Proseminar on computer-assisted mathematics

Session 1 - Introduction to Sage

Matrices in *Sage*

When we define a matrix in *Sage*, we can specify the ring or field in which we take the entries.

Let us for instance consider the matrix

$$\begin{pmatrix} 2 & 4 & 6 \\ 4 & 5 & 6 \\ 3 & 1 & 2 \end{pmatrix}$$

and declare it first as a matrix A with entries in $\mathbb{Q}$, then as a matrix B with entries the field with seven elements $\mathbb{F}_7$.

```
A = matrix( QQ, [[2,4,6],[4,5,6],[3,1,2]] )
show(A)
```

```
show(A.inverse())
```

$$\begin{pmatrix} -\frac{2}{9} & \frac{1}{9} & \frac{1}{3} \\ -\frac{5}{9} & \frac{7}{9} & -\frac{2}{3} \\ \frac{11}{18} & -\frac{5}{9} & \frac{1}{3} \end{pmatrix}$$

Judith Ludwig and Florent Schaffhauser
Heidelberg University, Summer semester 2024

Computer-assisted mathematics 2024

The seminar has two parts:

- A Sage part (16.04–28.05)
- A Lean part (04.06–23.07)

The seminar is collaborative and project-based.

To pass, attendance is mandatory
(unless excused in advance).

Programme of the Sage part of the seminar

After getting acquainted with Sage, me will focus on topics from linear algebra.

Topics of linear algebra to be covered in the seminar:

- Computer algebra systems. Representations of vectors and matrices.
- Row operations. Gaussian elimination. Row-reduced echelon form of a matrix.
- Invertible matrices. Elementary matrices. Determinant.
- Linear independence. Bases for the kernel and the image of a linear transformation.
- Rank-nullity theorem and the row space of matrix. Basis for the row space.
- Base change. Coordinates of a vector, matrix of a linear transformation.
- Eigenvalues and the characteristic polynomial. Diagonalisation.
- The Gram-Schmidt process. Least-square approximation.

However, feel free to suggest other topics for the final projects.

Schedule

#	Date	Topic	Speaker	Slides	Code
1	16/04	Introduction to Sage	Instructors		
2	23/04	Introduction to Git + Assignment #1	Instructors		
3	30/04	Kernels, images and diagonalisation	Working in pairs		
4	14/05	Least squares approximation	Working in pairs		
5	21/05	Project prepararation	Working in pairs		
6	28/05	Project presentation			
7	04/06	Introduction to Lean	Instructors		
8	11/06	Natural Number Game	Working in pairs		
9	18/06	Basic tactics + Assignment #2	Instructors		
10	02/07	Advanced tactics	Working in pairs		
11	16/07	Project preparation	Working in pairs		
12	23/07	Project presentation			

- Team up (soon!)
- Choose a project (later)

Practical organization

Minimal requirements

- At least one computer / tablet per team

Before 23.04

1. Create individual GitHub accounts
   https://github.com
2. Join the seminar's Zulip channel
   https://matematiflo.zulipchat.com

Advanced option (facultative)

- Install Sagemath (suggested version: 9.8)
  https://doc.sagemath.org/html/en/installation/index.html
- Install JupyterLab https://jupyter.org/install

