Método de Euler - EDO's

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Tabla de Contenidos


```
%load_ext autoreload

%autoreload 2

from src import ODE_euler, ajuste_polinomio, ajuste_exponencial, f_lineal_interpolate

import numpy as np
import matplotlib.pyplot as plt
import pandas as pd
```

```
The autoreload extension is already loaded. To reload it, use:
  %reload_ext autoreload
```

1 CONJUNTO DE EJERCICIOS

3. Utilice el método de Euler para aproximar las soluciones para cada uno de los siguientes problemas de valor inicial.

4. Aquí se dan las soluciones reales para los problemas de valor inicial en el ejercicio 3. Calcule el error real en las aproximaciones del ejercicio 3.

5. Utilice los resultados del ejercicio 3 y la interpolación lineal para aproximar los siguientes valores de $y(t)$. Compare las aproximaciones asignadas para los valores reales obtenidos mediante las funciones determinadas en el ejercicio 4.

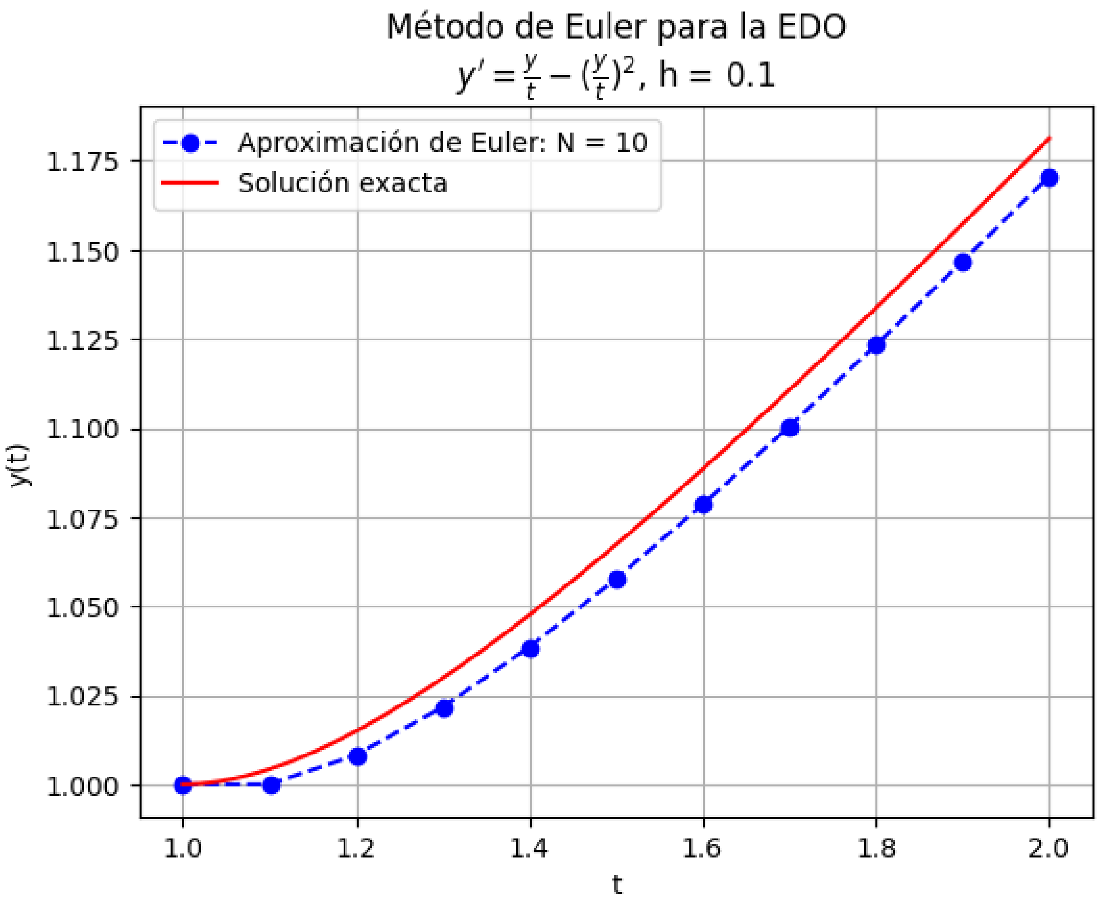
a.

$$y' = \frac{y}{t} - (\frac{y}{t})^2$$
$$1 \leq t \leq 2$$
$$y(1) = 1$$
$$h = 0.1$$

Solución:

$$y(t) = \frac{t}{1 + \ln(t)}$$

```
f = lambda t, y: y / t - (y / t) ** 2

exact_sol = lambda t: t / (1 + np.log(t))

a, b = 1, 2
y_t0 = 1
N = 10

ys, ts, h = ODE_euler(a=a, b=b, f=f, y_t0=y_t0, N=N)

t_exact = np.linspace(a, b, 100)
y_exact = exact_sol(t_exact)

plt.plot(
    ts,
    ys,
    label="Aproximación de Euler: N = 10",
    marker="o",
    linestyle="--",
    color="blue",
)
plt.plot(t_exact, y_exact, label="Solución exacta", linestyle="-", color="red")
plt.xlabel("t")
plt.ylabel("y(t)")
plt.title("Método de Euler para la EDO\n$y\' = \\frac{y}{t} - (\\frac{y}{t})^2$," + f" h = {
plt.legend()
plt.grid()
plt.show()
```



