Final Presentation

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Complex Systems Modelling in Economics

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Final project:

The Minority Game as a Model for Adaptive Competition and Market Dynamics

D. Challet and Y.-C. Zhang , Physica A 246, 407 (1997)

Problem:

Most of the economic theories are deductive in origin. However, most actions of real players are based on trial-and-error inductive thinking.

Proposal:

A simple model in which agents adapt by learning from past outcomes and make forecasts using only limited historical information.

Core idea:

1. If there are **more buyers** than sellers, prices rise → **sellers benefit** by selling at a higher price.
2. If there are **more sellers** than buyers, prices fall → **buyers benefit** by purchasing at a lower price.

→ Success comes from going against the crowd, **being in the minority.**

Objective

Explore the dynamics of the Minority Game by analyzing how volatility, predictability, and gains emerge from the interactions of agents.

Consider N agents that at each time step t_k decide whether to buy an asset $a_i(t_k) = +1$ or sell it $a_i(t_k) = -1$. Player used a finite set of ad hoc strategies to make their decision, based on the past record. Playes keeps a register with the accumulated virtual scores of the strategies $Q_{i,s}(t)$.

strategy

$$s_i(t) = \arg\max_s Q_{i,s}(t)$$

action

$$a_i \in \{-1,1\}$$

scores

$$Q_{i,s}(t+1) = Q_{i,s}(t) + g_{i,s}(t)$$

$$A(t) = \sum_{i=1}^{N} a_i(t)$$

attendance

At each time step:

1. Each agent i selects the strategy with the highest virtual score.
2. The agent follows the action suggested by the chosen strategy.
3. The global attendance is determined.
4. All strategies are scored based on their prediction, following the reward:

$$g_{i,s}(t) = -a_{i,s}(t) sign A(t)$$

Strategies S, histories μ and memory M.

- The input information of a strategy is: $\overrightarrow{\mu}(t_k) = [-signA(t_{k-1}), \ldots, -signA(t_{k-M})]$.
- We can represent the information of a history by an integer $\mu \in \{0,\ldots,2^M - 1\}$.
- A strategy is a map $\{-1, +1\}^M \to \{-1, +1\}$.

Number of history states combinations

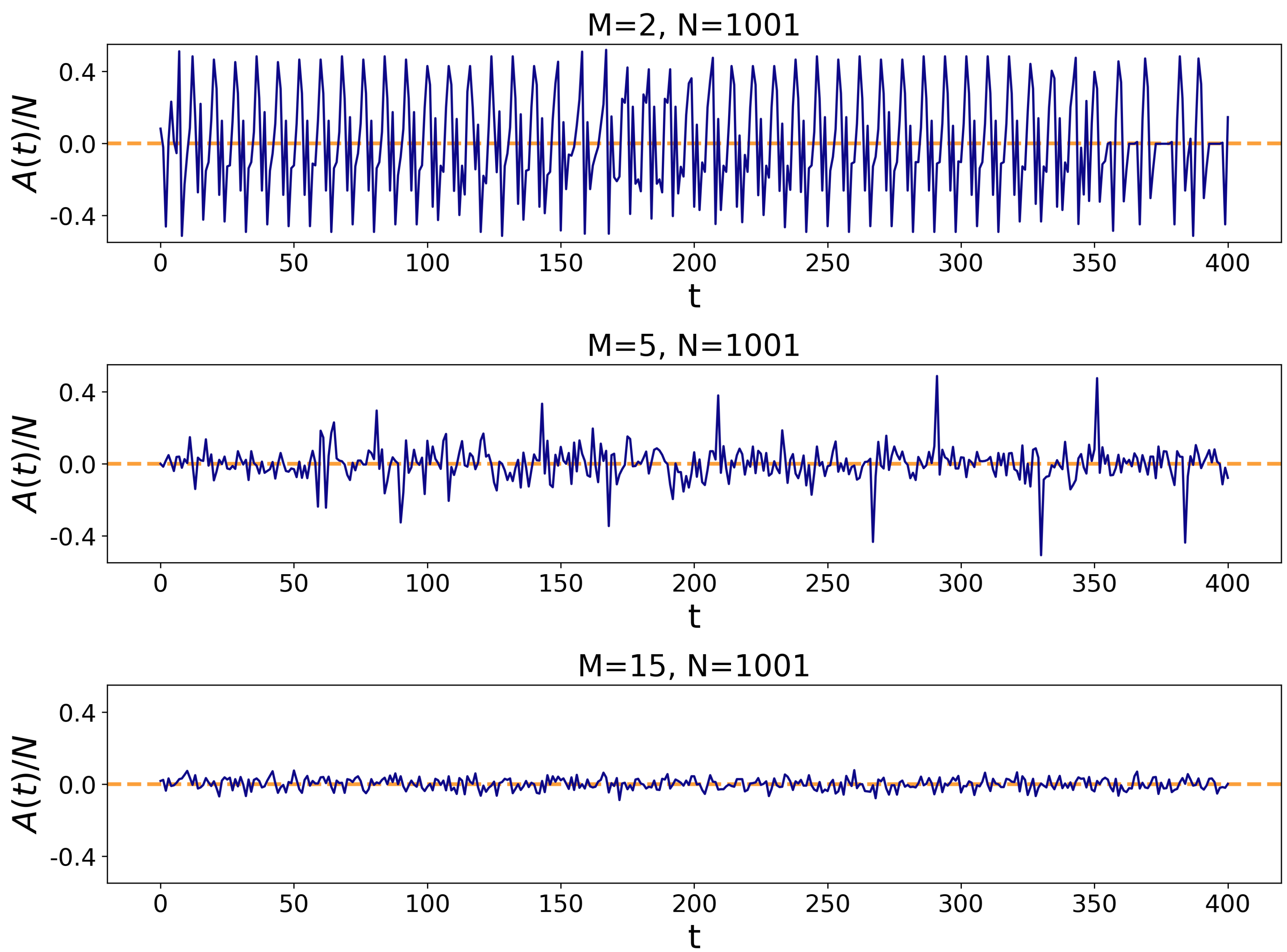
$P = 2^M$

History	Information	Prediction
$---$	0	Sell -1
$--+$	1	Buy $+1$
$-+-$	2	Sell -1
$-++$	3	Buy $+1$
$+--$	4	Buy $+1$
$+-+$	5	Sell -1
$++-$	6	Buy $+1$
$+++$	7	Sell -1

One of the possible strategies in the case of $M = 3$, the number of possible estrategies is $2^P = 2^{2^M}$.

