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| two coins a trick coin with two heads and a fair coin with one head and one tail. Grab one coin at random, and flip it $N$ times. Decide if it is the fair coin or the trick coin. The null hypothesis is that it is the fair coin. \*\*Decision Rule 1\*\*: If after $N$ flips there are no tails, then decision is the trick coin. If there is at least 1 tail then you know it is the fair coin. a. Using "Decision Rule 1", what is the lowest number of flips $N$ would you need in order to have a significance level less than 5% for this test? |  |
| N=5 what is the power of the test |  |
| Suppose $N=4$ is decided. How can you modify the decision process to have a significance level of exactly 5%? Does this change the power of the test? |  |
| Suppose if you guess correct you win \$100 (and if you're wrong you get nothing), but each flip of the coin costs \$10. What strategy would you use to maximize your expected profit from this game?  **Strategy**  For N=1 : Probability = 0.50 1-1/2 E(1)=0.50\*100-10=40  For N=2: Probability = 0.75 1-1/2^2 E(2)=0.75\*100-20=55  For N=3: Probability = 0.875 1-1/2^3 E(3)=0.875\*100-30=57.50  For N=4: Probability = 0.9375 1-1/2^4 E(4)=0.9375\*100-40=53.75 | .**Decision Rule (From Before)**:   * + If you see a tail, the coin must be fair (because the trick coin has two heads). You should **immediately stop** and guess it's the fair coin.   + If you see **only heads** after a certain number of flips, the probability that you're holding the trick coin increases with each additional head. However, at some point, the cost of flipping outweighs the value of additional information.   + E(N) = p(100 - 10N) - (1-p)(10N)=p\*100-10N   **Expected Profit=P(Correct guess after N flips)×100−N×10**.  **Fair Coin Assumption**: fair coin, probability flipping N consecutive heads is 0.5^N  **Posterior Probability of the Trick Coin**: After observing N consecutive heads, the probability that the coin is the trick coin can be computed as:  P(Trick Coin | N Heads)=1−P(N Heads if Fair Coin)=1-0.5^N |
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|  **Runs Test**: Tests for too many or too few runs (i.e., sequences of consecutive heads or tails). | A runs test, also known as the Wald–Wolfowitz runs test, was developed by mathematicians Abraham Wald and Jacob Wolfowitz. A runs test is a statistical analysis that helps determine the randomness of data by revealing any variables that might affect data patterns. |
|  **Chi-square Test**: Tests for the distribution of heads and tails (e.g., if heads and tails are not equally likely).   **Autocorrelation Test**: Tests for independence between different points in the sequence.   **Serial Test**: Looks for repeating patterns or structures.   **Frequency Test**: Checks if the proportion of heads and tails deviates from what is expected.   **Complexity Tests**: Assess the overall randomness by looking at the complexity or unpredictability of the sequence. |  |