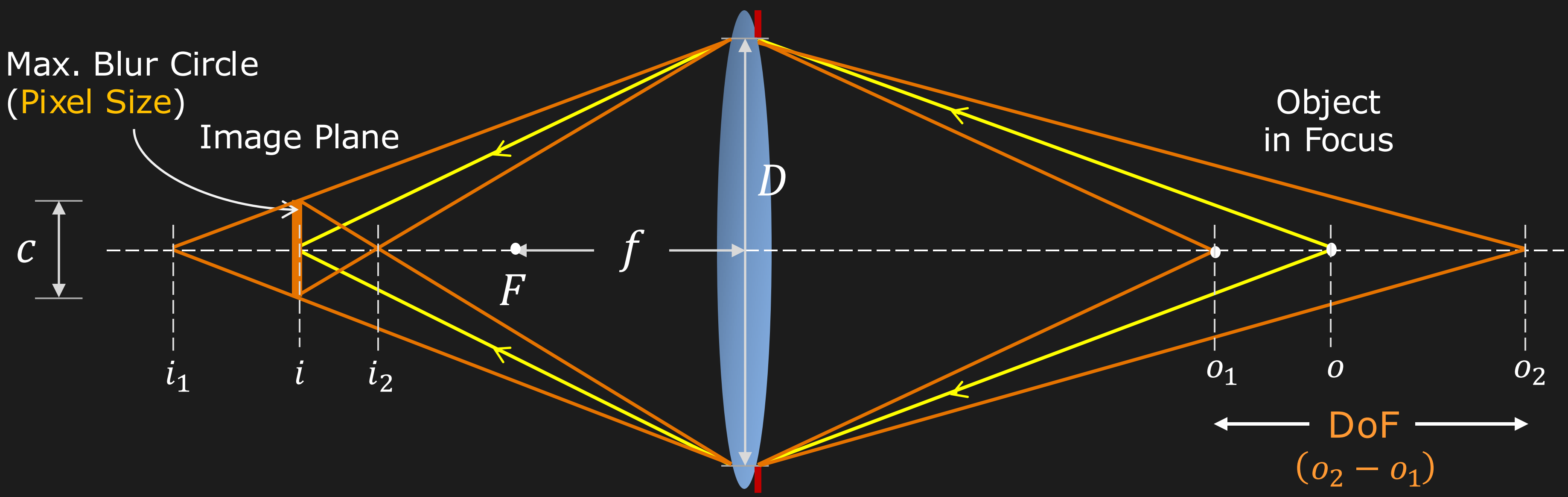
Depth of Field (DoF)

I.11


I.12

Range of object distances over which the image is "sufficiently well" focused, i.e., range over which blur b is less than pixel size.

Depth of Field (DoF)

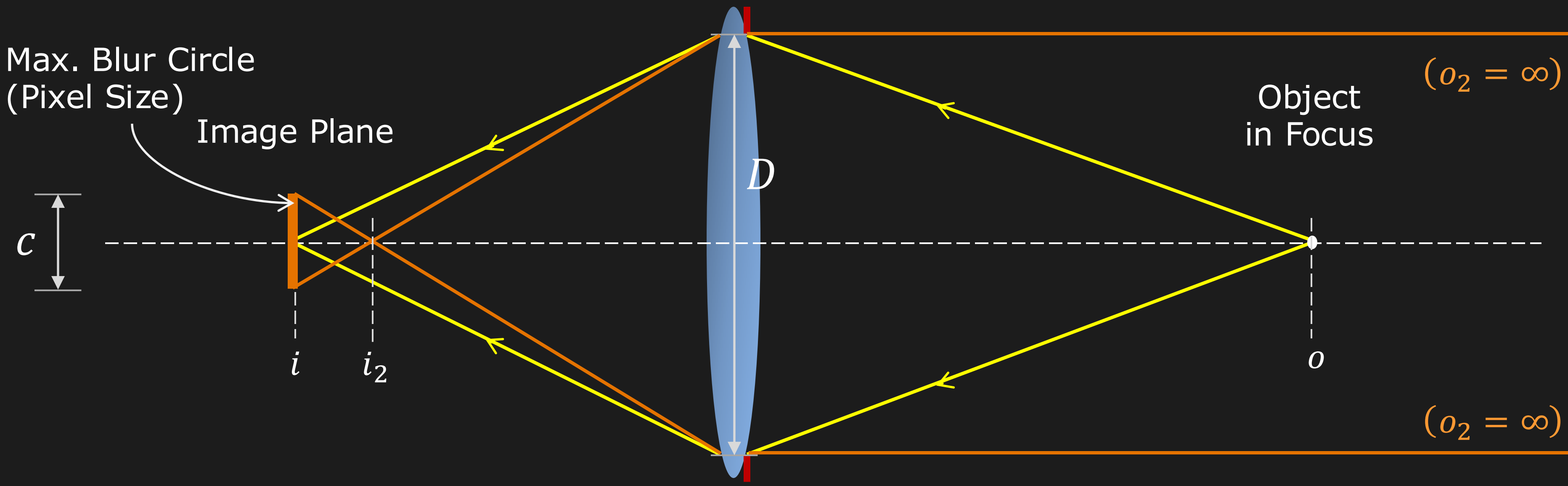


If o_1 and o_2 are the nearest and farthest distances respectively for which blur circle is maximum c, then:

$$c = \frac{f^2(o - o_1)}{N o_1 (o - f)} \qquad c = \frac{f^2(o_2 - o)}{N o_2 (o - f)}$$

Depth of Field: $\boxed{o_2 - o_1 = \dfrac{2 o f^2 c N (o - f)}{f^4 - c^2 N^2 (o - f)^2}}$

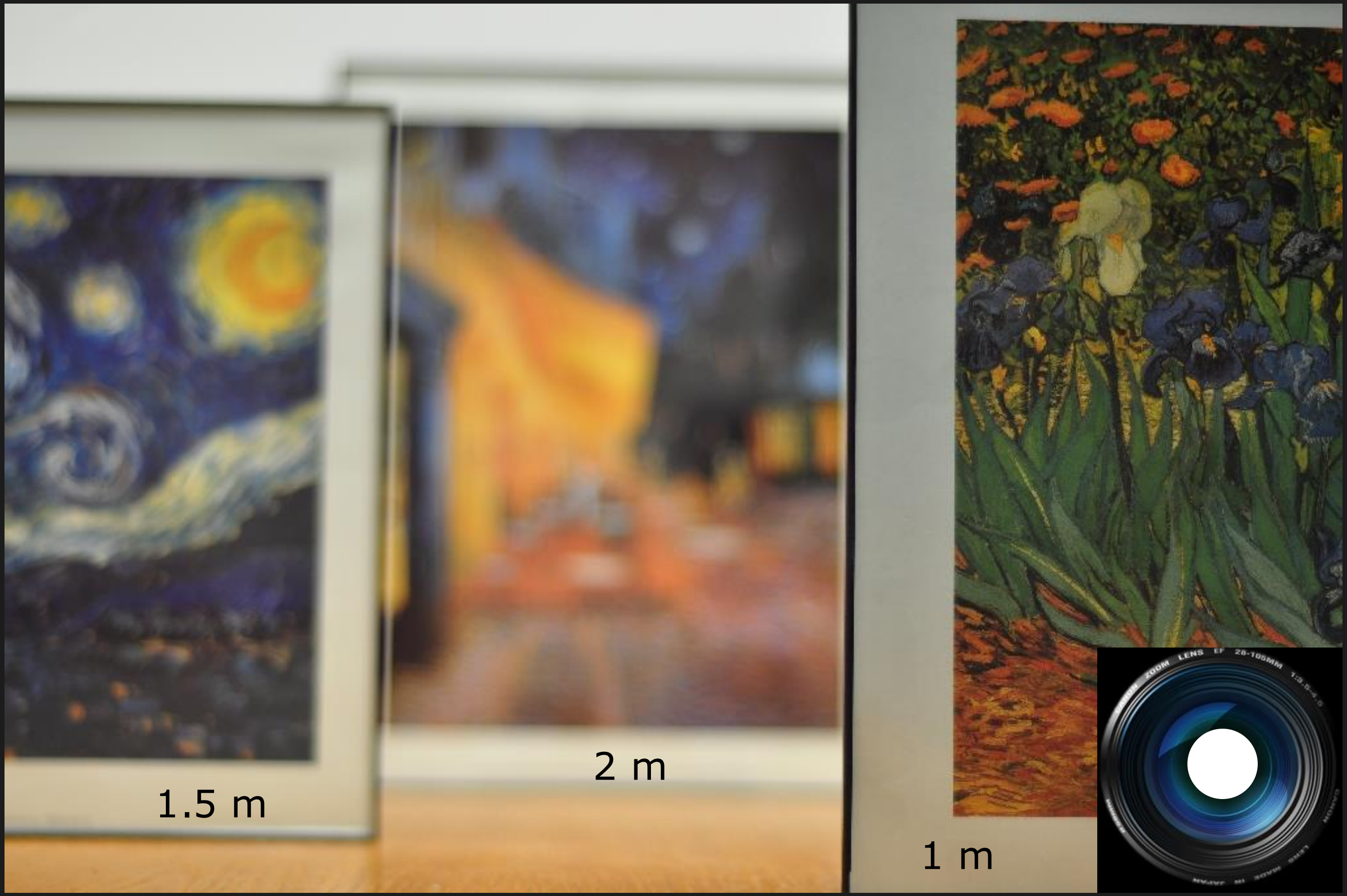
Hyperfocal Distance



The closest distance $o = h$ the lens must be focused to keep objects at infinity $(o_2 = \infty)$ acceptably sharp (blur circle $\leq c$).

