① $f(x) = \begin{cases} x^4\left(\sqrt{2} + \sin\frac{1}{x}\right), & x \in \mathbb{R}\setminus\{0\} \\ 0, & x = 0 \end{cases}$

Mo $a=0$ $\quad f'(0) = \lim\limits_{\substack{x\to 0 \\ (x\neq 0)}} \dfrac{f(x) - \overbrace{f(0)}^{0}}{x-0}$

$$\lim_{x\to 0} \frac{x^4\left(\sqrt{2}+\sin\frac{1}{x}\right)}{x} = \lim_{x\to 0} \underbrace{x^3}_{0}\cdot\underbrace{\left(\sqrt{2}+\sin\frac{1}{x}\right)}_{\text{korlátos}} = \boxed{0}$$

Formálisan $\quad 0 \leq \left|x^3\left(\sqrt{2}\cdot\sin\frac{1}{x}\right)\right| = |x^3|\cdot\left|\sqrt{2}+\sin\frac{1}{x}\right| \leq$
$|x|^3\left(\sqrt{2}+\underbrace{\left|\sin\frac{1}{x}\right|}_{\leq 1}\right) \leq |x|^3\cdot(\sqrt{2}+1)$
$\downarrow$
0
ha $x\to 0$

$\Rightarrow$ közrefogás $\lim\limits_{x\to 0} |f(x)| = 0$

Megj: a) $\nexists \lim\limits_{x\to 0}\left(\sin\frac{1}{x}\right) =$

Átviteli elv: legyen

$x_n := \dfrac{1}{2n\pi} \to 0 \ (n\to\infty) \quad n\in\mathbb{N}^+$

$\sin\left(\frac{1}{x_n}\right) = \sin 2n\pi = 0 \to \boxed{0} \ (\text{ha } n\to\infty)$

de: $\quad z_n := \dfrac{1}{\frac{\pi}{2}+2n\pi} \xrightarrow{\text{ha } (n\to\infty)} 0 \quad$ és $\sin\left(\frac{1}{z_n}\right) =$

$\sin\left(\frac{\pi}{2}+2n\pi\right) = 1$

$n\to\infty$

$$\lim_{n\to\infty} \sin\left(\frac{1}{2n}\right) = 0$$

$$\Rightarrow \nexists \lim_{x\to 0} \sin\frac{1}{x}$$

$\boxed{\text{2 Gyak}}$

① $f(x) = \sqrt{x^2-1} \quad x \in [1; +\infty]$

$\forall a \in (1; \infty) \to f \in D\{a\} \mid f'(a) = ?$

$$f'(a) = \lim_{x\to a} \frac{\sqrt{x^2-1} - \sqrt{a^2-1}}{x-a} =$$

$$\lim_{x\to a} \frac{(\sqrt{x^2-1} - \sqrt{a^2-1})(\sqrt{x^2-1} + \sqrt{a^2-1})}{(x-a)(\sqrt{x^2-1} + \sqrt{a^2-1})} =$$

$$\lim_{x\to a} \frac{x^2-1-(a^2-1)}{(x-a)(\sqrt{x^2-1} + \sqrt{a^2-1})} = \lim_{x\to a} \frac{\cancel{(x-a)}(x+a)}{\cancel{(x-a)}(\sqrt{x^2-1} + \sqrt{a^2-1})}$$

$$\lim_{x\to a} \frac{2a}{2\sqrt{a^2-1}} = \lim_{x\to a} \frac{a}{\sqrt{a^2-1}}$$

$\mathbb{R}$

$$\Rightarrow f \in D\{a\} \text{ és } f'(a) = \frac{a}{\sqrt{a^2-1}}$$

② Mechanikus deriválás

① $f(x) = 4x^3 - 2x^2 + x - 10 \qquad (x \in \mathbb{R}) \Rightarrow$

$f'(x) = (4x^3 - 2x^2 + x - 10)' =$

$$(4x^3)' + (-2x^2)' + (x)' + (-10)' = 4\cdot(x^3)' - 2\,(x^2)' + 1 + 0 =$$

$$\underbrace{12x^2 - 4x + 1} \quad (x \in \mathbb{R})$$

$$(x^{\alpha})' = \alpha x^{\alpha-1} \quad (\forall \alpha \in \mathbb{R},\ x>0)$$

