The pumping lemma

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Lemma

If L is regular, then $\exists k \in \mathbb{N}$ such that $\forall p \in L, |p| \geq k$ there exists ($\exists$) a splitting $p = uvw$ where $|uv| \leq k$ and $v \neq \varepsilon$ such that $\forall i \in \mathbb{N}$ it holds that $uv^i w \in L$.

Pumping lemma, contraposition

- If for all $k \in \mathbb{N}$ you can pick a word $p \in L$ with $|p| \geq k$
- such that however p is written as $p = uvw$ (subject to $|uv| \leq k$ and $v \neq \varepsilon$)
- you can find some $i \in \mathbb{N}$ such that $uv^i w \notin L$,
- then L is not regular.

An application

Question

Is $L_{\mathrm{pal}} = \{u \in \{a,b\}^* \mid u = u^{\mathrm{R}}\}$ *regular?*

- We get some $k \in \mathbb{N}$.
- We pick $a^k b a^k \in L_{\mathrm{pal}}$.
- If $uvw = a^k b a^k$, $|uv| \leq k$ and $v \neq \varepsilon$, then $v = a^l$ for some $1 \leq l \leq k$. So $uv^2w = a^{k+l} b a^k \notin L_{\mathrm{pal}}$.

Proposition

L_{pal} *is not regular.*

Your turn

Task: Prove that $\{a^n b^n \mid n \in \mathbb{N}\}$ is not regular.