The Cross-Section of Stock Returns: Rank-Based, Linear, and Non-Linear Models

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Summary

Introduction and motivations (1)

- Initial Research Project at Candriam: Mid-Term Abnormal Trends Detection
  - price-based
  - non-price based
  - alternative data-based
- Conclusions on price-based (Country and Sector Indices) approach:
  - Price-based leads to poor results (except for momentum and Hamed-Rao Mann-Kendall test)
  - Length of the outperformance ("abnormality") is short (1-3 months)
  - Market timing is more difficult that stock selection
- Project Update: price-based market timing (time-series) –> fundamental data-based stock selection (cross-section)

Introduction and motivations (2)

- Asset Pricing: Understanding the cross-section of expected asset (equity) returns
- Foundation: Capital Asset Pricing Model (**CAPM**) of **Sharpe (1964)** and **Lintner (1965)**
- Academic to Industry Transfer 1: **first index fund** (Bogle, Vanguard 500 fund, 1976)
- Questioning (**CAPM, EMH**):
  - Market risk is not the only driver of excess returns –> Arbitrage Pricing Theory **APT (Ross, 1976)**.
  - Discovering of anomalies –> **Value** (Basu, 1977) and **Size** (Banz, 1981)
- Development:
  - multi-factor models: **Fama and French (1993) three-factor model**, and **five-factor extension (Fama and French, 2015)**, incorporating **size, value, profitability**, and **investment** factors.
- Racing: "**factor zoo**" **Cochrane** (2011).

Introduction and motivations (3)

- Academic to Industry Transfer 2: **Smart Beta** (Research Affiliates, WisdomTree) circa 2011
- Recent Advances:
  - **ML –> Empirical Asset Pricing via Machine Learning** (Gu, Kelly and Xiu, 2019)
  - **DL –> Autoencoder Asset Pricing Models** (Gu, Kelly and Xiu, 2021)
  - ML and DL –> model **complex** and **non-linear relationships** between firm characteristics and expected returns.
- Aim of our study: Empirically assess, in the US (Russell 1000), which type of models lead to the best economic advantage (SR) in constructing factor portfolios
  - **Rank-based** (univariate sorts, sequential filtering)
  - **Linear-Models** (LASSO, Ridge, EN)
  - **Non-Linear Models** (RF, NN)

Data and Feature Engineering (1)

- Fundamental data comes from Bloomberg (BQL and API) for the Russell 1000 (RIY) Index.
- It consists of two sets of fundamental variables: i) "raw fields" that can be directlty retrieved from Bloomberg or with little transformations (change, ratios, volatility) and ii) aggregated scores (mean of ranks).
- Details on the computations for the transformed variables are left in the appendices.
- On the next slides, you can find the list of all the variables.

Data and Feature Engineering (2)

Raw Fields	Aggregated Scores
FREE_CASH_FLOW_MARGIN	QUALITY_SCORE
ROC_WACC_RATIO	GROWTH_SCORE
RETURN_COM_EQY	MOM_SCORE
NET_DEBT_TO_EBITDA	LOW_VOL_SCORE
CUR_MKT_CAP	SENTIMENT_SCORE
PX_TO_CASH_FLOW	VALUE_SCORE
PE_RATIO	TOTAL_SCORE
PX_TO_BOOK_RATIO	TOTAL_SCORE_MAHALANOBIS
HEADLINE_EV_TO_SALES	
HEADLINE_DVD_YIELD	
CF_ISSUE_COM_STOCK	
FCFE_PAYOUT	
OCF_OVER_TOT_ASSETS	
SALES_PER_SHARE	
FREE_CASH_FLOW_EQUITY_PER_SHARE	
FCFE_VARIABILITY	
ROA_CHANGE	
CUR_RATIO_CHANGE	
ASSET_TURNOVER_CHANGE	
LT_DEBT_TO_TOT_ASSET_CHANGE	
GROSS_MARGIN_CHANGE	
SALES_PER_SHARE_CHANGE	
FREE_CASH_FLOW_EQUITY_PER_SHARE_CH	
TARGET_PRICE_CHANGE	
EPS_CHANGE	
CONSENSUS_UPSIDE	

Table: Raw financial ratios and aggregated scores.

Data and Feature Engineering (3)

Raw Fields	Aggregated Scores
MOMENTUM_3M	
MOMENTUM_6M	
MOMENTUM_12M	
BETA	
VOLATILITY_1Y	
BUYBACK_YIELD	
EARNINGS_REVISION	
EARNINGS_SURPRISE	
NORMALIZED_ROA	
IDIOSYNCRATIC_VOLATILITY_1Y	
ALPHA	
NEW_EQ_ISSUANCE_LTM	
PFS	
IDIO_SHARPE	

Table: Raw financial ratios and aggregated scores.

