Exploring Cooperation in Deterministic and Stochastic Spatial Prisoner's Dilemma

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EXCELENCIA
MARÍA
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2023 - 2027

The problem of cooperation:

How cooperation can emerge and persist despite the evolutionary advantage often held by selfish behavior is a central question in evolutionary biology.

Source: Britannica

Source: Current Biology

Prisoners' Dilemma:

Two-player game in which each participant can choose to cooperate (C) or defect (D) in any encounter. The outcomes of the interaction are given by the payoff matrix:

$$\begin{array}{c|cc} & C & D \\ \hline C & R & S \\ D & T & P \end{array} \longrightarrow \begin{array}{c|cc} & C & D \\ \hline C & 1 & 0 \\ D & b & 0 \end{array}$$

$$T > R > P > S$$

b: Temptation to defect

*If we set $P = \epsilon$ with $0 < \epsilon \ll 1$, the findings are qualitatively not altered.

How cooperative states emerge and persist in the spatial Prisoner's Dilemma under deterministic and stochastic strategy update rules.

Deterministic Iterative Prisoner's Dilemma

Novak & May, Nature 359(6398), 1992

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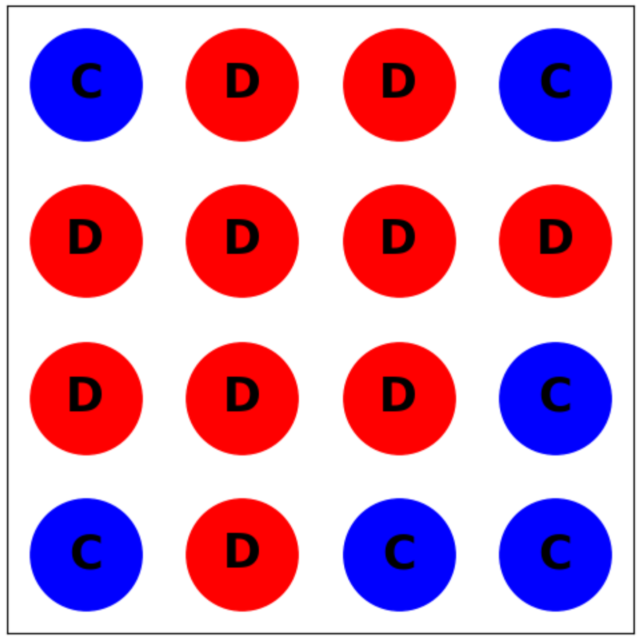
Initialize a $L \times L$ lattice with eight neighbors per site (Moore neighborhood) and periodic boundary conditions. Each site hosts a player who adopts one of the two strategies: cooperation (C) or defection (D).



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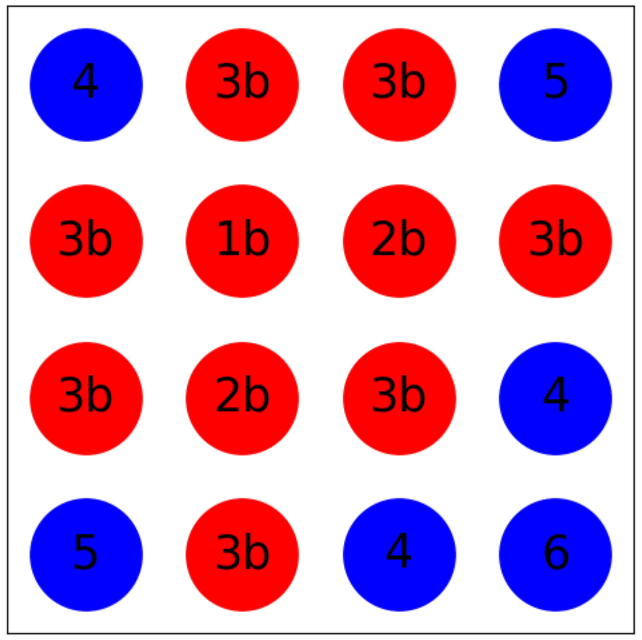

At each time step:

1. Each individual plays the game with their neighbors and with themselves.
2. The score will be the sum of the payoffs in those encounters.

Deterministic Model

Novak & May, Nature 359(6398), 1992

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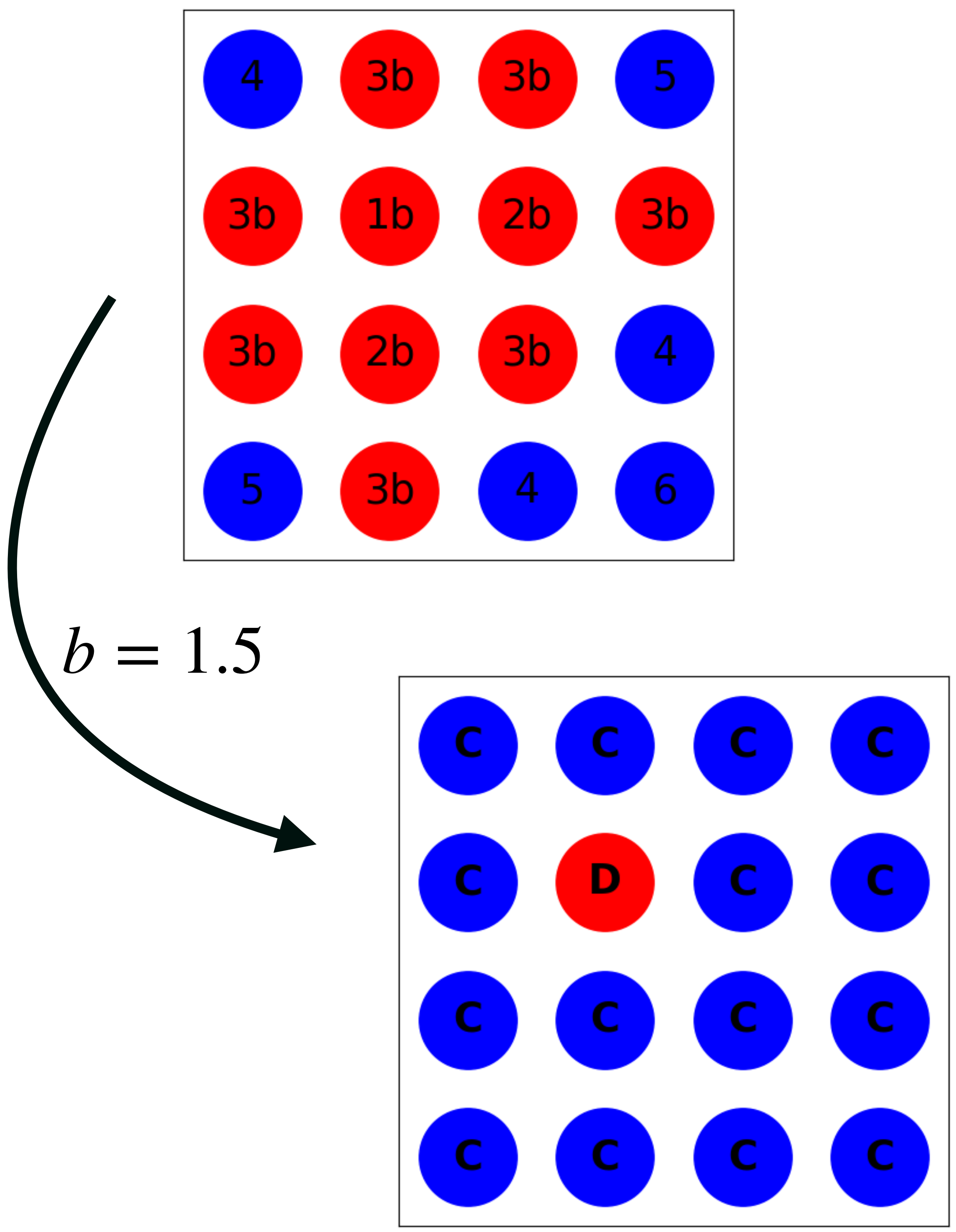
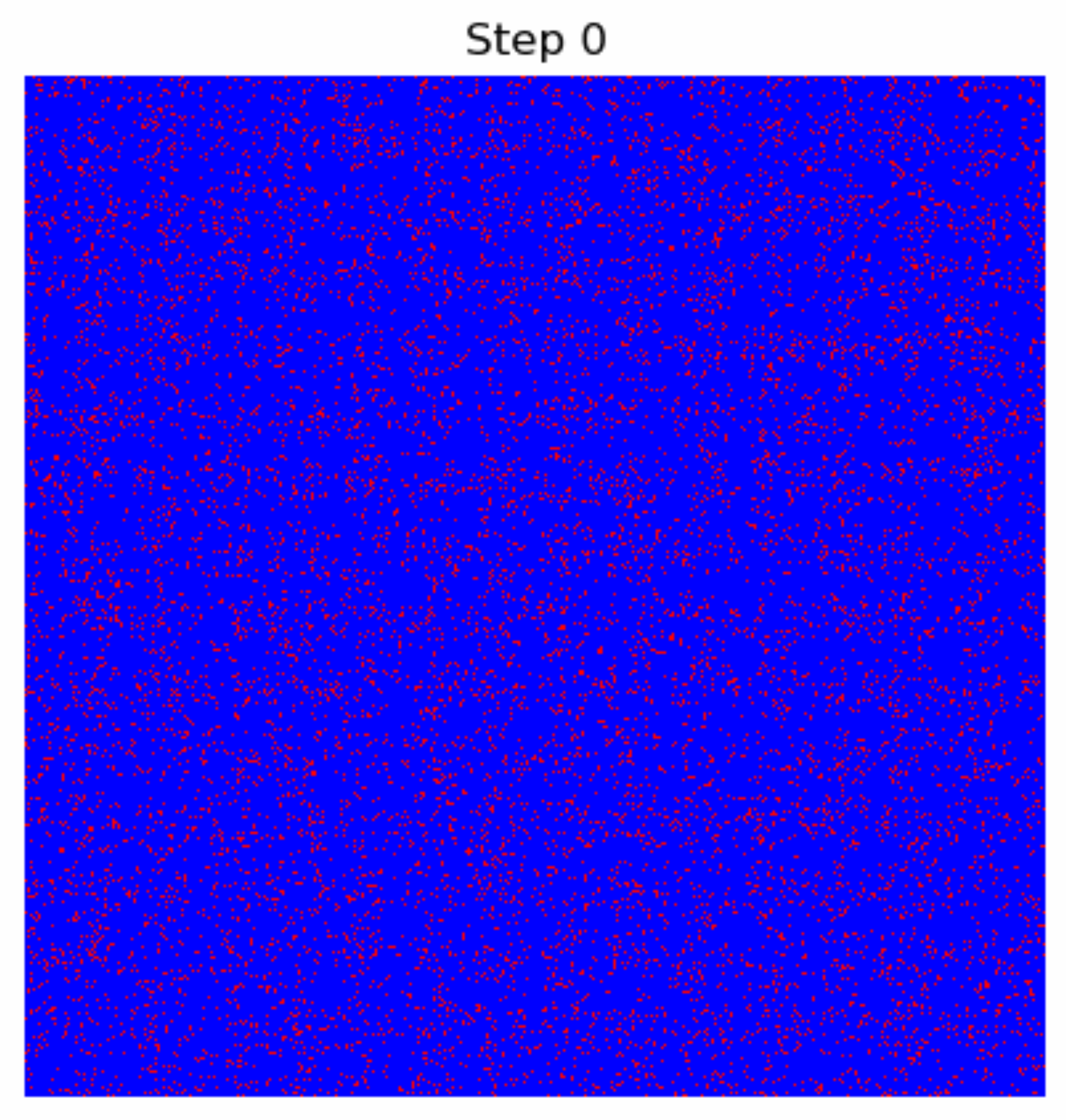


At each time step:

