

1. Take derivative of the following function:

$$f(x) = (x + x^3)^4 - x$$

SOLUTION:

$$f(x)' = 4(x + x^3)^3 * (1 + 3x^2) - 1$$

2. Linearize the following function at x_0 : $f(x) = x + 2x^2$

Write the equation of a line tangent at $x_0 = 3$:

a) First derive the form: $f(x_0 + \delta) = ?$

b) Then write the linear approximation of $f(x)$ at x_0 , i. e. , $f(x)|_{x_0} \approx ?$

SOLUTION:

a)

$$f(x_0 + \delta) \approx f(x_0) + df$$

$$df = (1 + 4x)|_{x_0} * \delta$$

$$f(x_0 + \delta) = 21 + 13\delta$$

b)

$$f(x)|_{x_0} \approx 21 + 13(x - x_0) =$$

$$= 21 + 13x - 39 =$$

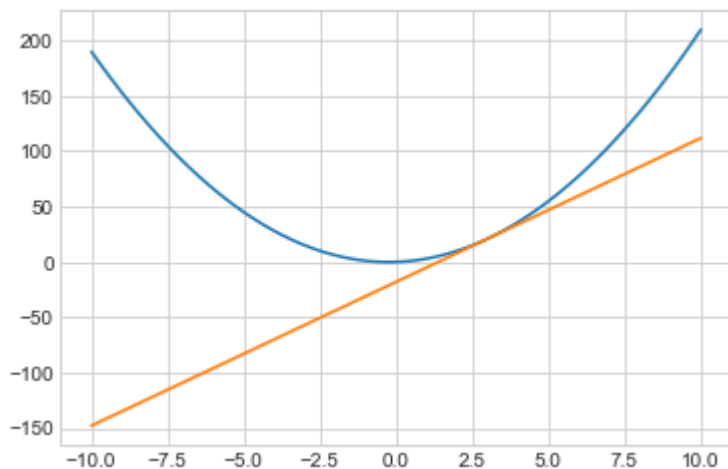
$$= -18 + 13x$$

In [19]:

```
import matplotlib.pyplot as plt
import numpy as np
plt.style.use('seaborn-whitegrid')
x = np.linspace(-10, 10, 100)
plt.plot(x, (x + 2*x**2))
plt.plot(x, (-18 + 13*x))
```

Out[19]:

[<matplotlib.lines.Line2D at 0x25972339280>]



3. Linearize the following function $f(x_1(\mathbf{p}), x_2(\mathbf{p}))$ for small changes in parameters \mathbf{p} , i.e., $\mathbf{p}_0 + \delta$, using the Jacobian:

$$f(x_1(\mathbf{p}), x_2(\mathbf{p})) = x_1^2(\mathbf{p}) + x_2^5(\mathbf{p})$$

with parameter vector defined as:

$$\mathbf{p} = [p_1, p_2]^T \text{ and with parameters of } f(x_1, x_2) \text{ defined as: } x_1(\mathbf{p}) = p_1, x_2(\mathbf{p}) = p_1^2 + p_2^2$$

SOLUTION:

$$f(x(\mathbf{p}_0 + \delta)) \approx f(x(\mathbf{p}_0)) + \nabla f^T|_{\mathbf{p}_0} * J * \delta$$

$$\nabla f^T|_{\mathbf{p}_0} = [2x_1, 5x_2^4]|_{\mathbf{p}_0}$$

$$J = \begin{bmatrix} 1 & 0 \\ 2p_1 & 2p_2 \end{bmatrix}$$

4. Assume a vector $x \in \mathbb{R}^d$ and a symmetric positive definite matrix $A \in \mathbb{R}^{d \times d}$.

Take the function $f(\mathbf{x}) = x^T A x * 3$ and compute its gradient: $\nabla f(x) = \frac{\partial f}{\partial x} = 3(A + A^T)x = 6Ax$