Time evolution of the attendance.

$$\boxed{A(t) = \sum_{i=1}^{N} a_i(t)}$$

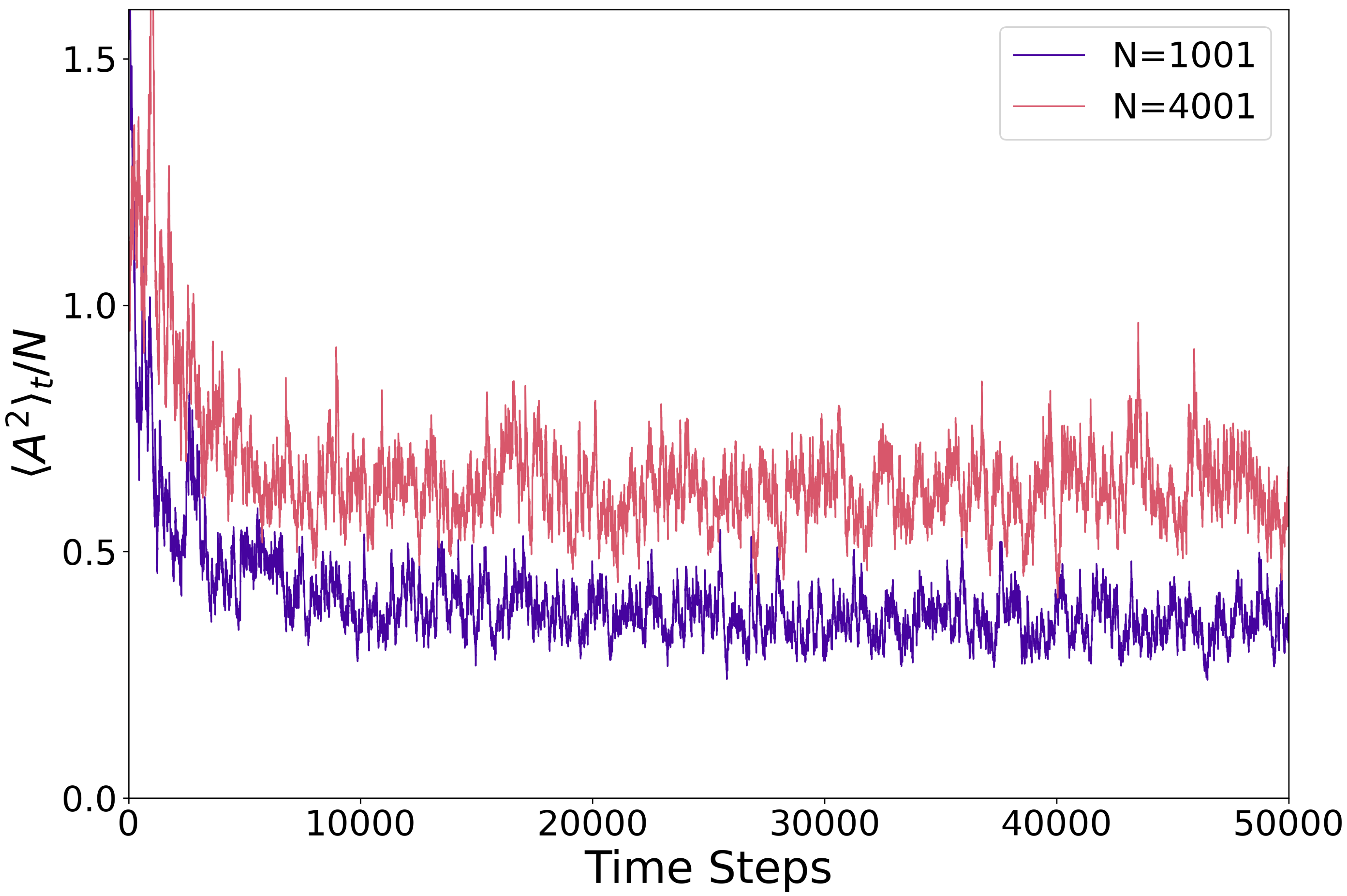


- Low M: large variations, system in a quasi-periodic state.
- Intermediate M: the system follows a chaotic course.
- Large M, fluctuations get narrowed.

Fixed: $S = 2$, $N = 1001$.

Time-dependent volatility: averaging the volatility over roughly $(1-\lambda)^{-1}$ last steps.

$$\boxed{\langle A^2\rangle_t = \lambda A^2(t) + (1-\lambda)\langle A^2\rangle_{t-1}}$$



$\langle A^2\rangle_t$ decreases from a high initial value until a certain value that depends on N and M.

Sign of self-organization, through adaptation.

