# The Minority Game as a Model for Adaptive Competition and Market Dynamics

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# Contents

## Abstract

The Minority Game is a simplified model that captures essential features of competition in environments with limited information. We implement numerical simulations of the game to understand the behavior of the volatility, predictability, and gains that emerge from agents' interactions. We identify that the behavior of the system is mainly determined by the scaling parameter $\alpha = 2^M/N$. By varying this parameter, we identify different regimes, including a crowded phase characterized by high volatility and poor resource management, an optimal regime centered around a critical value $\alpha_c$ where agents coordinate, and a random phase where the excess of information space leads to confusion. We characterize the phase transition using the predictability $\theta^2$, which serves as an order parameter separating efficient and inefficient regimes. Finally, we introduce producers-agents with fixed strategies, and analyze how they provide information to the system. As the producers' population increases, speculators begin to exploit producers' information, improving their gains. The results highlight how simple decision rules lead to complex market dynamics, including emergent patterns of coordination and inefficiency that arise from how agents process information in relation to the population scale.


# 1 Introduction

Most economic theories are deductive in origin. One assumes that each participant knows what is best for them, given that all other participants are equally intelligent in choosing their best actions. However, in the real world, most often their actual players' actions are based on trial-and-error inductive thinking, rather than deductive rationale [1]. As physicists, we would like to view a game with a large number of players, corresponding to a statistical system. Then, we can explore new approaches that appreciate the emerging collective phenomena in a novel way.

One of the prototypical agent-based models is the so-called Minority Game [1], which is the mathematical formulation of the 'El Farol Bar' model proposed by Brian Arthur [2]. This game embodies some basic market mechanisms while keeping the mathematical complexity to a minimum. The model is an example of a system where agents adapt by learning from past outcomes and make forecasts using only limited historical information. Agents try to differentiate themselves from their competitors, making it a model of global competition between adaptive agents.

In this work, we aim to explore the dynamics of the Minority Game by analyzing how volatility, efficiency, predictability, and strategic gains emerge from the interactions of agents. We begin by formally introducing the model. Then, we present the results in terms of volatility and predictability, highlighting how collective behavior changes with the system's parameters. Finally, we extend the model by introducing a new class of agents, called producers, and analyze how their presence provides exploitable information that speculators can use to improve their performance.

# 2 The Minority Game Model

The Minority Game is a repeated game involving $N$ agents who, at each time step $t$, independently decide between two possible actions: buy (+1) or sell (−1) a financial asset. The fundamental rule of the game is that agents who select the minority action are considered winners. The aggregate action, known as the attendance, is defined by:

$$A(t) = \sum_{i=1}^{N} a_i(t) \tag{1}$$

where $a_i(t) \in \{-1, +1\}$ is the action of agent $i$ at time $t$. The payoff of each agent depends on whether their action is the minority, i.e., the opposite sign of $A(t)$. When $A(t) > 0$, there are more buyers than sellers, the price increases, and agents who decided to sell benefit because they sell at a higher price than before. Conversely, when $A(t) < 0$, there are more sellers than buyers, and the price drops. As a result, agents who bought the asset benefit from acquiring it at a lower price. Thus, optimal performance is achieved by anticipating and avoiding the majority decision.

Agents do not observe others' actions, nor do they communicate or coordinate with one another. Instead, they base their choices on a shared public memory of the outcomes from the last $M$ rounds. This memory, or history, is encoded as a binary string of length $M$, where each entry represents the sign of the past attendances. Specifically, the history vector at time $t_k$ is given by $\vec{\mu}(t_k) = [-\text{sign}A(t_{k-1}), \ldots, -\text{sign}A(t_{k-M})]$. Since there are $2^M$ possible binary strings of length $M$, the total number of possible histories is $P = 2^M$. Then, we can represent the history at time $t$ by an integer index $\mu(t) \in \{0, \ldots, P-1\}$, which is obtained by interpreting the binary string $\vec{\mu}(t_k)$ as a number.

