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| I have two coins in my pocket - a trick coin with two heads and a fair coin with one head and one tail(s?). We'll play a game. I will grab one coin at random, and flip it $N$ times. After that you will have to decide if it is the fair coin or the trick coin. The null hypothesis is that it is the fair coin.  a. Using "Decision Rule 1", what is the lowest number of flips $N$ would you need in order to have a significance level less than 5% for this test? | \*\*Decision Rule 1\*\*:  If after $N$ flips there are no tails, then you decide it is the trick coin.  If there is at least 1 tail then you know it is the fair coin.  (1/2)^N=0.05  N\*ln(1/2)=ln(0.05)  N=ln(0.05)/ln(0.5)=4,32 ~5  (Flip 5 H/coin-true)=(1/2)5 <0.05 |
| b. Using $N$ from part a, what is the power of the test?  H0: coin is fair | The power of the test is the probability of correctly rejecting the null hypothesis  when the  alternative hypothesis is true.  P(reject H0/trick coin)  P(5 heads/trick coin) =1 |
| c. Suppose $N=4$ is decided. How can you modify the decision process to have a significance level of exactly 5%? Does this change the power of the test? | (1/2)4  fair coin 4 heads 0. = 0.0625  P(4 heads/fair coin)=0.0625  P(Reject H0​ | Fair coin)=P(All heads in 4 flips | Fair coin)×P(Reject H0​ if all heads)  P(All heads in 4 flips | Fair coin)×0.8=1/16​×0.8=0.05.  To achieve a significance level of exactly 5%, introduce randomization: If all flips are heads, reject the null hypothesis with 80% probability.  The power of the test remains unchanged at 100%, since with the trick coin, all flips will still be heads. |
| d. Extra Credit (2 points): Suppose if you guess correct you win \$100 (and if you're wrong you get nothing), but each flip of the coin costs \$10. What strategy would you use to maximize your expected profit from this game? | Nova presmetka, koja na sekoj cekor odlucuva dali ke prekine ili odi nataka, pri sto sobira pari od sekoj cekor  **Initial Decision Rule (0 flips):**   * **Expected value without any flips:** You have a 50% chance of either coin being selected randomly.   + Expected value = 0.5×100+0.5×0=50. * You could just guess without flipping, but that would only give an expected profit of 50−0=50.   **Decision After 1 Flip:**   * If you flip once, the coin will either show heads or tails.   + If you see **tails**, you know for sure it’s the **fair coin** (since the trick coin never shows tails), so you guess fair and win $100.   + If you see **heads**, the coin could still be either the **trick coin** or the **fair coin**, so you are uncertain.   + Cost for 1 flip = $10. * **Expected profit after 1 flip:**   + Probability of seeing tails (and knowing it’s the fair coin): 0.5.   + If tails: profit = $100 (win) - $10 (cost of flip) = $90.   + If heads: you remain uncertain, but you can still guess randomly (giving you a 50% chance of being right). * Expected profit for heads: 0.5×(100−10)=45. * Overall expected profit after 1 flip: 0.5×90+0.5×45=67.5.   **Decision After 2 Flips:**   * If you flip twice:   + If you see **at least one tail**, you know the coin is fair and win $100.   + If both flips are heads, you become more confident that it is the **trick coin**, but it’s still possible that you have the **fair coin**. * Cost for 2 flips = $20.   **Expected profit after 2 flips:**   * Probability of seeing at least one tail (and knowing it’s the fair coin) is 1−(0.5)2=0.75 * If at least one tail: profit = $100 (win) - $20 (cost) = $80. * If both flips are heads: you remain uncertain, but you can guess randomly.   + Expected profit for both heads: 0.5×(100−20)=40. * Overall expected profit after 2 flips: 0.75×80+0.25×40=70 |
|  | **Decision After 3 Flips:**   * With each additional flip, the probability that you are correct increases, but so does the cost. * **Expected profit after 3 flips:**   + Cost for 3 flips = $30.   + If you get at least one tail, profit = 100−30=70.   + If all 3 flips are heads, profit = 0.5×(100−30)=35. * Overall expected profit after 3 flips: 0.875×70+0.125×35=66.875.   Conclusion:  The expected profit is maximized **after 1 flip**, where the expected profit is $67.5. |