

# Applications of Amdahl's Law, & Properties I

## Lecture 15

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CS-210: Concurrency

16 March 2021



# What did we do in the last session?

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- Deadlock handling: An OS developer's perspective.
  - Avoidance
  - Detection and Recovery
- Resource Allocation Graphs.
- Introduction to Amdahl's law.

## Learning outcomes.

- ① To apply Amdahl's law to scenarios and determine appropriate actions for program optimisation.
- ② To define a property of a program.
- ③ To apply safety property in FSP model development and analyse the system.

## Outline.

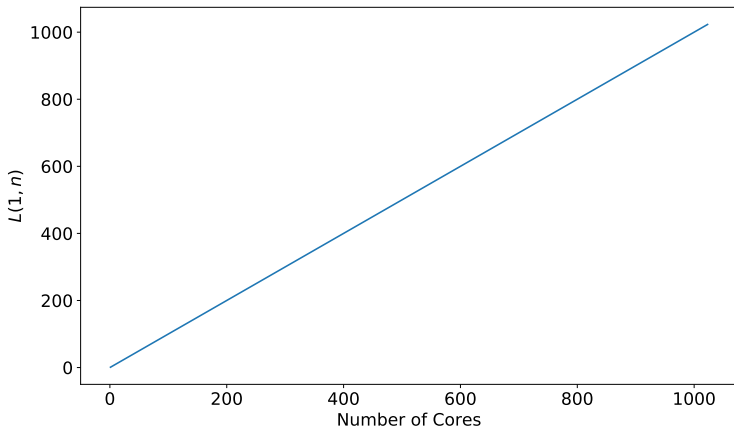
- ① Important insights from Amdahl's law.
- ② Properties of a program.
- ③ single lane bridge.
- ④ Another example: Washing Machine.

Given that:  $L(k, n) \leq \frac{1}{\frac{1}{k}\tilde{t}_s + \frac{1}{n}\tilde{t}_p}$ , what happens if  $\tilde{t}_s = 0$ ?

Here, we are assuming there are no serial part at all, and the parallel part is completely parallelisable.

# Important Observations Using Amdahl's Law

Given that:  $L(k, n) \leq \frac{n}{k}$ , a linear increase in speed up.

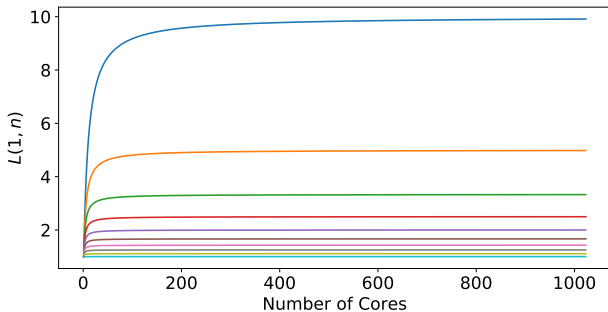
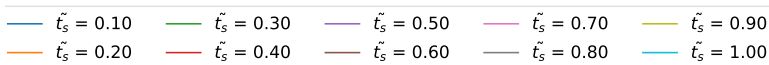


Given that:  $L(k, n) \leq \frac{1}{\frac{1}{k}\tilde{t}_s + \frac{1}{n}\tilde{t}_p}$ , what happens if we vary  $\tilde{t}_s \in \{0.1, \dots, 1\}$ ?

Here, we want to see how adding cores can affect the performance, given a certain amount of serial part in a program.

# Important Observations Using Amdahl's Law

Given that:  $L(1, n) \leq \frac{1}{\tilde{t}_s + \frac{1}{n}\tilde{t}_p}$ , the benefit diminishes as you add more and more processors, and depending on  $\tilde{t}_s$  it may not be beneficial to do this at all.



Given that:  $L(k, n) \leq \frac{1}{\frac{1}{k}\tilde{t}_s + \frac{1}{n}\tilde{t}_p}$ , what happens if we have *infinite* number of processors, i.e.  $n = \infty$ ?

Here, we want to see what is the upper bound of speed up for us given that we have unlimited number of cores.



Given that:  $L(k, n) \leq \frac{1}{\frac{1}{k}\tilde{t}_s + \frac{1}{n}\tilde{t}_p}$ , what happens if we have *infinite* number of processors, i.e.  $n = \infty$ ?