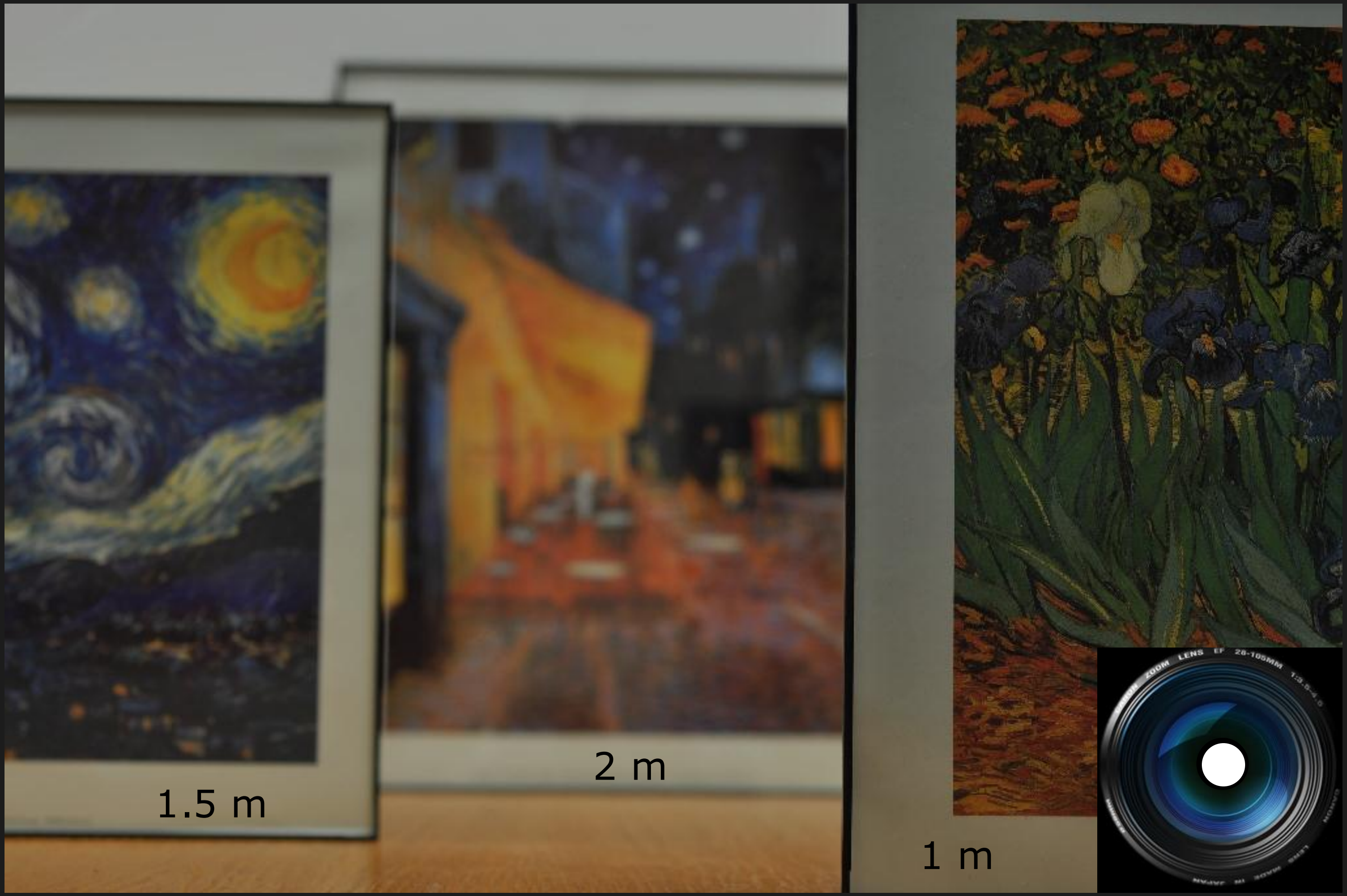
Hyperfocal Distance: $\boxed{h = \frac{f^2}{Nc} + f}$

Aperture Size: DOF vs. Brightness



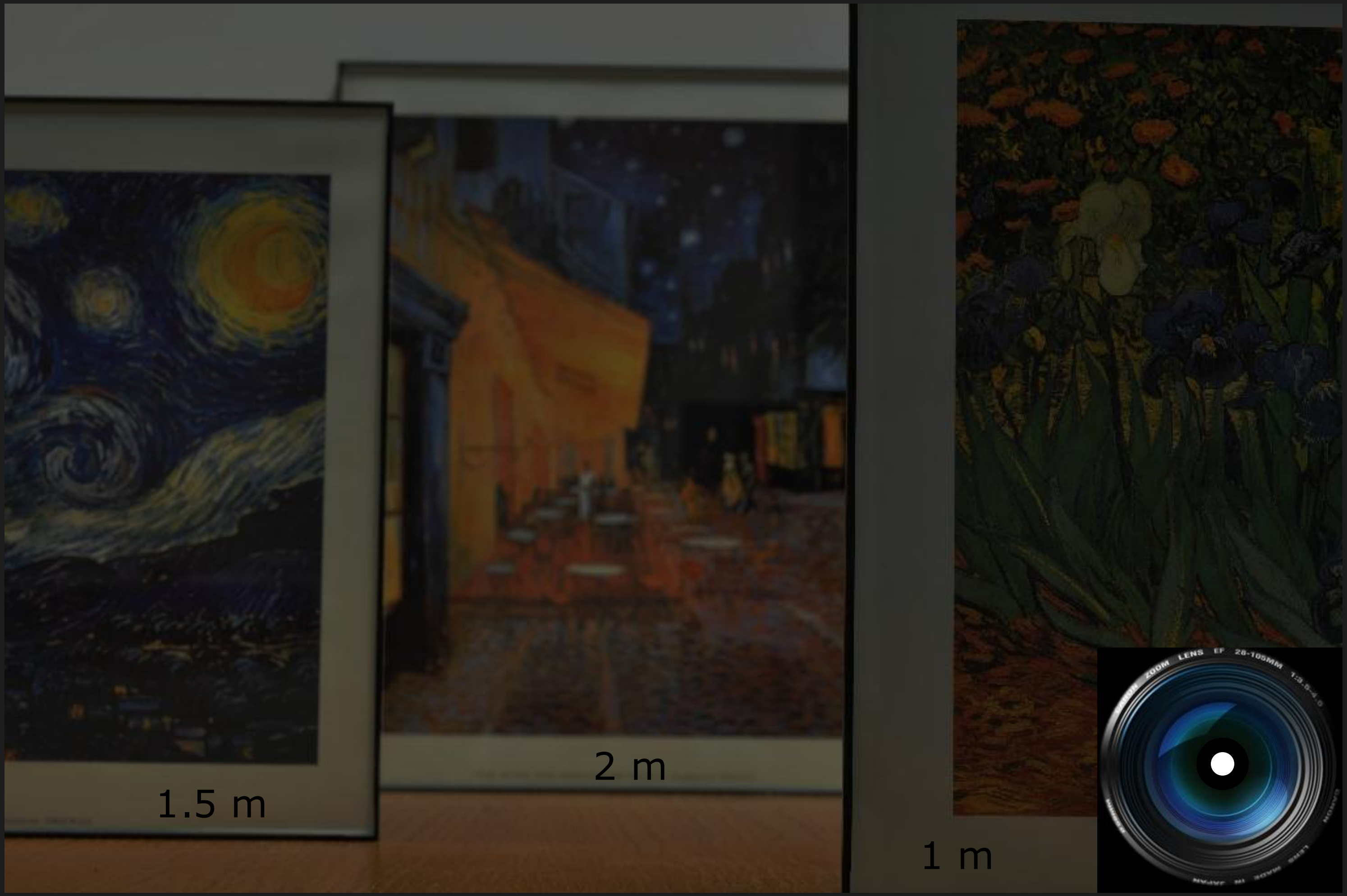
Focal Length 50 mm, Focus = 1 m, Aperture D = 25 mm, F-Number N = 2

