Apprenticeship Thesis - First Deliverable

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Contents

1 Introduction

1.1 Candriam presentation

Dexia Asset Management became Candriam after being acquired by New York Life Investments, a subsidiary of New York Life Insurance. This acquisition followed the 2011 dismantling of the Dexia group, triggered by the eurozone sovereign debt crisis.

Candriam retains its three management centers in Paris, Brussels, and Luxembourg, along with its original teams. In 2013, assets under management remained stable at €73 billion (on February 2025, AUM doubled from that time to reach €145 billion), with a target annual growth of 8% to 10% over the next five years.

The acquisition allows New York Life Insurance to increase its total assets under management to $500 billion (currently standing at $670 billion). The deal, announced in September 2013 for €380 million, was finalized on February 3 after an initial sale attempt to GSC Capital failed.

1.2 Mission and objectives

During my apprenticeship at Candriam, the main topic of study is: How to detect abnormal trends? The goal is to create investment strategies to monetize those trends. To address this issue, we decided, with my supervisors, to divide this issue into three smaller questions, all with a slightly different approach. Indeed, trying to identify a trend can be done through different scopes: observed trend (meaning that we wait for a trend to materialize) vs anticipated trend (meaning that we try to identify a trend before its realization, thanks to structural breaks or change-points) and with prices data vs non-price (macroeconomic, factor, alternative) data.

The first approach relies on observed trends and historical prices. The goal is to test and to find statistical procedures (tests, methods, models) allowing the identification of abnormal trends.

The second approach is based on anticipated trends and macroeconomic, factor and thematic data. The goal is to try to find relationships between prices and variables of these categories that may have caused structural breaks or change-points leading to trends.

The third approach is the same as the second but with alternative data (Volume Search on Google Trends).

1.3 Context

This work is done in Candriam Paris under the supervision of Olivier Clapt (Head of Multi-Asset Quantitative Research - MAQR) and Hugo Bergeret (Multi-Asset Quantitative Researcher). The MAQR team belongs to the broader Multi-Asset team. Around 200 persons are in Paris and other quant teams include the Fixed Income (2 persons), the CTA (3 persons), the Equity Market Neutral & Index Arbitrage (4 persons) as well as a last team on Equities in Brussels. Although some quant teams directly manage funds, the Fixed Income Quant and MAQR are in support of several funds and bring new tools, indicators and backtests to the discretionary portfolio managers (PM).

1.4 Tools

The main tool used to process data is Python. More specifically, Python is used through the VS Code IDE and I recently moved on PyCharm because of its better debugging capacity (according to me). I sometimes use Excel for small data management when it is faster than doing it on Python and the data come from Bloomberg. Along with the IDE, I use Git for the versioning. Standard tools (PowerPoint, Word) are used for communication and presentations, occasionally, for more formal communication (like this apprenticeship thesis), I use Overleaf.

1.5 Integration

Integration into the team was great. We have weekly meetings planned on Wednesday, which I find very useful because it allows i) to deliver results and keep up the pace and ii) to have regular feedback onto the work and its direction. There is no real gap from what I expected before beginning the internship. Maybe something new compared to my previous internship in equity quantitative research at Scientific Beta is that I have to do the literature review, which can be not so easy (especially on very large topic as the mission I am working on but nonetheless it is required and very useful) whereas at Scientific Beta the paper was already choose and I had to replicate it. Another difference is that, even if we base ourselves on academic papers for the methods and some ideas, the work is quite different from the one at Scientific Beta: we did not try to replicate and improve a paper but we use the tools from it to explore data and possible investment strategies. I think this main difference came from that fact that Scientific Beta is an index provider academically-led and Candriam is an asset management business, with shorter deadlines and directly manages money.

1.6 Assessment of the work done

All the work I have done is described in details in the following section of the report. The key competencies mobilized to carry out this work are rigorousness, summarizing capacity, research ability and of course knowledge of the technologies. One main difficulty I faced, was managing the different versions of the code (It's why I move onto Git). It is difficult to measure the impact of my work for now, but the work on the Mann-Kendall test should be put into production in the near future.

1.7 Self Assessment

On the personal skills side, my passion, curiosity, desire to understand and rigorousness are part of me. Passion because for me quantitative research is not just a job, it's broader and deeper than that. Even outside the working hours I am reading and learning new insights from paper that I find interesting to read, which is also a testimony of my curiosity. Thanks to this apprenticeship, I highly developed my desire to learn (which is required to carry out the work) and rigorousness: when coding, making an error is very very easy, which forces me to be rigorous and to check every action I made. It happens several times that when I have to do an algorithm that is not so easy for me, I go into Excel and do a toy example before doing it at scale with Python to check the results.

On the technical side, I learned Python syntax, algorithmic, project architecture, versioning and how to set up virtual environments. I still need to focus on learning the syntax to be quicker at coding and also on the algorithmic.

Lastly, on the managerial side, I strengthen my communication, presentation, summarizing and vulgarizing abilities. Indeed, when I have to show the results to non-quant people, it is important to be clear, synthetic and at the same time not too technical but giving the right amount of details and important hypotheses to interpret the results.

2 Identifying Abnormal Trends from prices

As mentioned in the introduction, this section relies on statistical techniques to detect trends. Before digging into specific techniques, I will try to provide a non-exhaustive list of filters, regime-switching models and statistical tests that I think could be useful for our objective.

For the filters, we can cite the Hodrick-Prescott (L2) filter, the Baxter-King filter (L2), the Kalman filter (L2) and the Hamilton filter (regression). You can refer to "Momentum Strategies with L1 filter" (see references) from Tung-Lam Dao (CFM) to see how it can be applied to create momentum strategies.

For the regime-switching models, we can think of Brutal Threshold Auto-regressive Models (TAR), Smooth TAR and Markov-switching Auto-regressive (MS-AR) models.

For the statistical tests, we can divide them into two categories: trend tests (TT) and change-point detection (CPD) tests. We can further subdivide them into non-parametric (no assumption about the distribution of the data) and parametric tests. The below table summarizes these categories and the corresponding tests.

Category	Trend Tests (TT)	CPD Tests
Non-Parametric	Mann-Kendall Theil-Sen Estimator	Mann-Whitney U Kolmogorov-Smirnov Rank-based CUSUM Pettitt's Mood's Median Bayesian Online CPD
Parametric	t-test for slope Cox-Stuart	Chow Bai-Perron Likelihood Ratio CUSUM

Table 1: Parametric and Non-Parametric Trend and CPD Tests

We decided to try first the Mann-Kendall test as it is a non-parametric test which is less restrictive than the parametric ones and because it is directly a test designed to detect trends.

2.1 Trend Tests and subsequent strategies

2.1.1 Hamed-Rao Mann-Kendall test

We will first dig into the Hamed-Rao Mann-Kendall (HR-MK) test that is a modified version of the Mann-Kendall test that accounts for autocorrelation. I will briefly present the original Mann-Kendall test but you can find all the details about the HR-MK test in "A modified Mann-Kendall trend test for autocorrelated data" (1997) from Khaled H. Hamed and A. Ramachandra Rao (see references).

The rank correlation test (Kendall, 1955) for two sets of observations $X = x_1, x_2, \ldots, x_n$ and $Y = y_1, y_2, \ldots, y_n$, is formulated as follows. The statistic S is calculated as in Eq. (1):

$$S = \sum_{i<j} a_{ij} b_{ij} \tag{1}$$

where

$$a_{ij} = \operatorname{sgn}(x_j - x_i) = \begin{cases} 1 & x_j < x_i \\ 0 & x_j = x_i \\ -1 & x_j > x_i \end{cases} \tag{2}$$

and b_{ij} is similarly defined for the observations in Y. Under the null hypothesis that X and Y are independent and randomly ordered, the statistic S tends to normality for large n, with mean and variance given by:

$$E(S) = 0 \tag{3}$$

$$\operatorname{var}(S) = n(n-1)\frac{(2n+5)}{18} \tag{4}$$

If the values in Y are replaced with the time order of the time series X, i.e. $1, 2, \ldots, n$, the test can be used as a trend test (Mann, 1945).

In this case, the statistic S reduces to that given in Eq. (5):

$$S = \sum_{i<j} a_{ij} = \sum_{i<j} \operatorname{sgn}(x_j - x_i) \tag{5}$$

with the same mean and variance as in eqns (3) and (4). Kendall (1955) gives a proof of the asymptotic normality of the statistic S. The significance of trends is tested by comparing the standardized test statistic

$$Z = \frac{S}{\sqrt{\mathrm{var}(S)}} \tag{6}$$

with the standard normal variate at the desired significance level.

The variance of S (in the original Mann-Kendall test - without autocorrelation correction) is computed by making some hypotheses (independence) on its terms notably by setting an expectation equals to 0. The correction brought by Hamed and Rao is that they released the hypothesis of independence thus correcting the expectation that was previously set to 0, which is no longer necessarily zero. Thus the variance on S is modified which also modify the assessment of statistical significance. For the derivation of the variance of S in presence of autocorrelation, refers to the paper of Hamed and Rao (see references).

In its original form, the only assumption that the MK requires is the independence of the data. As prices data exhibit strong autocorrelation it's why we're using the HR-MK. Below, we will compare the effectiveness of the HR-MK in detecting trends depending on the inputs we give them. We will pass 1) gross prices, 2) log prices, 3) detrended (with x-year moving average) log prices and 4) returns (at daily, weekly and monthly frequency) to the HR-MK test. For now and if not mentioned otherwise, we will use a rolling window of 80 data points for the test.

2.1.2 Hamed-Rao Mann-Kendall test and mom_3m and mom_6m: empirical results

To assess the HR-MK, we need some benchmarks. We choose the momentum 3 and 6 months, hereafter denoted mom_3m and mom_6m (including returns from the last month).

For the assets, we choose the SPX, KNY, TPX, CAC, DAX and UKX index as they display different price profiles: from very clear and upside trend for the SPX to less clear trends for the NKY and TPX. Data range for the prices is from 10-07-1987 to 26-11-2024 which is sufficiently long and encompass different market and economic regimes.

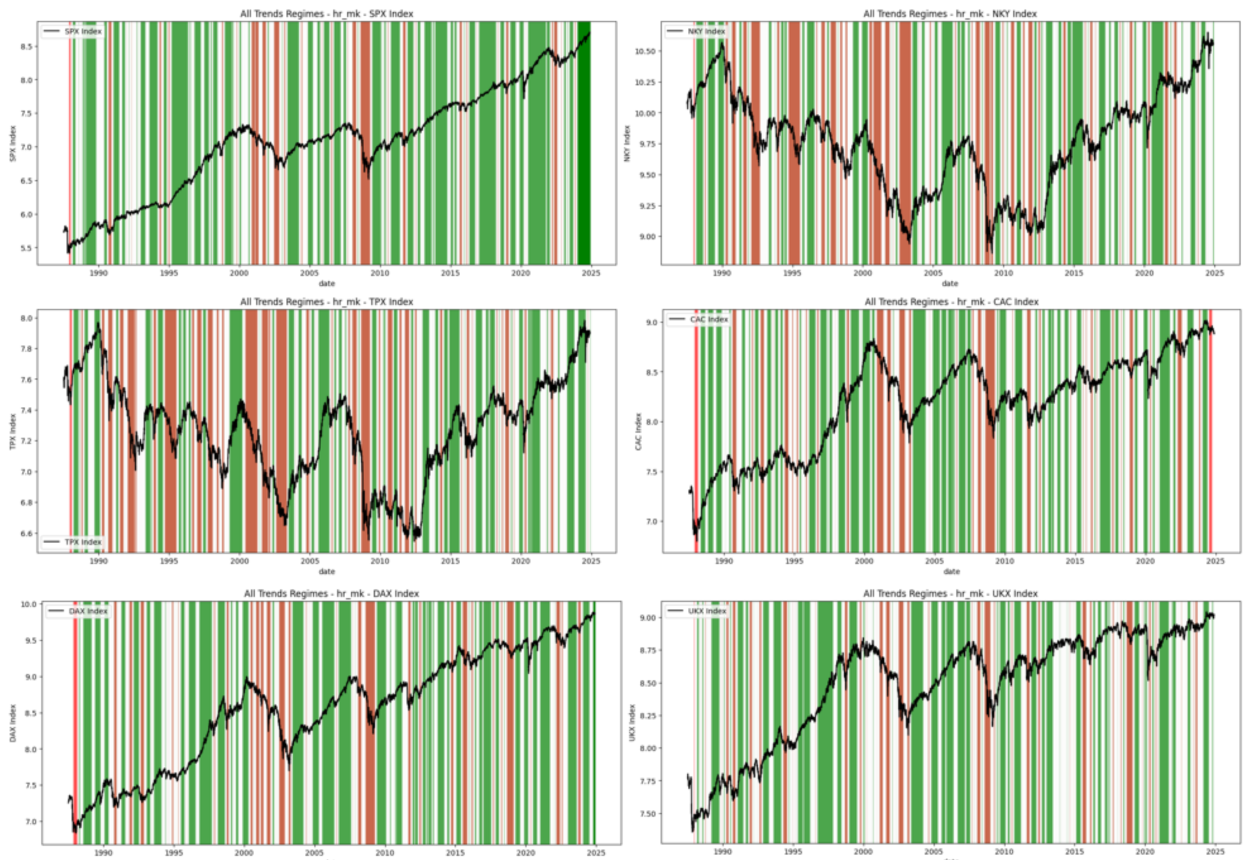
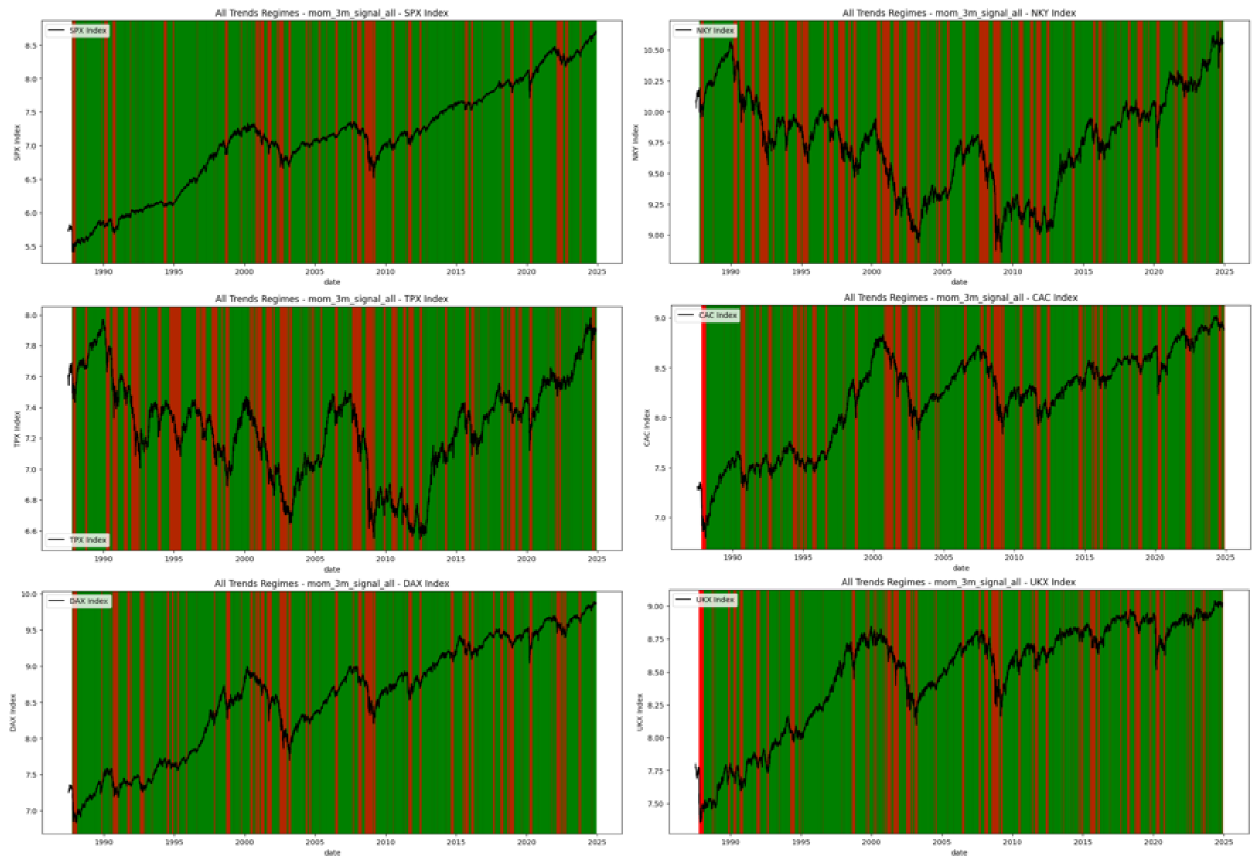
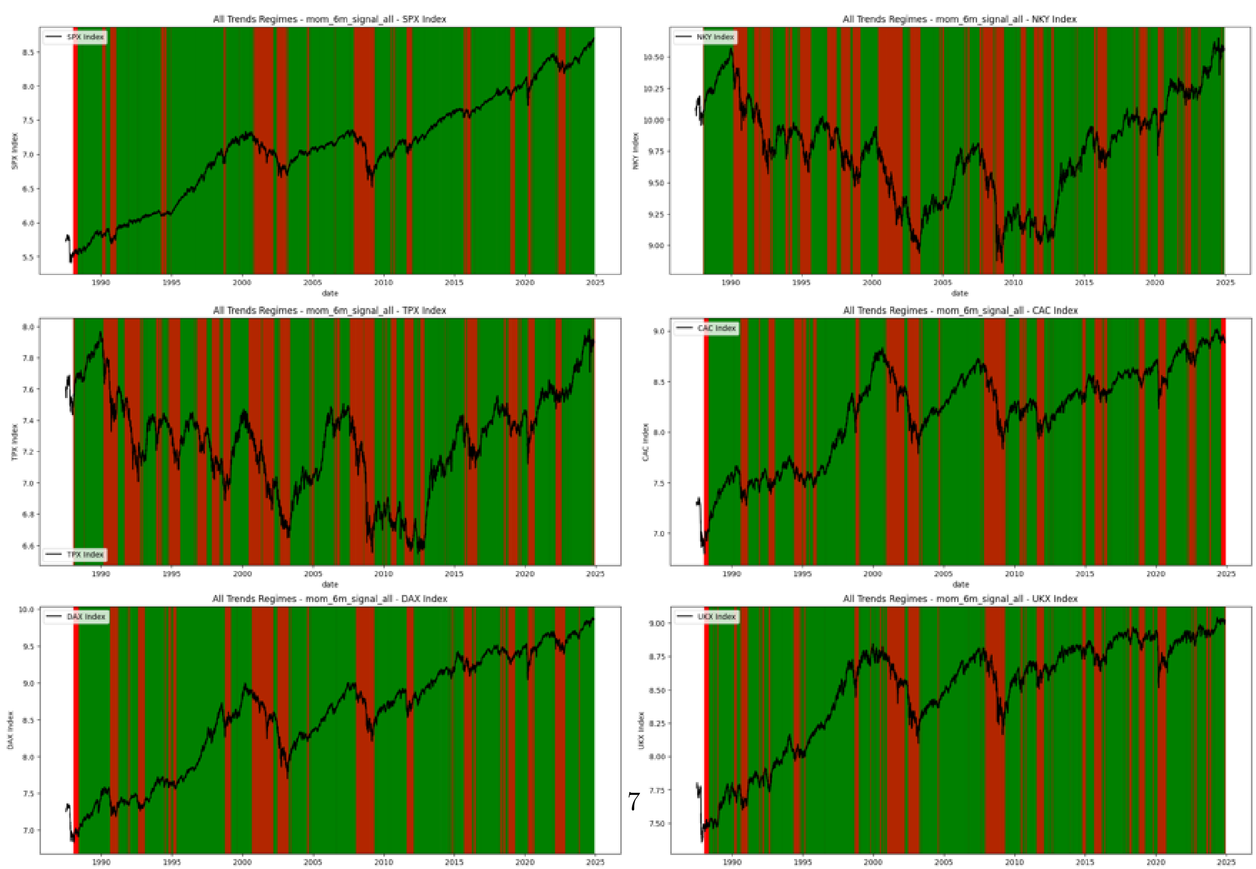


Figure 1: Trend regimes identified by HR-MK on log prices for various assets. Green represents an upside trend, red represents a downside trend and white represent no trend.



