**23 file:///D:/AAMatej/AA\_ fa23-midterm-in-class.html**

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| M1 Which variables **always** satisfy P(X=0) >0  **Choose ALL that apply!** | 1. Normal 2. Uniform discrete not olways 3. Exponential 4. Poisson 5. Geometri |
| M2 Let X=pnorm(rnorm(n)) for some large sample size n, what distribution will X have? **Choose ONE!**  ***Predavanje e deka so uniformna moze da se simuliraat site raspredelbi*** | 1. Normal 2. Uniform 3. Exponential 4. Poisson 5. Geometric |
| M3 Which of the following are NOT valid probability density functions? **Choose ALL that apply!**  **A**  **B**  **C**  **D** |  |
| M4 P(A) = 0.5 and P(B) = 0.6. Which of the following MUST be true? **Choose ONE!**  p(A)=0.5 p(B)=0.6 if P(A^B)=0 -> P(AUB)=0.5+0.6>1 not possible  **GTP e**  **A and B are not mutually exclusive**: While we know they can't be mutually exclusive (since their sum is greater than 1), that alone doesn’t imply anything specific about their other relationships.  Thus, **none of given options must be true based solely on the information provided**. | 1. A and B are independent 2. A and B are not independent 3. A and B are mutually exclusive 4. A and B are not mutually exclusive ??? 5. None of the above must be true |
| **MC5**- You have water temperatures from two different lakes in Wisconsin, and want to run a permutation test to see if there is evidence that the mean temperatures are different. You have 16 measurements from Lake A and 21 measurements from Lake B. Our test statistic is the difference of sample means. Why would you want to run the Monte Carlo simulation with 10,000 replicates rather than 100 to produce a distribution of test statistics? **Choose ONE!**  **GTP**  **a. 10,000 replicates lowers the standard error significantly.** | 1. 10,000 replicates lowers the standard error significantly 2. 10,000 replicates eliminates sampling bias due to flawed data collection 3. 10,000 replicates compensates for any difference in population variance 4. 10,000 replicates decreases α while maintaining the power of the test 5. None of these are valid reasons |
| GTP  **a. 10000 replicates lowers the standard error significantly**: The standard error is based on the original sample, not the number of replicates in a permutation test. The number of replicates affects the precision of the p-value estimate, not the standard error of the sample mean.  **b. 10,000 replicates eliminates sampling bias due to flawed data collection**: A permutation test assumes the data you have is a representative sample. Increasing the number of replicates in a Monte Carlo simulation doesn't fix biased data collection.  **c. 10,000 replicates compensates for any difference in population variance**: The permutation test inherently assumes that the null hypothesis is true, meaning the samples come from the same population, so population variance differences are not addressed by increasing the number of replicates.  **d. 10,000 replicates decreases α while maintaining the power of the test**: Increasing the number of replicates does not change the significance level (α) or power of the test. It only increases the precision of the estimate of the p-value. | |
| MC6 Which of the following are true of αα, β and power? **Choose ALL that apply!**  MC6. Which of the following are true of α, β and power? **Choose ALL that apply!**   * **b. β + power = 1**: This is correct. In hypothesis testing, the power of a test (1 - β) is the probability of correctly rejecting the null hypothesis when it is false. Thus, β (the probability of a Type II error) and power are complementary, and their sum equals 1. * **a. α + β = 1**: This is incorrect. α (the significance level) and β (the probability of a Type II error) are not directly related in this way. Their sum is not 1. * **c. As α increases, so does power**: This is true because increasing α (the significance level) makes the test more likely to reject the null hypothesis, thus increasing the power. However, this statement alone is incomplete without context, so **b** is a better choice. * **d. As α decreases, so does β**: This is incorrect. As α decreases, β typically increases, because lowering the chance of a Type I error (α) makes the test more conservative, which increases the chance of a Type II error (β).   **e. None of the above are true**: This is incorrect because **b** is true. | 1. α+β=1 2. β+power=1 3. As α increases, so does power 4. As α decreases, so does β 5. None of the above are true |
