Complement of a Vector Space

Input:

(i) dimension of the vector space $\mathbb{Q}^n$
(example: $n=3$)

(ii) family (non empty) of vectors in $\mathbb{Q}^n$

$\left(\text{ex.: } \begin{pmatrix} 1 \\ 2 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix} \Rightarrow V = \left\langle \begin{pmatrix} 1 \\ 2 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix} \right\rangle \right)$

Method:

(i) augment matrix of input vector with identity matrix:

$$\left(v_1 \mid \dots \mid v_n \mid \mathbb{1}_n \right)$$

ex
$$\left(\begin{array}{cc|ccc} 1 & 0 & 1 & 0 & 0 \\ 2 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 \end{array}\right)$$

$\Rightarrow$ matrix now has rang n

(ii) compute echelon form:

$$\left(\begin{array}{cc|ccc} 1 & 0 & 1 & 0 & 0 \\ 2 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 \end{array}\right) \cdot(-2) \rightsquigarrow \left(\begin{array}{cc|ccc} 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & -2 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 \end{array}\right) - \rightsquigarrow \left(\begin{array}{cc|ccc} 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & -2 & 1 & 0 \\ 0 & 0 & 2 & -1 & 1 \end{array}\right) \rightsquigarrow \left(\begin{array}{cc|ccc} 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & -2 & 1 & 0 \\ 0 & 0 & 1 & -\frac{1}{2} & \frac{1}{2} \end{array}\right)$$

(iii) pivot points:

ex
$$\left(\begin{array}{cc|ccc} 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & -2 & 1 & 0 \\ 0 & 0 & 1 & -\frac{1}{2} & \frac{1}{2} \end{array}\right)$$

(iv) basis vectors can be read as the original vectors $\Rightarrow$ basis of V: $\begin{pmatrix} 1 \\ 2 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}$ basis of V^c: $\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$

- gauß process preserves linear independence / dependence
- the vectors of the pivot points are obviously independent

<u>ex:</u>

$$\begin{pmatrix} 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & -2 & 1 & 0 \\ 0 & 0 & 1 & -\frac{1}{2} & \frac{1}{2} \end{pmatrix}$$

$\Rightarrow$ original vectors also independent