

# Proseminar on computer-assisted mathematics

## Session 1 – Introduction to Sage

### Matrices in Sage

When we define a matrix in Sage, we can specify the ring or field in which we take the entries.

Let us for instance consider the matrix

$$\begin{pmatrix} 2 & 4 & 6 \\ 4 & 5 & 6 \\ 3 & 1 & 2 \end{pmatrix}$$

and declare it first as a matrix  $A$  with entries in  $\mathbb{Q}$ , then as a matrix  $B$  with entries the field with seven elements  $\mathbb{F}_7$ .

```
A = matrix( QQ, [[2,4,6],[4,5,6],[3,1,2]] )  
show(A)
```

```
show(A.inverse())
```

$$\begin{pmatrix} -\frac{2}{9} & \frac{1}{9} & \frac{1}{3} \\ -\frac{5}{9} & \frac{7}{9} & -\frac{2}{3} \\ \frac{11}{18} & -\frac{5}{9} & \frac{1}{3} \end{pmatrix}$$

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Heidelberg University, Summer semester 2024

# Computer-assisted mathematics 2024

The seminar has two parts:

- A Sage part (16.04-28.05)
- A Lean part (04.06-23

The seminar is collaborative and project-based.

To pass, attendance is mandatory (unless excused in advance).

# Programme of the Sage part of the seminar

After getting acquainted with Sage, we will focus on topics from linear algebra.

Topics of linear algebra to be covered in the seminar:

- Computer algebra systems. Representations of vectors and matrices.
- Row operations. Gaussian elimination. Row-reduced echelon form of a matrix.
- Invertible matrices. Elementary matrices. Determinant.
- Linear independence. Bases for the kernel and the image of a linear transformation.
- Rank-nullity theorem and the row space of matrix. Basis for the row space.
- Base change. Coordinates of a vector, matrix of a linear transformation.
- Eigenvalues and the characteristic polynomial. Diagonalisation.
- The Gram-Schmidt process. Least-square approximation.

However, feel free to suggest other topics for the final projects.

# Schedule

#	Date	Topic	Speaker	Slides	Code
1	16/04	Introduction to Sage	Instructors		
2	23/04	Introduction to Git + Assignment #1	Instructors		
3	30/04	Kernels, images and diagonalisation	Working in pairs		
4	14/05	Least squares approximation	Working in pairs		
5	21/05	Project preparation	Working in pairs		
6	28/05	Project presentation			
7	04/06	Introduction to Lean	Instructors		
8	11/06	Natural Number Game	Working in pairs		
9	18/06	Basic tactics + Assignment #2	Instructors		
10	02/07	Advanced tactics	Working in pairs		
11	16/07	Project preparation	Working in pairs		
12	23/07	Project presentation			

- Team up (soon!)
- Choose a project (later)

# Practical organization

## Minimal requirements

- At least one computer / tablet per team

Before  
23.04

- 1 • Create individual GitHub accounts  
<https://github.com>
- 2 • Join the seminar's Zulip channel  
<https://matematiflo.zulipchat.com>

## Advanced option (facultative)

- Install Sagemath (suggested version: 9.8)  
<https://doc.sagemath.org/html/en/installation/index.html>
- Install JupyterLab <https://jupyter.org/install>

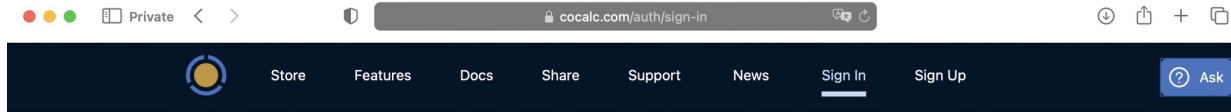
## Plan for today

- Go through a Sage tutorial

[https://matematiflo.github.com/SoSe\\_2024/notes/SageTutorial.pdf](https://matematiflo.github.com/SoSe_2024/notes/SageTutorial.pdf)

- When you are done with the tutorial, start using CoCalc to go through a Sage notebook


[https://matematiflo.github.com/SoSe\\_2024/notes/01\\_Matrices.ipynb](https://matematiflo.github.com/SoSe_2024/notes/01_Matrices.ipynb)



[Sign In](#) Password Reset



## Sign In to CoCalc

Sign in using your email address or a single sign-on provider.  Use your GitHub account.

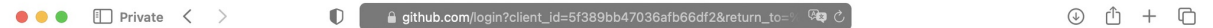


Institutional Single Sign-On:



Go to cocalc.com  
and sign in using  
your GitHub  
account.