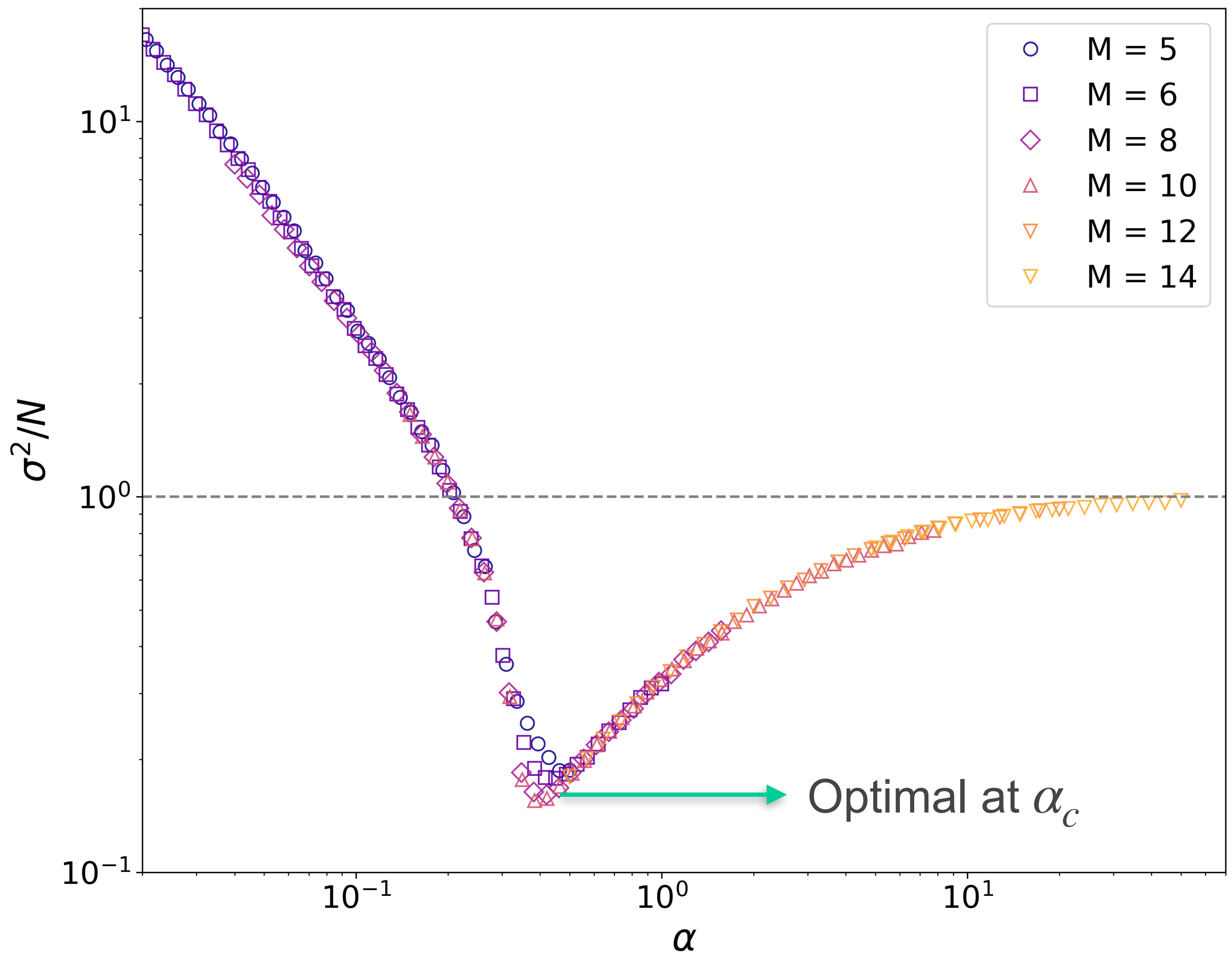
Fixed: $\lambda = 0.01$, $S = 2$, $M = 10$.

Volatility: time-averaged square of the attendance.

$$\sigma^2 = \lim_{T\to\infty} \frac{1}{T}\sum_{t=1}^{T} A(t)^2$$

Scaling parameter

$$\alpha = \frac{2^M}{N}$$



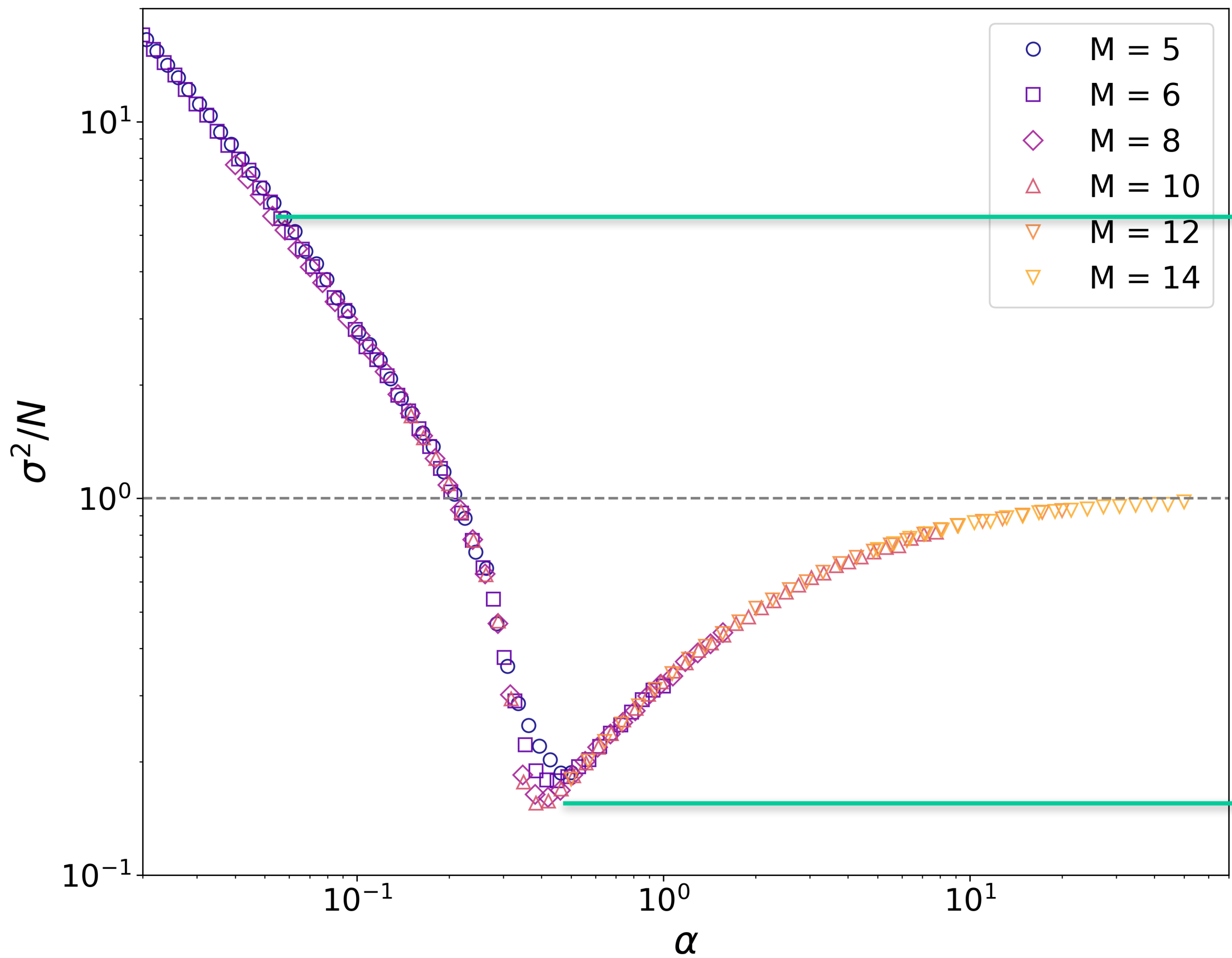
Fixed: $S = 2$.