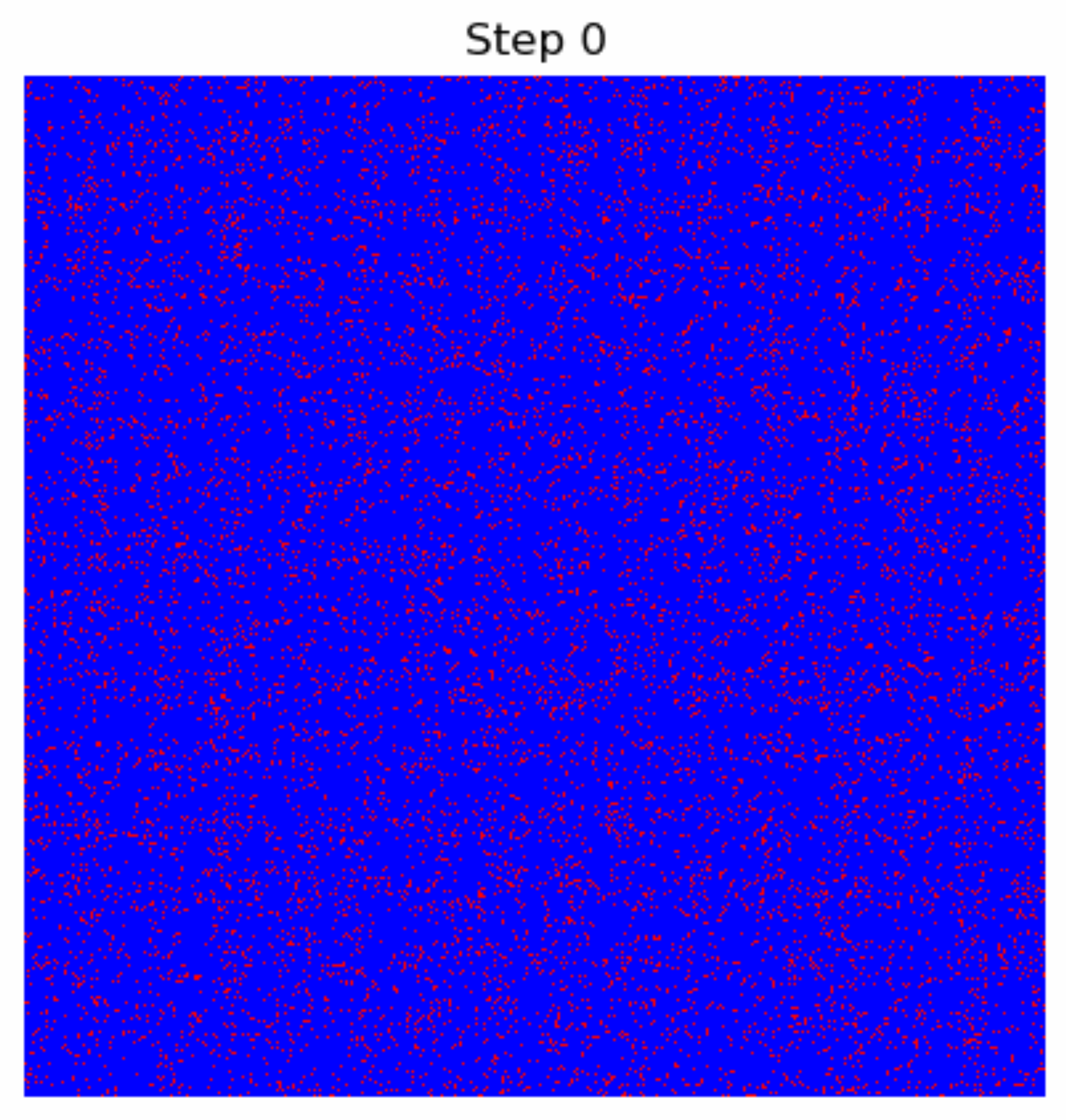
1. Each individual plays the game with their neighbors and with themselves.
2. The score will be the sum of the payoffs in those encounters.
3. Each lattice site is occupied by the player with the **highest score** among the owner of the site and its neighbors.

Simulations of the deterministic model for different b values.

$C \to C$ $D \to D$ $D \to C$ $C \to D$



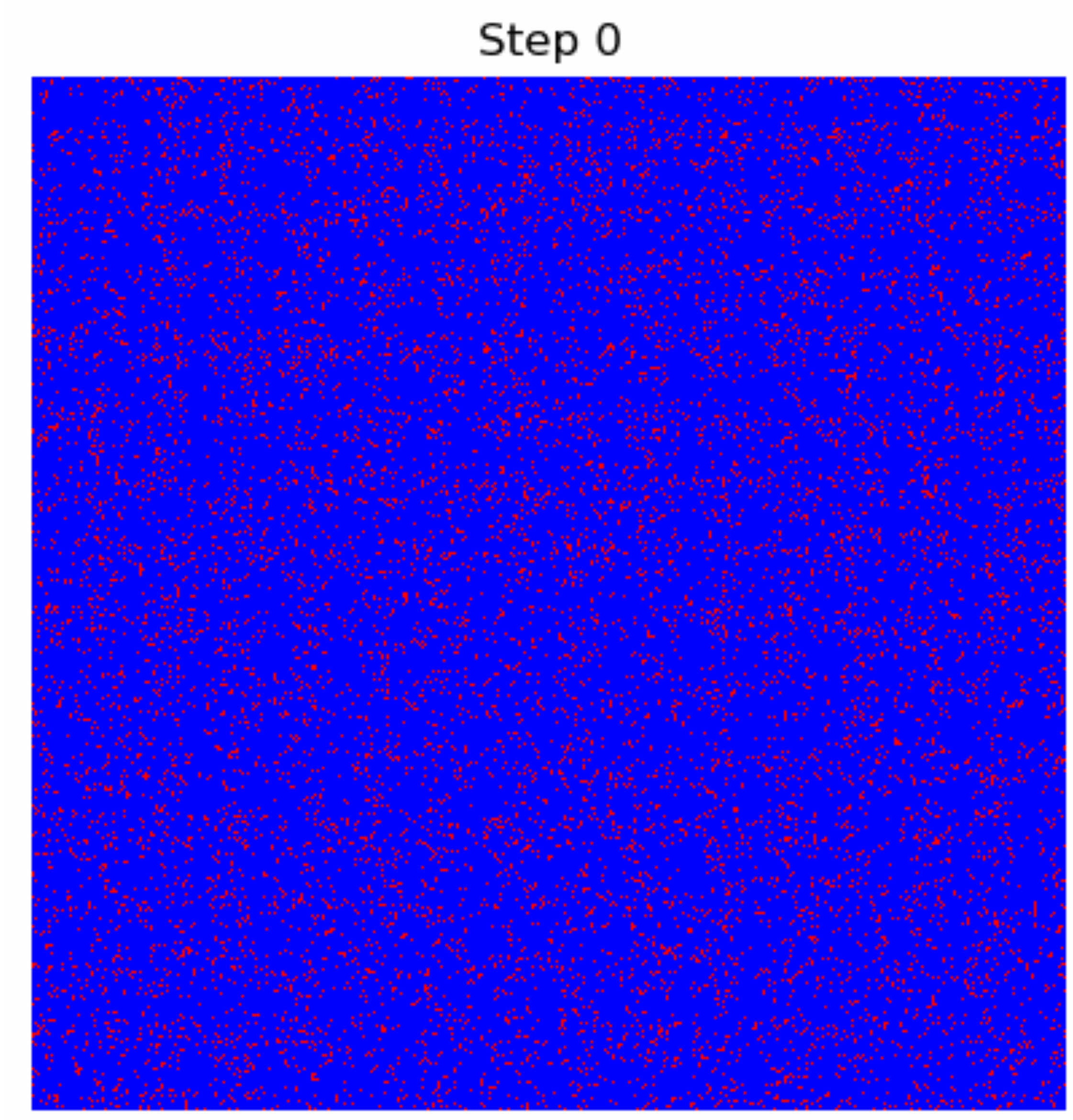
A few isolated defectors
$1 < b < 1.125$ (e.g., $b = 1.05$).



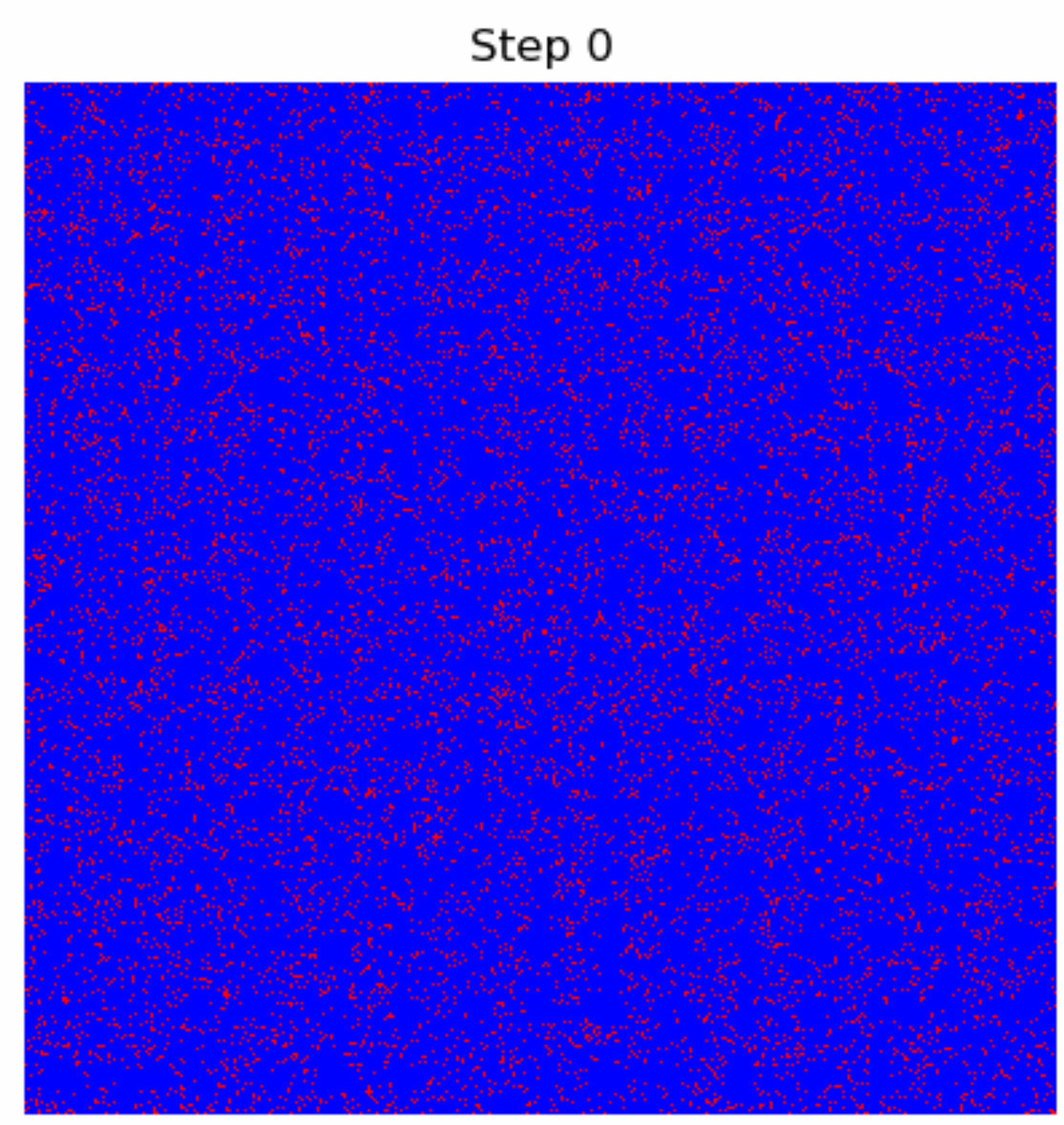
Interconnected lines of D with local oscillations
$1.125 \leq b < 1.8$ (e.g., $b = 1.78$).

Simulations of the deterministic model for different b values.

