

Nonlinear Voter Model

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School on Applications of Nonlinear Systems to Socio-Economic Complexity

Project presentation

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How many physicists does it take to change a ~~lightbulb~~ model?



José



Iago

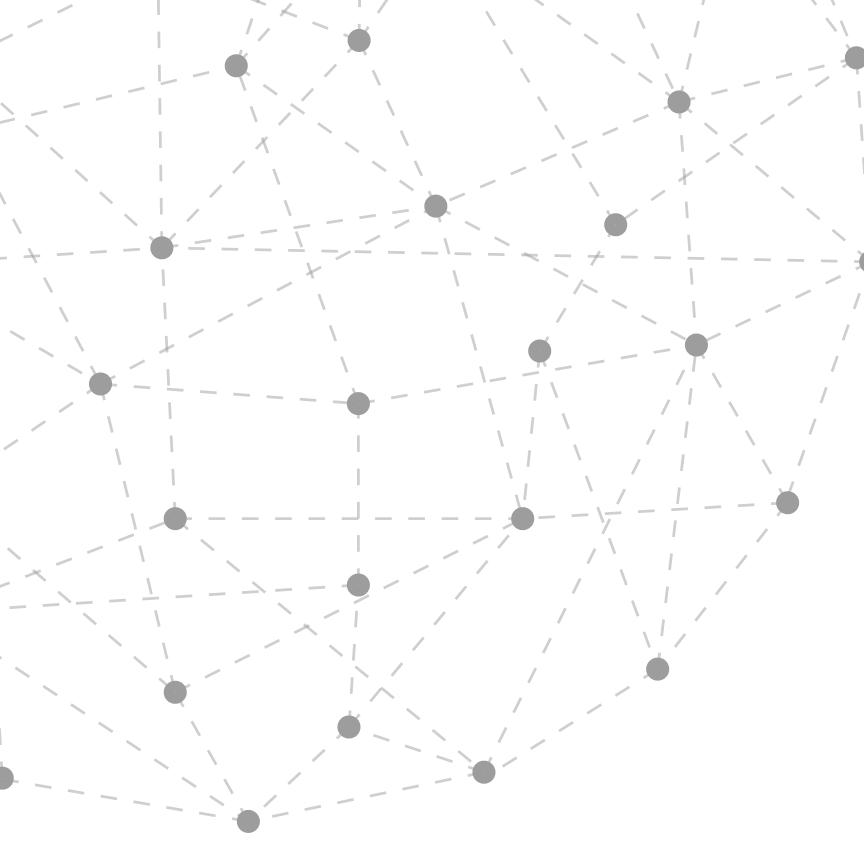


Mateo



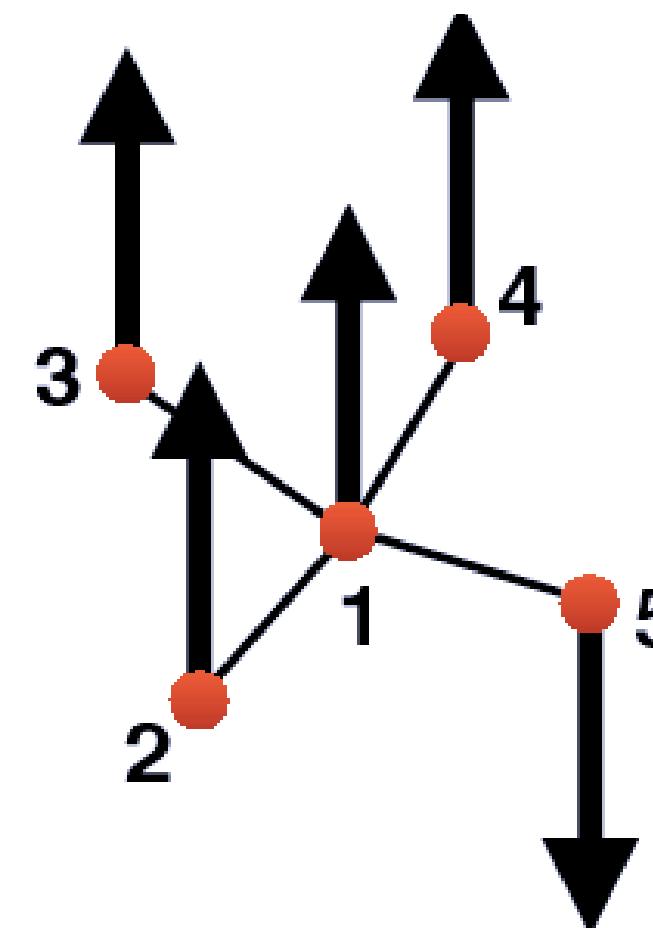
Wesley

Ans: only one, just implement it wrong!

