1. $f(x) = (\ln x)^{x+1} \quad (x > 1)$

$$\Updownarrow$$

$$\ln x > 0$$

1. mo. $f(x) = e^{\ln(\ln x)^{x+1}} = e^{x+1 \cdot \ln(\ln x)} \Rightarrow$

$$f'(x) = e^{(x+1)\cdot\ln(\ln x)} \cdot \left[\ln(\ln x) + (x+1)\frac{1}{\ln x}\cdot\frac{1}{x}\right] =$$

$$= (\ln x)^{x+1} \cdot \left(\ln(\ln x) + \frac{x+1}{x \cdot \ln x}\right)$$

2. mo. ln „mindkét oldal" $\Rightarrow \ln f(x) = \ln\left((\ln x)^{x+1}\right) \Rightarrow$

$$\Rightarrow \ln f(x) = (x+1)\cdot\ln(\ln x) \quad \Big|\, ()' \Rightarrow \frac{1}{f(x)}\cdot f'(x) =$$

$$\frac{f'(x)}{f(x)} = \ln(\ln x) + \frac{x+1}{x \cdot \ln x} \quad \Big|\cdot f(x)$$

$$f'(x) = f(x) \cdot \ln(\ln x) + \frac{x+1}{x \ln x}$$ logaritmikus deriválás

$$f(x) = \sqrt{\ln(\sin x)}$$

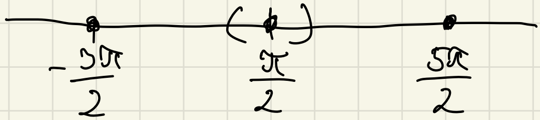
$$f'(x) = \frac{1}{2}\cdot\frac{1}{\sqrt{\ln(\sin x)}}\cdot\frac{1}{\sin x}\cdot\cos x = \frac{\mathrm{ctg}}{2\sqrt{\ln(\sin x)}}$$

Kik: $\begin{cases} \sin x > 0 \text{ és} \\ \ln(\sin x) \geq 0 = \ln 1 \end{cases} \Longrightarrow \exp(\ln(\sin x)) \geq \exp(\ln 1)$

$$\Updownarrow$$

$$\sin x \geq 1 > 0$$

$$\sin x = 1 \iff x = \frac{\pi}{2} + 2k\pi \quad (k \in \mathbb{Z})$$



izolált pont

nincs belsőpont $\Rightarrow$ f nem deriválható sehol

$D_{f'} = \emptyset$; $D_f = \left\{ \frac{\pi}{2} + 2k\pi \mid k \in \mathbb{Z} \right\}$

Feladat 3. $f(x) = \ln \frac{\sqrt{1+x}}{(x^2+1)^5} \quad (x > -1)$

$f' = ?$ érintő $(0, f(0))$-ban

$\forall x \in (-1, \infty) : f \in D\{x\}$ (két elemi függvény és műveletek) $\Rightarrow$

$f'(x) = \left(\frac{1}{2} \ln(x+1) - 5 \cdot \ln(x^2+1) \right)' =$

$\frac{1}{2} \cdot (\ln(x+1))' - 5 \cdot (\ln(x^2+1))' =$

$\frac{1}{2} \cdot \frac{1}{x+1} - 5 \cdot \frac{2x}{x^2+1} = \frac{1}{2(x+1)} - \frac{10x}{x^2+1} \quad \forall x > -1$

$0 \in (-1, \infty) \Rightarrow f \in D\{0\}$ és $f'(0) = \frac{1}{2} \Rightarrow$ Ja 0-ban érintő egyenlet

$y = f(0) + f'(0) \cdot (x - 0)$

$y = \ln 1 + \frac{1}{2} x$

$\boxed{y = \frac{1}{2} x \quad x \in \mathbb{R}}$

$$f(x) := \frac{\sqrt[3]{x-1} \cdot x^2}{(x-5)^3 \cdot (x+2)^7} \qquad x \in (5; +\infty)$$

