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① $$\int \frac{x+1}{x^2+2x+3}\cdot dx = \int \frac{(x^2+2x+3)'\cdot\frac{1}{2}}{x^2+2x+3}dx = \frac{1}{2}\cdot\int\frac{(x^2+2x+3)'}{x^2+2x+3}dx$$

$$= \frac{1}{2}\cdot\ln(x^2+2x+3)+C$$

② $$\int \frac{e^{2x}}{e^{2x}+1}dx = \int \frac{(e^{2x}+1)'}{e^{2x}+1}\cdot\frac{1}{2}\cdot dx =$$

$$= \frac{1}{2}\cdot\ln(e^{2x}+1)+C$$

③ $$\int x\cdot\sqrt{1-x^2}\,dx = \int -\frac{1}{2}\cdot(-x^2)'\cdot\sqrt{1-x^2} = -\frac{1}{2}\cdot\int(-x^2)'\cdot(1-x^2)^{\frac{1}{2}} =$$

$$= -\frac{1}{2}\cdot\frac{(1-x^2)^{\frac{3}{2}}}{3/2}+C$$

④ $$\int x^3\cdot(1-2x^4)^{2023}dx = \int -\frac{1}{8}\cdot(-2x^4)'\cdot(1-2x^4)^{2023}dx =$$

$$= -\frac{1}{8}\cdot\frac{(-2x^4)^{2024}}{2024}+c$$

(5) $$\int \frac{e^x}{\sqrt[3]{1+e^x}}\,dx = \int e^x \cdot (1+e^x)^{-\frac{1}{3}}\,dx =$$

$$= \frac{(1+e^x)^{2/3}}{2/3} + C$$

(6) $$\int \frac{2x+3}{x^2+2x+3}\,dx = \int \frac{2x+2+1}{x^2+2x+3}\,dx =$$

$$= \int \frac{(x^2+2x+3)'}{x^2+2x+3}\,dx + \int \frac{1}{x^2+2x+3}\,dx =$$

$$= \ln(x^2+2x+3) + \int \frac{1}{(x+1)^2+2}\,dx =$$

$$= \ln(x^2+2x+3) + \frac{1}{2} \cdot \int \frac{1}{\left(\frac{x+1}{\sqrt{2}}\right)^2+1}\,dx =$$

$$= \ln(x^2+2x+3) + \frac{1}{2} \cdot \frac{\operatorname{arctg}\left(\frac{x+1}{\sqrt{2}}\right)}{\frac{1}{\sqrt{2}}} =$$

$$= \ln(x^2+2x+3) + \frac{\sqrt{2}}{2} \cdot \operatorname{arctg}\left(\frac{x+1}{\sqrt{2}}\right) + C$$

⑦ $$\int \frac{x^2+3}{x^2-1}\,dx = \int \frac{x^2-1+4}{x^2-1}\,dx = \int 1-\frac{4}{x^2-1}\,dx =$$

$$x-4\cdot \operatorname{arccth}(x)+C$$

⑧ $$\int \frac{1}{(x^2+1)\cdot \operatorname{arctg}(x)}\,dx = \int \frac{\frac{1}{(x^2+1)}}{\operatorname{arctg}(x)}\,dx =$$

$$= \int \frac{(\operatorname{arctg}(x))'}{\operatorname{arctg}(x)}\,dx = \ln(\operatorname{arctg}(x))+C$$

⑨ $$\int \frac{\operatorname{tg}(x)}{(\ln(\cos x))^6}\,dx = \int \operatorname{tg}(x)\cdot(\ln(\cos x))^{-6}\,dx =$$

$$-\int -(\ln(\cos x))'\cdot(\ln(\cos x))^{-6}\,dx = -\frac{(\ln(\cos x))^{-5}}{-5}+C$$

(10) $\int \sqrt{\frac{1+x}{1-x}} + \sqrt{\frac{1-x}{1+x}}\,dx = \int \frac{\sqrt{1+x}}{\sqrt{1-x}} \cdot \frac{\sqrt{1+x}}{\sqrt{1+x}} + \int \frac{\sqrt{1-x}}{\sqrt{1+x}} \cdot \frac{\sqrt{1-x}}{\sqrt{1-x}} =$

$$\int \frac{1+x}{\sqrt{1-x^2}} + \int \frac{1-x}{\sqrt{1-x^2}} = \int \frac{2}{\sqrt{1-x^2}} = 2 \cdot \arcsin(x) + C$$

(11) $\int \frac{x^2}{1+x^2}dx = \int \frac{x^2+1-1}{1+x^2}dx = \int 1 - \frac{1}{1+x^2}\,dx =$

$$x - \operatorname{arctg}(x) + C$$

(12) $\int \frac{e^{3x}+1}{e^x+1}dx = \int \frac{\cancel{(e^x+1)}(e^{2x}-e^x+1)}{\cancel{(e^x+1)}}dx = \int e^{2x} - e^x + 1\,dx$

$$= \frac{e^{2x}}{2} - e^x + x + C$$

13 $$\int \sqrt{1-\sin(2x)}\,dx = \int \sqrt{\sin^2 x+\cos^2 x-2\sin x\cdot\cos x}\,dx =$$

$$\int \sqrt{(\sin x-\cos x)^2}\,dx = \int \sin x-\cos x = -\cos x-\sin x + C$$

14 $$\int \frac{e^{2x}}{\sqrt{1+3e^{2x}}}\,dx = \int e^{2x}\cdot\left(1+3e^{2x}\right)^{-\frac{1}{2}}\,dx =$$

$$\frac{1}{6}\int \left(3e^{2x}\right)'\cdot\left(1+3e^{2x}\right)^{-\frac{1}{2}}\,dx = \frac{1}{6}\cdot\frac{\left(1+3e^{2x}\right)^{\frac{1}{2}}}{1/2} + C$$

