

# Relational Model 4: Relational Calculus

Gary KL Tam

Department of Computer Science  
Swansea University

- Is Relational Calculus mathematics?

- Is Relational Calculus mathematics?
- Yes, based on a branch of Predicate Calculus (Logic).

- Is Relational Calculus mathematics?
- Yes, based on a branch of Predicate Calculus (Logic).
- Why should we learn it?

- Is Relational Calculus mathematics?
- Yes, based on a branch of Predicate Calculus (Logic).
- Why should we learn it?
- Because university teaches it !???

- Is Relational Calculus mathematics?
- Yes, based on a branch of Predicate Calculus (Logic).
- Why should we learn it?
- Because university teaches it !???

## Example Use

Microsoft Access: Query by example!

# Relational Algebra & Calculus

- Relational Algebra is procedural.
- Relational calculus is declarative

# Relational Algebra & Calculus

- Relational Algebra is procedural.
- Relational calculus is declarative
- It has been proven that Relational Calculus is **equivalent** to Relational Algebra in term of **expressive power**.



# Relational Algebra & Calculus

- Relational Algebra is procedural.
- Relational calculus is declarative
- It has been proven that Relational Calculus is **equivalent** to Relational Algebra in term of **expressive power**.
- The **upshot** of this is that no matter in what form a query (e.g. using SQL) is made, the DBMS should be able to ignore the steps implied by the formulation of the query and make decisions about how to fulfil that query in the most efficient manner.

# Relational Algebra & Calculus

- Relational Algebra is procedural.
- Relational calculus is declarative
- It has been proven that Relational Calculus is **equivalent** to Relational Algebra in term of **expressive power**.
- The **upshot** of this is that no matter in what form a query (e.g. using SQL) is made, the DBMS should be able to ignore the steps implied by the formulation of the query and make decisions about how to fulfil that query in the most efficient manner.
- This is the process known as **Query Optimisation**.

# Tuple Relational Calculus

- A non-procedural query language, where each query is of the form:  $\{t|P(t)\}$
- It is the set of all tuples  $t$  such that predicate  $P$  is true for  $t$
- $t$  is a **tuple variable**,  $t[A]$  denotes the value of tuple  $t$  on attribute  $A$
- $t \in r$  denotes that tuple  $t$  is in relation  $r$
- $P$  is a formula

# Predicate Calculus Formula

- Set of attributes and constants
- Set of comparison operators: (e.g.,  $<$ ,  $\leq$ ,  $=$ ,  $\neq$ ,  $\geq$ ,  $>$ )
- Set of connectives: and ( $\wedge$ ), or ( $\vee$ ), not ( $\neg$ )
- Implication ( $\Rightarrow$ ):  $x \Rightarrow y$ , if  $x$  is true, then  $y$  is true

$$x \Rightarrow y \equiv \neg x \vee y$$

# Predicate Calculus Formula

- Set of attributes and constants
- Set of comparison operators: (e.g.,  $<$ ,  $\leq$ ,  $=$ ,  $\neq$ ,  $\geq$ ,  $>$ )
- Set of connectives: and ( $\wedge$ ), or ( $\vee$ ), not ( $\neg$ )
- Implication ( $\Rightarrow$ ):  $x \Rightarrow y$ , if  $x$  is true, then  $y$  is true  
$$x \Rightarrow y \equiv \neg x \vee y$$
- Set of quantifiers:  
 $\exists t \in r(Q(t))$ : "there exists ( $\exists$ )" a tuple  $t$  in relation  $r$  such that predicate  $Q(t)$  is true.  
 $\forall t \in r(Q(t))$ :  $Q$  is true "for all ( $\forall$ )" tuples  $t$  in relation  $r$ .