Each agent is initially assigned $S$ deterministic strategies which make predictions based on the values of $\text{sign}A(t)$ in the previous $M$ steps. Specifically, a strategy is a mapping from each of the $P$ possible histories to a predicted action in $\{-1, +1\}$. Since there are two possible actions for each history, the total number of distinct strategies is $2^P$. Therefore, the collection of agents is highly heterogeneous, making it practically impossible to find two agents with identical strategies. Table 1 shows an example of a strategy for $M = 3$, illustrating how each history corresponds to a specific action.

| **History** | **Information** | **Prediction** |
|---|---|---|
| $---$ | 0 | Sell $-1$ |
| $--+$ | 1 | Buy $+1$ |
| $-+-$ | 2 | Sell $-1$ |
| $-++$ | 3 | Buy $+1$ |
| $+--$ | 4 | Buy $+1$ |
| $+-+$ | 5 | Sell $-1$ |
| $++-$ | 6 | Buy $+1$ |
| $+++$ | 7 | Sell $-1$ |

Table 1: One of the $2^P$ possible strategies for memory $M = 3$. The table show all $P = 2^M$ possible histories, the integer code $\mu = \{0, \ldots, 2^M - 1\}$, and the outputs regarding the given strategy.

Each strategy $s$ held by an agent $i$ is assigned a virtual score $Q_{i,s}(t)$ which quantifies its historical performance. After each round, all strategies of agent $i$, regardless of whether they were actually played, are updated based on how their prediction would have placed the agent in the minority. The strategies that would have led to a winning (minority) action are rewarded, while those that would have failed are penalized. The reward rule for a strategy $s$ at time $t$ can be formally expressed by defining the virtual gain assigned to the strategy of agent $i$ as:

$$g_{i,s}(t) = -a_{i,s}(t)\,\mathrm{sign}A(t) \tag{2}$$

In this formulation, the reward is proportional to the direction of the majority by considering just $\mathrm{sign}A(t)$. However, alternative payoffs are possible. For example, one could consider rewards proportional to the magnitude of $A(t)$.

The dynamics of the Minority Game proceed as follows. At the beginning of the game, each agent $i$ is randomly assigned $S \ll 2^p$ strategies. All strategy scores $Q_{i,s}$ are initialized to zero. At every time step:

1. Each agent $i$ selects the strategy $s_i$ with the highest virtual score, $s_i(t) = \arg\max_s Q_{i,s}(t)$.
2. The agent follows the action suggested by the chosen strategy $s_i$ for the current history $\mu(t)$.
3. The global attendance $A(t)$ is computed following equation 1.
4. All strategy scores $Q_{i,s}$ are updated according to whether the strategy would have resulted in a minority action. A typical update rule considering the reward given in equation 2 is:

$$Q_{i,s}(t+1) = Q_{i,s}(t) - a_{i,s} \cdot \mathrm{sign}A(t)$$

We refer to the system being in the asymptotic state after $100 \times 2^M$ time steps [3].

# 3 Results

## 3.1 Volatility

A natural starting point in analyzing the Minority Game is the time evolution of the global attendance, $A(t)$. Figure 1 (a)-(c) shows $A(t)$ as a function of time for a fixed number of agents $N = 1001$, number of strategies $S = 2$, and varying memory lengths $M = 2$, $M = 5$, and $M = 15$ (from top to bottom). Already, on a very qualitative level, we can distinguish different kinds of behavior. If $M$ is small enough ($M = 2$), the time series sweeps through an initial transient state and evolves towards a quasi-periodic state. On the contrary, if $M$ is increased ($M = 5$), the attendance follows a rather chaotic course. For large memory ($M = 15$), the fluctuations in attendance are considerably reduced.

To quantify how these fluctuations change over time, we consider a time-local version of the volatility, which is defined as the exponential moving average of the squared attendance. The time-local volatility is defined by:

$$\langle A^2 \rangle_t = \lambda A^2(t) + (1-\lambda)\langle A^2 \rangle_{t-1} \tag{3}$$

The parameter $\lambda$ controls the memory that influences recent observations. Figure 1 (d) shows the time evolution of $\langle A^2(t) \rangle_\lambda$ for a system with $S = 2$, $M = 10$ and $\lambda = 0.01$. We observe that the