$C \to C$ $D \to D$ $D \to C$ $C \to D$



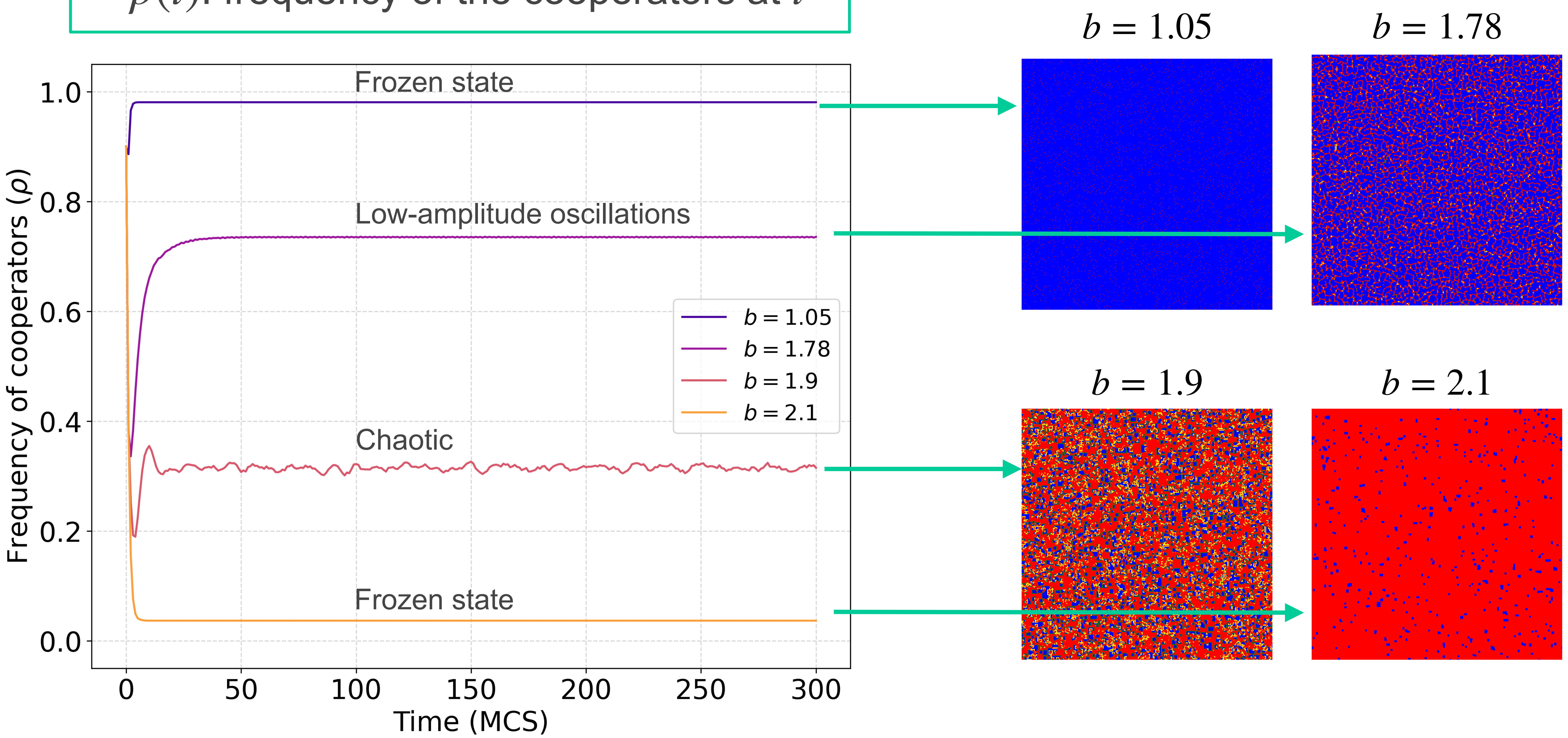
Dynamic coexistence regime
$1.80 \leq b < 2$ (e.g., $b = 1.9$).



Cooperators forming rectangular blocks resistant to defectors' invasions
$2 \leq b < 3$ (e.g., $b = 2.1$).

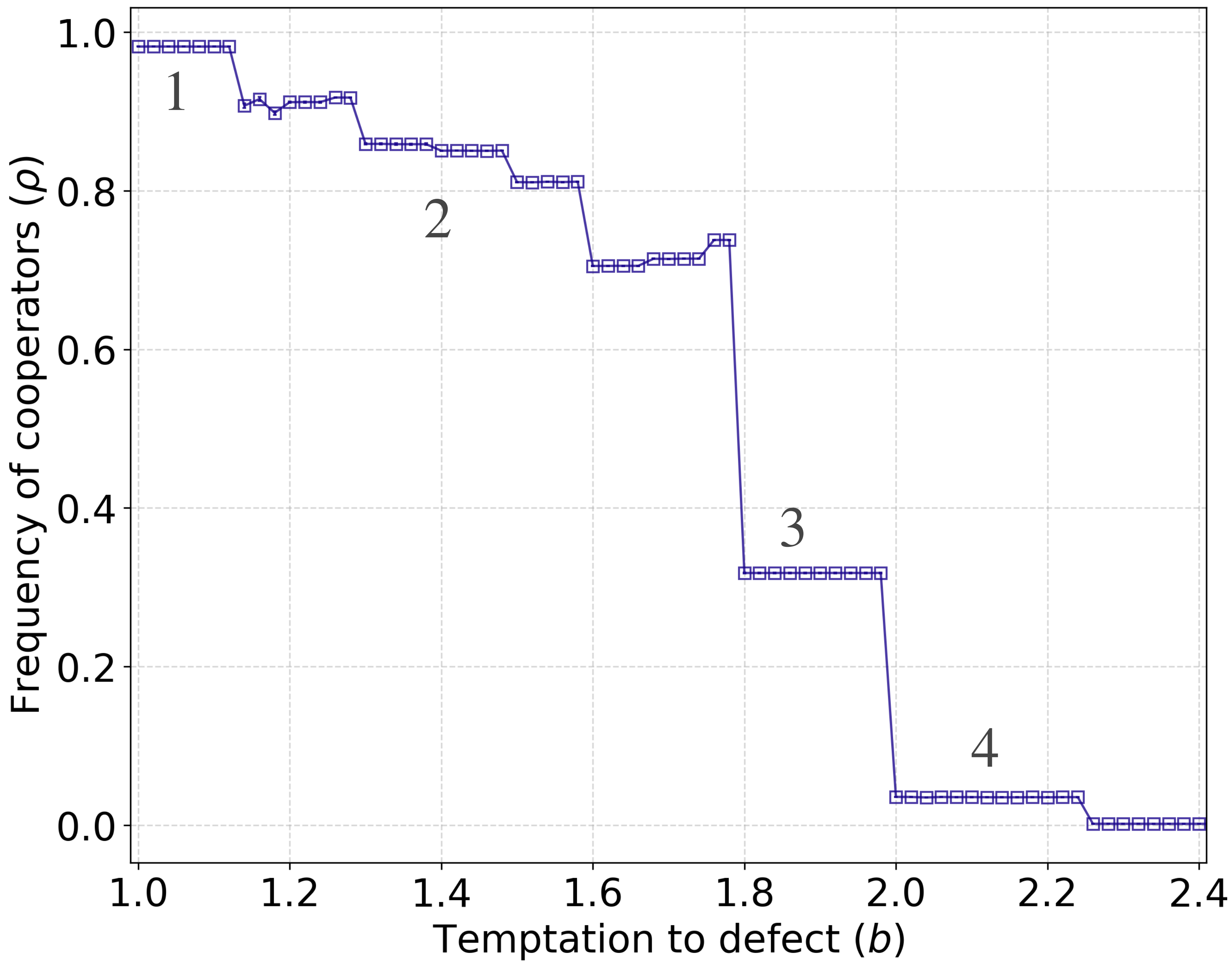
Time evolution of the deterministic system for different b values.

- $\rho(t)$: frequency of the cooperators at t



Asymptotic state of the deterministic system for different b values.

- ρ: frequency of the cooperators



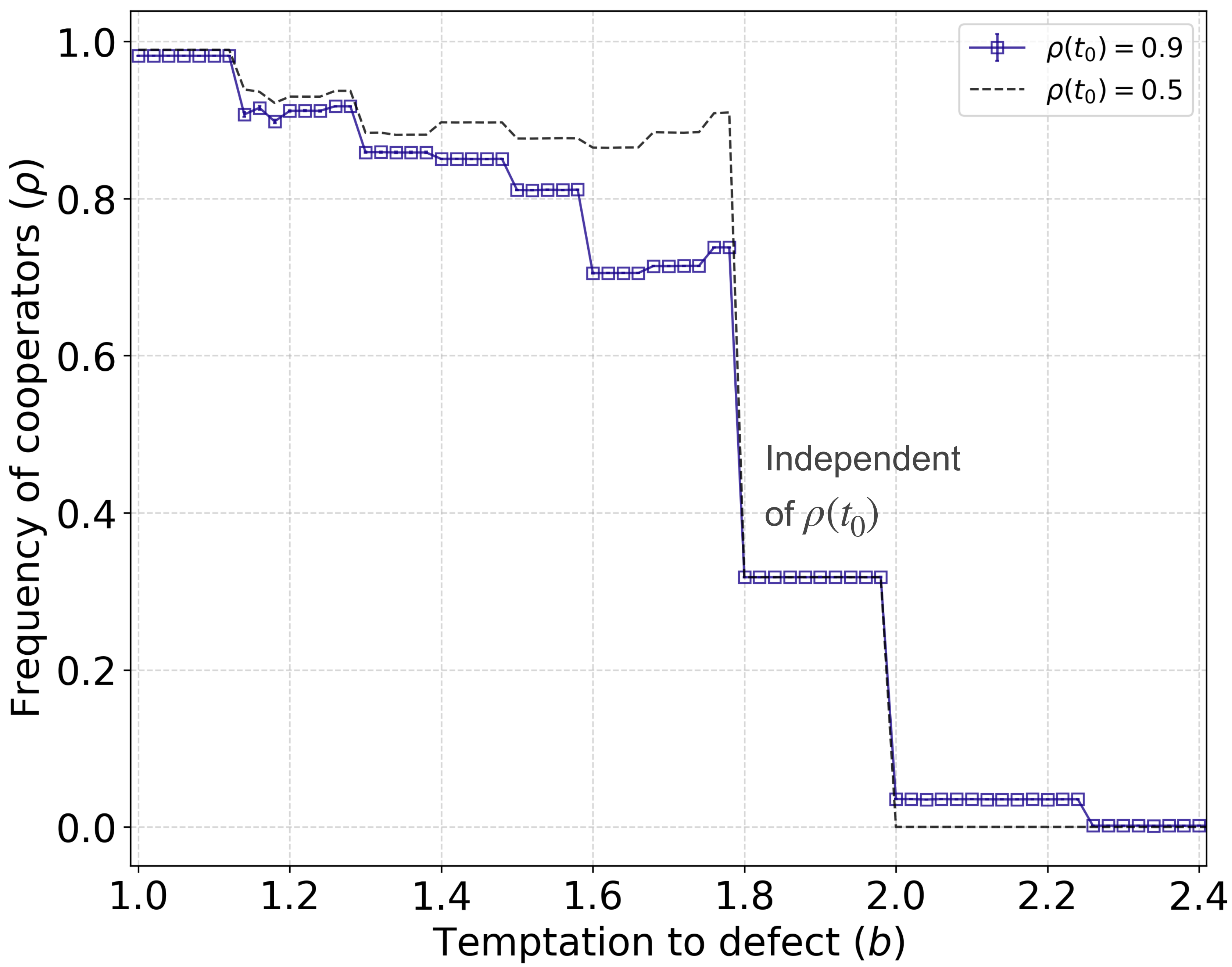
Different phases found:

1. Frozen state, with high ρ:
   $1 < b < 1.125.$
2. Local oscillations with different periodicity:
   $1.125 \leq b < 1.8.$
3. Chaotic behavior:
   $1.8 \leq b < 2.$
4. Frozen state, with low ρ*:
   $b \geq 2.$

*Beyond $b = 3$ cooperators extinguish.

Asymptotic state of the deterministic system for different initial conditons.

- $\rho(t_0)$: frequency of the cooperators at t_0



- Sensitivity to initial conditions in certain ranges of the parameter b.
- Abrupt changes slightly varying b.