(a) Trend regimes identified by mom_3m (on returns) for various assets.



(b) Trend regimes identified by mom_6m (on returns) for various assets.

Visually, we might say that the HR-MK (at 5% significance level) does a good jobs at identifying trends. mom_3m and mom_6m (computed from price returns) are always invested as the signal is constructed if mom_xm > 0 contrary to the HR-MK where there exist periods of no trend in white. Obviously, we also see that mom_6m exhibit longer signals than mom_3m.

These charts display "all" trends but we are interested in "abnormal" trends. Hence, we must define the abnormality. We define a positive (negative) abnormal trend if the test statistics (HR-MK) or z-score (mom_3m and mom_6m) is in the 9th (1st) decile. We use a rolling window of 10 years (10*252) to avoid look-ahead biais. We call these signals "abnormal" or "abn". We present these signals/regimes below.

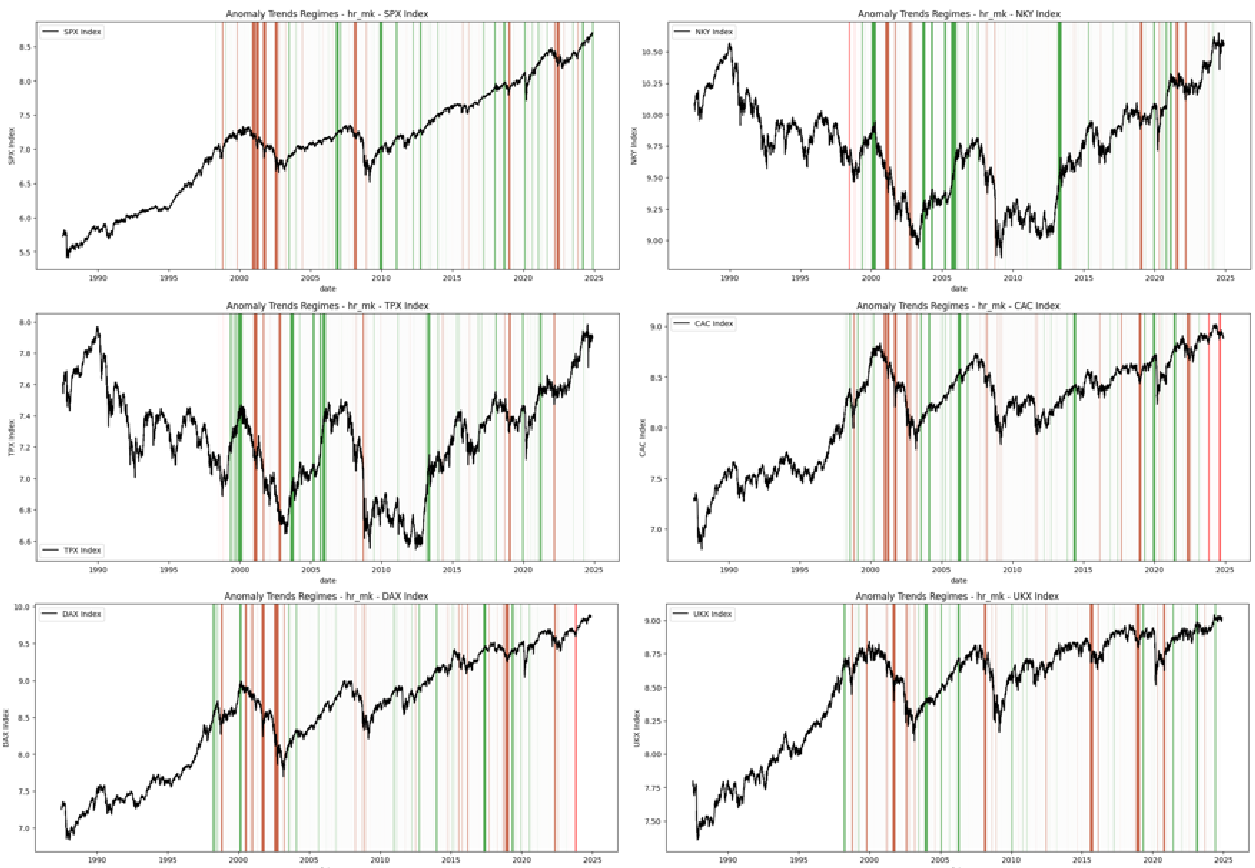
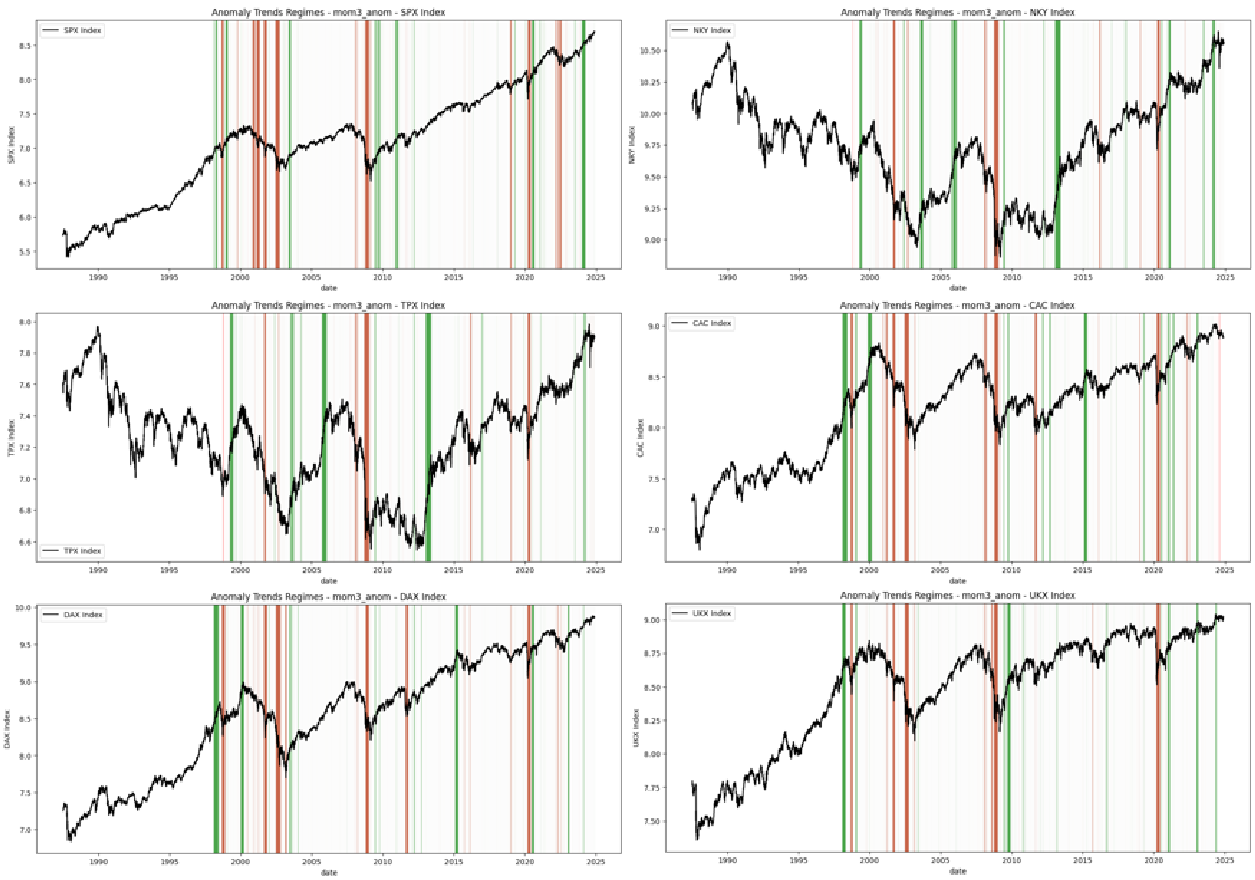
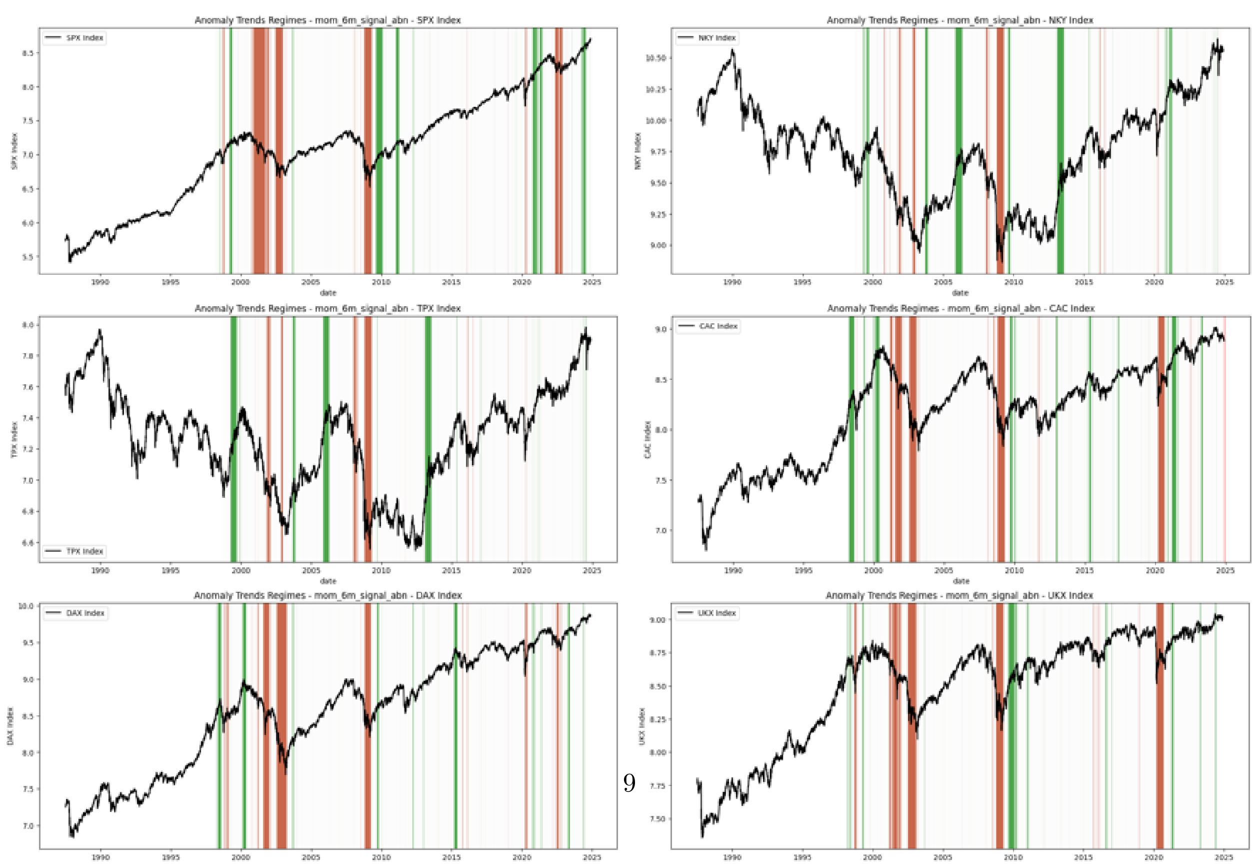


Figure 3: Trend regimes identified by abn HR-MK (on log prices) for various assets.

We can note that the signals occurred at a lower frequency, that is normal (by definition of the deciles) and what we desired to detect abnormal trends. We may also note that signal's length are shorter and may "turn off" once the abnormality trends began. It sorts of characterize the "starting change-point" of abnormal trends.



(a) Trend regimes identified by abn mom_3m (on price returns) for various assets.



(b) Trend regimes identified by abn mom_6m (on price returns) for various assets.

As we might want to profit from all the trend that started as abnormal (even if it becomes normal), we will finally present the third and last type of signal called "continuous abnormal" signals or "cont abn". We also conducted the analysis of these 3 signals ("all", "abn" and "cont abn") on different inputs for the HR-MK test: gross prices, detrended log prices with Simple Moving Average (SMA) 1Y, 2Y, 3Y and 5Y and on returns (at daily, weekly and monthly frequency). To avoid overloading the report with repetitive plots and results, I will provide a summary table of all of these configurations.

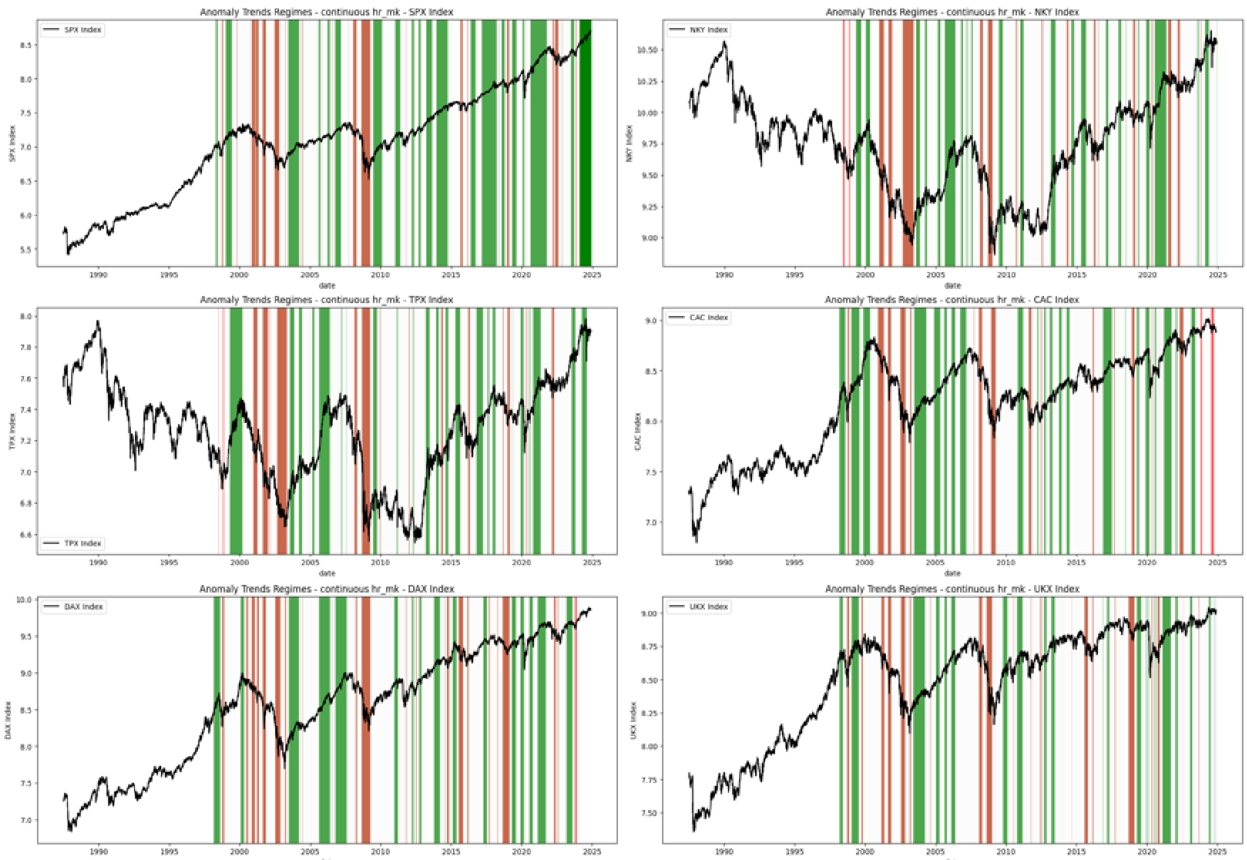
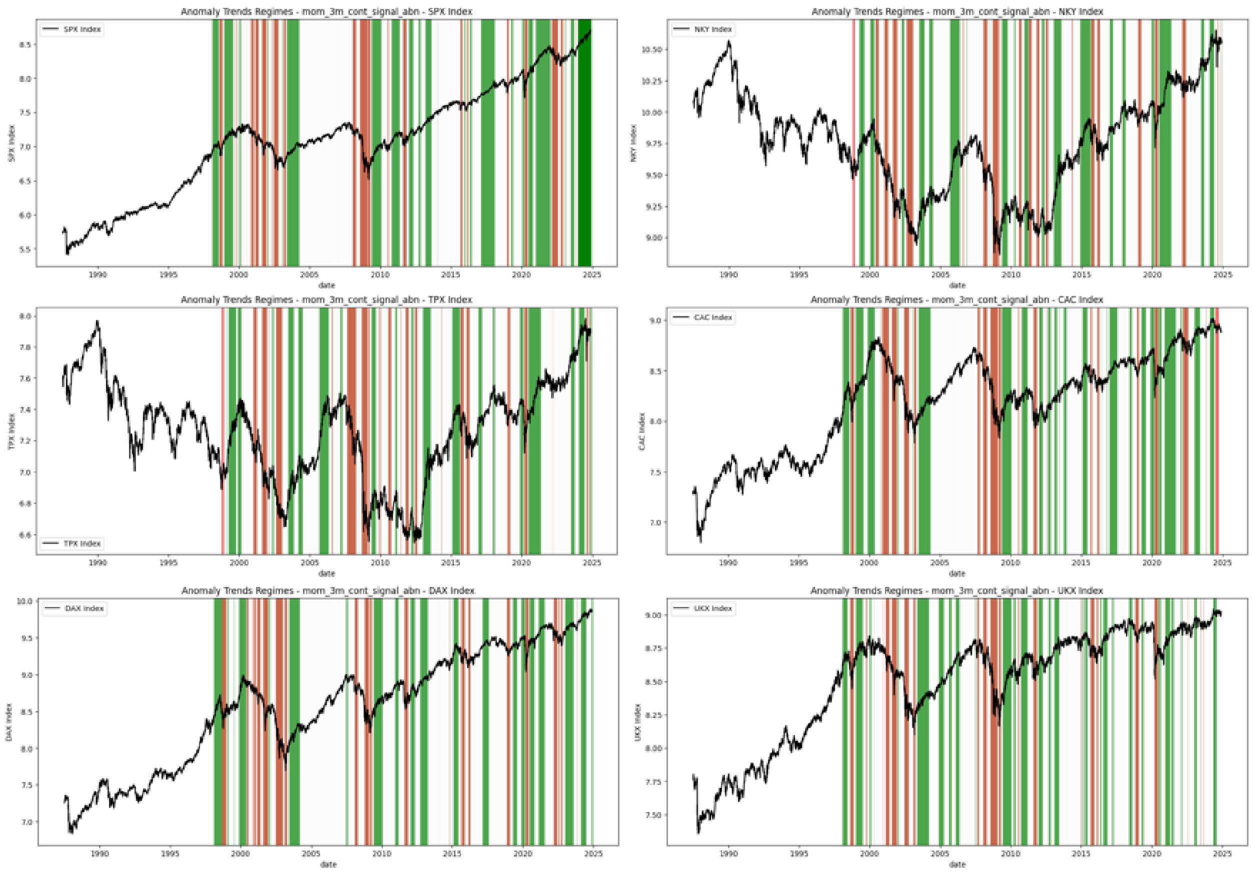
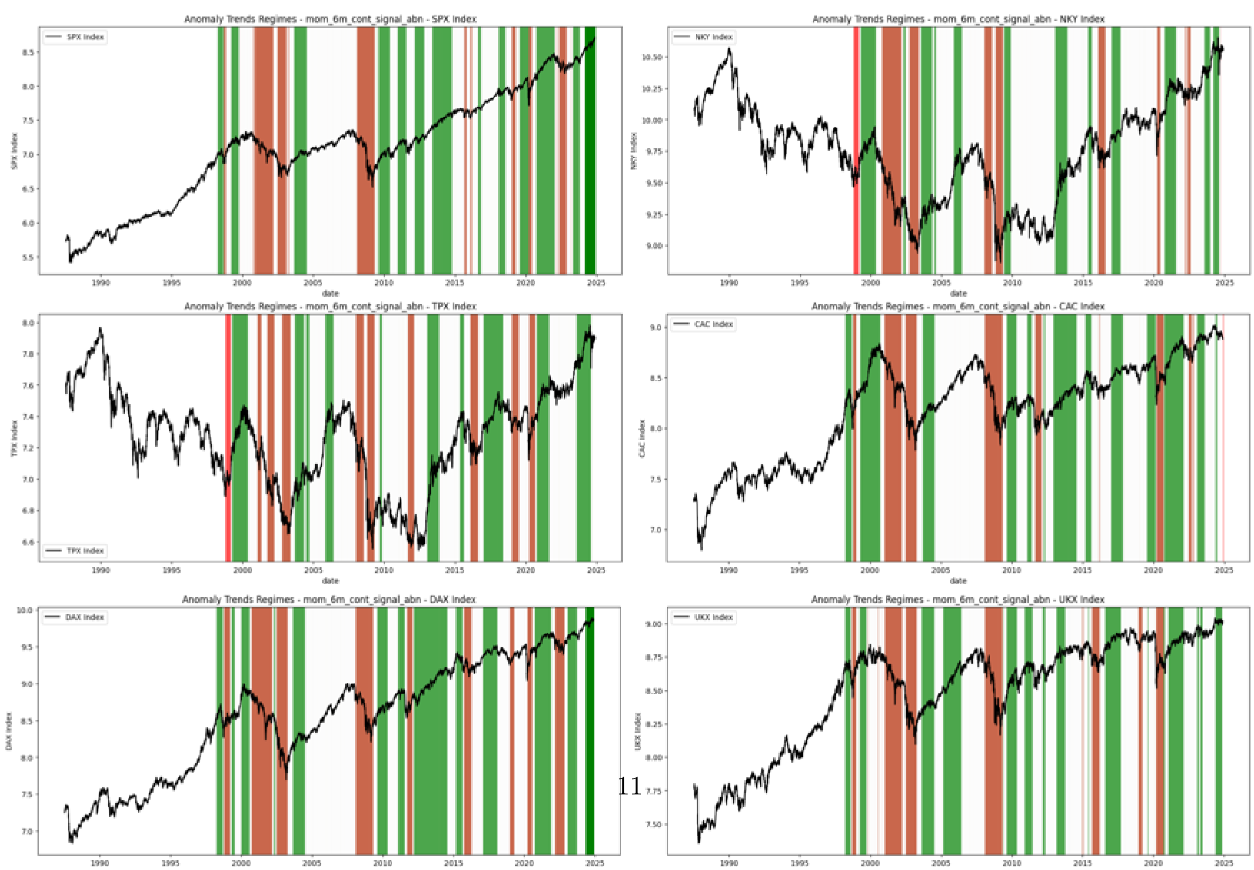