érintő $a=9$-ben $(9; f(9))$-ben

$$y = f(9) + f'(9) \cdot (x-9) \Rightarrow$$

$$\ln f(x) = \ln \frac{\sqrt[3]{x-1} \cdot x^2}{(x-5)^3 (x+2)^7} =$$

$$\ln\left(\sqrt[3]{x-1} \cdot x^2\right) - \ln\left((x-5)^3 \cdot (x+2)^7\right)$$

$$\frac{1}{3} \cdot \ln x-1 + 2 \cdot \ln x - 3 \ln x-5 - 7 \ln x+2 \overset{(\,)'}{\Rightarrow}$$

$$\frac{1}{3} \cdot \frac{1}{x+1} + 2 \cdot \frac{1}{x} - 3 \cdot \frac{1}{x-5} - 7 \cdot \frac{1}{x+2} =$$

$$\frac{1}{3x+3} + \frac{2}{x} - \frac{3}{x-5} - \frac{7}{x-2}$$

$$f(9) = \frac{\sqrt[3]{8} \cdot 9^2}{4^3 \cdot 11^7} = \frac{81}{2 \cdot 16 \cdot 11^7}$$

$$f'(9) = \frac{1}{24} + \frac{2}{9} - \frac{3}{4} - \frac{7}{11}$$

1. $f(x)=\sqrt{e^{2x-1}+1}$ $(x\in\mathbb{R})\Rightarrow \exists f^{-1}$; $f^{-1}\in D$; $(f^{-1})'(\sqrt{2})=?$

Mo. $I=\mathbb{R}$ nyílt intervallum, $f\in C$ $(\forall x\in D_f=\mathbb{R}: f\in C\{x\})$

$f=\sqrt{}\circ(\exp\circ(2\,\mathrm{id}-1)+1)\in C$ (fgy és műv. szab)

f szig mon nő: $f\in D$ (műv) és $f'(x)=$

$$\frac{1}{2}\left(e^{2x-1}+1\right)^{-\frac{1}{2}}\cdot\left(e^{2x-1}\right)\cdot 2=\frac{e^{2x-1}}{\sqrt{e^{2x-1}+1}}>0\ (\forall x\in\mathbb{R})\Rightarrow$$

$f\uparrow$ $\mathbb{R}$-en.

$f'(x)\neq 0$ $(\forall x\in\mathbb{R})\Rightarrow\exists f^{-1}: R_f\to\mathbb{R}$; $f^{-1}\in D\{b\}$ és

$$f^{-1\prime}(b)=\frac{1}{f'(a)}=\frac{1}{f'(f^{-1}(b))}$$

$$(f^{-1})'(\sqrt{2})=\frac{1}{f'(f^{-1}(\sqrt{2}))}=\frac{1}{f'(\frac{1}{2})}=\frac{1}{1/\sqrt{2}}=\boxed{\sqrt{2}}$$

$e^{2x-1}+1=2$ $\qquad\qquad$ $f(x)=\sqrt{2}\overset{!}{=}\sqrt{e^{2x-1}+1}$

$e^{2x-1}=1$

$2x-1=\ln 1$

$x=\frac{1}{2}\Rightarrow f^{-1}(\sqrt{2})=\frac{1}{2}$

Megj: „Most" megadható f^{-1} <u>explicite</u>

x	$-\infty$		$+\infty$
f'	$+$		$+$
f	1	$\xrightarrow{\text{szig. nő}}$	∞

$$\lim_{x \to -\infty} \sqrt{e^{2x-1}+1} = 1$$

$$\lim_{x \to +\infty} \sqrt{e^{2x-1}+1} = \infty$$

$$f \in C \Rightarrow \boxed{R_f = (1; +\infty)}$$

$$\Downarrow$$

$$\exists f^{-1} : (1; +\infty) \to \mathbb{R}; \text{ és } y \in (1; +\infty) \Rightarrow y = \sqrt{e^{2x-1}+1} \Leftrightarrow$$

$$0 < y^2 - 1 = e^{2x-1} \Rightarrow 2x - 1 = \ln(y^2-1)$$

$$x = \frac{\ln(y^2-1)+1}{2} = f^{-1}(y) \quad (y > 1)$$

$$f^{-1} \in D \text{ és } (f^{-1})'(y) = \frac{1}{2} \cdot \frac{1}{y^2-1} \cdot 2y = \underbrace{\frac{y}{y^2-1} \quad y > 1}$$

$$\text{Spec: } (f^{-1})'(\sqrt{2}) = \frac{\sqrt{2}}{2-1} = \underline{\underline{\sqrt{2}}} \checkmark$$

(3) $f(x) = \begin{cases} \dfrac{x}{1+e^{1/x}}, & \text{ha } x \in \mathbb{R} \setminus \{0\} \\ 0, & \text{ha } x = 0 \end{cases}$

