High Performance Nonlinear Controller Design for AC and DC Machines: Partial Feedback Linearization Approach

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Abstract This paper presents the design of a partial feedback linearization controller for speed control of DC and AC machines. The design of the controller is presented based on the nonlinear modeling of the plant. The model of the plant is obtained from differential dynamic equations. A stability analysis is presented to guarantee the stability of the proposed design. To validate the effectiveness of the controller, DC and AC machines systems are simulated. The performance of proposed controller is investigated with respect to different amount of load torque. Simulation studies show that the proposed controller provides the accuracy and high performance for both AC and DC machines.

Keywords Nonlinear controller, AC and DC machines and Partial Feedback Linearization.

1 Introduction

Electrical machines are the devices that convert electrical energy into mechanical energy or vice versa. Examples of electrical machines are transformers, generators and motors [1]. A motor converts electrical energy into mechanical energy and a generator converts mechanical energy into electrical energy. Both devices have basic two parts, stator and rotor. In a motor, mechanical force is created due to the electricity supply to both the stator and rotor windings and in a generator the mechanical energy is provided to the rotor by means of a prime mover. This force causes the rotor to rotate [1,2].

Two types of electrical machines such as electromagnetic and electrostatic machines are found in industries [1]. Electromagnetic machines are more commonly used. Electrostatic machines are used in high voltage and high current region. Electrical machines are classified into two categories depending on the type of input and the categories are AC and DC machines. Example of DC machines are separately excited, shunt wounded and series wounded. Each of the DC machine uses the Faradays left and right hand rules [2]. AC machine includes synchronous machine and induction machines. Synchronous machine uses magnetic locking principle where revolving field gets locked with the field produced by armature and the armature rotates. Induction machine uses Lenz law for motion of armature.

DC and AC machines are important for different applications such as in industry, robotics, nanotechnology [3], hard disk drive [4] and power system. DC machines have been widely used in many industrial applications such as electric vehicle, steel rolling, electric cranes, robotic manipulation and paper machines [5]. Because it has simple mechanical structure, it offers simple mechanism and continuous control. AC machines are used wherever the application depends on AC power from the national grid because they don't need commutators. They are particularly suitable for high power applications. It is mainly used for heavy industrial application, household appliances etc. [2,6].

An electrical machine comprises of rotor windings i.e. armature and stationary windings i.e. field coils. To perform the basic operation of DC machine, the power supply must be given to the stator windings as well as rotor windings through the carbon brushes that slide over a set of copper surfaces called a commutator. The commutator is mounted on the rotor windings of machine that helps to make a sliding switch and energizes particular portions of the armature. However in ac machines only the stator windings is fed with an AC power supply. The alternating flux is created around the stator

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windings because of supplying AC power that revolves with synchronous speed [1].

The speed of both AC and DC machines are related to the connection between the excitation winding and the rotor winding. Basically the speed is directly proportional to armature voltage and inversely proportional to the magnetic flux that produced by the stator windings. Adjusting the armature voltage or the field current the speed of electrical machine can be changed [7]. The controlling of speed of both AC and DC machines are important. Accurate control of speed is a great challenge for control engineers. This paper proposes a robust partial feedback linearisation controller to control the speed of AC and DC machines.

The rest of this paper is organised as follows. In section II a review of the related work is presented. Section III presents the preliminary definitions, Section IV shows the dynamics modeling and partial feedback linearization control law of DC and AC machines. Section V describes the stability analysis of the internal dynamics and Section VI presents the evaluation of the proposed controller. The paper is concluded in Section VII.

2 Related work

A number of researches have been done to model the dynamics of electrical machines. Modeling and control of electrical machines have been an area of active research over the last two decades. Different controllers have been proposed to control the speed of AC and DC machines, for example Proportional-Integral-Derivative (PID) controller is a popular controller in industries due to simple structure, low cost and easy to implementation [8–11]. It provides reliable performance for the system if PID parameter is identified properly. But it suffers due to lack of robustness.

Adaptive control technique such as sliding mode control is widely used to regulate the speed in control industry [12, 13]. The advantage of this control techniques is its simplicity, reduced-order system design and robustness against parameter variations and disturbances. Lyapunov stability theory is used for analysing the stability of plant associated with sliding mode control. To find the feedback gains and exhibits the desirable behavior of system, sliding mode control technique consists of two steps. First step is reaching phase and second step is sliding-mode phase. The first step ensures the driving the state from any initial state to reach the sliding manifold in finite time, while second step establishes sliding motion on the sliding manifold [14]. The major drawback of this control technique is state feedback control law which may not a continuous function of finite time.

Model reference adaptive control [15] is proposed to minimize the drawback of sliding mode control technique. In model reference adaptive control, it is applied to be controlled system with parameters varying or initially uncertain. The state feedback control law is changed due to the parameters uncertainty and produces desired performance. This control technique depends on the parameter of system model. In many practical applications, it is difficult to determine the parameters of model reference adaptive control due to the dynamics of the plant and unknown disturbances.

Self-tuning Artificial Neural Network (ANN) based controller [16, 17] is investigated to overcome the limitation of model reference adaptive control. In this process, the parameters of the plant is estimated that helps to determine unknown parameters. Self-tuning ANN based controller is composed of three parts namely a parameter estimator, a linear controller and a block that ensures the controller parameters from estimated parameters. In nonlinear system, this technique is related to linearization of model at operating time interval. This may produce error because of linearization of nonlinear model and cannot control the speed accurately.

Linear Quadratic Regulator (LQR) controller [18] has been proposed for speed control. In this controller the mean-value theorem is used to linearize the system model and reformulated with linear and nonlinear terms where the nonlinear terms are modelled as uncertainties in the design of robust control. The LQR controller has small operating region around equilibrium point and it may not be capable of preserving transient stability.

When the control of systems is difficult due to high nonlinearity, fuzzy logic controller (FLC) [19] can be designed. The advantages of FLC are robustness and easy to modification. It provides logical relation between the system and controller. Fuzzy logic controller comprises some membership values that have to divide into specific range which is known as fuzzification. A specify rule table is created to determine which output ranges are used. The controller results are calculated through the defuzzification of output ranges. The center of gravity method is the popular way to defuzzify the value. The performance of FLC depends on the expertization of the designer. A model predictive controller [20] is considered in for accurately control of speed. This controller is popular in control applications because of its ability to predict future output. However its operation principle is computationally complex.