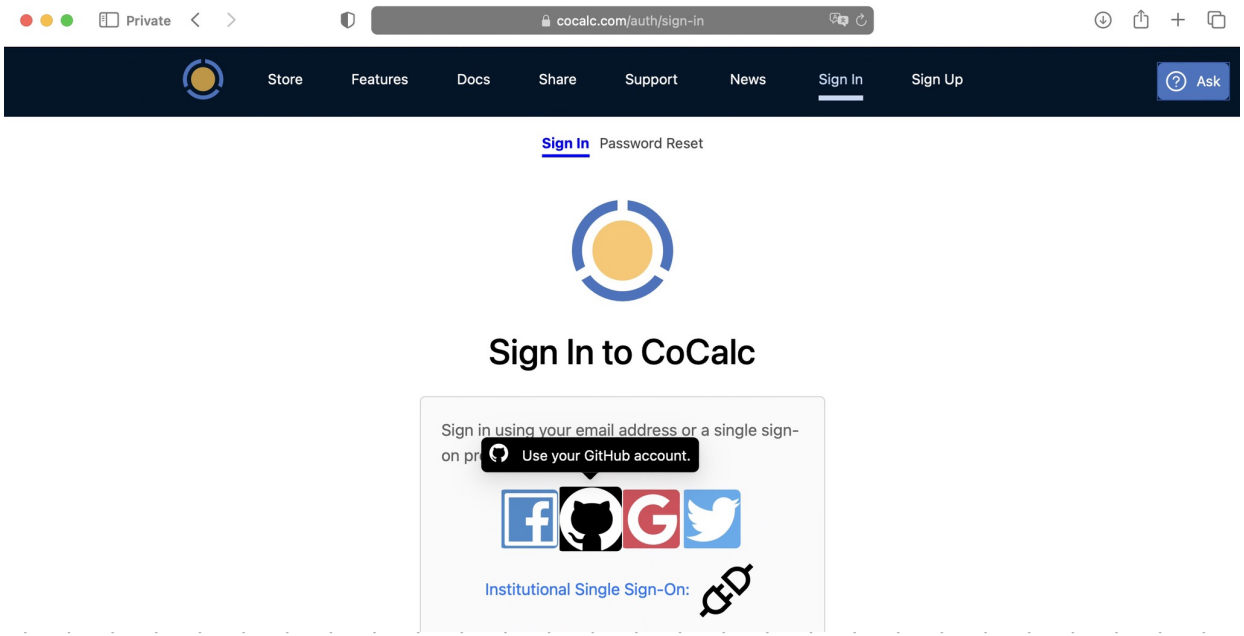
Plan for today

- Go through a Sage tutorial

  https://matematiflo.github.com/SoSe_2024/notes/SageTutorial.pdf

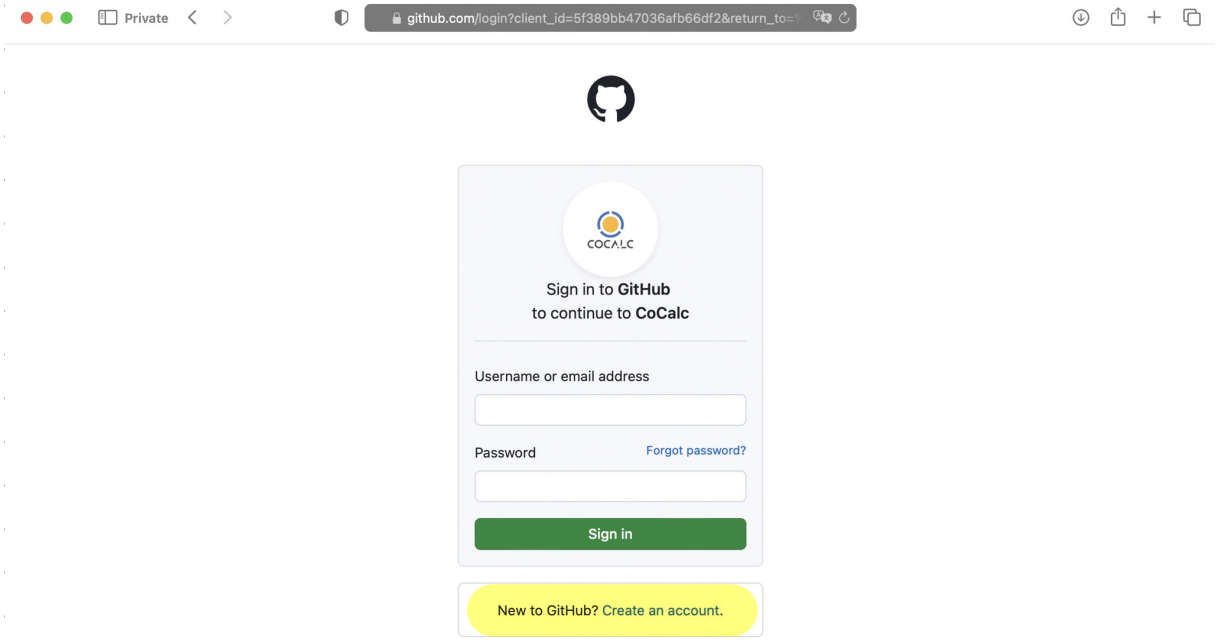
- When you are done with the tutorial, start using CoCalc to work on a Sage notebook