Error Real

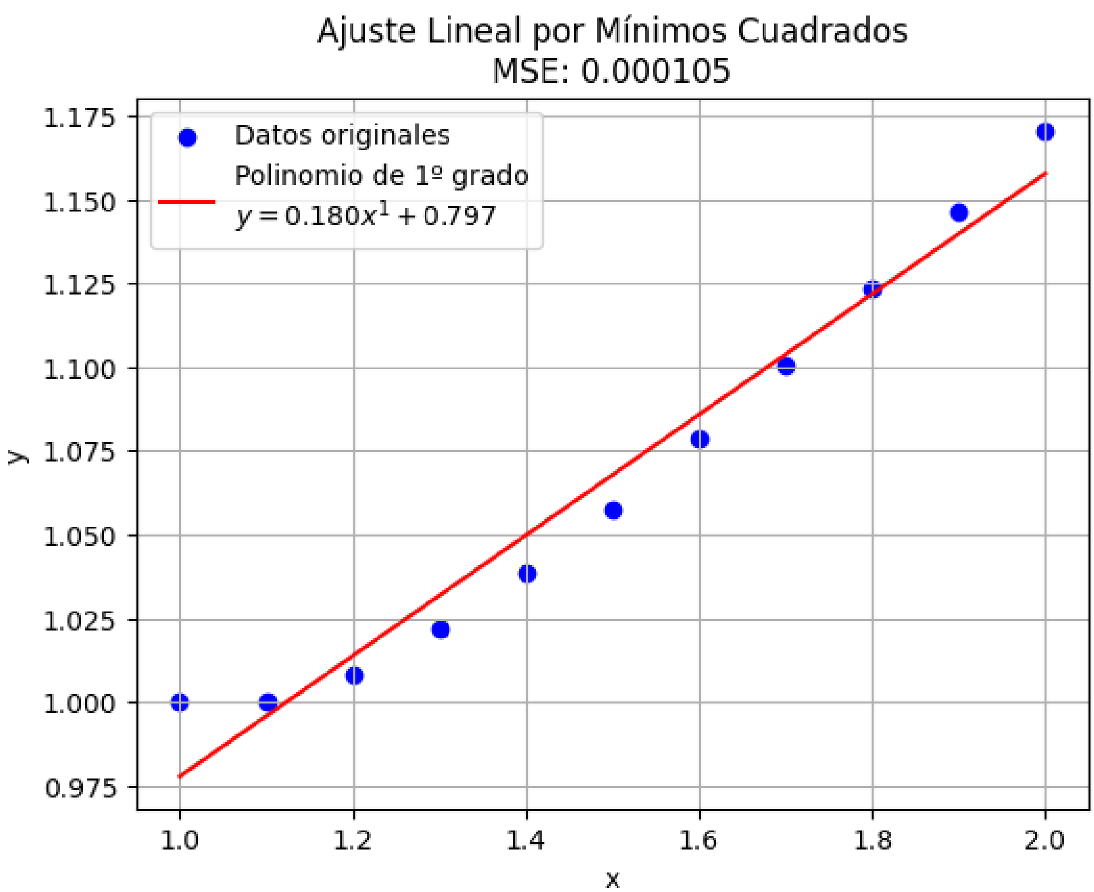
```
error = np.abs(exact_sol(ts) - ys)

results = pd.DataFrame(
    {"t_i": ts, "ŷ_i": ys, "y_i = y(t_i)": exact_sol(ts), "|y_i - ŷ_i|": error}
)

print(results)
```

```
     t_i       ŷ_i  y_i = y(t_i)  |y_i - ŷ_i|
0    1.0  1.000000      1.000000     0.000000
1    1.1  1.000000      1.004282     0.004282
2    1.2  1.008264      1.014952     0.006688
3    1.3  1.021689      1.029814     0.008124
4    1.4  1.038515      1.047534     0.009019
```

```
5    1.5   1.057668       1.067262      0.009594
6    1.6   1.078461       1.088433      0.009972
7    1.7   1.100432       1.110655      0.010223
8    1.8   1.123262       1.133654      0.010392
9    1.9   1.146724       1.157228      0.010505
10   2.0   1.170652       1.181232      0.010581
```

Interpolación Lineal

```
a, b = ajuste_polinomio(ts, ys, 1, 'Lineal')
```



1. $y(1.25)$

```
x = 1.25

y_exact = exact_sol(x)
```

```python
y_interpolate = f_lineal_interpolate(a, b, x)

print(f"Valor exacto en x = {x}: y = {y_exact:.6f}")
print(f"Valor interpolado en x = {x}: ŷ = {y_interpolate:.6f}")
print(f"Error absoluto: |y - ŷ| = {abs(y_exact - y_interpolate):.6f}")
```

```
Valor exacto en x = 1.25: y = 1.021957
Valor interpolado en x = 1.25: ŷ = 1.022729
Error absoluto: |y - ŷ| = 0.000772
```