Nonlinear feedback linearisation controller has been proposed in order to provide the speed control and stability of the system in large operating region in the presence of large disturbance [21, 22]. Feedback linearisation algebraically transforms a nonlinear system into a fully linearization so that linear control techniques can serve the stability of whole system. Direct Feedback linearization (DEL) [23] and exact feedback linearization (EFL) controllers have been proposed for nonlinear systems. The implementation of DFL and EFL controllers require all the parameters to measure for a system and it is essential to design differentiator which reduces the robustness of the controller.

Partial feedback linearization controller [24] does not require measuring all states. This controller reduces the number of differentiators to control the system. The reduction of the number of differentiators allows better stability compared to EFL. This paper presents the design of a partial feedback linearization controller for DC and AC machines.

3 Preliminary Definition

In this section definitions related to partial feedback linearization technique [25–29] are presented. Let a nonlinear system is defined as follows

$$\dot{x} = f(x) + g(x)u \tag{1}$$

$$y = h(x) \tag{2}$$

where $x \epsilon R^n$ is the state vector, $u \epsilon R^n$ is the control input vector, $y \epsilon R^n$ is the output vector, $f(x)$ and $g(x)$ are n dimensional vector fields in the state space, $h(x)$ is scalar function of x.

Definition 1: A nonlinear system defined in (1) and (2) where $f : D \to R^n$ and $g : D \to R^n$ are sufficiently smooth on a domain $D \subset R^n$ is said to be feedback linearization if there exist a difffeomorphism $T : D \to R^n$ such that $D_z = T(D)$ contains the origin and the change of variables $z = T(x)$ transforms the system into the form

$$\dot{z} = Az + B\gamma(x)(u - \alpha(x))$$

Definition 2: Let $h : R^n \to R$ is a smooth scalar function and $f : R^n \to R$ be a smooth vector field on R^n ,then the Lie derivative of $h(x)$ with respective to f is a scalar function defined by

$$L_f h(x) = \nabla h(x) f = \frac{\partial h(x)}{\partial x} f$$

The Lie derivative is interpreted as differentiation of the function $h(x)$ in the direction of the vector field f.

Definition 3: Let a nonlinear system is defined as in (1) and (2)

$$\dot{y} = L_f h(x) + L_g h(x)u \tag{3}$$

where $L_f h(x)$ and $L_g h(x)$ represent the Lie derivative of $h(x)$ with respective to $f(x)$ and $g(x)$ respectively. If $L_g h(x) = 0$ then the output do not appears and we have to differentiate repetitively as

$$y^r = L_f^r h(x) + L_g L_f^{r-1} h(x)u \tag{4}$$

If $L_g L_f^{r-1} h(x)u \neq 0$ then r is called the relative degree of the system.

Definition 4: A nonlinear coordinate transformation for a system which can be written as

$$z = \phi(x)$$

where z and x are the same dimensional vector and ϕ is the nonlinear function of x and the following two conditions are satisfied.

Condition 1: There exists an inverse transformation for all x i.e.

$$x = \phi^{-1}(z) \tag{5}$$

Condition 2: Each component of ϕ and ϕ^{-1} has continuous partial derivative up to any order which implies the differentiability of the function. Let

$$z_1 = h(x) = L_f^{1-1} h(x) = \frac{\partial h(x)}{\partial x} f$$

Substituting (1) into the above equation for x·

$$\dot{z}_1 = \frac{\partial h(x)}{\partial x} f + \frac{\partial h(x)}{\partial x} g(x)u \tag{6}$$

According to the definition of Lie derivative from (6)

$$\dot{z}_1 = L_f h(x) + L_g L^{1-1} h(x)u$$

Since the first derivative of the output is not influenced by input signal u i.e $L_g h(x) = 0$ and

$$\dot{z}_1 = L_f^{2-1} h(x) = z_2$$

$$\dot{z}_2 = L_f^{3-1} h(x) = z_3$$

$$\vdots$$

$$\dot{z}_{n-1} = L_f^{n-1} h(x) = z_n$$

$$\dot{z}_n = v = L_f^n h(x) + L_g L_f^{n-1} h(x)u$$

$$= L_f h(\phi^{-1}(z)) + L_g L_f^{n-1} h(\phi^{-1}(z))u \tag{7}$$

If $L_f h(\phi^{-1}(z))$ and $\beta(x) = L_g L_f^{n-1} h(\phi^{-1}(z))u$. Then we can write in (5) in the following way

$$v = \alpha(x) + \beta(x)u$$

$\beta(x)$ refers to the decoupling matrix. Then we can obtain a total of m equations in above form, which can be written as

$$\begin{bmatrix} z_1^{r_1} \\ \vdots \\ z_m^{r_m} \end{bmatrix} = \begin{bmatrix} L_f^{r_1} h_1(x) \\ \vdots \\ L_f^{r_m} h_m(x) \end{bmatrix} + \begin{bmatrix} L_{g_1} L_f^{r_1-h_1} & \dots & L_{g_1} L_f^{r_m-h_m} \\ L_{g_m} L_f^{r_1-h_1} & \dots & L_{g_m} L_f^{r_m-h_m} \end{bmatrix}$$

The matrix $\beta(x)$ is nonsingular (i.e the determinent of the matrix is not equal to zero),then the input transformation can be obtained as

$$u = \beta^{-1}(x)[v - \alpha(x)] \tag{8}$$

Substituting (7) into (8) results in a linear differential relation between the output z and u is

$$\begin{bmatrix} z_1^{r_1} \\ \vdots \\ z_m^{r_m} \end{bmatrix} = \begin{bmatrix} u_1 \\ \vdots \\ u_m \end{bmatrix}$$

Note that the above input output relation is decoupled, in addition to being linear.Therefore the fully linearised system with new co-ordinates

$$z = [z_1 \ \dots \ z_n]^T$$

where $n = r$ =relative degree

$$\dot{z} = Az + Bv \tag{9}$$

$$y = cz \tag{10}$$

where $A = \begin{bmatrix} 0\ 1\ 0 \dots 0 \\ 0\ 0\ 1 \dots \\ \vdots \\ 0\ 0\ 0\ \ 0\ \ 0 \end{bmatrix}$, $B = \begin{bmatrix} 0 \\ \vdots \\ 1 \end{bmatrix}$, $z = \begin{bmatrix} z_1 \\ \vdots \\ z_r \end{bmatrix}$

v is the control input for linearization system.