Figure 5: Trend regimes identified by cont abn HR-MK (on log prices) for various assets.

The aim of the cont abn signal was to extend the period length of the signals and we achieved what we wanted: for the long only signals (increasing trends, green bars) the average signal length across assets is 12 days for the abn HR-MK, 7 days for the abn mom_3m and 8 days for the mom_6m. When we look at the signal length for their cont abn versions, we get respectively: 78 days, 76 days and 156 days which are huge signal length extensions. The same conclusions hold whatever the type of inputs entered (mentioned above). It is worth noting that the HR-MK test performs very poorly on the returns whatever the frequency is. That is not surprising as the HR-MK is a trend test and there is no trends in returns which are (nearly) stationary.



(a) Trend regimes identified by cont abn mom_3m (on price returns) for various assets.



(b) Trend regimes identified by cont abn mom_6m (on price returns) for various assets.

2.1.3 Hamed-Rao Mann-Kendall test and mom_3m and mom_6m: assessing predictive power

Before backtesting the signals (results reported for the LO leg), it is interesting to study its predictive power. To do that, we scatter plotted the signal values (tstats for the HR-MK and z-scores for the momentum, at the date of the entry signal) on the x-axis vs its dynamic forward cumulative return (assuming 2 days lag) on the y-axis. Dynamic forward cumulative return is defined as follow: for example, if we have a signal of 10 days at date t, we take the signal values at date t as x and we take the cumulative return between date t+1 and t+1+1+10 as y. This "dynamic" version avoid setting a fixed window for the cumulative forward returns but can introduce a bias for trendy assets as the longer the signal lenght, the greater the cumulative return. To address this 'trend' bias in cumulative returns, we also plotted the inter-quartile range of the mom_3m (cumulative return) to spot the points outside this interval (I will present these results later). I will only present the scatter plot for the HR-MK cont signal, once again to avoid overloading the report but a summary table of the R squared for the three types of signal will be provided.

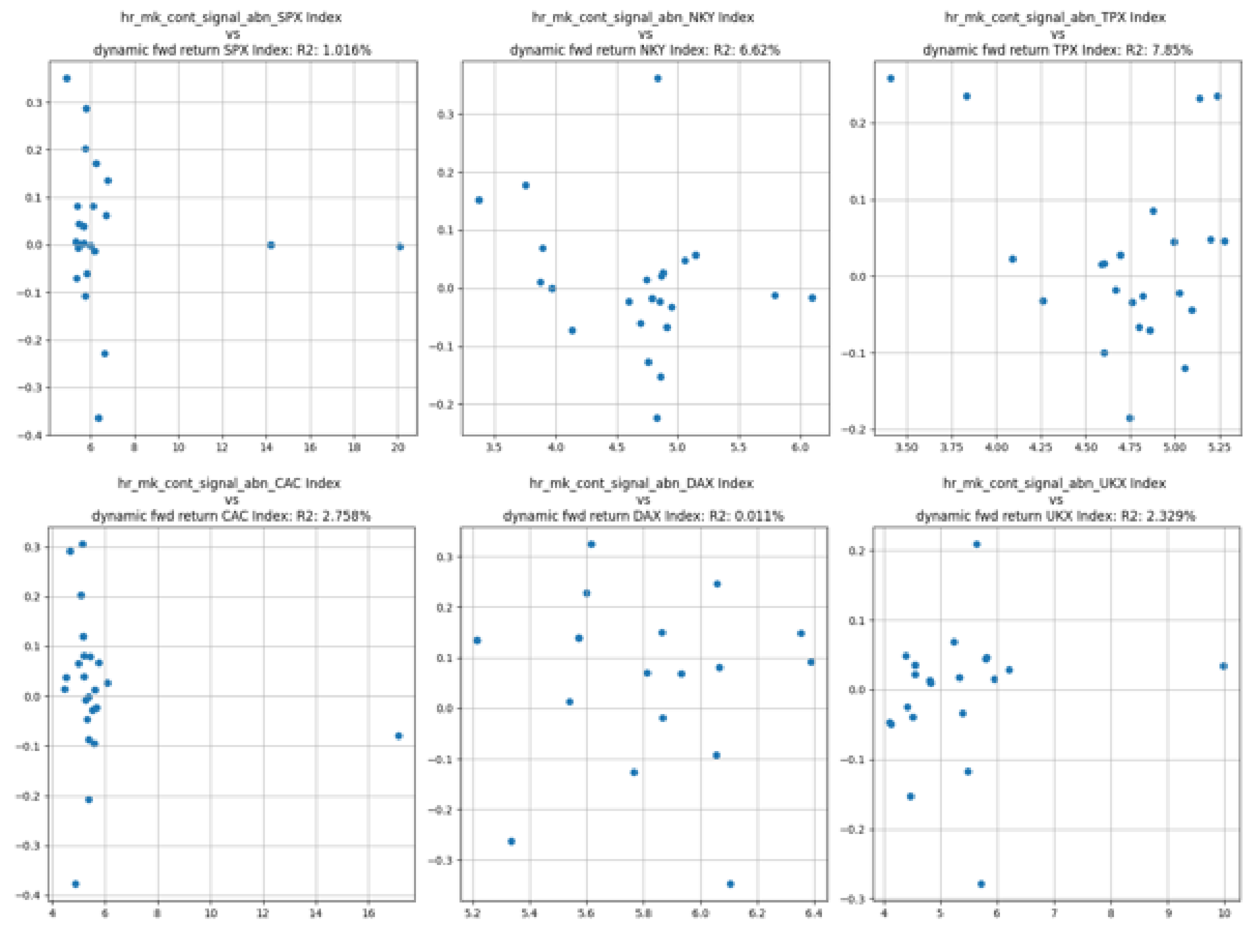


Figure 7: Scatter plot of HR-MK tstats (x-axis) vs forward cumulative return (y-axis) for various assets.

Before interpreting the R squared, it is important to remark that for some signals (especially the cont abn) there are few data points (sometimes only 12) that can biaised the R squared as we are running a regression on very small sample. Second remark is that if we look at the plots, even if we get a decent R squared, we can see negative dynamic forward cumulative returns although we have positive signals, implying a not so good hit ratio (not computed yet). Finally, a good companion to the hit ratio would have been the betas and their p-values to assess the direction and robustness of these relationships. Without these information, it is difficult to conclude for now but the results seem mixed.

	SPX Index	NKY Index	TPX Index	CAC Index	DAX Index	UKX Index
hr_mk_signal_all	0.296	2.585	1.287	1.723	1.590	0.375
hr_mk_signal_abn	1.148	1.858	6.153	1.284	0.483	1.750
hr_mk_cont_signal_abn	1.016	6.620	7.850	2.758	0.011	2.329
mom_3m_signal_all	1.139	0.315	0.810	1.029	0.693	3.129
mom_3m_signal_abn	4.967	2.067	1.170	0.288	0.438	0.903
mom_3m_cont_signal_abn	9.724	0.769	5.917	0.396	4.798	4.448
mom_6m_signal_all	1.709	0.020	0.125	2.584	0.856	9.569
mom_6m_signal_abn	4.630	10.341	5.439	2.157	0.083	0.701
mom_6m_cont_signal_abn	0.360	10.923	5.320	19.356	12.849	1.851

Table 2: Table showing the R squared from regressing dynamic forward cumulative returns on signal values for various indices.

One may ask: Are you sure the tstats from the HR-MK measure the strength of the trend? We provide below scatter plots of tstats from HR-MK test (x-axis) vs mom_2m (y-axis) for various assets. We can see a clear linear (sometimes concave) increasing relationship, allowing us to answer: yes the HR-MK tstats measure the strength of trends, strength defined as its underlying cumulative performance.

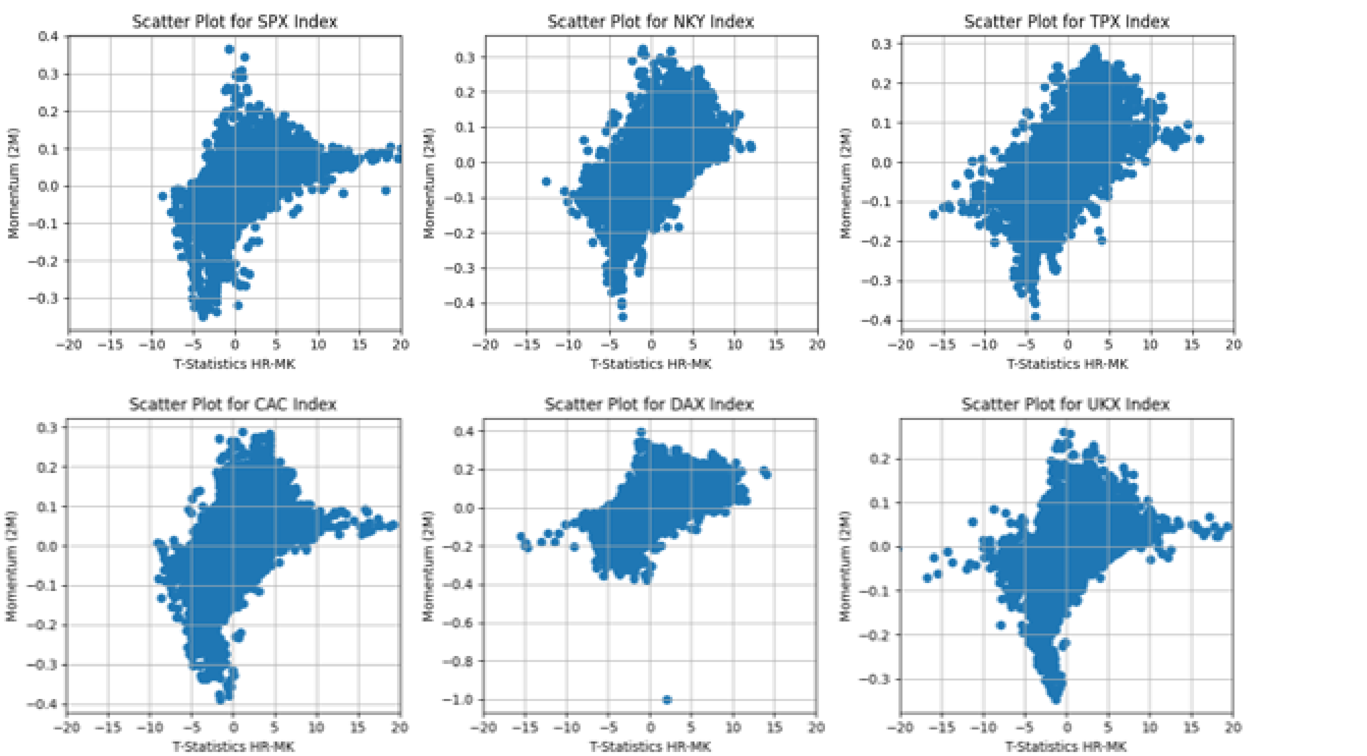


Figure 8: Scatter plot of HR-MK tstats (x-axis) vs mom_2m (y-axis) for various assets.

2.1.4 Hamed-Rao Mann-Kendall test and mom_3m and mom_6m: backtests

We present below the equity curves (equity curve represents non-compounded performance hence avoiding the ”start date effect”) only for the cont abn version of the signals as we are interesting in detecting abnormal trends, for various assets.

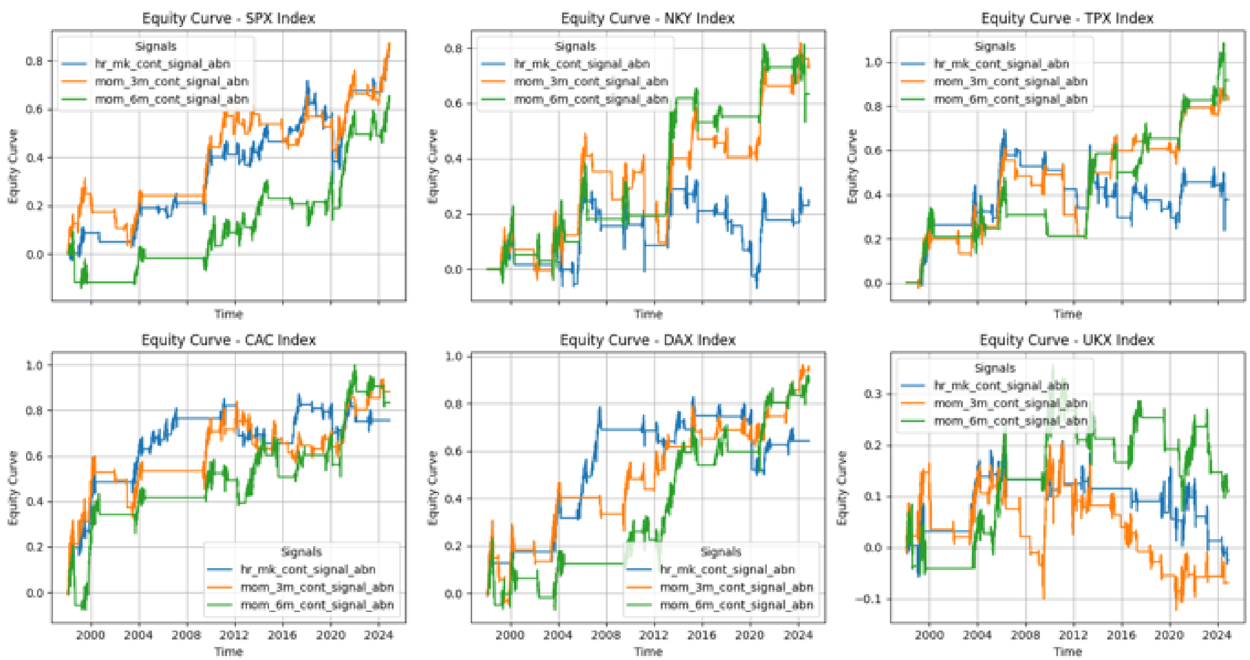


Figure 9: Equity Curves for cont abn signals and various assets.

Signal	Asset	Avg Perf/Year (%)	Annualized Volatility (%)	Risk Adjusted Return
mom_3m_cont_signal_abn	SPX Index	3.24	8.59	0.41
hr_mk_cont_signal_abn	SPX Index	3.22	9.11	0.38
mom_3m_cont_signal_abn	DAX Index	3.53	10.14	0.37
mom_3m_cont_signal_abn	TPX Index	3.40	10.53	0.35
mom_3m_cont_signal_abn	CAC Index	3.27	10.19	0.35
mom_3m_cont_signal_abn	TPX Index	3.08	9.88	0.33
hr_mk_cont_signal_abn	CAC Index	2.80	9.29	0.33
mom_6m_cont_signal_abn	DAX Index	3.35	11.32	0.32
mom_6m_cont_signal_abn	CAC Index	3.09	11.03	0.30
mom_6m_cont_signal_abn	SPX Index	2.43	9.07	0.29
hr_mk_cont_signal_abn	DAX Index	2.38	9.16	0.28
mom_3m_cont_signal_abn	NKY Index	2.72	10.76	0.27
mom_6m_cont_signal_abn	NKY Index	2.35	11.29	0.22
hr_mk_cont_signal_abn	TPX Index	1.40	10.73	0.14
hr_mk_cont_signal_abn	NKY Index	0.89	10.34	0.09
mom_3m_cont_signal_abn	UKX Index	0.41	7.89	0.06
hr_mk_cont_signal_abn	UKX Index	-0.09	6.79	-0.01
mom_3m_cont_signal_abn	UKX Index	-0.25	7.97	-0.03

Table 3: Performance comparison of different trend signals for various indices.