② $f(x) = \sqrt{x\sqrt{x\sqrt{x}}} \quad (x \in (0;+\infty))$

$$f'(x) = \left(\sqrt{x\sqrt{x\sqrt{x}}}\right)' = \left(x^{\frac{1}{2}} \cdot x^{\frac{1}{4}} \cdot x^{\frac{1}{8}}\right)' = \left(x^{\frac{7}{8}}\right)' =$$

$$\frac{7}{8} \cdot x^{-\frac{1}{8}} = \frac{7}{8\cdot\sqrt[8]{x}} \quad (x>0)$$

③ $\left(x^3 + \frac{1}{x^2} - \frac{1}{5x^5}\right)' = \quad x \in \mathbb{R}\setminus\{0\}$

$$3x^2 + (-2\cdot x^{-3}) - \frac{1}{5}\cdot -5\cdot x^{-6} =$$

$$3x^2 - \frac{2}{x^3} + \frac{1}{x^6} \qquad x \in \mathbb{R}\setminus\{0\}$$

④ $\left(x^a + a^x + ax + \frac{x}{a} + \frac{a}{x}\right)' \quad x>0,\ a>0$

$$a\cdot x^{a-1} + a^x \cdot \ln a + a\cdot 1 + \frac{1}{a}\cdot 1 + a \cdot -1\, x^{-2}$$

$$a\, x^{a-1} + a^x \ln a + a + \frac{1}{a} - \frac{a}{x^2}$$

$$(f\cdot g)' = f'\cdot g + g\cdot f'$$

5) $\left(\dfrac{x^5+2}{x^2+x+5}\right)' =$

$x \in \mathbb{R}$

Hf. Zápra lécin:

Házi: 1. $f(x) = \frac{2x^2 - 1}{x\sqrt{1+x^2}} \quad (0 < x \in \mathbb{R})$

$$\frac{2x^2}{x\sqrt{1+x^2}} - \frac{1}{x\sqrt{1+x^2}} = \left(\frac{2x}{\sqrt{1+x^2}} - \frac{1}{x\sqrt{1+x^2}}\right)'$$

$$\left(\frac{2x}{\sqrt{1+x^2}}\right)' - \left(\frac{1}{x\sqrt{1+x^2}}\right)' =$$

$$\frac{2\cdot\sqrt{1+x^2} - 2x\cdot\frac{1}{2}\cdot\frac{1}{\sqrt{1+x^2}}\cdot 2x}{1+x^2} - \frac{-1\cdot\left(x\cdot\sqrt{1+x^2}\right)'}{x^2(1+x^2)}$$

$$\sqrt{1+x^2} + x\cdot\frac{1}{2\cdot\sqrt{1+x^2}}\cdot 2x$$

$$\sqrt{1+x^2}\cdot\frac{2x^2}{2\sqrt{1+x^2}} =$$

$$\frac{\frac{1}{x^2}}{\sqrt{1+x^2}}$$

$$\frac{2\cdot\sqrt{1+x^2}-2x\cdot\frac{1}{2\cdot\sqrt{1+x^2}}\cdot 2x}{1+x^2} - 1\cdot\frac{\sqrt{\cancel{1+x^2}}\cdot\frac{\cancel{x^2}}{\cancel{\sqrt{1+x^2}}}}{\cancel{x^2}(1+x^2)}$$

$$1$$

$$+\ \frac{1}{1+x^2}$$

$$\frac{2\sqrt{1+x^2}-4x^2\cdot\frac{1}{2\sqrt{1+x^2}}+1}{1+x^2}$$

1. $f(x) = \frac{e^x}{e^x+1}$

$$\frac{e^x\cdot(e^x+1)-e^x\cdot e^x}{(e^x+1)^2} = \frac{\cancel{e^{2x}}+e^x-\cancel{e^{2x}}}{(e^x+1)^2} = \frac{e^x}{(e^x+1)^2}$$