- Low α (crowded regime): large fluctuations, agents are uncoordinated.
- At Intermediate α values, there is an optimal value α_c where agents coordinate.
- High α: excess of information leads to confusion - system approach toss-coin limit.

Optimal value for efficient use of shared information.

Volatility: time-averaged square of the attendance.

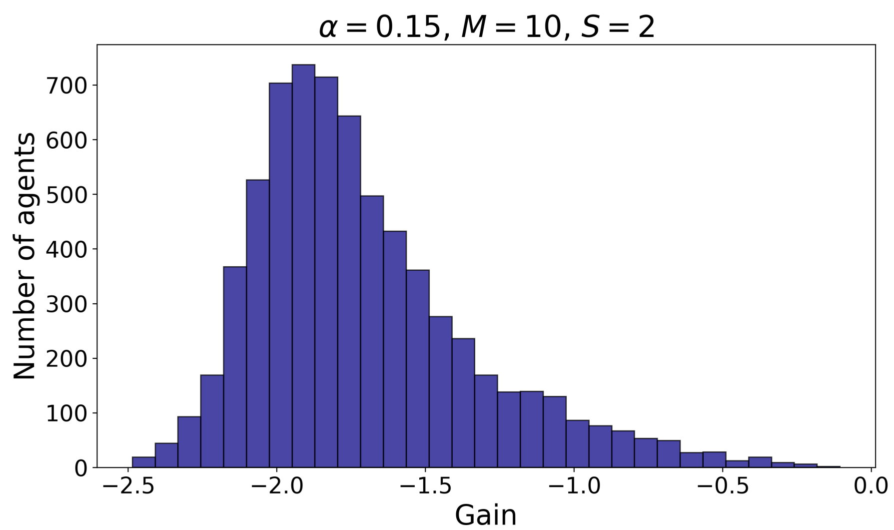
$$\sigma^2 = \lim_{T\to\infty} \frac{1}{T} \sum_{t=1}^{T} A(t)^2$$

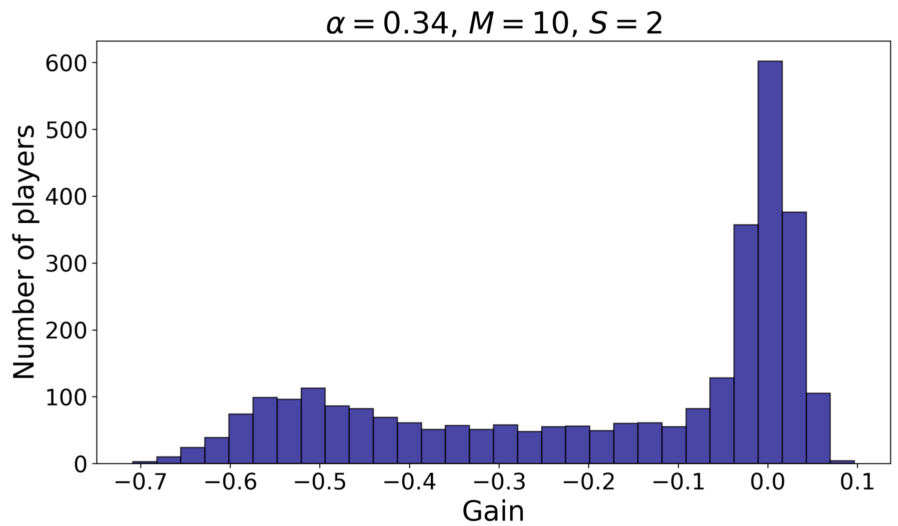


Fixed: $S = 2$.

Average gain per agent

$$\langle G_i \rangle = \lim_{T\to\infty} \frac{1}{T} \sum_{t=1}^{T} \left(-a_i(t) \cdot A(t)\right)$$