| dec 31 day  SA1 | At the end of each day, you count how many fish you caught and write this in your calendar. Let X = number of days in December you catch AT LEAST 1 fish. Assume each day is independent.   * What variable is X? * What is P(X=0)? (i.e what is the probability you don’t catch a single fish in the entire month?) =0.3 * What are the expectation and variance of X? * Suppose you repeat the exact same routine every day in January (assume the daily catch has the same distribution and ALL fishing days are independent). Note January also has 31 days. Let Y = number of days in January you catch at LEAST 1 fish. You want to estimate how many MORE days you catch something in January compared to December. What are the mean and variance of (Y-X)? |
| a.b   * The variable X represents the **number of days in December that you catch at least 1 fish**. Since each day’s outcome (catching at least 1 fish or catching no fish) can be viewed as a Bernoulli trial, X is the sum of 31 independent Bernoulli random variables, where the success probability is the probability of catching at least 1 fish on a given day.   The probability of catching at least 1 fish on any given day is:   * P(at least 1 fish)=P(1)+P(2)+P(3)+P(4)=0.25+0.20+0.15+0.10=0.70   So, X is a **Binomial random variable** with parameters n=31 (number of days) and p=0.70 (probability of catching at least 1 fish).  X∼Binomial(n=31,p=0.70)  b.P(X=0) is the probability that you don’t catch a single fish in the entire month of December. This would mean you catch no fish on each of the 31 days, so this is the probability of catching 0 fish every day raised to the power of 31.  P(X=0)=P(no fish on a given day)31=(P(0))31=(0.30)31 ≈2.04×10−16  This is an extremely small number, so the chance of catching no fish at all in the entire month is essentially zero.  **c.What are the expectation and variance of X?**  Since X follows a binomial distribution, the expected value E(X) and the variance Var(X)) are given by the formulas for a binomial random variable:  E(X)=np=31×0.70=21.7 Var(X)=np(1−p)=31×0.70×(1−0.70)=31×0.70×0.30=6.51:  **d. What are the mean and variance of Y−X?**  Let Y be the number of days in January you catch at least 1 fish. We are asked to compute the mean and variance of the difference Y−X.  Since both X and Y follow independent binomial distributions with the same parameters (n=31 and p=0.70   * E(X)=E(Y)=21.7 * Var(X)=Var(Y)=6.51   The difference Y−X is also a random variable. Since X and Y are independent:   * The **expected value** of Y−X:   E(Y−X)=E(Y)−E(X)=21.7−21.7=0   * The **variance** of Y−X is:   Var(Y−X)=Var(Y)+Var(X)=6.51+6.51=13.02 | |
| SA2 You are studying a rare disease that on average affects 2 out of every 1000 people in the general population. The current best available test for this disease has a type I error rate of about 5% and type II error rate of about 15%   1. If you test positive, what is your probability of having the disease? 2. If you test negative, what is your probability of not having the disease? | P(disease)=2/1000  P(test=positive/disease=negative)=0.5  P(test=negative/disease=positivetrue)=0.15  a . P(disease=true/test=positive)=1- P(disease=false/test=positive)=  (1-P(test=positive/disease=true))p(disease=true)/  ( P(test=positive)/disease=positive)\*P(disease)/  +P(test=positive/disease=negative)P(no disease))=  P(disease=negative/test=negative)=  1- P(disease=positivr/test=negative)  P(disease=positivr/test=negative)= (P(test=negative/disease=positive)\*P(disease)/  ( P(test=negative/disease=positive)\*P(disease)/  +P(test=negative/disease=negative)P(no disease))=  0.15\*2/1000/(0.15\*2/1000+(1-0.5)\*998/1000)) |
