1. Take derivative of the following function:

$f(x) = (x + x^3)^4 - x$

SOLUTION:

$f(x)' = 4(x + x^3)^3 * (1 + 3x^2) - 1$

2. Linearize the following function at $x_0 : f(x) = x + 2x^2$

Write the equation of a line tangent at $x_0 = 3$:

a) First derive the form: $f(x_0 + \delta) = ?$

b) Then write the linear approximation of $f(x)$ at $x_0, i.e., f(x)|x_0 \approx ?$

SOLUTION:

a)

$f(x_0 + \delta) \approx f(x_0) + df$

$df = (1 + 4x)|x_0 * \delta$

$f(x_0 + \delta) = 21 + 13\delta$

b)

$f(x)|x_0 \approx 21 + 13(x - x_0) =$

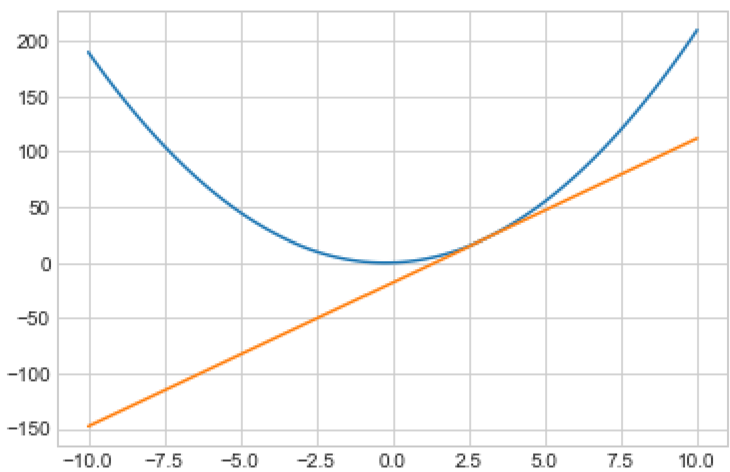
$= 21 + 13x - 39 =$

$= -18 + 13x$

In [19]:

```
import matplotlib.pyplot as plt
import numpy as np
plt.style.use('seaborn-whitegrid')
x = np.linspace(-10, 10, 100)
plt.plot(x, (x + 2*x**2))
plt.plot(x, (-18 + 13*x))
```

Out[19]: [<matplotlib.lines.Line2D at 0x25972339280>]



3. Linearize the following function $f(x_1(\boldsymbol{p}), x_2(\boldsymbol{p}))$ for small changes in parameters $\boldsymbol{p}$, i.e., $\boldsymbol{p}_0 + \delta$, using the Jacobian:

$f(x_1(\boldsymbol{p}), x_2(\boldsymbol{p})) = x_1^2(\boldsymbol{p}) + x_2^5(\boldsymbol{p})$

with parameter vector defined as:

$\boldsymbol{p} = [p_1, p_2]^T$ and with parameters of $f(x_1, x_2)$ defined as: $x_1(\boldsymbol{p}) = p_1, x_2(\boldsymbol{p}) = p_1^2 + p_2^2$

SOLUTION:

$f(x(\boldsymbol{p}_0 + \delta)) \approx f(x(\boldsymbol{p}_0) + \nabla f^T|\boldsymbol{p}_0 * J * \delta$

$\nabla f^T|\boldsymbol{p}_0 = [2x_1(\boldsymbol{p}), 5x_2^4(\boldsymbol{p})]|\boldsymbol{p}_0 = [2p_1, 5(p_1^2 + p_2^2)^4]$

$$J = \begin{bmatrix} 1 & 0 \\ 2p_1 & 2p_2 \end{bmatrix}$$

4. Assume a vector $x \in \R^d$ and a symmetric positive definite matrix $A \in \R^{dxd}$.

Take the function f(**x**) = $x^T Ax * 3$ and compute its gradient: $\nabla f(x) = \frac{\partial f}{\partial x} = 3(A + A^T)x = 6Ax$