- We then, computed cross-sectional z-scores and winsorized them at the 2nd and 98th percentiles.
- We also put a negative sign on the following ratios because economically higher means "bad" for the companies:

Data and Feature Engineering (4)

Table: List of Z-Scored Winsorized Financial Variables with negative sign

Variable Name
NET_DEBT_TO_EBITDA_ZSWINSO
CUR_MKT_CAP_ZSWINSO
PX_TO_CASH_FLOW_ZSWINSO
PE_RATIO_ZSWINSO
PX_TO_BOOK_RATIO_ZSWINSO
HEADLINE_EV_TO_SALES_ZSWINSO
CF_ISSUE_COM_STOCK_ZSWINSO
FCFE_VARIABILITY_ZSWINSO
LT_DEBT_TO_TOT_ASSET_CHANGE_ZSWINSO
BETA_ZSWINSO
VOLATILITY_1Y_ZSWINSO
IDIOSYNCRATIC_VOLATILITY_1Y_ZSWINSO

Ranked-Based Strategies - Univariate Sorts

- The Ranked-Based Long-Only (LO) strategies consist in going long the top n% of the ranked stocks according to a variable. This gives us the signals.
- The portfolios are constructed with an Equally-Weighting (EW) Scheme. Other Weighting Scheme (WS) may be of interest but this is beyond the scope of this study.
- We also performed "grid search" for the strategies' parameters such as the n%, AllIndustries or BestInIndustries and rebalancing frequencies but to avoid overhelming the presentation we'll just compare one strategy setting across all models: top 10% (deciles), selection of the stocks across the entire universe (AllIndustries) and rebalanced monthly assuming no TCs.

Ranked-Based Strategies - Univariate Sorts

- We run the backtests for univariate sorts on the 46 (we dropped NEW_EQ_ISSUANCE_LTM as this is a binary variable and BUYBACK_YIELD due to poor coverage) fundamental ratios. Start date is 31-01-2015, end date 31-05-2025.
- All the results, as well as plots and performance metrics are accessible through this [Excel Files] and this [folder] but to avoid overloading the presentation, we'll focus on the top 10 portfolios according to 2 metrics: the Information Ratio (IR) and the excess Sharpe Ratio (eSR).

Ranked-Based Strategies - Univariate Sorts - Results IR

top10	ir_EW
SALES_PER_SHARE_CHANGE_ZSWINSO_D8_AllIndustries_monthly	1.21
FREE_CASH_FLOW_MARGIN_ZSWINSO_D2_AllIndustries_monthly	1.20
FREE_CASH_FLOW_MARGIN_ZSWINSO_D1_AllIndustries_monthly	1.16
GROWTH_SCORE_ZSWINSO_D7_AllIndustries_monthly	1.10
FREE_CASH_FLOW_MARGIN_ZSWINSO_P5_AllIndustries_monthly	1.04
FCFE_PAYOUT_ZSWINSO_D5_AllIndustries_monthly	0.94
SALES_PER_SHARE_CHANGE_ZSWINSO_D6_AllIndustries_monthly	0.90
QUALITY_SCORE_ZSWINSO_P5_AllIndustries_monthly	0.87
FREE_CASH_FLOW_EQUITY_PER_SHARE_CHANGE_ZSWINSO_D7_AllIndustries_monthly	0.86
QUALITY_SCORE_ZSWINSO_D1_AllIndustries_monthly	0.85

Figure: Top 10 factors by IR (against Russell 1000 EW)

- The top 10 performing "factors" are mainly ratios related to profitability/quality (and growth in a lesser extent).

