

Proseminar on computer-assisted mathematics

Session 1 – Introduction to Sage

Matrices in Sage

When we define a matrix in Sage, we can specify the ring or field in which we take the entries.

Let us for instance consider the matrix

$$\begin{pmatrix} 2 & 4 & 6 \\ 4 & 5 & 6 \\ 3 & 1 & 2 \end{pmatrix}$$

and declare it first as a matrix A with entries in \mathbb{Q} , then as a matrix B with entries the field with seven elements \mathbb{F}_7 .

```
A = matrix( QQ, [[2,4,6],[4,5,6],[3,1,2]] )  
show(A)
```

```
show(A.inverse())
```

$$\begin{pmatrix} -\frac{2}{9} & \frac{1}{9} & \frac{1}{3} \\ -\frac{5}{9} & \frac{7}{9} & -\frac{2}{3} \\ \frac{11}{18} & -\frac{5}{9} & \frac{1}{3} \end{pmatrix}$$

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Heidelberg University, Summer semester 2024

Computer-assisted mathematics 2024

The seminar has two parts:

- A Sage part (16.04-28.05)
- A Lean part (04.06-23.07)

The seminar is collaborative and project-based.

To pass, attendance is mandatory (unless excused in advance).

Programme of the Sage part of the seminar

After getting acquainted with Sage, we will focus on topics from linear algebra.

Topics of linear algebra to be covered in the seminar:

- Computer algebra systems. Representations of vectors and matrices.
- Row operations. Gaussian elimination. Row-reduced echelon form of a matrix.
- Invertible matrices. Elementary matrices. Determinant.
- Linear independence. Bases for the kernel and the image of a linear transformation.
- Rank-nullity theorem and the row space of matrix. Basis for the row space.
- Base change. Coordinates of a vector, matrix of a linear transformation.
- Eigenvalues and the characteristic polynomial. Diagonalisation.
- The Gram-Schmidt process. Least-square approximation.

However, feel free to suggest other topics for the final projects.

Schedule

#	Date	Topic	Speaker	Slides	Code
1	16/04	Introduction to Sage	Instructors		
2	23/04	Introduction to Git + Assignment #1	Instructors		
3	30/04	Kernels, images and diagonalisation	Working in pairs		
4	14/05	Least squares approximation	Working in pairs		
5	21/05	Project preparation	Working in pairs		
6	28/05	Project presentation			
7	04/06	Introduction to Lean	Instructors		
8	11/06	Natural Number Game	Working in pairs		
9	18/06	Basic tactics + Assignment #2	Instructors		
10	02/07	Advanced tactics	Working in pairs		
11	16/07	Project preparation	Working in pairs		
12	23/07	Project presentation			

- Team up (soon!)
- Choose a project (later)

Practical organization

Minimal requirements

- At least one computer / tablet per team

Before
23.04

- 1 • Create individual GitHub accounts
<https://github.com>
- 2 • Join the seminar's Zulip channel
<https://matematiflo.zulipchat.com>

Advanced option (facultative)

- Install Sagemath (suggested version: 9.8)
<https://doc.sagemath.org/html/en/installation/index.html>
- Install JupyterLab <https://jupyter.org/install>

Plan for today

- Go through a Sage tutorial

https://matematiflo.github.com/SoSe_2024/notes/SageTutorial.pdf


- When you are done with the tutorial, start using CoCalc to work on a Sage notebook

https://matematiflo.github.io/SoSe_2024/notes/01_Matrices.ipynb


[Sign In](#) Password Reset



Sign In to CoCalc

Sign in using your email address or a single sign-on provider.  Use your GitHub account.



Institutional Single Sign-On: 

Go to cocalc.com
and sign in using
your GitHub
account.

You can also
create one on the
spot.