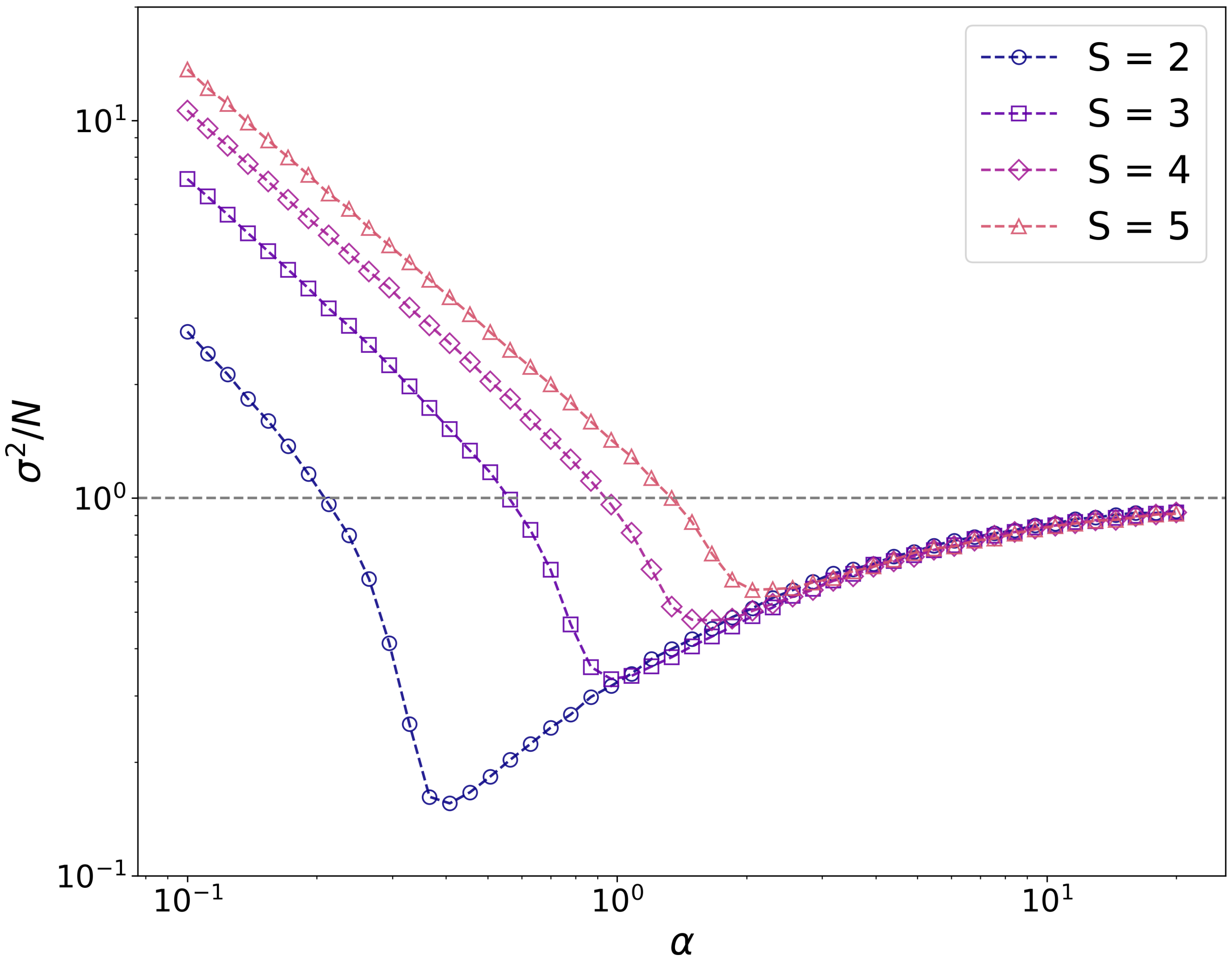


Volatility for different number of strategies.

$$\sigma^2 = \lim_{T\to\infty} \frac{1}{T} \sum_{t=1}^{T} A(t)^2$$

Scaling parameter

$$\alpha = \frac{2^M}{N}$$



Fixed: $M = 10$ while varying N.

Effects of increasing S:

- Shifts the crowded region to the right.
- The minimum of the volatility increases.

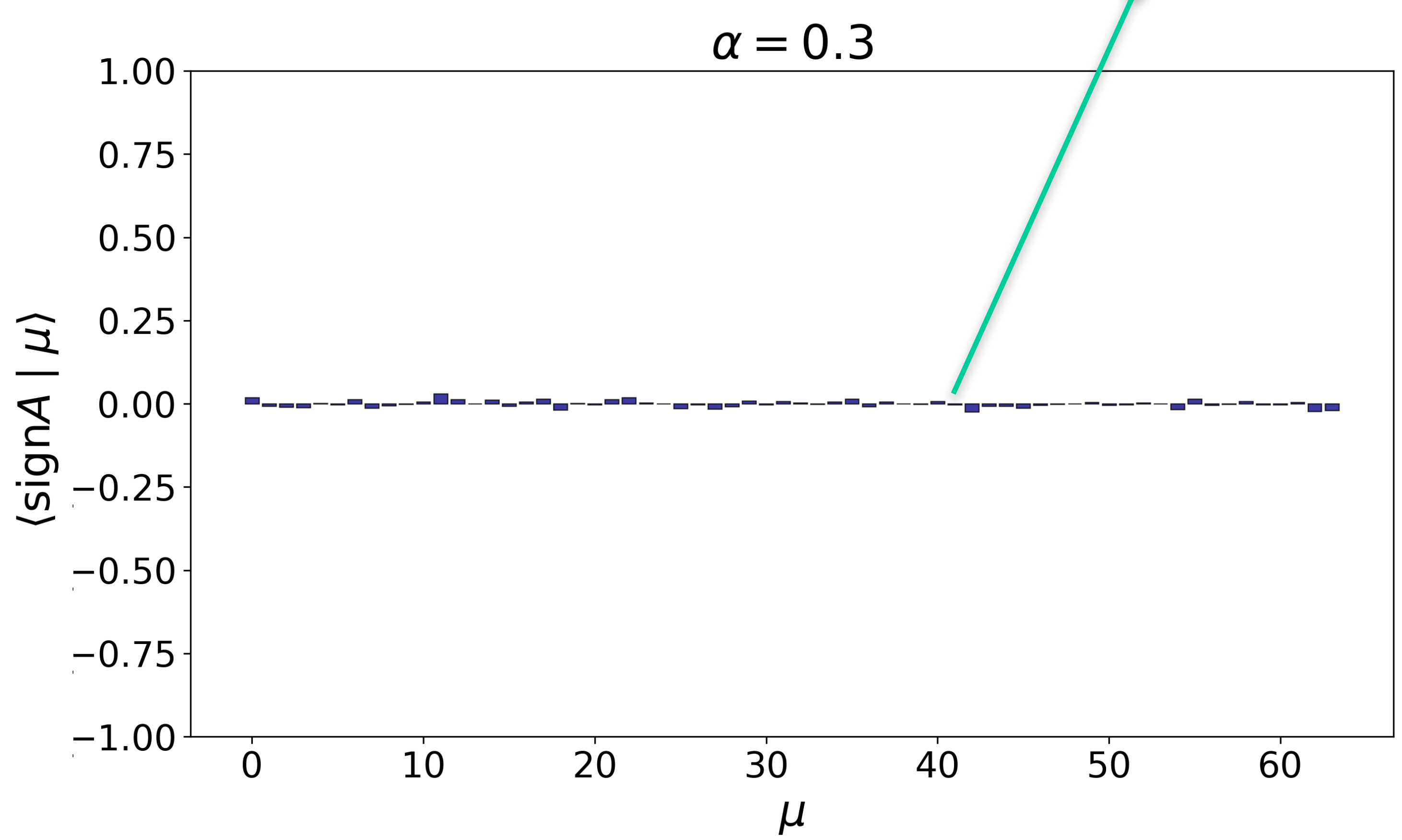
Harder for agents to coordinate effectively.

