1 General remarks

All answers can be put into a simple text file. Questions are enumerated **Q1**, **Q2**, and so on.

2 First recurrence

Consider the function $T(n)$ defined for $n \geq 1$ by the recurrence

$$\begin{aligned} T(1) &= 1 \\ T(n) &= T(\lceil \frac{n}{2} \rceil) + 2 \cdot T(\lfloor \frac{n}{2} \rfloor) + 2 + n. \end{aligned}$$

Q1 Compute by hand the values $T(1), T(2), \ldots, T(6)$ (showing the computations).

Q2 What is the asymptotic growth rate of $T(n)$?

Q3 What happens to $T(n)$ if we go from n to $2n$? (Recall the final part of the lectures/script for Week 04.)

Q4 And what happens if we go from n to $4n$?

Q5 Consider the code in `Recurrence1.java`, implementing the above $T(n)$. Make sure you understand the code. The value for the "default n" is $4.1 \cdot 10^{11}$ — what happens if that value is increased to $4.2 \cdot 10^{11}$?

Q6 Compile and run this program and check the predictions from Q3 and Q4. Especially consider small, medium and large n, and check whether with growing n the predictions become more precise (as to be expected for asymptotic analysis).

3 Second recurrence

Consider the function $T(n)$ defined for $n \geq 1$ by the recurrence

$$\begin{aligned} T(1) &= 1 \\ T(n) &= T(\lceil \frac{n}{2} \rceil) + T(\lfloor \frac{n}{2} \rfloor) + n. \end{aligned}$$

Q7 Compute by hand the values $T(1), T(2), \ldots, T(6)$ (showing the computations).

Q8 What is the asymptotic growth rate of $T(n)$?

Q9 Consider the code in `Recurrence2.java`, implementing the above $T(n)$. Make sure you understand the code. Show now experimentally, that $T(n)$ is NOT $O(n)$, that is, show experimentally that $T(n)$ grows MORE than linearly. If possible, try to refine that analysis.