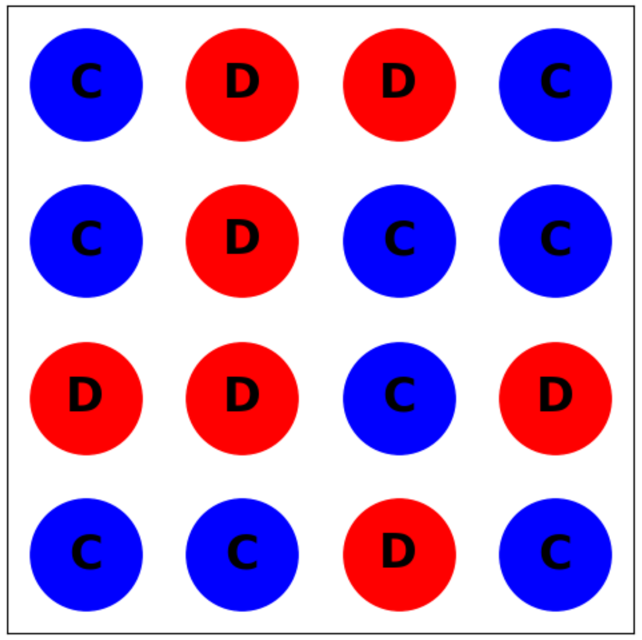
Motivations for a stochastic model.

Stochastic Iterative Prisoner's Dilemma

Szabó & Fath, Phys. Rep, 446(4-6), 2007

Szabó & Fath, Phys. Rep, 446(4-6), 2007

Initialize a $L \times L$ lattice with eight neighbors per site (Moore neighborhood) and periodic boundary conditions. Each site hosts a player who adopts one of the two strategies: cooperation (C) or defection (D).



Stochastic Model

Szabó & Fath, Phys. Rep, 446(4-6), 2007

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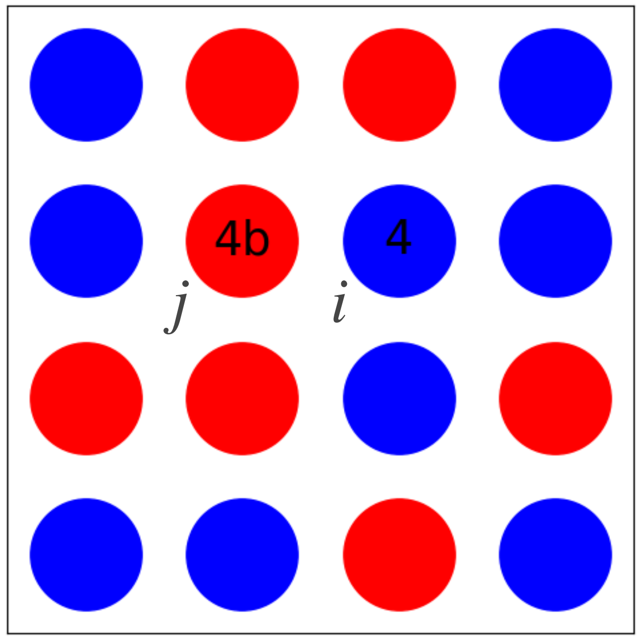

At each time step:

1. A player i and one of its neighbors j, with strategies s_i and s_j, are randomly selected.

2. Both individuals play the Prisoner's Dilemma game with all their immediate neighbors and themselves, and their total payoffs Π_i and Π_j are calculated.

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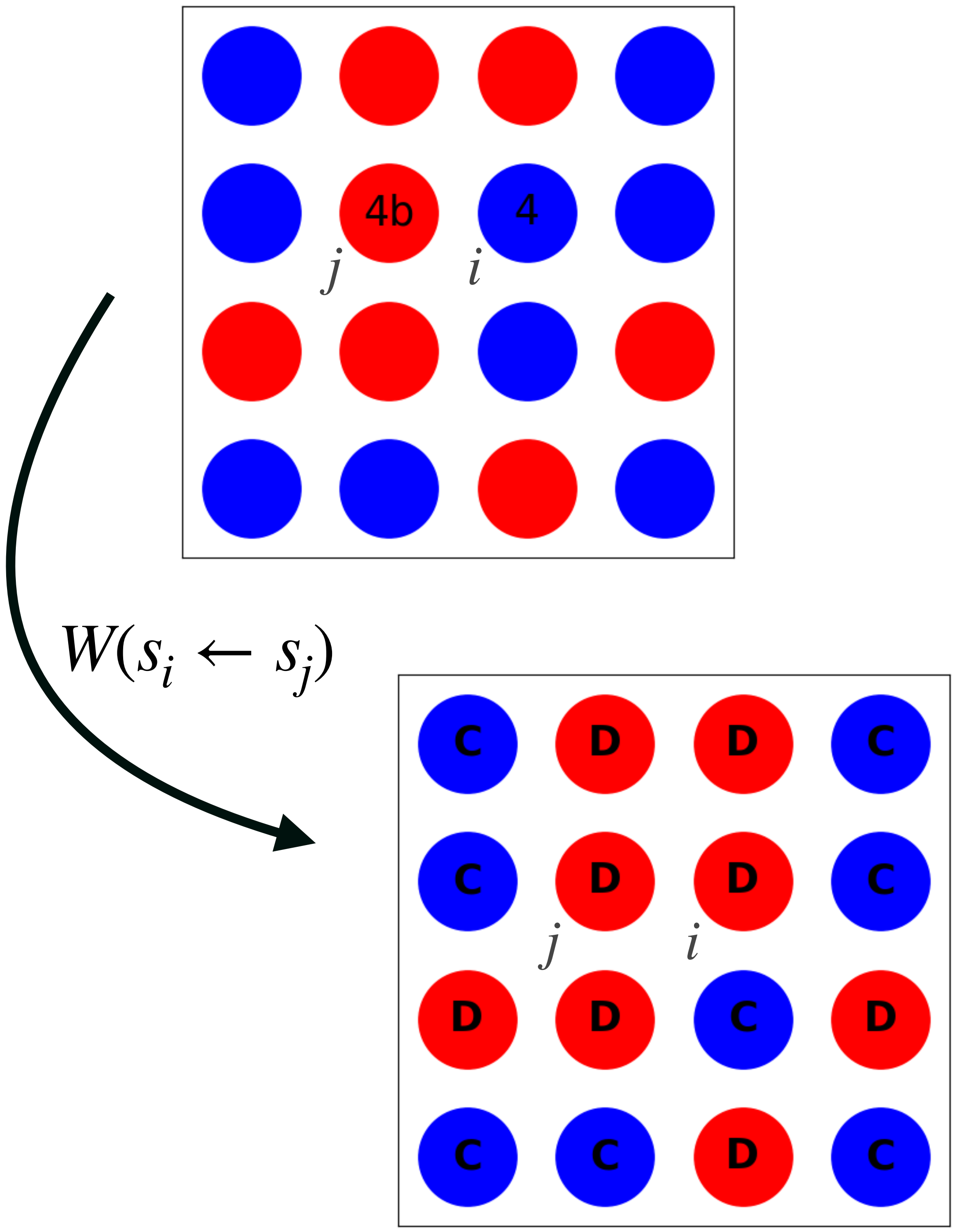


At each time step:

1. A player i and one of its neighbors j, with strategies s_i and s_j, are randomly selected.

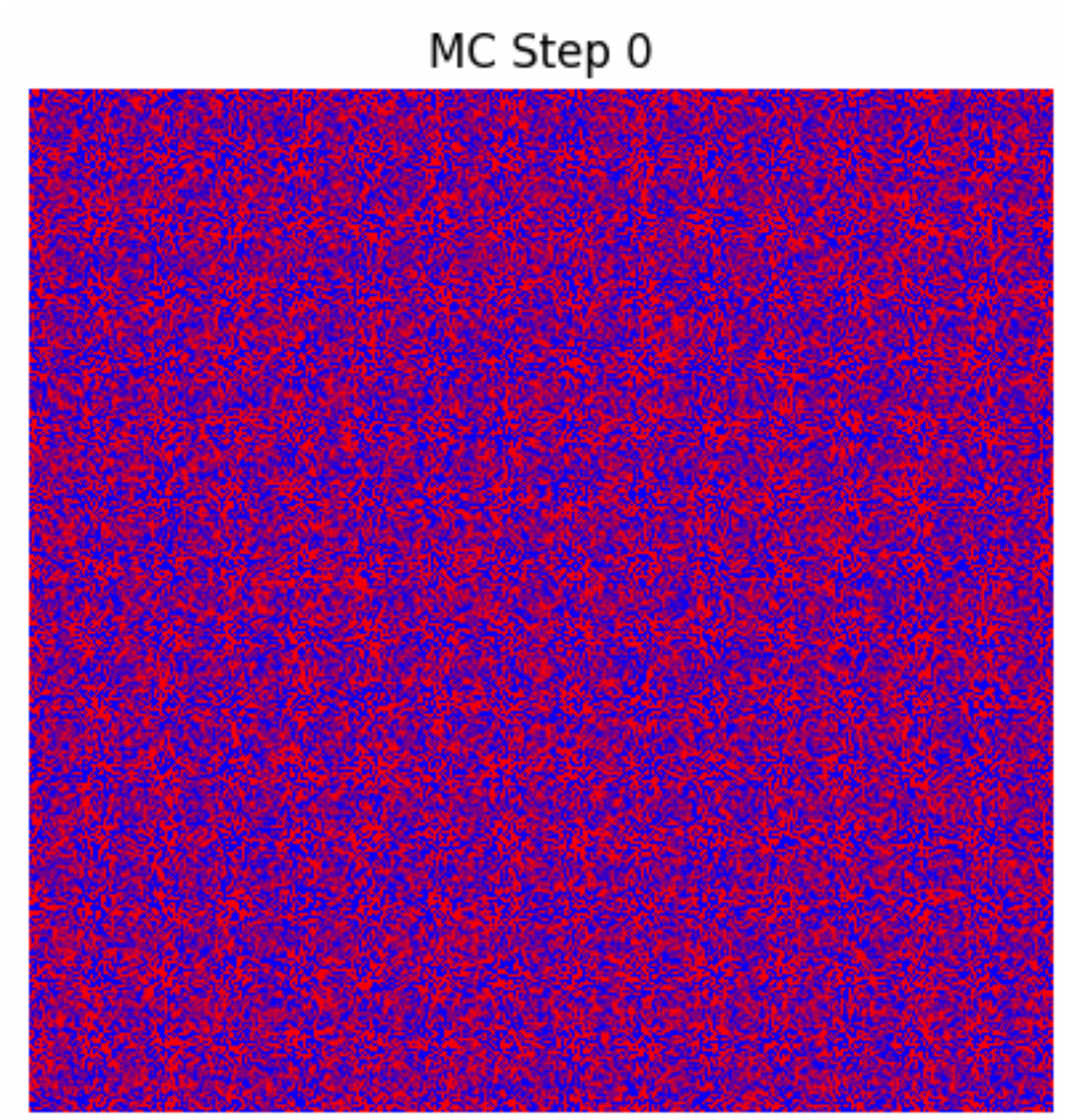
2. Both individuals play the Prisoner's Dilemma game with all their immediate neighbors and themselves, and their total payoffs Π_i and Π_j are calculated.

