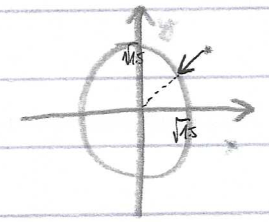
(3) Find formulas for projections $\mathbb{R}^2 \to K$ to the closest point for the following convex sets:

I) $x^2 + y^2 \leq 1.5$

$r = \sqrt{1.5}$

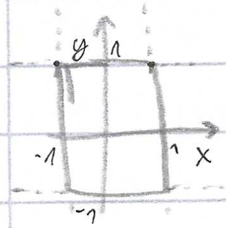


We notice that the closest ~~point~~ vector $\vec{v}^*$ for a ~~point~~ vector $\vec{v}$ outside the boundary has the same direction, but its ~~length~~ norm equals the value of radius.

The projection function takes normalized $\vec{v}$ and multiplies by $\sqrt{1.5}$.

$$\pi_K(\vec{v}) = \begin{cases} \vec{v} \text{ if } \vec{v} \in K, \\ \frac{\vec{v}}{\|\vec{v}\|} \cdot \sqrt{1.5} \text{ else} \end{cases}$$

II) $[-1,1] \times [-1,1]$



We divide the region into 8 subregions:

$$\pi_K(x,y) = \begin{cases} (-1,1) & \text{if } x < -1 \wedge y > 1, \\ (x,1) & \text{if } |x| \leq 1 \wedge y > 1, \\ (1,1) & \text{if } x > 1 \wedge y > 1, \\ (1,y) & \text{if } x > 1 \wedge |y| \leq 1, \\ (1,-1) & \text{if } x > 1 \wedge y < 1, \\ (x,-1) & \text{if } |x| \leq 1 \wedge y < -1, \\ (-1,-1) & \text{if } x < -1 \wedge y < -1, \\ (-1,y) & \text{if } x < -1 \wedge |y| \leq 1, \\ (x,y) & \text{otherwise} \end{cases}$$

Simpler version: $\pi_K(x,y) = \big(\min(\max(x,-1),1), \ \min(\max(y,-1,1) \big)$