Visually, the HR-MK cont abn signal does not seem to beat its benchmarks, maybe except for the SPX where it seems to be close to the mom_3m and for the UKX where it beats the mom_3m. To have a more formal comparison, a table of average performance by year, annualized volatility and Risk adjusted return (RAR) is provided. In terms of RAR, the HR-MK cont abn signal does not beat its benchmarks for any assets.

If we broaden this study to other inputs for the signals (gross prices, log prices, detrended log prices with SMA 1Y, 2Y, 3Y, 5Y and daily returns) we can see that the HR-MK cont abn signal ranked first in terms of RAR for 5 assets ou of 6, the SPX being the exception. If we try to find a discriminant input where the HR-MK works well, we can see that 4 HR-MK cont signal out of the 5 that beat the other signals, are computed from detrended prices, but the detrended period being variable (1Y, 2Y or 5Y).

Risk Adjusted Return Comparison across Inputs

Ranking	data source	signal	SPX Index	Ranking	data source	signal	NKY Index	Ranking	data source	signal	TPX Index
1	gross prices	mom_3m_cont_signal_abn	0.41	1	**returns**	hr_mk_cont_signal_abn	**0.34**	1	**detrended 5Y**	hr_mk_cont_signal_abn	**0.39**
2	log prices	mom_3m_cont_signal_abn	0.41	2	gross prices	mom_3_cont_signal_abn	0.27	2	gross prices	mom_6m_cont_signal_abn	0.35
3	detrended 1Y	mom_3m_cont_signal_abn	0.41	3	log prices	mom_3_cont_signal_abn	0.27	3	log prices	mom_6m_cont_signal_abn	0.35
4	detrended 2Y	mom_3m_cont_signal_abn	0.41	4	detrended 1Y	mom_3_cont_signal_abn	0.27	4	detrended 1Y	mom_6m_cont_signal_abn	0.35
5	detrended 3Y	mom_3m_cont_signal_abn	0.41	5	detrended 2Y	mom_3_cont_signal_abn	0.27	5	detrended 2Y	mom_6m_cont_signal_abn	0.35
6	detrended 5Y	mom_3m_cont_signal_abn	0.41	6	detrended 3Y	mom_3_cont_signal_abn	0.27	6	detrended 3Y	mom_6m_cont_signal_abn	0.35
7	returns	mom_3m_cont_signal_abn	0.41	7	detrended 5Y	hr_mk_cont_signal_abn	0.27	7	detrended 3Y	hr_mk_cont_signal_abn	0.35
8	gross prices	hr_mk_cont_signal_abn	0.39	8	detrended 5Y	mom_3_cont_signal_abn	0.27	8	detrended 5Y	mom_6m_cont_signal_abn	0.35
9	log prices	hr_mk_cont_signal_abn	0.38	9	returns	mom_3_cont_signal_abn	0.27	9	returns	mom_6m_cont_signal_abn	0.35
10	detrended 3Y	hr_mk_cont_signal_abn	0.32	10	gross prices	mom_6m_cont_signal_abn	0.22	10	gross prices	mom_3_cont_signal_abn	0.34
11	gross prices	mom_6m_cont_signal_abn	0.29	11	log prices	mom_6m_cont_signal_abn	0.22	11	log prices	mom_3_cont_signal_abn	0.34
12	log prices	mom_6m_cont_signal_abn	0.29	12	detrended 1Y	mom_6m_cont_signal_abn	0.22	12	detrended 1Y	mom_3_cont_signal_abn	0.34
13	detrended 1Y	mom_6m_cont_signal_abn	0.29	13	detrended 2Y	mom_6m_cont_signal_abn	0.22	13	detrended 2Y	mom_3_cont_signal_abn	0.34
14	detrended 2Y	mom_6m_cont_signal_abn	0.29	14	detrended 3Y	mom_6m_cont_signal_abn	0.22	14	detrended 3Y	mom_3_cont_signal_abn	0.34
15	detrended 3Y	mom_6m_cont_signal_abn	0.29	15	detrended 5Y	mom_6m_cont_signal_abn	0.22	15	detrended 5Y	mom_3_cont_signal_abn	0.34
16	detrended 5Y	mom_6m_cont_signal_abn	0.29	16	returns	mom_6m_cont_signal_abn	0.22	16	returns	mom_3_cont_signal_abn	0.34
17	returns	mom_6m_cont_signal_abn	0.29	17	detrended 3Y	hr_mk_cont_signal_abn	0.15	17	detrended 2Y	hr_mk_cont_signal_abn	0.31
18	detrended 2Y	hr_mk_cont_signal_abn	0.26	18	log prices	hr_mk_cont_signal_abn	0.09	18	gross prices	hr_mk_cont_signal_abn	0.19
19	detrended 1Y	hr_mk_cont_signal_abn	0.21	19	gross prices	hr_mk_cont_signal_abn	0.08	19	returns	hr_mk_cont_signal_abn	0.18
20	detrended 5Y	hr_mk_cont_signal_abn	0.16	20	detrended 1Y	hr_mk_cont_signal_abn	0.08	20	log prices	hr_mk_cont_signal_abn	0.14
21	returns	hr_mk_cont_signal_abn	-0.15	21	detrended 2Y	hr_mk_cont_signal_abn	0.06	21	detrended 1Y	hr_mk_cont_signal_abn	0.1

Ranking	data source	signal	CAC Index	Ranking	data source	signal	DAX Index	Ranking	data source	signal	UKX Index
1	**detrended 2Y**	hr_mk_cont_signal_abn	**0.39**	1	**detrended 1Y**	hr_mk_cont_signal_abn	**0.38**	1	**detrended 1Y**	hr_mk_cont_signal_abn	**0.29**
2	detrended 5Y	hr_mk_cont_signal_abn	0.38	2	gross prices	mom_3_cont_signal_abn	0.37	2	detrended 5Y	hr_mk_cont_signal_abn	0.11
3	gross prices	mom_3_cont_signal_abn	0.35	3	log prices	mom_3_cont_signal_abn	0.37	3	gross prices	mom_6m_cont_signal_abn	0.06
4	log prices	mom_3_cont_signal_abn	0.35	4	detrended 1Y	mom_3_cont_signal_abn	0.37	4	log prices	mom_6m_cont_signal_abn	0.06
5	detrended 1Y	mom_3_cont_signal_abn	0.35	5	detrended 3Y	mom_3_cont_signal_abn	0.37	5	detrended 1Y	mom_6m_cont_signal_abn	0.06
6	detrended 2Y	mom_3_cont_signal_abn	0.35	6	detrended 5Y	hr_mk_cont_signal_abn	0.37	6	detrended 2Y	mom_6m_cont_signal_abn	0.06
7	detrended 3Y	mom_3_cont_signal_abn	0.35	7	detrended 5Y	mom_3_cont_signal_abn	0.37	7	detrended 3Y	mom_6m_cont_signal_abn	0.06
8	detrended 5Y	mom_3_cont_signal_abn	0.35	8	returns	mom_3_cont_signal_abn	0.37	8	detrended 5Y	mom_6m_cont_signal_abn	0.06
9	returns	mom_3_cont_signal_abn	0.35	9	detrended 2Y	mom_3_cont_signal_abn	0.35	9	returns	mom_6m_cont_signal_abn	0.06
10	log prices	hr_mk_cont_signal_abn	0.33	10	gross prices	mom_6m_cont_signal_abn	0.32	10	gross prices	hr_mk_cont_signal_abn	-0.01
11	detrended 1Y	hr_mk_cont_signal_abn	0.32	11	gross prices	hr_mk_cont_signal_abn	0.32	11	log prices	hr_mk_cont_signal_abn	-0.01
12	gross prices	hr_mk_cont_signal_abn	0.31	12	log prices	mom_6m_cont_signal_abn	0.32	12	detrended 2Y	hr_mk_cont_signal_abn	-0.01
13	gross prices	mom_6m_cont_signal_abn	0.3	13	detrended 1Y	mom_6m_cont_signal_abn	0.32	13	detrended 3Y	hr_mk_cont_signal_abn	-0.01
14	log prices	mom_6m_cont_signal_abn	0.3	14	detrended 2Y	mom_6m_cont_signal_abn	0.32	14	gross prices	mom_3_cont_signal_abn	-0.03
15	detrended 1Y	mom_6m_cont_signal_abn	0.3	15	detrended 3Y	mom_6m_cont_signal_abn	0.32	15	log prices	mom_3_cont_signal_abn	-0.03
16	detrended 2Y	mom_6m_cont_signal_abn	0.3	16	detrended 5Y	mom_6m_cont_signal_abn	0.32	16	detrended 1Y	mom_3_cont_signal_abn	-0.03
17	detrended 3Y	mom_6m_cont_signal_abn	0.3	17	returns	mom_6m_cont_signal_abn	0.32	17	detrended 2Y	mom_3_cont_signal_abn	-0.03
18	detrended 5Y	mom_6m_cont_signal_abn	0.3	18	detrended 2Y	hr_mk_cont_signal_abn	0.29	18	detrended 3Y	mom_3_cont_signal_abn	-0.03
19	returns	mom_6m_cont_signal_abn	0.3	19	log prices	hr_mk_cont_signal_abn	0.28	19	detrended 5Y	mom_3_cont_signal_abn	-0.03
20	detrended 3Y	hr_mk_cont_signal_abn	0.26	20	detrended 3Y	hr_mk_cont_signal_abn	0.21	20	returns	mom_3_cont_signal_abn	-0.03
21	returns	hr_mk_cont_signal_abn	-0.16	21	returns	hr_mk_cont_signal_abn	0.03	21	returns	hr_mk_cont_signal_abn	-0.09

Figure 10: Comparison of Risk Adjusted Return across different inputs.

Reflecting on these results and on the procedure to build the cont abn signal, there is a sort of double counting for the abnormality with the detrented prices. In fact, the abnormal trend is characterized by both the input which is sometimes detrended and the deciles step. To avoid this double counting of abnormality, we re-conducted the analysis and the backtests on the detrended prices only, without the deciles step. In this configuration, there are no more the three types of signals (”all”, ”abn” and ”cont abn”) as the two latter where built on the deciles. So if latter these three types appear, they’re basically the same signals.

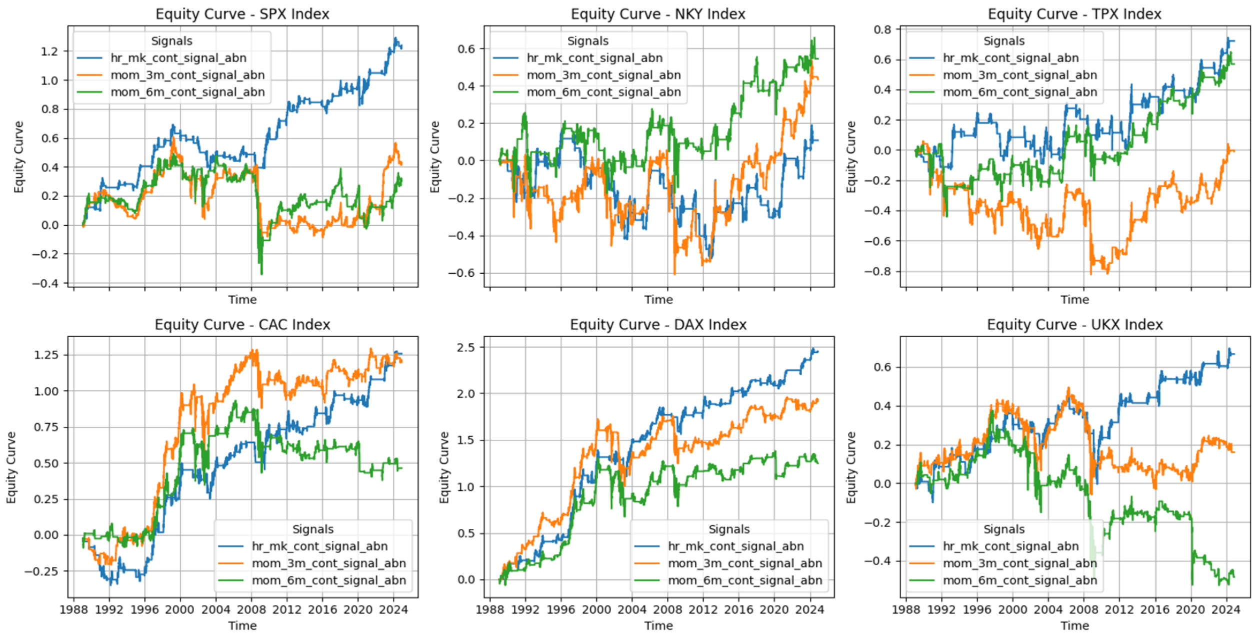


Figure 11: Equity Curves for detrended 1Y log prices (no deciles, same inputs for mom) across various assets.

Signals (detrended 1Y log prices, no deciles, same inputs for mom)	Assets	Avg Perf/Year (%)	Annualized Volatility (%)	Risk Adjusted Return
hr_mk_cont_signal_abn	**DAX Index**	**6.80**	**10.87**	**0.67**
mom_3m_cont_signal_abn	DAX Index	5.34	14.59	0.39
mom_6m_cont_signal_abn	DAX Index	3.46	15.05	0.25
hr_mk_cont_signal_abn	**SPX Index**	**3.44**	**8.27**	**0.45**
mom_3m_cont_signal_abn	SPX Index	1.19	11.83	0.11
mom_6m_cont_signal_abn	SPX Index	0.84	11.73	0.08
hr_mk_cont_signal_abn	**CAC Index**	**3.49**	**10.17**	**0.37**
mom_3m_cont_signal_abn	CAC Index	3.33	14.17	0.25
mom_6m_cont_signal_abn	CAC Index	1.29	13.83	0.10
hr_mk_cont_signal_abn	**UKX Index**	**1.85**	**7.80**	**0.25**
mom_3m_cont_signal_abn	UKX Index	0.45	11.50	0.04
mom_6m_cont_signal_abn	UKX Index	-1.34	11.70	-0.12
hr_mk_cont_signal_abn	**TPX Index**	**2.00**	**9.98**	**0.22**
mom_6m_cont_signal_abn	TPX Index	1.58	13.10	0.13
mom_3m_cont_signal_abn	TPX Index	-0.02	14.18	0.00
mom_6m_cont_signal_abn	NKY Index	1.51	14.94	0.11
mom_3m_cont_signal_abn	NKY Index	1.20	15.94	0.08
hr_mk_cont_signal_abn	**NKY Index**	**0.30**	**11.17**	**0.03**

Figure 12: Summary performance table grouped by asset for detrended 1Y log prices (no deciles, same inputs for mom).

With this configuration: detrended 1Y log prices as inputs, no deciles step and same inputs for the momentum, the HR-MK signals beat their benchmark for 5 assets out of 6 (the exception being the NKY).

The HR-MK test allows to improve the average non-compounded) performance by year by 111%, reduce the annualized volatility by 28% on average and increase the SR by 191% on average.

2.1.5 Hamed-Rao Mann-Kendall can beat mom_3m and mom_6m but why?

In this paragraph, we try to understand why the HR-MK test beat the mom_3m and mom_6m on some assets and for some inputs.

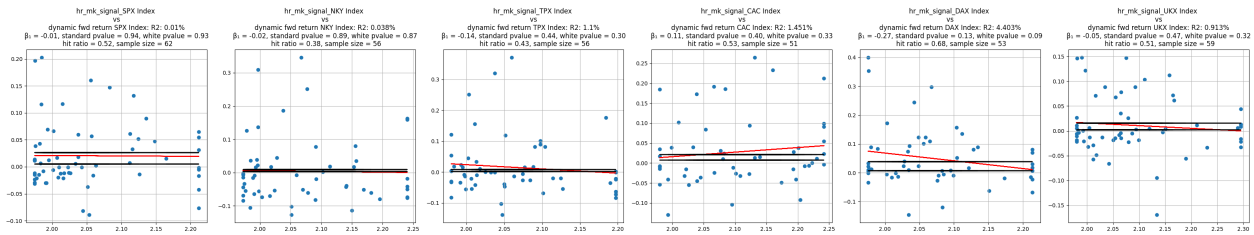


Figure 13: Dynamic Forward Analysis for hr_mk on de trended 1Y log prices (without deciles step).

In the above figure, we report the Dynamic Forward Analysis by asset. For each asset, we computed the R squared, the slope of the linear regression, its standard pvalues, its white corrected pvalues, the hit ratio and the sample size. We winsorized the signal values (x-axis) at 10% and 90% to avoid outliers that can distort the estimation of the linear regression.

We can see that the hit ratio is greater for the hr_mk than for the mom_3m for the SPX and DAX Index. But hit ratios are not a complete measure of performance as they represent the number of signals that lead to positive performance over all signals (that is a proportion) but it does not quantify the magnitude of the performance for each signals. It may explain why we have only two greater hit ratios for the hr_mk while of having 5 higher RAR out of 6 assets.

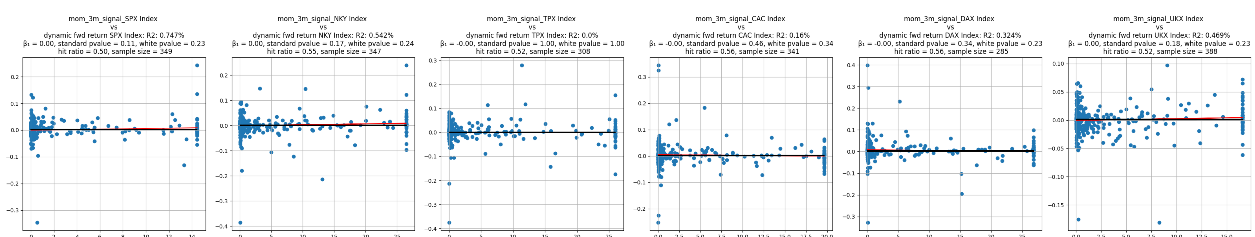


Figure 14: Dynamic Forward Analysis for mom_3m on de trended 1Y log prices (without deciles step).

2.2 Momentum Strategies, Disturbance (Mahalanobis Distance) Test and subsequent strategies

2.2.1 Stylized facts on cross-sectional momentum: Information Coefficient and momentum formation period

Before beginning this section, let's recall briefly our main mission: identifying abnormal trends to build on trend-following strategies. The term abnormal refers to the fact that these identifications and strategies must be opportunistic meaning that these strategies cannot be always invested, hence we want to time the market: a well-known and very difficult problem (much more difficult than selecting the best securities and staying invested, in my opinion).