$f \in D?$ és $f'(x) = ?$

Ha $x \in \mathbb{R} \setminus \{0\} \Rightarrow f \in D\{x\}$ (elemi fv. és műv. kapcsolat)

$x \neq 0$ $\frac{1}{x}$-hez

$1 + e^{1/x} \neq 0$ a nevezőben

és $f'(x) = \left(\dfrac{x}{1+e^{1/x}} \right)' =$

$$\frac{1 \cdot 1+e^{1/x} - x \cdot e^{1/x} \cdot \left(-\frac{1}{x^2}\right)}{\left(1+e^{1/x}\right)^2} =$$

$$= \frac{1+e^{1/x} + \frac{1}{x} \cdot e^{1/x}}{\left(1+e^{1/x}\right)^2} \quad (\forall x \in \mathbb{R} \setminus \{0\})$$

$$x = 0 \Rightarrow f'(0) \overset{\text{def}}{=} \lim_{x \to 0} \frac{\frac{x}{e^{1/x}+1} - 0}{x - 0} =$$

$$\underbrace{\lim_{x \to 0} 1 + e^{1/x}}$$ $=$

$$\lim_{x \to 0^+} \frac{1}{1+e^{1/x}} \qquad \lim_{x \to 0^-} \frac{1}{1+e^{1/x}}$$

$$\frac{1}{1+e^{+\infty}} = \frac{1}{+\infty} = \underline{\underline{0}} \qquad \frac{1}{1+e^{-\infty}} = \frac{1}{1}$$

$$\exists f'_d(0) = 0 \neq \exists f'_s(0) = 1$$

$$\Rightarrow \boxed{f \notin D\{0\}}$$

Teladabk

(1.) $\arccos\frac{1}{x}$

$$-\frac{1}{\sqrt{1-\left(\frac{1}{x}\right)^2}}\cdot \quad -\frac{1}{x^2}$$

$$-\frac{1}{\sqrt{1-\left(\frac{1}{x}\right)^2}}\cdot -\frac{1}{x^2} = \frac{1}{x^2\cdot\sqrt{1-\frac{1}{x^2}}}$$

2) 9l3 $\arcsin\frac{x}{2}$

$$\frac{1}{\sqrt{1-\left(\frac{x}{2}\right)^2}}\cdot\frac{1}{2} = \frac{1}{\sqrt{1-\frac{x^2}{4}}\cdot 2} = \frac{1}{\sqrt{\frac{4-x^2}{4}}\cdot 2} =$$

$$\frac{1}{\sqrt{4-x^2}\,\frac{1}{2}\cdot 2} = \frac{1}{\sqrt{4-x^2}}$$

3) 92x

$\arcsin(\sin x)$

$$\frac{1}{\sqrt{1-\sin x}}\cdot\cos x = \frac{\cos x}{\sqrt{1-\sin x}} =$$

4, 995 $\quad \operatorname{arctg} \frac{1+x}{1-x}$

$$\frac{1}{1+\left(\frac{1+x}{1-x}\right)^2} \cdot \frac{1\cdot(1-x)-(1+x)\cdot -1}{(1-x)^2} =$$

$$\frac{1}{1+\left(\frac{1+x}{1-x}\right)^2} \cdot \frac{2}{(1-x)^2} = \frac{2}{1+\frac{(1+x)^2}{(1-x)^2} \cdot (1-x)^2} =$$

$$\frac{2}{\frac{(1-x)^2+(1+x)^2}{(1-x)^2} \cdot (1-x)^2} = \frac{2}{x^2 \cancel{-2x}+1+x^2 \cancel{+2x}+1} =$$

$$\frac{2}{2x^2+2} = \frac{1}{x^2+1}$$

921

5, $\arccos \sqrt{1-x^2}$

$$-\frac{1}{\sqrt{1-(\sqrt{1-x^2})^2}} \cdot \frac{1}{2}\frac{1}{\sqrt{1-x^2}} \cdot -2x =$$

$$\frac{1}{\sqrt{1-(1-x^2)}} \cdot \frac{x}{\sqrt{1-x^2}} = \frac{x}{\sqrt{1-(1-x^2)} \cdot \sqrt{1-x^2}} =$$

$$\frac{x}{\sqrt{x^2}\cdot\sqrt{1-x^2}} = \frac{x}{\sqrt{x^2\cdot(1-x^2)}} = \frac{x}{\sqrt{x^2-x^4}} =$$

$$\frac{x}{\sqrt{x^2(1-x^2)}} = \frac{x}{|x|\cdot\sqrt{1-x^2}} = \frac{\operatorname{sgn}}{\sqrt{1-x^2}}$$