3. Player i adopts the strategy of the player j with probability:

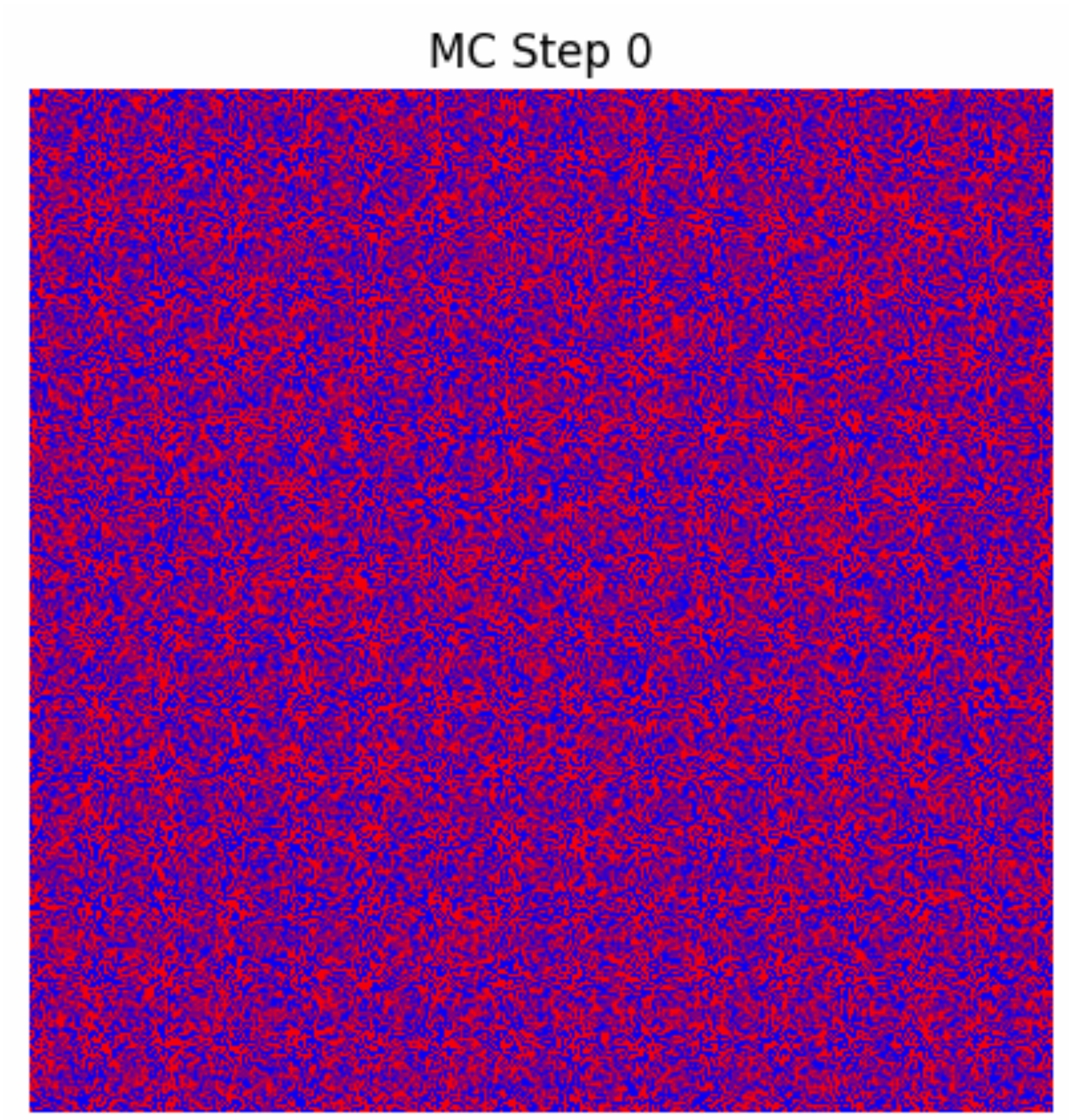
$$W(s_i \leftarrow s_j) = \frac{1}{1 + \exp\left(\frac{\Pi_i - \Pi_j}{K}\right)}$$

Simulation of the stochastic model for different b values.

Cooperators
Defectors



Completely cooperative state
$b < b_1^c$ (e.g., $b = 1.12$).

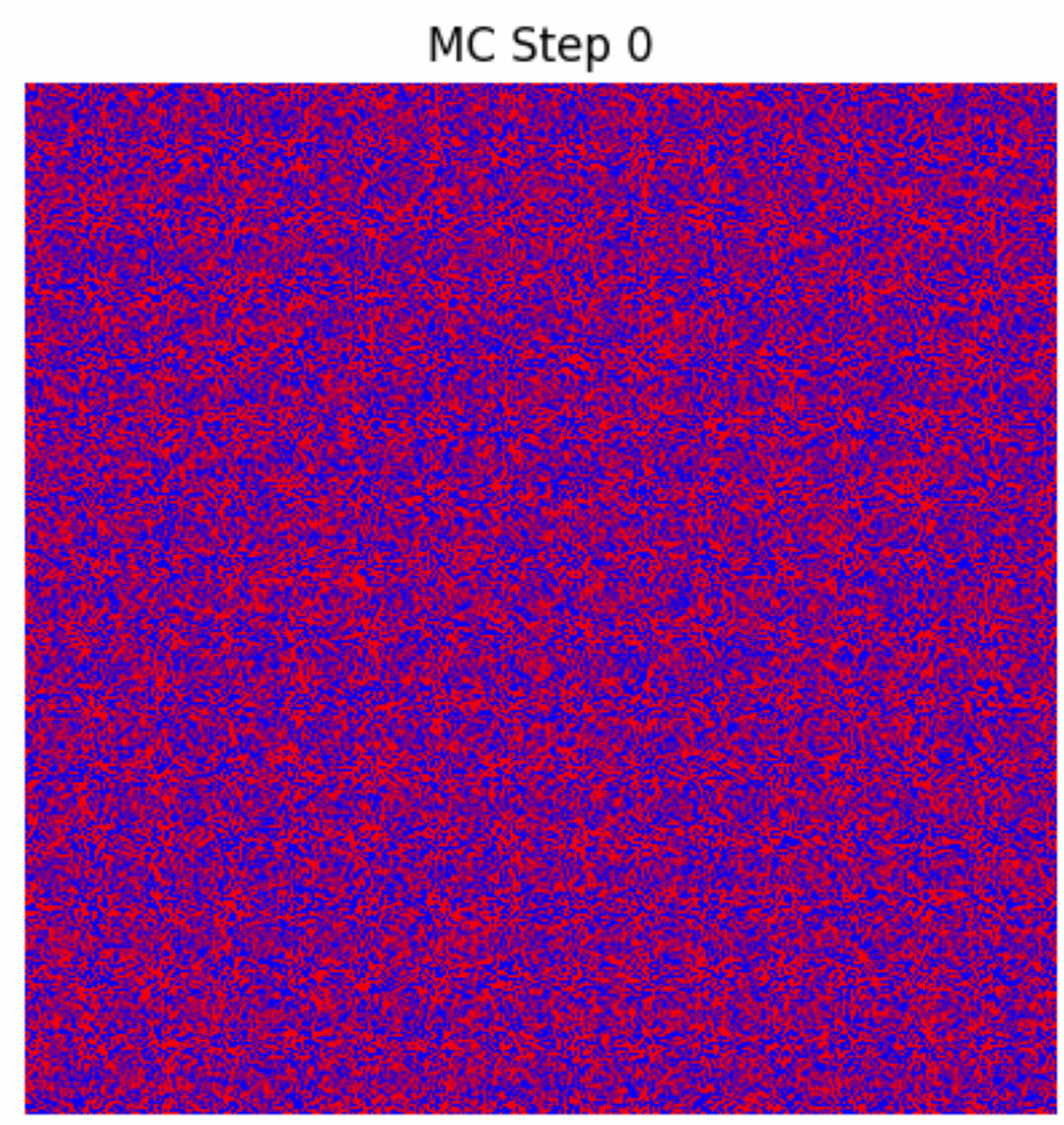


Coexistence of strategies (high ρ)
$b_1^c < b < b_2^c$ (e.g., $b = 1.2$).

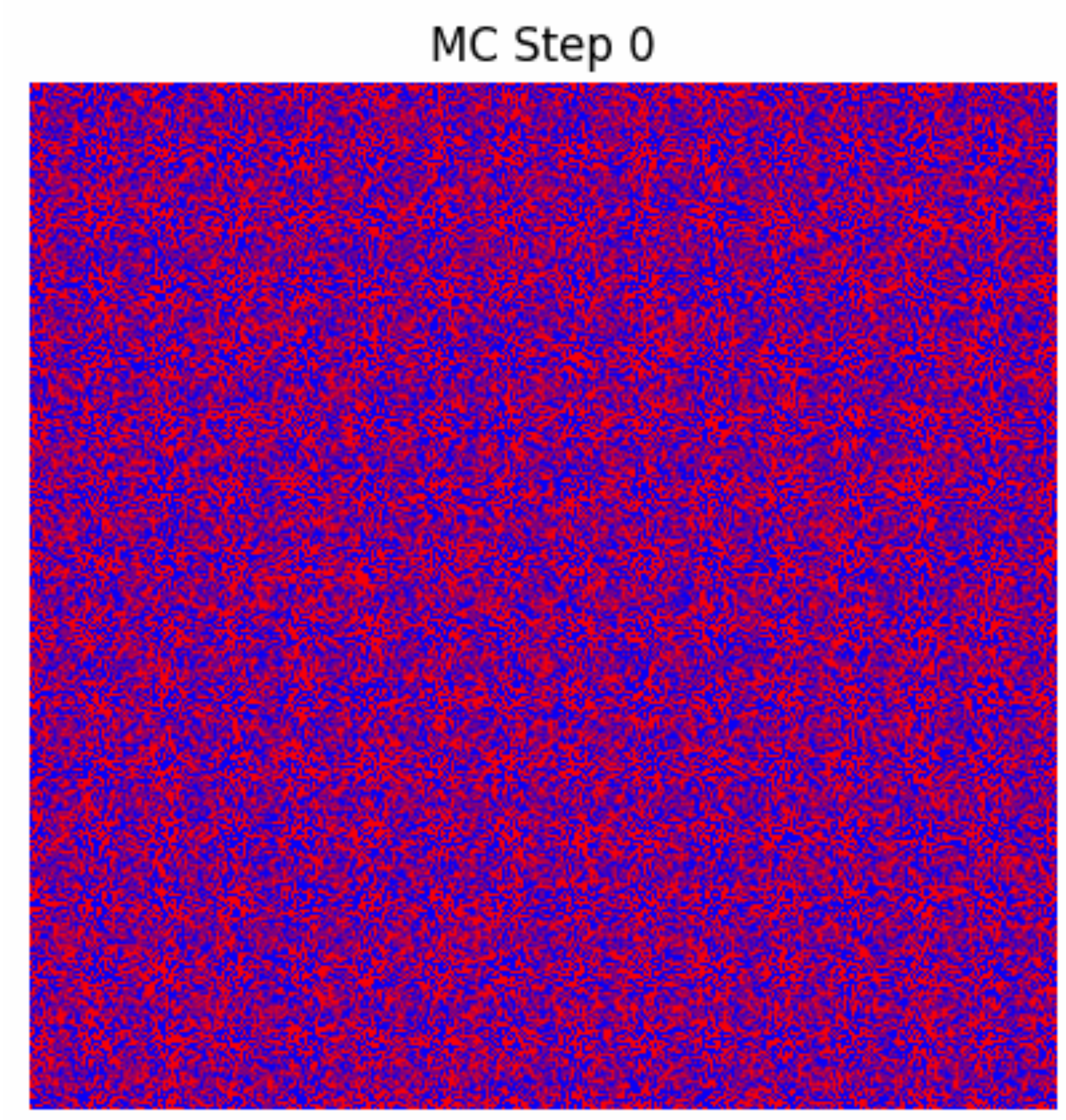
Fixed: $K = 0.1$

Simulation of the stochastic model for different b values.

Cooperators
Defectors



Coexistence of strategies (low ρ)
$b_1^c < b < b_2^c$ (e.g., $b = 1.35$).

