The derivation relation

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February 5, 2021

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Definition

Consider a grammar G specified by terminals Σ, non-terminals $\mathcal{N}$, start symbol S and set rules $\mathcal{R}$. The one-step derivation relation on $(\Sigma \cup \mathcal{N})^*$ is defined as:

$$\hookrightarrow := \{(uvw, uv'w) \mid u, v, w, v' \in (\Sigma \cup \mathcal{N})^* \ (v, v') \in \mathcal{R}\}$$

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Infix notation

We typically use infix notation for the (one-step) derivation relation, i.e. we write $u \hookrightarrow w$ for $(u, w) \in \hookrightarrow$ and $u \hookrightarrow\!\!\!\!\rightarrow w$ for $(u, w) \in \hookrightarrow\!\!\!\!\rightarrow$.

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The language defined by a grammar

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$$L(G) := \{w \in \Sigma^* \mid S \hookrightarrow\!\!\!\!\rightarrow w\}$$