## Example Query

loan

loan-number	branch-name	amount
L110	swansea	1530
L223	cardiff	2140
L331	neath	1000

Query 1:

Find the loan-number, branch-name and amount for loans of over £1200

## Example Query

loan

loan-number	branch-name	amount
L110	swansea	1530
L223	cardiff	2140
L331	neath	1000

Query 1:

Find the loan-number, branch-name and amount for loans of over £1200

Relational Algebra:  $\sigma_{\text{amount} > 1200}(\text{loan})$

## Example Query

loan

loan-number	branch-name	amount	
L110	swansea	1530	✓
L223	cardiff	2140	✓
L331	neath	1000	

### Query 1:

Find the loan-number, branch-name and amount for loans of over £1200

Relational Algebra:  $\sigma_{\text{amount} > 1200}(\text{loan})$

[Selection]

Step1: go to loan table

Step2: go through each tuple in loan

Step2-1: select the tuple with amount > 1200



## Example Query

loan

loan-number	branch-name	amount	
L110	swansea	1530	✓
L223	cardiff	2140	✓
L331	neath	1000	

Query 1:

Find the loan-number, branch-name and amount for loans of over £1200

Relational Calculus:  $\{t \mid t \in loan(t[amount] > 1200)\}$


## Example Query


loan	loan-number	branch-name	amount	
$t$ :	L110	swansea	1530	✓
$t$ :	L223	cardiff	2140	✓
	L331	neath	1000	


### Query 1:


Find the loan-number, branch-name and amount for loans of over £1200


Relational Calculus:  $\{t \mid t \in loan(t[amount] > 1200)\}$

$t$  is the result 

what is  $t$ ? 

| means such that 

$t$  is a tuple in loan 

$t[amount] > 1200$  

Note: RC defines the data (not steps)!

## Example Query

loan

loan-number	branch-name	amount
L110	swansea	1530
L223	cardiff	2140
L331	neath	1000

Query 2:

Find the loan number for each loan of an amount > £1200

## Example Query

loan	loan-number	branch-name	amount
	L110	swansea	1530
	L223	cardiff	2140
	L331	neath	1000

Query 2:

Find the loan number for each loan of an amount > £1200

Relational Algebra:  $\Pi_{\text{loan-number}} (\sigma_{\text{amount} > 1200} (\text{loan}))$

## Example Query

loan

loan-number	branch-name	amount
L110	swansea	1530
L223	cardiff	2140
L331	neath	1000

### Query 2:

Find the loan number for each loan of an amount > £1200

Relational Algebra:

$$\pi_{\text{loan-number}} (\sigma_{\text{amount} > 1200} (\text{loan}))$$

[Selection]

Step1: go to loan table

Step2: go through each tuple in loan

Step3: select the tuple with amount > 1200

[Projection]

Step1: go through each tuple in the above result

Step1-1: take loan-number

## Example Query

loan

loan-number	branch-name	amount
L110	swansea	1530
L223	cardiff	2140
L331	neath	1000

### Query 2:

Find the loan number for each loan of an amount > £1200

Relational Calculus:

$$\{t | \exists s \in loan(t[\text{loan-number}] = s[\text{loan-number}] \wedge s[\text{amount}] > 1200)\}$$

## Example Query

loan	loan-number	branch-name	amount
s:	L110	swansea	1530
s:	L223	cardiff	2140
	L331	neath	1000

$t$

### Query 2:

Find the loan number for each loan of an amount > £1200

### Relational Calculus:

$\{t \mid \exists s \in \text{loan} (t[\text{loan-number}] = s[\text{loan-number}] \wedge s[\text{amount}] > 1200)\}$

*t* is the result

what is *t*?

there exists a tuple *s* in loan

*t* is a tuple, but contains one attribute [loan-number]

is equal to, aka. == (in java)  
i.e., not assignment!

and  $s[\text{amount}] > 1200$

## Example Query

borrower

loan-number	cust-name
L110	Gary
L223	Maple
L331	Syrup

depositor

cust-name	acct-num
Gary	123
April	345
Dave	567

Query 3:

Find the names of all customers having a loan, an account, or both at the bank



## Example Query

borrower

loan-number	cust-name
L110	Gary
L223	Maple
L331	Syrup

depositor

cust-name	acct-num
Gary	123
April	345
Dave	567

### Query 3:

Find the names of all customers having a loan, an account, or both at the bank

Relational Algebra:

$$\Pi_{\text{cust-name}}(\text{borrower}) \cup \Pi_{\text{cust-name}}(\text{depositor})$$

Steps:

