Turing machines – adding tapes and symbols

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Recap

- A Turing machine has an unbounded tape (ie a double linked list) and a finite control (like a finite automaton).
- At any moment, one position on the tape is active and can be read and written to (often called read/write head).
- The Turing machine continues to run until it reaches a special halting state; there is one for answering “yes” and one for “no”.

A single tape is difficult to use

- Try describing a TM (with a single tape) that decides the language $\{ww \mid w \in \Sigma^*\}$.
- Its very cumbersome (but possible).
- It would be much easier if we had two tapes.

Lets have multiple tapes

- A two-tape TM has two unbounded tapes, with each having an independent read/write head.
- Thus, transitions now depend on the current state and two symbols, and yield the new state, two symbols to write and two directional commands.
- The input is initially on the first tape, the second tape starts completely blank ($\perp$).

Theorem

One-tape TMs and two-tape TMs have the exact same computational power.

Aren't we cheating with the alphabet?

Proposition

TMs using only $\Sigma \cup \{\perp\}$ *as alphabet are as powerful as TMs using a larger alphabet* $\Sigma' \supseteq \Sigma \cup \{\perp\}$.

Proof.

We can first space out the input such that after each cell containing an input character there are *k* blank cells. Then we can use binary encoding on those free cells to stand in for 2^k extra symbols. □