In this section, we'll try to identify change-points, with the idea that a change-point can occur before a starting trends. In other words, we are trying to find a trend-following strategy (time-series momentum).

Before digging into our testings, let's propose some stylized facts and baseline results of cross-sectional momentum. This will serve first, as a "benchmark" to see (or not) the additional value of time-series momentum and second, to remind us that cross-sectional momentum is quite strong and not so-easy to beat with the approach of market timing.

A measure to assess the quality of a signal is the **Information Coefficient (IC)**. The IC is defined as the correlation between the forecasted returns and the realized returns over a given period. In the following discussion, we define the IC as the correlation between the *signal values* (i.e., the numerical values that generate the signals) and the *1-month forward returns*:

$$\mathrm{IC}_t = \mathrm{corr}\left(S_t, R_{t+1}\right)$$

where:

- $S_t = (S_{1,t}, S_{2,t}, \ldots, S_{N,t})$ is the vector of signal values for each asset at time t,
- $R_{t+1} = (R_{1,t+1}, R_{2,t+1}, \ldots, R_{N,t+1})$ is the vector of 1-month forward returns,
- $\mathrm{corr}(S_t, R_{t+1})$ denotes the *cross-sectional Pearson correlation* between the signals and the forward returns.

To determine whether signals are better represented by numerical values or ranks, we also define the **rank-based Information Coefficient (IC_ranked)** as the Pearson correlation between the ascending ranks of the signal values and the 1-month forward returns:

$$\mathrm{IC}_t = \mathrm{corr}\left(\mathrm{rank}(S_t), R_{t+1}\right)$$

where:

- $\mathrm{rank}(S_t) = (\mathrm{rank}(S_{1,t}), \mathrm{rank}(S_{2,t}), \ldots, \mathrm{rank}(S_{N,t}))$ is the vector of ascending ranks of the signal values at time t,
- $R_{t+1} = (R_{1,t+1}, R_{2,t+1}, \ldots, R_{N,t+1})$ is the vector of 1-month forward returns,
- $\mathrm{corr}(\cdot, \cdot)$ denotes the *Pearson correlation* between the ranked signals and future returns.

Note: this version of the IC is sometimes called a **Rank IC**, and it corresponds to a *Spearman correlation* between the signals and the forward returns.

Lastly, we define a final alternative for the Information Coefficient (IC) called **IC_percentiles**. This measure assesses the quality of the signals, but instead of considering all assets, we focus only on the "tail" assets — those defined by some percentiles. This allows us to explore whether it is more effective to forecast returns based on extreme signal values (i.e., the tail assets) rather than the full sample, which might contain noise in the middle:

$$\mathrm{IC}_{\mathrm{percentiles},t} = \mathrm{corr}\left(S_{t,\mathrm{tail}}, R_{t+1}\right)$$

where:

- $S_{t,\mathrm{tail}}$ is the vector of signal values for the "tail" assets at time t. These assets are defined as the ones falling within specific percentiles (e.g., 10% lowest and 10% highest signals),
- $R_{t+1} = (R_{1,t+1}, R_{2,t+1}, \ldots, R_{N,t+1})$ is the vector of 1-month forward returns corresponding to the tail assets,
- $\mathrm{corr}(S_{t,\mathrm{tail}}, R_{t+1})$ denotes the Pearson correlation between the signal values of the "tail" assets and their forward returns.

Note: this version of the IC focuses on the extreme signal values (i.e., the tail of the distribution) to determine whether forecasting using these extreme values yields a higher correlation with future returns compared to using all assets in the sample.

This new alternative IC is available for both IC and IC_ranked, thus, at the end, we have four IC definitions: IC, IC_ranked, IC_percentiles,IC_ranked_percentiles.

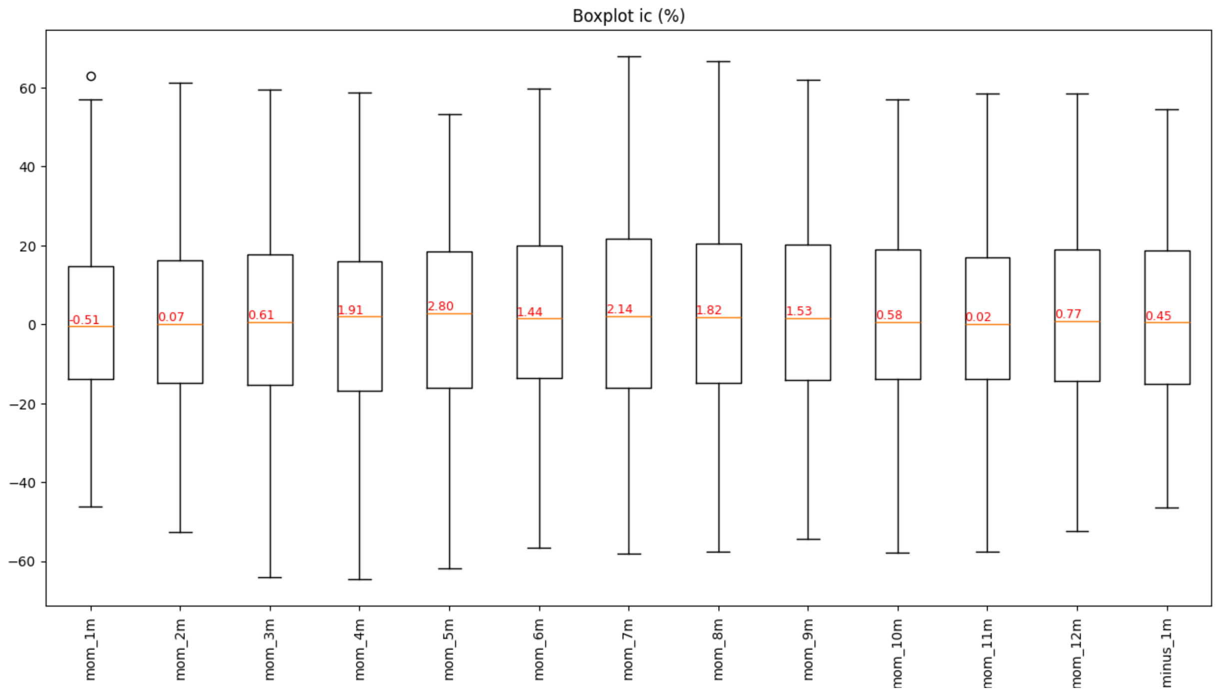


Figure 15: Information Coefficient of cross-sectional momentum strategies with varying window for the momentum. Momentum is based on excess monthly returns (gics3 - gics1)

[**These results are generated with script in main37**] The above chart displays the boxplot of ICs for varying momentum window ranging from 1-month to 12-month and computed based on excess returns (gics3 - gics1), the last being 12-month minus 1-month (the most recent month, to avoid the 1-month reversal effect). From this chart, we can conclude that the optimal momentum window is 12-month excluding the most recent month given the median IC at 2.71%.

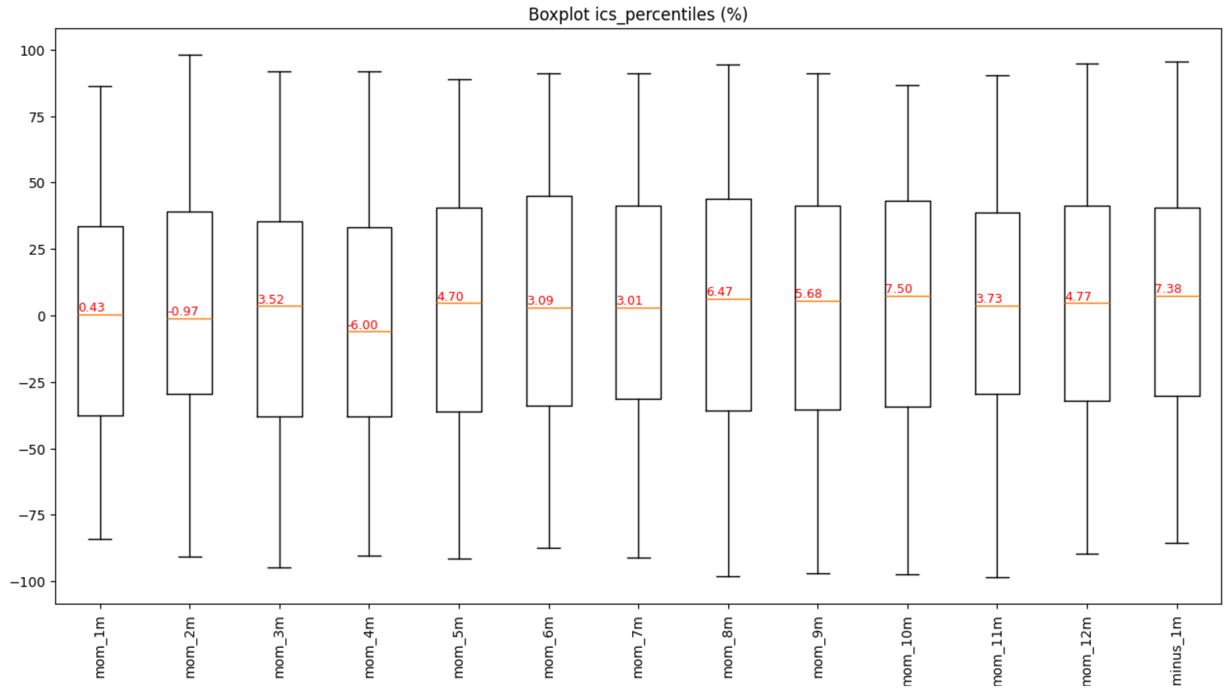


Figure 16: IC_percentiles (5,95) of cross-sectional momentum strategies with varying window for the momentum. Momentum is based on excess monthly returns (gics3 - gics1)

The above chart is very interesting because it leads to one main conclusion: it is by far (roughly 3 times) easier to predict the returns of the assets with extremes signals. I guess that's why the 'deciles' or 'quintiles' portfolio construction methods are used. We can also note that now, the best momentum window is 12-month.

For additional details, you can find below the charts for the IC_ranked and IC_ranked_percentiles. It is not clear (depends on the momentum window) wether ranks or numerical values should be preferred for computing the signals.

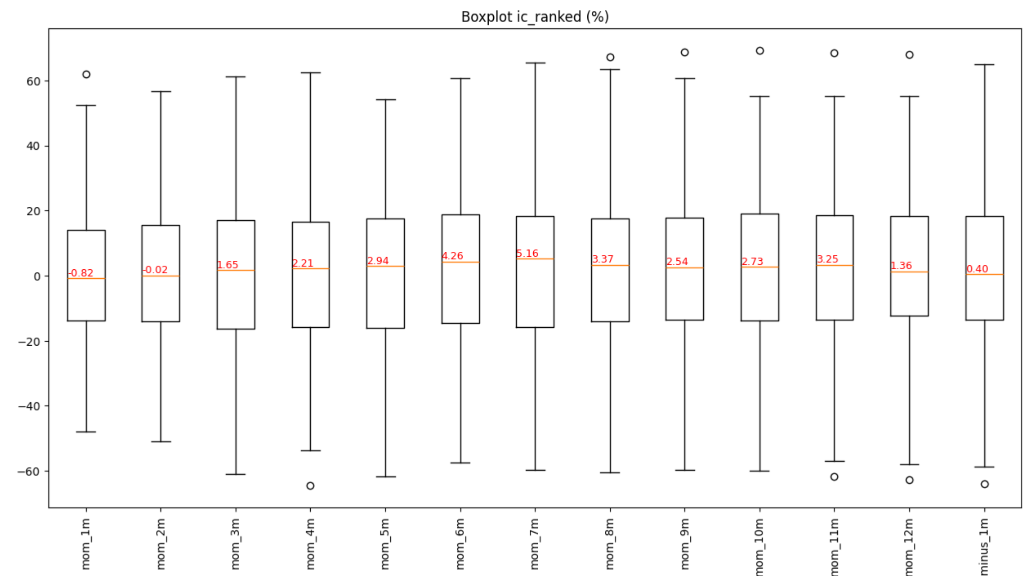


Figure 17: IC_ranked of cross-sectional momentum strategies with varying window for the momentum. Momentum is based on excess monthly returns (gics3 - gics1)

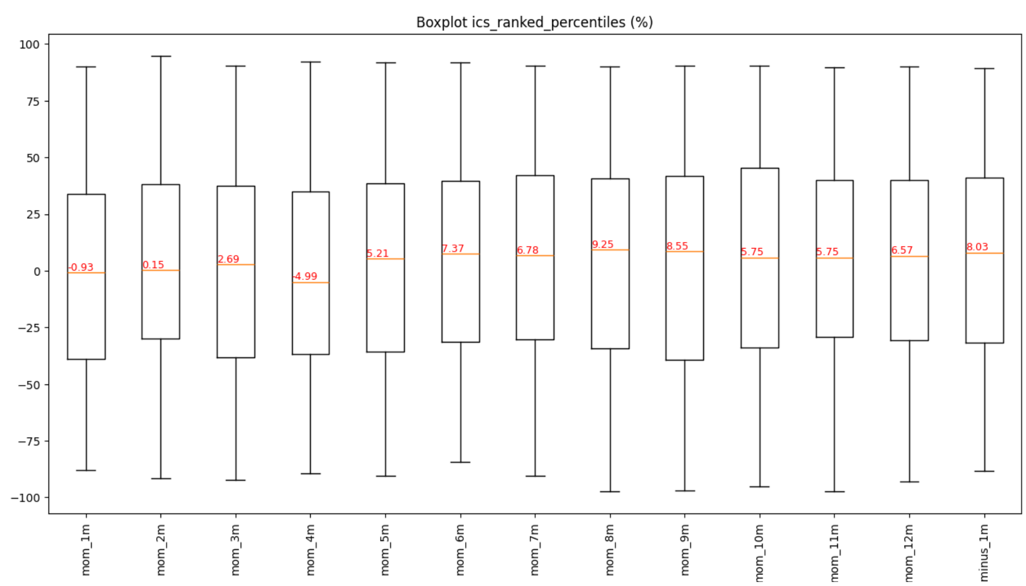


Figure 18: IC_ranked_percentiles (5,95) of cross-sectional momentum strategies with varying window for the momentum. Momentum is based on excess monthly returns (gics3 - gics1)

2.2.2 Some backtesting results on cross-sectional momentum strategies

[**These results are generated with the script main33, with input parameter sharpe_ratio_mom set to False.**] Now, we have identified that a good momentum window stands between 6-month and 12-month (well known stylized fact) and that selecting extreme signals percentiles (5,95) help in forecasting returns, we can run some basics cross-sectional momentum strategies.

Below you can find the equity curve (cumulative sum of returns) of a momentum 10-month cross-sectional strategy based on gics3 level S&1200 indices, forming portfolios based on (5,95) percentiles. Note that rebalancing is monthly and does not take into account transaction costs. Data is retrieved from bloomberg from 2006-05-31 to 2024-11-30. You can also find the excess equity curve which is the difference between the equity curve of the strategy and the one of its benchmark (S&P Global).

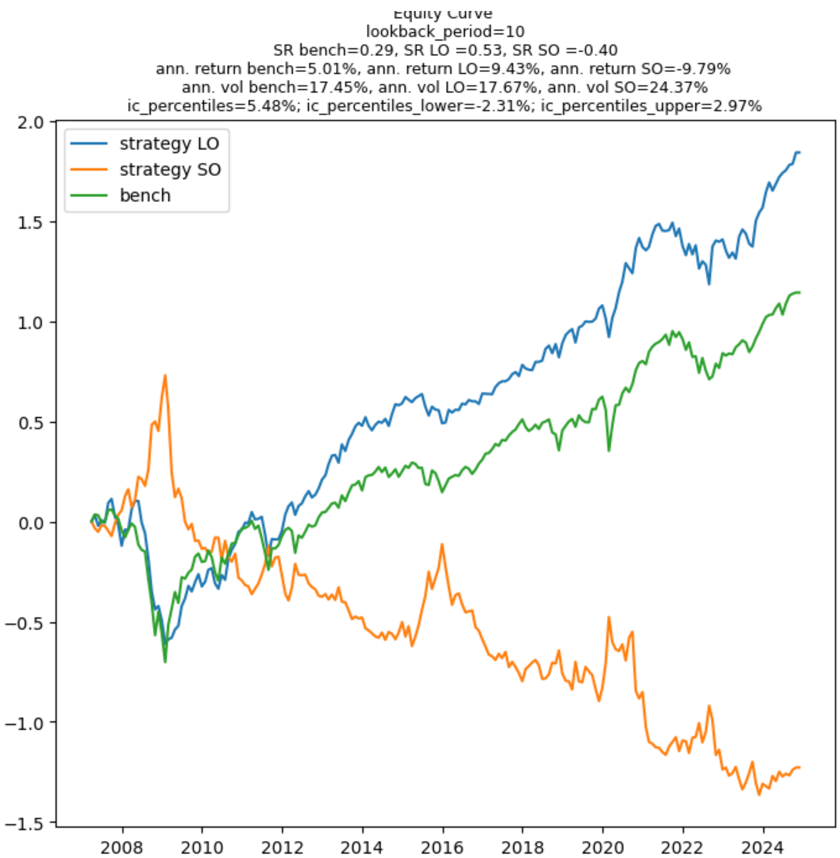


Figure 19: Equity curve of a 10-month cross-sectional momentum strategy. Portfolios are formed with percentiles (5,95).

Apart from the 2008-2011 period, the momentum strategy shows consistent outperformance (excess returns, as shown below). Sharpe Ratio (assuming rf=0) of the long leg of the strategy is 82% higher than the benchmark.

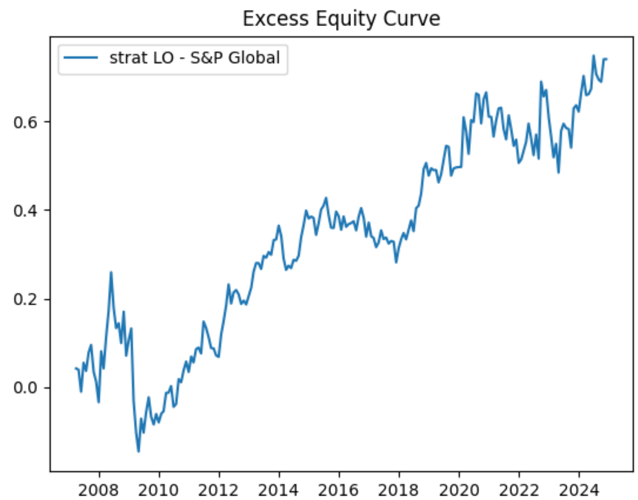


Figure 20: Excess Equity curve of a 10-month cross-sectional momentum strategy. Portfolios are formed with percentiles (5,95).

Although results are quite good, this strategy does not answer our problem because there is no detection of 'abnormal' trends. Before digging into the market-timing strategies, let's see a 'naive' strategy that defines the abnormality as the excess returns of the gics3 level S&P indices returns and their corresponding upper level, the gics1 indices. There is still no timing, but a sense of abnormal trends.

The results are shown below. Once again, the strategy outperform its benchmark, showing superior SR but it is worth noting that the SR of this strategy based on 'excess momentum' leads to smaller SR than the strategy based on 'returns'.