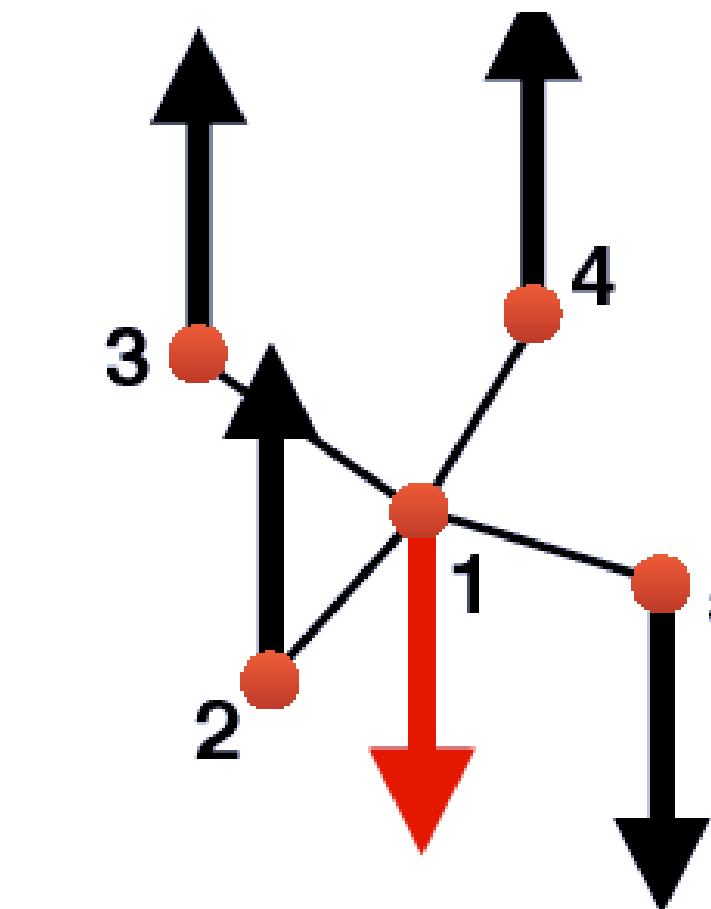


Voter model dynamics

Initial



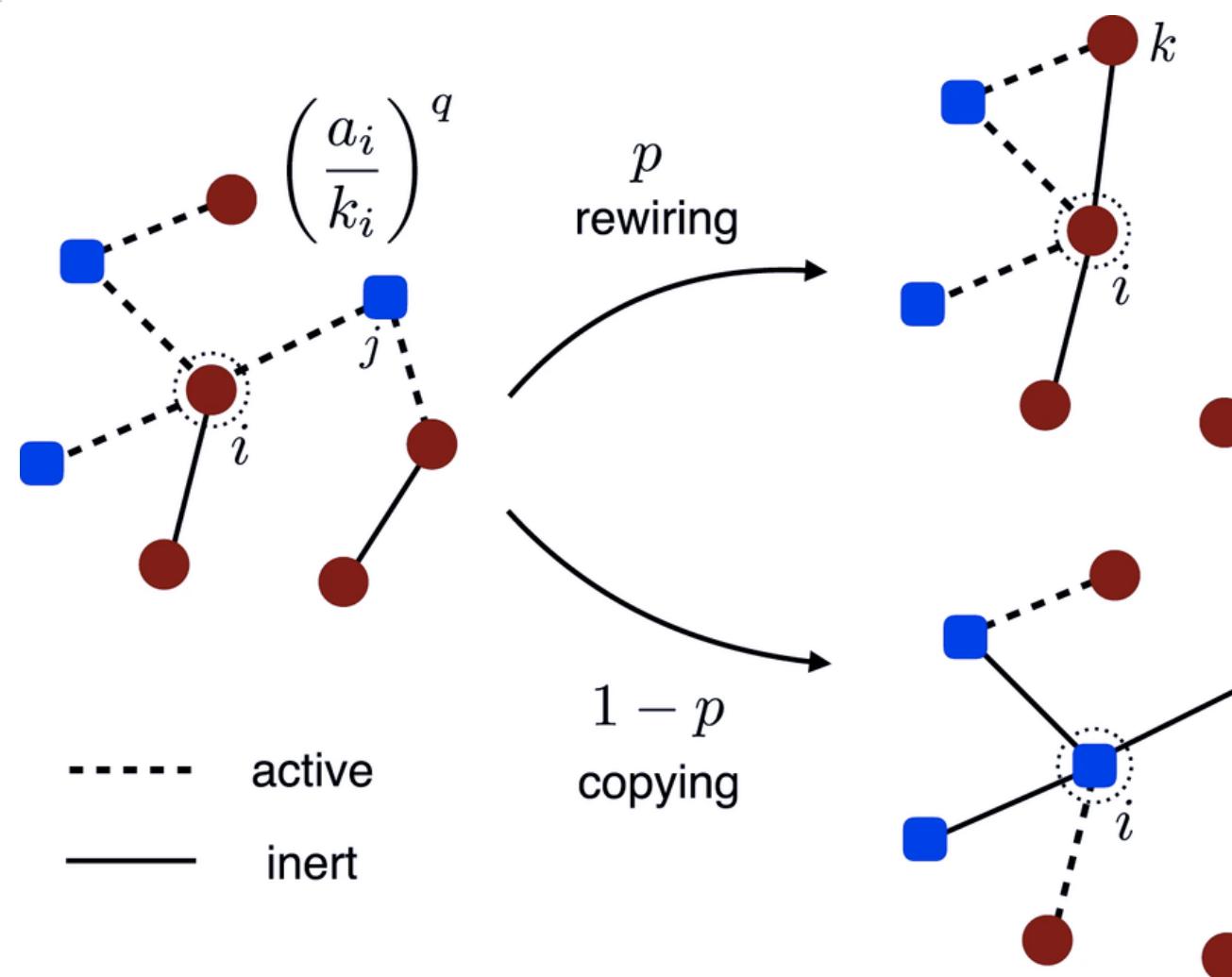
Final



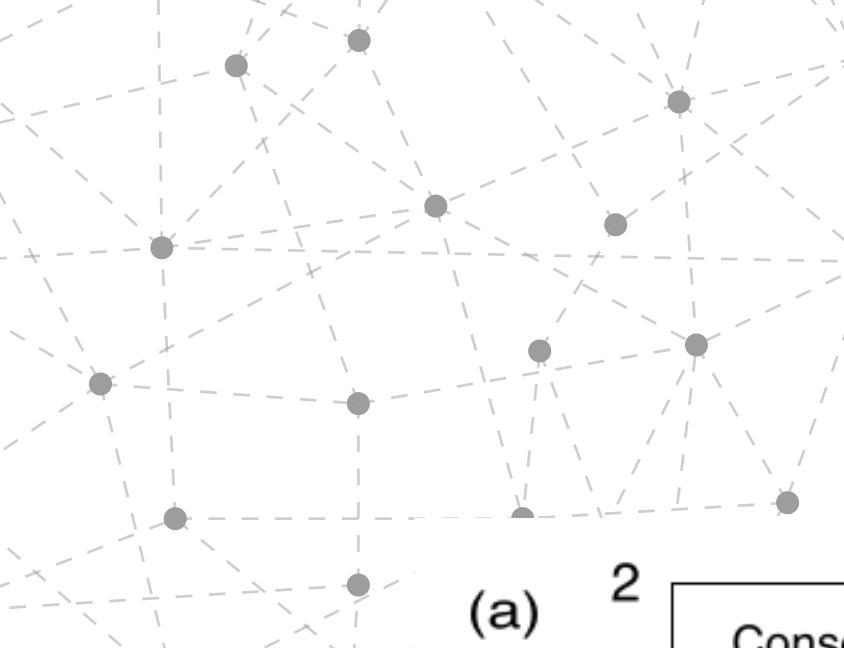
Coevolving Nonlinear Voter Model

Fragmentation transitions in a coevolving nonlinear voter model

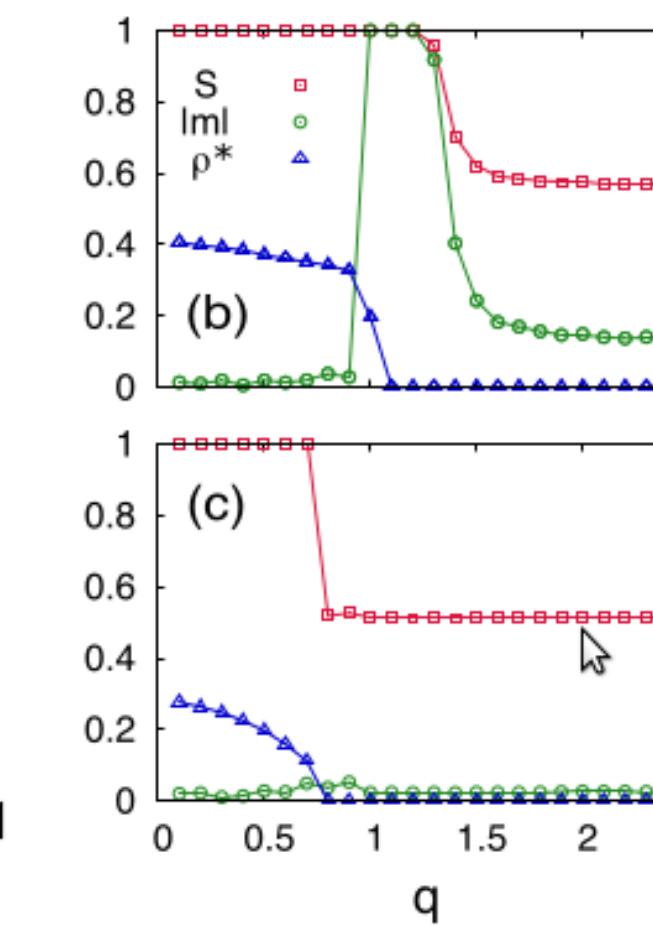
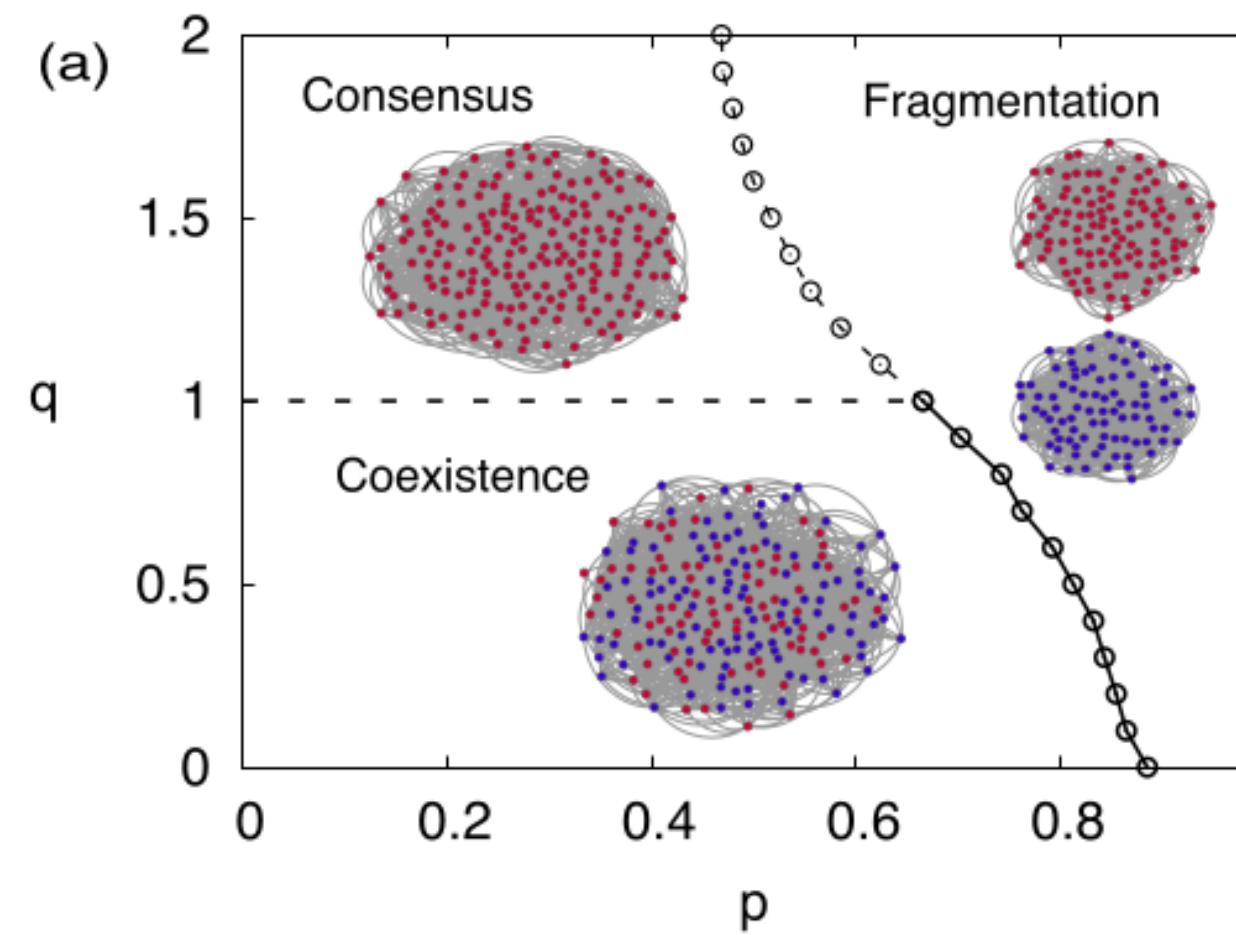
Byungjoon Min & Maxi San Miguel



Parameter	Meaning
p	Rate of rewiring or plasticity
q	Degree of nonlinearity
a_i	Number of active links connecting to i
k_i	Degree of i



Phase Diagram (paper)

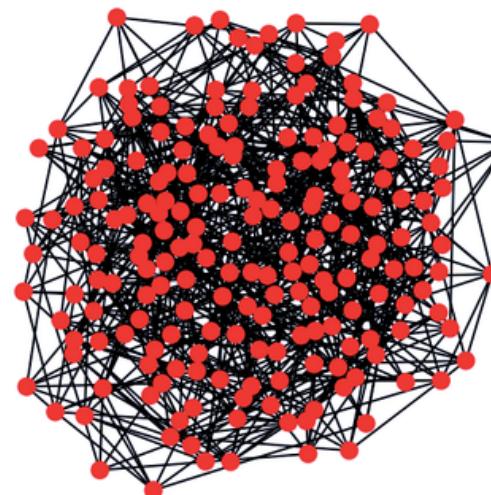


Parameter	Meaning
S	Size of the giant component
m	magnetization
ρ	density of active links

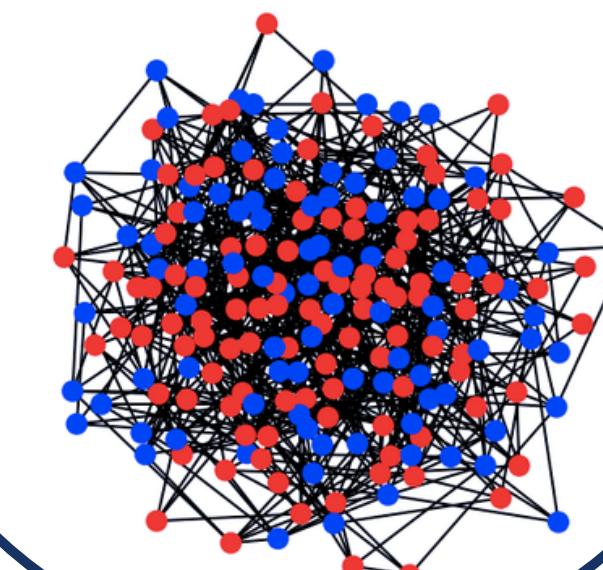
$$|m| = 0 \begin{cases} \rho \rightarrow 0 : & \text{fragmentation} \\ \rho > 0 : & \text{coexistence} \end{cases}$$