You can also  
create one on the  
spot.



Sign in to GitHub  
to continue to CoCalc

Username or email address

Password

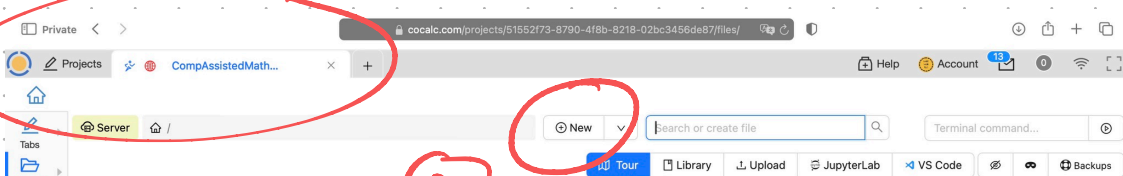
[Forgot password?](#)

Sign in

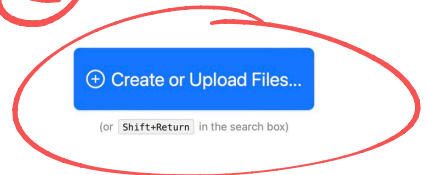
New to GitHub? [Create an account.](#)

Once logged in, <sup>①</sup> create a new project, <sup>②</sup> upload the notebook about matrices in Sage and start practicing!

①



②



Markdown

Markdown Whiteboard Slides Task List

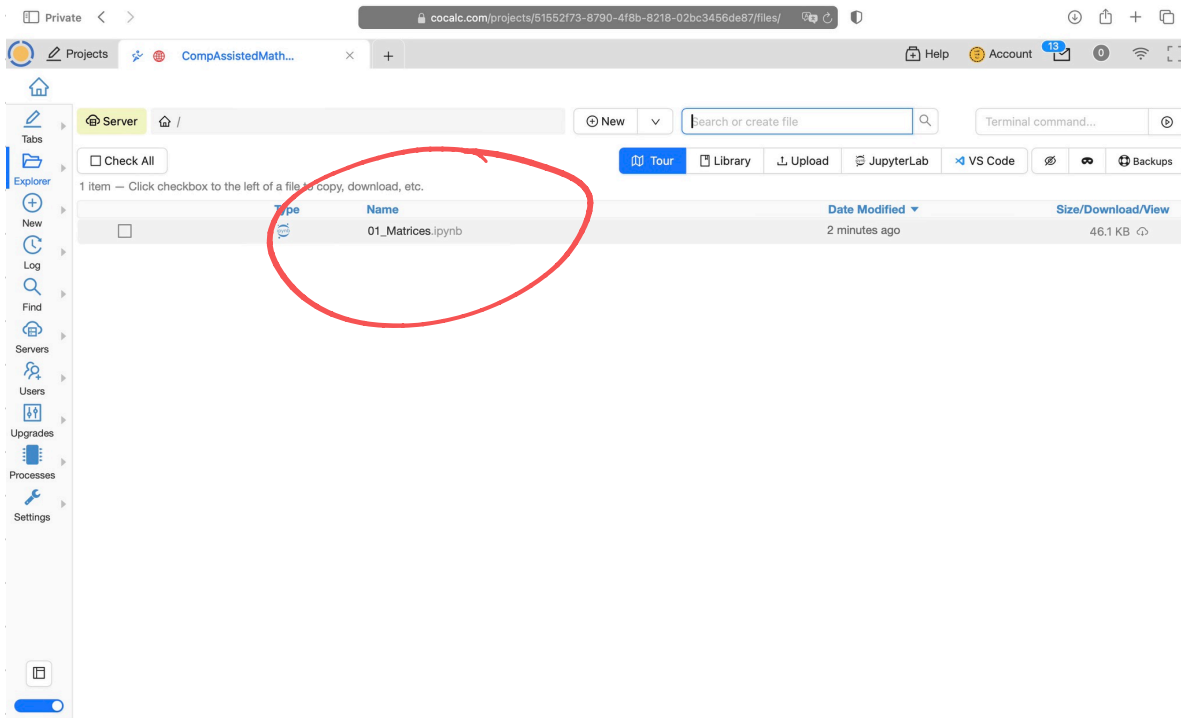
Upload Files Into Your Project

You can drop one or more files here or on the Explorer file listing. See [the docs](#) for more ways to get your files into your project.