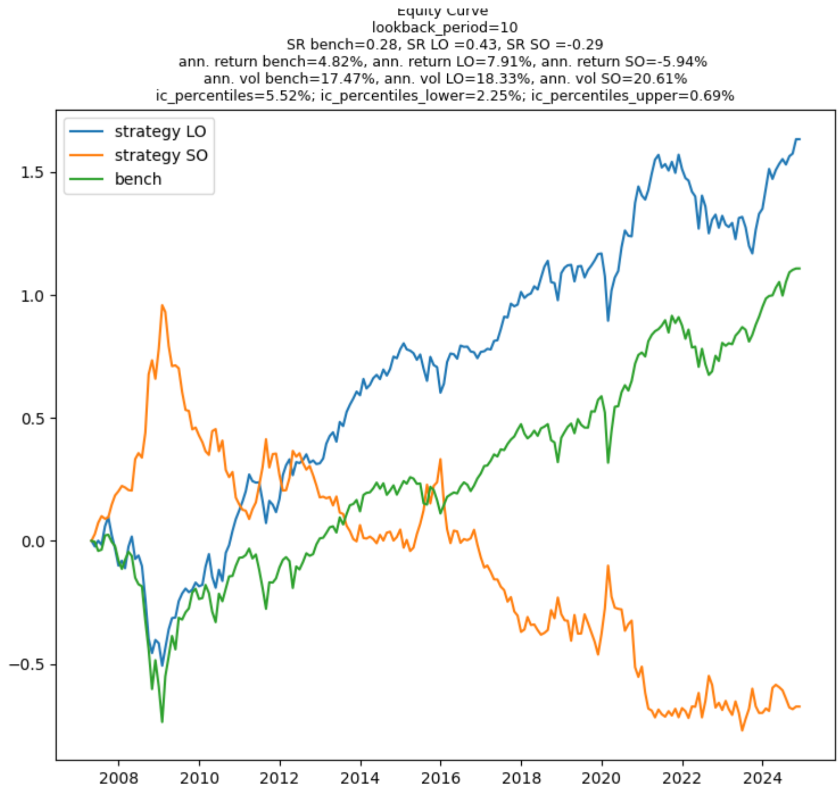


Figure 21: Equity curve of a 10-month cross-sectional momentum strategy based on 10-month gics3 mom minus 10-month gics1 momentum. Portfolios are formed with percentiles (5,95).

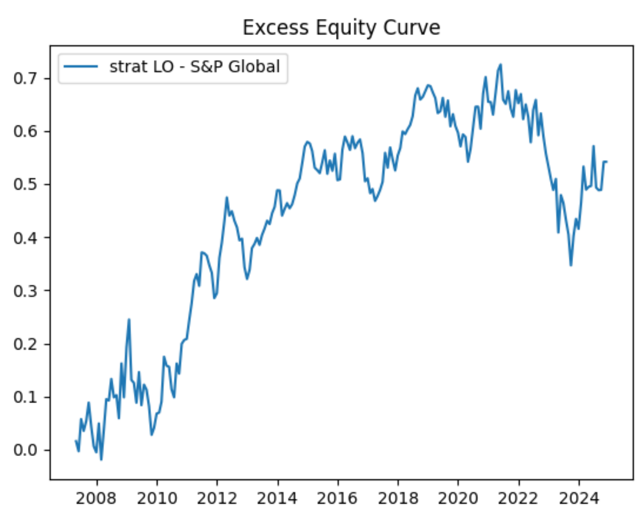


Figure 22: Equity curve of a 10-month cross-sectional momentum strategy based on 10-month gics3 mom minus 10-month gics1 momentum. Portfolios are formed with percentiles (5,95).

2.2.3 Some additional backtesting results on cross-sectional momentum strategies: using risk adjusted returns instead of momentum

[**These results are generated with the script main33, with input parameter sharpe_ratio_mom set to True.**] In this subsection, we explore the performance of cross-sectional momentum strategies that use Sharpe Ratio of the assets rather than their "gross" momentum (refered as to csmomsr). By doing that, we do not only select the best performing assets, we select the best performing assets on a risk-adjusted basis. We tried this strategy with two inputs: 1) the gics3 returns and 2) the gics3 momentum minus their corresponding gics1 momentum which we'll refer to as "excess gics3 over gics3 momentum". The second input is motivated by our goal: designing abnormal trend strategies. By doing that, we define the abnormality as the excess performance of an asset related to its upper level industry classification.

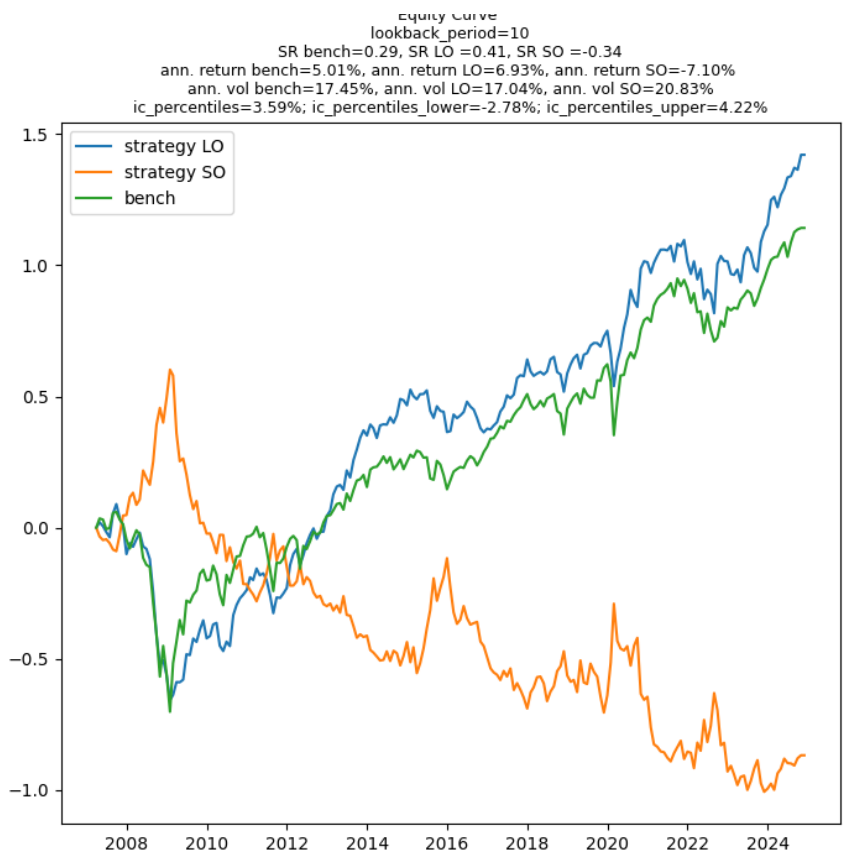


Figure 23: Equity curve of a 10-month csmomsr strategy using gics3 returns. Portfolios are formed with percentiles (5,95).

We can observe a positive trendy excess equity curve which is a good sign: our strategy near doubles the annualized return for an approximately same annualized volatility allowing the Sharpe Ratio to go from 0.29 to 0.53 which represents a +82.75% improvement.

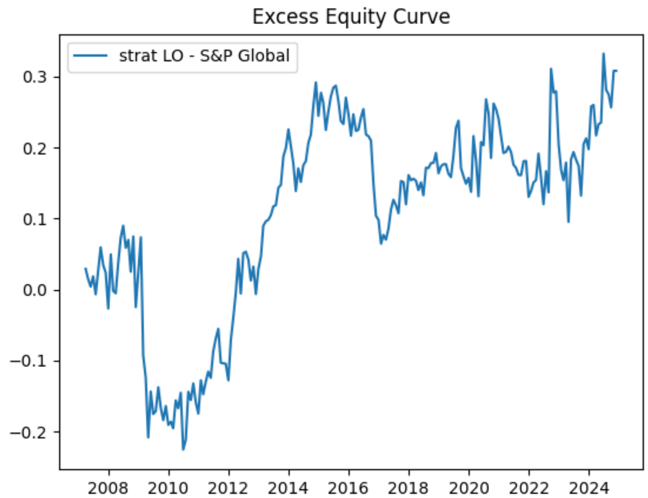


Figure 24: Excess Equity curve of a 10-month csmomsr strategy using gics3 returns. Portfolios are formed with percentiles (5,95).

Using the excess gics3 over gics3 10-month momentum instead of the gics3 returns lowers the performance: annualized return decreases from 9.43% to 7.91% and annualized volatility increases from 17.67% to 18.33% resulting in a 0.53-0.43=0.10 decrease in the SR. Nevertheless, even if the performance is lowered, outperformance of the benchmark is still achieved (SR benchmark = 0.28) and this strategy better suits our goal.

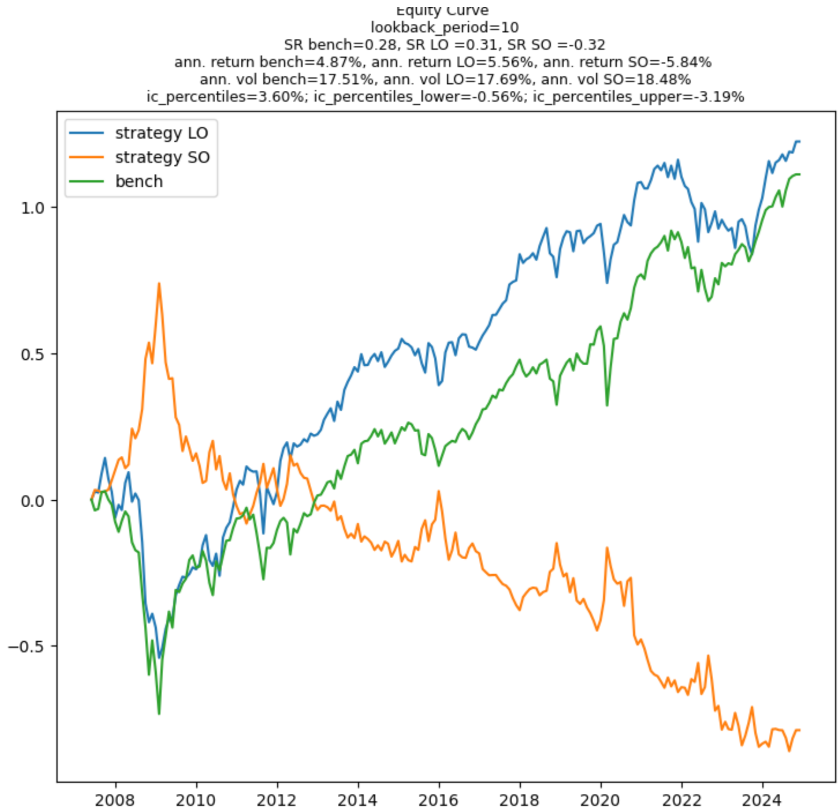


Figure 25: Equity curve of a 10-month csmomsr strategy using excess gics3 over gics3 momentum. Portfolios are formed with percentiles (5,95).

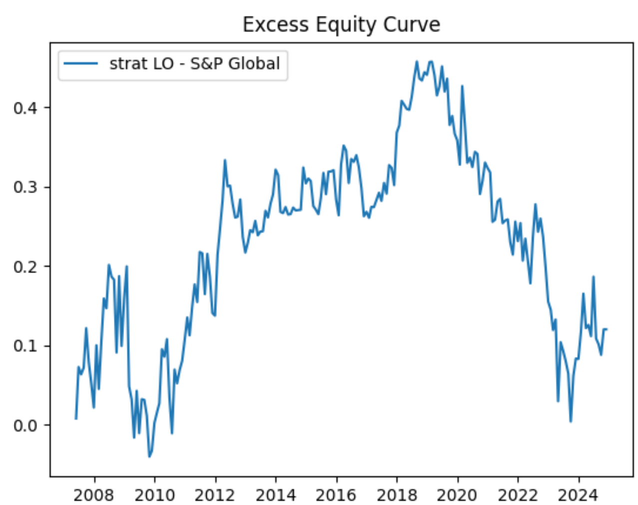


Figure 26: Excess Equity curve of a 10-month csmomsr strategy using excess gics3 over gics3 momentum. Portfolios are formed with percentiles (5,95).

2.2.4 Some additional backtesting results on cross-sectional momentum strategies: a simple regime approach

[**These results are generated with the script main29.**] The performance of cross-sectional momentum (csmom) strategies is time-varying. In Time-Varying Momentum Profitability (2010), the authors highlight that csmom strategies are more profitable during positive market regimes and low volatility regimes. The market is said to be in a positive state if its 36-month performance is positive. The market is said to be in a low volatility regime if its 12-month volatility is smaller than its 36-month volatility. The main table of the paper is shown below as reference.

Table 1: Momentum Profits, Market States and Market Volatility

Monthly returns of the momentum strategy are from the Ken French data library. Stocks are sorted into deciles using returns over the ranking period from month t-12 to month t-2, where month t is the holding period. The momentum profits or payoffs are measured by the holding month return differences between the equal-weighted portfolios of the winner and loser deciles. The monthly average momentum profits, denoted by MOM, are reported for the sample period and two subperiods. Market state is defined by the lagged three-year market return. Negative (positive) market states, or down (up) markets, are the times when the lagged three-year market return is negative (positive). A month is of high (low) volatility if the lagged 12-month market volatility is larger (smaller) than the lagged 36-month market volatility. An independent two-way sort for the months in the sample is carried out. Every month in the period is put into one of four categories depending whether the market state is positive or negative and whether the market volatility is high or low. The average monthly payoff for each of the four categories is reported. Robust t-statistics (in parentheses) are provided. All the payoffs are in percentage terms, with the monthly observations ranging from August 1929 to July 2009. The data and construction of the momentum strategy are used throughout all the other tables.

MOM	Positive Market State		Negative Market State	
	High Vol	Low Vol	High Vol	Low Vol
		August 1929 - July 2009		
0.79	0.89	1.56	−3.01	−1.29
(3.60)	(4.40)	(9.27)	(−1.94)	(−0.94)
		August 1929 - July 1969		
0.63	0.75	1.45	−3.25	−2.28
(1.95)	(3.31)	(5.97)	(−1.87)	(−1.75)
		August 1969 - July 2009		
0.95	1.00	1.70	−2.86	1.16
(3.33)	(3.29)	(8.75)	(−1.33)	(2.56)

Figure 27: Momentum Profits, Market States and Market Volatility.

Building on these results, we applied this "regime" momentum strategy on our gics3 universe. Due to the "regimes" specification, we invest only if we are in a positive and low volatility market. Hence, at some periods we are not invested. Instead of not being invested during these periods, we invest in the market. We note that, until now, this is the strategy with the highest SR standing at 0.84. The excess Equity Curve helps us seeing the periods where we have returns for our strategy: this is the periods corresponding to the non-plateau curve.

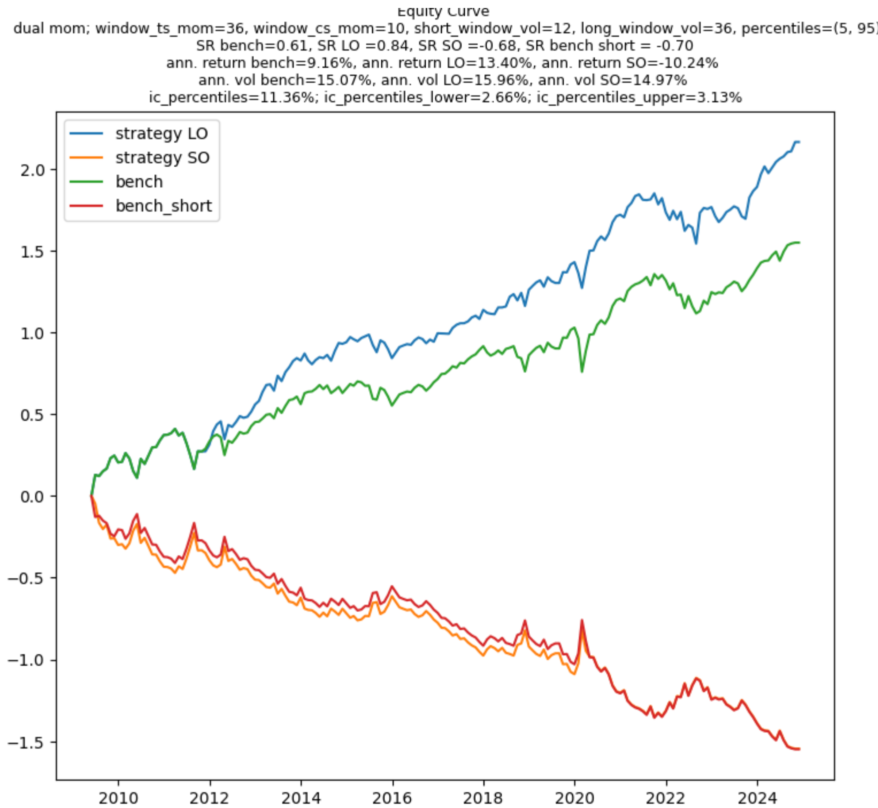


Figure 28: Equity Curve of Regime csmom strategy. Portfolios are formed based on (5,95) percentiles.

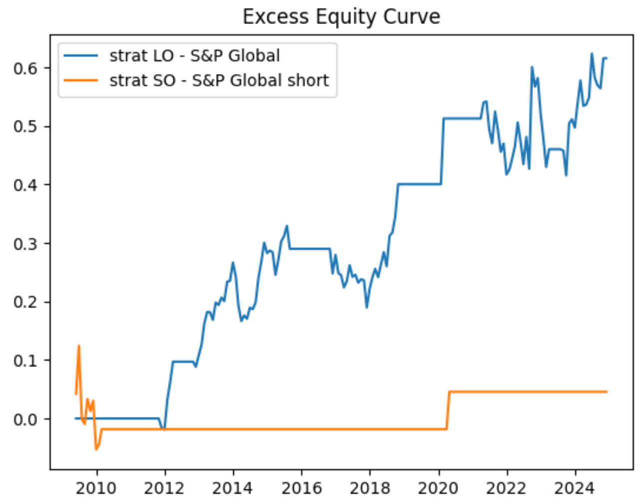


Figure 29: Excess Equity Curve of Regime csmom strategy. Portfolios are formed based on (5,95) percentiles.

The excess equity chart displays some interesting results: we can see an uptrend excess equity curve which is a sign of persistent and increasing outperformance. We can also note that the plateau parts of the curve represents negative market state and high volatility regime and hence, we invest in the market (h=0.0). Thus, this strategy also "indirectly" times the market: when we are in a negative market state (past 36-month performance negative) with high volatility (past 36-month volatility greater than its shorter 12-month volatility) we set our timing indicator h to 0.0. To allow comparison with a 100% time invested benchmark, instead of non-investing when h=0.0, we invest in the market. When h=1.0 indicating that we are in both a positive market state and low volatility regime, we select the assets based on a 10-month cross-sectional momentum strategy at the (5,95) percentiles. In this sense, this strategy managed to answer our problematic, which was "how to identify abnormal trends in assets?". Indeed, here, the abnormality is defined as the positive market and low volatility regimes.

We can try to add an abnormality layer to try to invest in the gics3 indices that outperforms their corresponding gics1 counterparts. To do that, the strategy is exactly the same as the one described above except for the input: we do not plug the gics3 monthly returns in the signal function but instead the excess returns defined as gics3 returns minus the corresponding gics1 returns.

