Flow of Newtonian Fluid through a Tesla Valve

Final project for Computer Solutions of Continuum Physics Problems

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Contents

Short Introduction to the Project

What Is a Tesla Valve?

- passive fixed geometry check valve
- suppresses the flow of fluid in the reverse direction (→)
- pressure drop in reverse (→) direction higher than in the forward direction (←)

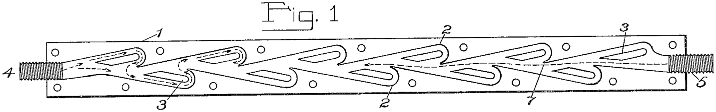


Figure: Cross-section of the valve [1].

Aims of the Project

- ✓ creation of mesh in the shape of the valve
- ✓ simulating a steady flow through the valve
- ✓ simulating a time-dependent flow
- ✓ computing the diodicity of the valve [2]
- × automatic generation of the mesh from an image [3]

$$\mathrm{Di} = \left(\frac{\Delta p_r}{\Delta p_f}\right)_Q$$

Stationary Navier-Stokes Equation

- Steady flow of and incompressible fluid

$$
\begin{aligned}
\rho\,\mathrm{div}(\boldsymbol{v}\otimes\boldsymbol{v}) = -\nabla p + \mathrm{div}\,(2\nu\mathbb{D}) \quad \text{in } \Omega,\\
\mathrm{div}\,\boldsymbol{v} = 0 \quad \text{in } \Omega,
\end{aligned}
$$

  $\Omega \subset \mathbb{R}^2$ is an open connected set representing a canal
  ρ given constant density
  μ given constant dynamic viscosity
  $\mathbb{S} = 2\mu\mathbb{D}$ shear stress
  $\mathbb{D}$ symmetric part of the velocity gradient

- Unknowns: pressure, velocity $(p, \boldsymbol{v})$ or pressure, velocity, stress $(p, \boldsymbol{v}, \mathbb{S})$.

Navier-Stokes equation

- Unsteady flow of and incompressible fluid

$$\begin{aligned} \frac{\partial \vec{v}}{\partial t} + (\nabla \vec{v})\vec{v} - \operatorname{div}(\nu \nabla \vec{v}) + \nabla p = 0 \quad & \text{in } \Omega, \\ \operatorname{div} \vec{v} = 0 \quad & \text{in } \Omega. \end{aligned}$$

- Unknowns: pressure p and velocity $\boldsymbol{v}$.

Initial and Boundary Conditions

- The boundary is partitioned as $\partial\Omega = \Gamma_{\text{in}} \cup \Gamma_{\text{out}} \cup \Gamma_{\text{wall}}$.
- We impose the following boundary conditions on $\boldsymbol{v}$:

$$
\begin{aligned}
\boldsymbol{v} &= 0 && \text{on } \Gamma_{\text{wall}} \text{ (wall of the canal)},\\
\boldsymbol{v} &= \boldsymbol{g} && \text{on } \Gamma_{\text{in}} \text{ (inlet)},\\
(\mathbb{T}\boldsymbol{n})_n &= 0 && \text{on } \Gamma_{\text{out}} \text{ (outlet)}.
\end{aligned}
$$

- As an initial condition of the time-dependent problem we use the steady solution.

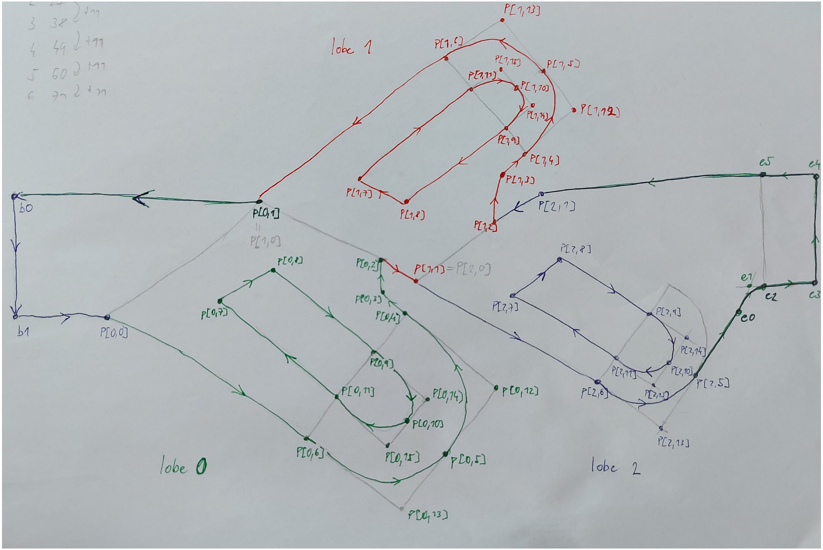
Coding

How Did I Progress?

