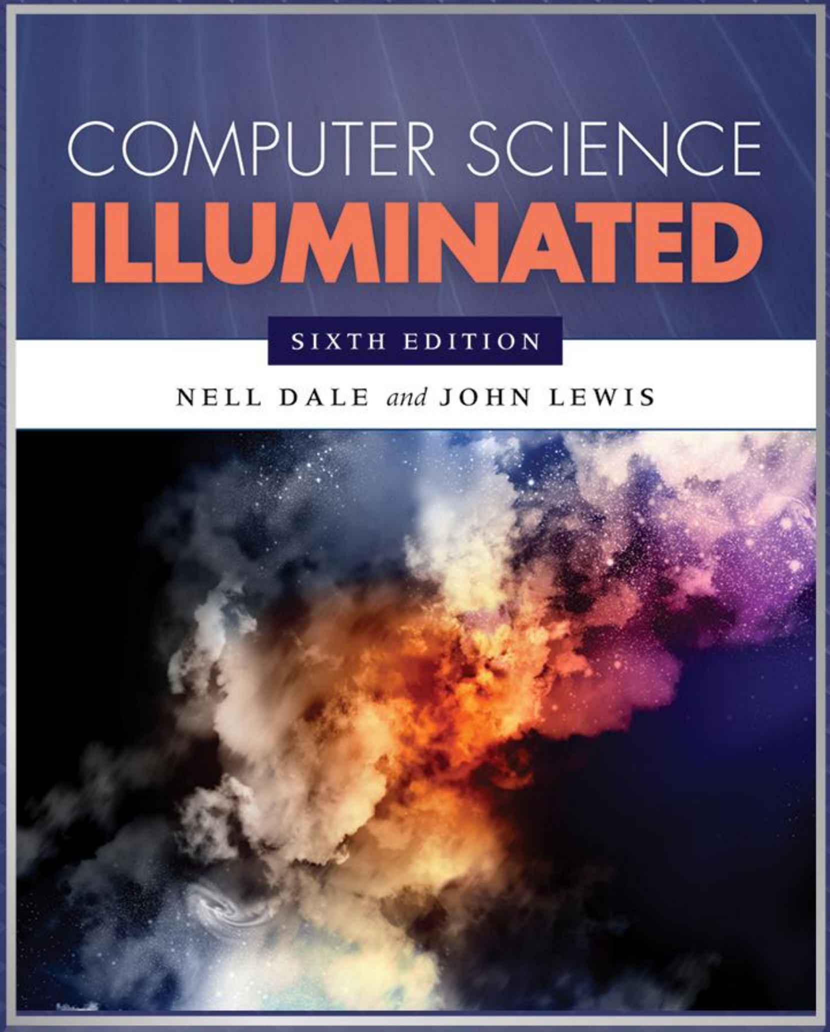
Binary Values and Number Systems



Chapter Goals

- Distinguish among categories of numbers
- Describe positional notation
- Convert numbers in other bases to base 10
- Convert base-10 numbers to numbers in other bases
- Describe the relationship between bases 2, 8, and 16
- Explain the importance to computing of bases that are powers of 2

Numbers

Natural Numbers

Zero and any number obtained by repeatedly adding one to it.

Examples: 100, 0, 45645, 32

Negative Numbers

A value less than 0, with a – sign

Examples: -24, -1, -45645, -32

Numbers

Integers

A natural number, a negative number

Examples: 249, 0, - 45645, - 32

Rational Numbers

An integer or the quotient of two integers

Examples: -249, -1, 0, 3/7, -2/5

Positional Notation

Positional Notation

How many ones are there in 642?

600 + 40 + 2 ?

Or is it

384 + 32 + 2 ?

Or maybe…

1536 + 64 + 2 ?

Positional Notation

Aha!