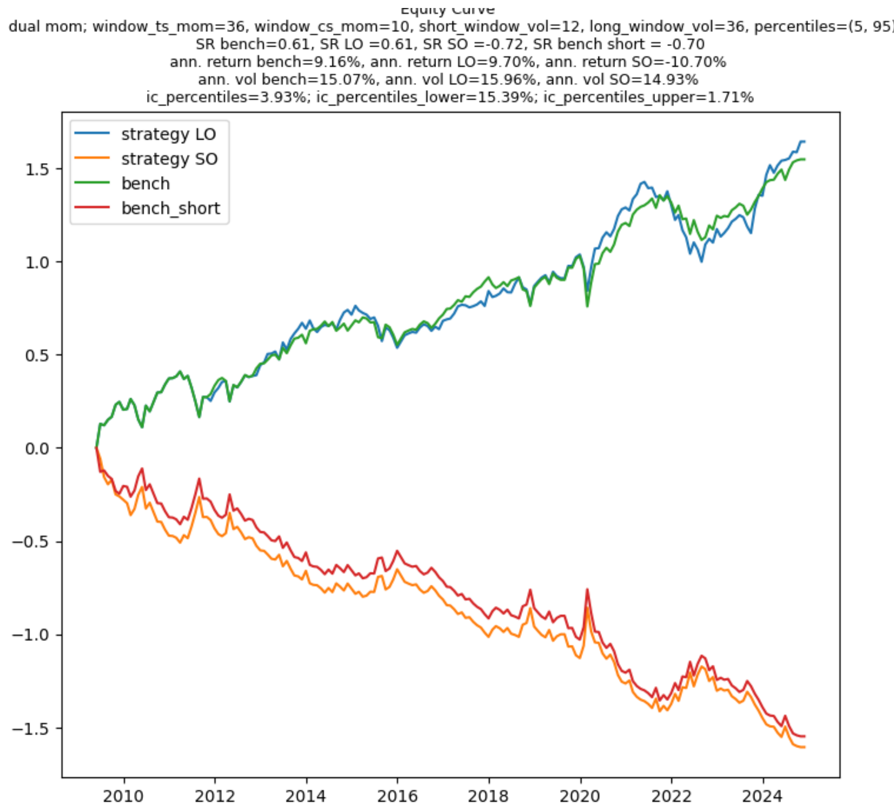


Figure 30: Equity Curve of Regime csmom strategy with excess returns as inputs. Portfolios are formed based on (5,95) percentiles.

Commenting briefly these results, we can see that annualized return and annualized volatility worsen and so the SR. We do not only decrease the outperformance versus the benchmark, we completely erase it as the SR of the benchark and our long portfolio are equal. We conclude that the Regime csmom strategy is best when signals are computed from gics3 returns.

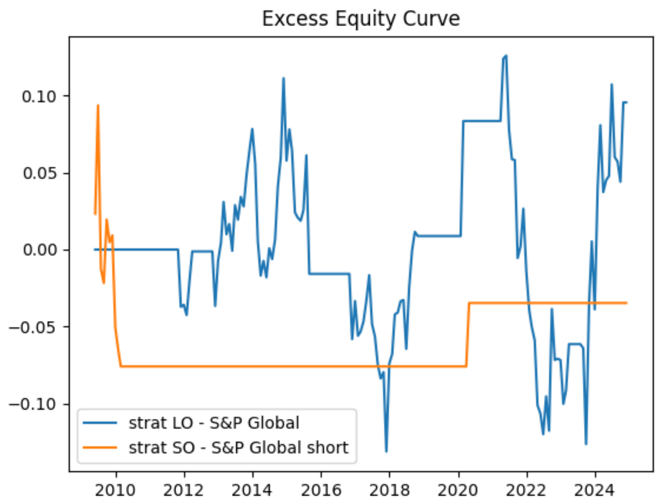


Figure 31: Excess Equity Curve of Regime csmom strategy with excess returns as inputs. Portfolios are formed based on (5,95) percentiles.

This strategy struggles to outperform the benchmark.

2.2.5 The Mahalanobis distance-based strategy: setting the path to market timing

[**These results are generated with the script main15.**] In this subsection, we'll explore the market timing strategy. The intuition of this strategy is to identify periods of "deviations" of assets' excess returns. These deviations will be characterized based on a statistical test built on the Mahalanobis distance. The result of this test is a binary vector h equals to 1.0 if the distance at time t is characterized as divergent. Once we identified the divergent periods, at these dates, we'll compute for each asset, its signed (to have the "sens") contribution to the Mahalanobis distance. From this step, the assets selection procedure is the same as the one above (long top percentiles). Hence, we're left with a "cross-sectional market timing strategy" in the sense that the statistical test serves as a market timing indicator and the assets are selected cross-sectionally.

Given the same dataframe (df) as in the above section, the procedure is given below:

- 1) On a rolling basis, we'll split the rolling window into two sub windows: the calibration window and the test window.
- 2) On the calibration window, the goal is to compute distances to estimate their mean and standard deviation which will be useful to normalize the distances in the test window. The Mahalanobis distance is computed between two matrices: x and mu. The matrix mu has the same number of columns as df (which is the number of assets) and its number of rows is equal to the paramater nb_recent_periods. The matrix x has the same number of columns as mu and its number of row is the "remaining" rows available in the calibration window, excluding mu' rows.
- 3) Once x and mu are defined, we can compute the Mahalanobis distance between them. We repeat this procedure on a rolling basis, inside the window calibration.
- 4) Once we have our several distances, we can compute their mean and standard deviation.
- 5) Now, we can perform the statistical test: on the test window, we select x and mu in the same vein as above and we compute their Mahalanobis distance. We normalize this distance thanks to the previously

estimated mean and standard deviation. We compute the p-value of this normalized distance using a standard normal distribution (see appendix why we use the standard normal distribution).

- 6) If the p-value is less than alpha, we set the binary vector h to 1.0 at time t, indicating a divergent period.
- 7) At the divergent periods identified, we compute the signed contribution of each asset to the distance and we long the top percentiles.
- 8) We repeat this procedure until reaching the last date of df.

We'll call this strategy the CrossSectionalMahalanobisTimed strategy (when h=0.0 we are invested in the market). Considering h=1.0 at all dates (no market-timing), we have a baseline strategy to assess our timing ability which we'll call the CrossSectionalMahalanobis strategy. We'll present the results first for the (non-timed strategy) CrossSectionalMahalanobis on excess gics3 returns (minus gics1 returns) and the same strategy but using gics3 retuns as input. Secondly, we'll present the results of the (timed strategy) CrossSectionalMahalanobisTimed both for excess gics3 returns and gics3 returns.

First, results below are the CrossSectionalMahalanobis strategy using excess gics3 returns.

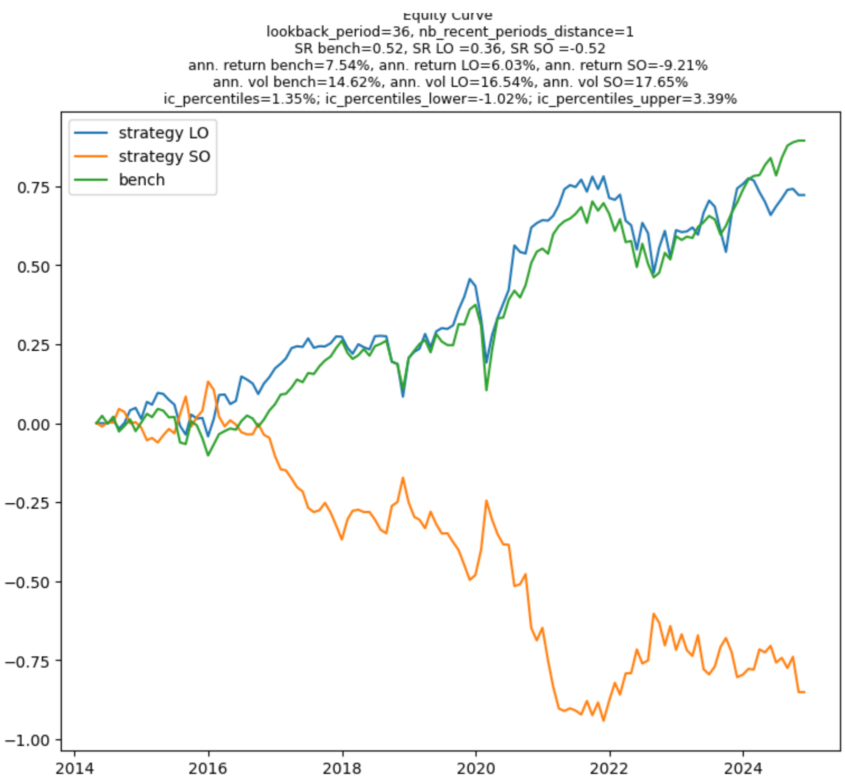


Figure 32: Equity Curve of the CrossSectionalMahalanobis strategy using excess gics3 returns. Portfolios are formed based on (5,95) percentiles.

We used a lookback period of 36 months and nb_recent_periods_distance of 1 month. From the above plot, we can see that our strategy is above its benchmark until late 2023. Unfortunately, the strategy exhibits a lower annualized return and volatility leading to a lower sharpe.

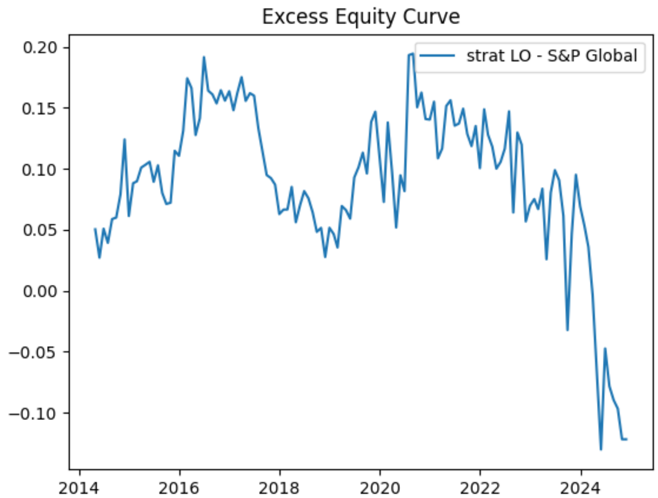


Figure 33: Excess Equity Curve of the CrossSectionalMahalanobis strategy using excess gics3 returns. Portfolios are formed based on (5,95) percentiles.

Always working with the CrossSectionalMahalanobis strategy but presenting results when using gics3 returns. In terms of SR and compared to CrossSectionalMahalanobis strategy when using excess gics3, we can see an improvement but if we look at the excess equity curve, we are underperforming the benchmark until mid 2020.

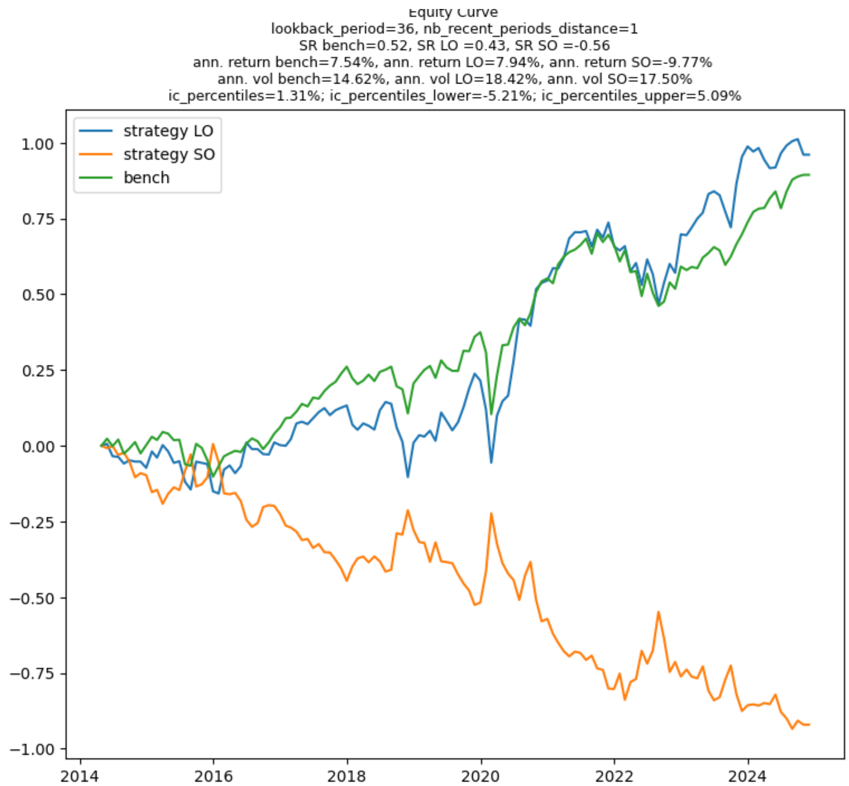


Figure 34: Equity Curve of the CrossSectionalMahalanobis strategy. Input is gics3 returns. Portfolios are formed based on (5,95) percentiles.

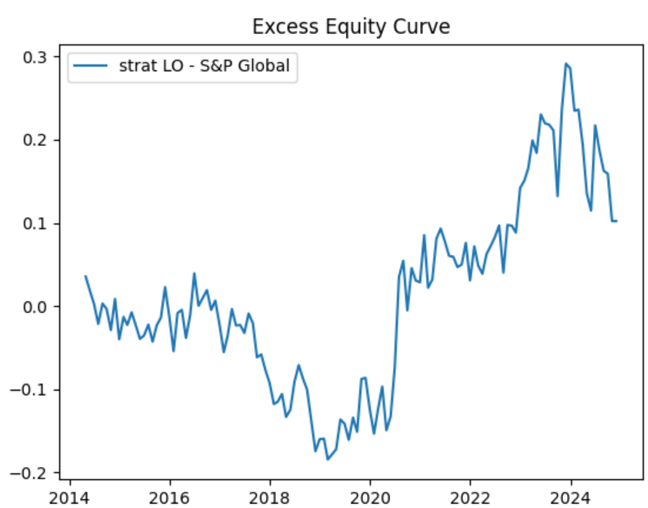


Figure 35: Excess Equity Curve of the CrossSectionalMahalanobis strategy. Input is gics3 returns. Portfolios are formed based on (5,95) percentiles.

To conclude on the CrossSectionalMahalanobis strategy, it is difficult to say whether we must use excess gics3 returns or gics3 returns as inputs. Comparatively to the benchmark, I would not say that this strategy is able to beat it.

[**These results are generated with the script main25.**] Secondly, we'll see the results of the CrossSectionalMahalanobisTimed strategy using both excess gics3 and gics3 returns. Below are displayed results of the CrossSectionalMahalanobisTimed strategy with inputs being excess gics3. For the specified parameters, we can see that the equity curve of our strategy is above the one of the benchmark except for 2020-2022 which corresponds to the recovery post COVID-19. Our strategy delivers both a better annualized return and annualized volatility, leading to a +19.64% in the Sharpe Ratio.

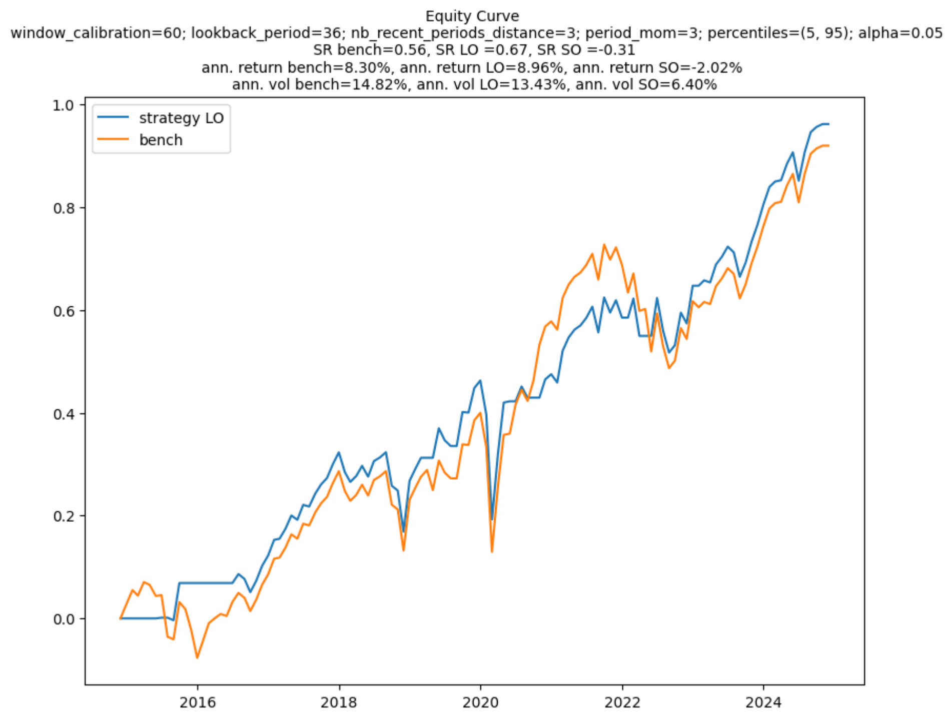


Figure 36: Equity Curve of the CrossSectionalMahalanobisTimed strategy. Input is excess gics3. Portfolios are formed based on (5,95) percentiles.

With the excess equity curve, we can spot the dates where our strategy is active (timing on, h=1.0). These periods corresponds to the non-plateau parts of the curve. The plateau parts of the curve means that there is no differences between our strategy returns and the returns of the benchmark, which corresponds to the dates where h=0.0.

Even if the gains are small versus the benchmark and that there is no uptrend excess equity curve, based on the SR and excess equity curve (compared to the CrossSectionalMahalanobis which is the non-timed version of this strategy and on the benchmark) we can conclude (little) timing ability.

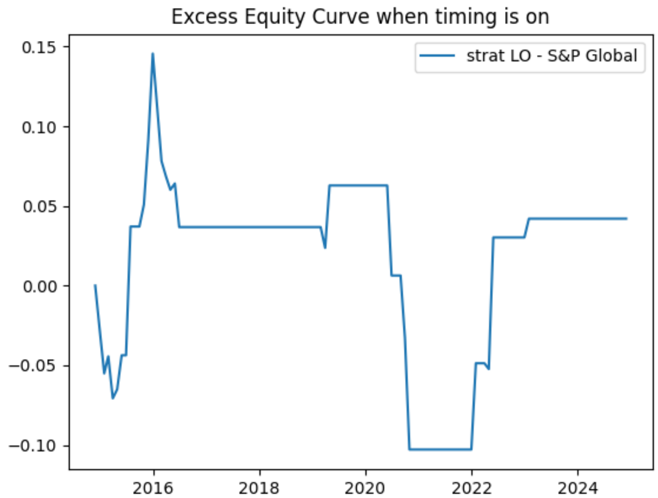


Figure 37: Excess Equity Curve of the CrossSectionalMahalanobisTimed strategy. Input is excess gics3. Portfolios are formed based on (5,95) percentiles.

Lastly, we present below the same strategy using gics3 returns as input instead of excess gics3 returns (minus gics1 returns).