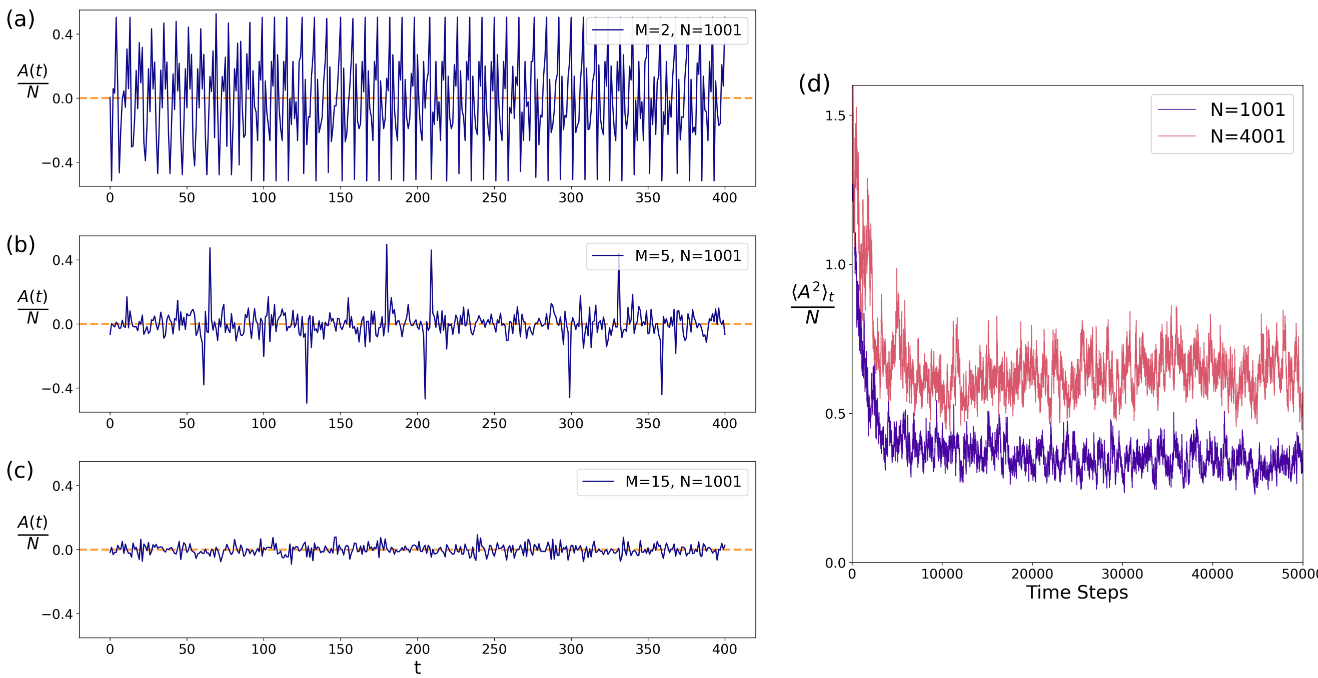



Figure 1: Panels (a)–(c) show the time evolution of the normalized attendance $A(t)/N$ for $S = 2$, $N = 1001$, and memory sizes $M = 2$, 5, and 15, from top to bottom. Panel (d) displays the local time average of $A^2$ as a function of time for fixed $\lambda = 0.01$, $S = 2$, and $M = 10$.

volatility decreases from a high initial value until a certain value that depends on $N$ and $M$. This is a sign of self-organization through adaptation, as agents learn from past decisions.

We now consider the global volatility to characterize the long-term collective behavior of the system. The global volatility is defined as the time-averaged square of the attendance:

$$\sigma^2 = \lim_{T\to\infty} \frac{1}{T} \sum_{t=1}^{T} A(t)^2. \tag{4}$$

Smaller values of $\sigma^2$ indicate coordination among agents, maximizing collective profits. In contrast, high values of $\sigma^2$ indicate uncoordinated behavior among players, resulting in reduced global benefits. A feature of the minority game is that its properties depend on the scaling parameter:

$$\alpha \equiv \frac{2^M}{N} = \frac{P}{N}. \tag{5}$$

Figure 2 shows us the normalized volatility $\sigma^2/N$ as a function of the scaling parameter $\alpha$ for different values of $M$ and $N$, with $S = 2$ strategies per agent. For small $\alpha$ values, when the number of agents is large relative to the strategy space, the system enters the crowded regime. If we fix $M$, we arrive at the crowded state by increasing $N$. The volatility in this regime shows large fluctuations, scaling as $\sim \alpha^{-1}$. The dashed horizontal line in the figure corresponds to the toss-coin limit, representing the volatility expected if all agents choose their actions randomly. When the volatility lies above this line, the system performs worse than a random system; in contrast, volatility values below this curve represent optimal systems, where agents are cooperating successfully [4]. Clearly, in the crowded regime, the system lacks coordination as the volatility exceeds the toss-coin limit.

Notably, increasing $\alpha$ reduces the volatility until an optimal value occurs at $\alpha_c \approx 0.34$. This marks the point of optimal efficiency. Beyond the critical $\alpha_c$ value, when the number of agents is small compared to the number of possible histories, the outcome appears random, very similar to what we would expect if agents were deciding whether to buy or not at random. The reason for this is that the information which agents receive about the past history is too complex for their limited processing analysis, and their behavior 'over-fits' the fluctuations of past attendance [3].