Average sign of the attendance given history μ.

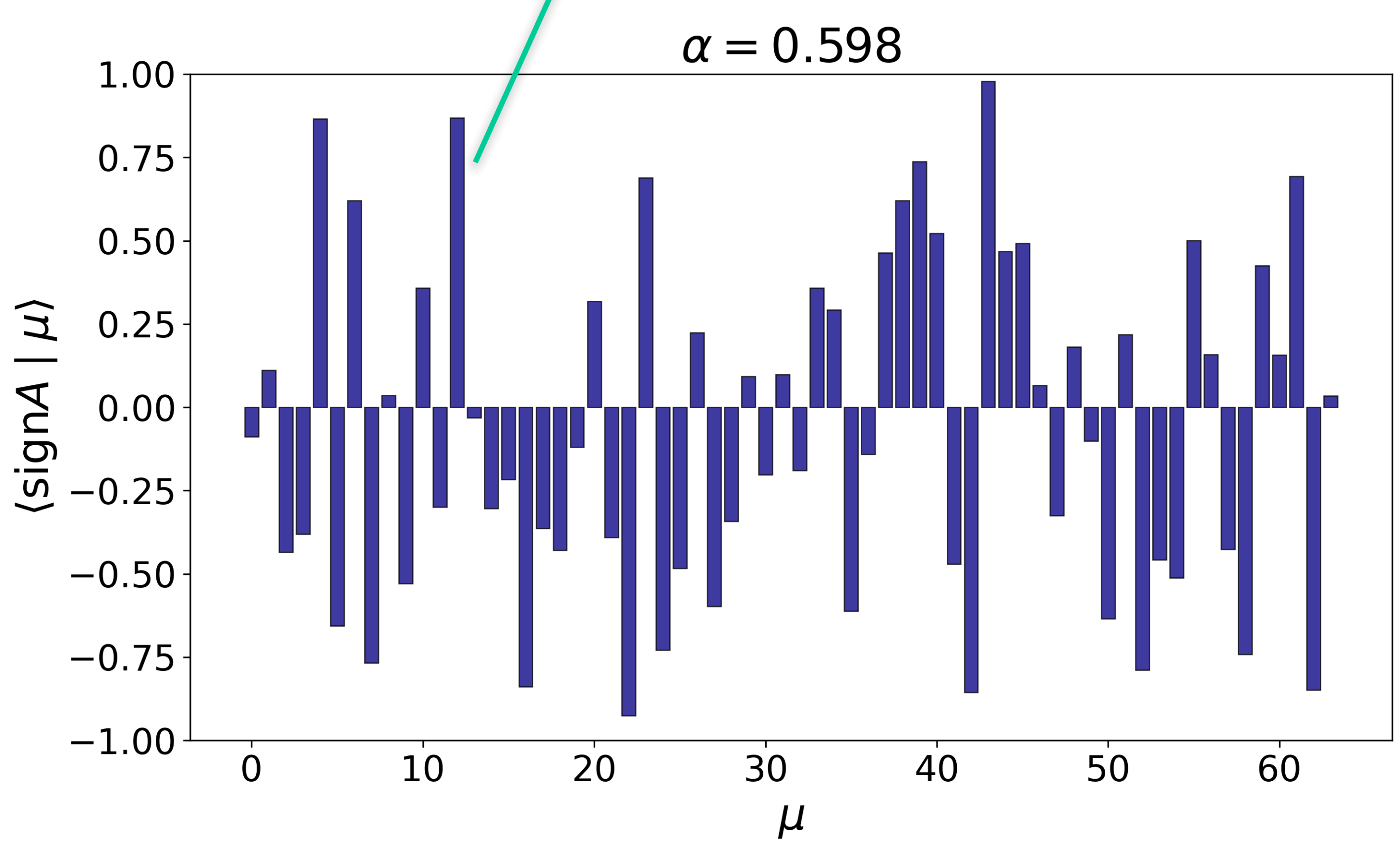
$$\boxed{\left\langle \text{sign } A \mid \mu \right\rangle = \lim_{T\to\infty} \frac{\sum_{t=1}^{T} \delta_{\mu,\mu(t)} \ \text{sign } A(t)}{\sum_{t=1}^{T} \delta_{\mu,\mu(t)}}}$$

Directional bias: predictable patterns.

Unbiased: no predictive information.



