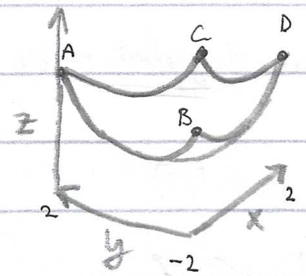
(2.) Let $f(x,y) = x^2 + e^x + y^2 - xy$. Function f restricted to $[-2,2] \times [-2,2]$ is Lipshitz, smooth and strongly convex. Find some corresponding (preferably optimal) constants L, α, β on K. Furthermore prove f is convex.

Sketch of function f:



We define points:

$A(-2, 2, f(-2,2))$
$B(-2, -2, f(-2,-2))$
$C(2, -2, f(2,-2))$
$D(2, 2, f(2,2))$

I) Function f is L lipshitz if $\forall x,y \in K$ we have:

$$\boxed{|f(x) - f(y)| \leq L \cdot \|x - y\|.}$$

We also know that the following holds:

$$\boxed{f \text{ is } L\text{-lipshitz iff } \|\nabla f\| \leq L.}$$

That means that to find the optimal L, we must find max $\|\nabla f\|$.
We know that the max of $\|\nabla f\|$ appear ~~somewhere~~ at one of the 4 corners of the domain, because $\|\nabla f\|$ ~~is~~ increases as we go further away from minima.

$$\nabla f = \begin{bmatrix} 2x + e^x - y \\ 2y - x \end{bmatrix}$$

A: $\|\nabla f(-2,2)\| = \sqrt{(-4 + e^{-2} - 2)^2 + (4+2)^2} = 8.39$

B: $\|\nabla f(-2,-2)\| = \sqrt{(-4 + e^{-2} + 2)^2 + (-4+2)^2} = 2.73$

C: $\|\nabla f(2,-2)\| = \sqrt{(4 + e^2 + 2)^2 + (-4-2)^2} = 14.67$

D: $\|\nabla f(2,2)\| = \sqrt{(4 + e^2 - 2)^2 + (4-2)^2} = 9.60$

We see that max $\|\nabla f\|$ is at point C and its value is 14.67. We set $L = 14.67$