Theorem

The product of two non-zero natural numbers is non-zero.

```
theorem mul_pos (a b : mynat) : a ≠ 0 → b ≠ 0 → a * b ≠ 0 :=
```

Proof :

```
begin
intros f h g,
cases b with k,
rw mul_zero at g,
exact h(g),
rw mul_succ at g,
have g2 := add_left_eq_zero(g),
exact f(g2),
end
```

Theorem

If $ab = 0$, then at least one of a or b is equal to zero.

```
theorem eq_zero_or_eq_zero_of_mul_eq_zero (a b : mynat) (h : a * b = 0) :
  a = 0 ∨ b = 0 :=
```

Proof :

```
begin
cases b,
right, refl,
rw mul_succ at h,
have h2:= add_left_eq_zero(h),
left,
exact h2,



end
```

Theorem

$ab = 0$, if and only if at least one of a or b is equal to zero.

```
theorem mul_eq_zero_iff (a b : mynat): a * b = 0 ↔ a = 0 ∨ b = 0 :=
```

Proof :

```
begin
split,
apply eq_zero_or_eq_zero_of_mul_eq_zero,
intro h,
cases h,
rw h, rw zero_mul, refl,
rw h, rw mul_zero, refl,
end
```

Theorem

If $a \neq 0, b$ and c are natural numbers such that $ab = ac$, then $b = c$.

```
theorem mul_left_cancel (a b c : mynat) (ha : a ≠ 0) : a * b = a * c → b = c
:=
```

Proof :

```
begin
induction c with d hd generalizing b,
intro h,
rw mul_zero at h,
rw mul_eq_zero_iff at h,
cases h with b,
exfalso,
exact ha(b)
exact h,
intro h,
cases b,
rw mul_zero at h,
symmetry at h,
rw mul_eq_zero_iff at h,
cases h with b,
exfalso, exact ha(b),
symmetry, exact h,
apply succ_eq_succ_of_eq,
rw mul_succ at h,
rw mul_succ at h,
rw add_right_cancel_iff at h,
have h1 :=hd(b),
apply h1,
exact h,
```