Fixed: $M = 6, N = 213, S = 2.$

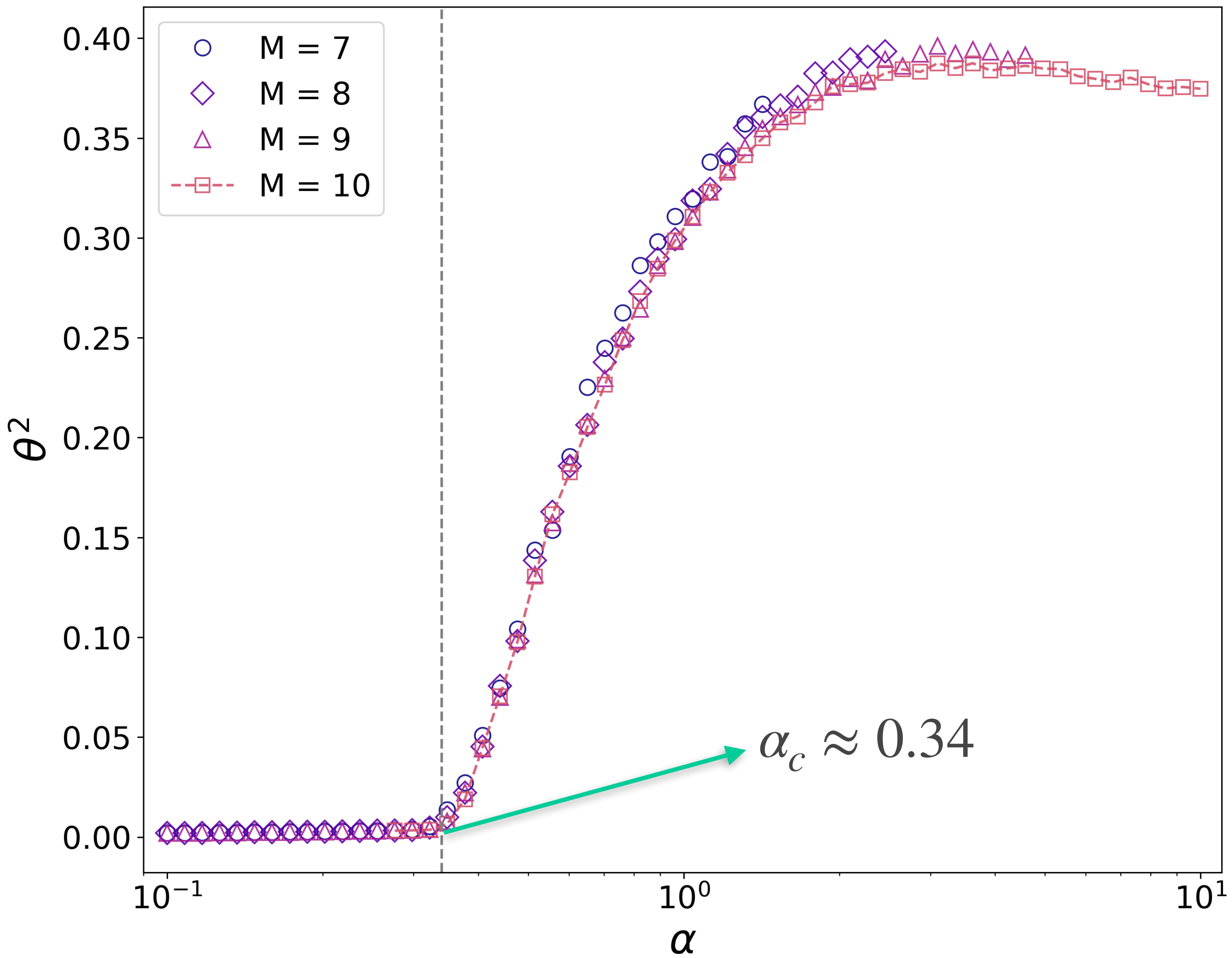


Fixed: $M = 6, N = 107, S = 2.$

Predictability:

$$\theta^2 = \frac{1}{2^M} \sum_{\mu=0}^{2^M-1} \left(\langle \operatorname{sign} A \mid \mu \rangle \right)^2$$

Scaling parameter $\alpha = \dfrac{2^M}{N}$



Fixed: $S = 2$.

- Low α: Efficient phase - no directional signal $\theta \sim 0$.
- Phase transition at $\alpha_c \approx 0.34$.
- High α: Predictable phase - system develops memory, leading to dependent directional bias.

θ^2 captures the phase transition between efficient and inefficient regimes.

Speculators and producers:

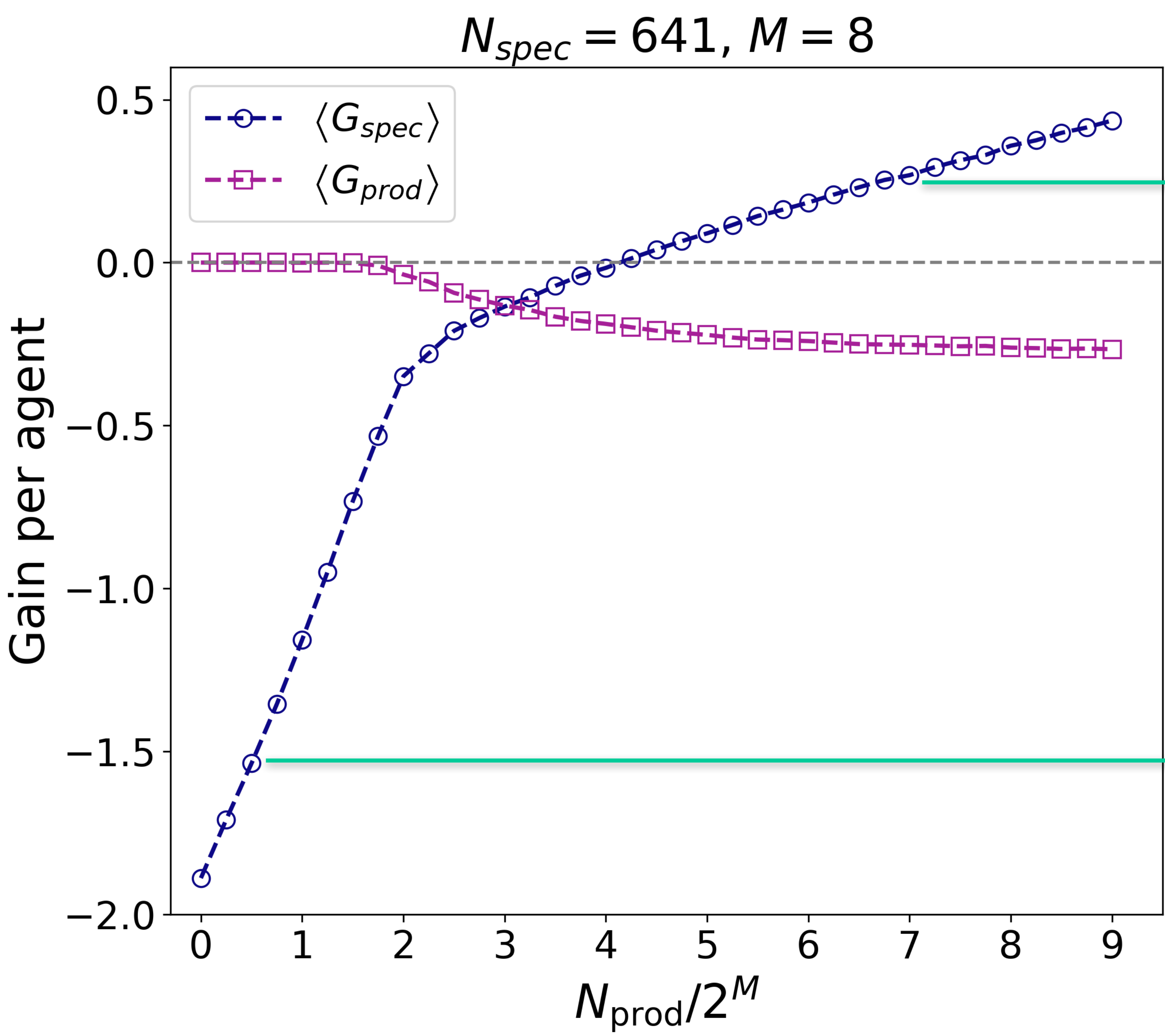
- **Speculators:** $S = 2$
- **Producers:** $S = 1$