  https://matematiflo.github.com/SoSe_2024/notes/01_Matrices.ipynb

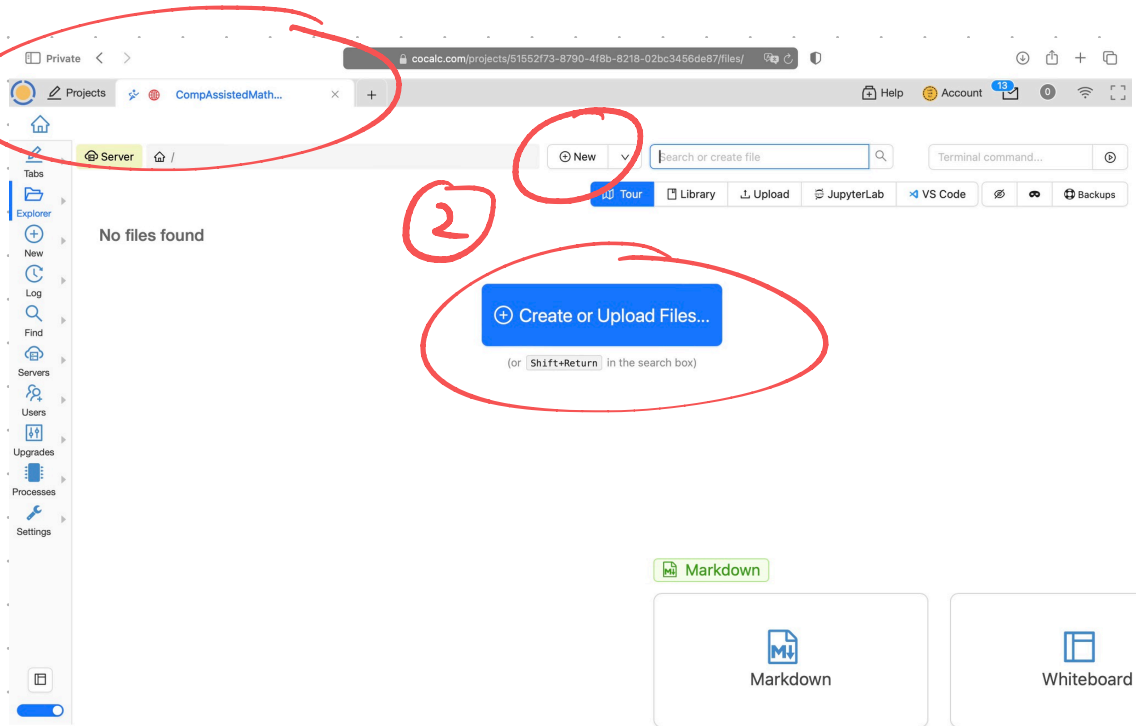


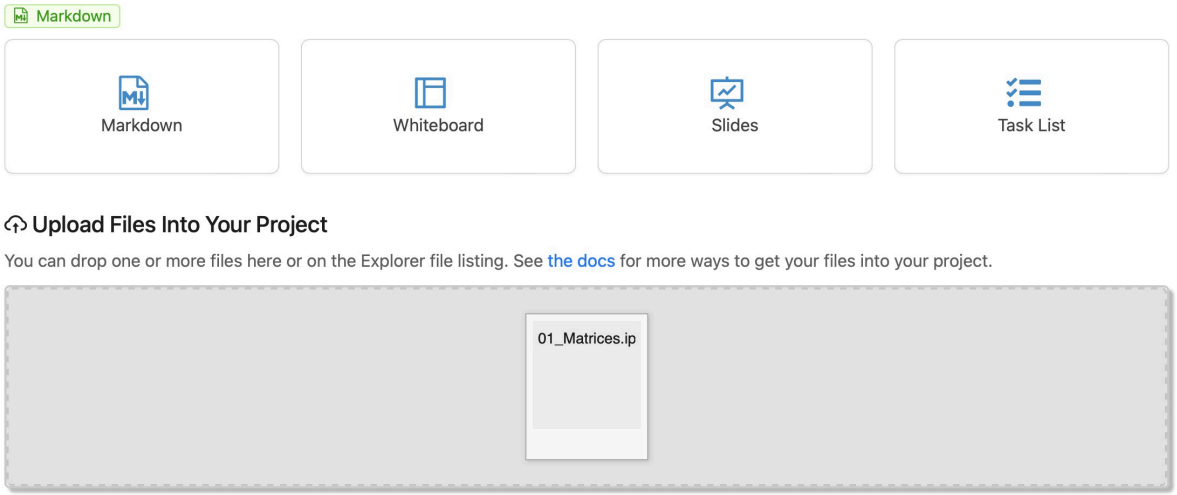
Go to cocalc.com and sign in using your GitHub account.

You can also create one on the spot.