6, 917

$$y = \sqrt{x} - \operatorname{arctg}\sqrt{x}$$

$$\frac{1}{2\sqrt{x}} - \frac{1}{1+(\sqrt{x})^2}\cdot\frac{1}{2\sqrt{x}} =$$

$$\frac{1}{2\sqrt{x}} - \frac{1}{(1+x)2\sqrt{x}} = \frac{1+x-1}{(1+x)2\sqrt{x}} =$$

$$\frac{x}{(1+x)2\sqrt{x}} = \frac{x}{\sqrt{x}}\cdot\frac{1}{2(1+x)} = \frac{\sqrt{x}}{2(1+x)}$$

9K5 [illegible]

7) $$\frac{1}{\sqrt{2}} \operatorname{arctg} \frac{\sqrt{2}}{x} = \frac{1}{\sqrt{2}} \cdot \frac{1}{1+\left(\frac{\sqrt{2}}{2}\right)^2} \cdot -\sqrt{2} \cdot \frac{1}{x^2} =$$

$$\frac{1}{\sqrt{2}} \cdot \frac{1}{1+\frac{2}{x^2}} \cdot \frac{-\sqrt{2}}{x^2} = \frac{-\cancel{\sqrt{2}}}{\cancel{\sqrt{2}} \cdot \left(1+\frac{2}{x^2}\right) x^2} =$$

$$\frac{-1}{x^2+\frac{2x^2}{x^2}} = -\frac{1}{x^2+2}$$

8, 878

$$e^x \cdot \left(x^2-2x+2\right)$$

$$e^x \cdot \left(x^2-2x+2\right) + e^x \cdot \left(2x-2\right)$$

$$e^x\left(x^2 \cancel{-2x} \cancel{+2} \cancel{+2x} \cancel{-2}\right) = e^x\left(x^2\right)$$

9, 951

$$\ln\left(e^x + \sqrt{1+e^{2x}}\right) =$$

$$\ln e^x \cdot \ln\sqrt{1+e^{2x}} = \frac{(e^x)'}{e^x} \cdot \frac{\left(\sqrt{1+e^{2x}}\right)'}{\sqrt{1+e^{2x}}} =$$

$$\frac{\frac{1}{2\sqrt{1+e^{2x}}}\cdot 2\cdot e^{2x}}{\sqrt{1+e^{2x}}} = \frac{\cancel{2}\cdot e^{2x}}{\cancel{2}\left(\sqrt{1+e^{2x}}\right)^2} =$$

$$\frac{e^{2x}}{1+e^{2x}} =$$

10

952

$$\operatorname{arctg}\left(x+\sqrt{1+x^2}\right)$$

$$\frac{1}{1+\left(x+\sqrt{1+x^2}\right)^2}\cdot 1+\frac{1}{\cancel{2}\sqrt{1+x^2}}\cdot\cancel{2}x =$$

$$\frac{1}{1+\left(x+\sqrt{1+x^2}\right)^2}\cdot\frac{\sqrt{1+x^2}+x}{\sqrt{1+x^2}} =$$

Órai feladatok

$$\begin{cases} ax^2+5x+b & x<0 \\ e^{ax}+b\sin(2x) & x\geq 0 \end{cases}$$

$2ax+5 \qquad (-\infty;0)$ műveleti szabályok, $f\in D\{x\}$

$a\cdot e^{ax}+b\cos(2x)\cdot 2 \qquad (0;\infty)$

$x=0 \quad f\in D\{0\} \Leftrightarrow (f\in C\{0\} \wedge f'_-(0)=f'_+(0))$

$$\lim_{0-0} f = \lim_{0+0} f$$

$$\left.\begin{aligned} \lim_{x\to 0-0} ax^2+5x+b &= b \\ \lim_{x\to 0+0} e^{ax}+b\sin(2x) &= 1 \end{aligned}\right\} b=1$$