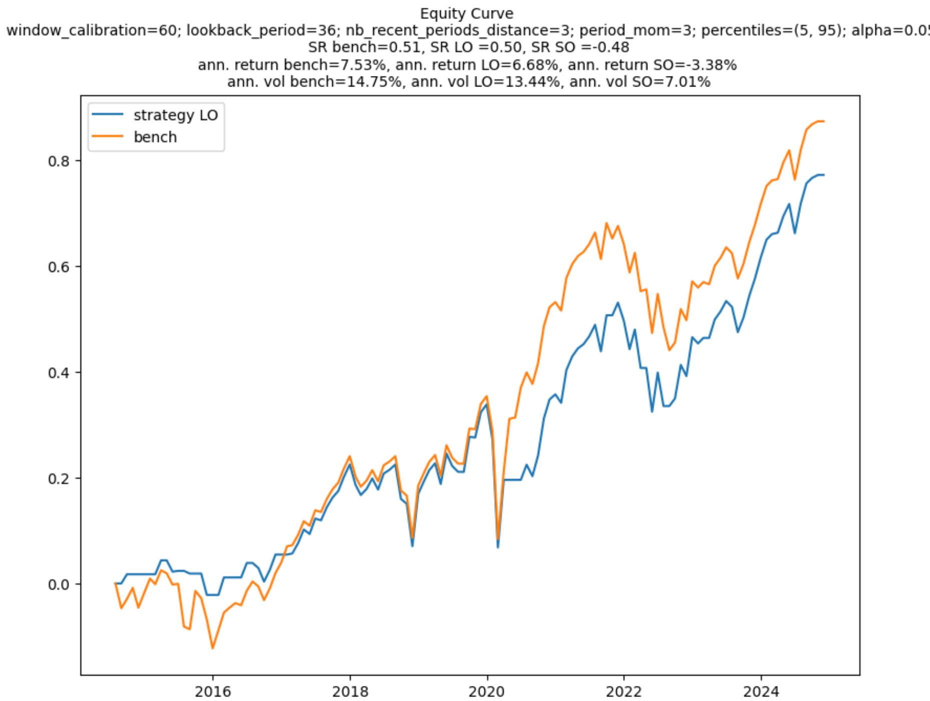


Figure 38: Equity Curve of the CrossSectionalMahalanobisTimed strategy. Input is gics3 returns. Portfolios are formed based on (5,95) percentiles.

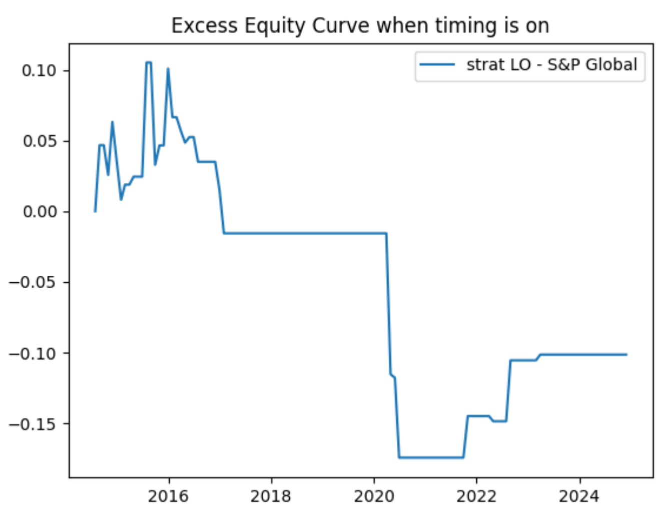


Figure 39: Excess Equity Curve of the CrossSectionalMahalanobisTimed strategy. Input is gics3 returns. Portfolios are formed based on (5,95) percentiles.

To conclude on the CrossSectionalMahalanobisTimed strategy, it is better using excess_gics3 returns as inputs.

To overall conclude on both the CrossSectionalMahalanobis and CrossSectionalMahalanobisTimed strategies, we can say that we have (little) timing ability, and that the best strategy for doing that is the CrossSectionalMahalanobisTimed when using excess gics3 returns.

2.2.6 Strategies, backtests summaries and conclusion

We summarize all the strategies studied in this section:

- Cross-Sectional Momentum
  - Momentum
    - gics3 returns
    - excess gics3 returns (minus gics1 returns)
  - Sharpe Ratio
    - gics3 returns
    - excess gics3 returns (minus gics1 returns)
- Regime Cross-Sectional Momentum
  - gics3 returns
  - excess gics3 returns (minus gics1 returns)
- Cross-Sectional Mahalanobis
  - gics3 returns
  - excess gics3 returns (minus gics1 returns)
- Cross-Sectional Mahalanobis Timed
  - gics3 returns
  - excess gics3 returns (minus gics1 returns)

We also summarize the results of these strategies based on their SR and the delta SR which is the SR of the strategy - the SR of the benchmark in the main concluding table below: [TO DO: ALIGN ALL STRATEGY DATES TO EASE COMPARISON]

Strategy			SR_strat	SR_bench	delta_SR
Cross-Sectional Momentum					
	Momentum	gics3	0.53	0.29	**0.24**
		excess gics3	0.43	0.29	0.14
	Sharpe Ratio				
		gics3	0.41	0.29	0.12
		excess gics3	0.31	0.29	0.02
Regime Cross-Sectional Momentum		gics3	0.84	0.61	0.23
		excess gics3	0.61	0.61	0
Cross-Sectional Mahalanobis		gics3	0.43	0.52	-0.09
		excess gics3	0.36	0.52	-0.16
Cross-Sectional Mahalanobis Timed		gics3	0.56	0.51	0.05
		excess gics3	0.71	0.56	0.15

Strategy		ic_median
Cross-Sectional Momentum	Momentum gics3	4.76%
	Momentum excess gics3	0.58%
	Sharpe Ratio gics3	5.68%
	Sharpe Ratio excess gics3	2.49%
Regime Cross-Sectional Momentum	gics3	6.77%
	excess gics3	6.67%
Cross-Sectional Mahalanobis	gics3	-2.93%
	excess gics3	0.29%
Cross-Sectional Mahalanobis Timed	gics3	3.91%
	excess gics3	-3.45%

To overall conclude on this part, we can say that market timing strategies are difficult. Indeed, the best strategy in terms of delta SR is the Cross-Sectional Momentum applied to the gics3 returns. Nevertheless, market timing is not impossible as shown by our Regime Cross-Sectional Momentum strategy applied to the gics3 returns which leads a delta SR of 0.23, very close to the one of the Cross-Sectional Momentum standing at 0.24. Finally, the strategy Cross-Sectional Mahalanobis Timed applied to excess gics3 returns gives more modest results albeit positive as highlighted by the delta SR of 0.11.

3 Asset prices' reactions to macroeconomic, factor and thematic changes

4 Identifying Abnormal Trends from alternative data

5 Conclusion

References

[1] Tung-Lam Dao, *Momentum Strategies with L_1 Filter*, 2014

[2] James D. Hamilton, *Why you should never use the Hodrick-Prescott filter*, NBER working paper 23429, 2017

[3] Khaled H. Hamed, A. Ramachandra Rao, *A modified Mann-Kendall trend test for autocorrelated data*, Journal of Hydrology, 1997

[4] R. K. Jaiswal, A. K. Lohani, H. L. Tiwari, *Statistical Analysis for Change Detection and Trend Assessment in Climatological Parameters*, 2015

[5] Kevin Q. Wang and Jianguo Xu, *Time-Varying Momentum Profitability*, 2010

[6] G. Antonacci, *Risk Premia Harvesting Through Dual Momentum*, 2017

6 Appendix

6.1 Appendix: Why using Ledoit-Wolf shrinkage estimation of the VCV matrix

Assessing Estimation Error Procedure

To empirically assess the estimation accuracy of the sample covariance and Ledoit-Wolf estimation we can follow this procedure:

- For n = 1 to N, with n being the sample size:
  - For i = 1 to I, with i being the number of simulations:
    - Simulate a "true" VCV matrix with parameters known, for example a multivariate normal distribution.
    - Compute the Frobenius and Spectral Error between both the estimated matrix via Ledoit-Wolf and sample covariance.
  - Compute the average errors across simulations and go to the next loop, n = n+1

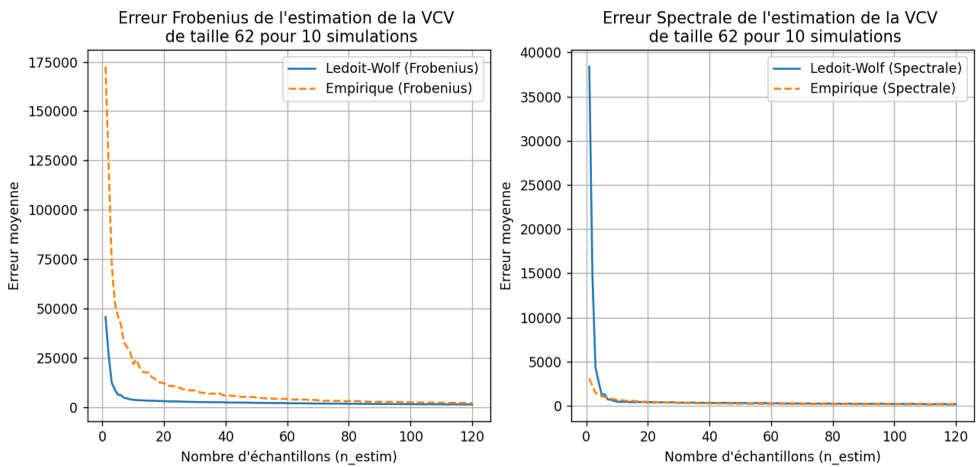


Figure 40: Frobenius and Spectral Error for the estimation of the VCV matrix depending on the type of estimation (Ledoit-Wolf or Empirical) and the number of sample data points.

Given a martix of size (n_estim, 62) we can see that for n_estim and the Frobenius Error, Ledoit-Wolf (LW) estimation is more accurate up to approximately n_estim = 80 than the sample covariance estimation. Although the difference is not that much when using Spectral Error, LW still estimate better the VCV matrix. It's why in our testings, we'll use the LW to estimate the VCV matrix with greater precision, especially if we want to use shorter sample size (time-series dimension) given our number of variables which is 62 and corresponds to the gics3 level sector returns.

Estimation of the Covariance Matrix: Ledoit-Wolf Shrinkage and Error Metrics

Let S denote the sample covariance matrix computed from n i.i.d. observations $X_1, \ldots, X_n \in \mathbb{R}^p$, and let Σ be the true covariance matrix.

Ledoit-Wolf Shrinkage Estimator

The Ledoit-Wolf shrinkage estimator is defined as:

$$\hat{\Sigma}_{\text{LW}} = (1 - \delta)S + \delta F$$

where:

- S is the sample covariance matrix,
- F is the shrinkage target matrix, typically chosen as

  $$F = \frac{\text{Tr}(S)}{p} I_p$$

  with $\text{Tr}(S)$ denoting the trace of S and I_p the $p \times p$ identity matrix,

- $\delta \in [0, 1]$ is the shrinkage intensity, optimally determined to minimize the mean squared error between $\hat{\Sigma}_{\text{LW}}$ and Σ.

Error Metrics

To assess the estimation quality, two matrix norms are considered:

Frobenius Norm Error The Frobenius norm error is defined as:

$$\text{Error}_F = \|\hat{\Sigma} - \Sigma\|_F = \sqrt{\sum_{i=1}^{p} \sum_{j=1}^{p} \left(\hat{\Sigma}_{ij} - \Sigma_{ij}\right)^2}$$

Spectral Norm Error The spectral norm error is defined as:

$$\text{Error}_{\text{spec}} = \|\hat{\Sigma} - \Sigma\|_2$$

where $\|\cdot\|_2$ denotes the operator norm (largest singular value). For symmetric matrices, this simplifies to:

$$\|\hat{\Sigma} - \Sigma\|_2 = \max_{1 \le i \le p} \left|\lambda_i \left(\hat{\Sigma} - \Sigma\right)\right|$$

with $\lambda_i(\cdot)$ the eigenvalues of the matrix.

6.2 Appendix: Mahalanobis Distance and Component Contributions

Let $x \in \mathbb{R}^p$ be a vector of observations, and $\mu \in \mathbb{R}^p$ the mean vector. Let $\Sigma \in \mathbb{R}^{p \times p}$ be the covariance matrix, assumed to be positive definite.

Mahalanobis Distance The Mahalanobis distance between x and μ is defined as:

$$d_M(x, \mu) = \sqrt{(x - \mu)^\top \Sigma^{-1} (x - \mu)}$$

where Σ^{-1} denotes the inverse of the covariance matrix.

Component Contributions

Define the deviation vector:

$$\Delta = x - \mu$$

The individual component contributions to the squared Mahalanobis distance are given by:

$$\text{contributions} = \Delta \odot (\Sigma^{-1} \Delta)$$

where $\odot$ denotes the element-wise (Hadamard) product.

Thus, the total squared distance can be decomposed as:

$$d_M(x, \mu)^2 = \sum_{i=1}^{p} \text{contributions}_i$$

Percentage Contributions

The percentage contribution of each component is defined as:

$$\text{contributions\_percent}_i = \frac{|\text{contributions}_i|}{\sum_{j=1}^{p} |\text{contributions}_j|} \times 100$$

6.3 Appendix: Mahalanobis distances distribution

For example, we provide below the rolling Mahalanobis distance overtime. We can see spikes at disturbed times which corresponds to economic events (2008 financial crisis, 2020 COVID19)

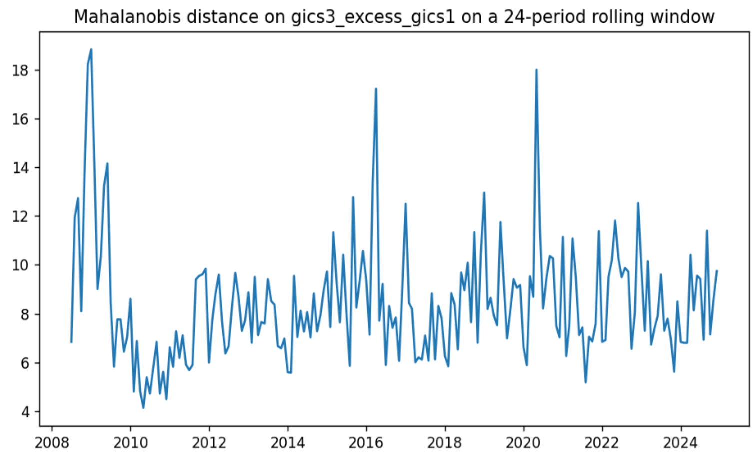


Figure 41: 24-period Rolling Mahalanobis Distance applied to monthly excess gics3 returns.

We can also see that approximately 45% of the disturbance comes from the top5 contributors to the distance out of 62 indices.

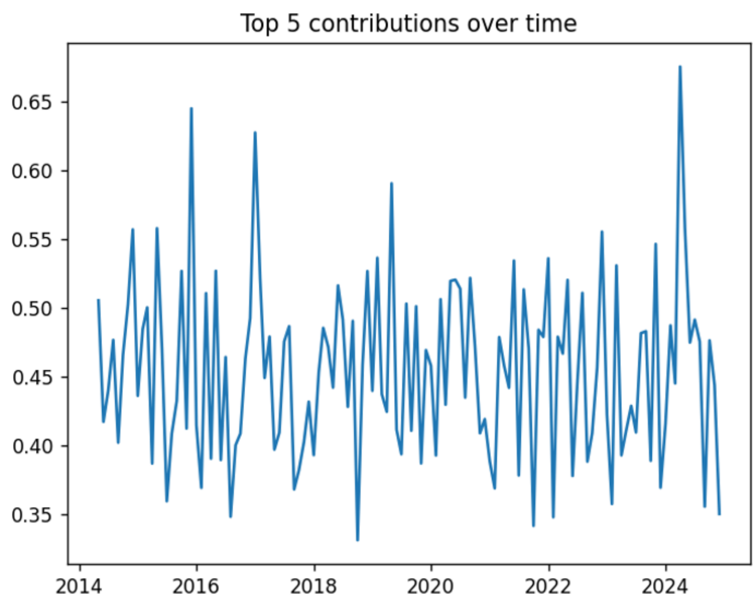


Figure 42: Contributions in % to the above rolling Mahalanobis Distance of the top5 contributors.

Finally, for simplicity and given the plot below, we use the normal distribution as the distances distribution. Note that we conducted more formal approach, like the Kolmogorov-Smirnoff test and the test was rejected for the distributions below meaning that mahalanobis distances do not exactly follow an exponential, log-normal or normal distribution.

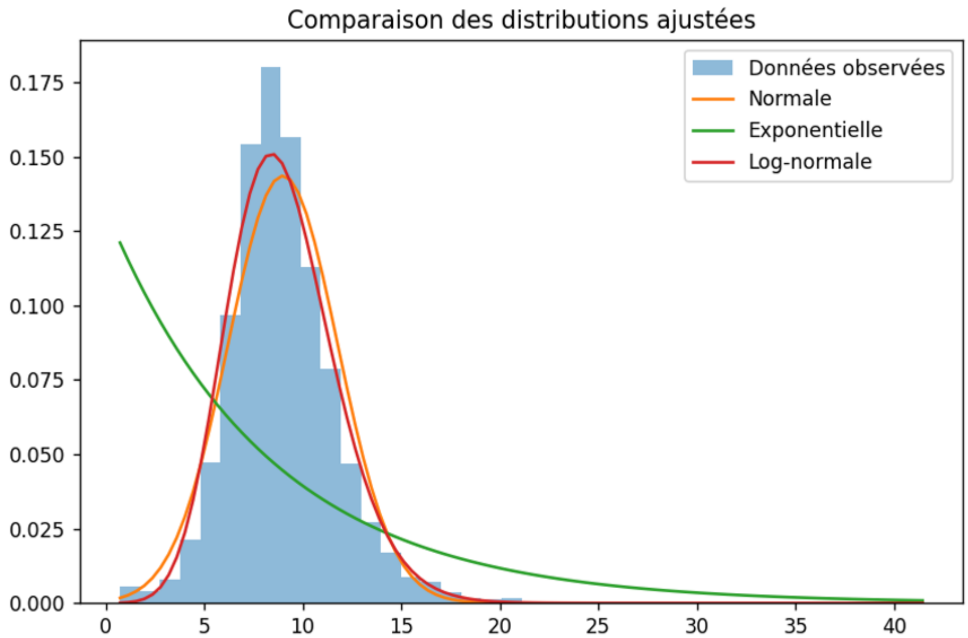


Figure 43: Empirical, exponential, log-normal and normal distributions of the mahalanobis distances.

6.4 Appendix: Should we filter prices?

In Momentum Strategies with L_1 Filter (2014), Tung-Lam Dao proposes some advantages of using filters (especially L_1 Filter) when dealing with daily prices. Building on this idea, we wanted to test if when using filtered monthly prices using L_2 Filter (Hodrick-Prescott, which is faster to compute as there is a closed-form formula unlike the L_1 filter) this allows us to have better performance on our strategies.

We provide below an example of the HP filter applied on a rolling-basis to avoid forward-looking bias. We conclude that for our set of data and frequency, the L_2 Filter did not allow to improve the Sharpe Ratio of our strategy.

Figure 44: HP Filter applied to a monthly gics3 index prices.