4 Dynamics Modeling and Partial Feedback Linearization Control Law

4.1 Induction Motor Model

Induction motors are mainly used to produce rotational motion and force in industries. This motor provides fast torque response and wide operating range of speed in the load variations . It is made of three-phase stator windings and a rotor winding. The complete model of induction motor [30, 31] in stator fixed a-b reference frame is given by

$$\frac{dI_{sa}}{dt} = -\gamma I_{sa} + \frac{k}{T_r}\phi_{ra} + p\omega k\phi_{rb} + \frac{1}{\sigma L_s}u_{sa}$$

$$\frac{dI_{sb}}{dt} = -\gamma I_{sb} + \frac{k}{T_r}\phi_{rb} - p\omega k\phi_{ra} + \frac{1}{\sigma L_s}u_{sb}$$

$$\frac{d\phi_{ra}}{dt} = \frac{M}{T_r}I_{sa} - \frac{1}{T_r}\phi_{ra} - p\omega\phi_{rb}$$

$$\frac{d\phi_{rb}}{dt} = \frac{M}{T_r}I_{sb} - \frac{1}{T_r}\phi_{rb} + p\omega\phi_{ra}$$

$$\frac{d\omega}{dt} = \frac{1}{j}(T - T_L)$$

where

M, L_s, L_r = Mutual, stator and rotor inductance

I_{sa}, I_{sb} = Stator current in reference frame a-b

u_{sa}, u_{sb} = Stator voltage in reference frame a-b

R_r, R_s = Stator and rotor resistance ; T_L = Load torque

T = Develop torque = $p\frac{M}{L_r}(\phi_{ra}I_{sb} - \phi_{rb}I_{sa})$

$\sigma = 1 - \frac{M^2}{L_s L_r}$

$T_r = \frac{L_r}{R_r}$ = rotor time constant

ω = Mechanical speed

$k = \frac{M}{\sigma L_s L_r}$

p = pole pair number ; j = Moment of inertia

$\gamma = \frac{R_s}{\sigma L_s} + \frac{R_r M^2}{\sigma L_s L_r^2}$

From (1) and (2)

$$u = [u_{sa}\ u_{sb}]^T, x = [I_{sa}\ I_{sb}\ \phi_{ra}\ \phi_{rb}\ \omega]^T$$

$$g(x) = \begin{bmatrix} \frac{1}{\sigma L_s} & 0 \\ 0 & \frac{1}{\sigma L_s} \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$f(x) = \begin{bmatrix} -\gamma I_{sa} + \frac{k}{T_r}\phi_{ra} + p\omega k\phi_{rb} \\ -\gamma I_{sb} + \frac{k}{T_r}\phi_{rb} - p\omega k\phi_{ra} \\ \frac{M}{T_r}I_{sa} - \frac{1}{T_r}\phi_{ra} - p\omega\phi_{rb} \\ \frac{M}{T_r}I_{sb} - \frac{1}{T_r}\phi_{rb} + p\omega\phi_{ra} \\ \frac{1}{j}(T - T_L) \end{bmatrix}$$

In the control of speed which involves the electromechanical torque and the square of the rotor flux leading to a nonlinear model. Let be the considered as

$$y = h(x) = \omega \tag{11}$$

Now differentiating (11) according to the Lie derivative

$$\dot{y} = L_f h(x) + L_{g_1} h(x) u_{sa} + L_{g_2} h(x) u_{sb} \tag{12}$$

Now

$$L_f h(x) = \frac{\partial h(x)}{\partial x} f \ = \ \frac{\partial \omega}{\partial x} f$$

$$= \begin{bmatrix} 0 \ 0 \ 0 \ 0 \ 1 \end{bmatrix} \begin{bmatrix} -\gamma I_{sa} + \frac{k}{T_r}\phi_{ra} + p\omega k\phi_{rb} \\ -\gamma I_{sb} + \frac{k}{T_r}\phi_{rb} - p\omega k\phi_{ra} \\ \frac{M}{T_r} I_{sa} - \frac{1}{T_r}\phi_{ra} - p\omega\phi_{rb} \\ \frac{M}{T_r} I_{sb} - \frac{1}{T_r}\phi_{rb} + p\omega\phi_{ra} \\ \frac{1}{j}(T - T_L) \end{bmatrix}$$

$$= \frac{1}{j}[p\frac{M}{L_r}(\phi_{ra} I_{sb} - \phi_{rb} I_{sa}) - T_L]$$

and

$$L_{g_1} h(x) = \frac{\partial h(x)}{\partial x} g_1 \ = \ \begin{bmatrix} 0 \ 0 \ 0 \ 0 \ 1 \end{bmatrix} \begin{bmatrix} \frac{1}{\sigma L_s} \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} = 0$$

$$L_{g_2} h(x) = \frac{\partial h(x)}{\partial x} g_2 \ = \ \begin{bmatrix} 0 \ 0 \ 0 \ 0 \ 1 \end{bmatrix} \begin{bmatrix} 0 \\ \frac{1}{\sigma L_s} \\ 0 \\ 0 \\ 0 \end{bmatrix} = 0$$

Since $L_{g_i} h(x) = 0$ for all i then we can apply the Lie derivative formula (4). Now the differentiating (12)

$$\ddot{y} = L_f^2 h(x) + L_{g_1} L_f h(x) u_{sa} + L_{g_2} L_f h(x) u_{sb}$$

Hence

$$L_f^2 h(x) \ = \ L_f(L_f h(x)) \ = \ \frac{\partial[\frac{1}{j}(T - T_L)]}{\partial x} f$$

$$= \frac{\partial(\frac{1}{j}[p\frac{M}{L_r}(\phi_{ra} I_{sb} - \phi_{rb} I_{sa})] - T_L)}{\partial x} f$$

$$= \frac{pM}{jL_r} \begin{bmatrix} -\phi_{rb} \ \phi_{ra} \ I_{sb} \ -I_{sa} \ 0 \end{bmatrix}$$
$$\begin{bmatrix} -\gamma I_{sa} + \frac{k}{T_r}\phi_{ra} + p\omega k\phi_{rb} \\ -\gamma I_{sb} + \frac{k}{T_r}\phi_{rb} - p\omega k\phi_{ra} \\ \frac{M}{T_r} I_{sa} - \frac{1}{T_r}\phi_{ra} - p\omega\phi_{rb} \\ \frac{M}{T_r} I_{sb} - \frac{1}{T_r}\phi_{rb} + p\omega\phi_{ra} \\ \frac{1}{j}(T - T_L) \end{bmatrix}$$