| 9 41 | Acc=41+46/41+46+9+4=87/100 |
| 46 4 | Type 1 false pozitiv/ |
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|  | SA3: a. for Trimax A the type 1 error rate is 9/50  b. For Trimax B the power is 46/50  c. Accuracy of Trimax C is (39+50)/100  d. It could be argued that a type 2 error is very bad, and Trimax C has no type 2 errors.  Trimax B has highest accuracy which is also desirable.  It would be hard to argue in favor of Trimax A. |
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**SP24-STAT340-Midterm-in-class-portion.pdf**

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| MC1. Which of the following random variables has the biggest mean ? If there are multiple tied for largest, **choose ALL that**  **Distributionsi, means, standard deviations** | **All ok**  Binomial(20,0.2) np  Normal(4,4²) 4  Geometric(0.2) (1-p)/p = 0.8/0.2=4  Poisson(4) lamda=4  Exponential(0.25) 1/lamda=1/0.25 |
| MC2.I have a funky die: it is weighted and it has the value 5 twice and no 6 (i.e. the faces are 1,2,3,4,5,5). The probabilities for are in order .  0.15, 0.15, 0.20, 0.20, 0.30 Suppose I roll the die once and represents the value I roll. Which of the following is the CDF of ? **Choose ONE!**  **1 2 3 4 5**  **0.15 0.15 0.20 0.20 0.30**  **Distributions, CDFs**  MC2: c; a is a pmf, b is a histogram of sample data, d has connected points which is inappropriate for a discrete distribution | **C** |
| MC3.If A,B, are independent events, which of the following is true?  **Choose ONE!**  a. P(A ∪ B) = P(A)P(B)  b. P(A ∩ B) = P(A) + P(B)  c. P(A ∩ B) = P(A|B)  d. P(A|B) = P(A)  e. P(A|B) = P(B|A) | Independence P(AB)=P(A)P(B)  P(A/B)=P(A) |
| MC4. Which of the following are true? **Choose ALL that apply!**  **Independence, mutual excusive rules**  **Rules variance, expected values** | **a.** If iX is a continuous random variable, then for any k P(X=k)=0 , .  b. We cannot use a normal distribution to model human heights because a normal random variable can take negative values.  **c.** If X,Y are two RVs that are **not independent**, then it’s **NOT** ALWAYS true that V(X+Y)=V((x)+V(Y)  d. If are two RVs that are not independent, then it’s NOT ALWAYS  e. Two events can be both independent and mutually exclusive. |
| MC5. Let , X~ N(10,22 ) which of the following is closest to P(X<12) ? Use the empirical rule. (Note that we use the notationN(μ,σ²) in STAT340). **Choose ONE!**  **Mean 10, 2sigmas=4 p~ 0.84 crtanje grafici**  <https://www.intmath.com/counting-probability/normal-distribution-graph-interactive.php> | a. 0.34  b 0.68  **c 0.84**  d 0.95  e 0.997 |
| MC6. You are a prosecutor for the SEC (Securities and Exchange Commission) suing companies for fraudulent financial reporting.Suppose we have the usual null (i.e. no fraud exists). Match each of the options below with one of these test outcomes:  TP, FP,TN, FN | 1. A fraudulent company is found guilty. \_\_\_**TP**\_\_\_\_\_\_ 2. A fraudulent company is found not guilty. \_**FN**\_\_\_\_\_\_\_\_ 3. An innocent company is found guilty. \_\_\_\_FP\_no\_ ?\_TN\_\_ 4. An innocent company is found not guilty. \_TN\_no \_?\_FP\_\_\_\_\_ |
| M7.Let represent the theoretical distribution assuming the null hypothesis of some statistic you have just computed.  Furthermore, suppose is symmetric around 0.  Which of the following is NOT a valid -value for some pair of null/alternative hypotheses you could test? **Choose ALL that apply!**  **Zapis ednostran/dvostran test** | d my Not valid b., d t can be negative ok is |
| M8,9.The following statement applies to both MC8 and MC9: Let be X1,…Xn an i.i.d. (i.e. independent and identically distributed) sample where Xi has μ mean and σ2 variance .  Which of the following shows the EXPECTED (i.e. typical) relationship between the sample variance S 2  and n? recol    **A**  Which of the following shows the EXPECTED (i.e. typical) relationship between the mean variance Var(xmean) and n? Recall    **E.** | **E**  **A** |