2. $y(1.93)$

```python
x = 1.93

y_exact = exact_sol(x)

y_interpolate = f_lineal_interpolate(a, b, x)

print(f"Valor exacto en x = {x}: y = {y_exact:.6f}")
print(f"Valor interpolado en x = {x}: ŷ = {y_interpolate:.6f}")
print(f"Error absoluto: |y - ŷ| = {abs(y_exact - y_interpolate):.6f}")
```

```
Valor exacto en x = 1.93: y = 1.164390
Valor interpolado en x = 1.93: ŷ = 1.145289
Error absoluto: |y - ŷ| = 0.019102
```

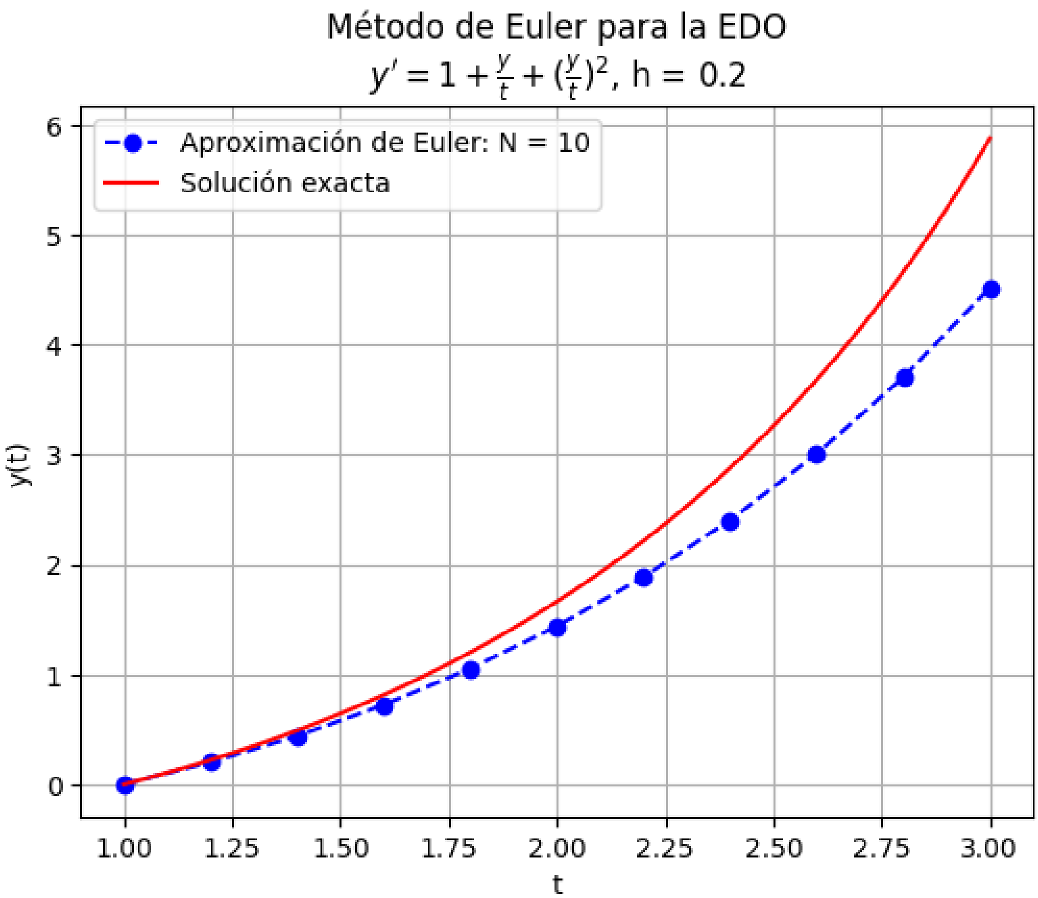
b.

$$y' = 1 + \frac{y}{t} + (\frac{y}{t})^2$$
$$1 \leq t \leq 3$$
$$y(1) = 0$$
$$h = 0.2$$

Solución

$$y(t) = t \cdot \tan(\ln(t))$$

```
f = lambda t, y: 1 + y / t + (y / t) ** 2

exact_sol = lambda t: t * np.tan(np.log(t))

a, b = 1, 3
y_t0 = 0
N = 10

ys, ts, h = ODE_euler(a=a, b=b, f=f, y_t0=y_t0, N=N)

t_exact = np.linspace(a, b, 100)
y_exact = exact_sol(t_exact)

plt.plot(
    ts,
    ys,
    label="Aproximación de Euler: N = 10",
    marker="o",
    linestyle="--",
    color="blue",
)
plt.plot(t_exact, y_exact, label="Solución exacta", linestyle="-", color="red")
plt.xlabel("t")
plt.ylabel("y(t)")
plt.title(
    "Método de Euler para la EDO\n$y' = 1 + \\frac{y}{t} + (\\frac{y}{t})^2$,"
    + f" h = {h}"
)
plt.legend()
plt.grid()
plt.show()
```