Step1: a projection on borrower

Step2: a projection on depositor

Step3: union of the above two results

cust-name
Gary
April
Dave
Maple
Syrup

## Example Query

borrower

loan-number	cust-name
L110	Gary
L223	Maple
L331	Syrup

depositor

cust-name	acct-num
Gary	123
April	345
Dave	567

Query 3:

Find the names of all customers having a loan, an account, or both at the bank

Relational Calculus:

$$\{t | \exists s \in \text{borrower} (t[\text{cust-name}] = s[\text{cust-name}]) \vee \exists u \in \text{depositor} (t[\text{cust-name}] = u[\text{cust-name}])\}$$

cust-name
Gary
April
Dave
Maple
Syrup

## Example Query

borrower	loan-number	cust-name
s:	L110	Gary
s:	L223	Maple
s:	L331	Syrup

*t*

depositor	cust-name	acct-num
u:	Gary	123
u:	April	345
u:	Dave	567

*t*

### Query 3:

Find the names of all customers having a loan, an account, or both at the bank

### Relational Calculus:

there exists a tuple *s* in borrower

$$\{t | \exists s \in \text{borrower} (t[\text{cust-name}] = s[\text{cust-name}]) \vee \exists u \in \text{depositor} (t[\text{cust-name}] = u[\text{cust-name}])\}$$

there exists a tuple *u* in depositor

*t* is the cust-name of *s* or *u*

cust-name
Gary
April
Dave
Maple
Syrup

## Example Query

borrower

loan-number	cust-name
L110	Gary
L223	Maple
L331	Syrup

loan

loan-number	branch-name	amount
L110	swansea	1530
L223	cardiff	2140
L331	neath	1000

Query 4:

Find the names of all customers having a loan at the Neath branch

## Example Query

borrower	loan-number	cust-name	loan	loan-number	branch-name	amount
	L110	Gary		L110	swansea	1530
	L223	Maple		L223	cardiff	2140
	L331	Syrup		L331	neath	1000

### Query 4:

Find the names of all customers having a loan at the Neath branch

Relational Algebra:

$$\Pi_{\text{cust-name}} (\sigma_{\text{branch-name}='neath'} (\text{loan} \bowtie \text{borrower}))$$

Steps:

Step1: natural join loan and borrower on ALL common attributes

Step2: for all tuple from step1, select tuple with branch-name = 'neath'

Step3: pick the cust-name column

## Example Query

borrower

loan-number	cust-name
L110	Gary
L223	Maple
L331	Syrup

loan

loan-number	branch-name	amount
L110	swansea	1530
L223	cardiff	2140
L331	neath	1000

Query 4:

Find the names of all customers having a loan at the Neath branch

$\Pi_{\text{cust-name}} (\sigma_{\text{branch-name}='neath'} ($

loan-number	cust-name	branch-name	amount
L110	Gary	swansea	1530
L223	Maple	cardiff	2140
L331	Syrup	neath	1000

$)$

$\Pi_{\text{cust-name}} ($

loan-number	cust-name	branch-name	amount
L331	Syrup	neath	1000

$)$

cust-name
Syrup

## Example Query

borrower

loan-number	cust-name
L110	Gary
L223	Maple
L331	Syrup

loan

loan-number	branch-name	amount
L110	swansea	1530
L223	cardiff	2140
L331	neath	1000

Query 4:

Find the names of all customers having a loan at the Neath branch

Relational Calculus:

$$\{t | \exists s \in \text{borrower} (t[\text{cust-name}] = s[\text{cust-name}]) \wedge \\ \exists u \in \text{loan} (u[\text{loan-number}] = s[\text{loan-number}] \wedge u[\text{branch-name}] = \text{'neath'})\}$$

## Example Query

borrower	loan-number	cust-name
	L110	Gary
	L223	Maple
	s: L331	Syrup

loan	loan-number	branch-name	amount
	L110	swansea	1530
	L223	cardiff	2140
	u: L331	neath	1000

Query 4:

Find the names of all customers having a loan at the Neath branch

Relational Calculus:

$$\{t | \exists s \in \text{borrower}(t[\text{cust-name}] = s[\text{cust-name}]) \wedge \exists u \in \text{loan}(u[\text{loan-number}] = s[\text{loan-number}] \wedge u[\text{branch-name}] = \text{'neath'})\}$$