642 is 600 + 40 + 2 in BASE 10

The base (or Radix) of a number determines the number of different digit symbols (numerals) and the values of digit positions

Positional Notation

Continuing with our example…

642 in base 10 *positional notation* is:

Hundreds (10^2)	Tens (10^1)	Units (10^0)
6	4	2

$$6 \times 10^2 = 6 \times 100 = 600$$
$$+ 4 \times 10^1 = 4 \times 10 = 40$$
$$+ 2 \times 10^0 = 2 \times 1 = 2 \quad = 642 \text{ in base } 10$$

This number is in base 10

The power indicates the position of the number

Positional Notation

As a formula:

$$d_n * R^{n-1} + d_{n-1} * R^{n-2} + ... + d_2 * R^1 + d_1 * R^0$$

R is the radix (base) of the number

n is the number of digits in the number

d_i is the digit in the i^{th} position in the number

$$642 \text{ is } 6 * 10^2 + 4 * 10 + 2 * 1$$

Positional Notation

What if 642 has the base of 13?

$$
\begin{array}{rcrcr}
6 \times 13^2 & = & 6 \times 169 & = & 1014 \\
+\ 4 \times 13^1 & = & 4 \times 13 & = & 52 \\
+\ 2 \times 13^0 & = & 2 \times 1 & = & 2 \\
& & & = & 1068 \text{ in base } 10
\end{array}
$$

642 in base 13 is equivalent to 1068 in base 10

Binary

Decimal is base 10 and has 10 digit symbols:

0,1,2,3,4,5,6,7,8,9

Binary is base 2 and has 2 digit symbols:

0,1

For a number to exist in a given base, it can only contain the digits in that base, which range from 0 up to (but not including) the base.

What bases can these numbers be in? 122, 198, 178, G1A4

Bases Higher than 10

How are digits in bases higher than 10 represented?

With distinct symbols for 10 and above.

Base 16 has 16 digits:
0,1,2,3,4,5,6,7,8,9,A,B,C,D,E, and F

Converting Other Bases to Decimal

Converting Octal to Decimal

What is the decimal equivalent of the octal number 642?

$$
\begin{aligned}
6 \times 8^2 &= 6 \times 64 &= 384 \\
+\ 4 \times 8^1 &= 4 \times 8 &= 32 \\
+\ 2 \times 8^0 &= 2 \times 1 &= 2 \\
& &= 418 \text{ in base } 10
\end{aligned}
$$

Converting Hexadecimal to Decimal

What is the decimal equivalent of the hexadecimal number DEF?

$$
\begin{array}{rcrcr}
D \times 16^2 & = & 13 \times 256 & = & 3328 \\
+\ E \times 16^1 & = & 14 \times 16 & = & 224 \\
+\ F \times 16^0 & = & 15 \times 1 & = & 15 \\
 & & & = & 3567 \text{ in base 10}
\end{array}
$$

Remember, the digit symbols in base 16 are 0,1,2,3,4,5,6,7,8,9,A,B,C,D,E,F

Converting Binary to Decimal

What is the decimal equivalent of the binary number 1101110?

1×2^6	=	1×64	=	64
$+ 1 \times 2^5$	=	1×32	=	32
$+ 0 \times 2^4$	=	0×16	=	0
$+ 1 \times 2^3$	=	1×8	=	8
$+ 1 \times 2^2$	=	1×4	=	4
$+ 1 \times 2^1$	=	1×2	=	2
$+ 0 \times 2^0$	=	0×1	=	0