Error Real

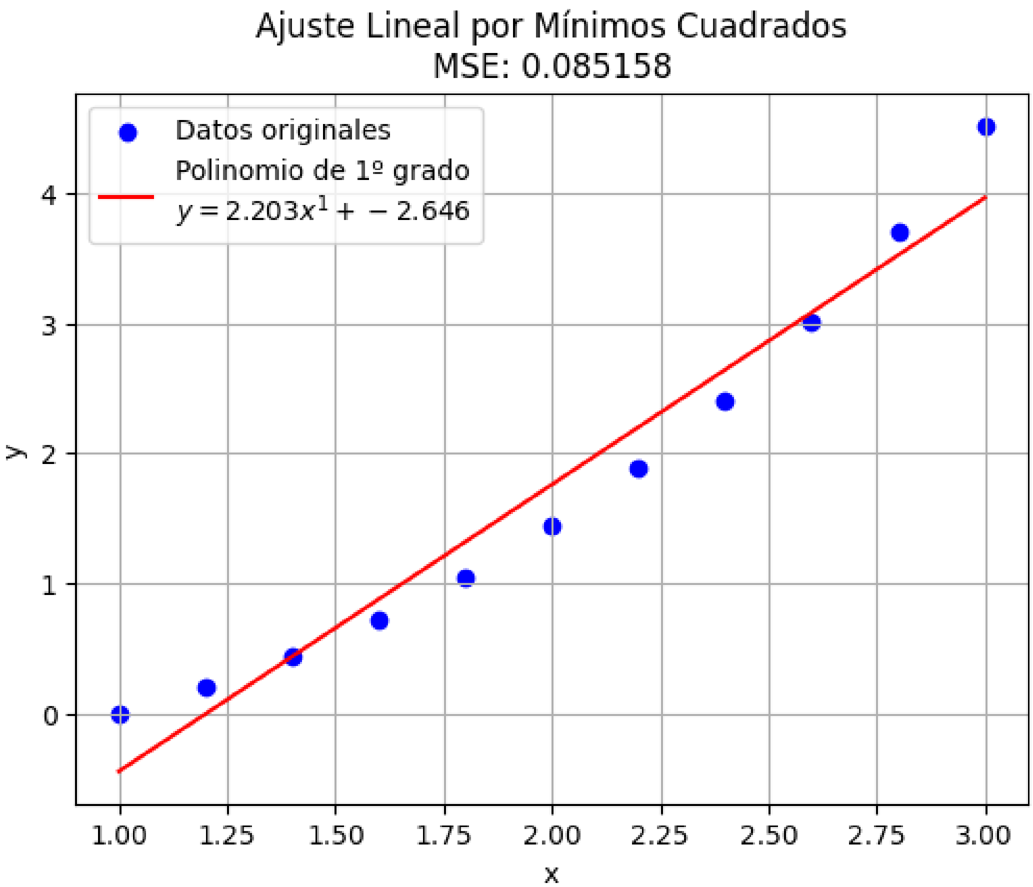
```
error = np.abs(exact_sol(ts) - ys)

results = pd.DataFrame(
    {"t_i": ts, "ŷ_i": ys, "y_i = y(t_i)": exact_sol(ts), "|y_i - ŷ_i|": error}
)

print(results)
```

```
     t_i       ŷ_i  y_i = y(t_i)  |y_i - ŷ_i|
0    1.0  0.000000      0.000000     0.000000
1    1.2  0.200000      0.221243     0.021243
2    1.4  0.438889      0.489682     0.050793
3    1.6  0.721243      0.812753     0.091510
4    1.8  1.052038      1.199439     0.147401
```

```
5    2.0   1.437251       1.661282      0.224031
6    2.2   1.884261       2.213502      0.329241
7    2.4   2.402270       2.876551      0.474282
8    2.6   3.002837       3.678475      0.675638
9    2.8   3.700601       4.658665      0.958064
10   3.0   4.514277       5.874100      1.359823
```

Interpolación Lineal

```python
a, b = ajuste_polinomio(ts, ys, 1, "Lineal")
```



1. $y(2.1)$

```python
x = 2.1

y_exact = exact_sol(x)
```

```python
y_interpolate = f_lineal_interpolate(a, b, x)

print(f"Valor exacto en x = {x}: y = {y_exact:.6f}")
print(f"Valor interpolado en x = {x}: ŷ = {y_interpolate:.6f}")
print(f"Error absoluto: |y - ŷ| = {abs(y_exact - y_interpolate):.6f}")
```

```
Valor exacto en x = 2.1: y = 1.924962
Valor interpolado en x = 2.1: ŷ = 1.979697
Error absoluto: |y - ŷ| = 0.054735
```