| SA1  Suppose you are going to spend the afternoon in the UW Arboretum doing some bird watching. Describe a random variable related to your bird-watching afternoon that could be modeled using each of the following. Make sure to state and justify any assumptions you make! | 1. A uniform distribution   Appearance of various birds is equal probable   1. A binomial distribution   Probability in n watching seeing papiga among all other kinds   1. A geometric distribution   Number of failures before seeing papiga   1. A normal distribution   Chose one kind of bird and se how high they flie |
| SA2 In a bag of 48 rainbow candies I get the following     1. I really like blue and orange. If I eat a candy at random and I really like it, what is the probability that it is orange? 2. I decide to grab and eat candies at random until I get one that I really like. I decide to model the number of candies I get before getting one I really like as a geometric random variable with p=12/48. Is this a good model? Why or why not? 3. Define the colors blue and purple to be “dark” colors, and the rest to be non-dark. Is getting a dark candy mutually exclusive with getting a color I really like? Explain why or why not. 4. Is getting a dark candy independent from getting a color I really like? Explain why or why not. | 1. C(8,1)C(40,1)/C(48,1) P(Orange | I like it) = 8/(8+4) 2. A geometric is not appropriate because p changes as you eat candies. neznaev 3. **Not mutualy** ecl. Like blue that is also dark   **like** blue orange  **dark** blue purple   1. Dark 16/48 like 12/48 dark ^like=4/48   16/48\*12/48 =4/48  16\*12/48=4  12/3=4 yes |
| SA2 candies conditional probabilities are used. But you are taking the candies without replacement from the bag can use **hypergeometric distribution**. | |
| SA3 A medical supplier Trimax wants to build a better rapid COVID antigen test. They have three models: Trimax-A, Trimax-B andTrimax-C. They take 50 individuals who are confirmed to have COVID-19 and 50 individuals who are confirmed to not haveCOVID-19 and ask them to take all three tests. The null hypothesis in each test is that the person does not have COVID. A“FALSE” result is a negative result (i.e. no COVID). Here are the results. | 1. Calculate the Type 1 error rate of Trimax-A **9/50**   **False pozitive**   1. Calculate the **power of Trimax**-**B** (A) **1-False-nega** 2. Calculate the **accuracy** (i.e. proportion of all results that are correct) of Trimax-C. **89/100** 3. Which test do you think they should market, and why?   Biggest acc. 87/100 **90/100** 89/100    **Power for B 46/50 ? 4/50 II error? ?? for A** |

340\_Midterm in class .html

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| MC1You run a test with a significance level of 0.05, and calculate a p-value of 0.134. Which of the following is a valid conclusion?   1. The test provides conclusive evidence that the null hypothesis is true. 2. The test tells us that there’s only a 13.4% chance that the null hypothesis is true. 3. The test suggests that our significance level was set too low, we should re-run the test with a higher significance level. 4. The test tells us if H0 were true there’s a 13.4% chance data like ours (or more extreme) would be observed. 5. The test provides evidence that there is a 86.4% chance that the alternative hypothesis is correct. | d? |
| MC2 A Monte Carlo simulation of 10k iterations resulted in the below distribution of test statistics (see plot). Which of the following test and statistic would give the lowest p-value? (Remember left-tailed means Ha uses < and right-tailed means >)   1. t2, as a two-tailed test 2. t1, as a right-tailed test 3. t3, as a left-tailed test 4. t4, as a left-tailed test 5. t1, as a two-tailed test |  |
