

# CS-175 Alternative Assessment - 1903585

## Question 1

- (a)  $\{1, 2, 4\} \in X$
- $1 \in X$  by rule 1 ( $1 \in X$ )
  - $2 \in X$ :  $1 \in X$  by rule 1,  $(1 \times 2) \in X \therefore 2 \in X$
  - $4 \in X$ :  $1 \in X$  by rule 1,  $(1 + 3) \in X \therefore 4 \in X$
- (b)  $\{3, 6, 9\} \notin X$
- If  $3 \in X$  then  $(3 - 3) \in X \rightarrow 0 \in X$ ,  $0 \notin X \therefore 3 \notin X$  by rule 3 (the membership cannot be established from the rules 1 And 2)
  - If  $6 \in X$  then  $(6 - 3) \in X \rightarrow 3 \in X$ ,  $3 \notin X$  by a.  $\therefore 6 \notin X$  by rule 3 (the membership cannot be established from the rules 1 And 2)
  - If  $9 \in X$  then  $(9 - 3) \in X \rightarrow 6 \in X$ ,  $6 \notin X$  by b.  $\therefore 9 \notin X$  by rule 3 (the membership cannot be established from the rules 1 And 2)
- (c)  $X$  is a set of natural numbers except numbers that are divisible by three. Numbers divisible by three cannot be a part of this set because their membership cannot be established from the rules 1 and 2

## Question 2

- (a)
- The first player takes any number of coins, leaving either one pile left or two piles with unequal numbers of coins. In the first case, second player takes the remaining pile and wins. In the second case, the second player takes such number of coins from the second pile so that the number of coins in both piles is equal again.
  - The first player takes such number of coins from one pile so that the number of coins in both piles is equal. The second player is therefore in losing position as explained in a.
- (b) With the assumption there cannot be an empty pile:
- With only one coin in each pile, the first player is in a winning position; the winning strategy is simply taking one coin (/pile)
  - Otherwise, the winning strategy is to follow the “normal” NIM strategy. This repeats until there is one of two scenarios:
    - There’s only one pile remaining; the player takes coins leaving the pile with only one coin
    - There’s one pile containing two or more coins and one pile with only one remaining coin; the player takes the full bigger pile, again, leaving only one coin

## Question 3

- (a)
- Alice as the first player will perform an action from the state  $Cl_{0'}$  to the state  $Cl_1$ . Bob will make an action from the state  $Cl_1$  to the state  $Cl_2$ . From there, Bob can therefore copy any move that Alice makes for  $n$  moves, leaving him in the  $Cl_0$  state. Alice as an “Attacker” is therefore a loser.
  - The difference between  $Cl_{0'}$  and  $Cl_0$  is that from the state  $Cl_{0'}$ , we can make an action that will get us to a state from which we can do an infinite number of tick actions. That is not true for the state  $Cl_0$ , since the sequence of the tick actions from this state must terminate.
- (b)
- The statement (ii) can be expressed. Expressed in modal logic  $M$ :  
 $P \models \langle \text{buttonPressed} \rangle [-] \text{false}$

P – stands for process

**buttonPressed** – represents the emergency button being pressed

Therefore, pressing the emergency button is an action that will get us to a state from which we cannot perform any further action (represented by [-])

- b. The statement (i) cannot be expressed, since we do not have any actions that would make up a transition system

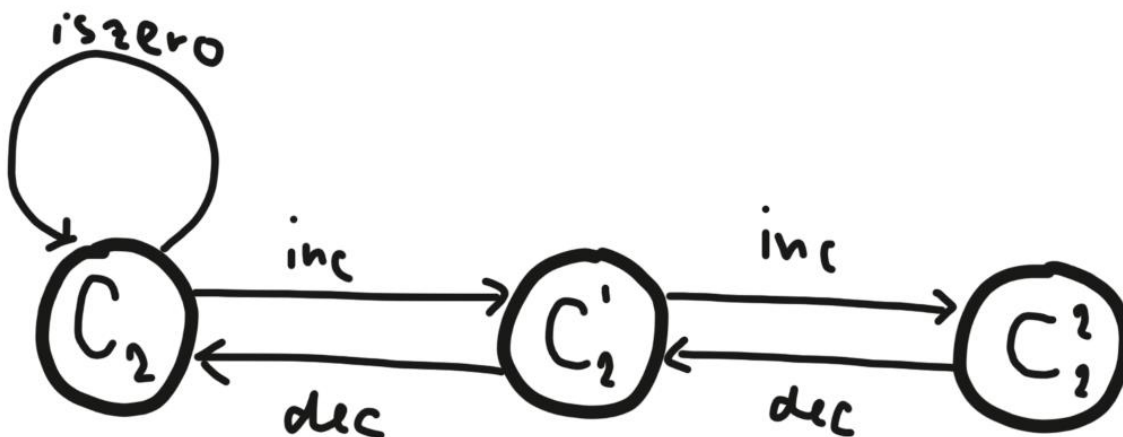
#### Question 4

(a) Actions = {iszero, inc, dec}

States =  $\{C_2, C_2^1, C_2^2\}$

Transitions =  $\{(C_2, \text{iszero}, C_2), (C_2, \text{inc}, C_2^1), (C_2^1, \text{inc}, C_2^2), (C_2^1, \text{dec}, C_2), (C_2^2, \text{dec}, C_2^1)\}$

(b) –



(c)  $C_2 \models \langle \text{iszero} \rangle \text{true} \wedge \langle \text{inc} \rangle \text{true} \wedge [\text{dec}] \text{false}$

$C_2^1 \models \langle \text{inc} \rangle \text{true} \wedge \langle \text{dec} \rangle \text{true} \wedge [\text{iszero}] \text{false}$

$C_2^2 \models \langle \text{dec} \rangle \text{true} \wedge [\text{inc}] \text{false} \wedge [\text{iszero}] \text{false}$

(d)

a.  $C_3 \equiv \text{iszero}.C_3 + \text{inc}.C_3^1$

$C_3^1 \equiv \text{inc}.C_3^2 + \text{dec}.C_3$

$C_3^2 \equiv \text{inc}.C_3^3 + \text{dec}.C_3^1$

$C_3^3 \equiv \text{dec}.C_3^2$

b.

