Intro to DSP - MUSIC 320 Homework 1 (100 Points)

Handed out on: Septemebr 22$^{\text{th}}$, 2025 - 3:30 pm

Due on: October 6$^{\text{th}}$, 2025 - 1:30 pm

Submission instructions

Please submit homework using Canvas's assignment submission system (in the Assignments tab). Select the associated link in that tab and proceed with your submission.

You must **turn in 2 items**:

- **A write-up (with all the instructed answers and figures)** in PDF format. Scanned and typed submission are both acceptable.
- **All your code and data files in a single compressed** file (zip, tar or tar.gz) containing the jupyter notebook with all the answers to the lab exercises and any other files requested in the homework instructions.

For your files, use the following **naming convention**: `hwX_suid.type` where X is the homework number and suid is your Stanford ID. For example, the write-up for homework 1 will be `hw1_mab.pdf` and the code files `hw1_mab.zip/hw1_mab.tar.gz/hw1_mab.tar`.

Required readings - Please make sure to leave yourself enough time during the week to get through the week's reading assignment prior to class. Is is important to have spent some time with the material prior to class in order to grasp the topic at hand and to fully participate in the class discussion. Please note that your ability to participate in discussion and answer simple questions will influence your classroom participation grade.

Theory Problems (65 pts.)

1. (15 pts) [Sinusoids] Define $x(t)$ as

$$x(t) = 2\sin\left(\omega_0 t - \frac{\pi}{4}\right) + \cos(\omega_0 t)$$

   (a) Express $x(t)$ in the form $x(t) = A\cos(\omega_0 t + \phi)$, where $\phi \in [-\pi, \pi)$ is in radians and $A > 0$.

   (b) Does the previous result depend on some special property of the two sinusoids combined, or can any two sinusoids be combined into a single sinusoid like this? Under what conditions can two different sinusoids be combined like this?

   (c) Find a complex valued signal $\tilde{x}(t)$ such that $x(t) = Re\{\tilde{x}(t)\}$

2. (15 pts) The initial phase of a sinusoid can be related to a time shift as follows:

$$x(t) = A\cos(2\pi f_0 t + \phi) = A\cos(2\pi f_0(t - t_1))$$

   Assuming that the period of the sinusoidal wave is $T_0 = 12$ sec, please find the initial phase ϕ for the following values of the time shift t_1 :

   (a) $t_1 = -3$ sec

   (b) $t_1 = 0$ sec

(c) $t_1 = 3$ sec

(d) $t_1 = 7$ sec

3. (15 pts)

   (a) For a sinusoid with a period $T_0 = 1/10$ seconds, what is the frequency f_0 in Hz? What is the frequency ω_0 in radians per second?

   (b) Define $x(t)$ as

   $$x(t) = A\sin[\omega(t-\tau)]$$

   Write an expression for the phase in terms of the frequency ω and time delay τ.

   (c) For $x(t)$ defined as above, find the phase $\phi(t)$ at $t = 0$ for a time delay of $\tau = .25$ seconds and the frequency obtained for part (a).

4. (10 pts) [AM] An amplitude modulated (AM) cosine wave is represented by

   $$x(t) = [12 + 7\sin(\pi t - \pi/3)]\cos(13\pi t)$$

   Use complex sinusoids to show that $x(t)$ can be expressed as

   $$x(t) = A_1\cos(\omega_1 t + \phi_1) + A_2\cos(\omega_2 t + \phi_2) + A_3\cos(\omega_3 t + \phi_3)$$

   where $\omega_1 < \omega_2 < \omega_3$. What are the values A_i, ϕ_i, and ω_i for $i = 1, 2, 3$?

5. (10 pts) [Taylor series] The Taylor series of e^x, $\cos(x)$, $\sin(x)$ for x near 0 can be written as follows:

   $$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!}$$

   $$\sin(x) = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{(2n+1)!}$$

   $$\cos(x) = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n}}{(2n)!}$$

   Using the terms up to x^{10} show that these Taylor series are consistent with Euler's Identity

   $$e^{jx} = \cos(x) + j\sin(x)$$

Lab Assignments (35 Points)
Please see the attached Jupyter notebook.

Reading Assignment
Chapters 1, 2, 3, 4 from the textbook, J. O. Smith, "Mathematics of the DFT".