| MC3 A consulting company is investigating whether some new project management software will help workers be more productive than the old software. Let the null hypothesis be that the new software makes no difference, and the alternative be that the new software improves productivity. For each blank below, choose one of the following:  **A: Type I error, B: Type II error, or C: Correct conclusion** | 1. The new software DOES make workers more efficient, and they conclude that it DOES \_\_C\_\_\_ 2. The new software DOES make workers more efficient, and they conclude that it does NOT \_II\_\_B\_\_ 3. The new software does NOT make workers more efficient, and they conclude that it DOES \_FP\_\_I A\_\_ 4. The new software does NOT make workers more efficient, and they conclude that it does NOT \_\_C\_\_ |
| MC4 Which is the correct ending for this statement with regards to Monte Carlo testing and estimation? | “Increasing the number of Monte Carlo runs…”   1. …gives more consistent results. 2. …lowers the chance of a Type 1 Error. 3. …increases the power of a hypothesis test. 4. …results in lower p-values. 5. …can compensate for a small amount of data. |
| **MC5 MC6 next two questions are based on the following plot.**  Four tests are considered to test the hypothesis H0:θ=0,  H1:θ>0, at a significance level of α=0.05. The power graph above give v.s.s the rejection rate of each test depending on the effect size (the true θ value). The nominal α=0.05 value is shown with a horizontal line. | **MC5**: Test 2 is ***always better*** than which other test?   1. Test 2 2. Test 3 3. Test 4 4. All other tests 5. None of the other tests   **MC6**: Which test fails to control the type I error rate?   1. Test 1 2. Test 2 3. Test 3 4. Test 4 5. None of the tests fails |
| SA1 You work for a semiconductor manufacturing facility. Due to the high complexity of the manufacturing process, each chip has a 30% chance of being defective. Let X=1 if the chip is functional, and X=0 if the chip is defective.  A Bernoulli with p=0.70 god 1-p=0.30 bad  B. E(X)=p  C. V(X)=p(1-p)=).7\*0.3=0.21  D. Binomial (N,p) | 1. What kind of random variable is this? Specify both the name and any necessary parameter(s). 2. Compute the expectation of this random variable. Show your work. 3. Compute the variance of this random variable. Show your work. 4. You manufacture 100 of these chips. Let Y=the number of chips that are functional. What kind of random variable is this? Again, specify the name and any necessary parameter(s) |
| SA2 Let X be a Poisson variable with λ=20.   1. What is the expectation of this random variable? You may state this without computation. (*Hint: think about what*λ*means.*) 2. Give an example of an experiment that you can model using this random variable (be specific in your wording). This is an open ended question, so many correct responses are possible. 3. What is the probability of observing X=24.5 as an outcome? Give a numeric value, and briefly explain your answer. 4. What is another random variable that you can use to approximately model a Poisson? (There are two acceptable answers here). Just the name of a variable is sufficient, no need to provide parameters. | 1. λ=20 2. arrival of # cars on crossing with same rate 3. P(k)= λ ^k e^(- λ)/k! =20^k \*e^(-20)/k! p(k)=0 ?? 4. Normal binomial |
| Recall the random variable defined in homework 1, where the pdf is f(x)=2x for 0≤x≤1 and 0 everywhere else. Also recall the corresponding cdf is F(x)=x2  0≤x≤1. Below is shown a plot of the pdf for  pdf\_x = **function**(x) ifelse(0<=x & x<=1,2\*x,0)  ggplot() + geom\_function(fun=pdf\_x, n=1e5) +  coord\_fixed(ratio=.5) + theme\_minimal() +  xlim(c(-1,2)) + ylim(-1,3) + labs(x='x', y='PDF')  Let the functions dx, px, qx represent the pdf, cdf, and inverse cdf functions respectively for this random variable. Find the following, and make sure you show your work or write a brief explanation of your answers:   1. P(X=0.5) = 0 for continuous rv 2. dx(0.5) = 1 is y value 3. px(0.5) = 0.5\*0.5/2 4. qx(0.5) = CDF=x\*2=0.5 x= sqrt(0.5) |  |
| Match the following pdfs to their corresponding cdfs.   * pdf A matches cdf \_\_\_B\_\_ * pdf B matches cdf \_\_\_C\_ * pdf C matches cdf \_\_\_D\_\_ * pdf D matches cdf \_\_\_F\_\_ * pdf E matches cdf \_\_\_A\_\_ * pdf F matches cdf \_\_\_E\_ |  |