Here, we want to see what is the upper bound of speed up for us given that we have unlimited number of cores.

$\frac{1}{n}\tilde{t}_p = 0$ , so,  $L(k, n) \leq \frac{k}{\tilde{t}_s}$ . This is the upper bound of improvement.

# Any questions?

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What is the upper bound of speed up due to 12 cores when your program has 40% of parallelisable part?

Please go to [www.menti.com](http://www.menti.com) and use the code **3521 0273**.

What is the upper bound of speed up due to 12 cores when your program has 40% of parallelisable part?

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$\Rightarrow$  Here,  $k = 1$ ,  $n = 12$ ,  $\tilde{t}_p = 0.4$  and  $\tilde{t}_s = 1 - \tilde{t}_p = 0.6$ . So, the upper bound is  $L(k, n) \leq \frac{1}{0.6 + \frac{1}{12}0.4} \approx 1.58$ .

You come across a program that has 20% sequential part. Now, you are told that any of the following would require the same amount of time and person hours:

- ① Double performance for sequential part, and triple the performance of the parallelisable part.
- ② Only quadruple the performance of parallelisable part.
- ③ Only quadruple the serial part. Which one of the above option would you go for?

Please go to [www.menti.com](https://www.menti.com) and use the code **1427 6652**.

You come across a program that has 20% sequential part. Now, you are told that any of the following would require the same amount of time and person hours:

- ❶ Double performance for sequential part, and triple the performance of the parallelisable part.
- ❷ Only quadruple the performance of parallelisable part.
- ❸ Only quadruple the serial part. Which one of the above option would you go for?

Please go to [www.menti.com](http://www.menti.com) and use the code **1427 6652**.

⇒ Each of the above has the following performance gain respectively: 2.72, 2.17 and 1.18. So, you should go for the first option.

# Any questions?

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## Properties of a Program



A property is an attribute of a program that is true for every possible execution of that program, i.e. at all possible states.

Properties of interest for concurrent programs fall into two categories:

**Safety** property asserts that nothing bad (e.g. interference or deadlock) happens during execution. This must be true for all states.

**Liveness** property asserts that something good (e.g. a result, fairness and no restriction to progress) eventually happens. This is eventually true. We will generally concern ourselves with Liveness.

**Safety** property holds in all states – nothing bad happens.

**Liveness** property eventually holds – something good happens.

Examples.

- Safety:
  - Deadlock free
  - Mutual exclusion
- Liveness
  - A result.
  - Fairness.
  - Progress.

To check the safety property of a process  $P$ , we define a deterministic process property  $Q$ , and do the following:

- 1 Create a composite process  $R = (P \parallel Q)$  .
- 2 Check whether  $R$  has paths to an error state or not (much like testing).

# Car Park Example Revisited

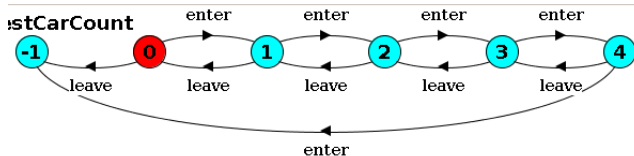
```
Entrance = (enter -> Entrance).  
Exit = (leave -> Exit).  
Controller(Capacity=4) = Spaces[Capacity],  
Spaces[spaceLeft:0..Capacity] =  
  (when spaceLeft>0 enter -> Spaces[spaceLeft-1]  
   |when spaceLeft<Capacity leave -> Spaces[spaceLeft+1]).  
||CarPark = (Entrance || Controller || Exit).  
// Property for checking the number of total cars  
property TotalCars = TotalCars[0],  
TotalCars[i:0..4] = (enter -> TotalCars[i+1] | leave ->  
TotalCars[i-1]).  
||TestCarCount = (CarPark || TotalCars).
```

In this case, there will be no errors, as we have used conditions to guard against these. What would happen if the guards were not there?