2. $f(x) = 2^{x^3}$

2. $f(x) = \frac{1}{x}+\sqrt{1+\frac{1}{x^2}} = -\frac{1}{x^2}+\frac{1}{2}\cdot\frac{1}{\sqrt{1+\frac{1}{x^2}}}\cdot -2\,\frac{1}{x^3} =$

$$-\frac{1}{x^2}+\frac{1}{2\cdot\sqrt{1+\frac{1}{x^2}}}\cdot\frac{-2}{x^3} =$$

2. $y = \frac{2x}{1-x^2}$ $\quad \frac{2\cdot(1-x^2) - 2x\cdot(-2x)}{1^2 - 2x^x - x^4} = \frac{2+2x^2}{(1-x^2)^2} =$

$$\frac{2\cdot(1+x^2)}{(1-x^2)^2}$$

3. $\frac{1+x-x^2}{1-x+x^2}$ $\quad \frac{(1-2x)\cdot(1-x+x^2) - (1+x-x^2)\cdot(-1+2x)}{(1-x+x^2)^2} =$

$$\frac{-\cancel{2x}+\cancel{2x^2}-\cancel{2x^3}+1-\cancel{x}+\cancel{x^2}+1+\cancel{x}-\cancel{x^2}-\cancel{2x}-\cancel{2x^2}+\cancel{2x^3}}{(1-x+x^2)^2} = \frac{-4x+2}{(1-x+x^2)^2}$$

4 (85a) $x\sqrt{1+x^2}$

$$1\cdot\sqrt{1+x^2} + x\cdot\left(\sqrt{1+x^2}\right)'$$

$$\frac{1}{2}\cdot\frac{1}{\sqrt{1+x^2}}\,(2x)$$

$$\sqrt{1+x^2} + x\cdot\frac{1}{2\sqrt{1+x^2}}\cdot 2x$$

$$\sqrt{1+x^2} + \frac{\cancel{2}x^2}{\cancel{2}\sqrt{1+x^2}} = \sqrt{1+x^2} + \frac{x^2}{\sqrt{1+x^2}} =$$

$$\frac{\sqrt{1+x^2}\,\sqrt{1+x^2}+x^2}{\sqrt{1+x^2}} = \frac{1+x^2+x^2}{\sqrt{1+x^2}} = \frac{1+2x^2}{\sqrt{1+x^2}}$$

5) (853), $\sqrt[3]{x^2} - \frac{2}{\sqrt{x}}$

$$\left(\sqrt[3]{x^2}\right)' - \left(\frac{2}{\sqrt{x}}\right)' = \frac{2}{3} x^{-\frac{1}{3}} - \left(2 \cdot x^{-\frac{1}{2}}\right)'$$

$$\frac{-2 \cdot \frac{1}{2\sqrt{x}}}{x}$$

$$\frac{2}{3} \cdot \frac{1}{\sqrt[3]{x}} + \frac{1}{\sqrt{x} \cdot x}$$

$$\frac{2}{3 \cdot \sqrt[3]{x}} + \frac{1}{x \cdot \sqrt{x}}$$

6) 862 $\cos 2x - 2\sin x$

$$(\cos 2x)' - (2\sin x)' = -\sin 2x \cdot 2 \quad -2 \cdot \cos x$$

$$-2 \cdot \sin(2x) - 2\cos x$$

7) 851

$$x + \sqrt{x} + \sqrt[3]{x}$$

$$(x)' + (\sqrt{x})' + (\sqrt[3]{x})' = 1 + \frac{1}{2\sqrt{x}} + \frac{1}{3} \cdot \frac{1}{\sqrt[3]{x^2}} =$$