Sign in to GitHub
to continue to CoCalc

Username or email address

Password

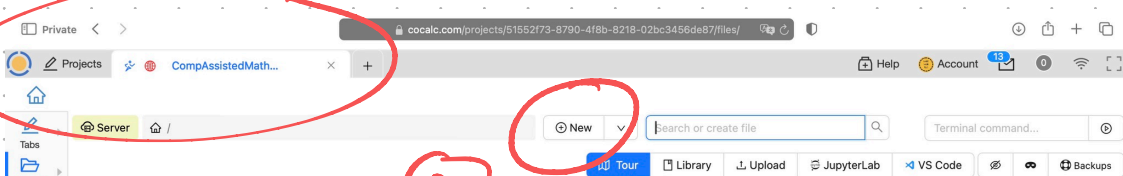
[Forgot password?](#)

Sign in

New to GitHub? [Create an account.](#)

Once logged in, ^①create a new project, ^②upload the notebook about matrices in Sage and start practicing!

①



②

Markdown

Whiteboard

Slides

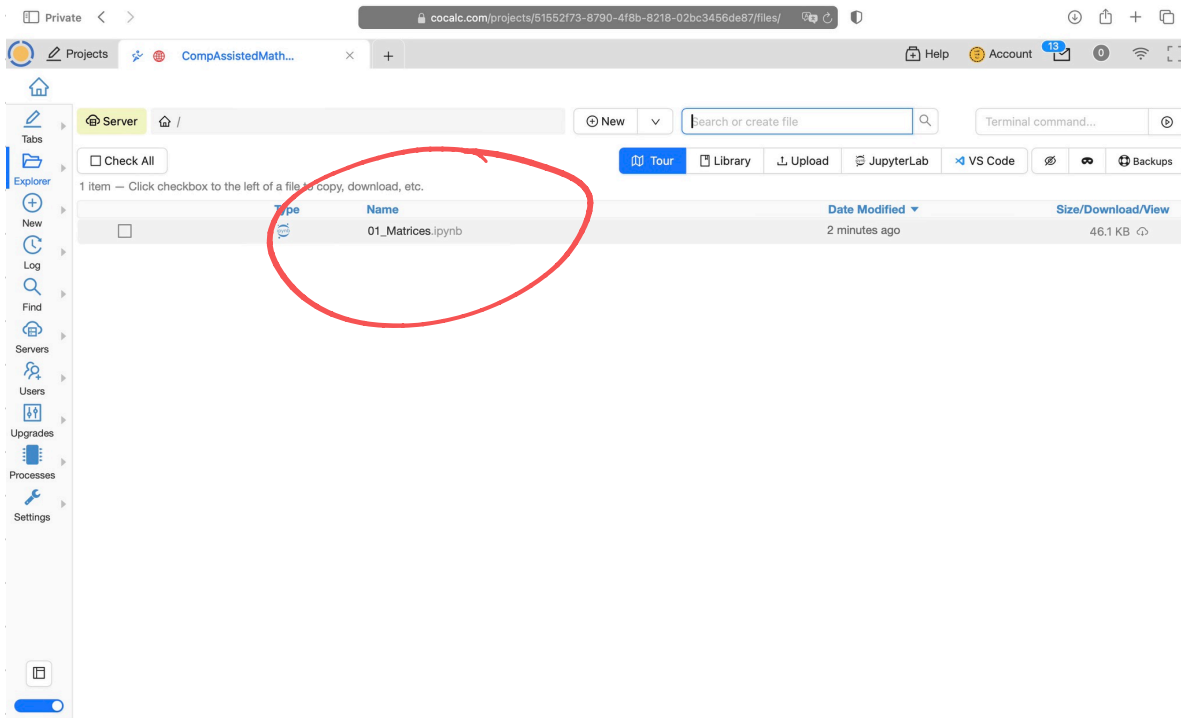
Task List

Upload Files Into Your Project

You can drop one or more files here or on the Explorer file listing. See [the docs](#) for more ways to get your files into your project.