Our results in a regular random network with $k = 8$

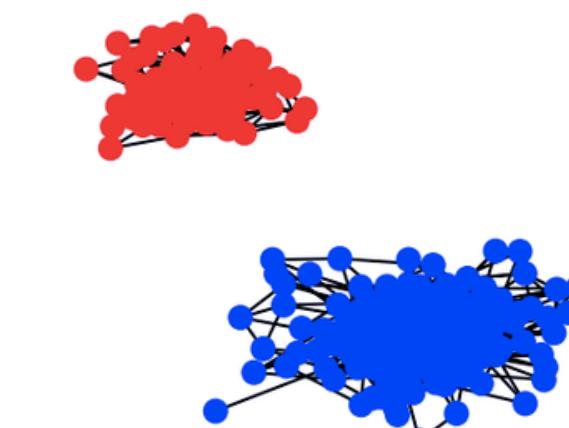
$(p,q) = (0.2, 2)$

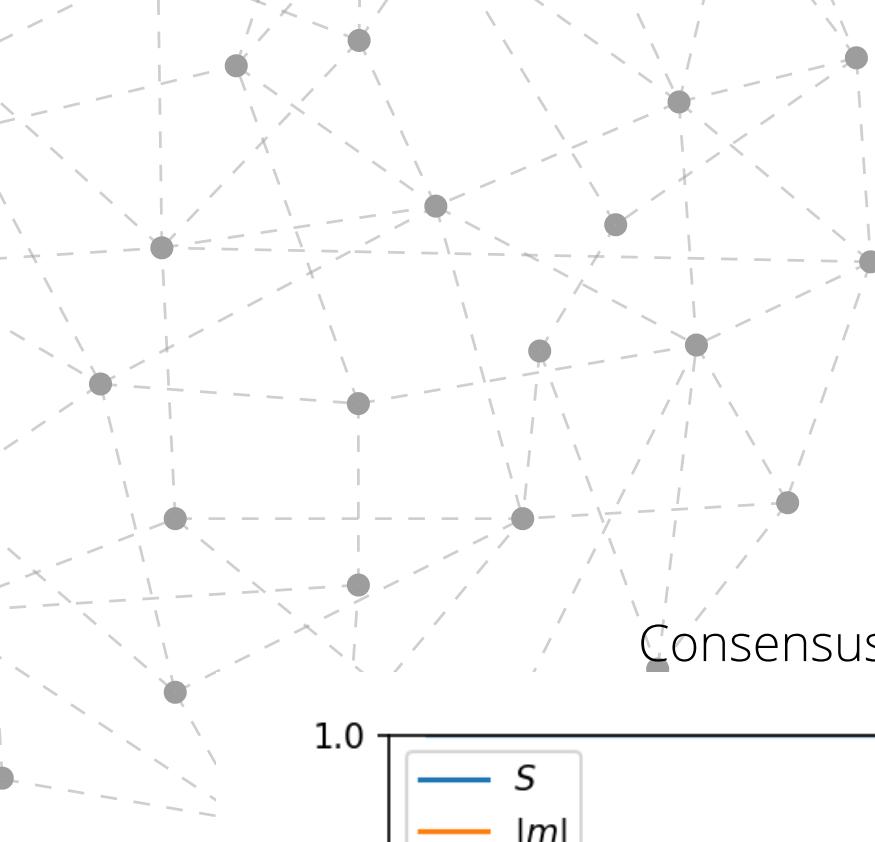


$(p,q) = (0.2, 0.5)$

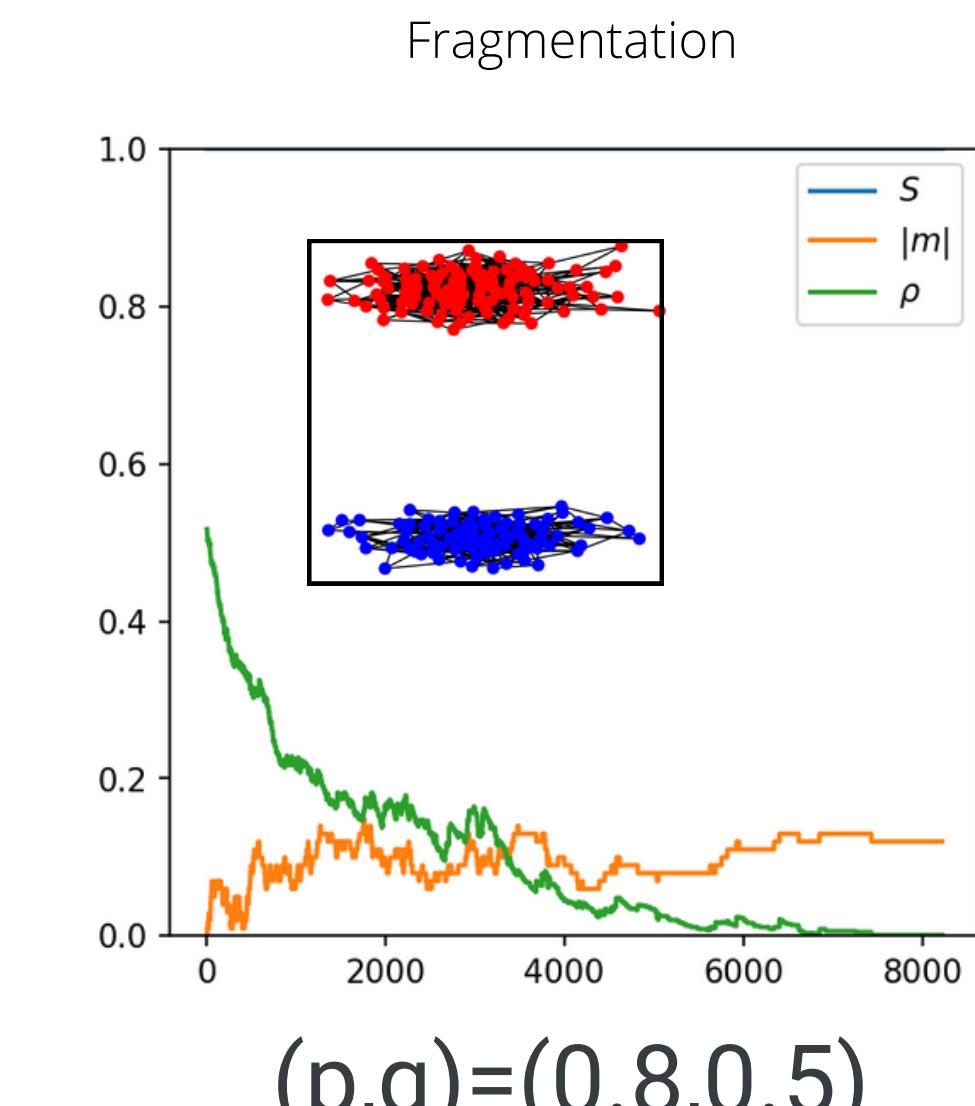
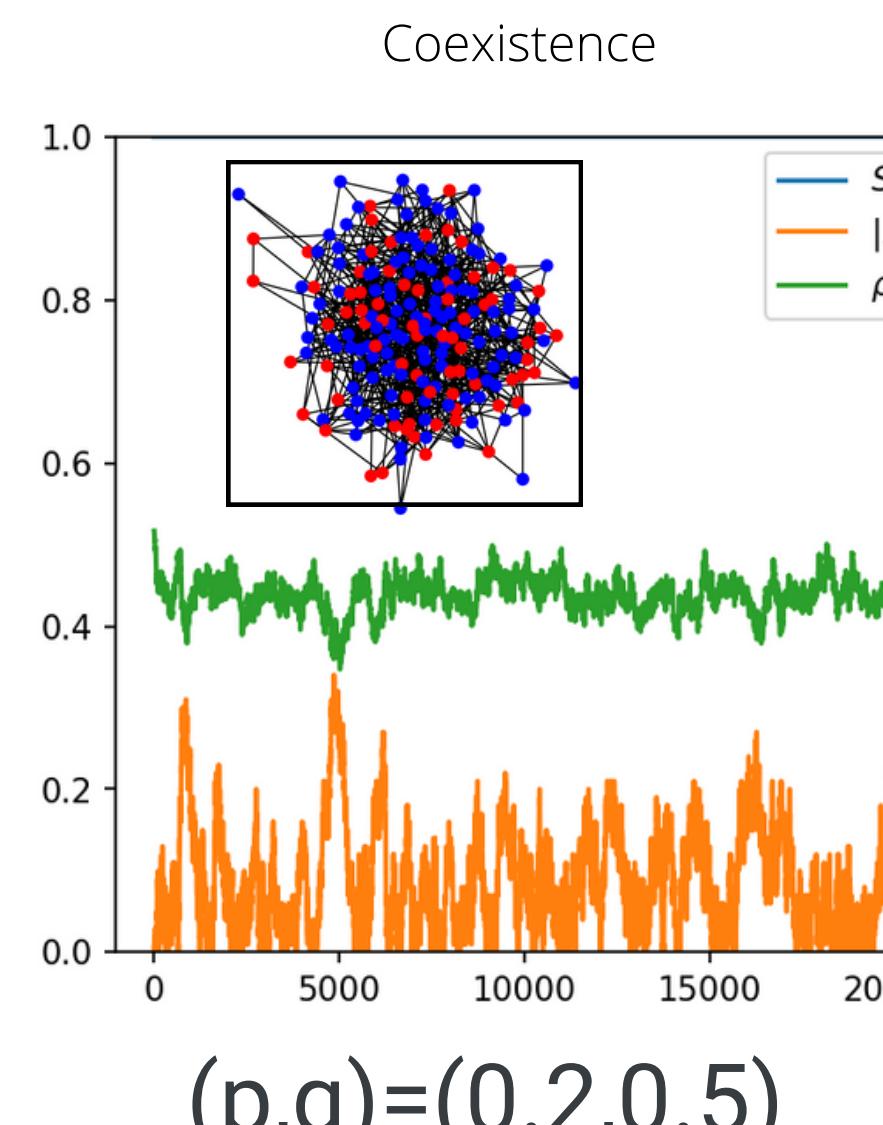
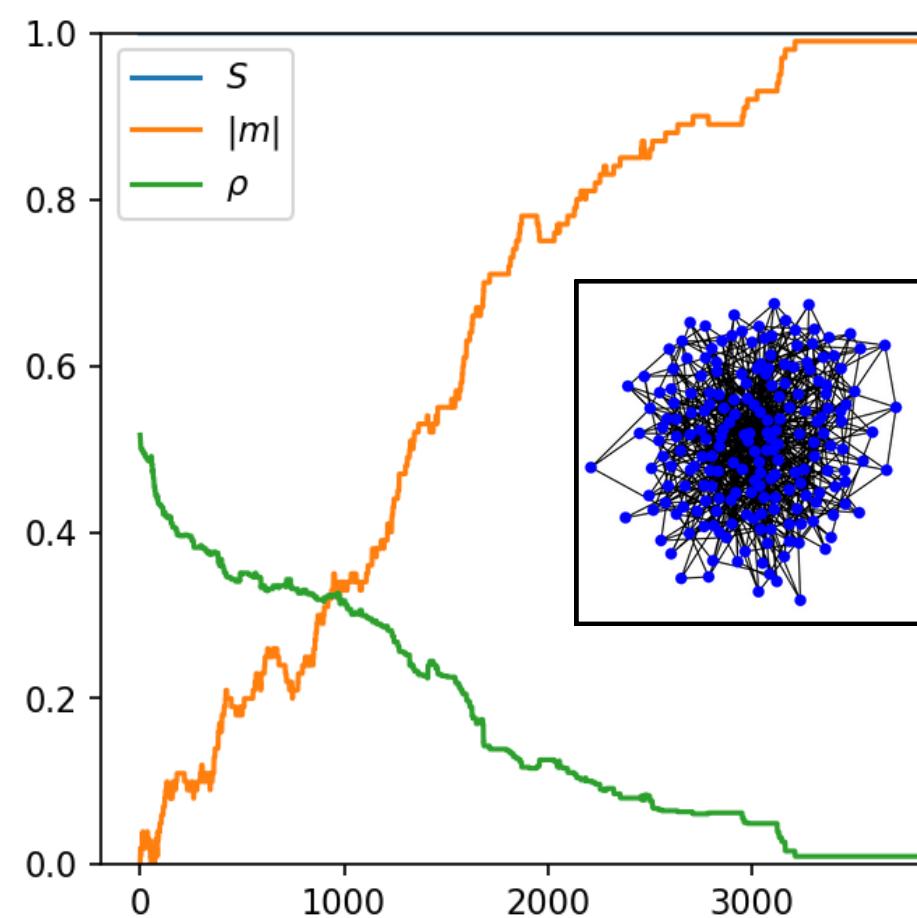