- → automatic mesh generation from an image unsuccessful
- → Netgen proved to be a better option
- → solving the steady flow using mixed formulation but no pressure drop
- → moving to Navier-Stokes which showed the expected pressure drop
- → solving the steady flow using velocity-pressure formulation, again no pressure drop
- → increased the Reynolds number $\Rightarrow$ difference in pressure drops between forward and reverse mode

Note: ChatGPT very useful for code translation from FEniCS to Firedrake and for debugging.

netgen_mesh() – The Most Challenging Part



Results

Velocity

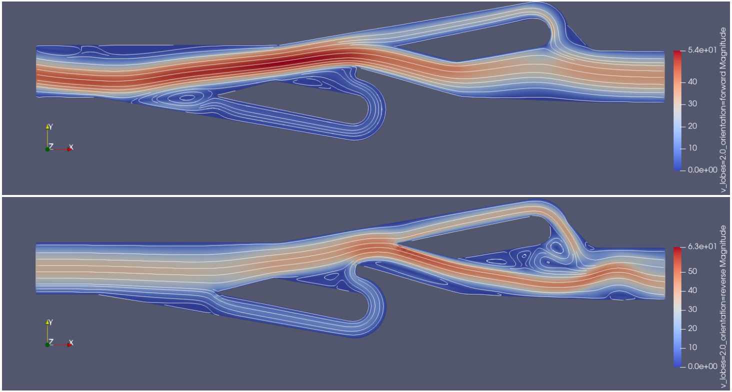


Figure: The top picture shows the forward mode ($\leftarrow$), the bottom picture shows the reverse mode ($\rightarrow$), both at $t = 20$ and $\mathrm{Re} = 685.9$.

Pressure

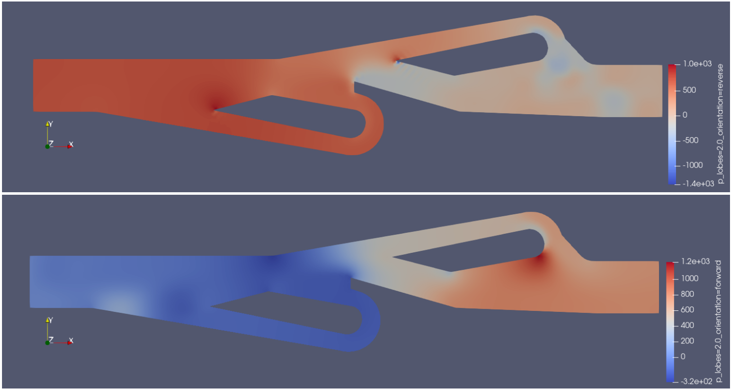


Figure: The top picture shows the reverse mode ($\rightarrow$), the bottom picture shows the forward mode ($\leftarrow$), again at $t = 20$ and $\mathrm{Re} = 685.9$.

Pressure Drop

Lobes	Mode	Pressure Drop	Diodicity
2	reverse	722	0.95
	forward	763	
4	reverse	2141	1.40
	forward	1530	
6	reverse	3123	1.48
	forward	2116	
8	reverse	5600	2.08
	forward	2686	

Table: Pressure drop for valves of different length at $t = 20$ and $\mathrm{Re} = 685.9$. Note that t was chosen arbitrarily and different choice would produce slightly different results.

Possible Problems

- pressure may have a singularity in the corners of the domain
- solution oscillations visible for higher Reynolds number

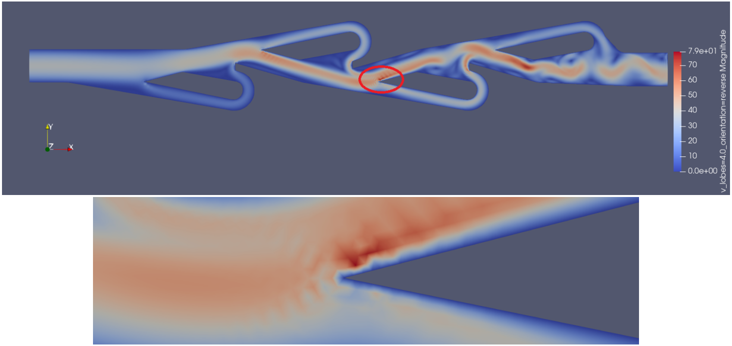


Figure: Velocity at $t = 20$, $\mathrm{Re} = 685.9$, reverse mode ($\rightarrow$). We can see the velocity spike and produce "ripples" around sharp corners.

Conclusion

Solved problems:

- ✓ mesh generation using Netgen
- ✓ computing the steady flow
- ✓ computing the time-dependent flow
- ✓ the valve shows diodicity greater than 1

Unsolved problems:

- × automatic mesh generation from an image

References

[1] Nikola Tesla. Tesla valve cross-section. https://patents.google.com/patent/US1329559. 1920.

[2] Tesla valve – diodicity. https://en.wikipedia.org/wiki/Tesla_valve#Diodicity.

[3] Nanomesh documentation. https://nanomesh.readthedocs.io/en/latest/.