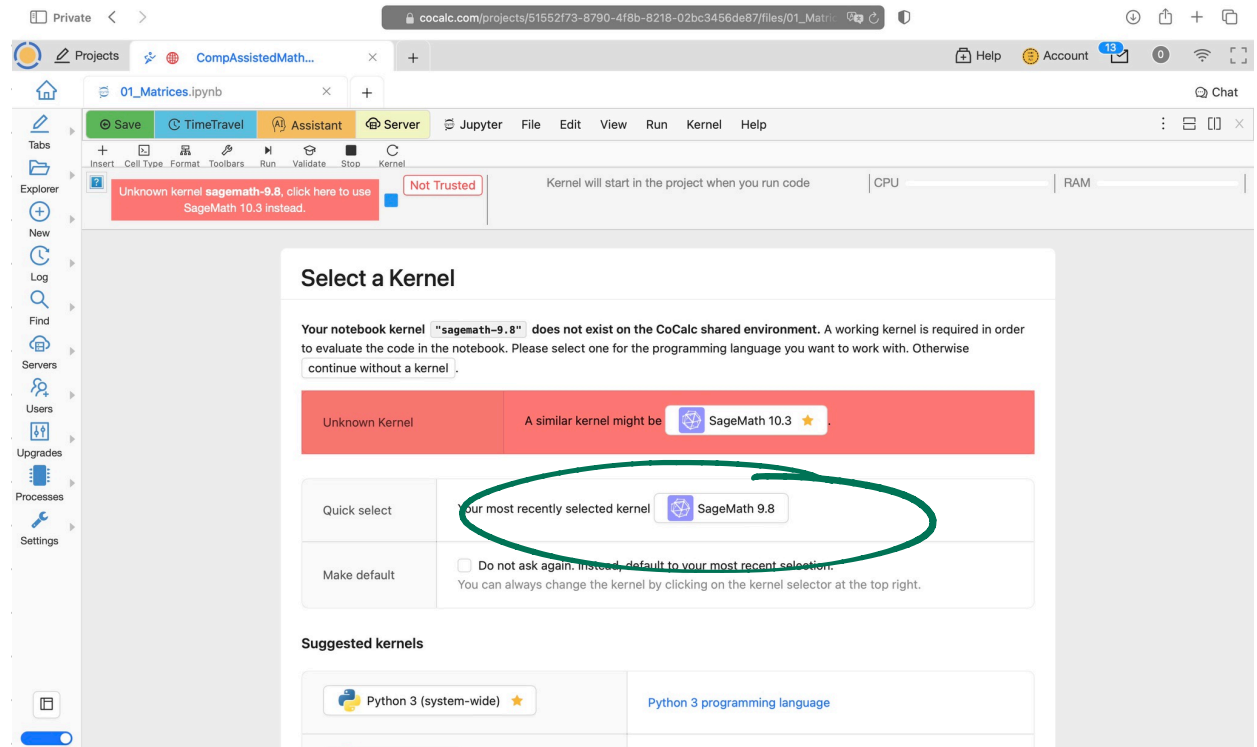
01_Matrices.ip

At the bottom of the page



You can then open the uploaded file

We recommend using the SageMath 9.8 kernel (it is ok to trust the kernel and the notebook, we promise!)



You can now start working on the notebook!

The screenshot shows a web browser window with a Jupyter notebook interface. The browser address bar shows a URL from cocalc.com. The notebook interface includes a top toolbar with buttons for Save, TimeTravel, Assistant, Server, Jupyter, File, Edit, View, Run, Kernel, and Help. Below this is a sub-toolbar with Insert, Cell Type, Format, Toolbars, Run, Validate, Stop, and Kernel. The notebook title is '01_Matrices.ipynb'. The left sidebar contains navigation icons for Explorer, New, Log, Find, Servers, Users, Upgrades, Processes, and Settings. The main content area displays the notebook text:

Basic operations

Matrices in Sage

When we define a matrix in Sage, we can specify the ring or field in which we take the entries.
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and declare it first as a matrix A with entries in \mathbb{Q} , then as a matrix B with entries the field with seven elements \mathbb{F}_7 .

The notebook shows an input cell with the following code:

```
In [5]: 1 A = matrix( QQ, [[2,4,6],[4,5,6],[3,1,2]] )
        2 show(A)
```

The output of the cell is:

```
Out[5]: 
$$\begin{pmatrix} 2 & 4 & 6 \\ 4 & 5 & 6 \\ 3 & 1 & 2 \end{pmatrix}$$

```

At the bottom right of the output area, it says '0.068 seconds', 'Explain', 'Format', 'Copy', and '5'.