Pumping Lemma for context-free languages

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Proving that a language is not context-free

- We mentioned that $\{a^n b^n c^n \mid n \in \mathbb{N}\}$ is not context-free,
- and that neither is $\{ww \mid w \in \Sigma^*\}$,
- but how do we prove this?
- By using the Pumping Lemma for Context-free Languages!

The lemma

Lemma

If L is context-free, then there is some $k \in \mathbb{N}$ such that for every $p \in L$ with $|p| \geq k$ there is a splitting $p = uvwxy$ where $|vx| \geq 1$ and $|vwx| \leq k$ such that for all $i \in \mathbb{N}$ it holds that $uv^iwx^iy \in L$.

Pumping Lemma for context-free languages, contraposition

Consider a language L.

1. If for every $k \in \mathbb{N}$ we can chose some $p \in L$ with $|p| \geq k$,
2. such that for every splitting $p = uvwxy$ such that $|vx| \geq 1$ and $|vwx| \leq k$,
3. we can find some $i \in \mathbb{N}$ with $uv^i wx^i y \notin L$,
4. then L is not context-free.