$$= \frac{pM}{jL_r}(\gamma I_{sa}\phi_{rb} - \frac{k}{T_r}\phi_{ra}\phi_{rb} - p\omega\phi_{rb}^2 - \gamma\phi_{ra} I_{sb} + \frac{k}{T_r}\phi_{ra}\phi_{rb} - p\omega phi_{ra}^2 + \frac{M}{T_r} I_{sa} I_{sb} - \frac{1}{T_r}\phi_{ra} I_{sb} - p\omega\phi_{rb} I_{sb} - \frac{M}{T_r} I_{sa} I_{sb} + \frac{1}{T_r} I_{sa}\phi_{rb} - p\omega\phi_{ra} I_{sa}$$

$$= \frac{pM}{jL_r}[\gamma(I_{sa}\phi_{rb} - \phi_{ra} I_{sb} - p\omega k(\phi_{ra}^2 + \phi_{rb}^2) + \frac{1}{T_r}(I_{sa}\phi_{rb} - \phi_{ra} I_{sb}) - p\omega(\phi_{ra} I_{sa} + \phi_{rb} I_{sb})$$

$$= \frac{pM}{jL_r}[(\frac{1}{T_r} + \gamma)(I_{sa}\phi_{rb} - \phi_{ra} I_{sb}) - p\omega k(\phi_{ra}^2 + \phi_{rb}^2) - p\omega (\phi_{ra} I_{sa} + \phi_{rb} I_{sb})$$

and

$$L_{g_1} L_f h(x) = \frac{\partial[\frac{1}{j}(T - T_L)]}{\partial x} g_1$$

$$= \frac{\partial[\frac{1}{j}(\frac{pM}{L_r}(\phi_{ra} i_{sb} - \phi_{rb} i_{sa})) - T_L]}{\partial x} g_1$$

$$= \frac{pM}{jL_r} \begin{bmatrix} -\phi_{rb} \ \phi_{ra} \ I_{sb} \ -I_{sa} \ 0 \end{bmatrix} \begin{bmatrix} \frac{1}{\sigma L_s} \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$= \frac{pM}{jL_r} - \phi_{rb}\frac{1}{\sigma L_s} = \frac{M}{\sigma L_s L_r} - \phi_{rb}\frac{p}{j} = -\frac{1}{j} pk\phi_{rb}$$

$$L_{g_2} L_f h(x) = \frac{\partial[\frac{1}{j}(\frac{pM}{L_r}(\phi_{ra} i_{sb} - \phi_{rb} i_{sa})) - T_L]}{\partial x} g_2$$

$$= \frac{pM}{jL_r} \begin{bmatrix} -\phi_{rb} \ \phi_{ra} \ I_{sb} \ -I_{sa} \ 0 \end{bmatrix} \begin{bmatrix} 0 \\ \frac{1}{\sigma L_s} \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$= \frac{pM}{jL_r}\phi_{ra}\frac{1}{\sigma L_s} = \frac{M}{\sigma L_s L_r}\phi_{ra}\frac{p}{j} = \frac{1}{j} pk\phi_{ra}$$

Hence the relative degree of the system is 2. Now the original x states are transformed into z states through nonlinear co-ordinate transformation by choosing

$$z_1 = h(x) = \omega$$

$$\dot{z_1} = z_2 = L_f h(x) + L_{g_1} h(x) u_{sa} + L_{g_2} h(x) u_{sb} = L_f h(x)$$

$$\dot{z_2} = z_3 = v = L_f^2 h(x) + L_{g_1} L_f h(x) u_{sa} + L_{g_2} L_f h(x) u_{sb}$$

According to (9) and (10)

$$A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, c = \begin{bmatrix} 1 \ 0 \end{bmatrix}, z = \begin{bmatrix} z_1 \\ z_2 \end{bmatrix}$$

Hence the partial feedback linearization law

$$u = \frac{v - L_f^2 h(x)}{[L_{g_1} L_f h(x) \ L_{g_1} L_f h(x)]}$$

4.2 Series Motor Model

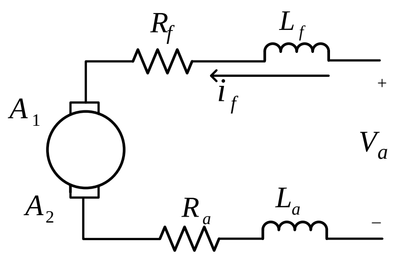

Fig. 1: Equivalent Circuit of Series DC Motor

A series motor is one in which the field circuit is connected in series with armature circuit. The Equivalent circuit of series DC motor is shown in Fig 1. The dynamic model of a series DC motor [32,33] is given below:

$$\frac{di_a}{dt} = \frac{1}{L_a}(v_a - i_a(R_p + R_a) + R_p i_f - v_b)$$

$$\frac{di_f}{dt} = \frac{1}{L_f}(-i_f(R_p + R_f) + R_p i_a)$$

$$\frac{d\omega}{dt} = \frac{1}{j}(T_e - B\omega - T_L)$$

where

$i_a, i_f =$ Armature and field current

$v_a =$ Applied voltage ; $v_b =$ back EMF $= k i_f \omega$

$R_a, R_f =$ Armature and field resistance ;$T_L =$ Load torque

$T_e =$ Develop torque $= k i_a i_f$

$\omega =$ Mechanical speed ; $B =$ Viscous damping co-efficient

$L_a, L_f =$ Armature and field inductance

R_p is the resistance which is placed in parallel with field windings for field weaking.
For (1) and (2)

$$x = [i_a \; i_f \; \omega]^T \; ; \; u = v_a$$

$$f(x) = \begin{bmatrix} \frac{1}{L_a}\{-i_a(R_p + R_a) + R_p i_f - v_b\} \\ \frac{1}{L_f}\{-i_f(R_p + R_f) + R_p i_a\} \\ \frac{1}{j}(T_e - B\omega - T_L) \end{bmatrix} ; g(x) = \begin{bmatrix} \frac{1}{L_a} \\ 0 \\ 0 \end{bmatrix}$$

This model is nonlinear because it provides the product of two variables of i_a and i_f.
Now be considered (11)

$$y = h(x) = \omega$$

Differentiating (11) according to the Lie derivative for this system is as given below

$$\dot{y} = L_f h(x) + L_g h(x) v_a \tag{13}$$

Now

$$L_f h(x) = \frac{\partial h(x)}{\partial x} f = \frac{\partial \omega}{\partial x} f$$

$$= [0 \; 0 \; 1] \begin{bmatrix} \frac{1}{L_a}\{-i_a(R_p + R_a) + R_p i_f - v_b\} \\ \frac{1}{L_f}\{-i_f(R_p + R_f) + R_p i_a\} \\ \frac{1}{j}(T_e - B\omega - T_L) \end{bmatrix}$$