= 110 in base 10

Converting Binary to Octal

- Mark groups of *three* (from right)
- Convert each group

10101011	10	101	011
	2	5	3

10101011 is 253 in base 8

Converting Binary to Hexadecimal

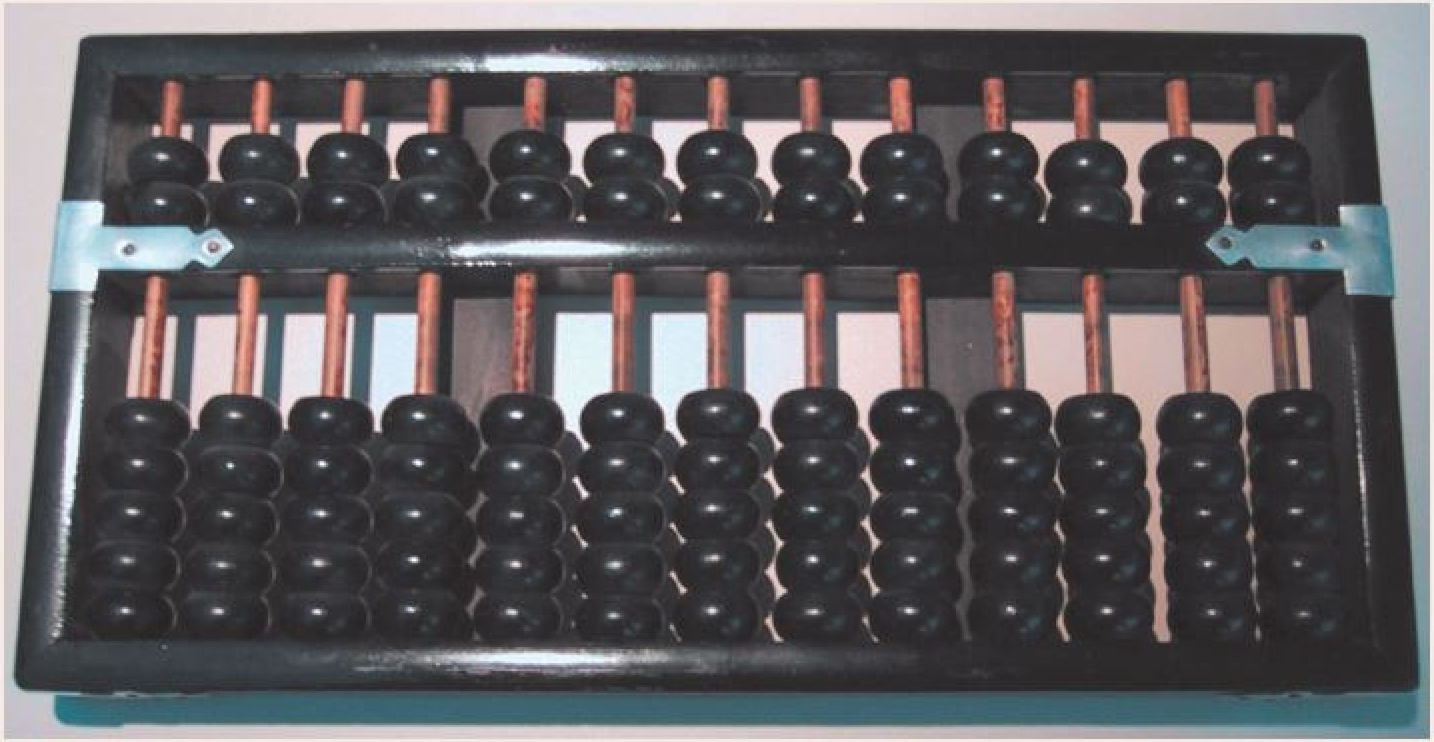
- **Mark groups of *four* (from right)**
- **Convert each group**

10101011 **1010** **1011**

A **B**

10101011 is AB in base 16

Abacus



Courtesy of Theresa DiDonato

Converting Decimal to Other Bases

Converting Bases to Other Bases

Converting number in base *x* to base *y*:

Repeated division of dividend with target base, storing the remainder and using the quotient as the dividend for the next iteration. Reading off the remainders in reverse order.

Converting Bases to Other Bases

Algorithm for converting number in base 10 to other bases

```
While (value is not zero)
      Divide value by the new base
      Store remainder
      Replace value with the quotient
Read remainders in reverse order
```

Converting Decimal to Binary

What is 294 (base 10) in base 2?

Try it!

Converting Decimal to Binary

Converting $294_{base(10)}$ gives:

294/2 = 147	Divide this way	Remainder: 0	Read this way
147/2 = 73		Remainder: 1	
73/2 = 36		Remainder: 1	
36/2 = 18		Remainder: 0	
18/2 = 9		Remainder: 0	
9/2 = 4		Remainder: 1	
4/2 = 2		Remainder: 0	
2/2 = 1		Remainder: 0	
1/2 = 0		Remainder: 1	

Reading off remainders in reverse:

$294_{base(10)} = 100100110_{base(2)}$

Converting Decimal to Hexadecimal

What is 294 (base 10) in base 16?

Try it!

Converting Decimal to Hexadecimal

Converting $294_{base(10)}$ gives:

294/16 = 18	Divide this way	**Remainder: 6**	Read this way
18/16 = 1		**Remainder: 2**	
1/16 = 0		**Remainder: 1**	

Reading off remainders in reverse:

$294_{base(10)} = 126_{base(16)}$

Converting Base *x* to Base *y*

What is 3AF (base 16) in base 4?

Try it!

