PART 1.

HOMEWORK 1:

Prove equivalence 1. <=> 4. of Proposition 2.3 in notes

Proposition 2.3: Let f be a function and let Γ be its graph. The following are equivalent

(1) f is convex: $f(tx + (1-t)y) \leq t \cdot f(x) + (1-t) f(y)$

(4) For each $x_1 \dots x_k \in D$ and $\alpha_1, \dots \alpha_k \in [0,1]$, where $\sum_i^k \alpha_i = 1$ we have

$$f\left(\sum_i^k \alpha_i \cdot x_i\right) \leq \sum_1^k \alpha_i f(x_i)$$

(1 => 4) Proof by induction:

I) We prove for a base case: $k = 2$

$$f(\alpha x_1 + \alpha_2 \cdot x_2) \leq \alpha_1 \cdot f(x_1) + \alpha_2 \cdot f(x_2)$$

= Because $\alpha_1 + \alpha_2 = 1$

$$f(\alpha_1 \cdot x_1 + (1-\alpha_1) x_2) \leq \alpha_1 \cdot f(x_1) + (1-\alpha_1) \cdot f(x_2)$$

This is the definition of a convex function so we proved for $k = 2$.