$(p,q) = (0.8, 0.5)$

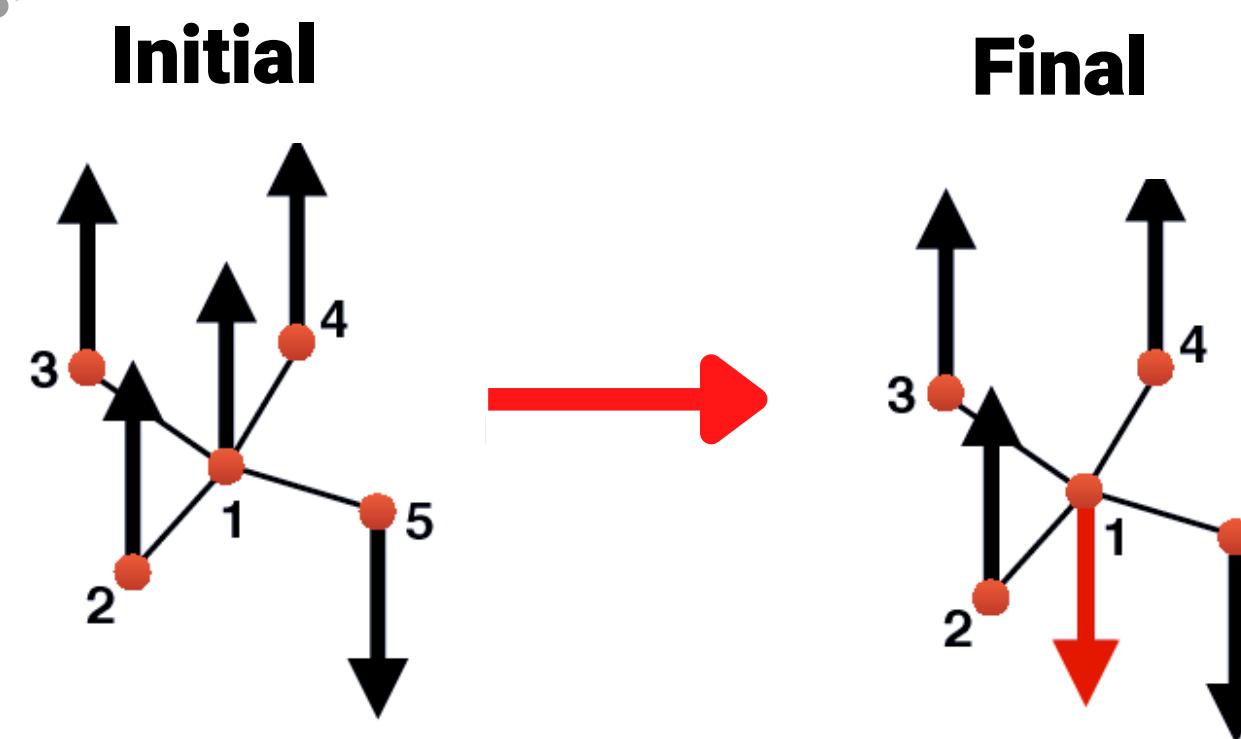




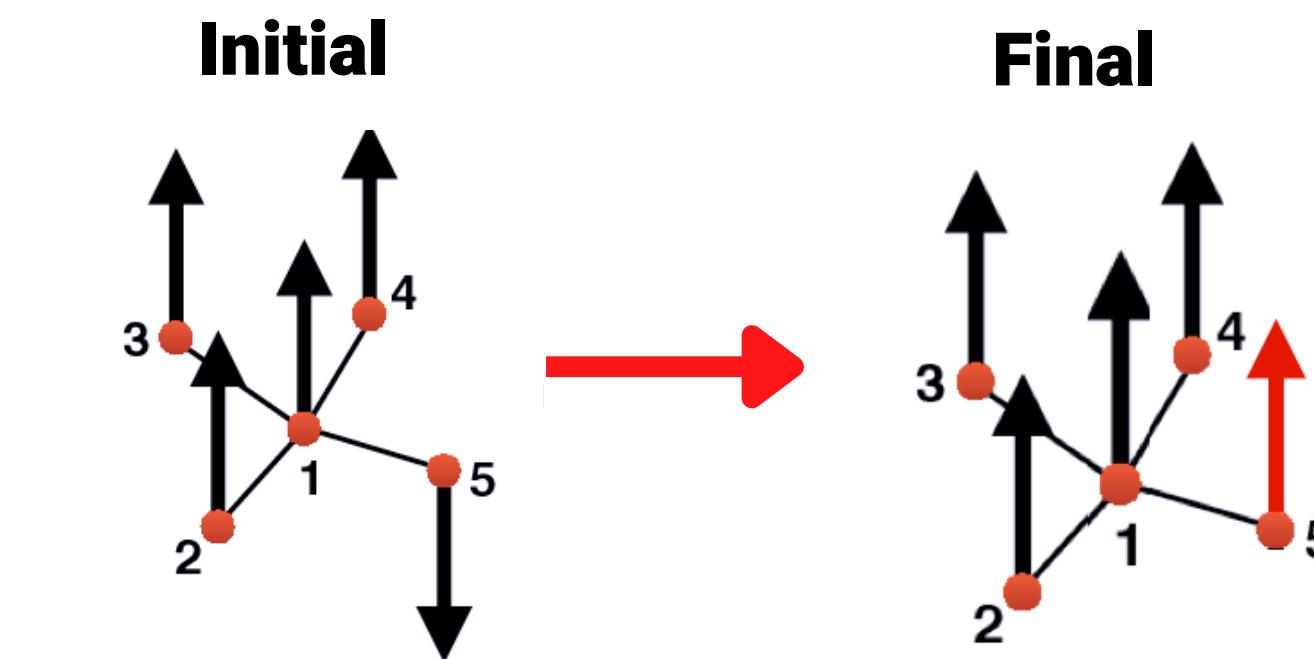
Results in a Barabasi-Albert network



Changing the order dynamics: reverse voter model



Original Model



Reverse Model

Motivation

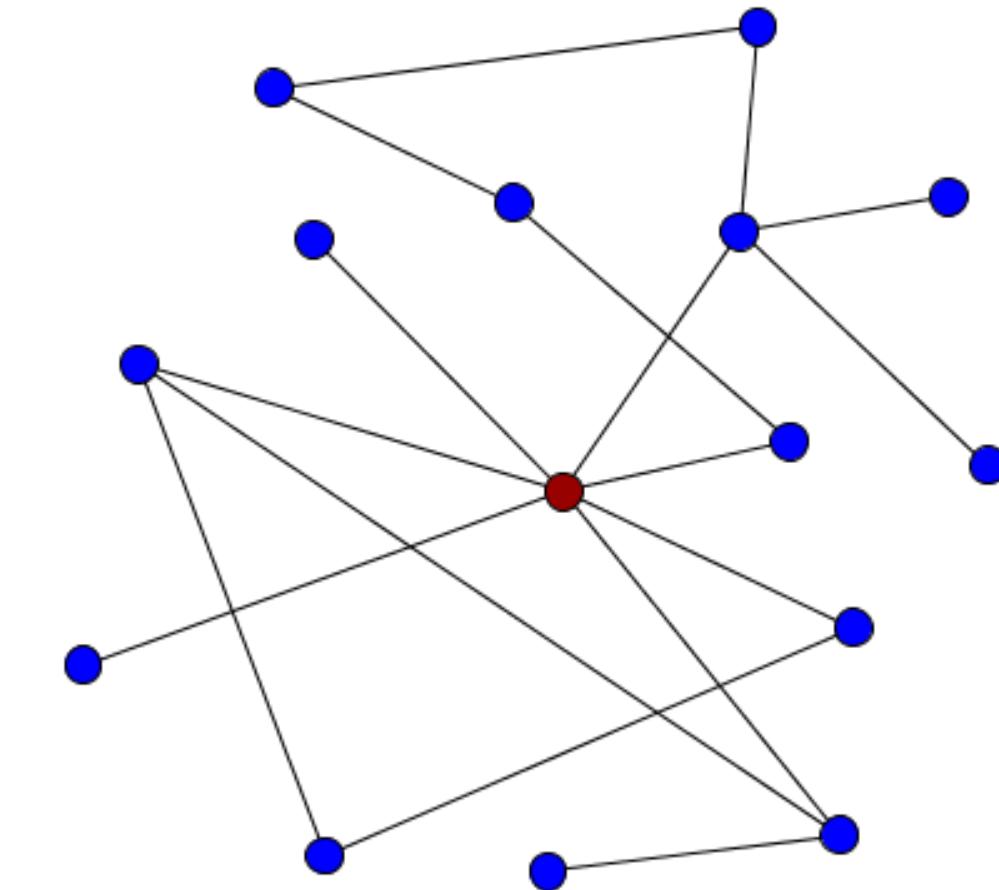
Effect of network topology on the ordering dynamics of voter models

Claudio Castellano

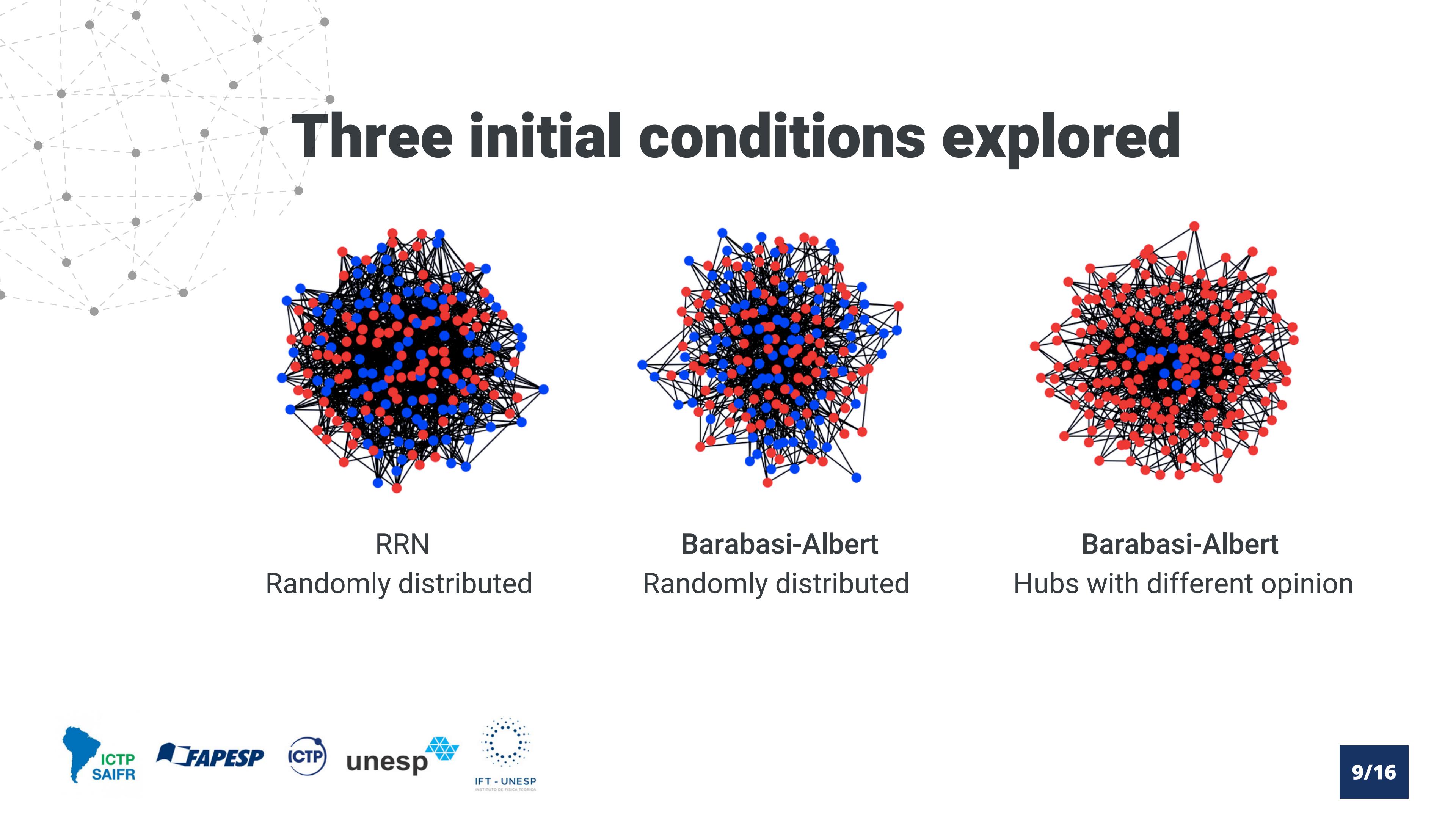
Dipartimento di Fisica, Università di Roma “La Sapienza” and Center for Statistical Mechanics and Complexity, INFN Unità Roma 1, P.le A. Moro 2, 00185 Roma, Italy

Abstract. We introduce and study the reverse voter model, a dynamics for spin variables similar to the well-known voter dynamics. The difference is in the way neighbors influence each other: once a node is selected and one among its neighbors chosen, the neighbor is made equal to the selected node, while in the usual voter dynamics the update goes in the opposite direction. The reverse voter dynamics is studied analytically, showing that on networks with degree distribution decaying as $k^{-\nu}$, the time to reach consensus is linear in the system size N for all $\nu > 2$. The consensus time for link-update voter dynamics is computed as well. We verify the results numerically on a class of uncorrelated scale-free graphs.

