II) We assume it holds for ~~n~~ $k=n$

$$f\left(\sum_i^n \alpha_i \cdot x_i\right) \leq \sum_i^n \alpha_i \cdot f(x_i)$$

and prove for $k=n+1$

Inductive step: $f\left(\sum_i^{n+1} \alpha_i \cdot x_i\right) \leq \sum_i^{n+1} \alpha_i \cdot f(x_i)$

We want

$f\left(\sum_i^{n+1} \alpha_i \cdot x_i\right)$ to be similar to convex definition so we can use our inequality hypothesis.

$\boxed{\text{NOTE: } \sum_1^n \frac{\alpha_i}{t} = 1}$

We define: $t = 1 - \alpha_{n+1}$, and rewrite:

convexity ~~hypothesis~~

$$f\left(\sum_i^{n+1} \alpha_i \cdot x_i\right) = f\left(t \cdot \sum_i^n \frac{\alpha_i}{t} \cdot x_i + (1-t) \cdot x_{n+1}\right) \leq t \cdot f\left(\sum_i^n \frac{\alpha_i}{t} \cdot x_i\right) + (1-t) f(x_{n+1}) \leq$$

we assume it holds for $k=n$

$$\leq t \cdot \sum_i^n \frac{\alpha_i}{t} \cdot f(x_i) + (1-t) \cdot f(x_{n+1}) =$$

we bring back t in the sum and use ~~t~~ $\cdot a_{n+1} = 1-t$

$$= \sum_i^n \alpha_i \cdot f(x_i) + \alpha_{n+1} \cdot f(x_{n+1}) = \underline{\underline{\sum_i^{n+1} \alpha_i \cdot f(x_i)}}$$

~~4 => 1 We can prove this just by ~~setting~~ $k-2$ coefficients to 0.~~

using a case where

are set