$$1 + \frac{1}{2\sqrt{x}} + \frac{1}{3\sqrt[3]{x^2}}$$

8 (852) $\frac{1}{x}+\frac{1}{\sqrt{x}}+\frac{1}{\sqrt[3]{x}}$

$$\left(\frac{1}{x}\right)'+\left(\frac{1}{\sqrt{x}}\right)'+\left(\frac{1}{\sqrt[3]{x}}\right)' =$$

$$-\frac{1}{x^2}-\frac{1}{2\sqrt{x^3}}-\frac{1}{3\sqrt[3]{x^4}}$$

$$-\frac{1}{x^2}-\frac{1}{2x\cdot\sqrt{x}}-\frac{1}{3x\sqrt[3]{x}}$$

9, 895

$$\ln\left(x+\sqrt{x^2+1}\right)$$

$$\frac{\left(x+\sqrt{x^2+1}\right)'}{x+\sqrt{x^2+1}} = \quad 1+\frac{1}{2}\frac{1}{\sqrt{x^2+1}}\cdot 2x$$

$$\frac{1+\frac{1}{\cancel{2}\sqrt{x^2+1}}\cdot\cancel{2}x}{x+\sqrt{x^2+1}} = \frac{1+\frac{x}{\sqrt{x^2+1}}}{x+\sqrt{x^2+1}} = \frac{\frac{\cancel{x+\sqrt{x^2+1}}}{\sqrt{x^2+1}}}{\cancel{x+\sqrt{x^2+1}}} =$$

$$\frac{1}{\sqrt{x^2+1}}$$

$$\ln x+\ln\sqrt{x^2+1}$$

$$\frac{1}{x}+\frac{\frac{1}{2}\cdot\frac{1}{\sqrt{x^2+1}}\cdot 2x}{\sqrt{x^2+1}} = \frac{1}{x}+\frac{\frac{x}{\sqrt{x^2+1}}}{\sqrt{x^2+1}}$$

$$\frac{1}{x}+\frac{x}{x^2+1}$$

$$\frac{(x^2+1)}{x^3+x^2} \quad \frac{x^2}{} : \frac{x^4+x^2}{x^3 \cdot 2^1}$$

$$\frac{1+x^2}{x}$$

10 | 890 $\frac{1}{4} \cdot \ln \frac{x^2-1}{x^2+1}$

$$\frac{1}{4} \cdot \frac{\left(\frac{x^2-1}{x^2+1}\right)'}{\frac{x^2-1}{x^2+1}} = \frac{1}{4} \cdot \frac{\frac{2x\cdot(x^2+1)-(x^2-1)\cdot 2x}{x^4+2x^2+2}}{\frac{x^2-1}{x^2+1}} =$$

$$\frac{1}{4} \cdot \frac{2x^3+2x-(2x^3-2x)}{x^4+2x^2+2} \cdot \frac{x^2+1}{x^2-1} = \frac{1}{4} \cdot \frac{4x}{\cancel{(x^2+1)^2}} \cdot \frac{\cancel{x^2+1}}{x^2-1} =$$

$$\frac{1}{4} \cdot \frac{4x}{x^2+1\cdot(x^2-1)} = \frac{x}{x^4\cancel{-x^2+x^2}-1} = \boxed{\frac{x}{x^4-1}}$$