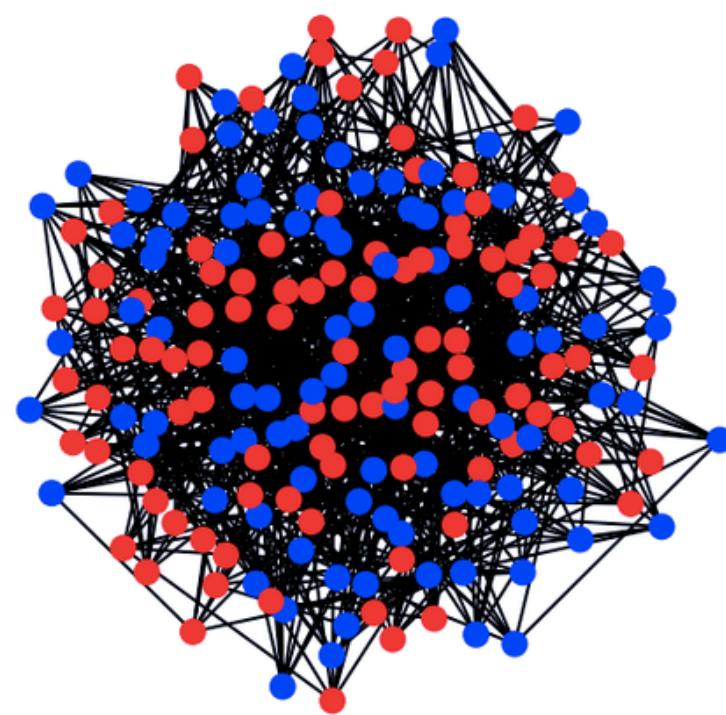
Claudio Castellano, AIP Conference Proceedings 779, 114-120 (2005)
<https://doi.org/10.1063/1.2008600>



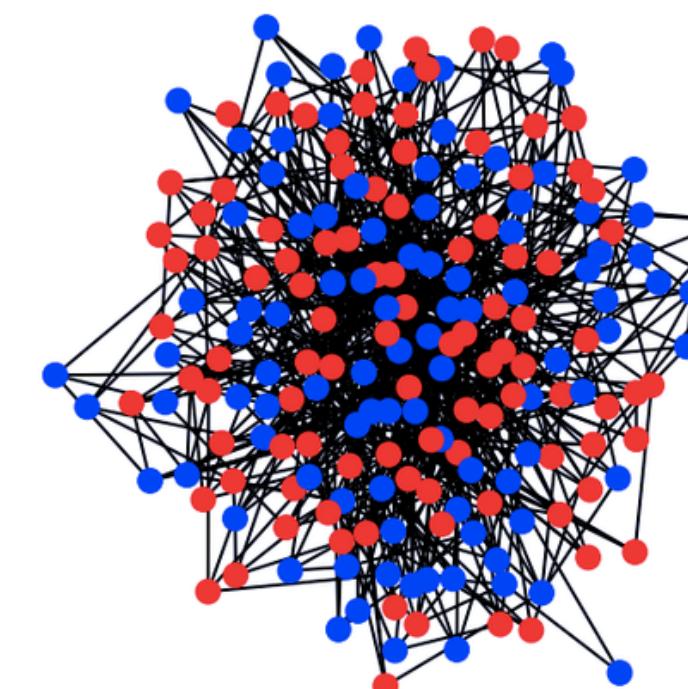
Will the presence of hubs favor the order in the modified coevolving model?



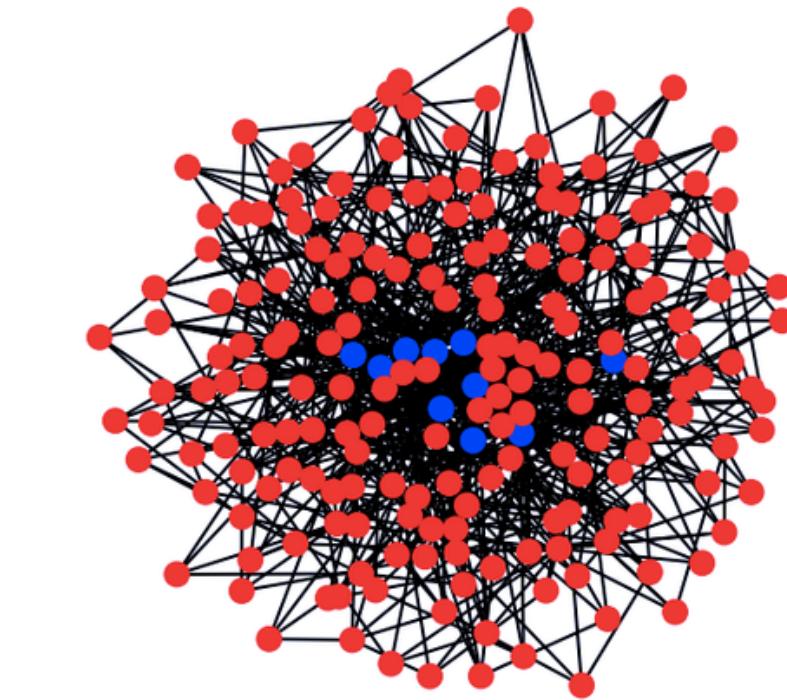
Three initial conditions explored



RRN
Randomly distributed

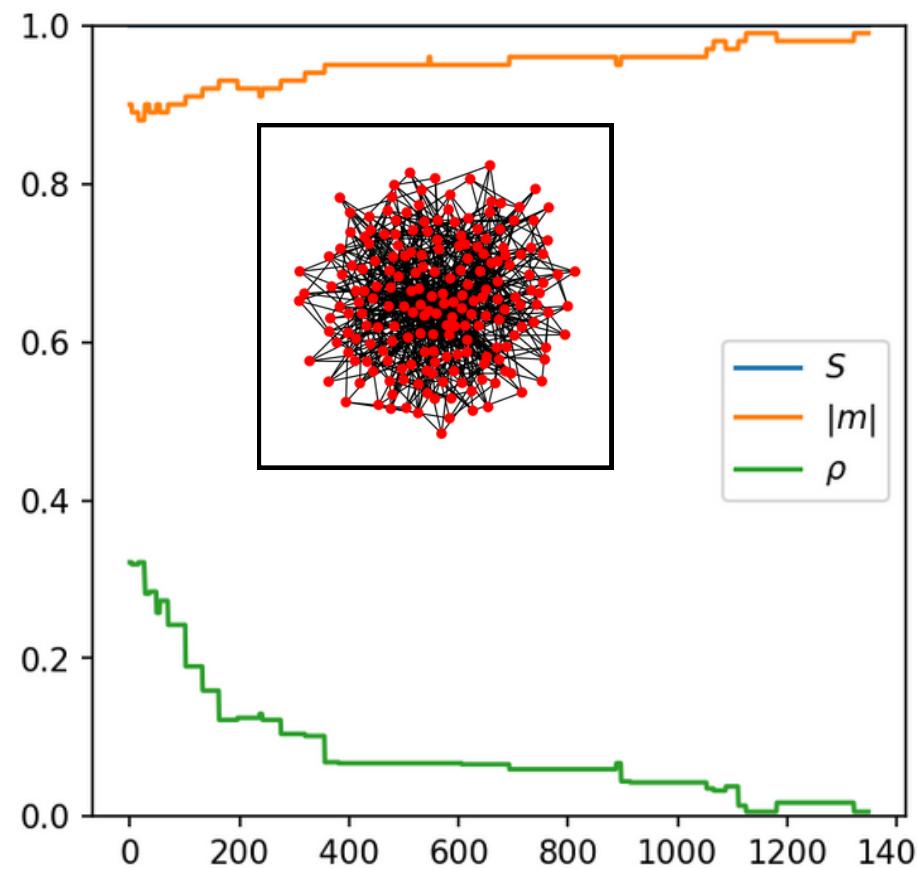
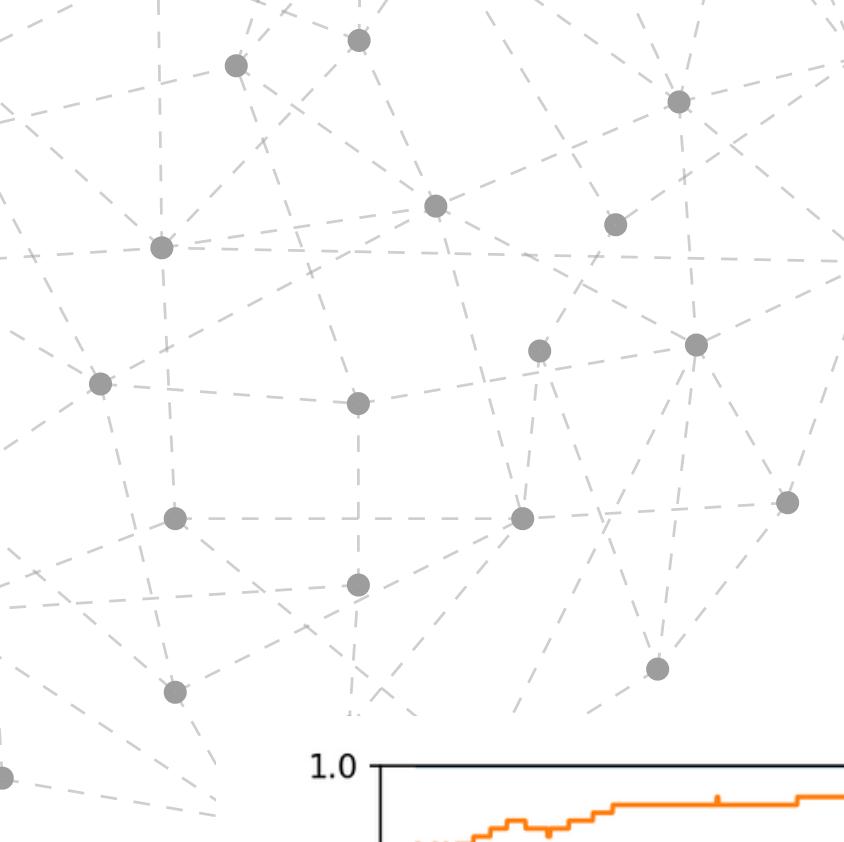