Average gain per agent for:

- Speculators: $\langle G_{\text{spec}} \rangle$
- Producers: $\langle G_{\text{prod}} \rangle$

Average gain per agent

$$\langle G_i \rangle = \lim_{T \to \infty} \frac{1}{T} \sum_{t=1}^{T} \left(-a_i(t) \cdot A(t) \right)$$



Speculators exploit producers' information, improving their gains.

Speculators compete over a limited amount of information and perform poorly.

Speculators and producers:

- **Speculators:** $S = 2$
- **Producers:** $S = 1$

Average gain per agent for:

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Average gain per agent

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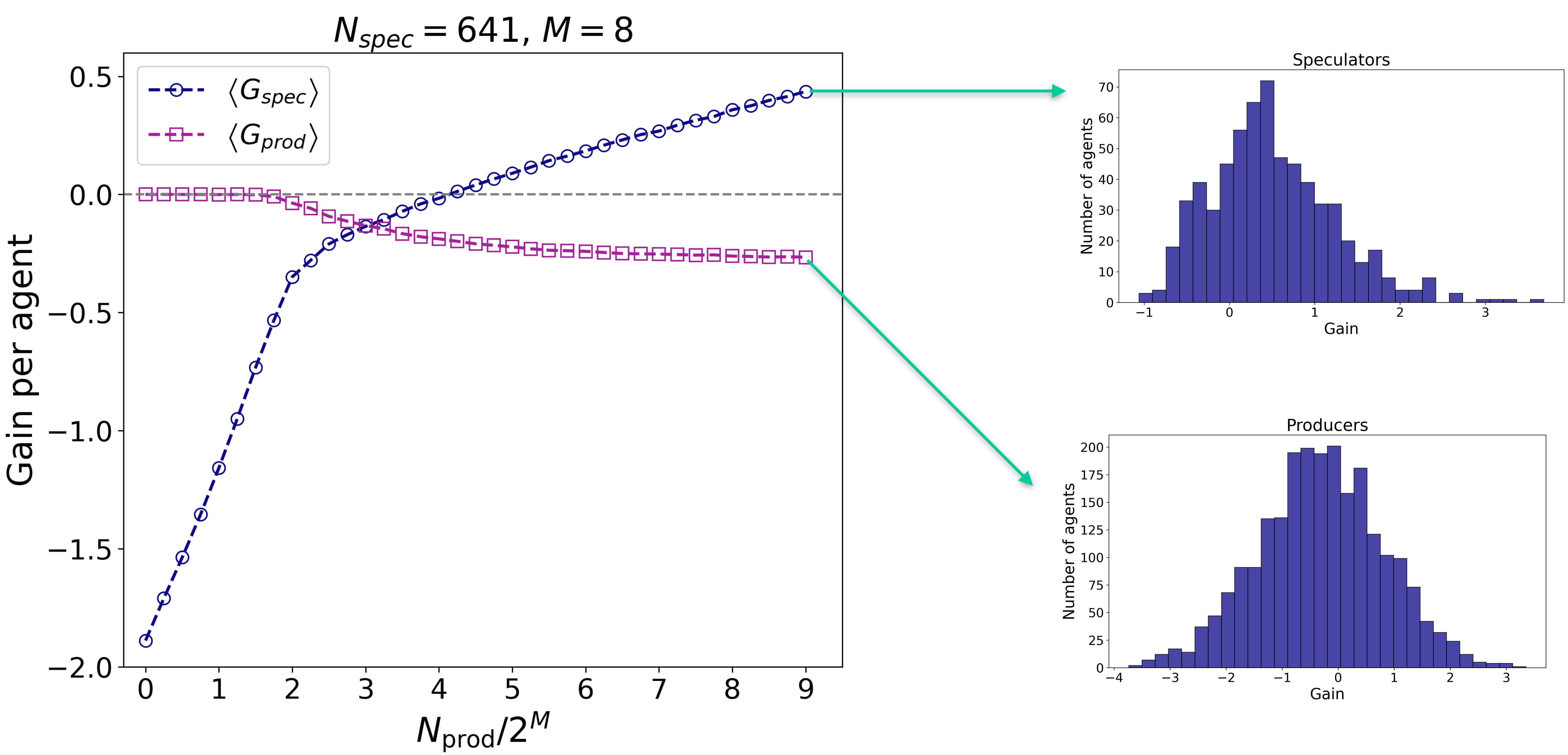

Summary & Conclusions

- We have implemented the **Minority Game** as a model for adaptive competition in financial markets.
- In the crowded regime, the system shows high **volatility (σ^2)** and a lack of coordination. Around the critical point α_c, agents self-organize optimally. For large α, the excess of information leads to uncoordinated and random behavior.
- **Predictability θ^2** serves as an **order parameter**, vanishing in the efficient regime and increasing beyond the critical point α_c, revealing predictability in the system.
- **Speculators** can **enhance** their **gains** by exploiting external information provided by **producers**.
- **Future Work:**
  - Consider noisy agents.
  - Communication between agents.

Main References

[1] Arthur, W. B. (1994). Inductive reasoning and bounded rationality. The American economic review, 84(2), 406-411.

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[3] Challet, D., Marsili, M., & Zhang, Y. C. (2004). Minority games: interacting agents in financial markets. OUP Oxford

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