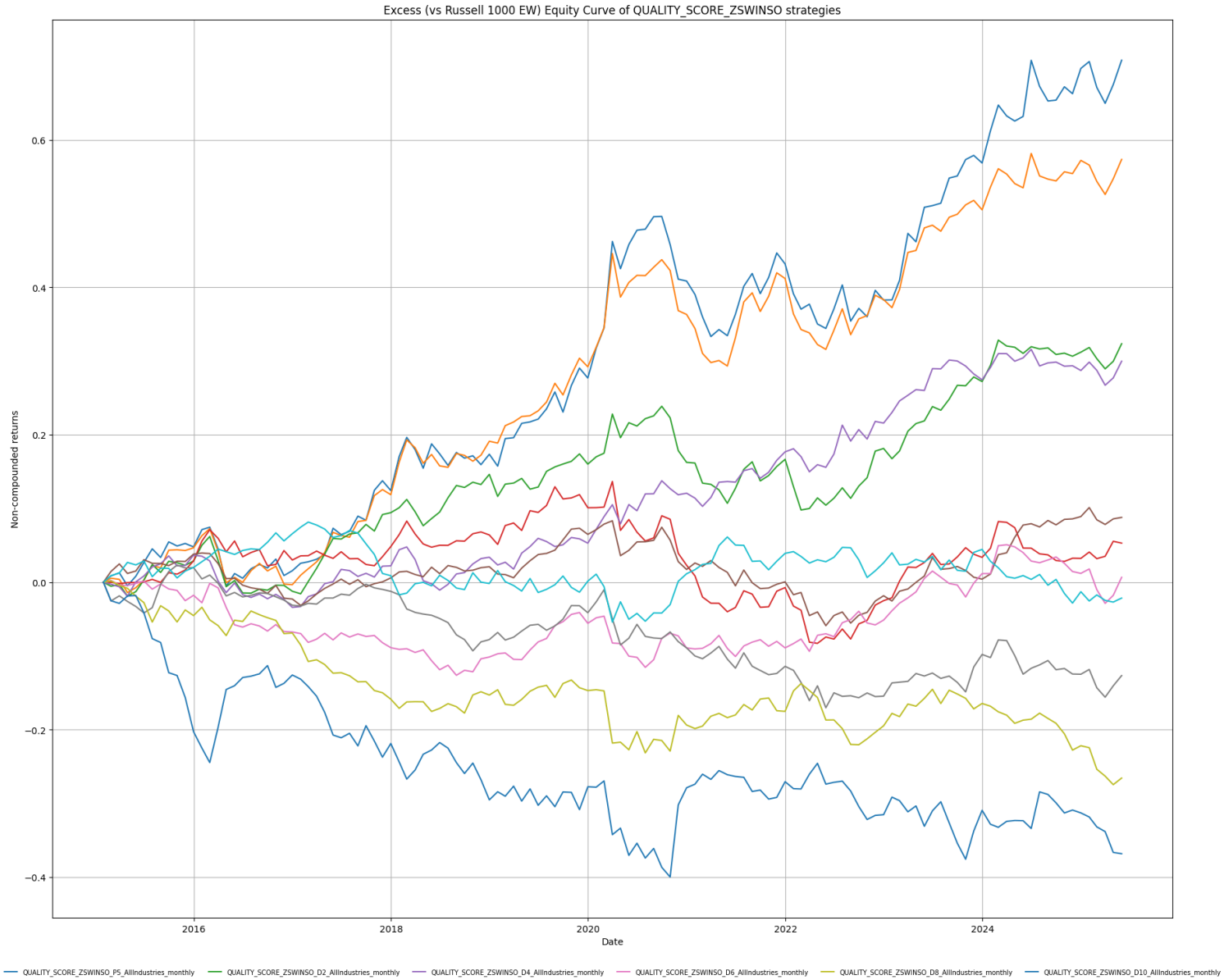
Ranked-Based Strategies - Univariate Sorts - Results eSR

	top_excess_ann_sr_EW
QUALITY_SCORE_ZSWINSO_P5_AllIndustries_monthly	0.49
QUALITY_SCORE_ZSWINSO_D1_AllIndustries_monthly	0.45
FREE_CASH_FLOW_MARGIN_ZSWINSO_D2_AllIndustries_monthly	0.42
FREE_CASH_FLOW_MARGIN_ZSWINSO_D1_AllIndustries_monthly	0.41
ROC_WACC_RATIO_ZSWINSO_P5_AllIndustries_monthly	0.38
IDIO_SHARPE_ZSWINSO_D2_AllIndustries_monthly	0.37
TOTAL_SCORE_ZSWINSO_D9_AllIndustries_monthly	0.35
FREE_CASH_FLOW_MARGIN_ZSWINSO_P5_AllIndustries_monthly	0.35
OCF_OVER_TOT_ASSETS_ZSWINSO_D1_AllIndustries_monthly	0.34
ROC_WACC_RATIO_ZSWINSO_D1_AllIndustries_monthly	0.34

Figure: Top 10 factors by eSR (against Russell 1000 EW)

- The top performing factor is the QUALITY_SCORE, followed by profitability metrics.

Ranked-Based Strategies - Univariate Sorts - excess EC



Ranked-Based Strategies - Sequential Filtering

- In the previous subsection, we presented results from univariate sort portfolios.
- In this subsection, we present results for sequential filtering. We refer to sequential filtering as a process where we first rank the top n% stocks, then we further rank these top n% stocks on another ratio and so on. By doing that, we avoid averaging ratios which can result in a stock being selected if it has a strong ratio that can offset another low ratio. In the sequential filtering, the stocks being selected must pass all the filters.