$f'_-(0)=2\cdot 0+5=5$

$f'_+(0)=a\cdot 1+2b=2b+a \Rightarrow 2\cdot 1+a=5$

$a=3$

$b=1 \quad a=3$

$f'(0)=5$

$$f'(x) := \begin{cases} 6x+5 \\ 3\cdot e^{3x}+2\cos 2x \end{cases}$$

$$f: \mathbb{R}\to\mathbb{R} \quad f(x) := \begin{cases} a\cdot e^{2x}+b\cdot\cos x & (x<0) \\ x^4+5x^2-4x+1 & (x\geq 0) \end{cases}$$

$x\in(-\infty;0)$ műveleti szabályok alapján $f\in D\{x\}$ és

$$f'(x) = 2ae^{2x}-b\cdot\sin(x)$$

$x\in(0;\infty)$ deriválhatósága rendkívül m.sz. a. $f\in D\{x\}$ és

$$f'(x) = 4x^3+10x-4$$

$x=0$, akkor

$$f\in D\{0\} \iff f\in C[0] \wedge f'_-(0)=f'_+(0)$$

$f\in C[0]$, ha $\quad \lim\limits_{0-0} f = \lim\limits_{0+0} f$

$$\lim_{x\to 0-0} a+b \qquad \lim_{x\to 0+0} 1 \quad \Rightarrow a+b=1$$

$f'_-(0) = f'_+(0)$ teljesül, ha

$2a = -4$

$a = -2$ $\Longrightarrow$ $a + b = 1$

$-2 + b = 1$

$b = 3$

$a = -2 \quad b = 3$

Kérdés: $f'(0) = -4$

$$f'(x) = \begin{cases} -4 \cdot e^{2x} - 3 \cdot \sin x & x < 0 \\ 4x^2 + 10x - 4 & x \geq 0 \end{cases}$$

$$f(x) = \begin{cases} e^{1-\cos^3(x)} & \text{ha } x < 0 \\ 2 \cdot \ln(x+1) + 1 & \text{ha } x \geq 0 \end{cases}$$

$x < 0$ esetén de. va. mi. sz. $f \in D\{x\}$

$$f'(x) = e^{1-\cos^3(x)} \cdot -3 \cdot \cos^2(x) \cdot -\sin(x)$$

$x > 0$ esetén:

$$f'(x) = 2 \frac{1}{x+1}$$

$x=0$,

$f \in C[0]$

$$1 = \lim_{0-0} f = \lim_{0+0} f = 1$$

tetszőleges L esetén folytonos a függvény

$$0 = f'_-(0) = f'_+(0) = L$$

$$L=0$$

$f'(0)=0$

$$f'(x) = \begin{cases} e^{1-\cos^3(x)} \cdot -3\cdot\cos^2(x) \cdot -\sin(x) & x\neq 0 \\ 0 & x=0 \end{cases}$$

$$f(x)\begin{cases} ax^2+1 & x<2 \\ x^2-bx-3 & x\geq 2 \end{cases}$$

na $x<2$, alleo a da. va. w. sa. u.a! $f\in D\{x\}$

$$f'(x)=2ax$$

na $x>2$, alleo . " — $f\in D\{x\}$

$$f'(x)=2x-b$$

na $x=2$, allor

$$x\in D\{2\},\Leftrightarrow x\in C[2]\wedge f'_-(2)=f'_+(2)$$

- $f\in C[2]$, na $4a+1=\lim\limits_{2-0} f=\lim\limits_{2+0} f=4-2b-3$
- $f'_-(2)=f'_+(2)\Rightarrow 4a=f'_-(2)=f'_+(2)=4-b$

$$\left.\begin{aligned} 4a+1&=1-2b \\ 4a&=4-b \end{aligned}\right\} b=4-4a$$

$$4a+1=1-2(4-4a)$$
$$4a+1=1-8+8a$$
$$8=4a$$
$$\underline{\underline{a=2}}$$

$$b=4-8$$
$$\underline{\underline{b=-4}}$$

$f'(x) = 8$, deci

$$f'(x) \begin{cases} 4x & x<2 \\ 8 & x=2 \\ 2x+4 & x>2 \end{cases}$$

$$f(x) = \begin{cases} x\cdot\cos(x) + k\cdot\sin x + 1 & x \leq 0 \\ p\cdot e^x + x^2 + 3x & x > 0 \end{cases}$$