# Car Park Example Revisited

```

Entrance = (enter -> Entrance).
Exit = (leave -> Exit).
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```

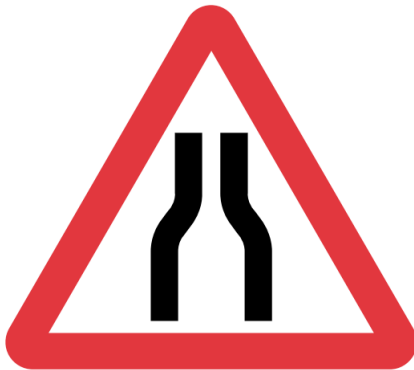


# Any questions?

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Do you know this symbol?



Source: ISBN 9780115528552 Please go to [www.menti.com](http://www.menti.com) and use the code 6571 3500.

# Single Lane Bridge Problem

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Usually we use it to show that we have a narrow lane ahead...



Source: [geograph.org.uk](http://geograph.org.uk)



# Single Lane Bridge Problem

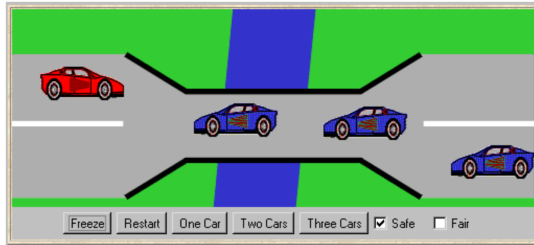
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These lanes can be tricky. We need to ensure safety.



Source: [birminghammail.co.uk](http://birminghammail.co.uk)

# Single Lane Bridge Problem



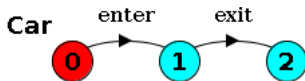
Source: Chapter 7, Magee & Kramer.

A bridge over a river is only wide enough to permit a single lane of traffic. Consequently, cars can only move concurrently if they are moving in the same direction. A safety violation occurs if two cars moving in the different directions enter the bridge at the same time.

- Actions: enter and exit.
- Processes:
  - Active: Car, Convoy, Cars.
  - Passive: Bridge.
- Safety properties: EntranceOrder, ExitOrder and SingleCarOnBridge.

# Single Lane Bridge Problem

Car = (enter -> exit -> STOP).

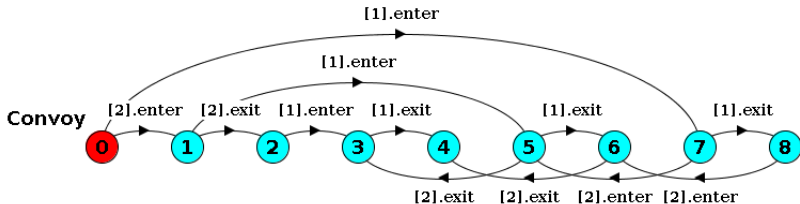


The Car model is simple: it just enters and then exits the bridge. Note that this is only the building block for Convoy and Cars.

# Single Lane Bridge Problem

A Convoy is a queue of cars.

```
const N = 2
range T = 0..N
range ID = 1..N // lower ID is for the first car on the queue.
Car = (enter -> exit -> STOP).
||Convoy = ([ID]:Car). // composing multiple cars with IDs.
```

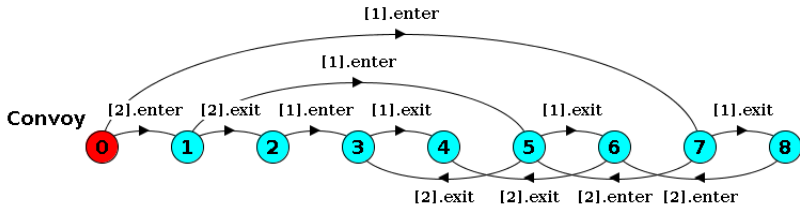


We now have two cars: [1] and [2], each with alphabets enter and exit.  
Any issues? Please go to [www.menti.com](http://www.menti.com) and use the code **78 73 05 3**

# Single Lane Bridge Problem

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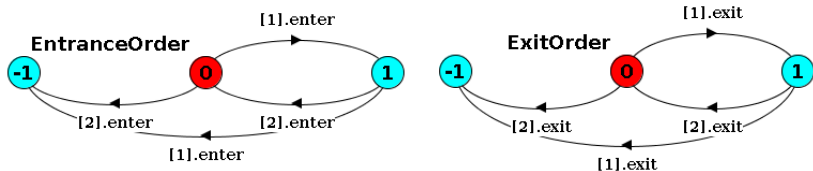


We now have two cars: [1] and [2], each with alphabets enter and exit. Clearly violates the order of entrance and exit.

# Single Lane Bridge Problem

We will define a couple of safety properties for *entrance* and *exit*.

