Mathematics 2, Part 3	2022/23
Homework 5	
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The homework consists of three problems. The solutions are to be submitted to the appropriate mailbox on Učilnica before the exam, but preferably in a week. The solutions should contain a clear and well explained proofs, procedures, explanations, etc.

(1) Let V be a real vector space with an inner product f and a linear transformation $\tau : V \to V$ such that

$$f(\tau(v), w) = -f(v, \tau(w))$$

for all $v, w \in V$. Show that τ has all real eigenvalues equal to 0.

(2) Find an inner product g on $\mathbb{R}_2[x]$ for which basis $\mathcal{B} = \{1, 1 + x, 1 + x^2\}$ is reciprocal to itself. Write an explicit formula for g.

(3) In Homework 4 you proved $\alpha(p) = \int_0^1 p(x)\, dx$ is a linear form on $\mathbb{R}_2[x]$. For $p, q \in \mathbb{R}_2[x]$ let

$$h(p, q) = p(-1)q(-1) + p(0)q(0) + p(1)q(1)$$

be an inner product on $\mathbb{R}_2[x]$. (This was proved in the classroom, no need to verify here as well.) Find a polynomial $r \in \mathbb{R}_2[x]$ such that $\alpha(p) = h(p, r)$ for all $p \in \mathbb{R}_2[x]$.