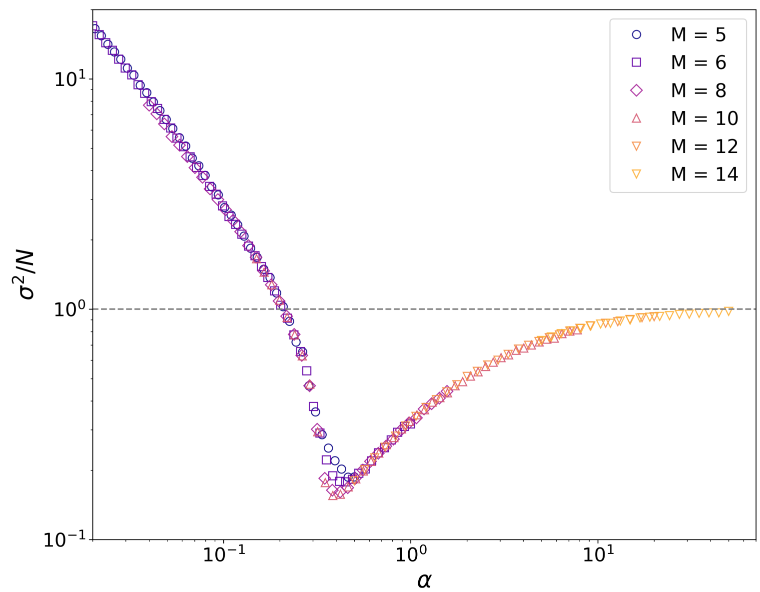


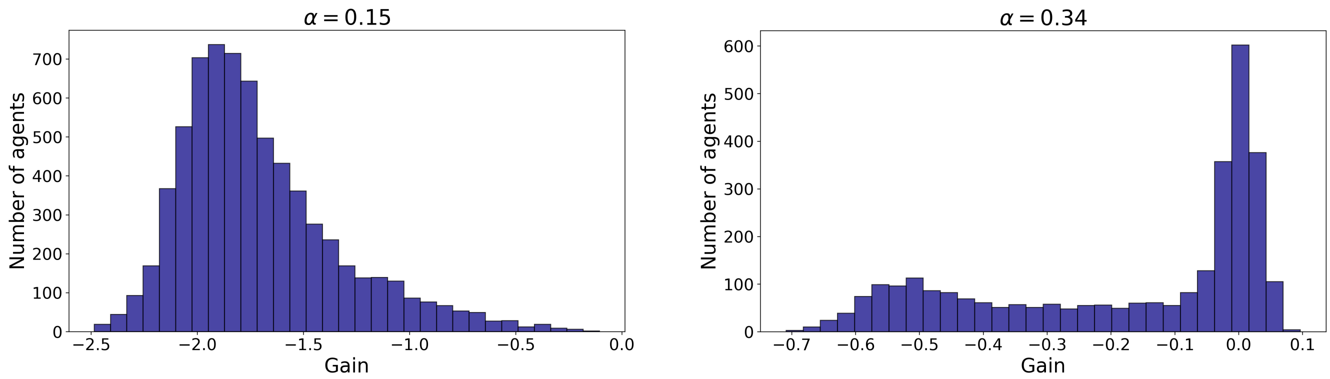

Figure 2: Normalized volatility as a function of the control parameter $\alpha$ for several combinations of memory $M$ and number of agents $N$, considering $S = 2$ strategies. The symbols correspond to different values of memory length $M$. The horizontal line indicates a fully random game. Results averaged over $100 \times 2^M$ time steps after thermalization and over 100 realizations.

To evaluate the performance of individual agents, we compute the average gain over time as:

$$\langle G_i \rangle = \lim_{T\to\infty} \frac{1}{T} \sum_{t=1}^{T} -a_i(t) \cdot A(t), \tag{6}$$

which represents the agent's ability to profit by consistently anticipating and avoiding the majority.

Then, we can better understand the distribution of wealth among the agents. Figure 3 shows two representative examples for $\alpha = 0.15$, corresponding to the crowded regime, and for $\alpha = 0.34$, corresponding to the optimal regime. For the case of the crowd regime, we observe a long-tail behavior. Although most agents are losing higher quantities, there are a few agents that almost do not present losses. In contrast, in the optimal regime, we observe that most agents organize around zero gain, and some even achieve positive profits. A fraction of agents still experience negative gains, reflecting their inability to coordinate effectively with the rest of the population. However, these losses are reduced in comparison to the crowded regime, indicating an overall improvement.




