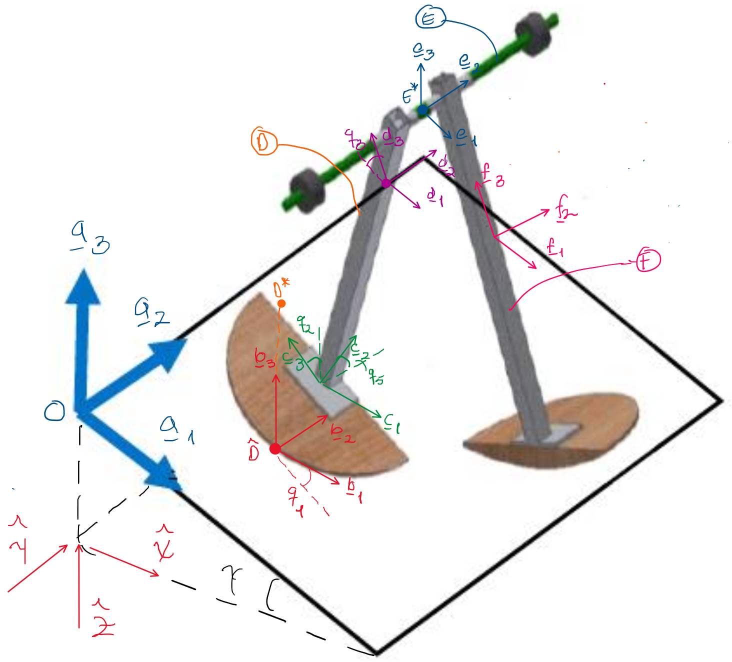
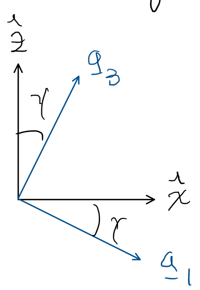


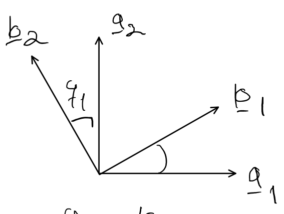
De manera general, se tiene que el plano sobre el que se mueven los pies semiesféricos puede estar inclinado un ángulo γ, es decir:



$$q_3 = \cos\gamma\,\hat{z} + \operatorname{sen}\gamma\,\hat{x}$$
$$q_1 = \cos\gamma\,\hat{x} - \operatorname{sen}\gamma\,\hat{z}$$

γ	q_1	q_3
$\hat{x}$	cos	sen
$\hat{z}$	-sen	cos

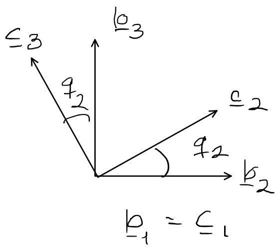
Ahora, se tiene que para determinar la orientación del marco D se necesitan de dos marcos de referencia intermedios (B y C):



$q_3 = b_3$

q_1	b_1	b_2
q_1	cos	- sen
q_2	sen	cos

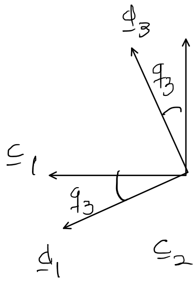
$$[{}^{A}R^{B}] = \begin{bmatrix} \cos q_1 & -\sin q_1 & 0 \\ \sin q_1 & \cos q_1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$



$b_1 = c_1$

q_2	c_2	c_3
b_2	cos	- sin
b_3	sin	cos

$$[{}^{B}R^{C}] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos q_2 & -\sin q_2 \\ 0 & \sin q_2 & \cos q_2 \end{bmatrix}$$