Aperture Size: DOF vs. Brightness



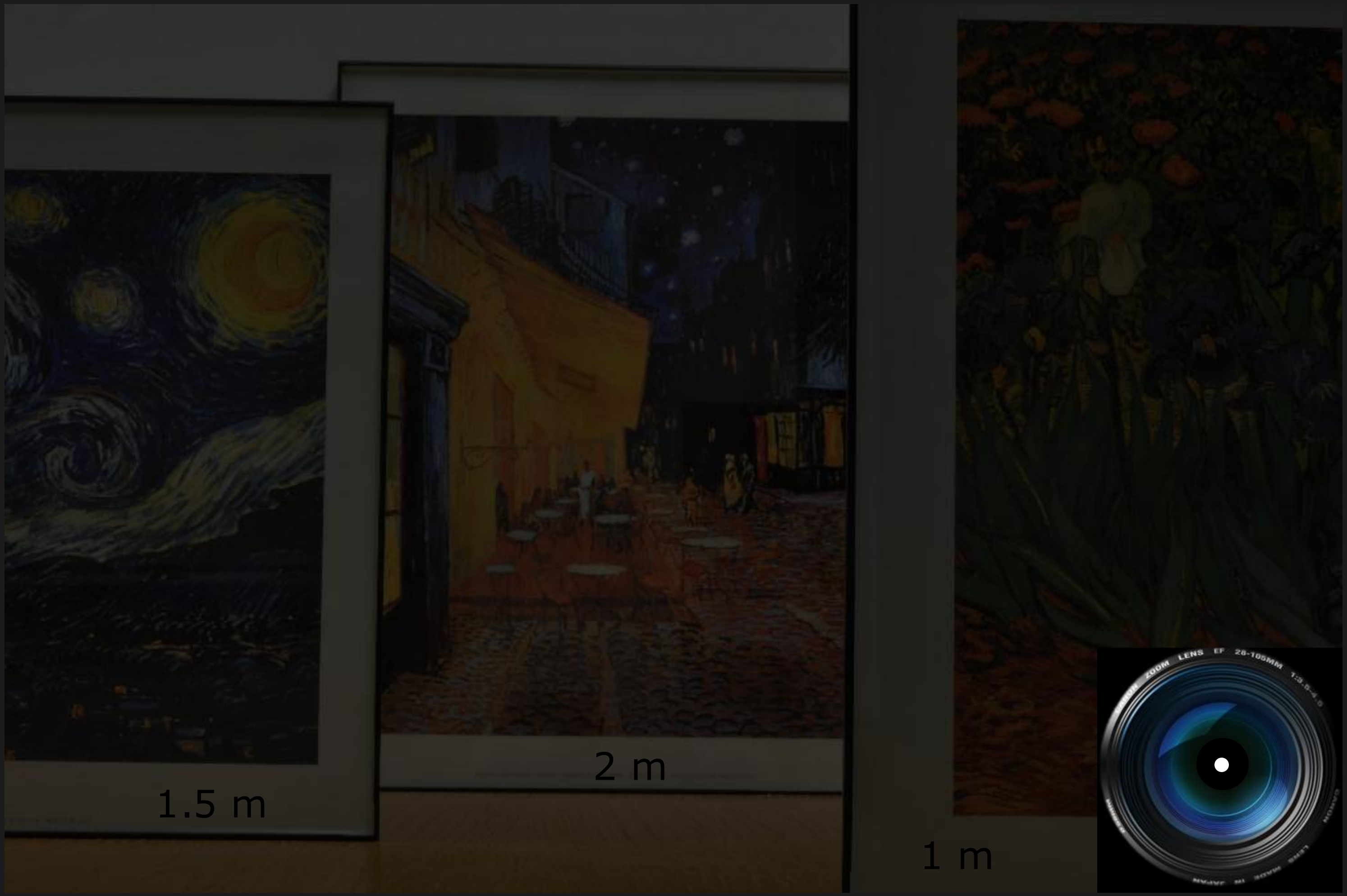
Focal Length 50 mm, Focus = 1 m, Aperture D = 12.5 mm, F-Number N = 4

Aperture Size: DOF vs. Brightness



Focal Length 50 mm, Focus = 1 m, Aperture D = 6.25 mm, F-Number N = 8

Aperture Size: DOF vs. Brightness



Focal Length 50 mm, Focus = 1m, Aperture D = 3.125mm, F-Number N = 16

Aperture Size: DOF vs. Brightness

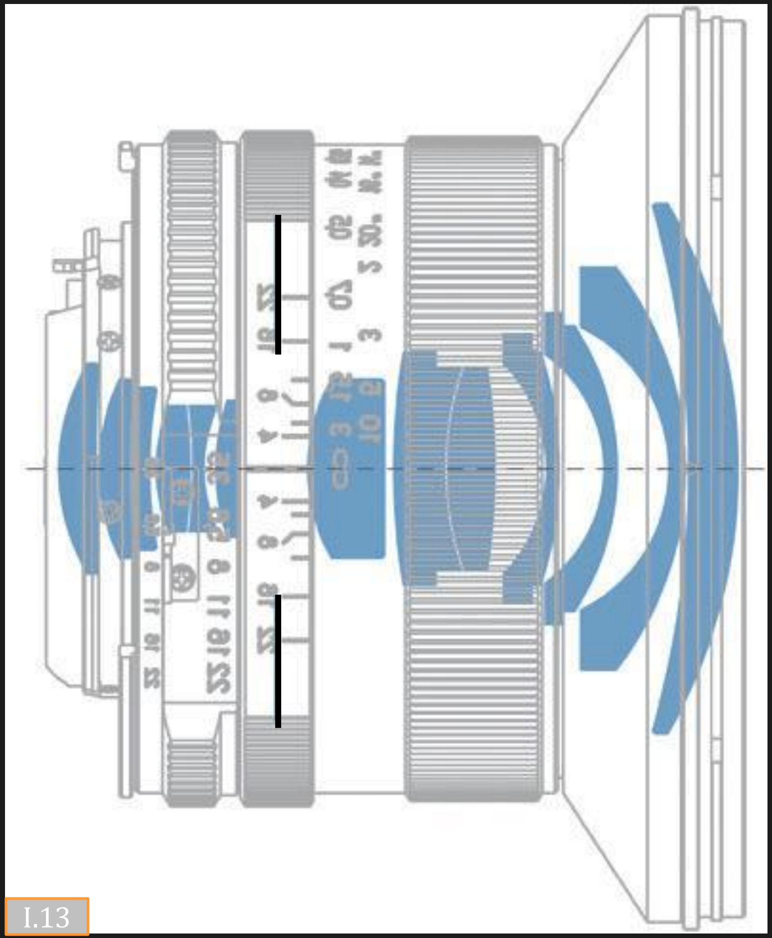
Large Aperture (Small F-Number)