(a) (b)

Figure 3: Distribution of gains per agent for (a) $\alpha = 0.15$ and (b) $\alpha = 0.34$. Results are obtained using fixed parameters $M = 10$ and $S = 2$, with $N = 6827$ agents in (a) and $N = 3011$ agents in (b). Each distribution is computed over $100 \times 2^M$ time steps after thermalization.

We can now analyze the effects of increasing the number of strategies $S$ available to each agent. Figure 4 shows the normalized volatility as a function of the scaling parameter $\alpha$ for different values of $S$, with memory length fixed at $M = 10$ and varying $N$. As a result, we observe that an increase in $S$ shifts the crowded regime to the right. Additionally, the minimum volatility increases as more strategies are considered. We found that providing players with more strategies to choose from does not improve their performance. On the contrary, increasing $S$ makes it harder for agents to coordinate effectively. Nevertheless, we observe that the curves for $S = 2$ and $S > 2$ are qualitatively similar. Then, without loss of generality, we can continue our analysis to the case of $S = 2$.

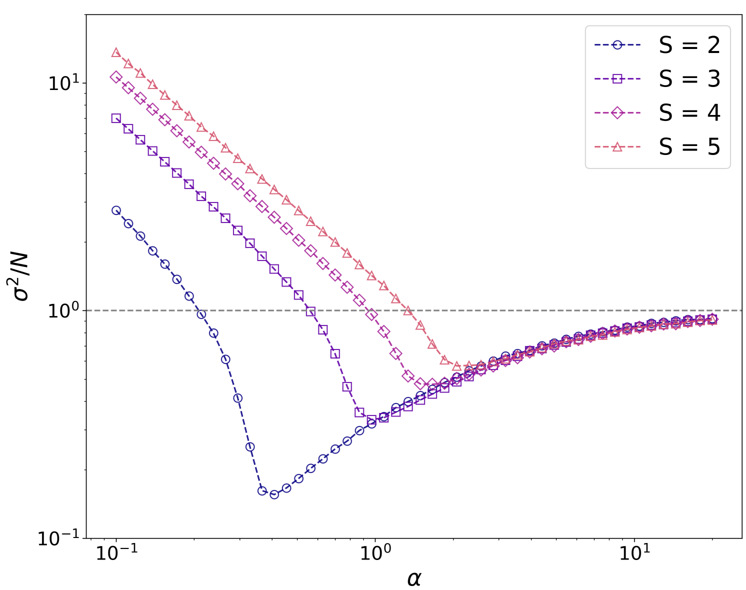



Figure 4: Normalized volatility as a function of $\alpha$ for different values of the number of strategies $S$, considering $M = 10$, and varying $N$. Results averaged over $100 \times 2^M$ time steps after thermalization and over 100 realizations.

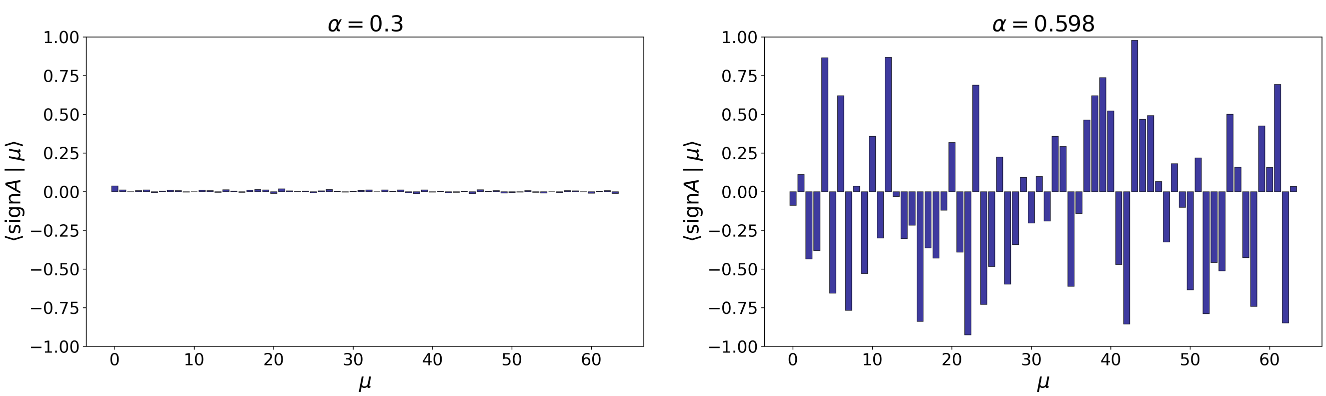
## 3.2 Efficiency and Predictability

While volatility captures the degree of collective fluctuations in agents' actions, it does not reveal whether the system exhibits predictable patterns that agents could exploit to obtain profits. In financial markets, a system is considered efficient if no information is left in the price signal. This means that you cannot use freely accessible information and profit from it. In a similar spirit, we can investigate the efficiency of the agents in extracting and destroying the information stored in the sequence of attendance [5]. We consider the average sign of the attendance with fixed memory pattern $\mu$ as:

$$\langle \mathrm{sign} A \mid \mu \rangle = \lim_{T\to\infty} \frac{\sum_{t=1}^{T} \delta_{\mu,\mu(t)} \,\mathrm{sign} A(t)}{\sum_{t=1}^{T} \delta_{\mu,\mu(t)}}, \tag{7}$$

where $\delta_{\mu,\mu(t)}$ is the Kronecker delta that selects when the public history at time $t$, $\mu(t)$, equals one of the possible histories $\mu$. This quantity measures whether, on average, the attendance has a preferred direction when a given history occurs. If $\langle \mathrm{sign} A \mid \mu \rangle = 0$, the next choice of the agents given a history $\mu$ will be unpredictable. In contrast, the next choice given $\mu$ will be predictable up to some degree if the conditional average of the attendance is not zero, i.e. $\langle \mathrm{sign} A \mid \mu \rangle \neq 0$ [6].

Figure 5 (a) shows how, for $\alpha = 0.3$, the choices of the agents are completely random given a history $\mu$, leading to a symmetric distribution. On the other hand, Figure 5 (b) shows the results for $\alpha = 0.598$, where a clear asymmetry indicates that some histories are associated with non-zero average attendance, proving the presence of predictable patterns that agents could potentially exploit.




(a) (b)

Figure 5: Histogram of the average sign of attendance given a history $\mu$ for (a) the symmetric phase with $N = 213$ and $M = 6$, such that $\alpha = 0.3$ and (b) asymmetric phase with $N = 107$ and $M = 6$, such that $\alpha = 0.598$. Results computed considering $1000 \times 2^M$ time steps after thermalization.

To quantify the overall amount of information remaining in the system, we define a global measure of efficiency based on the squared average of the conditional signs of attendance [7]. This quantity, known as the predictability, is computed by averaging over all $2^M$ possible histories:

$$\theta^2 = \frac{1}{2^M} \sum_{\mu=0}^{2^M-1} \left( \langle \mathrm{sign} A \mid \mu \rangle \right)^2 . \tag{8}$$

The value of $\theta^2$ measures the amount of remaining information in the time series. If agents have succeeded in fully adapting to the available information, then $\langle \mathrm{sign} A \mid \mu \rangle = 0$ for all histories $\mu$, leading to $\theta^2 = 0$. On the contrary, $\theta^2 > 0$ indicates that there is potentially exploitable information. In this sense, $\theta^2$ captures the predictability of the system, where lower values correspond to more efficient systems.

Figure 6 shows the behavior of $\theta^2$ as a function of $\alpha$, for different values of memory length $M$. We observe that in the crowded phase, $\alpha < \alpha_c$, the system is efficient, as all information is eliminated from the signal. For higher values of $\alpha$ ($\alpha > \alpha_c$), $\theta^2$ continuously increases, indicating the emergence of predictable patterns. This transition from $\theta^2 = 0$ to $\theta^2 > 0$ marks a qualitative change in the collective behavior of the agents. Therefore, $\theta^2$ can be considered as an order parameter, revealing a phase transition occurring at $\alpha_c \approx 0.34$. The regime where $\theta^2 = 0$ is referred to as the symmetric phase, in which the system is fully efficient and the outcomes are unpredictable given the history. In contrast, the asymmetric phase corresponds to $\theta^2 > 0$, indicating that some histories are associated with biased outcomes, revealing predictability in the system [8].