Hence

$$L_g h(x) = \frac{\partial h(x)}{\partial x} g = [0 \; 0 \; 1] \begin{bmatrix} \frac{1}{L_a} \\ 0 \\ 0 \end{bmatrix} = 0$$

Since $L_g h(x) = 0$, Again differentiating (13) according to (4) can be obtained as follows

$$\ddot{y} = L_f^2 h(x) + L_g L_f h(x) v_a$$

Now

$$L_f^2 h(x) = L_f(L_f h(x)) = \frac{\partial[\frac{1}{j}(k i_a i_f - T_L - B\omega)]}{\partial x} f$$

$$= \frac{1}{j} \begin{bmatrix} k i_f & k i_a & -B \end{bmatrix} \begin{bmatrix} \frac{1}{L_a}\{-i_a(R_p + R_a) + R_p i_f - v_b\} \\ \frac{1}{L_f}\{-i_f(R_p + R_f) + R_p i_a\} \\ \frac{1}{j}(T_e - B\omega - T_L) \end{bmatrix}$$

$$= \frac{1}{j}[\frac{k}{L_a}(-i_f i_a(R_p + R_a) + R_p i_f^2 - v_b i_f) + \frac{k}{L_f}(R_p i_a^2 - i_a i_f \\ (R_p + R_f)) - \frac{B}{j}(\frac{1}{j}(T_e - B\omega - T_L)]$$

$$= \frac{1}{j}[k R_p(\frac{i_f^2}{L_a} + \frac{i_a^2}{L_f}) - T_e(\frac{R_p + R_a}{L_a} + \frac{R_p + R_f}{L_f}) - \frac{k i_f}{j L_a} v_b \\ - \frac{B}{j}(T_e - B\omega - T_L)]$$

and

$$L_g L_f h(x) = \frac{1}{j} \begin{bmatrix} k i_f & k i_a & -B \end{bmatrix} \begin{bmatrix} \frac{1}{L_a} \\ 0 \\ 0 \end{bmatrix} = \frac{k i_f}{j L_a}$$

Hence the relative degree of the system is 2. Now the original states are transformed into states through nonlinear co-ordinate transformation by choosing

$$z_1 = h(x) = \omega$$

$$\dot{z_1} = z_2 = L_f h(x) + L_g h(x) v_a = L_f h(x)$$

$$\dot{z_2} = z_3 = v = L_f^2 h(x) + L_g L_f h(x) v_a$$

Then

$$A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, c = \begin{bmatrix} 1 & 0 \end{bmatrix}, z = \begin{bmatrix} z_1 \\ z_2 \end{bmatrix}$$

and the partial feedback linearization law for this system is

$$u = \frac{v - L_f^2 h(x)}{L_g L_f h(x)}$$

4.3 Separately Excited DC Motor Model (SEDM)

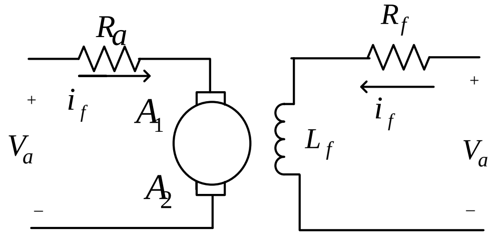

Fig. 2: Equivalent Circuit of Separately Excited DC Motor

Separately excited DC motor is an electromagnetic type DC motor. When the two windings of the motor are separated, this gives rise to a separately excited. The equivalent circuit of a separately excited is given number of Fig 2. The complete model of a separately excited DC motor [5, 6] is as follows:

$$\frac{di_a}{dt} = \frac{1}{L_a}(v_a - i_a R_a - v_b)$$

$$\frac{di_f}{dt} = \frac{1}{L_f}(v_f - i_f R_f)$$

$$\frac{d\omega}{dt} = \frac{1}{j}(T_e - B\omega - T_L)$$

where

$i_a, i_f =$ Armature and field current

$v_a =$ Armature voltage ; $v_b =$ back EMF $= k i_f \omega$

$R_a, R_f =$ Armature and field resistance ;$T_L =$ Load torque

$T_e =$ Develop torque $= k i_a i_f$

$\omega =$ Mechanical speed ; $B =$ Viscous damping co-efficient

$L_a, L_f =$ Armature and field inductance

For (1) and (2)

$$x = [i_a \ i_f \ \omega]^T \ ; \ u = [v_a \ v_f]^T$$

$$f(x) = \begin{bmatrix} \frac{1}{L_a}(-i_a R_a - v_b) \\ -\frac{1}{L_f} i_f R_f \\ \frac{1}{j}(T_e - B\omega - T_L) \end{bmatrix} ; \ g(x) = \begin{bmatrix} \frac{1}{L_a} & 0 \\ 0 & \frac{1}{L_f} \\ 0 & 0 \end{bmatrix}$$

Since two variables are multiplied with each other, so this model is known as nonlinear model.
Now be considered (11)

$$y = h(x) = \omega$$

Now differentiating (11) according to the Lie derivative

$$\dot{y} = L_f h(x) + L_{g_1} h(x) v_a + L_{g_2} h(x) v_f \tag{14}$$

Hence

$$L_f h(x) = \frac{\partial h(x)}{\partial x} f = \frac{\partial \omega}{\partial x} f$$

$$= \begin{bmatrix} 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \frac{1}{L_a}(-i_a R_a - v_b) \\ -\frac{1}{L_f} i_f R_f \\ \frac{1}{j}(T_e - B\omega - T_L) \end{bmatrix} \begin{bmatrix} \frac{1}{L_a} & 0 \\ 0 & \frac{1}{L_f} \\ 0 & 0 \end{bmatrix}$$

$$= \frac{1}{j}(T_e - B\omega - T_L)$$

$$L_{g_1} h(x) = \frac{\partial h(x)}{\partial x} g_1 = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \frac{1}{L_a} \\ 0 \\ 0 \end{bmatrix} = 0$$

$$L_{g_2} h(x) = \frac{\partial h(x)}{\partial x} g_2 = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ \frac{1}{L_f} \\ 0 \end{bmatrix} = 0$$