Pe $x>0$, deci de. ecu. am. sa $f \in D\{x\}$

$$f'(x) = \cos x - x\cdot\sin x + k\cdot\cos(x)$$

Pe $x<0$, avem de. ca au sa $f \in D\{x\}$

$$f'(x) = p e^x + 2x + 3$$

Pe $x=0$, avem

$$f \in D\{0\} \Longleftrightarrow f \in C[0] \wedge f'_-(0) = f'_+(0)$$

- $f \in C[0]$: $1 = \lim\limits_{x\to 0-0} x\cdot\cos(x) + k\cdot\sin x + 1 =$

$$\lim_{x\to 0+0} p\cdot e^x + x^2 + 3x = p$$

$$\overset{!!}{p = 1}$$

$$\cdot \quad 1+\alpha = f'_-(0) = f'_+(0) = \beta + 3$$

$$\Downarrow$$

$$1 + \alpha = \beta + 3$$

$$1 + \alpha = 4$$

$$\alpha = 3$$

$f'(0) = 4$, tehát $f'(x)$ az $\mathbb{R}$-en, és $\alpha = 3$ $\beta = 1$

$$f'(x) \begin{cases} 4 \cdot \cos x - x \cdot \sin x & x < 0 \\ 4 & x = 0 \\ e^x + 2x + 3 & x > 0 \end{cases}$$

$$y = f'(a) \cdot (x - a) + f(a)$$

$$f'\left(-\frac{\pi}{2}\right) = 4 \cdot \cos -\frac{\pi}{2} - -\frac{\pi}{2} \cdot \sin -\frac{\pi}{2}$$

$$-\frac{\pi}{2}$$

$$(x - a) = x - \frac{\pi}{2}$$

$$f(a) = -\frac{\pi}{2} \cdot \cos -\frac{\pi}{2} + \alpha \cdot \sin -\frac{\pi}{2} + 1$$

$$0 + \alpha \cdot -1 + 1 = -\alpha + 1$$

$$\underline{y = -\frac{\pi}{2} \cdot \left(x - \frac{\pi}{2}\right) + -\alpha + 1}$$

11, 932

$$\ln\left(\arccos\frac{1}{\sqrt{x}}\right) = \frac{\frac{-1}{\sqrt{1-\left(\frac{1}{\sqrt{x}}\right)^2}} \cdot -\frac{1}{2\sqrt{x^3}}}{\arccos\frac{1}{\sqrt{x}}} =$$

$$\frac{1}{\sqrt{1-\frac{1}{x}} \cdot 2\sqrt{x^3}} = \frac{1}{2\sqrt{x^3\left(1-\frac{1}{x}\right)}} = \frac{1}{2\sqrt{x^3 - x^2}} =$$

$$\frac{1}{2x\sqrt{x-1} \cdot \arccos\frac{1}{\sqrt{x}}}$$

/12 880

$$e^x\left(1+\operatorname{ctg}\frac{x}{2}\right) = e^x \cdot \left(1+\operatorname{ctg}\frac{x}{2}\right) + e^x \cdot -\frac{1}{\sin^2\frac{x}{2}} \cdot \frac{1}{2}$$

$$e^x\left(1+\operatorname{ctg}\frac{x}{2} + -\frac{1}{\sin^2\frac{x}{2}} \cdot \frac{1}{2}\right) =$$

$$e^x\left(1+\frac{\cos\frac{x}{2}}{\sin\frac{x}{2}} - \frac{1}{2\cdot\sin^2\frac{x}{2}}\right)$$

$$e^x\left(\frac{2\sin^2\frac{x}{2} + 2\cdot\sin\frac{x}{2}\cdot\cos\frac{x}{2} - 1}{2\sin^2\frac{x}{2}}\right)$$

/13 848

$$\frac{(2-x^2)(3-x^3)}{(1-x)^2} = \frac{6-2x^3-3x^2+x^5}{x^2-2x+1} =$$

$$\frac{5x^4-6x-6x^2\cdot(1-x)^2-(6-2x^3-3x^2+x^5)\cdot(2x-2)}{(1-x)^4}$$

$$\frac{5x^4-6x-\cancel{6x^2}-5x^5+\cancel{6x^2}+6x^3-\cancel{6x^2}+4x^4+6x^3+-x^6+12-4x^3-\cancel{6x^2}+2x^5}{(1-x)^4}=$$

$$\frac{-6x+12x^3+4x^4+2x^5-x^6+12}{(1-x)^4}$$

14 $$\operatorname{arctg}\frac{x^2}{a}=\frac{1}{1+\left(\frac{x^2}{a}\right)^2}\cdot\frac{1}{a}\cdot 2x=$$

