Non-deterministic finite automata: Definition

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Right-linear grammars

Definition

A grammar is *right-linear*, if all rules are of the form $T \to \varepsilon$ or $T \to aR$ for $T, R \in \mathcal{N}$ and $a \in \Sigma$.

We shall also allow the form $T \to a$ as abbreviation for $T \to aQ$, $Q \to \varepsilon$ for a fresh non-terminal Q.

Definition

A grammar is *left-linear*, if all rules are of the form $T \to \varepsilon$ or $T \to Ra$ for $T, R \in \mathcal{N}$ and $a \in \Sigma$.

Theorem

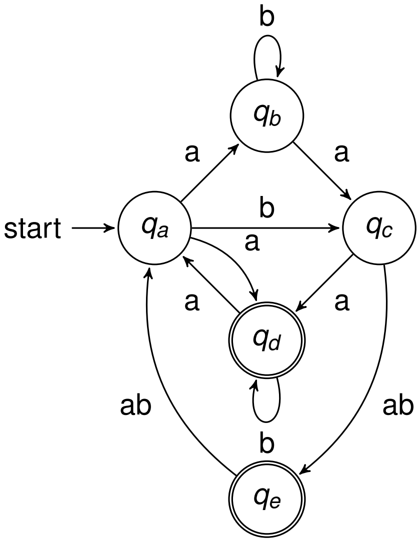
Right-linear and left-linear grammars describe the same languages; the regular languages.

Defining non-deterministic finite automata

Definition

A non-deterministic finite automaton over an alphabet Σ is given by a set V of states, a transition relation $\delta \subseteq V \times \Sigma \times V$, a start state $s \in V$ and a set of accepting (or final) states $F \subseteq V$.

A finite state automaton[1]



[1] adapted from an example by Till Tantau

A run of an automaton

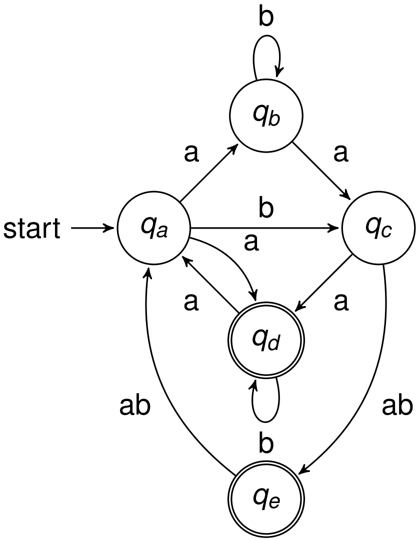
Definition

A non-deterministic finite automaton over an alphabet Σ is given by a set V of states, a transition relation $\delta \subseteq V \times \Sigma \times V$, a start state $s \in V$ and a set of accepting (or final) states $F \subseteq V$.

Definition

A *run* of an automaton $\mathcal{A}$ on a word $w \in \Sigma^*$ is a word $v_0 v_1 \ldots v_{|w|} \in V^*$ such that $v_0 = s$ and $\forall i < |w| \; (v_i, w_i, v_{i+1}) \in \delta$. If $v_{|w|} \in F$, it is *accepting*, otherwise it is rejecting. An automaton accepts a word $w \in \Sigma^*$ iff there exists an accepting run on w. We let $L(\mathcal{A})$ be the language of all words accepted by $\mathcal{A}$.

A finite state automaton[2]



abba $\Rightarrow$ **no**

baaba $\Rightarrow$ **yes**

[2]adapted from an example by Till Tantau

Reversal of a language

Definition

Given a word $w \in \Sigma^*$, let its reversal $w^{\mathrm{R}} \in \Sigma^*$ be defined by $|w^{\mathrm{R}}| = w$ and $w^{\mathrm{R}}(i) = w(|w| - i - 1)$.

(This literally says that w^{R} is w read backwards.)

Definition

Given a language $L \subseteq \Sigma^*$, let $L^{\mathrm{R}} := \{w^{\mathrm{R}} \mid w \in L\}$.

Theorem

If L is regular, then so is L^{R}.

Outlook

- Deterministic automata would be even nicer.
- Regular languages are closed under union, intersection, interleaving.
- Regular expressions!