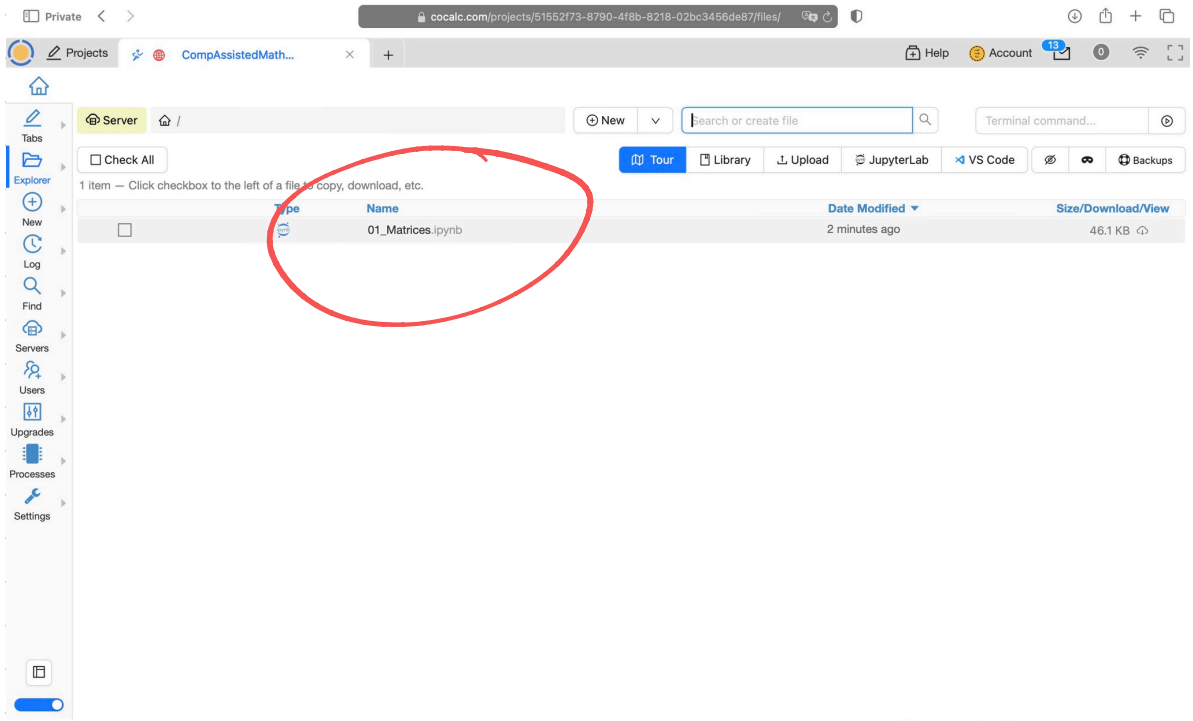


Once logged in, create a new project, upload the notebook about matrices in Sage and start practicing!

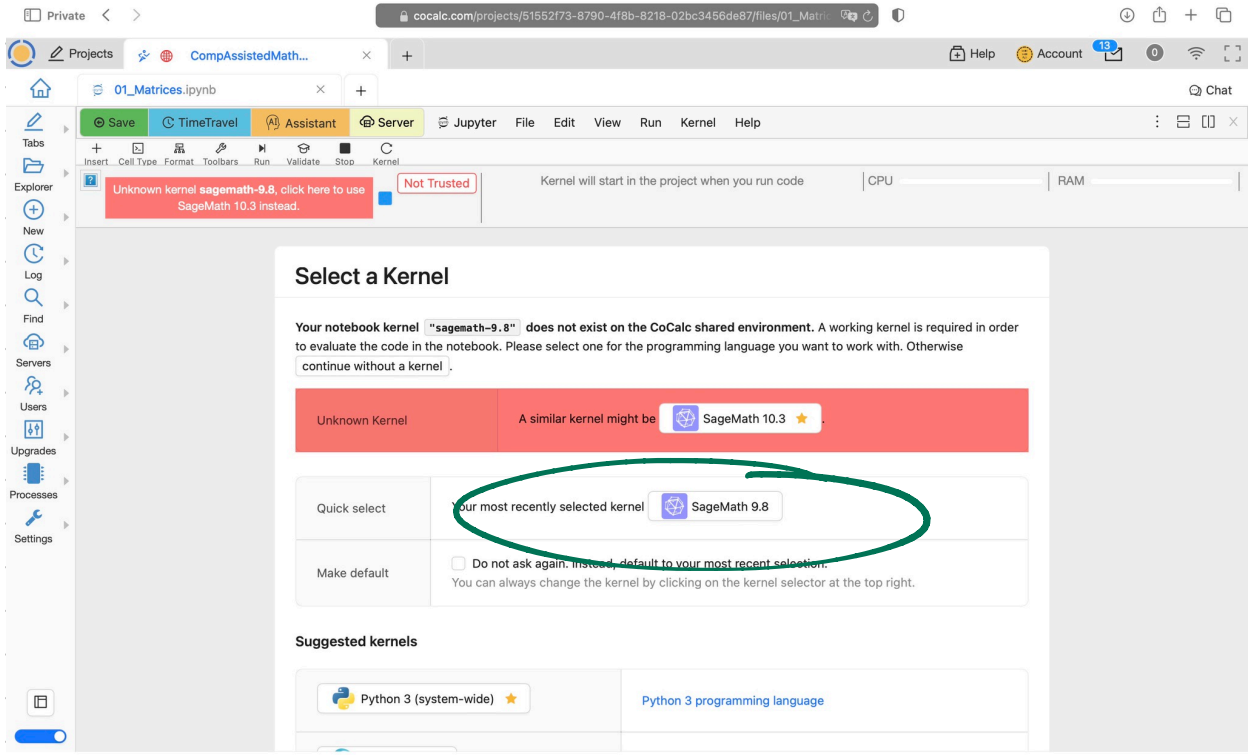




At the bottom of the page



You can then open the uploaded file



We recommend using the SageMath 9.8 kernel

(it is ok to trust the kernel and the notebook, we promise!)

You can now start working on the notebook!