```
property EntranceOrder = EntranceOrder[1],
EntranceOrder[id:ID] = ([id].enter ->
EntranceOrder[id%N+1]).
property ExitOrder = ExitOrder[1],
ExitOrder[id:ID] = ([id].exit -> ExitOrder[id%N+1]).
||CheckConvoy = (Convoy || EntranceOrder || ExitOrder).
```



The properties ensures that any invalid sequence of actions is leading to ERROR.

# Single Lane Bridge Problem

We will define a couple of safety properties for *entrance* and *exit*.

```
property EntranceOrder = EntranceOrder[1],  
EntranceOrder[id:ID] = ([id].enter ->  
EntranceOrder[id%N+1]).  
property ExitOrder = ExitOrder[1],  
ExitOrder[id:ID] = ([id].exit -> ExitOrder[id%N+1]).  
||CheckConvoy = (Convoy || EntranceOrder || ExitOrder).
```

**Composing...**

```
property EntranceOrder violation.  
property ExitOrder violation.  
potential DEADLOCK
```

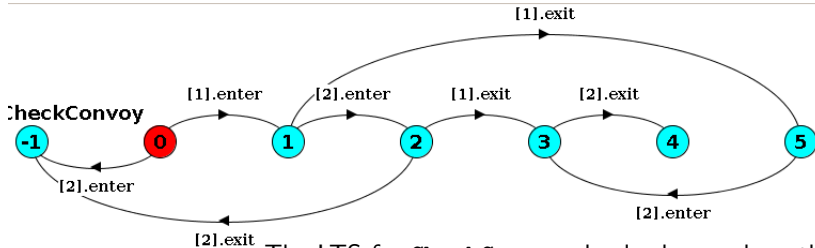
While composing CheckConvoy, we can clearly see an example trace to ERROR. Note that the deadlock here is simply the end of program, so nothing to worry about.



# Single Lane Bridge Problem

We will define a couple of safety properties for *entrance* and *exit*.

```
property EntranceOrder = EntranceOrder[1],
EntranceOrder[id:ID] = ([id].enter ->
EntranceOrder[id%N+1]).
property ExitOrder = ExitOrder[1],
ExitOrder[id:ID] = ([id].exit -> ExitOrder[id%N+1]).
||CheckConvoy = (Convoy || EntranceOrder || ExitOrder).
```

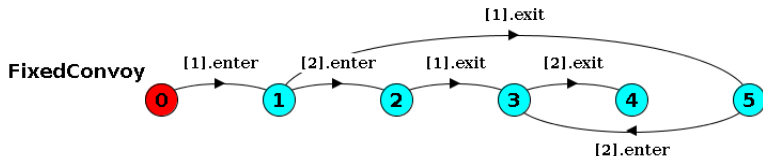


The LTS for `CheckConvoy` clearly shows where the problems are.

# Single Lane Bridge Problem

We will create a couple of processes that allow us to model *entrance* and *exit* and associated order of events.

