II) Function f is β-smooth if $\forall x, y \in K$:

$$\boxed{\|\nabla f(x) - \nabla f(y)\| \leq \beta \|x - y\|}$$

We also know the following holds, since f is at least twice differentiable:

$$\boxed{\text{All eigenvalues of } \nabla^2 f \text{ lie on } [0, \beta].}$$

To find the optimal β, we must find the largest eigenvalue of Hessian.

III) Function f is α-strongly convex if $f(x) - \frac{\alpha}{2}\|x\|^2$ is convex.

We also know the following holds, since f is at least twice differentiable:

$$\boxed{\text{All eigenvalues of } \nabla^2 f \text{ lie on } [\alpha, \infty)}$$

To find optimal α, we must find the smallest eigenvalue of Hessian.

$$H = \nabla^2 f = \begin{bmatrix} 2+e^x & -1 \\ -1 & 2 \end{bmatrix}$$

Find eigenvalues: $\det(\nabla^2 f - \lambda I) = \begin{vmatrix} 2+e^x-\lambda & -1 \\ -1 & 2-\lambda \end{vmatrix} = (2+e^x-\lambda)(2-\lambda) - 1 = 0$

$$(4 - 2\lambda) + (2e^x - \lambda e^x) + (-2\lambda + \lambda^2) - 1 = 0$$

$$\lambda^2 - \lambda(4 + e^x) + 2e^x + 3 = 0$$

$$\lambda_1 = \frac{4 + e^x + \sqrt{4 + e^{2x}}}{2}$$

$$\lambda_2 = \frac{4 + e^x - \sqrt{4 + e^{2x}}}{2}$$

We observe that value of eigenvalues change depending on x value.
We choose λ_1 and set $x = 2$ to get max eigenvalue and
we choose λ_2 and set $x = -2$ to get min eigenvalue:

$$\beta = \frac{4 + e^2 + \sqrt{4 + e^4}}{2} = 9.52 \qquad \alpha = \frac{4 + e^{-2} - \sqrt{4 + e^{-4}}}{2} = 1.07$$

IV) We know that f is convex, since it is strongly convex. Also all eigenvalues of Hessian are positive.