Barabasi-Albert
Randomly distributed

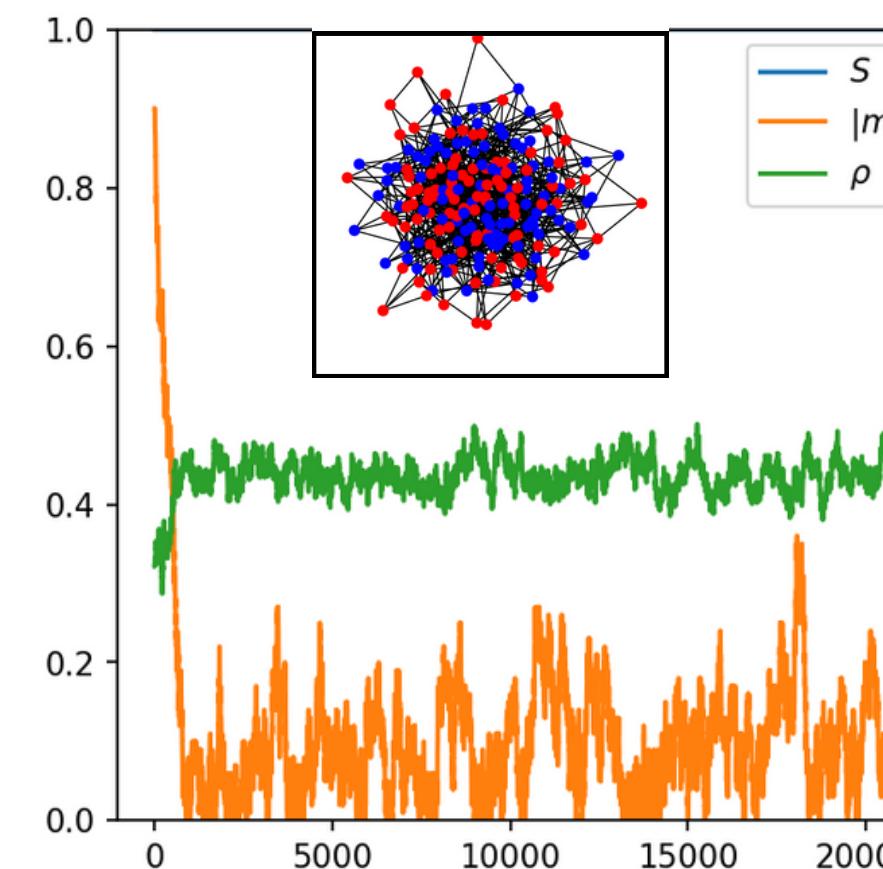


Barabasi-Albert
Hubs with different opinion

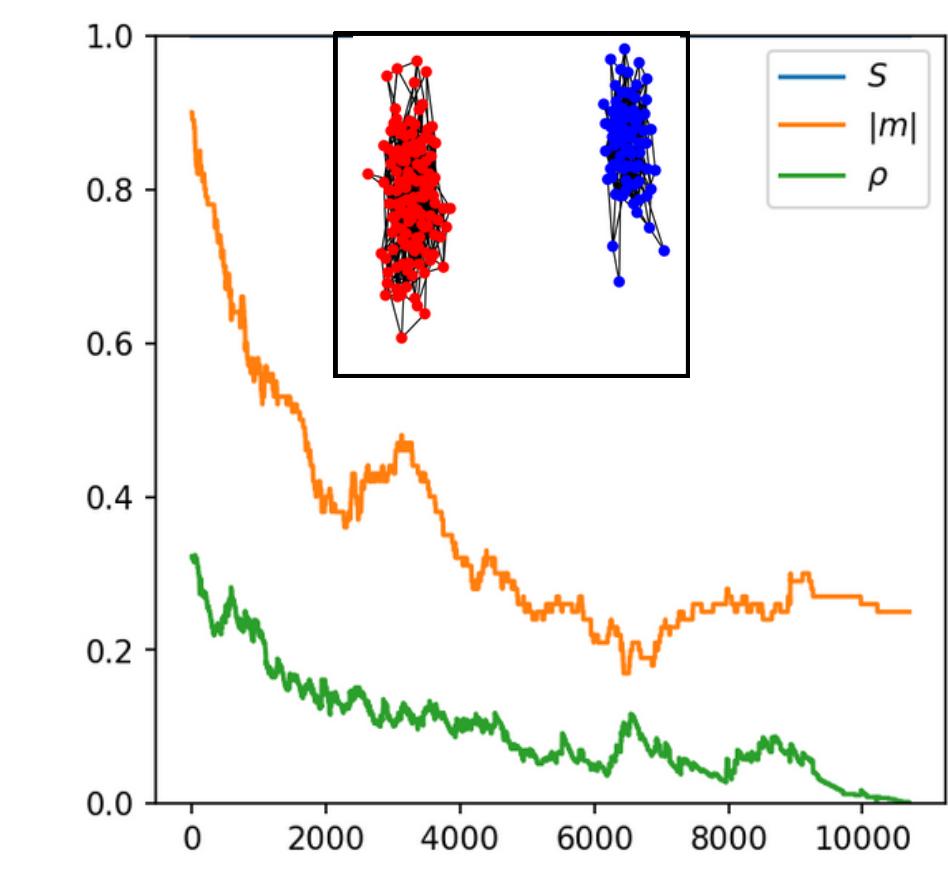
Original Model in the Barabasi-Albert network, with the hubs having a different opinion (blue)



$(p,q)=(0.2,2)$

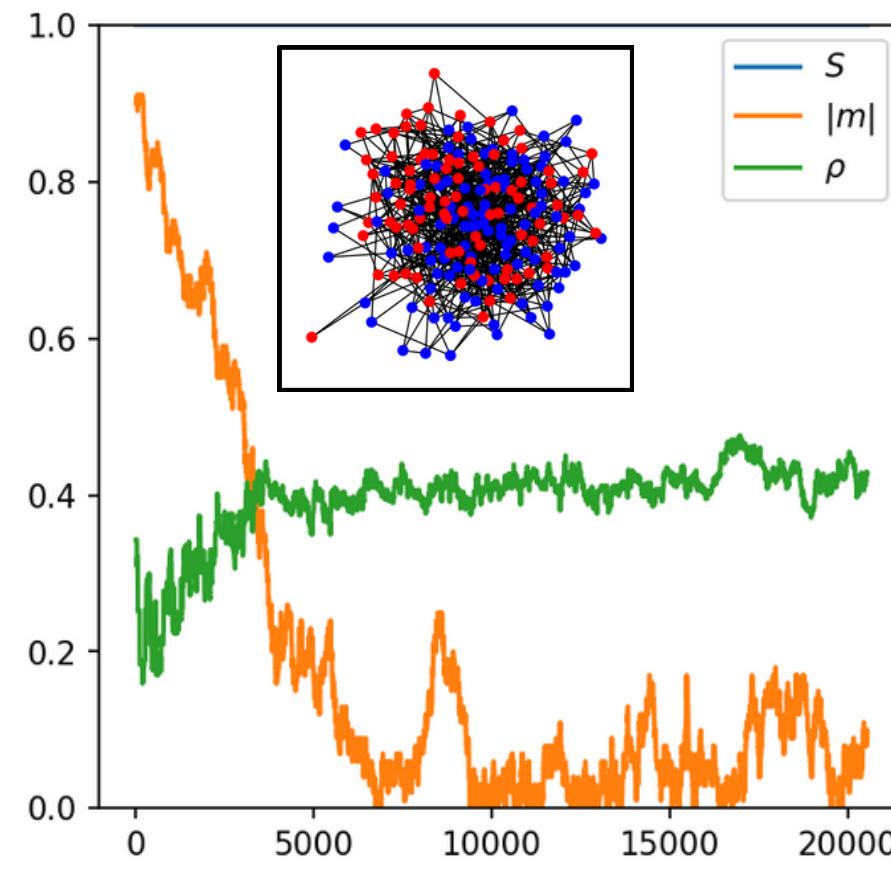
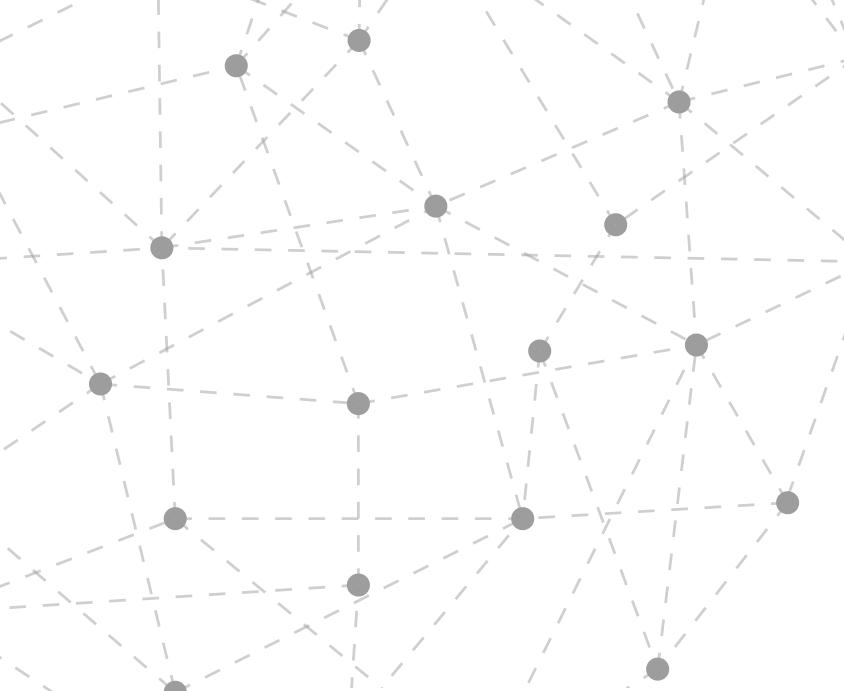