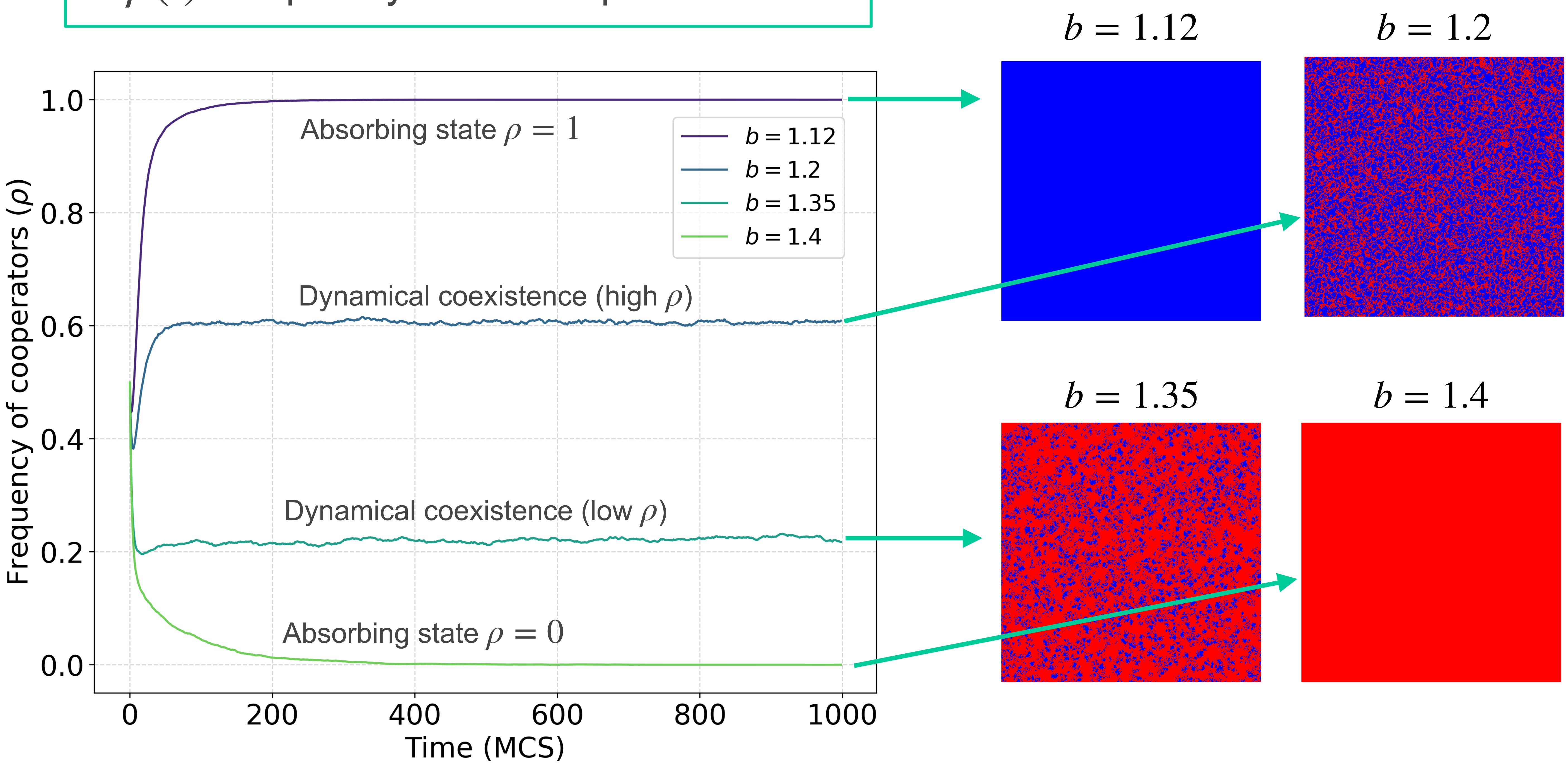


Extinction of cooperators
$b > b_2^c$ (e.g., $b = 1.4$).

Fixed: $K = 0.1$

Time evolution of the stochastic system for different b values.

- $\rho(t)$: frequency of the cooperators at t



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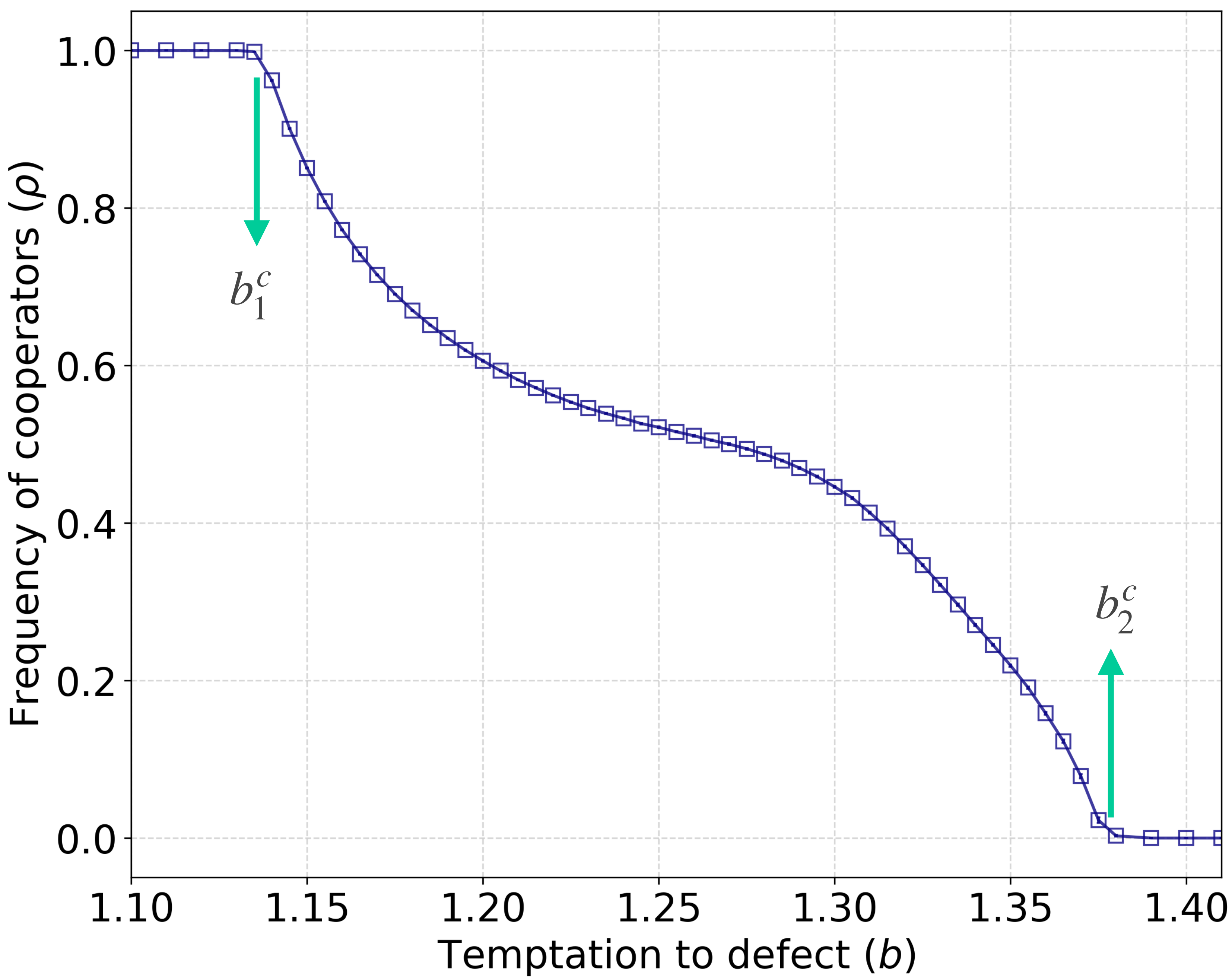
Asymptotic state of the deterministic system for different b values.

- ρ: frequency of the cooperators



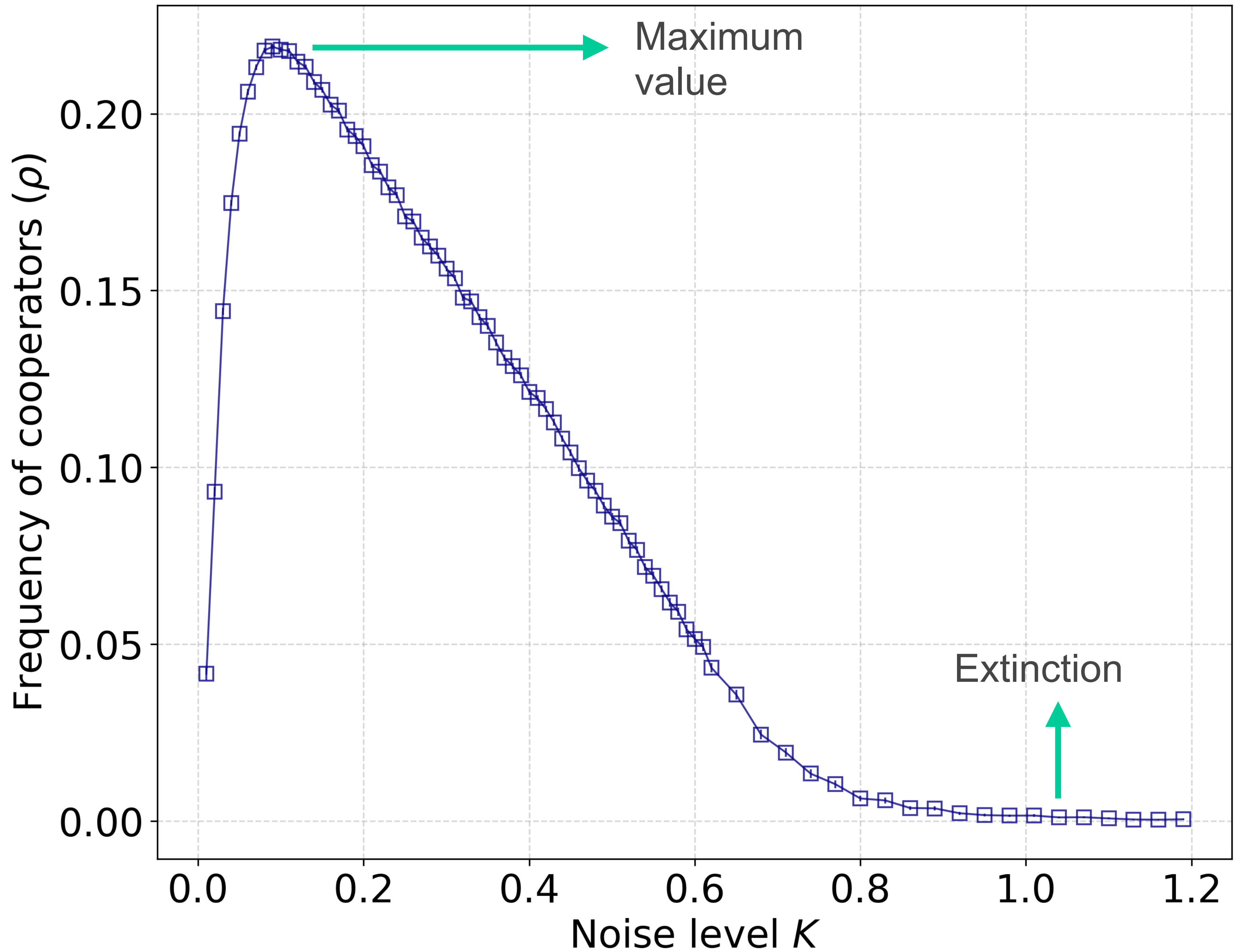
Fixed: $K = 0.1$

Significant changes to the dynamics of cooperation:

- Smooth transition from complete cooperation at b_1^c to the extinction of cooperators at b_2^c.
- For $K = 0.1$, transitions occur at $b_1^c \approx 1.13$ and $b_2^c \approx 1.38$.

Asymptotic state of the deterministic system for different K values.

- ρ: frequency of the cooperators



There exists an optimal level of noise K that maximizes cooperation.

Fixed: $b = 1.35$

- We have implemented both **deterministic** and **stochastic** versions of the iterative Prisoner's Dilemma on a regular lattice.

- In the classical and deterministic model, the system is **highly sensitive** to the temptation to defect b. **Small changes in b** can lead to different dynamics, including **static configurations**, low-amplitude **oscillations**, or even **chaotic** patterns.

- The stochastic model exhibits a **monotonic transition** as a function of b, **from** a **completely cooperative** state **to** an **extinction of cooperation**, which occurs **at lower values of b** than in the deterministic case. There is an **optimal value K** that can **enhance cooperative** behavior.

- **Future Work:**
    - Consider other network topologies.
    - Incorporating inter-species interactions.

Main References

[1] Hauert, C., & Szabó, G. (2005). Game theory and physics. American Journal of Physics, 73(5), 405-414.