Since $L_{g_i} h(x) = 0$ for all i then we can apply the Lie derivative formula (4). Now the differentiating (14)

$$\ddot{y} = L_f^2 h(x) + L_{g_1} L_f h(x) v_a + L_{g_2} L_f h(x) v_f$$

$$L_f^2 h(x) = L_f(L_f h(x)) = \frac{\partial[\frac{1}{j}(k i_a i_f - T_L - B\omega)]}{\partial x} f$$

$$= \frac{1}{j} \begin{bmatrix} k i_f & k i_a & -B \end{bmatrix} \begin{bmatrix} \frac{1}{L_a}(-i_a R_a - v_b) \\ -\frac{1}{L_f} i_f R_f \\ \frac{1}{j}(T_e - B\omega - T_L) \end{bmatrix}$$

$$= \frac{1}{j}[\frac{k}{L_a} i_f(-R_a i_a - v_b) - \frac{k}{L_f} R_f i_f i_a - \frac{B}{j}(T_e - B\omega - T_L)$$

$$= \frac{k}{jL_a} i_f(-R_a i_a - v_b) - \frac{R_f}{jL_f} T_e - \frac{B}{j^2}(T_e - B\omega - T_L)$$

Hence

$$L_{g_1}L_fh(x) = \frac{1}{j}\left[ki_f\ ki_a\ -B\right]\begin{bmatrix}\frac{1}{L_a}\\0\\0\end{bmatrix} = \frac{ki_f}{jL_a}$$

and

$$L_{g_2}L_fh(x) = \frac{1}{j}\left[ki_f\ ki_a\ -B\right]\begin{bmatrix}0\\\frac{1}{L_f}\\0\end{bmatrix} = \frac{ki_a}{jL_a}$$

Since $L_{g_i}L_fh(x) = 0$ at least one i, according to the Definition (3), the relative degree of the system is 2. Now the original x states are transformed into z states through nonlinear co-ordinate transformation by choosing

$$z_1 = h(x) = \omega$$

$$\dot{z_1} = z_2 = L_fh(x) + L_{g_1}h(x)v_a + L_{g_2}h(x)v_f = L_fh(x)$$

$$\dot{z_2} = z_3 = v = L_f^2h(x) + L_{g_1}L_fh(x)v_a + L_{g_2}L_fh(x)v_f$$

$$A = \begin{bmatrix}0\ 1\\0\ 0\end{bmatrix}, B = \begin{bmatrix}0\\1\end{bmatrix}, c = \begin{bmatrix}1\ 0\end{bmatrix}, z = \begin{bmatrix}z_1\\z_2\end{bmatrix}$$

Hence the partial feedback linearization law

$$u = \frac{v - L_f^2h(x)}{\left[L_{g_1}L_fh(x)\ L_{g_1}L_fh(x)\right]}$$

4.4 Shunt Motor Model

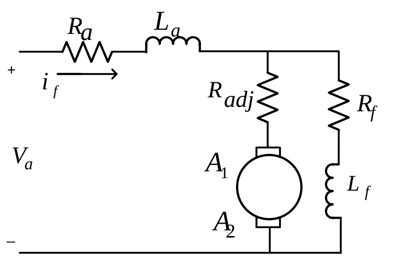

Fig. 3: Equivalent Circuit of Shunt DC Motor

The shunt motor is different from the series motor in that the field winding is connected in parallel with the armature instead of in series. Since the field winding is placed in parallel with the armature, it is called a shunt winding and the motor is called a shunt motor. The equivalent circuit of a shunt motor is given number of Fig 3. For the convenience of calculation making the armature inductance is taken to zero $L_1 = 0$ so that current flowing through armature is

$$i_a = \frac{v_T - v_b}{R_a}$$

The dynamic model of shunt motor [34–36] is given below:

$$\frac{di_f}{dt} = \frac{1}{L_f}(v_T - i_f(R_{adj} + R_f))$$
$$\frac{d\omega}{dt} = \frac{1}{j}(T_e - B\omega - T_L)$$
$$\frac{d\theta}{dt} = \omega$$

where

v_T = Terminal voltage ; v_b = back EMF $= ki_f\omega$

i_a, i_f = Armature and field current

R_a, R_f = Armature and field resistance ; T_L = Load torque

T_e = Develop Torque $= \frac{ki_f}{R_a}v_T - \frac{k^2i_f^2\omega}{R_a}$

ω = Mechanical speed ; B = Viscous damping co-efficient

L_a, L_f = Armature and field inductance

R_{adj} = Adjustable resistance ; θ = angular position

For (1) and (2)

$$x = [i_f\ \omega\ \theta]^T = [x_1\ x_2\ x_3]^T\ ;\ u = v_T$$

$$f(x) = \begin{bmatrix}-A_2x_1\\-A_4x_1^2x_2 - A_5x_2 - A_6T_L\\x_2\end{bmatrix}; g(x) = [A_1\ A_3x_1\ 0]^T$$

This model is nonlinear because it involves the square of variable as well as the product of two variables.

where

$$A_1 = \frac{1}{L_f}; A_2 = \frac{R_adj + R_f}{L_a}; A_3 = \frac{k}{jL_a}; A_4 = \frac{k^2}{jR_a}$$

$$A_5 = \frac{B}{j}; A_6 = \frac{1}{j}$$

$$y = h(x) = \omega = x_2$$

Hence

$$\dot{y} = L_fh(x) + L_gh(x)u$$

Now

$$L_fh(x) = \frac{\partial h(x)}{\partial x}f = \frac{\partial x_2}{\partial x}f$$

$$= \begin{bmatrix}0 & 1 & 0\end{bmatrix} \begin{bmatrix} -A_2x_1 \\ -A_4x_1^2x_2 - A_5x_2 - A_6T_L \\ x_2 \end{bmatrix}$$

$$= -A_4x_1^2x_2 - A_5x_2 - A_6T_L$$

and

$$L_gh(x) = \frac{\partial h(x)}{\partial x}g = \frac{\partial x_2}{\partial x}g$$

$$= \begin{bmatrix}0 & 1 & 0\end{bmatrix} \begin{bmatrix} A_1 \\ A_3x_1 \\ 0 \end{bmatrix} = A_3x_1$$