$(p,q)=(0.2,0.5)$

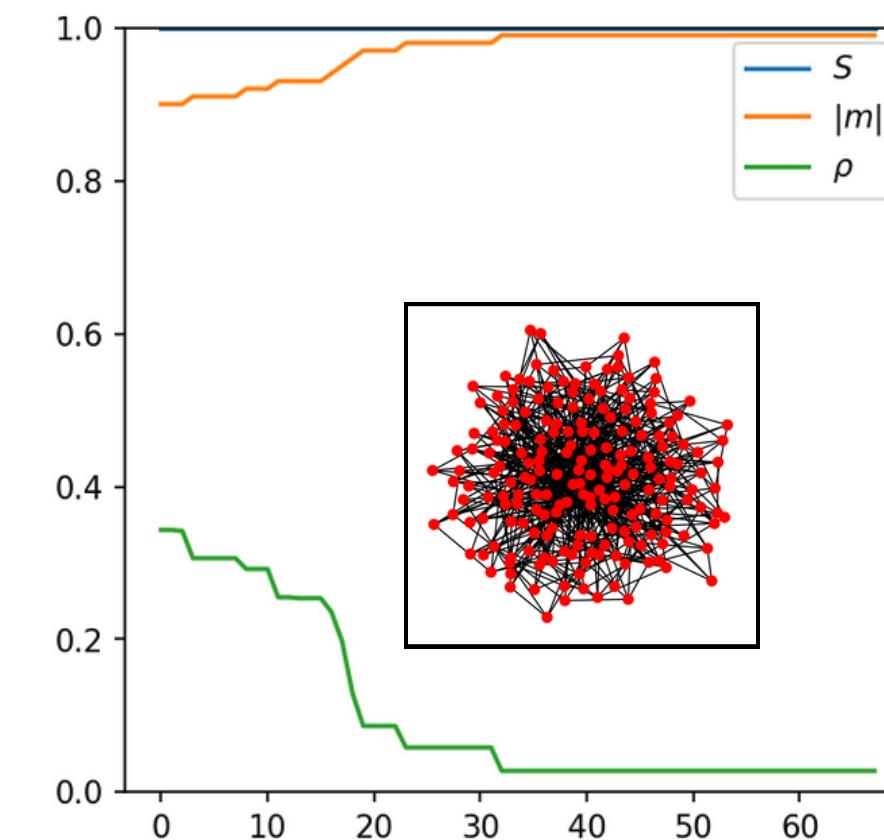


$(p,q)=(0.8,0.5)$

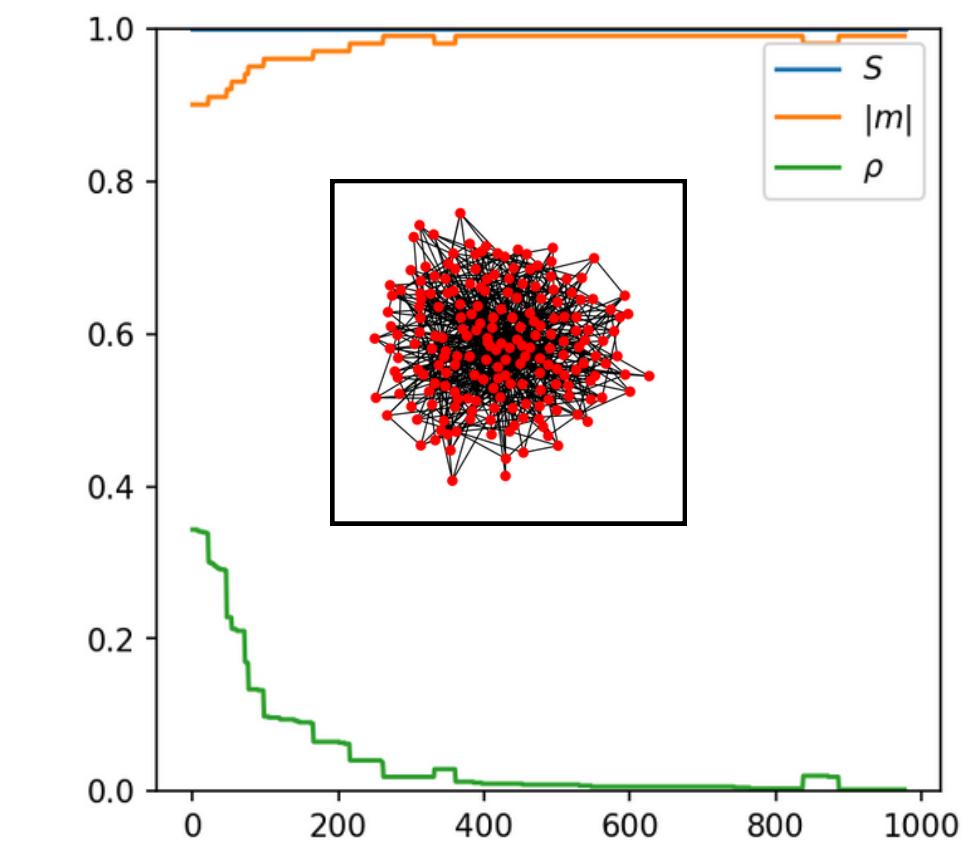
Reverse Model in the Barabasi-Albert network, with the hubs having a different opinion (blue)