```
Entrance = Entrance[1],  
Entrance[i:ID] = ([i].enter -> Entrance[i%N+1]).  
Exit = Exit[1],  
Exit[i:ID] = ([i].exit -> Exit[i%N+1]).  
||FixedConvoy = ([ID]:Car || Entrance || Exit).  
||CheckFixedConvoy = (FixedConvoy || EntranceOrder ||  
ExitOrder). // for checking the properties.  
||Cars = ({west, east}:FixedConvoy). // creating the Cars.
```



# Single Lane Bridge Problem

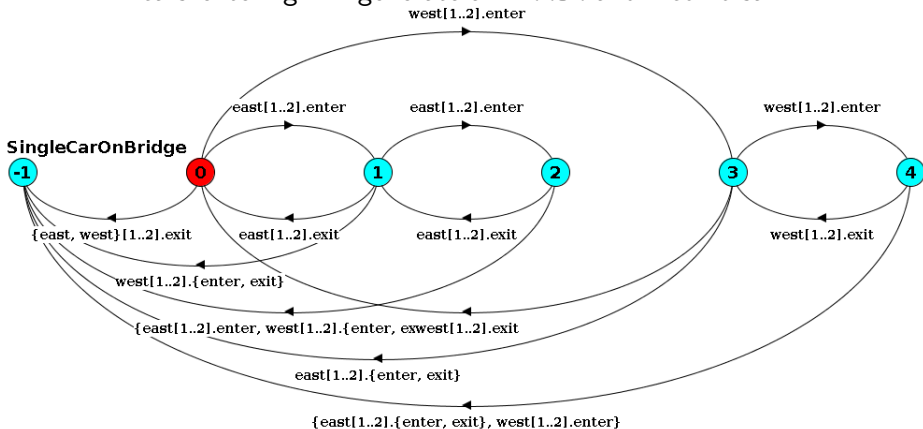
We will now create the property that check that there is only one car on the bridge.

```
property SingleCarOnBridge = (west[ID].enter -> CountWest[1]
                              | east[ID].enter -> CountEast[1]),
CountWest[i:ID] = (west[ID].enter -> CountWest[i+1]
                  | when i>1 west[ID].exit -> CountWest[i-1]
                  | when i==1 west[ID].exit -> SingleCarOnBridge),
CountEast[i:ID] = (east[ID].enter -> CountEast[i+1]
                  | when i>1 east[ID].exit -> CountEast[i-1]
                  | when i==1 east[ID].exit -> SingleCarOnBridge).
|| CheckCars = (Cars | SingleCarOnBridge).
```

Trace to property violation in SingleCarOnBridge:  
east.1.enter  
west.1.enter  
- - - - -

# Single Lane Bridge Problem

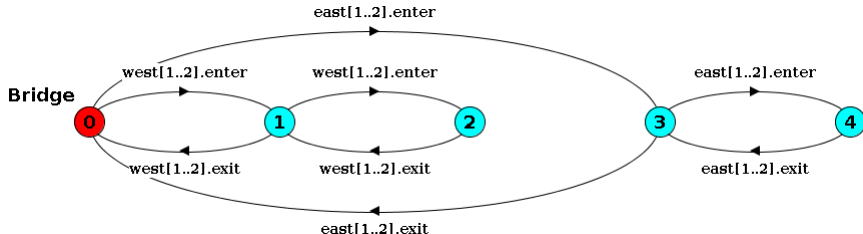
The LTS shows that once we have allowed *east cars* to go ahead, any *west cars* entering will generate an ERROR and *vice versa*.



# Single Lane Bridge Problem

It is time for us to model the Bridge.

```
Bridge = Bridge[0][0],  
Bridge[nWest:T][nEast:T] = (  
when (nWest == 0 && nEast<N) east[ID].enter -> Bridge[nWest][nEast + 1]  
| when (nEast>0) east[ID].exit -> Bridge[nWest][nEast - 1]  
| when (nEast==0 && nWest<N) west[ID].enter -> Bridge[nWest+1][nEast]  
| when (nWest>0) west[ID].exit -> Bridge[nWest-1][nEast]  
).  
  
||SingleLane = (Cars || Bridge).  
||CheckSingleLane = (SingleLane || SingleCarOnBridge).
```



# Any questions?

---



```
public class Bridge {  
    private int westCount = 0;  
    private int eastCount = 0;  
    public synchronized void westEnter()  
        throws InterruptedException{  
        while (eastCount > 0) wait();  
        westCount += 1; notifyAll();  
    }  
    public synchronized void westExit(){  
        westCount -= 1;  
    }  
    public synchronized void eastEnter()  
        throws InterruptedException{  
        while (westCount > 0) wait();  
        eastCount += 1; notifyAll();  
    }  
    public synchronized void eastExit(){  
        westCount += 1;  
    }  
}
```

A simple Bridge class without any queues for cars from west and east; just using simple counters.

