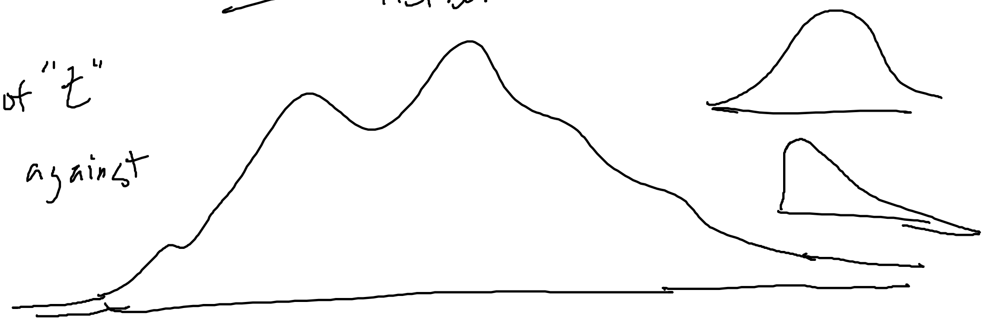
How to calculate 2-Tailed p-values

In Theory - Test statistic T under H_0 has *some* distribution



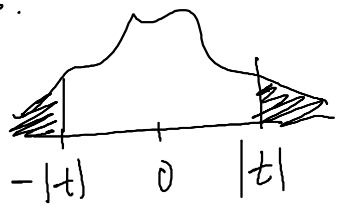
2-Tailed test idea:
very Low or very High values of "t" would be considered evidence against H_0.

dilemma: t can only be big or small (it can't be in both tails at once)

Approaches to handle this:

1) Simply take $|t|$ and $-|t|$ to put t in both tails.

$$p = P(T \le -|t|) + P(T \ge |t|)$$



☆ assumes T distribution is symmetric & centered at 0.

1a) (similar)

absolute value the distr. of T

$$p = P(|T| \ge |t|)$$

This basically makes the same assumptions.

2) take the one tailed p-value and double it

idea is t could have just as well been equally extreme in the other direction.

* Does not assume symmetry or center at 0
* works well for MC tests

in R:

$$2 * \min\left(\text{mean}(T_s >= t),\ \text{mean}(T_s <= t)\right)$$

assuming T_s is a vector of simulated t values
t is the observed test stat.

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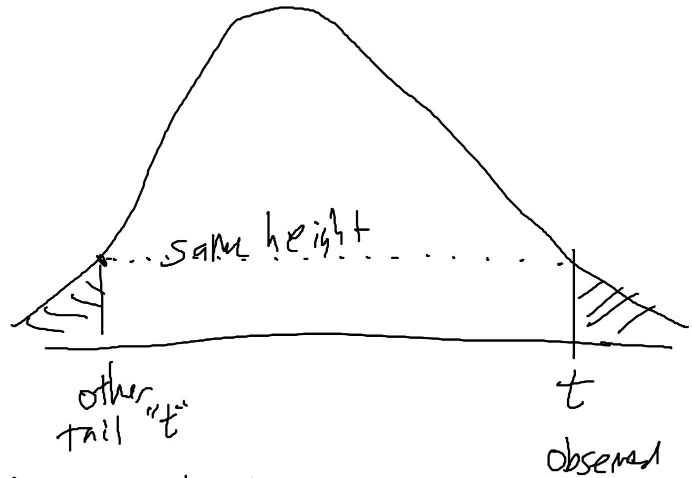
3) (more complicated)

find a value in the opposite tail with equal "likelihood"



* more complicated
* does not work well with MC methods

* based on ideas from Fisher

$X_i \sim \text{Bernoulli}(p)$

x	$P(X=x)$
0	$1-p$
1	p

$$E(X_i) = 0(1-p) + 1(p)$$
$$= p$$

$$\bar{X} = \frac{X_1 + X_2 + \cdots + X_n}{n} = \frac{1}{n}\left(X_1 + X_2 + \cdots + X_n\right)$$

$$E(\bar{X}) = E\left(\frac{1}{n}\left(X_1 + X_2 + \cdots + X_n\right)\right)$$
$$= \frac{1}{n}E\left(X_1 + X_2 + \cdots + X_n\right)$$
$$= \frac{1}{n}\left(EX_1 + EX_2 + \cdots + EX_n\right)$$
$$= \frac{1}{n}(\underbrace{p + p + \cdots + p}_{n \text{ of them}}) = \frac{1}{\cancel{n}}(\cancel{n}p) = p$$

X_1 (first widget) $E(X_1) = p$

$\bar{X}$ (mean for n) $E(\bar{X}) = p$

What is the variance of a Bern(p) ?

x	$P(X=x)$	$(x-\mu)^2$
0	$1-p$	$(0-p)^2$
1	p	$(1-p)^2$

$(-p)^2 = p^2$

$$Var(X) = E(X-\mu)^2 = (1-p)(-p)^2 + p(1-p)^2$$

$$= p(1-p)(\cancel{p} + 1 - \cancel{p})$$

$$= p(1-p)$$