11 | 905 $-\frac{\cos x}{2\cdot \sin^2 x} + \ln\sqrt{\frac{1+\cos x}{\sin x}}$

$$-1 \cdot \frac{-\sin x \cdot 2\cdot \sin^2 x - \cos x \cdot 2 \cdot \sin 2x}{4\cdot \sin^4 x} = \frac{2\cdot \sin^3 x + \cos x \cdot 2 \cdot \sin 2x}{4 \cdot \sin 4x}$$

$$\frac{\sin^3 x + \cos x \cdot 2 \cdot \sin x \cdot \cos x}{\sin^4 x}$$

$$\frac{\sin^2 x + 2\cdot \cos x}{2 \cdot \sin^3 x} + \frac{\left(\sqrt{\frac{1+\cos x}{\sin x}}\right)'}{\sqrt{\frac{1+\cos x}{\sin x}}}$$

$$\frac{1}{4} \cdot \ln \frac{x^2-1}{x^2+1}$$

$$\frac{1}{4} \cdot \ln x^2-1 - \ln x^2+1$$

$$\frac{1}{4} \cdot \frac{2x}{x^2-1} - \frac{2x}{x^2+1} = \frac{(x+1)^2 \cdot (x-1) - 2x(x+1) \cdot (x-1)}{4(x+1)^2(x-1)} =$$

$$\frac{x^3+2x^2+x-x^2-2x-1 - -2x^3+2x}{4} =$$

$$\frac{1}{2} \cdot \frac{1}{\sqrt{\frac{1+\cos x}{\sin x}}} \cdot \frac{-\sin x \cdot \sin x - (1+\cos x) \cdot \cos x}{\sin^2 x} \cdot \frac{1}{\sqrt{\frac{1+\cos x}{\sin x}}}$$

$$\frac{1}{2} \cdot \frac{1}{\sqrt{\frac{1+\cos x}{\sin x}}} \cdot \frac{-\sin^2 x - \cos x - \cos^2 x}{\sin^2 x} \cdot \frac{1}{\sqrt{\frac{1+\cos x}{\sin x}}}$$

$$\frac{\sin^2 x + 2 \cdot \cos x}{2 \cdot \sin^3 x} + \frac{1}{2} \cdot \frac{1}{\sqrt{\frac{1+\cos x}{\sin x}}} \cdot \frac{-\sin^2 x - \cos x - \cos^2 x}{\sin^2 x} \cdot \frac{1}{\sqrt{\frac{1+\cos x}{\sin x}}}$$

$$\frac{\sin^2 x + 2\cos x}{2 \cdot \sin^3 x} + \frac{-\sin^2 x - \cos^2 x - \cos x}{2 \cdot \frac{1+\cos x}{\sin x} \cdot \sin^2 x}$$

$$\frac{\sin^2 x + 2\cos x}{2 \cdot \sin^3 x} + \frac{-\sin^2 x - \cos^2 x - \cos x}{2 \cdot (1+\cos x) \cdot \sin x}$$

$$\frac{(\sin^2 x + 2\cos x)(1+\cos x) + (-\sin^2 x - \cos^2 x - \cos x) \cdot \sin^2 x}{2 \cdot \sin^3 x \cdot (1+\cos x)} =$$

$$\frac{\sin^2 x + \sin^2 x \cdot \cos x + 2\cos x + 2\cos^2 x - \sin^4 x - \cos^2 x \cdot \sin^2 x - \cos x \cdot \sin^2 x}{2 \cdot \sin^3 x \, (1+\cos x)}$$

11, 884

$$\sqrt{x+1} - \ln\left(1 + \sqrt{x+1}\right)$$

$$\frac{1}{2} \cdot \frac{1}{\sqrt{x+1}} \cdot 1 - \frac{\left(1+\sqrt{x+1}\right)'}{1+\sqrt{x+1}} =$$

$$\frac{1}{2\sqrt{x+1}} - \frac{\frac{1}{2\sqrt{x+1}}}{1+\sqrt{x+1}} = \frac{1}{2\sqrt{x+1}} - \frac{1}{2\sqrt{x+1}\cdot\left(1+\sqrt{x+1}\right)} =$$

$$\frac{1}{2\sqrt{x+1}} - \frac{1}{2\sqrt{1+1}} \cdot \frac{1}{1+\sqrt{x+1}}$$

$$\frac{1+\sqrt{x+1}-1}{2\sqrt{x+1}\cdot\left(1+\sqrt{x+1}\right)} = \frac{\cancel{\sqrt{x+1}}}{2\cancel{\sqrt{x+1}}} = \frac{1}{2\left(1+\sqrt{x+1}\right)}$$