```
public class BridgeQueue {  
    private Queue<Integer> westQueue;  
    private Queue<Integer> eastQueue;  
    private int westCount;  
    private int eastCount;  
    BridgeQueue(){  
        westQueue = new LinkedList<Integer>();  
        eastQueue = new LinkedList<Integer>();  
        westCount = 0;  
        eastCount = 0;  
    }  
}
```

An improved version, where we use queues instead to hold cars from west and east. Note that by using linked list first-in-first-out queue implementation, we are making sure that an element that enters the queue first, exits first.



```
public synchronized void westEnter()  
    throws InterruptedException{  
    while (eastCount > 0)  
        wait();  
    westCount += 1;  
    westQueue.add(westCount);  
    System.out.println("Added to the west queue: " + westCount);  
    notifyAll();  
}  
public synchronized void westExit(){  
    westCount -= 1;  
    westQueue.remove();  
    System.out.println("Removed from the west queue: " + westCount);  
}
```

It has methods for *westEnter* and *westExit*, adding and removing from the shared *westQueue* respectively.

```
public synchronized void eastEnter()  
    throws InterruptedException{  
    while (westCount > 0)  
        wait();  
    eastCount += 1;  
    eastQueue.add(eastCount);  
    System.out.println("Added to the east queue: " + eastCount);  
    notifyAll();  
}  
public synchronized void eastExit(){  
    westCount += 1;  
    eastQueue.remove();  
    System.out.println("Removed from the east queue: " + eastCount);  
}
```

Similar methods for *eastEnter* and *eastExit*, adding and removing from the shared *eastQueue* respectively.

```
public class WestCar implements Runnable {  
    private BridgeQueue bridge;  
    private String name;  
    WestCar(String name, BridgeQueue bridge){  
        this.bridge = bridge;  
        this.name = name;  
    }  
    ..  
}
```

Cars are coming from west and east. So, we are going to implement each west and east as Threads. Here, we have the WestCar implementation: it has the shared BridgeQueue instance, and also a name.

```
@Override
public void run() {
    Random random = new Random();
    int randomInt = 0;
    while (true){
        try {
            System.out.println(name + " is adding to west queue.");
            bridge.westEnter();
            randomInt = random.nextInt(1000); // upto 1 sec
            Thread.sleep(randomInt);
            bridge.westExit();
            System.out.println(name + " has removed from west queue.");
        } catch (InterruptedException ex) {
            System.out.println(name + " was interrupted. ");
            break;
        }
    }
}
```

When this WestCar is run, it randomly queues a car, and waits for a bit before removing it from the queue.

```
public class EastCar implements Runnable{  
    private BridgeQueue bridge;  
    private String name;  
    EastCar(String name, BridgeQueue bridge){  
        this.bridge = bridge;  
        this.name = name;  
    }  
}
```

EastCar is the same in principle, but works on the shared eastQueue.

```
@Override
public void run() {
    Random random = new Random();
    int randomInt = 0;
    while (true){
        try {
            System.out.println(name + " is adding to east queue.");
            bridge.eastEnter();
            randomInt = random.nextInt(1000); // upto 1 sec
            Thread.sleep(randomInt);
            bridge.eastExit();
            System.out.println(name + " has removed from east queue.");
        } catch (InterruptedException ex) {
            System.out.println(name + " was interrupted. ");
            break;
        }
    }
}
```

EastCar is the same in principle, but works on the shared eastQueue.

```
public static void main(String[] args)
    throws InterruptedException {
    BridgeQueue bridge = new BridgeQueue();
    WestCar westCarA = new WestCar("WA", bridge);
    WestCar westCarB = new WestCar("WB", bridge);
    EastCar eastCarA = new EastCar("EA", bridge);
    EastCar eastCarB = new EastCar("EB", bridge);
    Thread twa = new Thread(westCarA);
    Thread twb = new Thread(westCarB);
    Thread tea = new Thread(eastCarA);
    Thread teb = new Thread(eastCarB);
    twa.start(); twb.start(); tea.start(); teb.start();
    Thread.sleep(5000); // sleep for a while
    twa.interrupt(); twb.interrupt(); tea.interrupt(); teb.interrupt();
    twa.join(); twb.join(); tea.join(); teb.join();
    System.out.println("Program has ended.");
}
```

In the main program, we just create the BridgeQueue and create four different threads to work on it.

