Verifier and Spoiler games

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Predicate logic

Definition

- A multi-sorted structure is given by some sets (the sorts), some relations on those sets and functions between those sets.
- A *term* is an expression built from variables, constants and functions in a way that type-checks.
- A relation applied to suitable terms is a formula, and we can built further formula in the forms $\phi \wedge \psi$, $\phi \vee \psi$, $\neg\phi$, $\exists x \in X\ \phi(x)$ and $\forall x \in X\ \phi(x)$. item<4-> A sentence is a formula where variables x only appear inside the scope of a quantifier $\forall x \in X$ or $\exists x \in X$.

Example

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$\forall n \in \mathbb{N} \exists m \in \mathbb{N}\ (n + k) < m$ is a formula. If k is a constant, it is a sentence.

The verifier/spoiler game

Definition

Given a predicate logic sentence L, we construct the verifier/spoiler as follows:

1. If L is of the form $R(c_1, \ldots, c_n)$, then it is a leaf which belongs to W iff $R(c_1, \ldots, c_n)$ is true.
2. If $L = \phi \wedge \psi$, then L has two children labelled ϕ and ψ, and L belongs to V_1 (spoiler acts).
3. If $L = \phi \vee \psi$, then L has two children labelled ϕ and ψ, and L belongs to V_0 (verifier acts).
4. If $L = \neg\phi$, then L has a single child ϕ, and the two players swap roles from ϕ onwards.

The verifier/spoiler game

Definition (continued)

Given a predicate logic sentence L, we construct the verifier/spoiler as follows:

1. If $L = \exists x \in X\ \phi(x)$, then L has a child $\phi(c)$ for each $c \in X$, and L belongs to V_0 (verifier acts).
2. If $L = \forall x \in X\ \phi(x)$, then L has a child $\phi(c)$ for each $c \in X$, and L belongs to V_1 (spoiler acts).

Theorem

A sentence is true iff Player 0 (verifier) has a winning strategy in the verifier/spoiler game. Otherwise, Player 1 (spoiler) has a winning strategy.