Ranked-Based Strategies - Sequential Filtering

- Sequential Filtering (SF) slightly allows to outperform univariate sorts: 0.53 (1.03) e(SR) vs 0.49 (0.97) e(SR).

Linear Models Strategies - Setup (1)

$$r_{i,t+1} - rf_{t+1} = \mathbb{E}_t \left(r_{i,t+1} - rf_{t+1} \right) + \varepsilon_{i,t+1} \tag{1}$$

$$r_{i,t+1} - rf_{t+1} = g^*(z_{i,t}) + \varepsilon_{i,t+1} \tag{2}$$

$$r_{i,t+1} - rf_{t+1} = \alpha + \sum_{k=1}^{K} \beta_k \, z_{i,t}^{(k)} + \varepsilon_{i,t+1} \tag{3}$$

- Functional form: equations (1) and (2) describe an asset's excess return without assuming any specific functional form.
- Linearity: equation (3) is specific to the linear models category (our subsection).
- Objective: find parameters that maximize the out-of-sample (OOS) explanatory power.

Linear Models Strategies - Setup (2)

- Sample Splitting: Expanding Training, Validation and Test
- For 10Y of data we used 3Y of training initially (increasing monthly), 2Y of validation and 1 month of test
- Models are re-estimated each month but hyperparameters are only tuned once, at the beginning

Linear Models Strategies - LASSO

- Model fails to select features
- Hence, each stock has the same predicted excess return, the intercept

Linear Models Strategies - Ridge

- Ridge solves the LASSO's issue as it keeps all the features
- But the OOS performance is nearly the same as the one of a naive model always predicting excess returns of 0.0

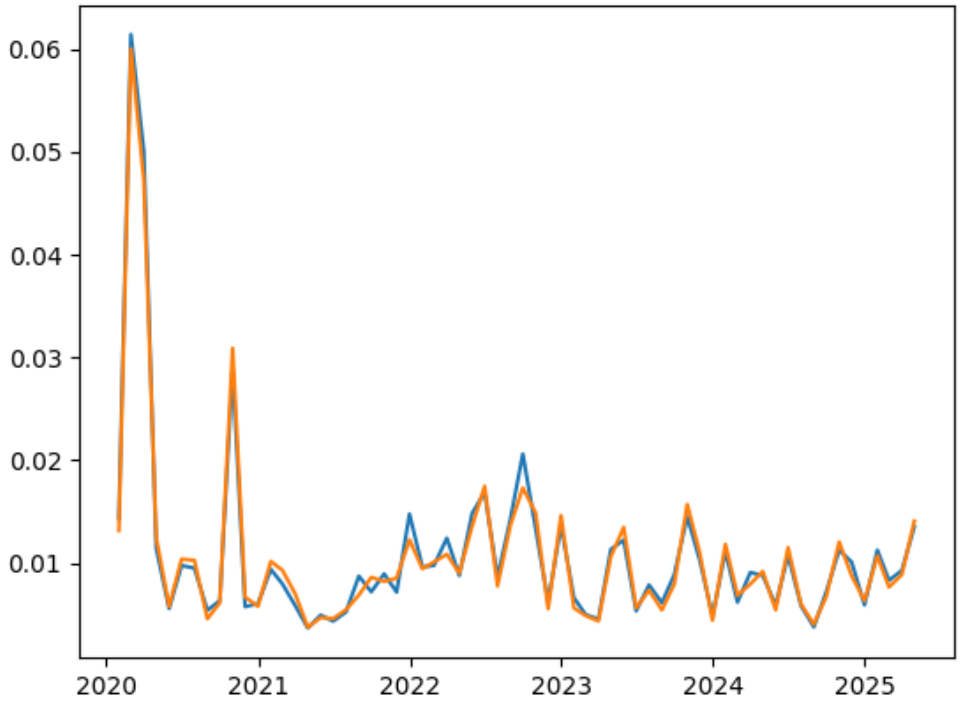


Figure: OOS R-squared of Ridge (orange) and Naive (blue) models

Linear Models Strategies - EN

- ...

Non-Linear Models Strategies - (TRM)

Non-Linear Models Strategies - (MSM)

Non-Linear Models Strategies - (RF)

Non-Linear Models Strategies - (NN)

Conclusion

- In this project, we studied 3 approaches for constructing portfolios:
  - Model-free (univariate sorts, SF)
  - Linear Models
  - Non-Linear Models
- From the results we have at this moment, we can say that the model-free approach offer simplicity, interpretability and good economic performance.
- Next Steps: add sector dummies, try classification task (?), try to predict (the rank?) at longer horizon (3-6 months?).