[2] Hauert, C., & Szabó, G. (2024). Spontaneous symmetry breaking of cooperation between species. *PNAS nexus*, *3*(9), pgae326.

[3] Nowak, M. A. (2006). Evolutionary dynamics: exploring the equations of life. Harvard university press.

[4] Nowak, M. A., & May, R. M. (1992). Evolutionary games and spatial chaos. *nature*, *359*(6398), 826-829.

[5] Nowak, M. A., & May, R. M. (1993). The spatial dilemmas of evolution. International Journal of bifurcation and chaos, 3(01), 35-78.

[6] Szabó, G., & Fath, G. (2007). Evolutionary games on graphs. *Physics reports*, *446*(4-6), 97-216.

[7] Szabó, G., & Tőke, C. (1998). Evolutionary prisoner’s dilemma game on a square lattice. *Physical Review E*, *58*(1), 69.

THANK YOU

for your attention

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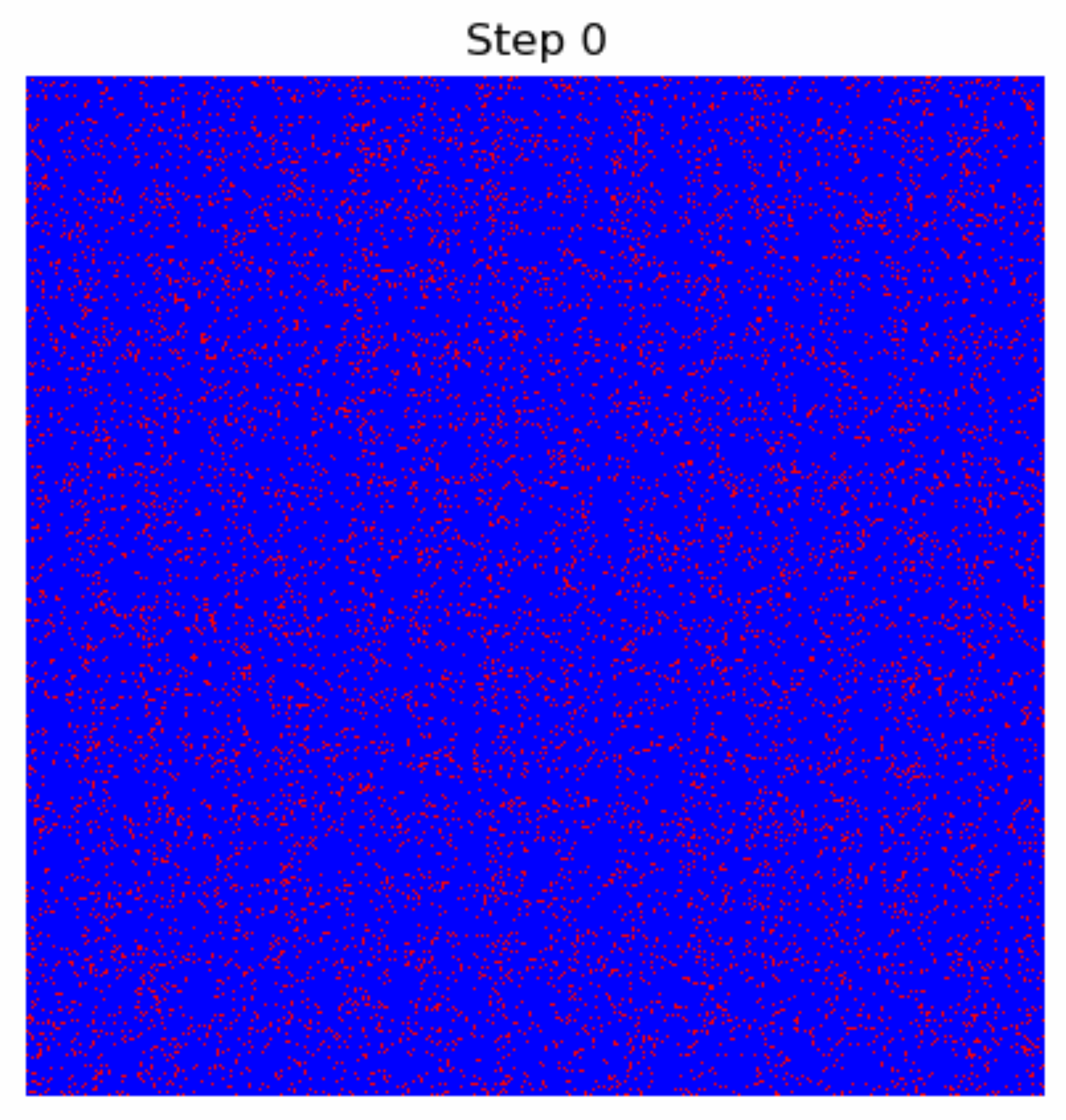
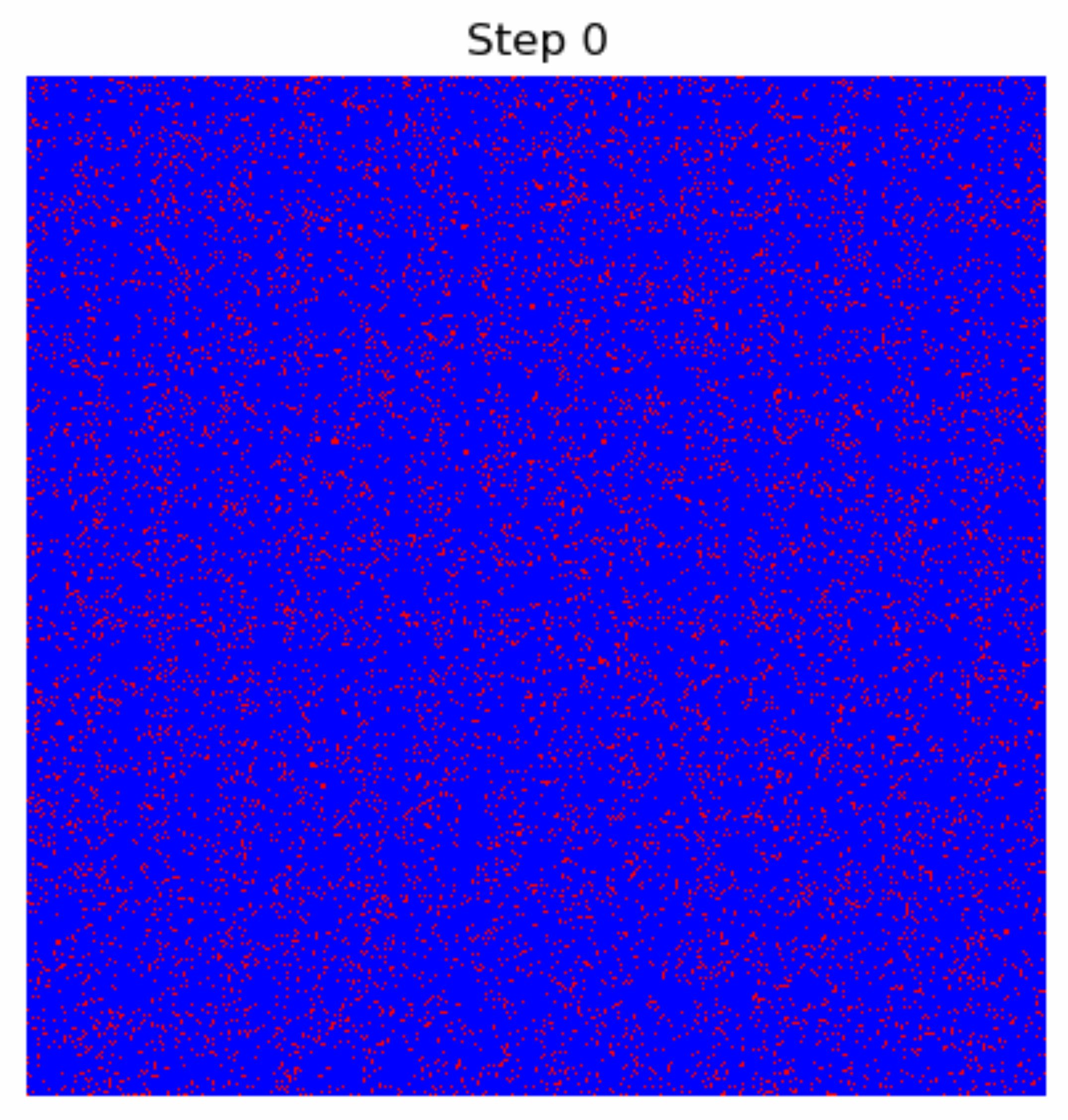
Extra Slides

Simulations of the deterministic model for different b values.

$C \to C$ $D \to D$ $D \to C$ $C \to D$



Interconnected lines of D with local oscillations $1.125 \leq b < 1.8$ (e.g., $b = 1.13$).



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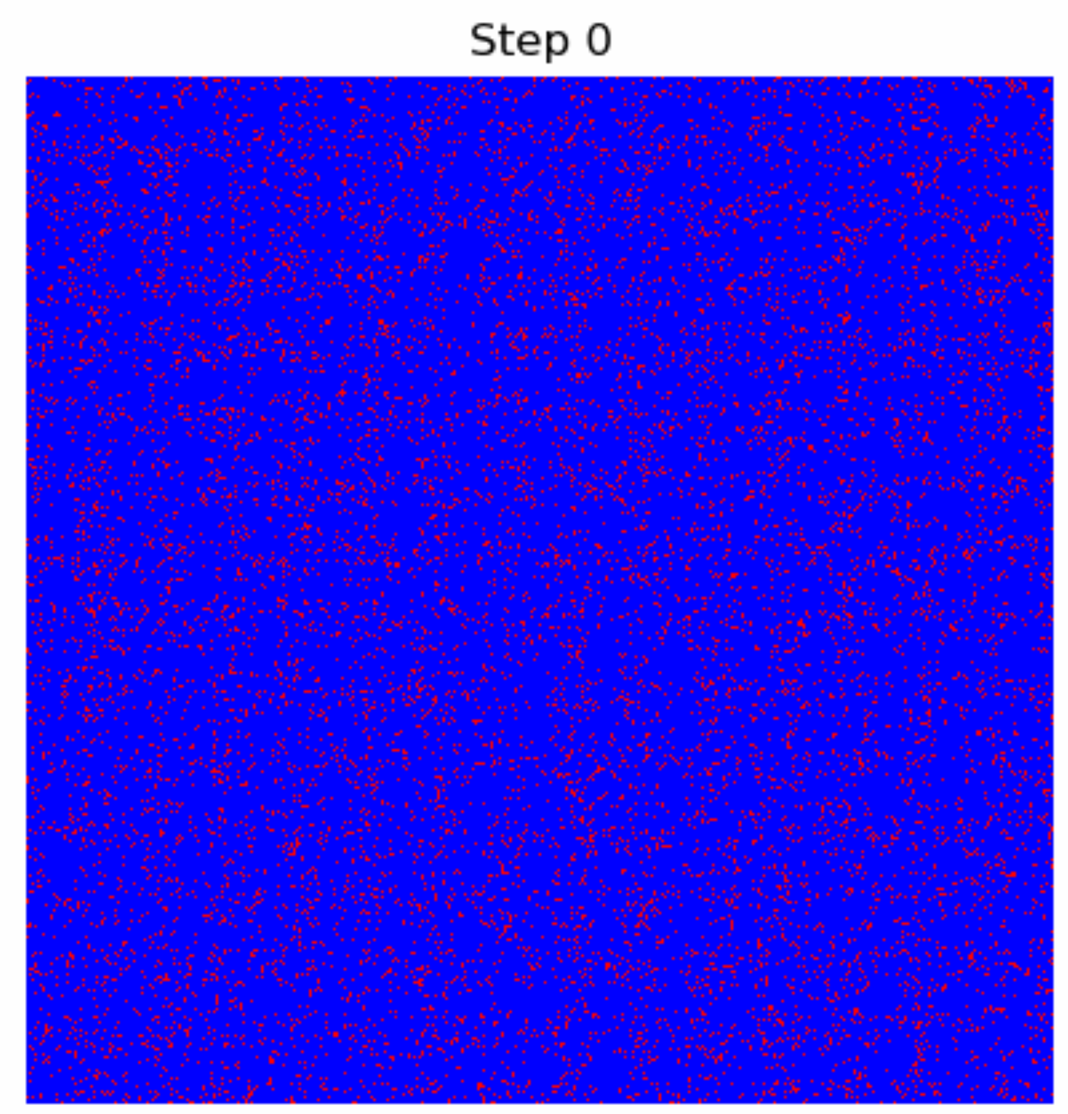

