A little bit of game theory

Arno Pauly

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The definition

Definition

- A finite two-player win/lose perfect information sequential game is given by a finite tree $T = (V, E)$, a partition of the vertices $V = V_0 \cup V_1$, and a subset of leaves W.
- A strategy σ_i for Player i selects for each vertex $v \in V_i$ an outgoing edge.
- A pair of strategies (σ_0, σ_1) induces a *play*, a path from the root to a leaf ℓ_{σ_0,σ_1}. If $\ell_{\sigma_0,\sigma_1} \in W$, Player 0 wins, otherwise Player 1 wins.
- A winning strategy is a strategy σ_i such that for all strategies σ_{1-i}, Player i wins the induced play.

Winning strategies

Theorem

In every finite two-player win/lose perfect information sequential game, one player has a winning strategy (we call this determined).

Example ((not really))

Chess.

Non-example: Rock,paper,scissors