~~11~~2 $f(x) = \sin\left(\sqrt{x^3+1}\right)$

gsol 1

$$\cos\left(\sqrt{x^3+1}\right) \cdot \frac{1}{2} \cdot \frac{1}{\sqrt{x^3+1}} \cdot 3x^2 = \cos\left(\sqrt{x^3+1}\right) \cdot \frac{3x^2}{2\sqrt{x^3+1}}$$

$$\frac{3x^2 \cdot \cos\left(\sqrt{x^3+1}\right)}{2 \cdot \sqrt{x^3+1}}$$

/13

$$x^2 \cdot \sqrt[3]{x} \qquad 2x \cdot \sqrt[3]{x} + x^2 \cdot \frac{1}{\sqrt[3]{x^2}} \cdot \frac{1}{3}$$

(13 g) d) $\quad x^2 \cdot \sqrt[3]{x} \Longrightarrow x^2 \cdot x^{\frac{1}{3}} = x^{\frac{7}{3}} = \frac{7}{3} \cdot x^{\frac{4}{3}}$

$$2x \cdot \sqrt[3]{x} + x^2 \cdot \frac{1}{3} \cdot \frac{1}{\sqrt[3]{x^2}} =$$

$$2x \cdot \sqrt[3]{x} + \frac{x^2}{3 \cdot \sqrt[3]{x^2}}$$

$$\frac{(2x \cdot \sqrt[3]{x}) \cdot (3 \cdot \sqrt[3]{x^2}) + x^2}{3 \cdot \sqrt[3]{x^2}} = \frac{6x \cdot x + x^2}{3 \cdot \sqrt[3]{x^2}} = \frac{7x^2}{3 \cdot \sqrt[3]{x}}$$

24) $\quad \ln \frac{\sqrt{1+x}}{(x^2+1)^5}$

$$\ln \sqrt{1+x} - \ln (x^2+1)^5$$

$$\frac{\frac{1}{2} \cdot \frac{1}{\sqrt{1+x}} \cdot 1}{\sqrt{1+x}} - \frac{5 \cdot (x^2+1)^4 \cdot 2x}{(x^2+1)^5} =$$

$$\frac{1}{2 \cdot (1+x)} - \frac{10x}{x^2+1} =$$

15 $\ln \frac{(2x^2+3)^4}{\sqrt{x+1}}$

$\ln(2x^2+3)^4 - \ln\sqrt{x+1}$

$\frac{4(2x^2+3)^3 \cdot 4x}{(2x^2+3)^4} - \frac{\frac{1}{2} \cdot \frac{1}{\sqrt{x+1}}}{\sqrt{x+1}} = \frac{16x}{(2x^2+3)} - \frac{1}{2x+2} = \cdot$

$f'(x_0) \cdot (x - \cancel{x_0}) + f(x_0)$

$-\frac{1}{2}x + \ln 81$

$f(x) = \sqrt{x}\, e^{-x}$

$f'(x) = \frac{1}{2\sqrt{x}} \cdot e^{-x} + \sqrt{x} \cdot -e^{-x}$

$-\frac{1}{2e} \cdot (x-1) + \frac{1}{e}$

16, gsdl/k

$$\ln(x^2 \cdot e^x) = \ln x^2 + \ln e^x = \frac{2x}{x^2} + 1 = \frac{2}{x} + 1$$