$\underline{c}_2 = \underline{d}_2$

q_3	$\underline{d}_1$	$\underline{d}_3$
$\underline{c}_1$	cos	sin
$\underline{c}_3$	-sin	cos

$$[{}^{C}R^{D}] = \begin{bmatrix} \cos q_3 & 0 & \sin q_3 \\ 0 & 1 & 0 \\ -\sin q_3 & 0 & \cos q_3 \end{bmatrix}$$

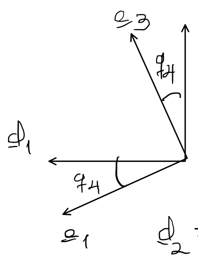
De esta forma, se obtiene que:

$$[{}^{A}R^{D}] = [{}^{A}R^{B}][{}^{B}R^{C}][{}^{C}R^{D}] = \begin{bmatrix} \cos q_1 & -\sin q_1 & 0 \\ \sin q_1 & \cos q_1 & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos q_2 & -\sin q_2 \\ 0 & \sin q_2 & \cos q_2 \end{bmatrix}\begin{bmatrix} \cos q_3 & 0 & \sin q_3 \\ 0 & 1 & 0 \\ -\sin q_3 & 0 & \cos q_3 \end{bmatrix}$$

$$[{}^{A}R^{D}] = \begin{bmatrix} \cos q_1 & -\sin q_1 \cos q_2 & \sin q_1 \sin q_2 \\ \sin q_1 & \cos q_1 \cos q_2 & -\cos q_1 \sin q_2 \\ 0 & \sin q_2 & \cos q_2 \end{bmatrix}\begin{bmatrix} \cos q_3 & 0 & \sin q_3 \\ 0 & 1 & 0 \\ -\sin q_3 & 0 & \cos q_3 \end{bmatrix}$$

$$[{}^{A}R^{D}] = \begin{bmatrix} c_1c_3 - s_1s_2s_3 & -s_1c_2 & c_1s_3 + s_1s_2c_3 \\ s_1c_3 + c_1s_2s_3 & c_1c_2 & s_1s_3 - c_1s_2c_3 \\ -c_2s_3 & s_2 & c_2c_3 \end{bmatrix}$$

Ahora, entre el marco D y el E existe una coordenada generalizada q_4 representando la rotación de la varilla de la cadera con respecto a la pierna de la derecha:



$\underline{d}_2 = \underline{e}_2$

q_4	$\underline{e}_1$	$\underline{e}_3$
$\underline{d}_1$	cos	sin
$\underline{d}_3$	-sin	cos

$$[{}^{D}R^{E}] = \begin{bmatrix} \cos q_4 & 0 & \sin q_4 \\ 0 & 1 & 0 \\ -\sin q_4 & 0 & \cos q_4 \end{bmatrix}$$

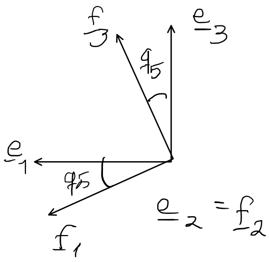
Así, se obtiene que:

$$[{}^{A}R^{E}] = [{}^{A}R^{D}][{}^{D}R^{E}] = \begin{bmatrix} c_1c_3 - s_1s_2s_3 & -s_1c_2 & c_1s_3 + s_1s_2c_3 \\ s_1c_3 + c_1s_2s_3 & c_1c_2 & s_1s_3 - c_1s_2c_3 \\ -c_2s_3 & s_2 & c_2c_3 \end{bmatrix}\begin{bmatrix} c_4 & 0 & s_4 \\ 0 & 1 & 0 \\ -s_4 & 0 & c_4 \end{bmatrix}$$

$$[{}^{A}R^{E}] = \begin{bmatrix} c_1c_3c_4 - s_1s_2s_3c_4 - c_1s_3s_4 - s_1s_2c_3s_4 & -s_1c_2 & c_1c_3s_4 - s_1s_2s_3s_4 + c_1s_3c_4 + s_1s_2c_3c_4 \\ s_1c_3c_4 + c_1s_2s_3c_4 - s_1s_3s_4 + c_1s_2c_3s_4 & c_1c_2 & s_1s_3s_4 + c_1s_2s_3s_4 + s_1s_3c_4 - c_1s_2c_3c_4 \\ -c_2s_3c_4 - c_2c_3s_4 & s_2 & -c_2s_3s_4 + c_2c_3c_4 \end{bmatrix}$$