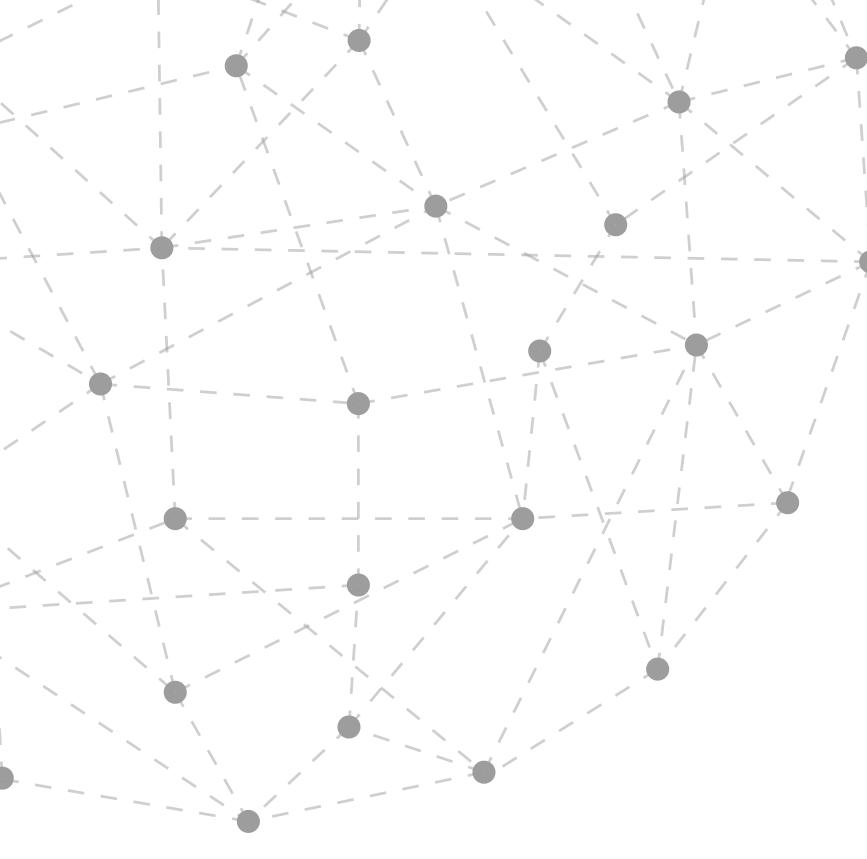
$(p,q)=(0.2,2)$



$(p,q)=(0.2,0.5)$

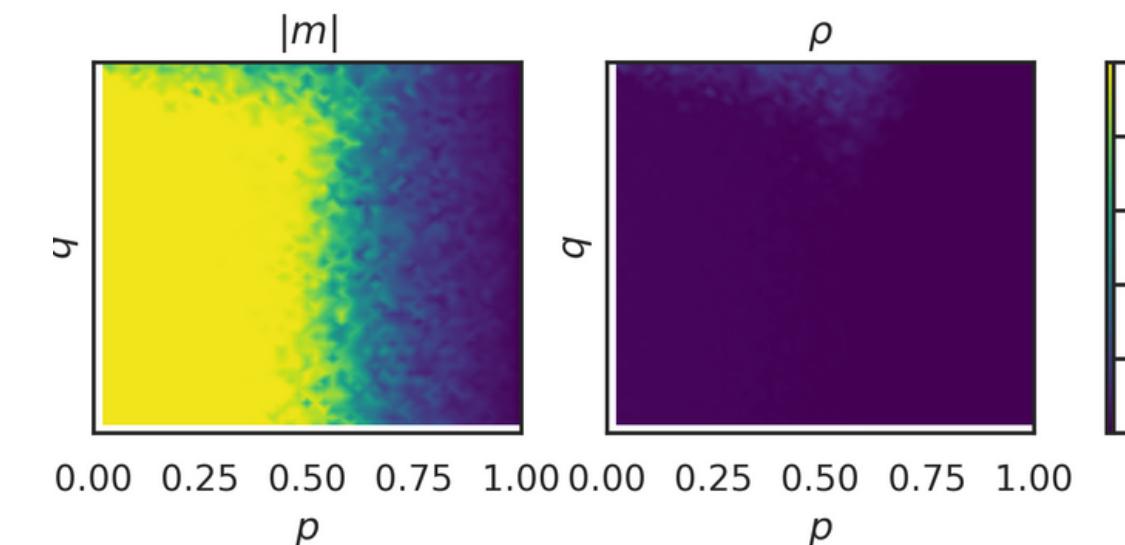


$(p,q)=(0.8,0.5)$

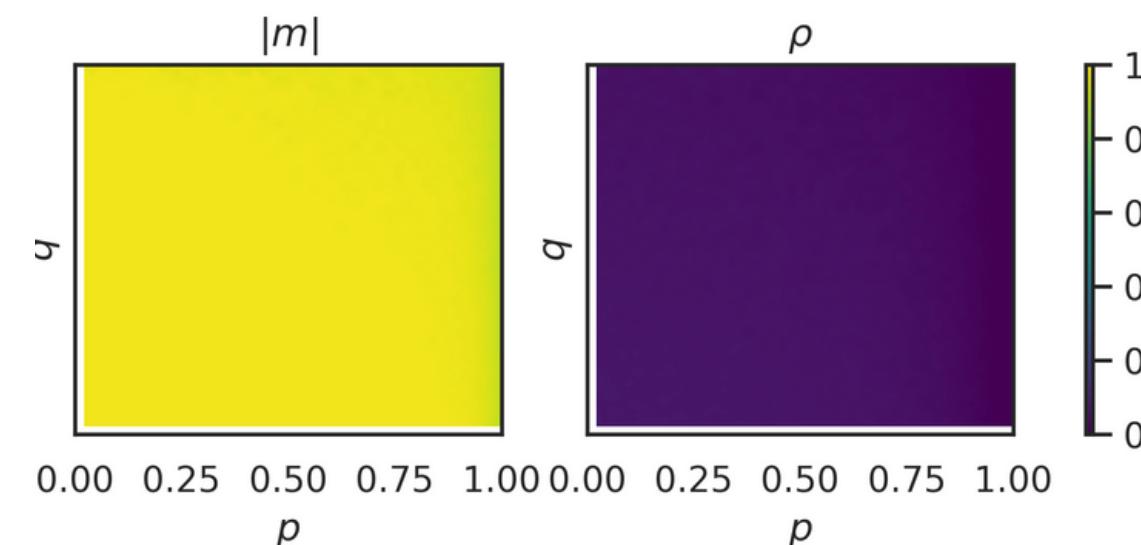


A new phase diagram for the Reverse Voter Model

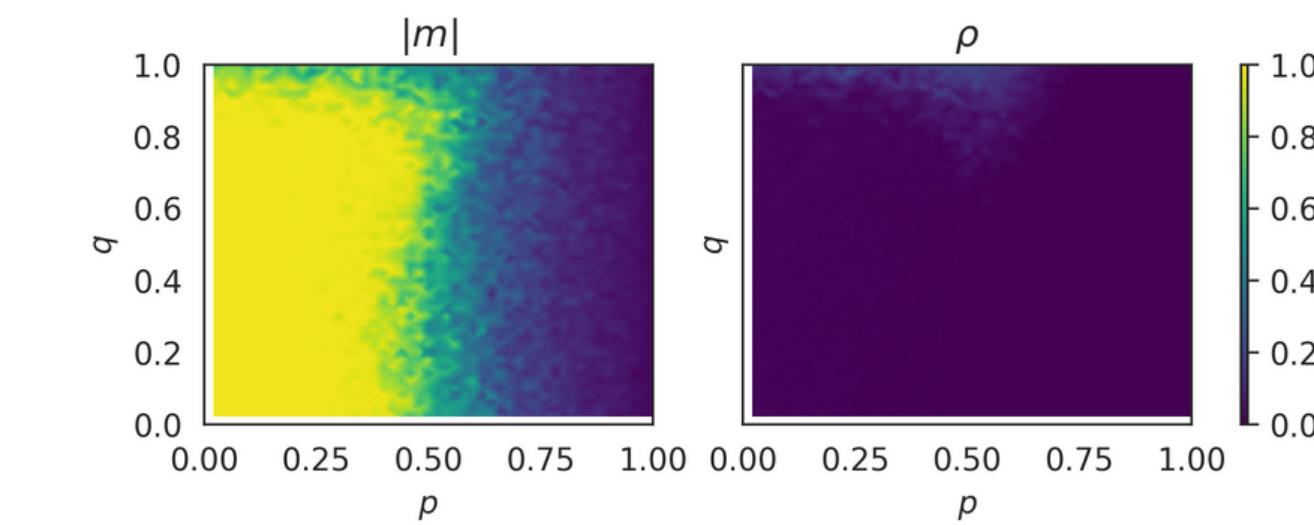
RNN



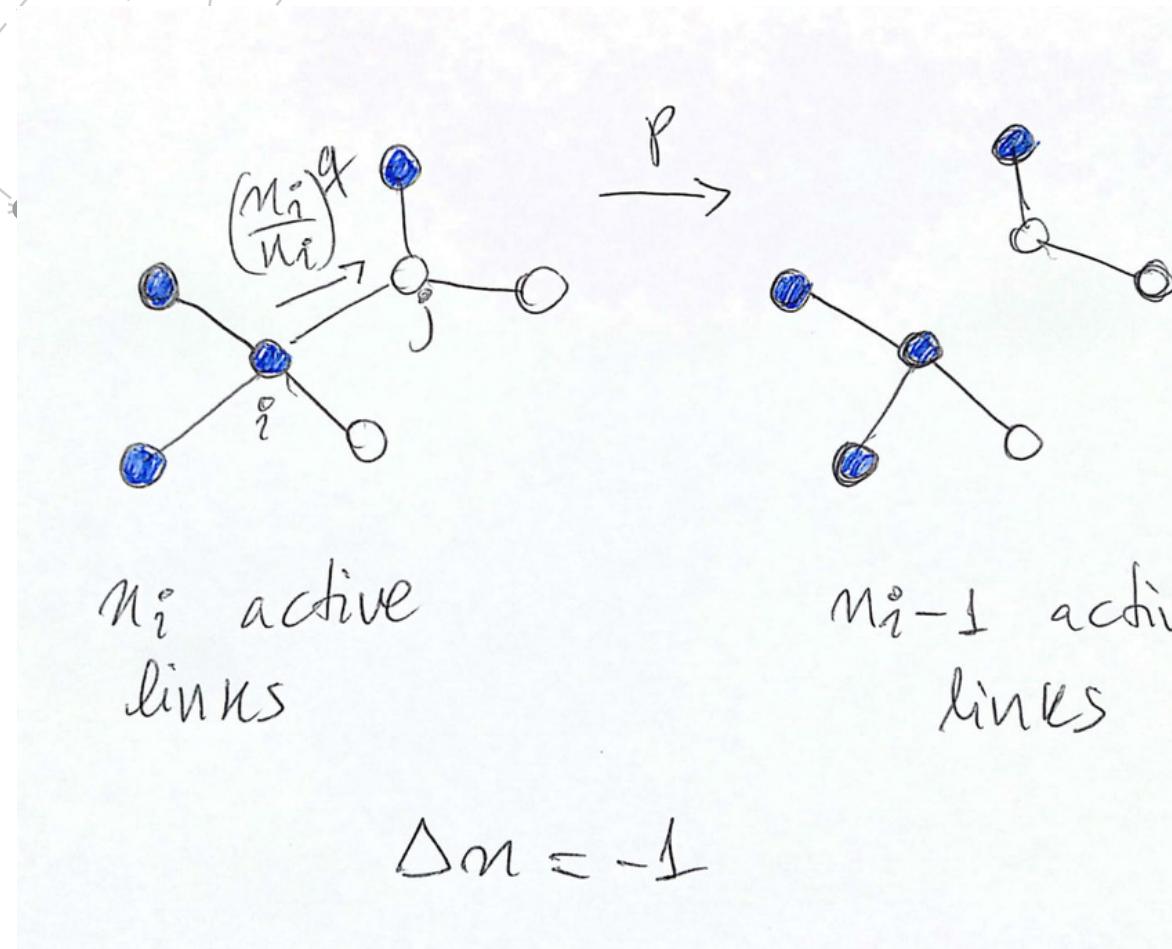
BA (hubs with different opinion)



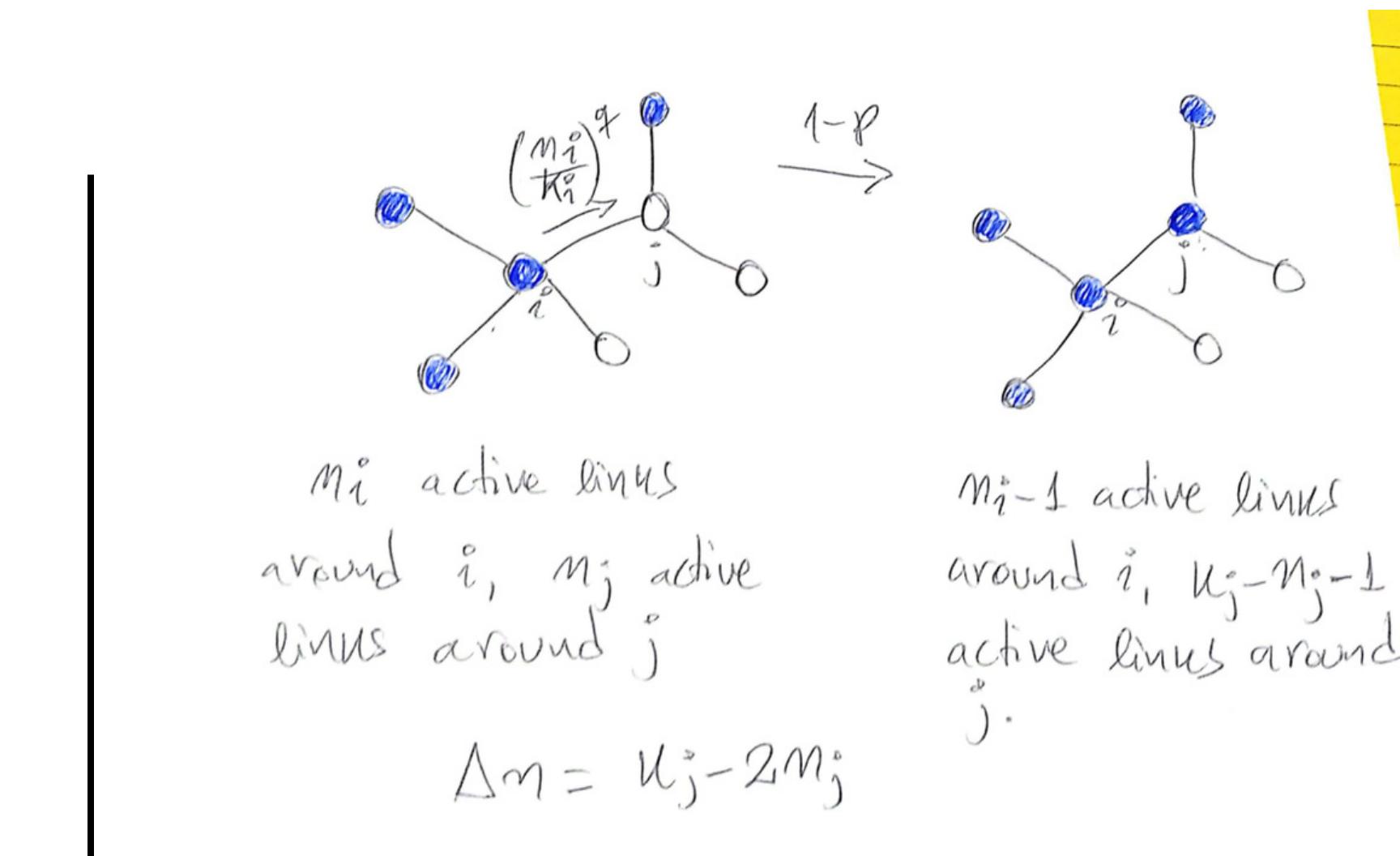
BA (random initial condition)



Master equation approach



Rewiring



Copying

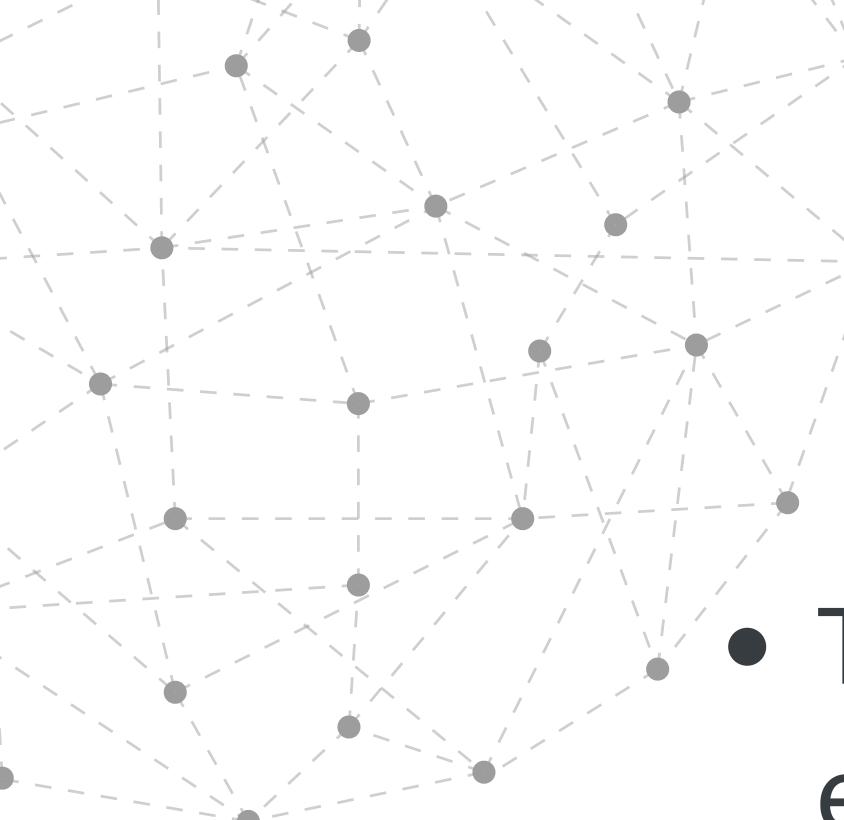
Master equation approach

$$\frac{df}{dt} = \sum_{S=\pm 1} (-\sigma_S) p \left[\sum_n \frac{P(n)}{\mu_1} \kappa \langle n^q \rangle_{nS} \right] (\theta)$$

$$\times \left\{ (1-p) \left\langle \frac{k^2}{\mu_1} \right\rangle_{\theta} \left[1 - (1-p) \frac{f}{\sigma_S} \right] + p - 1 \right\}$$

$$\langle n^q \rangle_{nS} = \sum_{m=1}^n m^q \frac{n!}{m!(n-m)!} \left(\frac{p}{2\sigma_S} \right)^m \left[1 - \frac{p}{2\sigma_S} \right]^{n-m}$$

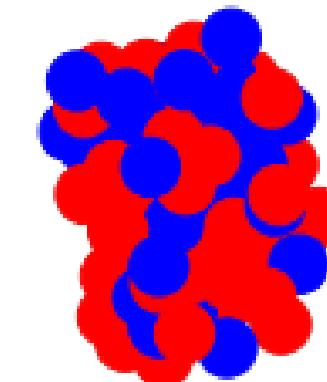
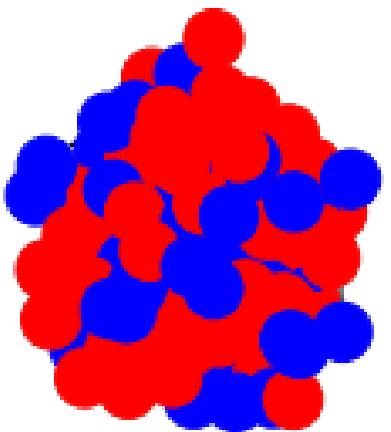
$$\sigma_S + \bar{\sigma}_S = 1$$



Conclusions

- The ordering dynamics can change the results in co-evolving networks
- Hubs are not always able to disseminate their opinion to the rest of the network.
- It is possible to obtain fragmentation on both models, but it depends on the initial opinion states.

Next steps... Pacific Polarization (or almost)



lago



ICTP
SAIFF



fapesp

