CS_175: Modelling Computing Systems 2 **Exercise 1**

Due: Problem Session in Week 3 (Thursday 13 February 2020)

1. Consider the set $X \subseteq \mathbb{N}$ defined as follows.

   (a) $1 \in X$.

   (b) If $n \in X$ then $(n \times 3) \in X$ and $(n+4) \in X$.

   (c) Nothing is in X unless its membership can be established from the above.

   Give three elements of $\mathbb{N}$ which are elements of X, and three elements of $\mathbb{N}$ which are not elements of X, explaining for each one why it is or is not an element.

   Give a complete description of the set X.

2. Recall that binary trees are defined inductively by the following BNF equation:

   $$t \;::=\; \star \;|\; N(t_1, t_2)$$

   That is, a binary tree is either a leaf $\star$ or a node N with two subtrees t_1 and t_2.

   Give an inductive definition of the function *nodecount*(t) which computes the number of internal nodes in the binary tree t, where the definition of a binary tree is as given in Example 8.7 (page 211).

   Use this function to verify that $\mathit{nodecount}\Big(N\Big(N\big(\star, N(\star,\star)\big), N(\star,\star)\Big)\Big) \;=\; 4.$

   > ***Hint:*** *Following the inductive definition of binary trees, your inductive definition of nodecount*(t) *should have a clause for* $\star$ *and a clause for* $N(t_1, t_2)$*; that is, it should look as follows:*
   >
   > $$\mathit{nodecount}(\star) = \cdots$$
   >
   > $$\mathit{nodecount}\big(N(t_1, t_2)\big) = \cdots$$