Hence the relative degree of the system is 1. Now the original x states are transformed into z states through nonlinear co-ordinate transformation by choosing

$$z_1 = y = h(x) = \omega = x_2$$

$$\dot{z_1} = z_2 = v = -A_4x_1^2x_2 - A_5x_2 - A_6T_L + A_3x_1$$

$$A = [0], B = [1], c = [1], z = [z_1]$$

and the partial feedback linearization law

$$u = \frac{v + A_4x_1^2x_2 + A_5x_2 + A_6T_L}{+A_3x_1}$$

4.5 Permanent Magnet (PM) Stepper Motor Model

A stepper motor is a unique type of DC motor that rotates in fixed steps of a certain number of degrees. Step size can range from 0.9 to 90 . The size of each step is determined by N_r (Number of rotor teeth). The formula for the step size of the two phases PM stepper motor is given by

$$\theta_s = \frac{90}{N_r} \text{ deg}$$

It consists of a rotor and stator. In this case, the rotor is a permanent magnet and the stator is consists of electromagnets. The dynamic models of the two phases Permanent Magnet stepper motor [35, 37] is given below:

$$\frac{di_a}{dt} = \frac{1}{L}[v_a - Ri_a + k\omega \sin(N_r\theta)]$$

$$\frac{di_b}{dt} = \frac{1}{L}[v_b - Ri_b - k\omega \cos(N_r\theta)]$$

$$\frac{d\theta}{dt} = \frac{1}{j}[-ki_a \sin(N_r\theta) + ki_b \cos(N_r\theta) - B\omega]$$

$$\frac{d\theta}{dt} = \omega$$

where
i_a, i_b and v_a, v_b are the current and voltage in phase A and B respectively. L and R are the self inductance and resistance of each phases windings. k is the motor constant and j is the rotor inertia. N_r is the number of rotor teeth.

$B =$ Viscous friction constant

$\theta =$ motor position and

$\omega =$ rotor speed

For convenience of calculation let

$$A_1 = \frac{R}{L}, A_2 = \frac{k}{L}, A_3 = \frac{k}{j}, A_4 = \frac{B}{j}, A_5 = N_r, \\ u_1 = \frac{v_a}{L} \text{ and } u_2 = \frac{v_b}{L}$$

For using (1) and (2)
where
$x = [i_a\ i_b\ \omega\ \theta]^T = [x_1\ x_2\ x_3\ x_4]^T$, $u = [u_1\ u_2]^T$

$$f(x) = \begin{bmatrix} -A_1x_1 + A_2x_3 \sin(A_5x_4) \\ -A_1x_2 - A_2x_3 \cos(A_5x_4) \\ -A_3x_1 \sin(A_5x_4) + A_3x_2 \cos(A_5x_4) - A_4x_3 \\ x_3 \end{bmatrix};$$

$$g(x) = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$$

and

$$y = h(x) = \omega$$

Now

$$\dot{y} = L_fh(x) + L_{g_1}h(x)u_1 + L_{g_2}h(x)u_2 \tag{15}$$

Hence

$$L_fh(x) = \frac{\partial h(x)}{\partial x}f$$

$$= \begin{bmatrix}0 & 0 & 1 & 0\end{bmatrix} \begin{bmatrix} -A_1x_1 + A_2x_3 \sin(A_5x_4) \\ -A_1x_2 - A_2x_3 \cos(A_5x_4) \\ -A_3x_1 \sin(A_5x_4) + A_3x_2 \cos(A_5x_4) - A_4x_3 \\ x_3 \end{bmatrix}$$

$$= -A_3x_1 \sin(A_5x_4) + A_3x_2 \cos(A_5x_4) - A_4x_3$$

and

$$L_{g_1}h(x) = \frac{\partial h(x)}{\partial x}g_1 = \frac{\partial x_3}{\partial x}g_1 = \begin{bmatrix}0 & 0 & 1 & 0\end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} = 0$$

$$L_{g_2}h(x) = \frac{\partial h(x)}{\partial x}g_2 = \frac{\partial x_3}{\partial x}g_2 = \begin{bmatrix}0 & 0 & 1 & 0\end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix} = 0$$

Since $L_g h(x) = 0$ Then again differentiating (15) according to (4) can be obtained as follows

$$\ddot{y} \;=\; L_f^2 h(x) + L_{g_1} L_f h(x) u_1 + L_{g_2} L_f h(x) u_2$$

Now

$$L_f^2 h(x) = L_f(L_f h(x)$$

$$= \frac{\partial[-A_3 x_1 \sin(A_5 x_4) + A_3 x_2 \cos(A_5 x_4) - A_4 x_3]}{\partial x} f$$

$$= \begin{bmatrix} -A_3 \sin(A_5 x_4) \\ A_3 \cos(A_5 x_4) \\ 0 \\ -A_3 A_5 [x_1 \cos(A_5 x_4) + x_2 \sin(A_5 x_4)] \end{bmatrix}^T$$

$$\begin{bmatrix} -A_1 x_1 + A_2 x_3 \sin(A_5 x_4) \\ -A_1 x_2 - A_2 x_3 \cos(A_5 x_4) \\ -A_3 x_1 \sin(A_5 x_4) + A_3 x_2 \cos(A_5 x_4) - A_4 x_3 \\ x_3 \end{bmatrix}$$

$$= -A_1 A_3 x_1 \sin(A_5 x_4) - A_2 A_3 x_3 \sin^2(A_5 x_4) - A_1 A_3 x_2 \cos(A_5 x_4) - A_2 A_3 x_3 \cos^2(A_5 x_4) - A_3 A_5 x_3 [x_1 \cos(A_5 x_4) + x_2 \sin(A_5 x_4)$$

$$= A_1 A_3 x_1 \sin(A_5 x_4) - A_2 A_3 x_3 - A_1 A_3 x_2 \cos(A_5 x_4) - A_3 A_5 x_3 x_1 \cos(A_5 x_4) - A_3 A_5 x_3 x_2 \sin(A_5 x_4)$$

$$= A_2 A_3 x_3 - A_3 \sin(A_5 x_4)(A_1 x_1 + A_5 x_2 x_3) - A_3 \cos (A_5 x_4)(A_1 x_2 + A_5 x_1 x_3)$$

$$= A_3[A_2 x_3 - \sin(A_5 x_4)(A_1 x_1 + A_5 x_2 x_3) - \cos(A_5 x_4) (A_1 x_2 + A_5 x_1 x_3)]$$

$$= -\frac{k}{j}[\frac{k}{L}\omega + (\frac{R}{L} i_a + N_r i_b \omega) \sin(N_r \theta) + (\frac{R}{L} i_b + N_r i_a \omega) \cos(N_r \theta)$$

and

$$L_{g_1} L_f h(x) = L_{g_1}(\frac{\partial h(x)}{\partial x} f)$$

$$= \begin{bmatrix} -A_3 \sin(A_5 x_4) \\ A_3 \cos(A_5 x_4) \\ 0 \\ -A_3 A_5 [x_1 \cos(A_5 x_4) + x_2 \sin(A_5 x_4)] \end{bmatrix}^T \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$= -A_3 \sin(A_5 x_4) = -\frac{k}{j} \sin(N_r \theta)$$