Converting Base *x* to Base *y*

Converting $3AF_{base(16)}$ gives:

3AF/4 = 943/4 = EB	Divide this way	Remainder: 3	Read this way
EB/4 = 235/4 = 3A		Remainder: 3	
3A/4 = 58/4 = E		Remainder: 2	
E/4 = 14/4 = 3		Remainder: 2	
3/4 = 0		Remainder: 3	

Reading off remainders in reverse:

$3AF_{base(16)} = 32233_{base(4)}$

Arithmetic in Binary

Arithmetic in Binary

There are only 2 digit symbols in binary:
  0 and 1

Be careful when doing arithmetic in a given base, remember which base you are in!

  1 + 1 = 2
  1 + 1 = 10
  5 + 5 = 10
  5 + 5 = error, digit 5 does not exist

Arithmetic in Binary

Addition of two numbers is relatively intuitive

1 + 1 in base 2 is 0 with a carry 1

```
  1 0 1 0 1 1 1
+ 1 0 0 1 0 1 1
```

Arithmetic in Binary

Addition of two numbers is relatively intuitive

1 + 1 in base 2 is 0 with a carry 1

```
         1
 1 0 1 0 1 1 1
+1 0 0 1 0 1 1
--------------
             0
```

Carry Values

Arithmetic in Binary

Addition of two numbers is relatively intuitive

1 + 1 in base 2 is 0 with a carry 1

```
          1 1
  1 0 1 0 1 1 1
 +1 0 0 1 0 1 1
 --------------
            1 0
```

Carry Values

Arithmetic in Binary

Addition of two numbers is relatively intuitive

1 + 1 in base 2 is 0 with a carry 1

```
       1 1 1        <- Carry Values
  1 0 1 0 1 1 1
 +1 0 0 1 0 1 1
 --------------
          0 1 0
```

Arithmetic in Binary

Addition of two numbers is relatively intuitive

1 + 1 in base 2 is 0 with a carry 1

```
   1 1 1 1
 1 0 1 0 1 1 1
+1 0 0 1 0 1 1
--------------
       0 0 1 0
```

Carry Values

Arithmetic in Binary

Addition of two numbers is relatively intuitive

1 + 1 in base 2 is 0 with a carry 1

```
     1 1 1 1 1        <- Carry Values
   1 0 1 0 1 1 1
 + 1 0 0 1 0 1 1
 ---------------
         0 0 0 1 0
```

Arithmetic in Binary

Addition of two numbers is relatively intuitive

1 + 1 in base 2 is 0 with a carry 1

```
 - 1 1 1 1 1
  1 0 1 0 1 1 1
 +1 0 0 1 0 1 1
 --------------
      1 0 0 0 1 0
```

Carry Values

Arithmetic in Binary

Addition of two numbers is relatively intuitive

1 + 1 in base 2 is 0 with a carry 1

```
1  1 1 1 1 1
 1 0 1 0 1 1 1
+1 0 0 1 0 1 1
--------------
 0 1 0 0 0 1 0
```

Carry Values

Arithmetic in Binary

Addition of two numbers is relatively intuitive

1 + 1 in base 2 is 0 with a carry 1

```
  1   1 1 1 1 1
    1 0 1 0 1 1 1
  + 1 0 0 1 0 1 1
  ---------------
  1 0 1 0 0 0 1 0
```

Carry Values

Subtracting Binary Numbers

Remember borrowing in decimal subtraction?

When subtracting B from A, if A_i is less than B_i, then borrow 1 from A_{i+1} and add the value of the base (R from slide 9)

Subtracting Binary Numbers

Remember borrowing in decimal subtraction?
Apply that concept here:

```
   2 3 4
 - 1 7 6
 -------
```

Subtracting Binary Numbers

Remember borrowing in decimal subtraction?

Apply that concept here:

$$\begin{array}{r} 2\phantom{\,{}^{1}4} \\ 2\,\cancel{3}\,{}^{1}4 \\ -\;1\;7\;6 \\ \hline 8 \end{array}$$

Subtracting Binary Numbers

Remember borrowing in decimal subtraction?
Apply that concept here:

$$\begin{array}{r} 1\,{}^{1}2\ \ \ \\ \cancel{2}\ \cancel{3}\ {}^{1}4 \\ -\ 1\ 7\ 6 \\ \hline 5\ 8 \end{array}$$