- Bright Image or Small Exposure Time
- Shallow Depth of Field

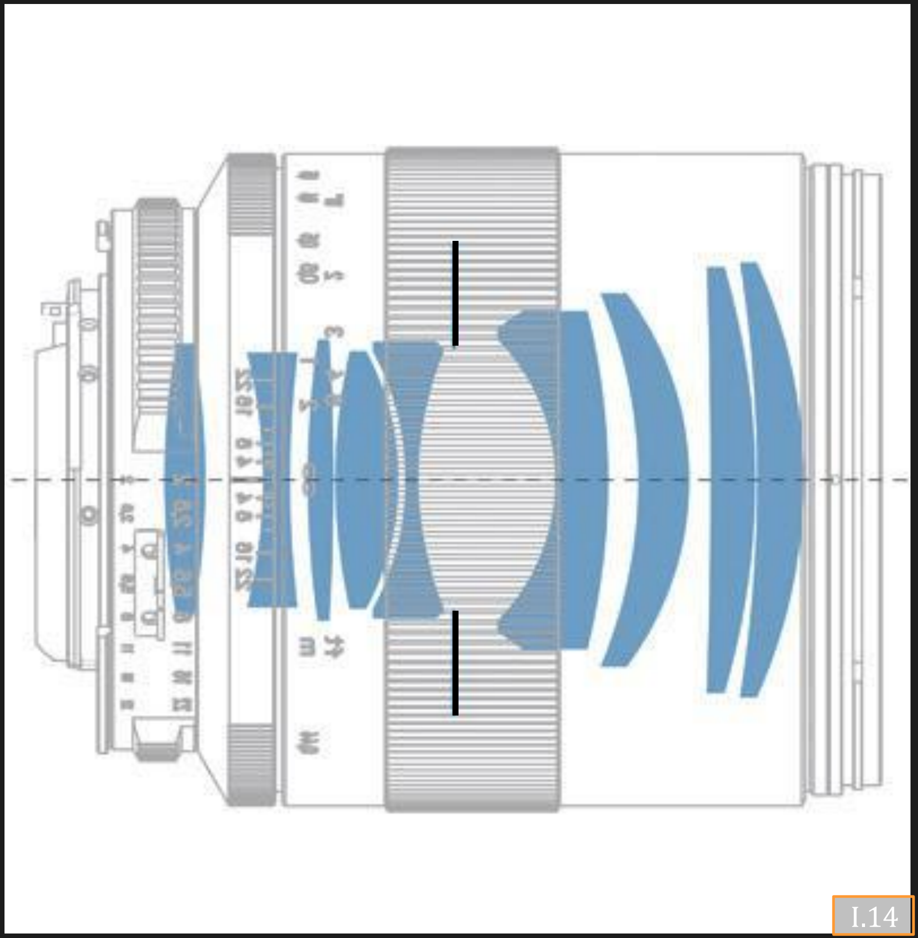
Small Aperture (Large F-Number)

- Dark Image or Long Exposure Time
- Large Depth of Field

Compound Lenses

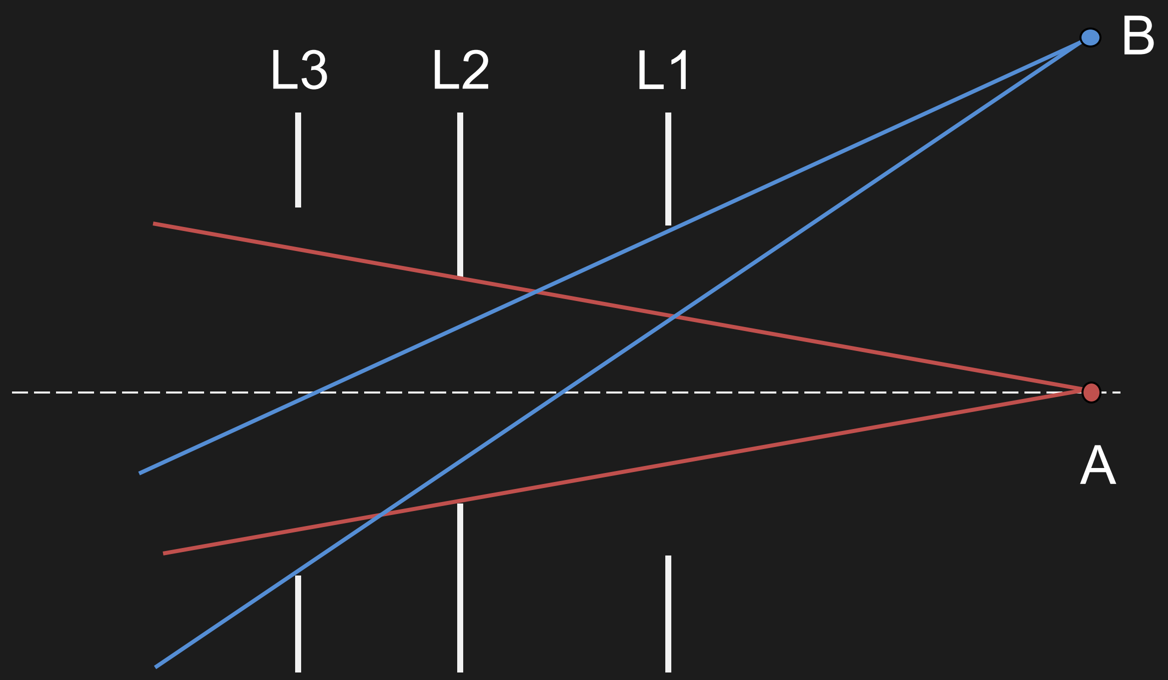


Zeiss 18mm F3.5



Zeiss 100mm F2

Vignetting



More light passes through L3 from point A than point B.

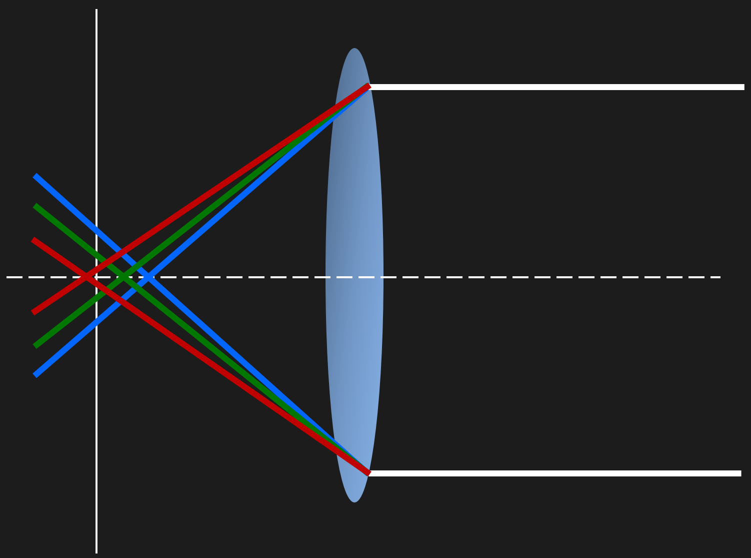
Results in a smooth fall-off in brightness from A to B.

Vignetting

Brightness fall-off (Vignetting) in image of a White Wall

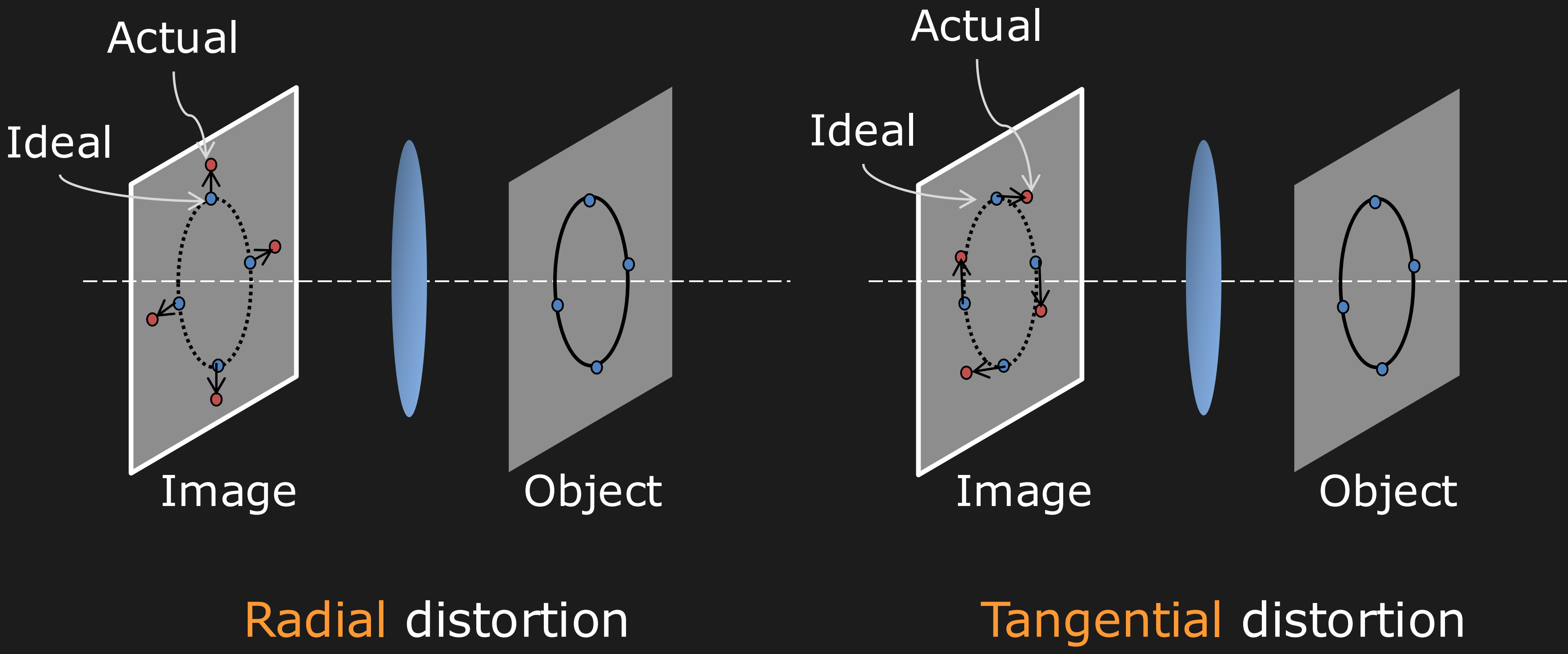
Brightness fall-off (Vignetting) in image of a Natural Scene

Chromatic Aberration



Refractive index (and hence focal length) of lens is different for different wavelengths.