1. $y(2.75)$

```python
x = 2.75

y_exact = exact_sol(x)

y_interpolate = f_lineal_interpolate(a, b, x)

print(f"Valor exacto en x = {x}: y = {y_exact:.6f}")
print(f"Valor interpolado en x = {x}: ŷ = {y_interpolate:.6f}")
print(f"Error absoluto: |y - ŷ| = {abs(y_exact - y_interpolate):.6f}")
```

```
Valor exacto en x = 2.75: y = 4.394170
Valor interpolado en x = 2.75: ŷ = 3.411467
Error absoluto: |y - ŷ| = 0.982703
```

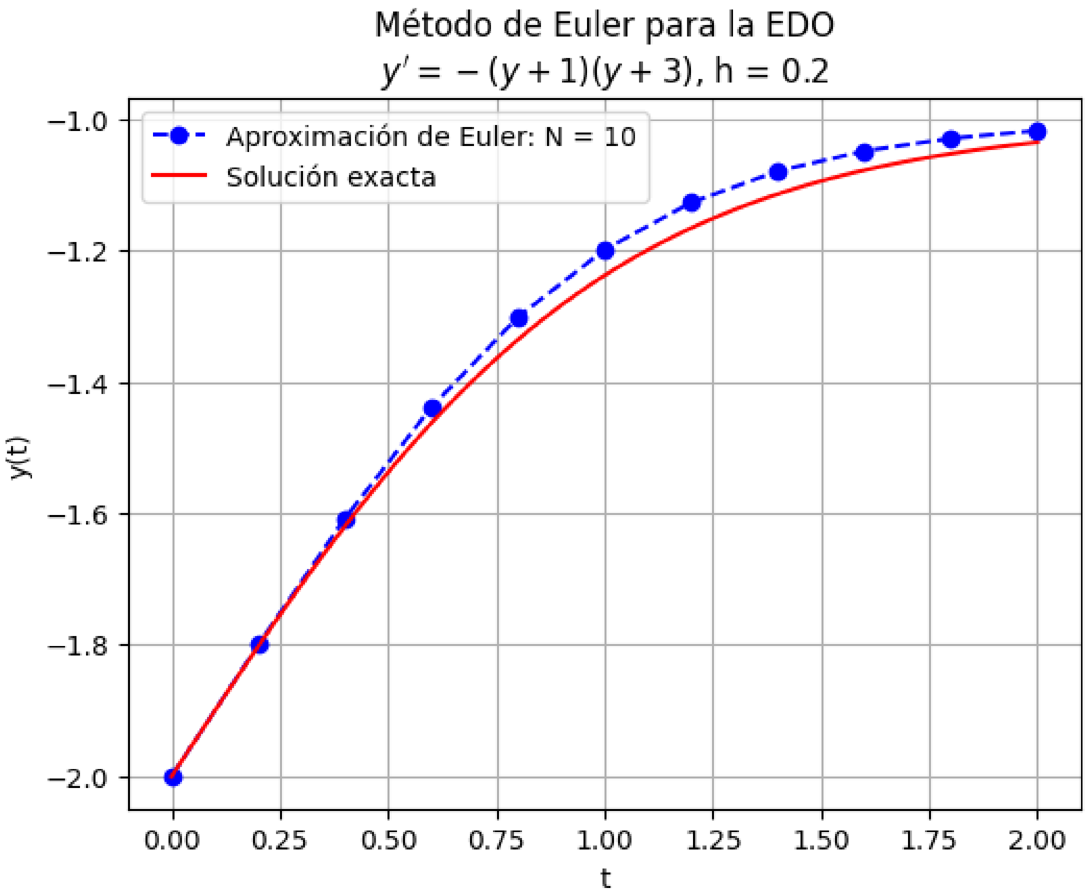
c.

$$y' = -(y+1)(y+3)$$
$$0 \leq t \leq 2$$
$$y(0) = -2$$
$$h = 0.2$$

Solución

$$y(t) = -3 + \frac{2}{1+e^{-2t}}$$

```python
f = lambda t, y: - (y + 1) * (y + 3)

exact_sol = lambda t: - 3 + 2 / (1 + np.exp(- 2 * t))

a, b = 0, 2
y_t0 = -2
N = 10

ys, ts, h = ODE_euler(a=a, b=b, f=f, y_t0=y_t0, N=N)

t_exact = np.linspace(a, b, 100)
y_exact = exact_sol(t_exact)

plt.plot(
    ts,
    ys,
    label="Aproximación de Euler: N = 10",
    marker="o",
    linestyle="--",
    color="blue",
)
plt.plot(t_exact, y_exact, label="Solución exacta", linestyle="-", color="red")
plt.xlabel("t")
plt.ylabel("y(t)")
plt.title("Método de Euler para la EDO\n$y' = - (y + 1) (y + 3)$," + f" h = {h}")
plt.legend()
plt.grid()
plt.show()
```