$$[{}^{A}R^{E}] = \begin{bmatrix} c_1c_{3+4} - s_1s_2s_{3+4} & -s_1c_2 & c_1s_{3+4} + s_1s_2c_{3+4} \\ s_1c_{3+4} + c_1s_2s_{3+4} & c_1c_2 & s_1s_{3+4} - c_1s_2c_{3+4} \\ -c_2s_{3+4} & s_2 & c_2c_{3+4} \end{bmatrix}$$

Además, se tiene que la pierna izquierda puede rotar alrededor de la barra de la cadera, esto representado a través de la coordenada generalizada q_5



q_5	f_1	f_3
e_1	cos	sin
e_3	−sin	cos

$$[{}^E R^F] = \begin{bmatrix} \cos q_5 & 0 & \sin q_5 \\ 0 & 1 & 0 \\ -\sin q_5 & 0 & \cos q_5 \end{bmatrix}$$

Así,

$$[{}^A R^F] = [{}^A R^E][{}^E R^F] = \begin{bmatrix} c_1 c_{3+4} - s_1 s_2 s_{3+4} & -s_1 c_2 & c_1 s_{3+4} + s_1 s_2 c_{3+4} \\ s_1 c_{3+4} + c_1 s_2 s_{3+4} & c_1 c_2 & s_1 s_{3+4} - c_1 s_2 c_{3+4} \\ -c_2 s_{3+4} & s_2 & c_2 c_{3+4} \end{bmatrix} \begin{bmatrix} \cos q_5 & 0 & \sin q_5 \\ 0 & 1 & 0 \\ -\sin q_5 & 0 & \cos q_5 \end{bmatrix}$$

$$[{}^A R^F] = \begin{bmatrix} c_1 c_{3+4+5} - s_1 s_2 s_{3+4+5} & -s_1 c_2 & c_1 s_{3+4+5} + s_1 s_2 c_{3+4+5} \\ s_1 c_{3+4+5} + c_1 s_2 s_{3+4} & c_1 c_2 & s_1 s_{3+4+5} - c_1 s_2 c_{3+4+5} \\ -c_2 s_{3+4+5} & s_2 & c_2 c_{3+4+5} \end{bmatrix}$$

Velocidades Angulares

$${}^A\underline{\omega}^B = \dot{q}_1 \underline{a}_3$$

$${}^A\underline{\omega}^C = {}^A\underline{\omega}^B + {}^B\underline{\omega}^C = \dot{q}_1 \underline{a}_3 + \dot{q}_2 \underline{b}_1 = \dot{q}_1 \underline{a}_3 + \dot{q}_2 [{}^A R^B] \underline{b}_1$$

$${}^A\underline{\omega}^D = {}^A\underline{\omega}^B + {}^B\underline{\omega}^C + {}^C\underline{\omega}^D = \dot{q}_1 \underline{a}_3 + \dot{q}_2 [{}^A R^B] \underline{b}_1 + \dot{q}_3 [{}^A R^C] \underline{c}_2$$

$${}^A\underline{\omega}^E = {}^A\underline{\omega}^B + {}^B\underline{\omega}^C + {}^C\underline{\omega}^D + {}^D\underline{\omega}^E = \dot{q}_1 \underline{a}_3 + \dot{q}_2 [{}^A R^B] \underline{b}_1 + \dot{q}_3 [{}^A R^C] \underline{c}_2 + \dot{q}_4 [{}^A R^D] \underline{d}_2$$

$${}^A\underline{\omega}^F = \dot{q}_1 \underline{a}_3 + \dot{q}_2 [{}^A R^B] \underline{b}_1 + \dot{q}_3 [{}^A R^C] \underline{c}_2 + \dot{q}_4 [{}^A R^D] \underline{d}_2 + \dot{q}_5 [{}^A R^E] \underline{e}_2$$