Subtracting Binary Numbers

Remember borrowing in decimal subtraction?
Apply that concept here:

$$\begin{array}{r} 1\,{}^{1}2\;\; \\ \cancel{2}\,\cancel{3}\,{}^{1}4 \\ -\ 1\ 7\ 6 \\ \hline 0\ 5\ 8 \end{array}$$

Subtracting Binary Numbers

Same concept for subtraction in other base:

```
    1 0 1 0 1 1 1
-     1 1 1 0 1 1
  ---------------
```

Subtracting Binary Numbers

Same concept for subtraction in other base:

```
   1 0 1 0 1 1 1
-    1 1 1 0 1 1
---------------
               0
```

Subtracting Binary Numbers

Same concept for subtraction in other base:

```
   1 0 1 0 1 1 1
-    1 1 1 0 1 1
   -------------
             0 0
```

Subtracting Binary Numbers

Same concept for subtraction in other base:

```
  1 0 1 0 1 1 1
-   1 1 1 0 1 1
---------------
          1 0 0
```

Subtracting Binary Numbers

Same concept for subtraction in other base:

$$
\begin{array}{cccccccc}
 & & & 0 & & & & \\
 & 1 & 0 & \cancel{1} & {}^{1}0 & 1 & 1 & 1 \\
- & & 1 & 1 & 1 & 0 & 1 & 1 \\
\hline
 & & & & 1 & 1 & 0 & 0
\end{array}
$$

Subtracting Binary Numbers

Same concept for subtraction in other base:

$$
\begin{array}{cccccccc}
 & 0 & & 0 & & & & \\
 & \cancel{1} & {}^{1}0 & \cancel{1} & {}^{1}0 & 1 & 1 & 1 \\
- & & 1 & 1 & 1 & 0 & 1 & 1 \\
\hline
 & & & & 1 & 1 & 0 & 0
\end{array}
$$

Subtracting Binary Numbers

Same concept for subtraction in other base:

$$
\begin{array}{ccccccccc}
 & 0 & 1 & {}^{1}0 & & & & & \\
 & \cancel{1} & {}^{1}\cancel{0} & \cancel{1} & {}^{1}0 & 1 & 1 & 1 \\
- & & 1 & 1 & 1 & 0 & 1 & 1 \\
\hline
 & & & 1 & 1 & 1 & 0 & 0
\end{array}
$$

Subtracting Binary Numbers

Same concept for subtraction in other base:

$$
\begin{array}{ccccccc}
0 & 1 & {}^{1}0 & & & & \\
\cancel{1} & {}^{1}\cancel{0} & \cancel{1} & {}^{1}0 & 1 & 1 & 1 \\
- & 1 & 1 & 1 & 0 & 1 & 1 \\
\hline
 & 0 & 1 & 1 & 1 & 0 & 0
\end{array}
$$

Subtracting Binary Numbers

Same concept for subtraction in other base:

$$
\begin{array}{cccccccc}
 & 0 & 1 & {}^{1}0 & & & & \\
 & \cancel{1} & {}^{1}\cancel{0} & \cancel{1} & {}^{1}0 & 1 & 1 & 1 \\
- & & 1 & 1 & 1 & 0 & 1 & 1 \\
\hline
 & 0 & 0 & 1 & 1 & 1 & 0 & 0
\end{array}
$$

Counting in Power-of-2 Bases

BINARY	OCTAL	DECIMAL
0	0	0
1	1	1
10	2	2
11	3	3
100	4	4
101	5	5
110	6	6
111	7	7
1000	10	8
1001	11	9
1010	12	10

Binary Numbers and Computers

Computers have storage units called **binary digits** or **bits**

Low Voltage = 0	
High Voltage = 1	all bits have 0 or 1

… or the other way around, but we don't need to worry about that

Binary and Computers

Byte

8 bits

The number of bits in a word determines the word length of the computer, which is usually a multiple of 8

- **32-bit machines**
- **64-bit machines etc.**

Ethical Issues

The FISA Court

What does the United States Foreign Intelligence Surveillance Court do?

When did most people first hear of the FISA Court?

What checks and balances are there between the FISA Court and other government entities?

What is the stated intent of the FISA Court?

Who am I?

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Can you tell the person sitting next to you three things about me?

Do you know?

?

What concept makes positional notation possible?

What three sets can pre-school children identify?

What words represent the third set?

How does an abacus work?

How does bi-quinary work?