Error Real

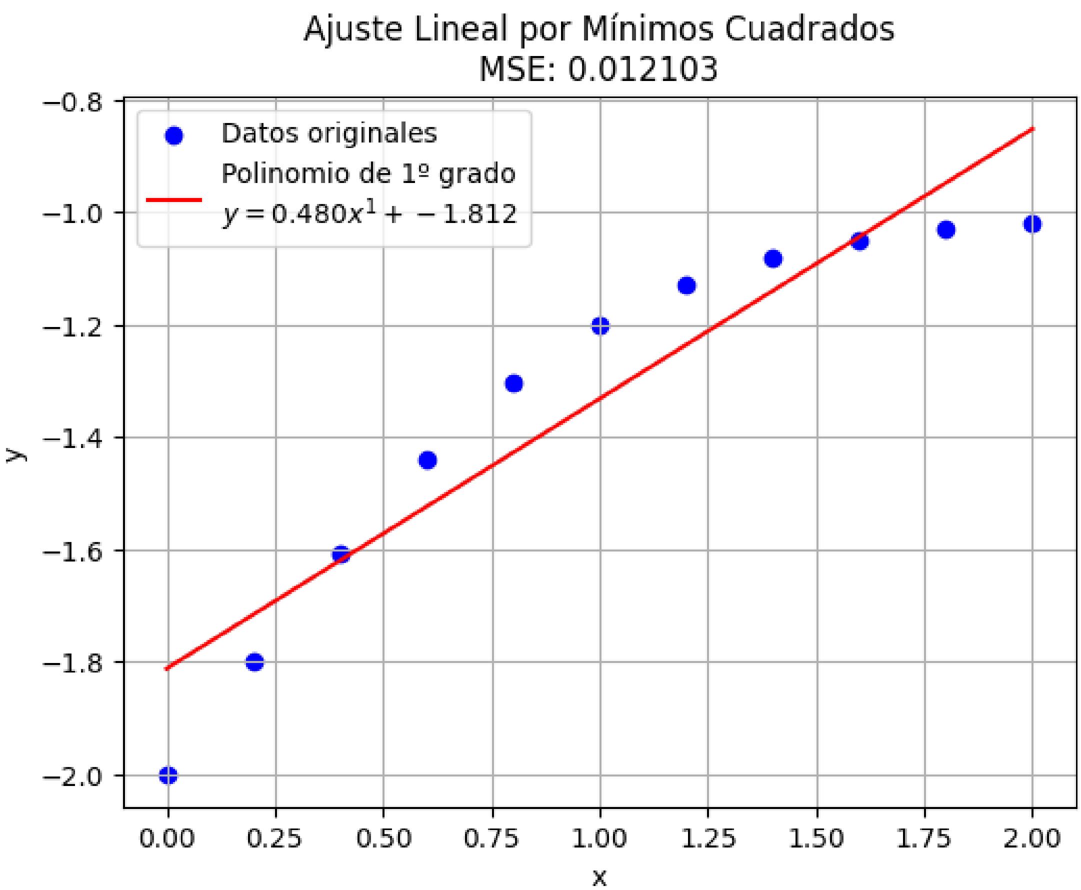
```
ts = np.array(ts)
error = np.abs(exact_sol(ts) - ys)

results = pd.DataFrame(
    {"t_i": ts, "ŷ_i": ys, "y_i = y(t_i)": exact_sol(ts), "|y_i - ŷ_i|": error}
)

print(results)
```

```
    t_i       ŷ_i  y_i = y(t_i)  |y_i - ŷ_i|
0   0.0 -2.000000     -2.000000     0.000000
1   0.2 -1.800000     -1.802625     0.002625
2   0.4 -1.608000     -1.620051     0.012051
3   0.6 -1.438733     -1.462950     0.024218
```

```
4   0.8 -1.301737     -1.335963     0.034226
5   1.0 -1.199251     -1.238406     0.039155
6   1.2 -1.127491     -1.166345     0.038854
7   1.4 -1.079745     -1.114648     0.034903
8   1.6 -1.049119     -1.078331     0.029212
9   1.8 -1.029954     -1.053194     0.023240
10  2.0 -1.018152     -1.035972     0.017821
```

Interpolación Lineal

```
a, b = ajuste_polinomio(ts, ys, 1, "Lineal")
```



1. $y(1.3)$

```python
x = 1.3

y_exact = exact_sol(x)

y_interpolate = f_lineal_interpolate(a, b, x)

print(f"Valor exacto en x = {x}: y = {y_exact:.6f}")
print(f"Valor interpolado en x = {x}: ŷ = {y_interpolate:.6f}")
print(f"Error absoluto: |y - ŷ| = {abs(y_exact - y_interpolate):.6f}")
```

```
Valor exacto en x = 1.3: y = -1.138277
Valor interpolado en x = 1.3: ŷ = -1.188040
Error absoluto: |y - ŷ| = 0.049763
```

