CS_175: Modelling Computing Systems 2 **Exercise 2**

Due: Problem Session in Week 5 (Thursday 27 February 2020)

1. Consider the following game of TAKE-POWER-OF-2: Starting with a pile of n coins, two players alternately remove a number of coins which is a power of 2. That is, a player may take 1 coin, or 2 coins, or 4 coins, or 8 coins, or 16 coins, or 2^k coins for any k. The player who takes the last coin wins.

   (a) For each number n from 1 to 10, explain who has the winning strategy in TAKE-POWER-OF-2 starting from a pile of n coins. In the cases in which the first player has the winning strategy, state how many coins the first player should take.

   (b) Generalize the above by explaining who has the winning strategy in TAKE-POWER-OF-2 starting from a pile of n coins for an arbitrary n.

2. The game of DAYS-OF-THE-YEAR is played by two players who take turns naming a date of the year starting from January 1st. On any move a player may increase the month or the day but not both. Thus, for example, the first player can start the game by naming any day in January (apart from the 1st), or the 1st of any month of the year (apart from January). The player who names December 31st wins.

   Work out who has the winning strategy for this game. For a start, you can note that there is a winning move from any date in December (apart from the 31st), as well as from the 31st of any month (apart from December).