Geometric Distortion



Due to lens imperfections

Geometric Distortion Correction

Radial (Barrel) distortion

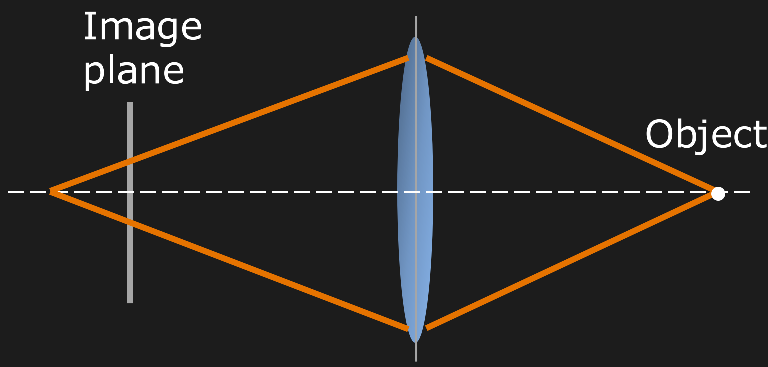
Undistorted image

When Geometric Distortion is Useful

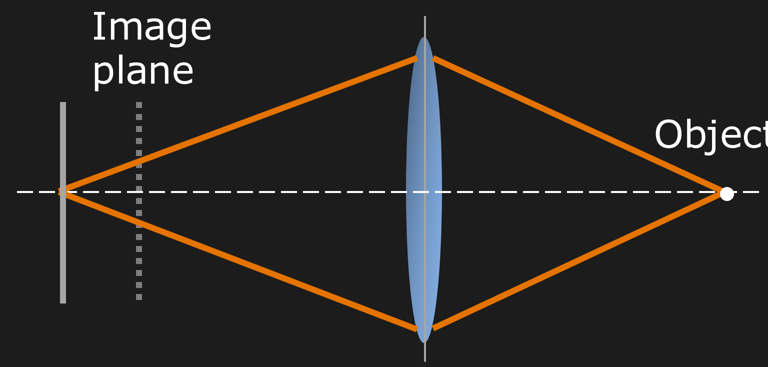
Fisheye Lens

Radial Distortion in Fisheye Lens

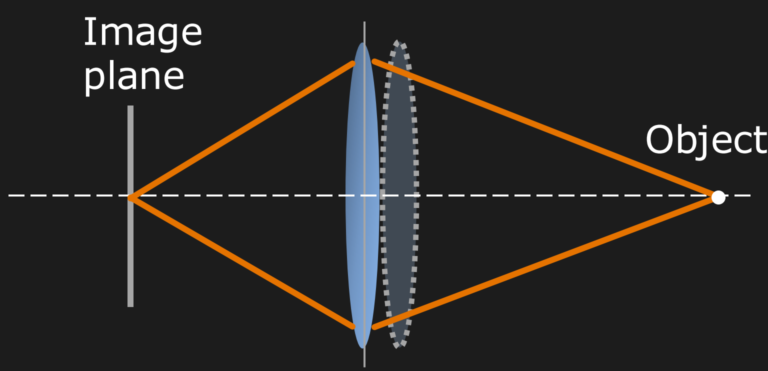
Focusing



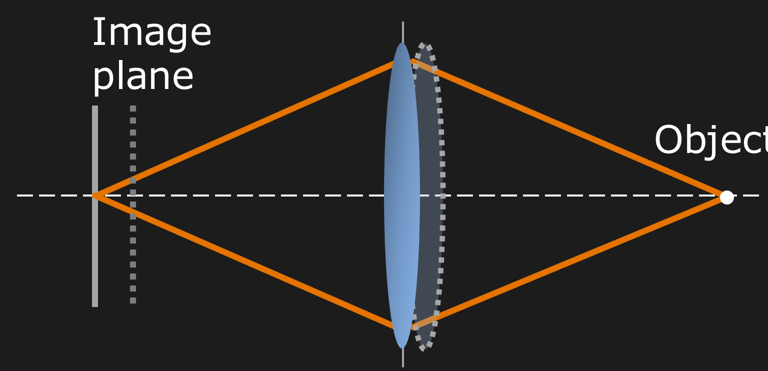
Defocused System



Move the image plane

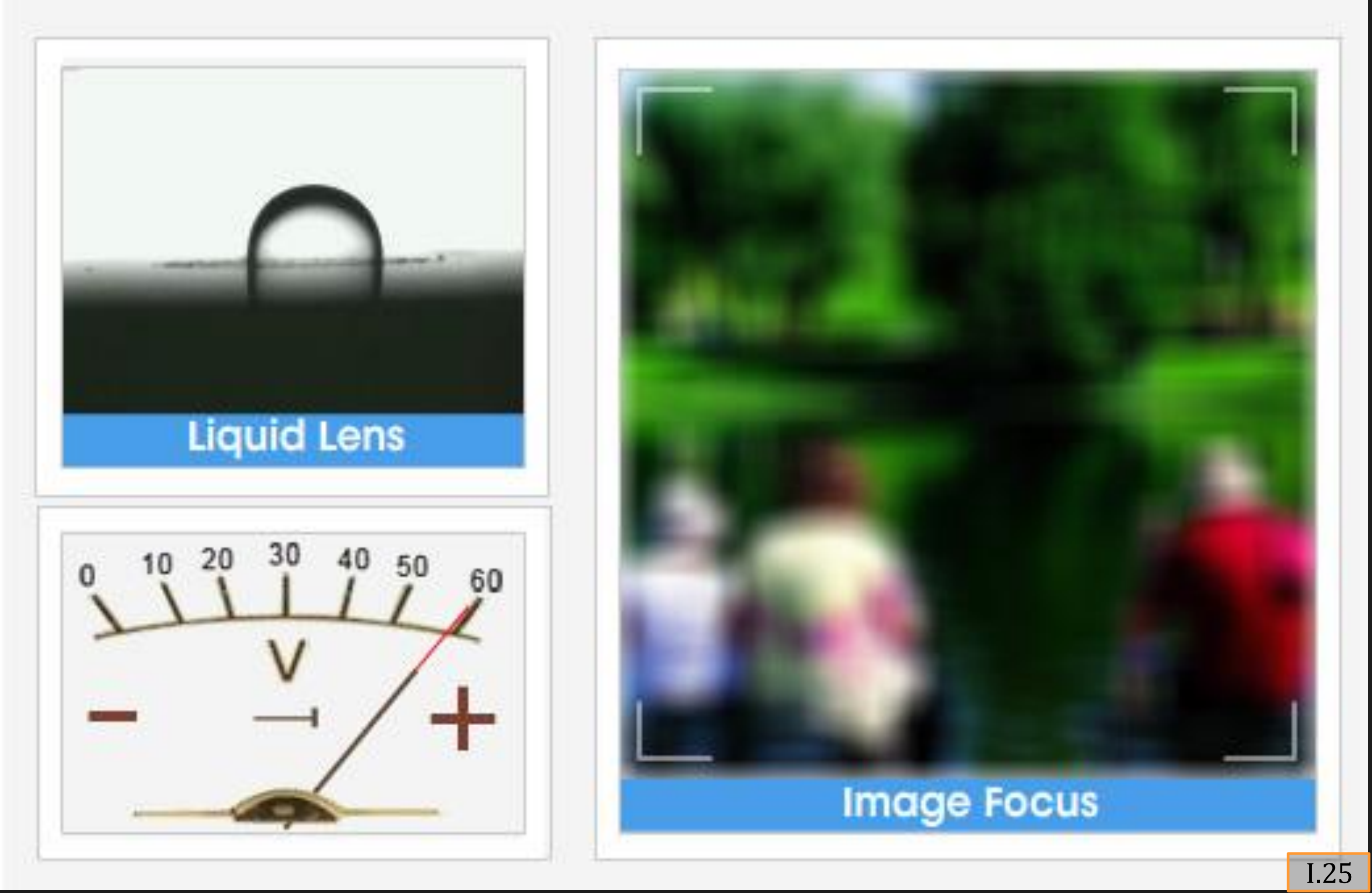


Move the lens



Move both lens and image plane

Liquid Lens



I.25

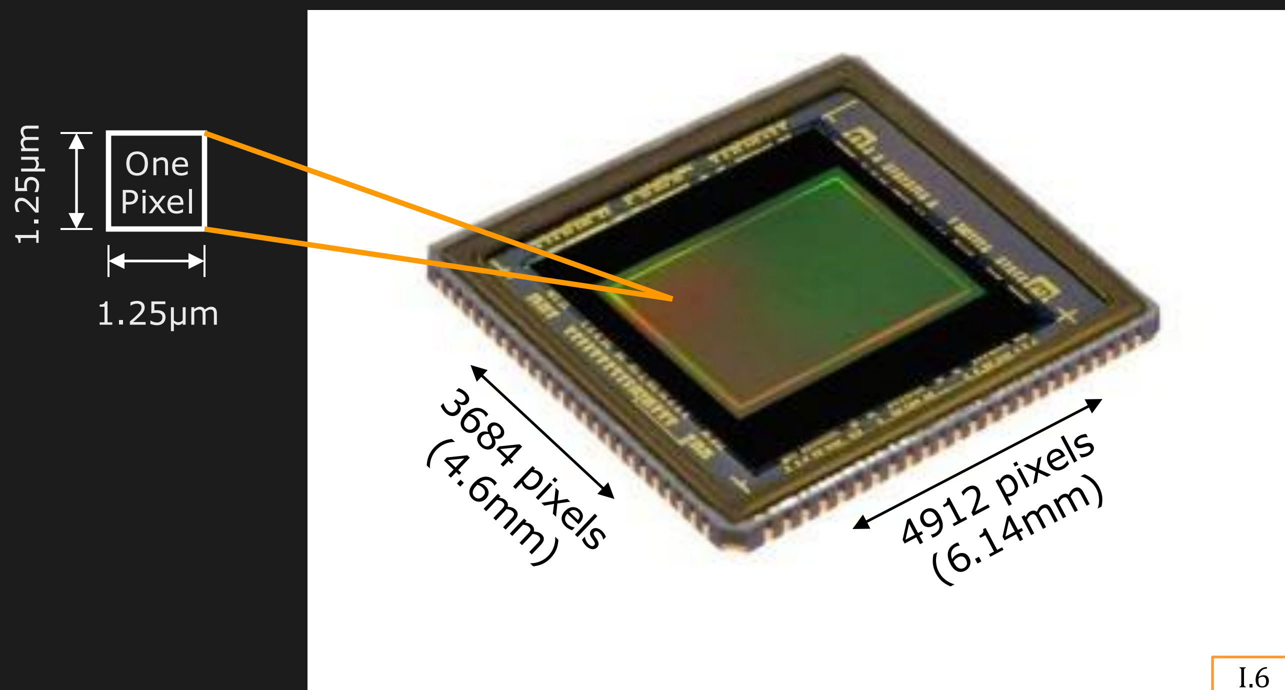
The shape and hence the focal length of the liquid lens can be precisely controlled by applying a voltage.

Image Sensing

Capturing or Recording of Images

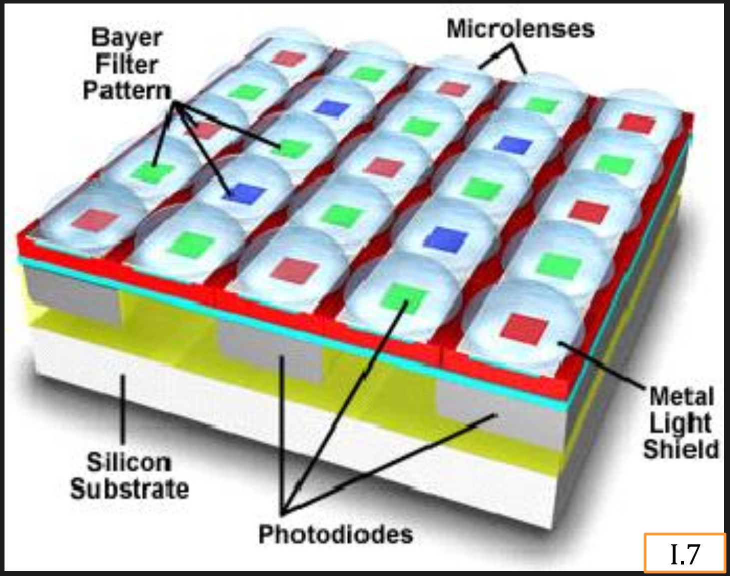
Need to convert Optical Images to Digital Images (*numbers*) for computer representation and use.

