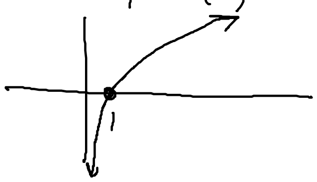
Lets consider the Probability (proportion) of having diabetes for a given value of glu

$$odds(E) = \frac{Pr[E]}{1-Pr[E]}$$

$y = \ln(x)$



$Prob \in (0,1)$

$Odds \in (0,\infty)$

$\ln(odds) \in (-\infty,\infty)$

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say $\ln(odds) = z$

$$odds = e^z$$

$$Pr[Y=1] = \frac{e^z}{1+e^z}$$

$$= \frac{e^{-z}}{e^{-z}} \frac{e^z}{1+e^z} = \frac{1}{e^{-z}+1}$$

$odds = \frac{p}{1-p}$

$(1-p)odds = p$

$odds - p \cdot odds = p1$

$odds = p(1+odds)$

$p = \frac{odds}{1+odds}$

$\hat{z} = \ln(\text{odds}(Y=1))$

$\hat{y} = \sigma(\hat{z})$

$\hat{z} = \hat{\beta}_0 + \hat{\beta}_1 X_1 + \cdots + \hat{\beta}_p X_p$

consider $X_1 = x^*$

$X_2 = x^* + 1$

$\hat{z}_1 = \hat{\beta}_0 + \hat{\beta}_1 x^*$

$\hat{z}_2 = \hat{\beta}_0 + \hat{\beta}_1 (x^* + 1)$

$= \hat{\beta}_0 + \hat{\beta}_1 x^* + \hat{\beta}_1$

$\hat{z}_2 - \hat{z}_1 = \hat{\beta}_1$

$\hat{\beta}_1$ is the expected change in <u>log odds</u> for a <u>1</u> unit increase in X.

$\hat{z}_2 = \log(\text{odds}(Y=1 \mid X = x^*+1))$

$\hat{z}_1 = \log(\text{odds}(Y=1 \mid X = x^*))$

$\hat{\beta}_1 = \hat{z}_2 - \hat{z}_1 = \log(\sim) - \log(\sim)$

$= \log\left(\frac{\sim}{\sim}\right) = \log\left(\frac{\text{odds}(Y=1 \mid X=x^*+1)}{\text{odds}(Y=1 \mid X=x^*)}\right)$

$= \log(\text{odds ratio for a 1 unit increase in } X)$

<u>or</u>

$e^{\hat{\beta}_1} =$ predicted odds ratio for a 1 unit incr. in X