Aceleraciones Angulares

$${}^A\underline{\alpha}^B = \ddot{q}_1 \underline{a}_3 \qquad {}^B\underline{\alpha}^C = \ddot{q}_2 \underline{b}_1 \qquad {}^C\underline{\alpha}^D = \ddot{q}_3 \underline{c}_2 \qquad {}^D\underline{\alpha}^E = \ddot{q}_4 \underline{d}_2 \qquad {}^E\underline{\alpha}^F = \ddot{q}_5 \underline{e}_2$$

$${}^A\underline{\alpha}^C = {}^A\underline{\alpha}^B + {}^B\underline{\alpha}^C + {}^A\underline{\omega}^B \times {}^B\underline{\omega}^C$$

$${}^A\underline{\alpha}^D = {}^A\underline{\alpha}^B + {}^B\underline{\alpha}^C + {}^C\underline{\alpha}^D + {}^A\underline{\omega}^B \times {}^B\underline{\omega}^C + {}^A\underline{\omega}^C \times {}^C\underline{\omega}^D$$

$${}^A\underline{\alpha}^E = {}^A\underline{\alpha}^B + {}^B\underline{\alpha} + {}^C\underline{\alpha}^D + {}^A\underline{\omega}^B \times {}^B\underline{\omega}^C + {}^A\underline{\omega}^C \times {}^C\underline{\omega}^D + {}^A\underline{\omega}^D \times {}^D\underline{\omega}^E$$

$${}^A\underline{\alpha}^F = {}^A\underline{\alpha}^B + {}^B\underline{\alpha} + {}^C\underline{\alpha}^D + {}^A\underline{\omega}^B \times {}^B\underline{\omega}^C + {}^A\underline{\omega}^C \times {}^C\underline{\omega}^D + {}^A\underline{\omega}^D \times {}^D\underline{\omega}^E + {}^A\underline{\omega}^E \times {}^E\underline{\omega}^F$$

Velocidades y Aceleraciones de puntos

Se considerarán los siguientes puntos:

- D^*: Centro de la esfera que se encuentra en los pies del caminador
- $\hat{D}$: Punto de la esfera que entra en contacto con el plano del piso
- E^*: Centro de masa de la barra en la cadera
- R^* y I^*: Centros de masa de las piernas derecha e izquierda, respectivamente

Las dimensiones se extrajeron de la Tesis de Marín [1]:

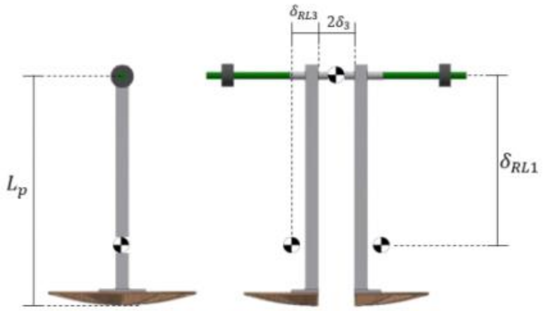


Ilustración 6. Propiedades Geométricas

Simbolo	Cantidad	Valor
l_p	Longitud de cada pierna	0.4418 m
m_p	Masa de cada pierna	0.302 Kg
m_H	Masa de la cadera	1.027 Kg
R_esf	Radio de la superficie de contacto	489.639 mm
$\delta 3$	Distancia	35 mm
$\delta RL3$	Distancia	30.236 mm
$\delta RL1$	Distancia	316.887 mm

