From non-deterministic to determinsitic

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Non-deterministic vs deterministic finite automata

Definition

A non-deterministic finite automaton over an alphabet Σ is given by a set V of states, a transition relation $\delta \subseteq V \times \Sigma \times V$, a start state $s \in V$ and a set of accepting (or final) states $F \subseteq V$.

Definition

A deterministic finite automaton over an alphabet Σ is given by a set V of states, a transition function $\delta : V \times \Sigma \to V$, a start state $s \in V$ and a set of accepting (or final) states $F \subseteq V$.

The run of an automaton

Definition

A deterministic finite automaton over an alphabet Σ is given by a set V of states, a transition function $\delta : V \times \Sigma \to V$, a start state $s \in V$ and a set of accepting (or final) states $F \subseteq V$.

Definition

The *run* of a deterministic automaton $\mathcal{A}$ on a word $w \in \Sigma^*$ is $q_0 q_1 \ldots q_{|w|-1} \in V^*$ where $q_0 = s$ and $q_{i+1} = \delta(q_i, w_i)$. If $q_{|w|-1} \in F$, it is *accepting*, otherwise it is rejecting. An automaton accepts a word $w \in \Sigma^*$ iff its run on w is accepting. We let $L(\mathcal{A})$ be the language of all words accepted by $\mathcal{A}$.

A technicality

A state in a deterministic finite automaton has outgoing edges with all labels – but we usually don't draw dead ends.

Equivalence

Theorem

Deterministic and non-deterministic finite automata recognize the same languages.

The powerset construction

Definition

By $\mathcal{P}(\mathbf{X})$ we denote the powerset of the set $\mathbf{X}$,
i.e. $\mathcal{P}(\mathbf{X}) = \{S \mid S \subseteq \mathbf{X}\}$.

Proof.

Given a non-deterministic finite automaton with states V and transition relation δ, let the powerset construction automaton be the deterministic finite automaton with:

- states $\mathcal{P}(\mathbf{X})$
- transition function Δ where $\Delta(S, a) = \{q \in V \mid \exists r \in S \; (r, a, q) \in \delta\}$
- initial state $\{s\}$ (where s is the initial state of the original automaton)
- final states $\{S \mid S \cap F \neq \emptyset\}$ (where F is the set of final states of the original automaton)

$\square$

A consequence

Definition
Given a language L, let $L^C = \{w \in \Sigma^* \mid w \notin L\}$.

Corollary
If L is regular, then so is L^C.

A heads-up

- Determinizing a non-determinsitic finite automaton is a very natural exam question.
- Practise it!
- Check your answers by trying out a few words to see whether both automata give the same answer.