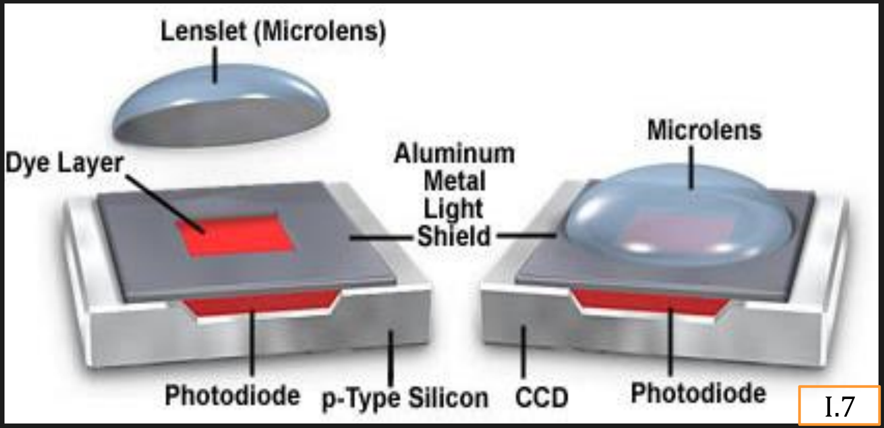
Image Sensor: A Closer Look



18 Megapixels

Image Sensor: A Closer Look

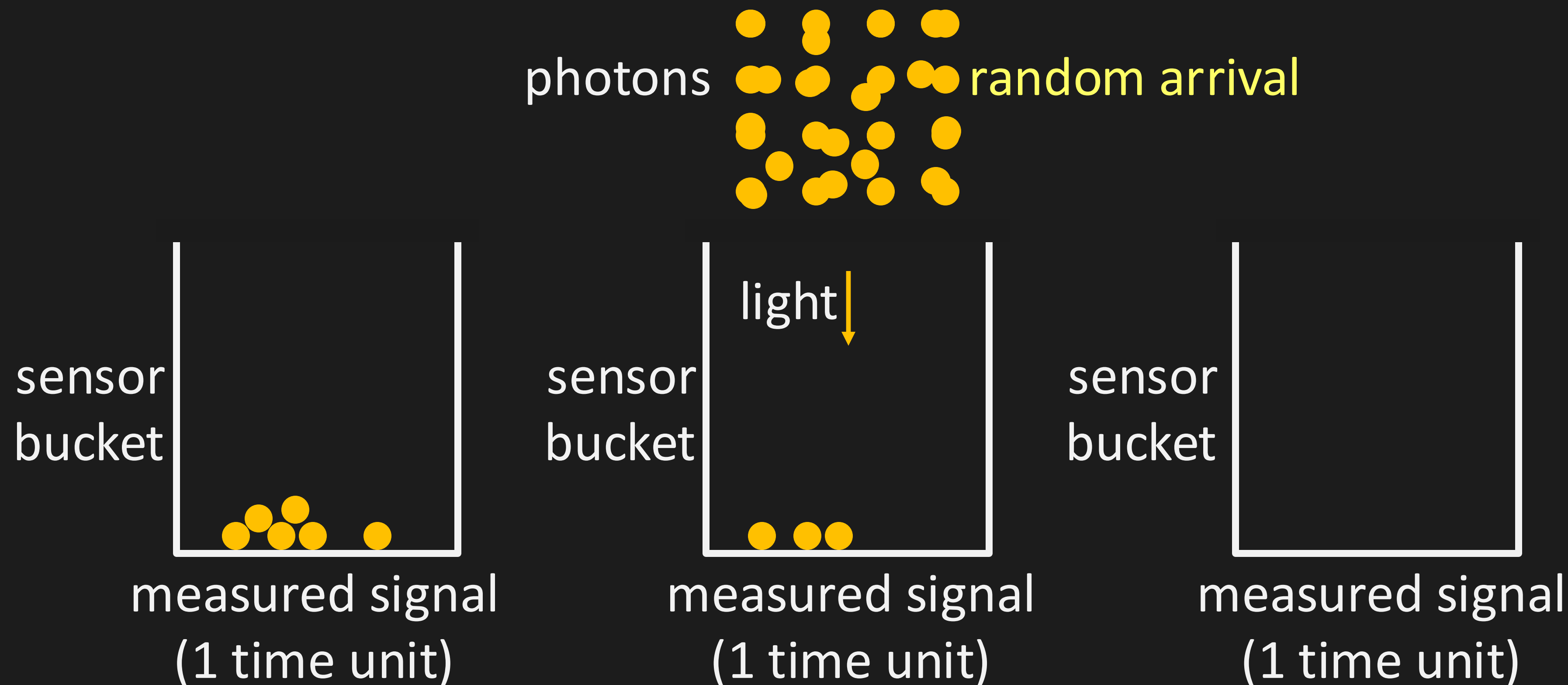




Noise in Image Sensors

- Photon Shot Noise (Scene Dependent)
  - Quantum nature of light
  - Random arrival of photons
- Readout Noise (Scene Independent)
  - Electronic Noise: Pre analog-to-digital conversion
  - Quantization Noise: Post analog-to-digital conversion
- Other Sources (Scene Independent)
  - Dark Current Noise: Thermally generated electrons
  - Fixed Pattern Noise: Defective pixels

Photon Noise: Random Arrival Of Photons



arrival of light is a discrete random Process

mean photon flux (per unit time)= 3 Photons

Photon Noise

measured value

true (mean) value

photon noise (random variable)