15 $$\int \frac{e^{x}}{\sqrt[3]{1+e^{x}}}\,dx = \int e^{x}\cdot\left(1+e^{x}\right)^{-\frac{1}{3}}\,dx = \frac{\left(1+e^{x}\right)^{\frac{2}{3}}}{2/3} + C$$

16 $$\int \frac{9x^2}{\sqrt{2-3x^3}}\,dx = \int 9x^2\cdot\left(2-3x^3\right)^{-\frac{1}{2}}\,dx = -\int\left(-3x^3\right)'\cdot\left(2-3x^3\right)^{-\frac{1}{2}}\,dx =$$

$$-2\cdot\left(2-3x^3\right)^{\frac{1}{2}} + C$$

(17) $\int \frac{2x-5}{\sqrt[4]{(x^2-5x+13)^3}}\,dx = \int 2x-5 \cdot (x^2-5x+13)^{-\frac{3}{4}}\,dx =$

$$= \int (x^2-5x+13)' \cdot (x^2-5x+13)^{-\frac{3}{4}}\,dx =$$

$$= \frac{(x^2-5x+13)^{1/4}}{1/4} + C$$

(18) $\int x^2 \cdot e^{2x}\,dx = \int x^2 \cdot \left(\frac{e^{2x}}{2}\right)' dx =$

$$= \frac{1}{2} \cdot x^2 \cdot e^{2x} - \underbrace{\int x \cdot e^{2x}\,dx}_{} =$$

$$\int x \cdot \frac{e^{2x}}{2}\,dx =$$

$$= \frac{1}{2} \cdot x \cdot e^{2x} - \int \frac{e^{2x}}{2}\,dx =$$

$$= \frac{1}{2} \cdot x^2 \cdot e^{2x} - \frac{1}{2} \cdot x \cdot e^{2x} - \frac{1}{2} \cdot \frac{e^{2x}}{2} + C$$

(19) $$\int x^2 \cdot \sin(5x)\,dx = \int x^2 \cdot \left(\frac{\cos(5x)}{5}\right)' dx =$$

$$= \frac{1}{5} \cdot x^2 \cdot \cos(5x) - \underbrace{\int \frac{1}{5} \cdot 2x \cdot \cos(5x)\,dx}_{\frac{1}{5} \cdot \int 2x \left(\frac{\sin(5x)}{5}\right)' dx} =$$

$$\underbrace{\frac{1}{5} \cdot \int 2x \left(\frac{\sin(5x)}{5}\right)' dx}_{2x \cdot \frac{1}{5} \cdot - \sin(5x) - \int 2 \cdot \frac{1}{5} \cdot (\sin 5x)\,dx} =$$

$$2x \cdot \frac{1}{5} \cdot - \sin(5x) - \int 2 \cdot \frac{1}{5} \cdot (\sin 5x)\,dx =$$

$$\frac{1}{5} \cdot x^2 \cdot \cos(5x) + \frac{1}{25} \cdot 2x \cdot \sin(5x) + \frac{2}{25} \cdot \frac{\cos(5x)}{5} + C$$

(20) $$\int x \cdot \ln^2(x^3)\,dx = \int \left(\frac{x^2}{2}\right)' \cdot \ln^2(x^3)\,dx =$$

$$\frac{x^2}{2} \cdot \ln^2(x^3) - \int \frac{\cancel{x^2}}{\cancel{2}} \cdot \cancel{2} \cdot \ln(x^3) \cdot \frac{3x^{\cancel{2}}}{x^{\cancel{3}}}\,dx =$$

$$\frac{x^2}{2} \cdot \ln^2(x^3) - \int \ln(x^3) \cdot 3x\,dx =$$

$$\frac{x^2}{2} \cdot \ln^2(x^3) - \left(\ln(x^3) \cdot \frac{3x^2}{2} - \int \frac{3x^2}{2} \cdot \frac{3x^2}{x^3}\right) =$$

$$\frac{x^2 \cdot \ln^2(x^3)}{2} - \left(\frac{\ln(x^3) \cdot 3x^2}{2} - \int \frac{9x}{2}\right) =$$

$$\frac{x^2 \cdot \ln^2(x^3)}{2} - \frac{\ln(x^3) \cdot 3x^2}{2} + \frac{9x^2}{4} + C$$

(21) $\int \frac{\ln(x)}{\sqrt{x}}\,dx = \int \ln(x)\cdot x^{-\frac{1}{2}}\,dx = \int \ln(x)\left(\frac{x^{\frac{1}{2}}}{1/2}\right)' =$

$= \ln(x)\cdot \frac{2x^{\frac{1}{2}}}{1} - \int 2\cdot x^{\frac{1}{2}}\cdot \frac{1}{x} =$

$= 2\cdot \ln(x)\cdot x^{\frac{1}{2}} - 2\cdot \int x^{-\frac{1}{2}} =$

$= 2\cdot \ln(x)\cdot x^{\frac{1}{2}} - 4\cdot x^{\frac{1}{2}} + C$

(22) $\int e^{2x}\cdot \sin^2(x)\,dx =$