2. $y(1.93)$

```python
x = 1.93

y_exact = exact_sol(x)

y_interpolate = f_lineal_interpolate(a, b, x)

print(f"Valor exacto en x = {x}: y = {y_exact:.6f}")
print(f"Valor interpolado en x = {x}: ŷ = {y_interpolate:.6f}")
print(f"Error absoluto: |y - ŷ| = {abs(y_exact - y_interpolate):.6f}")
```

```
Valor exacto en x = 1.93: y = -1.041267
Valor interpolado en x = 1.93: ŷ = -0.885689
Error absoluto: |y - ŷ| = 0.155578
```

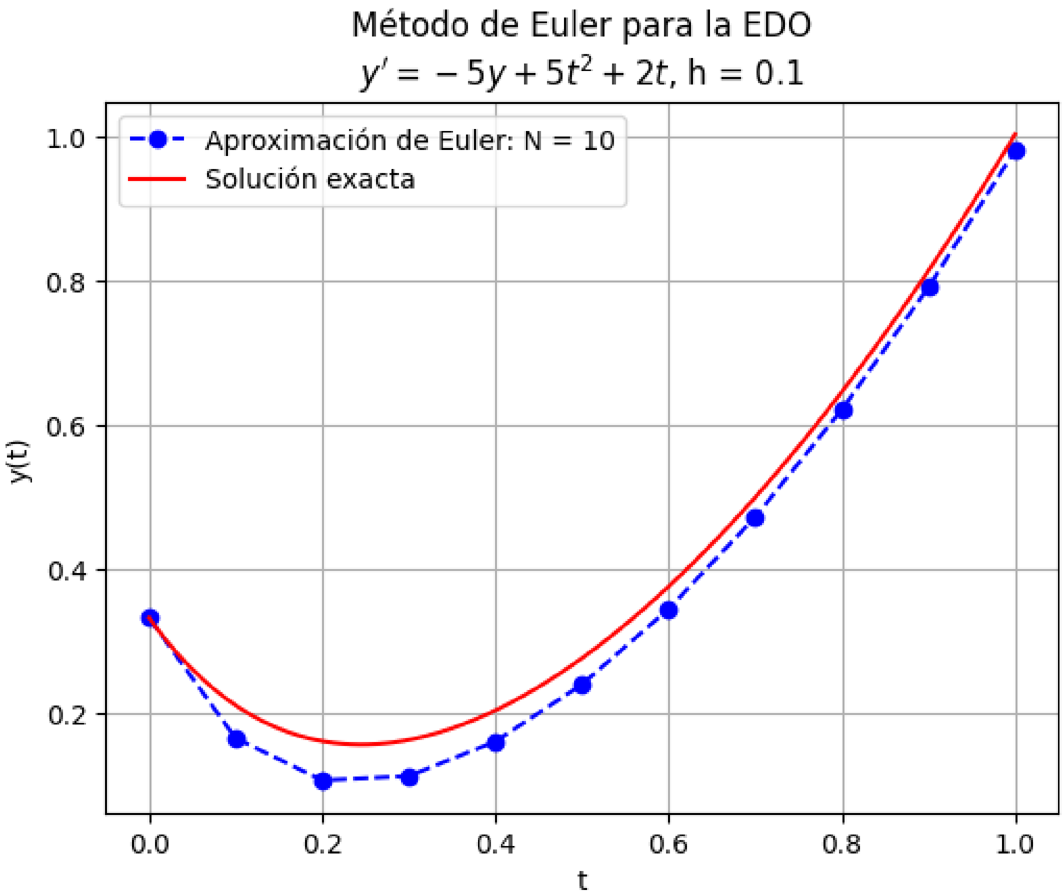
d.

$$y' = -5y + 5t^2 + 2t$$
$$0 \leq t \leq 1$$
$$y(0) = \frac{1}{3}$$
$$h = 0.1$$

Solución

$$y(t) = t^2 + \frac{1}{3}e^{-5t}$$

```
f = lambda t, y: -5 * y + 5 * t**2 + 2 * t

exact_sol = lambda t: t**2 + (1 / 3) * np.exp(-5 * t)

a, b = 0, 1
y_t0 = 1 / 3
N = 10

ys, ts, h = ODE_euler(a=a, b=b, f=f, y_t0=y_t0, N=N)

t_exact = np.linspace(a, b, 100)
y_exact = exact_sol(t_exact)

plt.plot(
    ts,
    ys,
    label="Aproximación de Euler: N = 10",
    marker="o",
    linestyle="--",
    color="blue",
)
plt.plot(t_exact, y_exact, label="Solución exacta", linestyle="-", color="red")
plt.xlabel("t")
plt.ylabel("y(t)")
plt.title(f"Método de Euler para la EDO\n$y' = -5y + 5t^2 + 2t$, h = {h}")
plt.legend()
plt.grid()
plt.show()
```