## Example Query

borrower	loan-number	cust-name	loan	loan-number	branch-name	amount
	L110	Gary		L110	swansea	1530
	L223	Maple		L223	neath	2140
	L331	Syrup		L331	neath	1000

Query 5:

Find the names of all customers who have a loan at the 'neath' branch, but no account at any branch of the bank

depositor	
cust-name	acct-num
Gary	123
Maple	345
Dave	567

## Example Query

borrower	loan-number	cust-name	loan	loan-number	branch-name	amount
	L110	Gary		L110	swansea	1530
	L223	Maple		L223	neath	2140
	L331	Syrup		L331	neath	1000

Query 5:

Find the names of all customers who have a loan at the 'neath' branch, but no account at any branch of the bank

depositor	
cust-name	acct-num
Gary	123
Maple	345
Dave	567

Relational Algebra:

$$\Pi_{\text{cust-name}} (\sigma_{\text{branch-name}='neath'} (\text{loan} \bowtie \text{borrower})) - \Pi_{\text{cust-name}} (\text{depositor})$$

cust-loan-in-neath
Maple
Syrup

$$- \left( \begin{array}{c} \text{cust-acct} \\ \text{Gary} \\ \text{Maple} \\ \text{Dave} \end{array} \right)$$

Results:

Syrup

## Example Query

borrower	loan-number	cust-name	loan	loan-number	branch-name	amount
	L110	Gary		L110	swansea	1530
	L223	Maple		L223	neath	2140
	L331	Syrup		L331	neath	1000

### Query 5:

Find the names of all customers who have a loan at the 'neath' branch, but no account at any branch of the bank

depositor	
cust-name	acct-num
Gary	123
Maple	345
Dave	567

Relational Calculus:

$$\{t \mid \exists s \in \text{borrower}(t[\text{cust-name}] = s[\text{cust-name}]) \wedge \\ \exists u \in \text{loan}(u[\text{loan-number}] = s[\text{loan-number}] \wedge u[\text{branch-name}] = \text{'neath'}) \\ \wedge \neg \exists v \in \text{depositor}(t[\text{cust-name}] = v[\text{cust-name}])\}$$

## Example Query

borrower	loan-number	cust-name	loan	loan-number	branch-name	amount
	L110	Gary		L110	swansea	1530
s:	L223	Maple	u:	L223	neath	2140
s:	L331	Syrup	u:	L331	neath	1000

Query 5:

Find the names of all customers who have a loan at the 'neath' branch, but no account at any branch of the bank

	depositor
cust-name	acct-num
Gary	123
v: Maple	345
Dave	567

Relational Calculus:

$$\{t \mid \exists s \in \text{borrower}(t[\text{cust-name}] = s[\text{cust-name}]) \wedge \\ \exists u \in \text{loan}(u[\text{loan-number}] = s[\text{loan-number}] \wedge u[\text{branch-name}] = \text{'neath'}) \\ \wedge \neg \exists v \in \text{depositor}(t[\text{cust-name}] = v[\text{cust-name}])\}$$

# Microsoft Access: Query-By-Example

Field:	ShipCountry	Company Name	ExtendedPrice
Table:			
Total:	Group By	Group By	Sum
Sort:			
Show:			
Criteria:	"Canada"		<10000
or:	"UK"		<10000

Ship Country	Company Name	SumOfExtendedPrice
Canada	Bottom-Dollar Markets	\$28,025.51
Canada	Laughing Bacchus	\$522.50
Canada	Mère Paillarde	\$37,123.65
UK	Around the Horn	\$14,602.15
UK	B's Beverages	\$7,383.90

Ship Country	Company Name	SumOfExtendedPrice
Canada	Laughing Bacchus	\$522.50
UK	B's Beverages	\$7,383.90

Base on:

Relational calculus theory

Usage:

**Define what you want**

***Not how* you get it**

DBMS generate the query code for you.

For quick exploration

No knowledge of SQL

# Declarative Programming

- CS-205



- A different way of thinking.

## SQL is built on both:

Relational Algebra (Procedure, step by step)

Relational Calculus (Declarative)

You declare what you want.