```
run:
WB is adding to west queue.
EA is adding to east queue.
WA is adding to west queue.
EB is adding to east queue.
Added to the west queue: 1
Added to the west queue: 2
Removed from the west queue: 1
WA has removed from west queue.
WA is adding to west queue.
Added to the west queue: 2
Removed from the west queue: 1
WA has removed from west queue.
WA is adding to west queue.
Added to the west queue: 2
Removed from the west queue: 1
WB has removed from west queue.
WB is adding to west queue.
Added to the west queue: 2
Removed from the west queue: 1
WB has removed from west queue.
WB is adding to west queue.
Added to the west queue: 2
Removed from the west queue: 1
WB has removed from west queue.
WB is adding to west queue.
Added to the west queue: 2
WB was interrupted.
EA was interrupted.
WA was interrupted.
EB was interrupted.
Program has ended.
```

This is the output of the program. Is there anything surprising in the output?



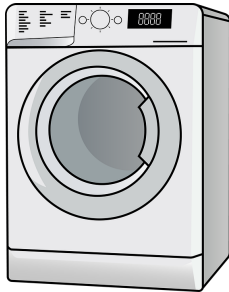
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WA has removed from west queue.
WA is adding to west queue.
Added to the west queue: 2
Removed from the west queue: 1
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WB is adding to west queue.
Added to the west queue: 2
Removed from the west queue: 1
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WB is adding to west queue.
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WB was interrupted.
EA was interrupted.
WA was interrupted.
EB was interrupted.
Program has ended.
```

This is the output of the program. Is there anything surprising in the output? EA and EB never gets to go through the bridge and add to the queue, as cars continue to come from the west. This is called *starvation*, i.e. some of the threads never get to do anything, and keep waiting on others.

# Any questions?

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You are given a scenario for a *washing machine*: in this machine, you can **turn on** and **open the door** to **load it**, then **close the door**, and **start** the process. The process includes: **washing** clothes, **rinse**ing them, and finally **drying** them before returning to the initial state. At this point, you should be able to **open** the door and Once you have started the washing process, at any state, you should be able to **pause** the machine, which takes you to the Paused state, and you would use **unpause** to resume operations.

There is a safety *cycle* property: wash must come before rinse, and both must happen before dry. Produce FSP code for the model and the property.

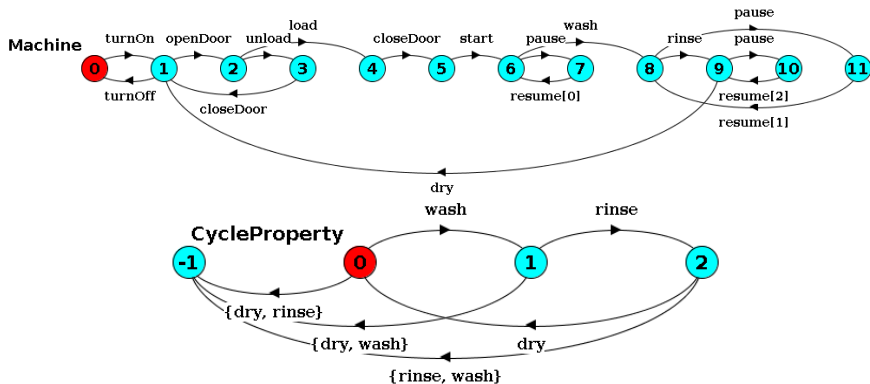
# Working through the problem

---

```
property CycleProperty = (wash -> rinse -> dry -> CycleProperty).

Machine = (turnOn -> Initial),
Initial = (openDoor -> (load -> closeDoor -> start -> WashProcess[0]
    | unload -> closeDoor -> Initial)
    | turnOff -> Machine),
WashProcess[i:0..2] = (
    when i==0 wash -> WashProcess[i+1]
  | when i==1 rinse -> WashProcess[i+1]
  | when i==2 dry -> Initial
  | pause -> Pause[i]),
Pause[i:0..2] = (resume[i] -> WashProcess[i]).

|| CycleTest = (Machine || CycleProperty).
```



- Safety property asserts cases that must be true for all states throughout the LTS.
- Explicitly describing these properties help us ensure that we are not able to reach a **bad** state, and LTSA will automatically tell us if we did reach a **bad** state during compilation.