01\_Matrices.ip

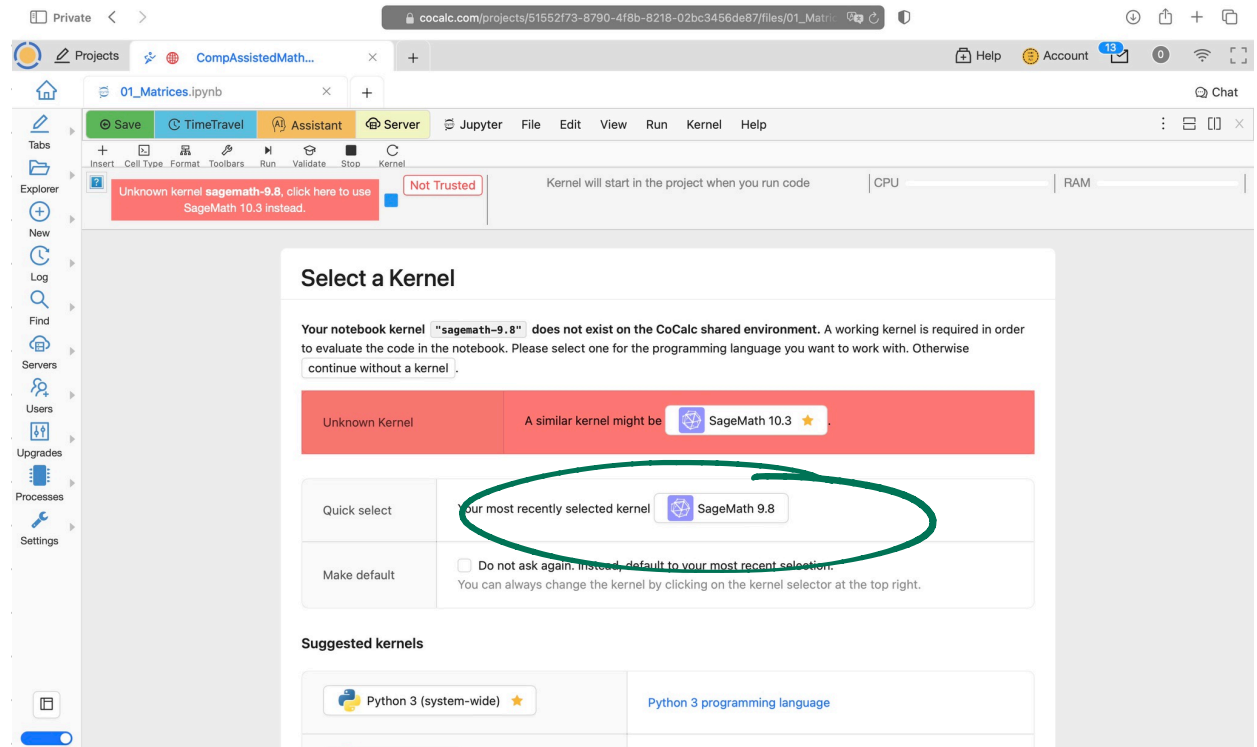
At the bottom of the page





You can then open the uploaded file

We recommend using the SageMath 9.8 kernel (it is ok to trust the kernel and the notebook, we promise!)



You can now start working on the notebook!

The screenshot shows a web browser window with a Jupyter notebook interface. The browser address bar shows a URL from cocalc.com. The notebook interface includes a sidebar with navigation icons, a top toolbar with buttons like Save, TimeTravel, Assistant, and Server, and a main content area. The notebook is titled '01\_Matrices.ipynb' and is using SageMath 9.8. The kernel status is 'idle (halt...)'. The main content area displays the title 'Basic operations' and 'Matrices in Sage'. Below the title, there is a paragraph explaining how to define a matrix in Sage, followed by a 3x3 matrix. The matrix is defined as 
$$\begin{pmatrix} 2 & 4 & 6 \\ 4 & 5 & 6 \\ 3 & 1 & 2 \end{pmatrix}$$
. Below the matrix, there is a paragraph explaining how to declare the matrix  $A$  with entries in  $\mathbb{Q}$ , and then as a matrix  $B$  with entries in the field with seven elements  $\mathbb{F}_7$ . The notebook shows an input cell with the following code: 

```
In [5]: 1 A = matrix( QQ, [[2,4,6],[4,5,6],[3,1,2]] )
        2 show(A)
```

 The output of the code is displayed below the input cell: 

```
Out[5]: 
$$\begin{pmatrix} 2 & 4 & 6 \\ 4 & 5 & 6 \\ 3 & 1 & 2 \end{pmatrix}$$

```