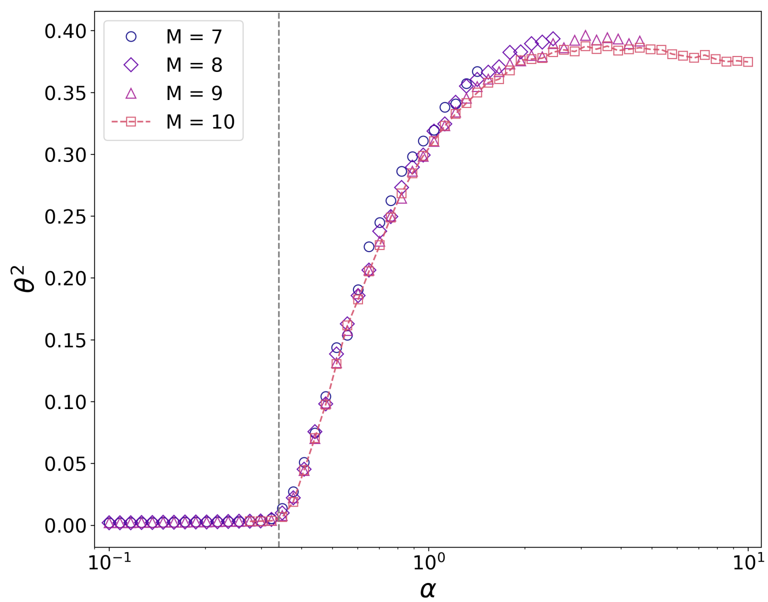



Figure 6: Predictability $\theta^2$ as a function of $\alpha$ for different values of memory $M$, and fixed $S = 2$. Results computed considering $100 \times 2^M$ times steps after thermalization, and averaging over 100 realizations.

## 3.3 Producers and speculators

We now introduce a new class of agents known as producers, which represent those agents that neither try to speculate on the market nor look at it in detail to make a direct benefit. Instead, they use it for other business purposes. They do not adapt their actions based on the state of the market but instead follow a fixed strategy.

We define $N_{\mathrm{prod}}$ as the number of producers in the model, and $N_{\mathrm{spec}}$ as the number of speculators (the ones considered in the previous sections). The total number of agents is thus $N = N_{\mathrm{prod}} + N_{\mathrm{spec}}$.

Producers can be formally described as agents who use only a single, fixed strategy ($S = 1$), whereas speculators can have $S \geq 2$ strategies [4]. In our implementation, we fix $S = 2$ strategies for the speculators. To capture the similarity between the two strategies for each speculator, we introduce a correlation parameter $c$, defined as the average fraction of predicted actions that the strategies agree on. The standard Minority Game corresponds to the independent case $c = 1/2$, while having only one strategy is obtained with $c = 1$. The other very special case is $c = 0$, where all agents have two strategies that always predict in the opposite way.

To assess how the agent types perform, we compute the average gain per agent for each group: $\langle G_{\text{spec}} \rangle$ for speculators and $\langle G_{\text{prod}} \rangle$ for producers. Figure 7 shows $\langle G_{\text{spec}} \rangle$ and $\langle G_{\text{prod}} \rangle$ as a function of the normalized number of producers, $N_{\text{prod}}/2^M$, for the fixed parameters $M = 8$, $S = 2$ and $N_{\text{spec}} = 641$. When the number of producers is low, the system is in the symmetric phase for the given parameters. In this regime, speculators compete over a limited amount of information and perform poorly, while producers do not obtain resources due to the unpredictability of outcomes. As the number of producers increases, the system transitions into the asymmetric phase, where residual information becomes exploitable and speculators start to profit. In contrast, producers experience a slight decline in gain as they become the source of the information.