Tabla 1.Propiedades Geométricas

Punto $\hat{D}$

$$^A\underline{V}^{\hat D} = {}^A\underline{V}^{D*} + {}^A\underline{\omega}^{D} \times \underline{r}^{\hat D/D*} \qquad \underline{r}^{\hat D/D*} = -R\,\underline{b}_3$$

$$^A\underline{V}^{D*} = \dot q_x \underline{a}_1 + \dot q_y \underline{a}_2 + {}^A\underline{\omega}^{B} \times \underline{r}^{\hat D/D*}$$

$$^A\underline{V}^{\hat D} = \dot q_x \underline{a}_1 + \dot q_y \underline{a}_2 + {}^A\underline{\omega}^{B} \times \underline{r}^{\hat D/D*} + {}^A\underline{\omega}^{D} \times \underline{r}^{\hat D/D*}$$

$$= \dot q_x \underline{a}_1 + \dot q_y \underline{a}_2 + (\dot q_1 \underline{b}_3) \times (-R\underline{b}_3) + (\dot q_1 \underline{b}_3 + \dot q_2 \underline{b}_1 + \dot q_3 \underline{c}_2) \times (-R\underline{b}_3)$$

$$\dot q_x \underline{a}_1 + \dot q_y \underline{a}_2 + (\dot q_2 \underline{b}_1 + \dot q_3 \underline{c}_2) \times (-R\underline{b}_3) = 0 \quad \Rightarrow \text{Rodadura}$$

$$(\dot q_2 \underline{b}_1) \times (-R\underline{b}_3) = \dot q_2 R\,\underline{b}_2$$

$$(\dot q_3 \underline{c}_2) \times (-R\underline{b}_3)$$

$$-\dot q_3 R(\cos q_2 \underline{b}_2 + \sin q_2 \underline{b}_3) \times \underline{b}_3 = -\dot q_3 R \cos q_2 \underline{b}_1$$

De la matriz de rotación $[^BR^C]$:

$$\underline{c}_2 = \cos q_2 \underline{b}_2 + \sin q_2 \underline{b}_3$$

$$\underline{c}_3 = -\sin q_2 \underline{b}_2 + \cos q_2 \underline{b}_3$$

Así,

$$\dot q_x \underline{a}_1 + \dot q_y \underline{a}_2 + \dot q_2 R\,\underline{b}_2 - \dot q_3 R \cos q_2 \underline{b}_1 = 0$$

Por la matriz de rotación $[^AR^B] \Rightarrow$

$$\underline{b}_1 = \cos q_1 \underline{a}_1 + \sin q_1 \underline{a}_2$$

$$\underline{b}_2 = -\sin q_1 \underline{a}_1 + \cos q_1 \underline{a}_2$$

$$\dot q_x \underline{a}_1 + \dot q_y \underline{a}_2 + \dot q_2 R(-\sin q_1 \underline{a}_1 + \cos q_1 \underline{a}_2) - \dot q_3 R \cos q_2 (\cos q_1 \underline{a}_1 + \sin q_1 \underline{a}_2) = 0$$

$$(\dot q_x - \dot q_2 R \sin q_1 - \dot q_3 R \cos q_2 \cos q_1)\underline{a}_1 + (\dot q_y + \dot q_2 R \cos q_1 - \dot q_3 R \cos q_2 \sin q_1)\underline{a}_2 = 0$$

$$\dot q_x - \dot q_2 R \sin q_1 - \dot q_3 R \cos q_2 \cos q_1 = 0 \qquad \dot q_y + \dot q_2 R \cos q_1 - \dot q_3 R \cos q_2 \sin q_1 = 0$$

$$\dot q_x = \dot q_2 R \sin q_1 + \dot q_3 R \cos q_2 \cos q_1 \qquad \dot q_y = -\dot q_2 R \cos q_1 + \dot q_3 R \cos q_2 \sin q_1$$

[1] : Marín Lancheros, D. (2017). *Modelamiento dinámico en 3D y diseño de observador para un caminador bípedo pasivo.* Uniandes

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