$$I_{meas} = I_{true} + \sigma_{photon}$$

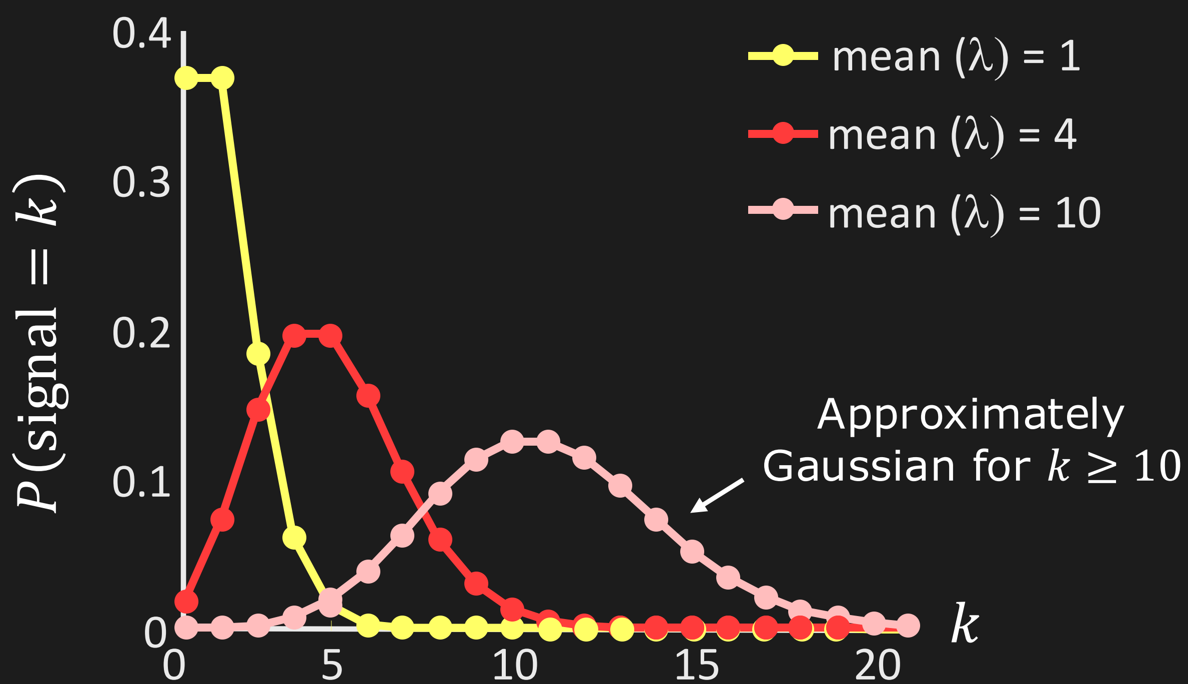
variance

$$var\left(\sigma_{photon}\right) = I_{true}$$

standard deviation

$$std\left(\sigma_{photon}\right) = \sqrt{I_{true}}$$

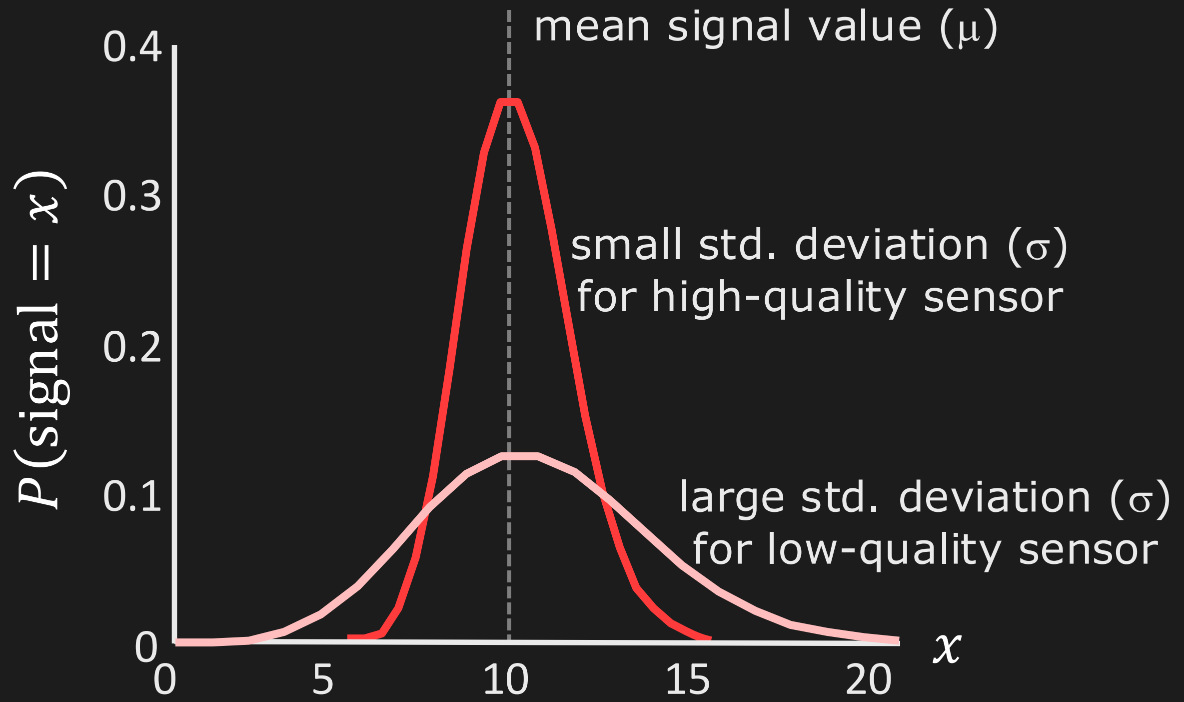
Photon Noise: Poisson Distribution



$$P(\text{signal} = k) = \frac{\lambda^k e^{-\lambda}}{k!}$$

$Var\,[\text{signal}] = Mean\,[\text{signal}] \Rightarrow$ Scene Dependent Noise

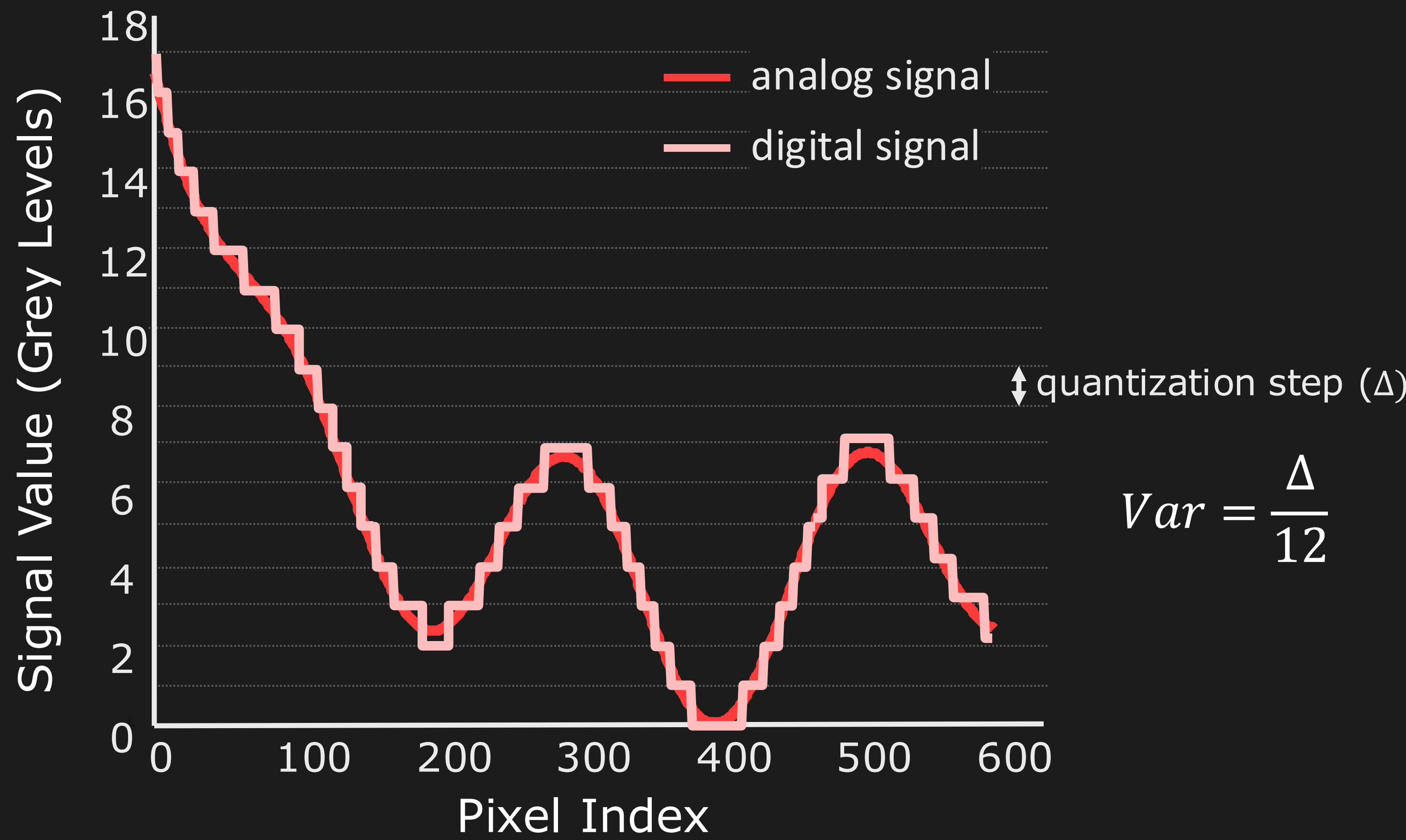
Read Noise: Gaussian Distribution



$$\boxed{P(\text{signal} = x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}}$$

Depends on Sensor Quality (Scene Independent)

Quantization Noise



$$Var = \frac{\Delta}{12}$$

Negligible in Modern Sensors

Due to High Intensity Resolution (12-14 bits)

Other Noise Sources

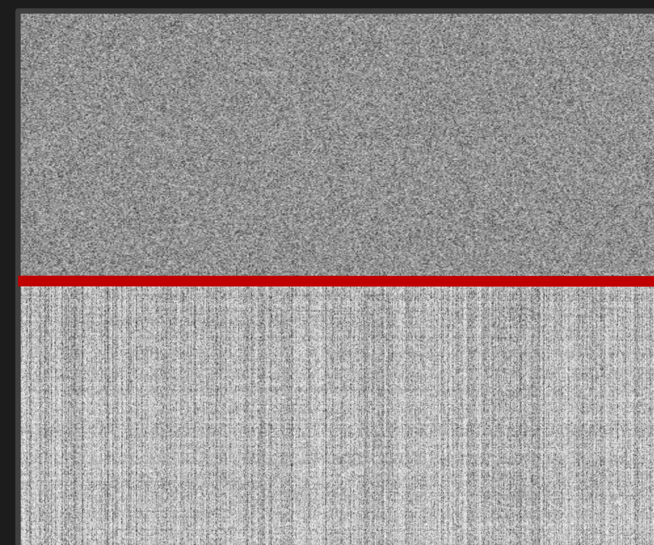
Dark Current Noise

Follows Poisson Distribution

Significant Only for Long (>2 min) Exposures (Astronomy)

Fixed Pattern Noise