17, gsdl(i

$$= \left(\sqrt[3]{x+\sqrt{x}}\right)^{-1} =$$

$$-1 \cdot \frac{1}{\left(\sqrt[3]{x+\sqrt{x}}\right)^2} \; \frac{1}{3} \cdot \frac{1}{(x+\sqrt{x})^{2/3}} \left(1 + \frac{1}{2\sqrt{x}}\right)$$

$$-\frac{1}{3} \cdot \frac{1 + \frac{1}{2\sqrt{x}}}{(x+\sqrt{x})^{2/3} \cdot (x+\sqrt{x})^{2/3}} = -\frac{1}{3} \cdot \frac{1 + \frac{1}{2\sqrt{x}}}{(x+\sqrt{x})^{4/3}}$$

$$\frac{-\frac{1}{3} - \frac{1}{6}\sqrt{x}}{(x+\sqrt{x})^{4/3}} = \frac{-1}{3(x+\sqrt{x})^{4/3}} - \frac{\sqrt{x}}{6(x+\sqrt{x})^{4/3}}$$

1f / gd / g

$$(x+2)^8 \cdot (x+3)^6$$

$$8(x+2)^7 \cdot (x+3)^6 + (x+2)^8 \cdot 6 \cdot (x+3)^5$$

$$(x+3)^5 \cdot (x+2)^7 \cdot \left((x+3)\cdot 8 + 6\cdot(x+2)\right) =$$

$$(x+3)^5 \cdot (x+2)^7 \cdot (8x+24+6x+12)$$

$$(14x+36)$$

18/ $f(x) = \frac{x+3}{3x^2+1}$

$$\frac{3x^2+1-(x+3)(6x)}{9x^4+6x^2+1} = \frac{3x^2+1-(6x^2+18x)}{(3x^2+1)^2}$$

$$\lim_{x\to 1} \frac{\frac{x+3}{3x^2+1}-1}{x-1} = \frac{(x+3)-(3x^2+1)}{(x-1)(3x^2+1)} = -1\,\frac{3x^2+1-(x+3)}{(x-1)(3x^2+1)} =$$

$$-1\,\frac{3x^2-x-2}{(x-1)(3x^2+1)} = -1\,\frac{\cancel{(x-1)}(x+4/6)}{\cancel{(x-1)}(3x^2+1)} = -1\,\frac{x+4/6}{3x^2+1} =$$

$$-1 \cdot \frac{10/6}{4} = -1 \cdot \frac{10}{24} = -\frac{10}{24} \Rightarrow \mathbb{R}$$

19/ $$f(x) := \frac{x}{\sqrt{1+x}}$$

$$\frac{\sqrt{1+x} - x \cdot \frac{1}{2} \cdot \frac{1}{\sqrt{1+x}} \cdot 1}{1+x} = \frac{\sqrt{1+x} - \frac{x}{2\sqrt{1+x}}}{1+x} =$$

$$\frac{\sqrt{1+x}}{1+x} - \frac{x}{2(1+x)^{3/2}} \cdot \qquad \frac{2}{4} - \frac{3}{2(4)^{3/2}}$$

120

$$\frac{2}{4} - \frac{3}{2 \cdot 8} =$$

$$\frac{8}{16} - \frac{3}{16} = \frac{5}{16}$$

$$\frac{x}{e^x+1} =$$

$$\frac{e^x+1 - x \cdot e^x}{e^{2x} + 2 \cdot e^x + 1} \quad \frac{e^x(1+e^{-x} - x)}{e^x(e^x+2+e^{-x})} = \frac{1+e^{-x}-x}{e^x+2+e^{-x}}$$

$$\frac{1+1}{1+2+1} = \frac{2}{4} = \frac{1}{2} \qquad \frac{1+\frac{1}{e}-1}{e+2+\frac{1}{e}} = \frac{1}{e} \cdot \frac{1}{e+2+\frac{1}{e}} =$$

$$\frac{1}{e^2+2e+1} = \frac{1}{(e+1)^2}$$

$\sin^2 x \cdot \cos^2 x$

$2 \cdot \sin x \cdot \cos x \cdot \cos^2 x + \sin^2 x \cdot 2 \cdot \cos x \cdot - \sin x$

1 h

3 k

4 c

7 f

d l

$e^{0,1x+3}$

$e^{g} = f$

$g = 0,1x+3$

$e^{0,1x+3}$