Error Real

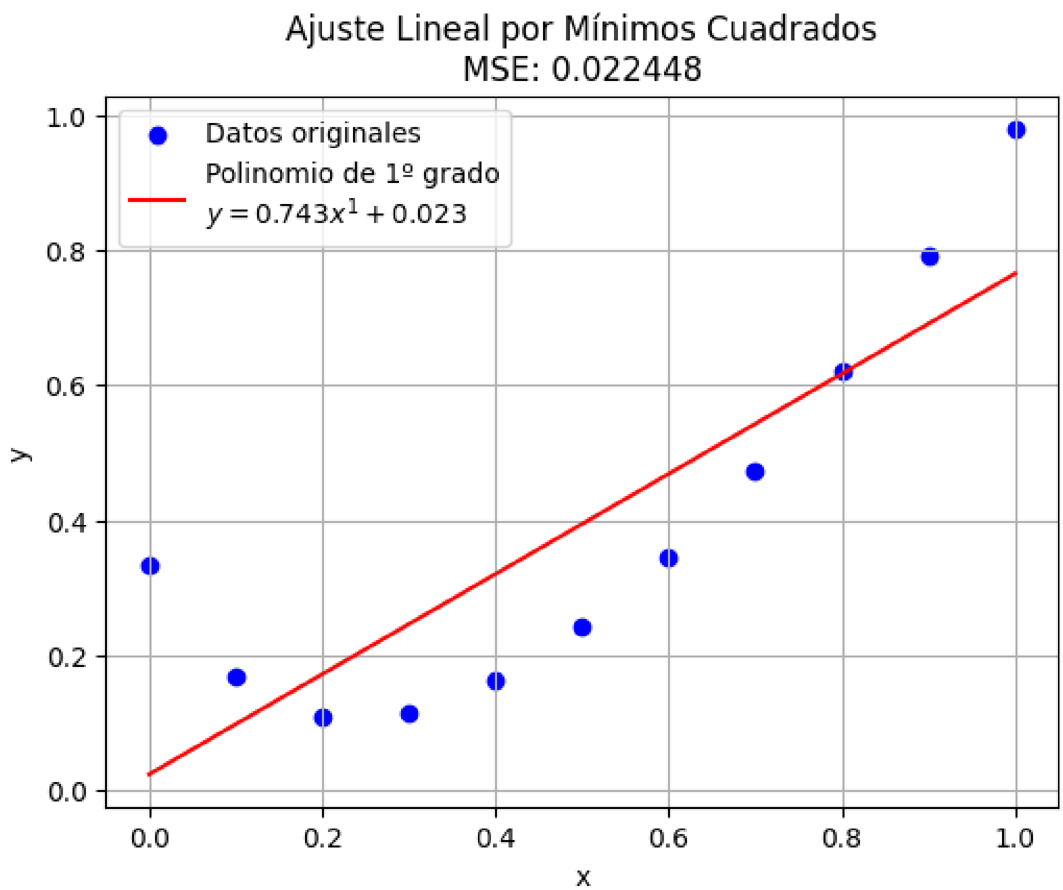
```
ts = np.array(ts)
error = np.abs(exact_sol(ts) - ys)

results = pd.DataFrame(
    {"t_i": ts, "ŷ_i": ys, "y_i = y(t_i)": exact_sol(ts), "|y_i - ŷ_i|": error}
)

print(results)
```

```
    t_i       ŷ_i  y_i = y(t_i)  |y_i - ŷ_i|
0   0.0  0.333333      0.333333     0.000000
1   0.1  0.166667      0.212177     0.045510
2   0.2  0.108333      0.162626     0.054293
3   0.3  0.114167      0.164377     0.050210
```

4	0.4	0.162083	0.205112	0.043028
5	0.5	0.241042	0.277362	0.036320
6	0.6	0.345521	0.376596	0.031075
7	0.7	0.472760	0.500066	0.027305
8	0.8	0.621380	0.646105	0.024725
9	0.9	0.790690	0.813703	0.023013
10	1.0	0.980345	1.002246	0.021901

Interpolación Lineal

```
a, b = ajuste_polinomio(ts, ys, 1, "Lineal")
```



1. $y(0.54)$

```python
x = 0.54

y_exact = exact_sol(x)

y_interpolate = f_lineal_interpolate(a, b, x)

print(f"Valor exacto en x = {x}: y = {y_exact:.6f}")
print(f"Valor interpolado en x = {x}: ŷ = {y_interpolate:.6f}")
print(f"Error absoluto: |y - ŷ| = {abs(y_exact - y_interpolate):.6f}")
```

```
Valor exacto en x = 0.54: y = 0.314002
Valor interpolado en x = 0.54: ŷ = 0.423923
Error absoluto: |y - ŷ| = 0.109922
```

2. $y(0.94)$

```python
x = 0.94

y_exact = exact_sol(x)

y_interpolate = f_lineal_interpolate(a, b, x)

print(f"Valor exacto en x = {x}: y = {y_exact:.6f}")
print(f"Valor interpolado en x = {x}: ŷ = {y_interpolate:.6f}")
print(f"Error absoluto: |y - ŷ| = {abs(y_exact - y_interpolate):.6f}")
```

```
Valor exacto en x = 0.94: y = 0.886632
Valor interpolado en x = 0.94: ŷ = 0.721048
Error absoluto: |y - ŷ| = 0.165584
```

GitHub: Tarea12 - @mateobtw18