Random Noise



Fixed Pattern Noise

Can be Reduced by Dark Frame Subtraction

Sensor Dynamic Range

$$\text{Dynamic Range} = 20 \log\left(\frac{B_{max}}{B_{min}}\right)$$ decibels (dB)

B_{max}: The maximum possible photon energy (full potential well)

B_{min}: The minimum detectable photon energy (in the presence of noise)

Sensor	$B_{max}:B_{min}$	dB
Human Eye	1,000,000:1	120
HDR Display	200,000:1	106
Digital Camera	4096:1	72.2
Film Camera	2948:1	66.2
Digital Video	45:1	33.1

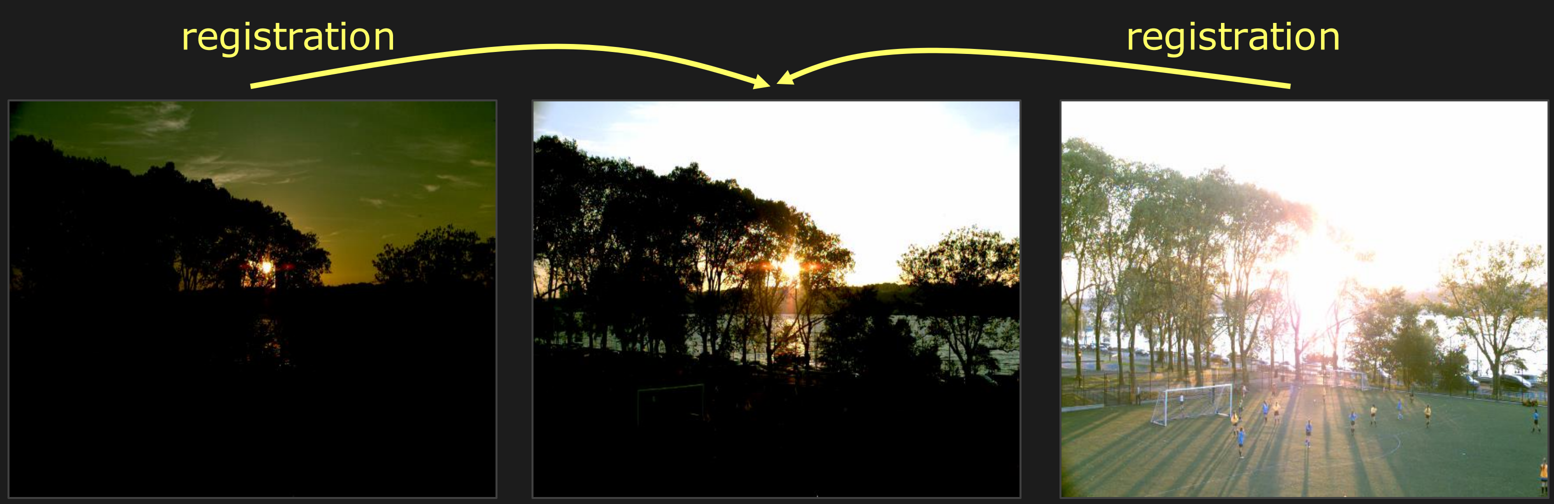
Overcoming Low Dynamic Range

Low Dynamic Range

Overcoming Low Dynamic Range

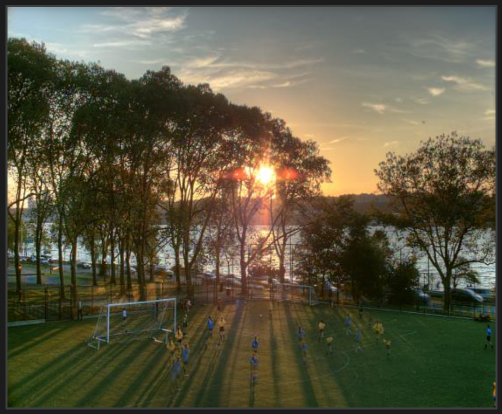
Low Dynamic Range

Exposure Bracketing for HDR Capture



captured images

Exposure Bracketing for HDR Capture



HDR image

Camouflage in Cuttlefish

Cuttlefish detects the patterns in the environment and changes the appearance of its skin to camouflage itself.

[Hanlon 2007]

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