$$\frac{2x}{\left(1+\frac{x^4}{a^2}\right)}\cdot\frac{1}{a}=\frac{2x}{\frac{a^2+x^4}{a^2}}\cdot\frac{1}{a}=\frac{2xa^2}{a^2+x^4}\cdot\frac{1}{a}=$$

$$\frac{2xa}{a^2+x^4}$$

15 854

$$y = x\sqrt{1+x^2} = 1\cdot\sqrt{1+x^2} + x\cdot\frac{1}{2\sqrt{1+x^2}}\cdot 2x =$$

$$\sqrt{1+x^2} + \frac{\cancel{2}x^2}{\cancel{2}\sqrt{1+x^2}} = \sqrt{1+x^2} + \frac{x^2}{\sqrt{1+x^2}} =$$

$$\frac{1+x^2+x^2}{\sqrt{1+x^2}} = \frac{2x^2+1}{\sqrt{1+x^2}}$$

Z11

$$f(x) \begin{cases} e^{ax+b}\cdot \operatorname{sh} x & \text{ha } x \leq 0 \\ a\cdot \operatorname{arctg}(2x)+b & x>0 \end{cases}$$

ha $x<0$, [illegible] $f \in D(x)$

$$f'(x) = a\cdot e^{ax+b}\cdot \operatorname{sh} x + e^{ax+b}\cdot \operatorname{ch} x$$

ha $x>0$, — " —

$$f'(x) = a\cdot\frac{1}{1+4x^2}\cdot 2 = \frac{2a}{1+4x^2}$$

ha $x=0$, akkor

$$f(0-0) = f(0+0) \wedge f'_-(x) = f'_+(x)$$

$$0 = \lim_{x \to 0-0} e^{ax+b} \cdot \operatorname{sh} x = \lim \operatorname{arctg}(ax) + b = b$$

$b = 0$

$$e^b = f'_-(x) = f'_+(x) = \frac{2a}{1}$$

$$e^b = \frac{2a}{1} \Rightarrow 1 = \frac{2a}{1}$$

$$a = \frac{1}{2}$$

$a = \frac{1}{2}$, $b = 0$ paraméterek esetén kéz derivált

R: e

$$f(x) = \frac{a+\cos x}{e^x} \qquad x<0$$

$$a\cdot\sin x + b\,\text{arctg}\,x + \frac{x^2-10x+4}{b} \qquad x\geq 0$$

$x<0$, allora – c ——

$$f'(x) = \frac{-\sin x\cdot e^x - (a+\cos x)\cdot e^x}{e^{2x}}$$

$x>0$

$$f'(x) = a\cdot\cos x + b\cdot\frac{1}{1+x^2} + \frac{(2x-10)}{b}$$

$x=0$ $\qquad f\in C(0)$

$$a+1 = \lim_{x\to 0^-}\frac{a+\cos x}{e^x} = \lim_{x\to 0^+} a\cdot\sin x + b\,\text{arctg}\,x + \frac{x^2-10x+4}{b} = \frac{4}{b}$$

$$a+1 = \frac{4}{b}$$

$$-(a+1) = f'_-(0) = f'_+(0) = a+b-\frac{10}{b}$$

$$\left.\begin{aligned} -a-1 &= a+b-\frac{10}{b} \\ a+1 &= \frac{4}{b} \end{aligned}\right\} \to b = \frac{4}{a+1}$$

$$-a-1 = a+\frac{4}{a+1} - 10 \cdot \frac{a+1}{4}$$

$$-2a-1 = \frac{4}{a+1} - \frac{10a+10}{4}$$

$$-8a-4 = \frac{16}{a+1} - 10a - 10$$

$$-\cancel{8a^2} - 4a - 8a - 4 = 16 - \cancel{10a^2} - 10a - 10a - 10$$

$$2a^2 + 8a - 10 = 0 \Rightarrow \frac{-8 \pm \sqrt{64 - 4 \cdot 2 \cdot -10}}{4}$$

$$\frac{-8 \pm 12}{4} = 1$$

$$\to -5$$

$$-\frac{b}{x+1} = -b \cdot \frac{1}{x+1} = \left(-b \cdot (x+1)^{-1}\right)' =$$

$$= (-)\, b \cdot (-1) \frac{1}{(x+1)^2} = +\frac{b}{(x+1)^2}$$