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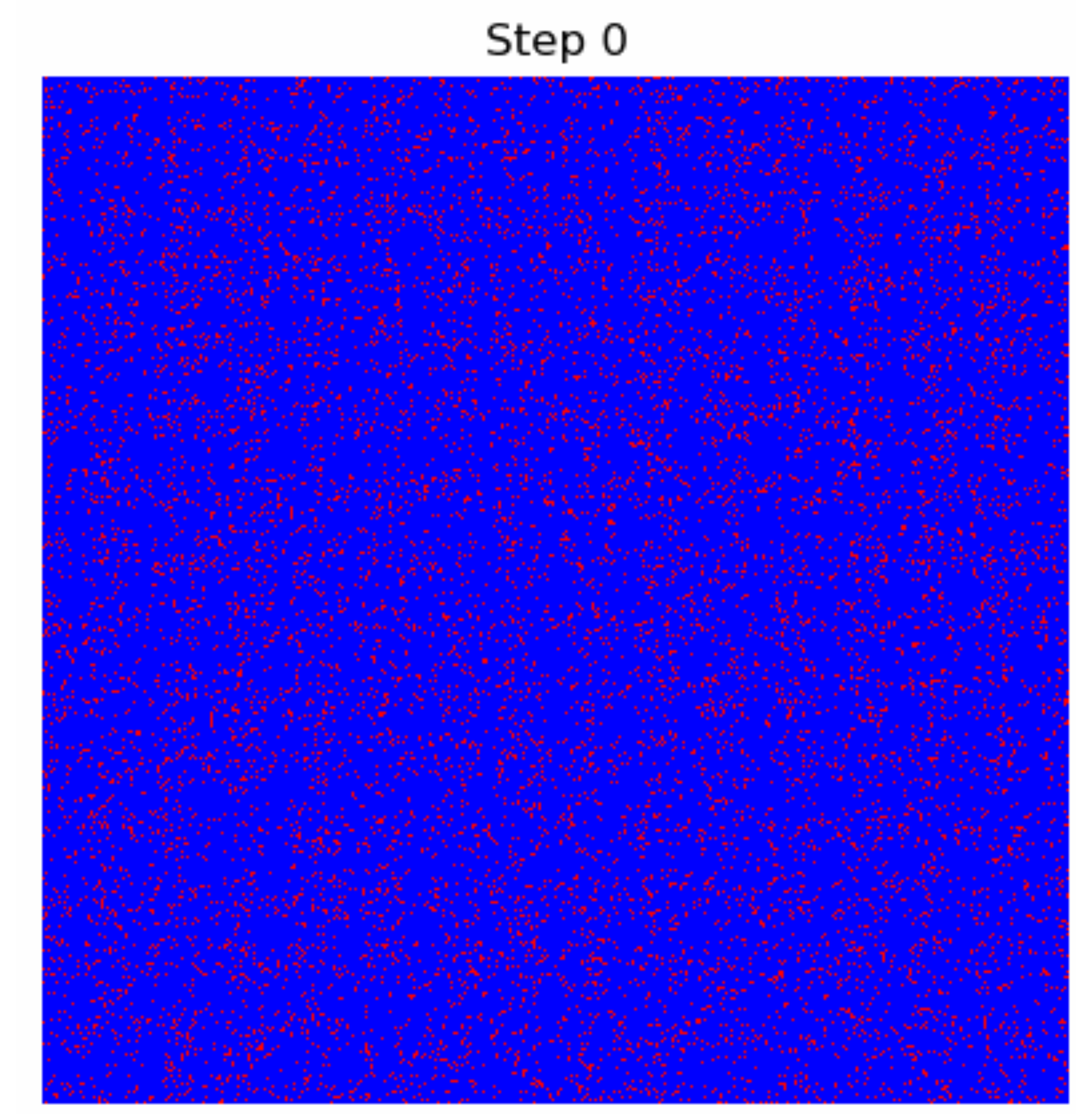
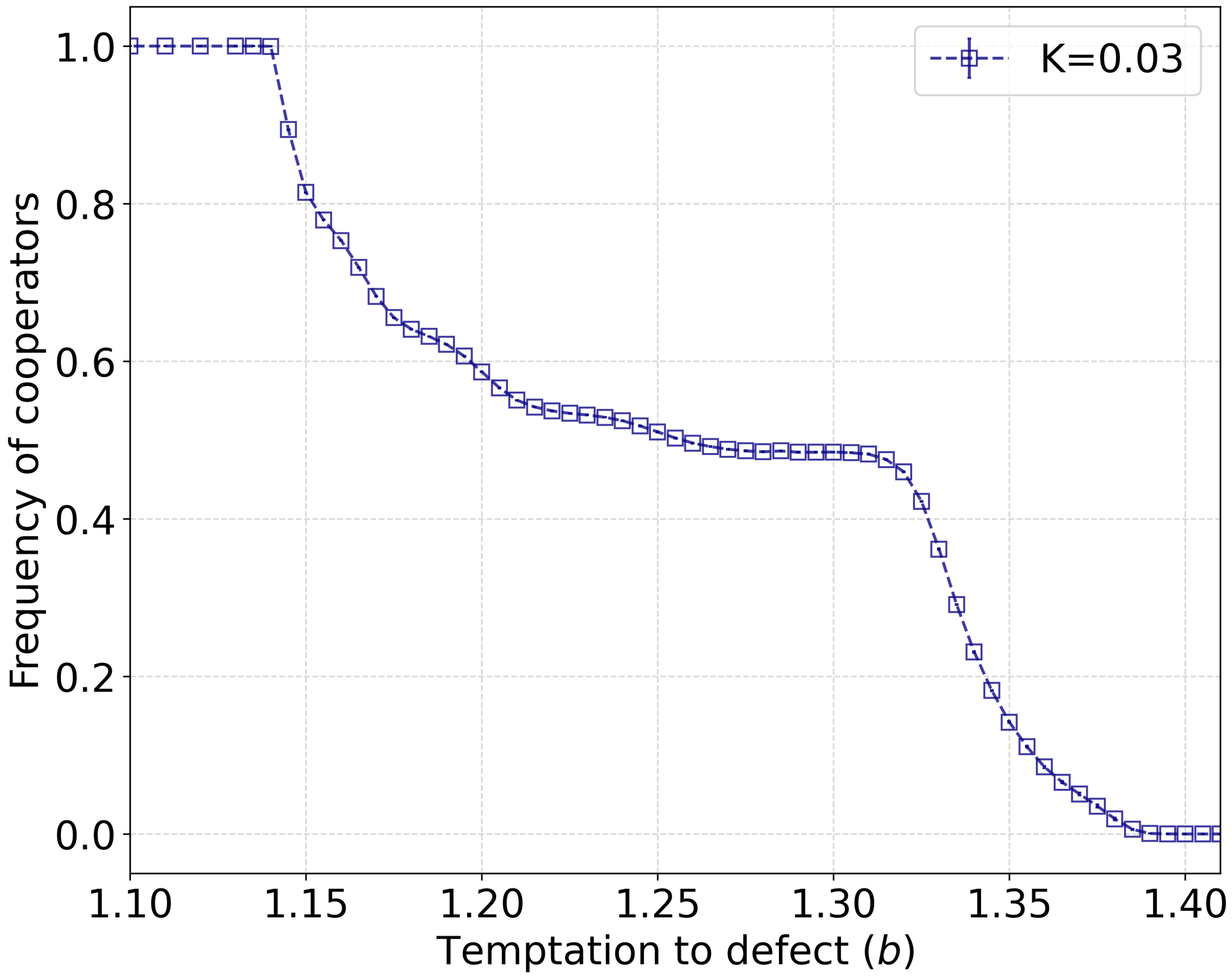


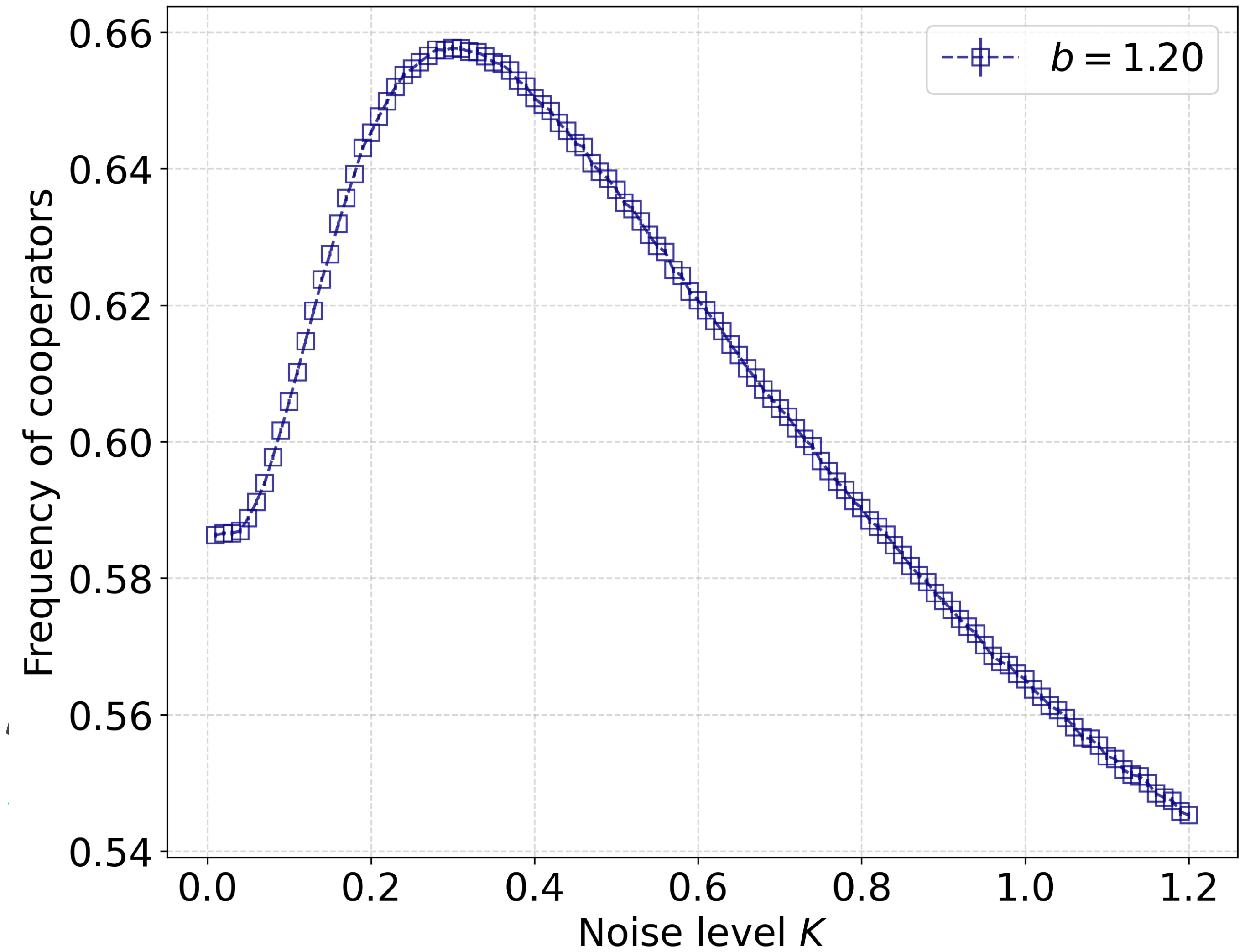
Extinction of cooperators
$b \ge 3$ (e.g., $b = 3$).

Asymptotic state of the deterministic system for different K and b values.

- ρ: frequency of the cooperators



Fixed: $K = 0.03$



Fixed: $b = 1.20$