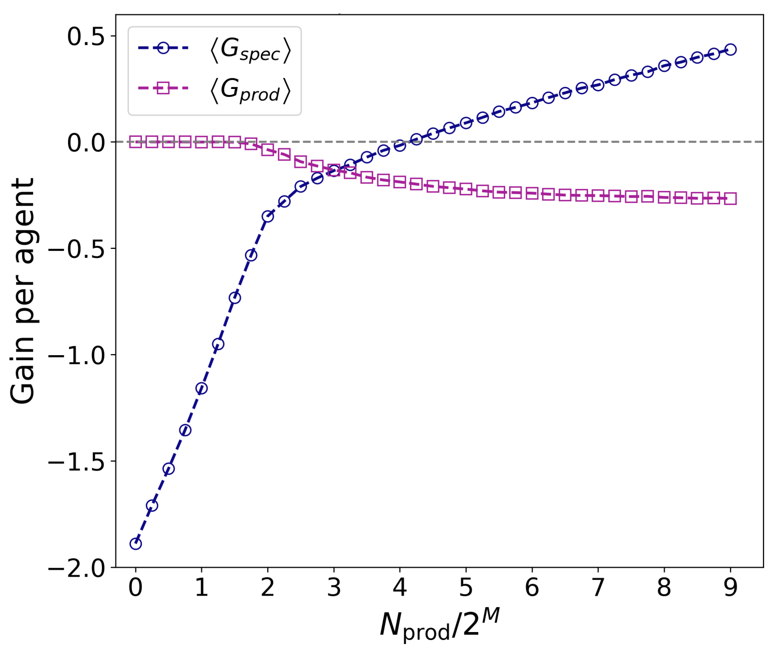
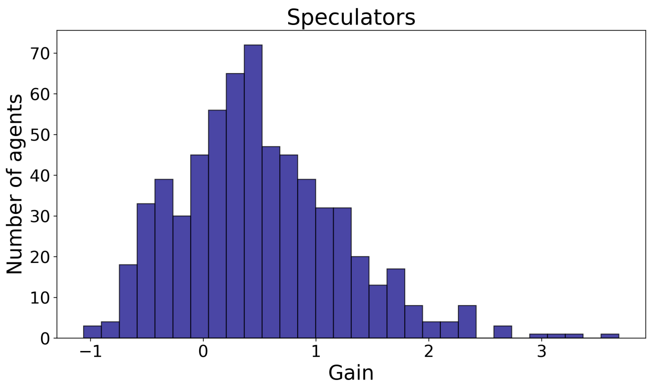
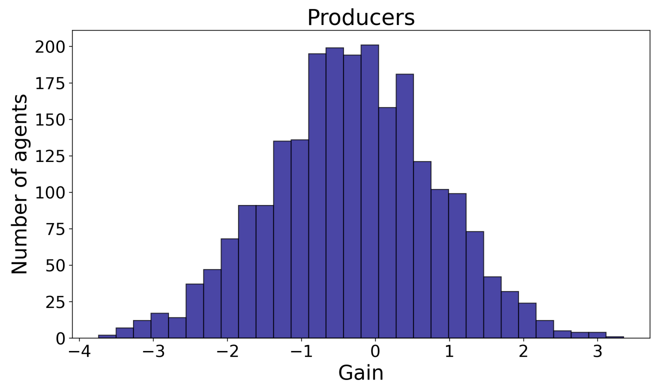



Figure 7: Average gain for producers and speculators as a function of the number of producers. The number of speculators is fixed at $N_{\text{spec}} = 641$. Results computed for fixed $c = 0$, $M = 8$, and $S = 2$, considering $100 \times 2^M$ times steps after thermalization and averaging over 1000 realizations.

We further analyze the distribution of the gains in the regime where speculators profit from the information injected by producers. Figure 8 shows the distribution of gains per agent for (a) speculators and (b) producers for a case where the number of producers is significantly larger than the size of the strategy space given by $N_{\text{spec}} = 641$, $N_{\text{prod}} = 2304$, and $M = 8$, such that $N_{\text{prod}}/2^M = 9$. The gain distribution for speculators shows a high frequency of agents with low and even negative gains, while a few speculators achieve higher profits. The overall shape appears to follow a longtailed distribution. Further analysis would be valuable for statistically characterizing the inequality in outcomes and assessing the presence of heavy-tailed behavior. In contrast, for the producers, the distribution resembles a Gaussian centered on their average net loss.




(a)




(b)

Figure 8: Distribution of gains per player for (a) speculators and (b) producers. Results are obtained for $N_{\text{spec}} = 641$, $N_{\text{prod}} = 2304$, and $M = 8$, such that $N_{\text{prod}}/2^M = 9$. Each distribution is computed over $100 \times 2^M$ time steps after thermalization, considering the fixed parameters $c = 0$ and, $S = 2$.

# 4 Conclusions

In this work, we have implemented and analyzed the Minority Game, an agent-based model that includes premises of competition in financial markets, where players taking the minority action are rewarded. Through numerical simulations, we investigate the interplay between the different features of the model, such as the memory, number of strategies, and population sizes, as well as considering different kinds of players.

Our finding shows that the model undergoes a phase transition governed by the scaling parameter $\alpha = 2^M/N$. For low $\alpha$ values, the system remains in a crowded regime with high volatility and a lack of coordination among the players. As $\alpha$ increases, there is an optimal value $\alpha_c$ where agents coordinate and reduce the overall volatility. Nevertheless, further increasing $\alpha$ leads to random behavior, indicating that an excess of information leads to confusion.

The predictability $\theta^2$ serves as an order parameter, capturing the phase transition between efficient and inefficient collective dynamics. Finally, introducing producers, who are agents with a unique fixed strategy, highlights how external information can be exploited by speculators, thereby enhancing their gains.

Overall, the minority game illustrates how simple inductive rules can lead to complex market phenomena, including self-organization, efficiency breakdowns, and the emergence of non-trivial gains. Future work could explore the role of noisy agents to represent bounded rationality or external perturbations [4]. Additionally, introducing communication or local interactions among agents may provide new insights into how coordination emerges [9].

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