$$L_{g_2} L_f h(x) = L_{g_2}(\tfrac{\partial h(x)}{\partial x} f)$$

$$= \begin{bmatrix} -A_3 \sin(A_5 x_4) \\ A_3 \cos(A_5 x_4) \\ 0 \\ -A_3 A_5 [x_1 \cos(A_5 x_4) + x_2 \sin(A_5 x_4)] \end{bmatrix}^T \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}$$

$$= A_3 \cos(A_5 x_4) = \frac{k}{j} \cos(N_r \theta)$$

Hence the relative degree of the system is 2. Now the original x states are transformed into z states through nonlinear co-ordinate transformation by choosing

$$z_1 = h(x) = \omega$$

$$\dot{z_1} = z_2 = L_f h(x) + L_{g_1} h(x) u_1 + L_{g_2} h(x) u_2 = L_f h(x)$$
$$\dot{z_2} = z_3 = v = L_f^2 h(x) + L_{g_1} L_f h(x) u_1 + L_{g_2} L_f h(x) u_2$$

According to (9) and (10)

$$A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, c = \begin{bmatrix} 1 & 0 \end{bmatrix}, z = \begin{bmatrix} z_1 \\ z_2 \end{bmatrix}$$

Hence the partial feedback linearization law

$$u = \frac{v - L_f^2 h(x)}{\left[L_{g_1} L_f h(x) \; L_{g_1} L_f h(x)\right]}$$

5 Stability Analysis of Internal Dynamics

In this section we present the stability analysis of the plant using partial feedback linearization controller. The analysis shows that the proposed controller is stable for all AC and DC machines. Here let η is a stability factor. The output function is chosen in such a way that the output equation $y = h(x) = 0$ at $x = x_0$. Therefore the output $y = h(x)$ is the actually dynamic deviation of the practical output from the output at an equilibrium point [29, 38]. In the use of controlling that means to impose dynamic deviation of the output of the system keeps zero at any times,

$$y = h(x) = 0 \quad 0 \leq t \leq \infty$$

From the previous steps, it has been seen that the first r order equation are transformed into linear equation. To check the stability, it is essential to calculate the rest of $n-r$ order equations through proper transformation. Here $\eta_1 \ldots \eta_{n-r}$ needs to be selected in such a way that it is satisfied

$$L_{g_1} \eta_i(x) \ldots L_{g_N} \eta_i(x) = 0$$

If it is satisfied, then $\dot{\eta(x)} = L_f \eta_i(x)$ where $i = 1, 2, \quad n-r$. In general, the control law needs to be chosen so that

$$\lim_{t \to \infty} h_i(x) \to 0$$

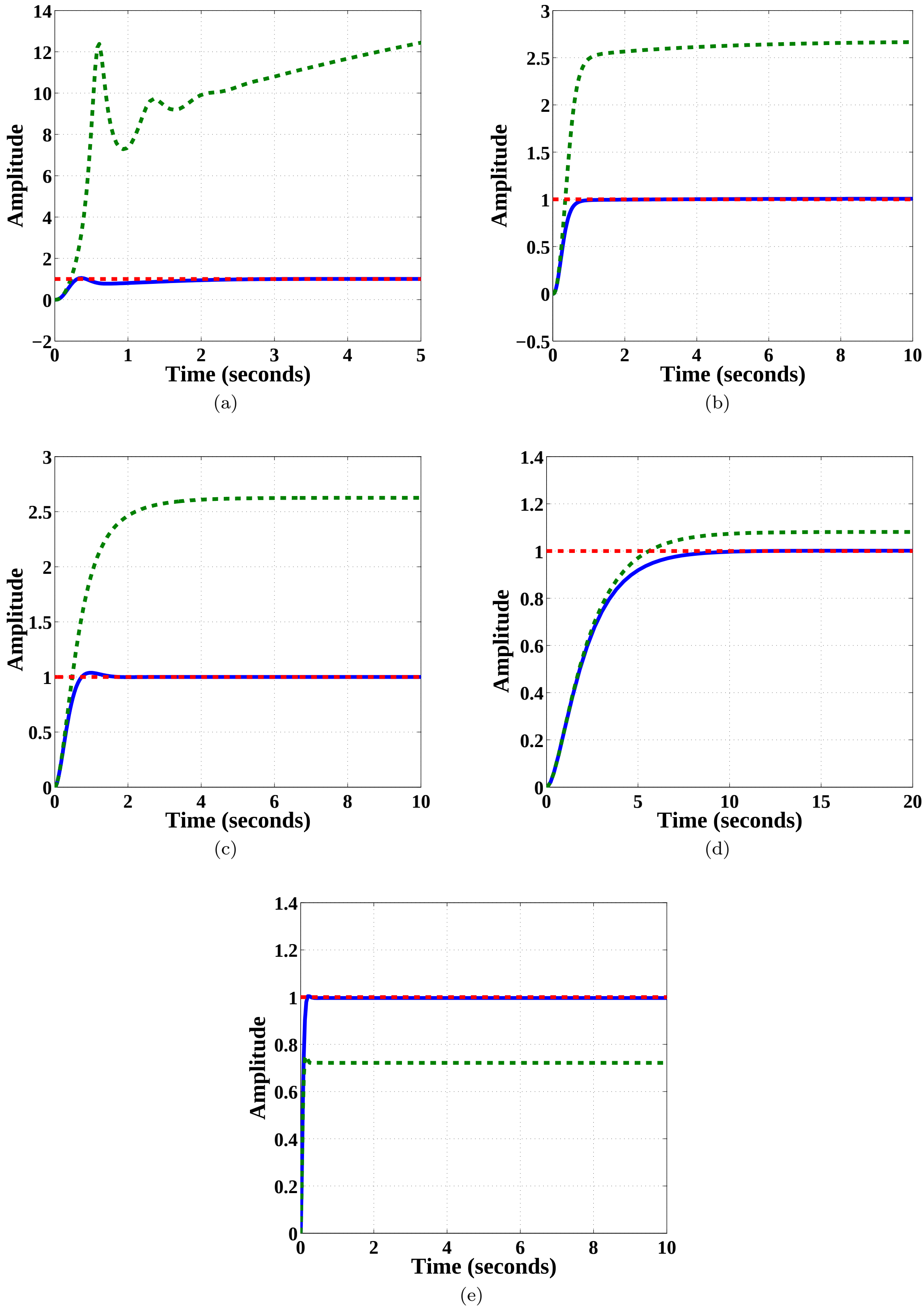

Fig. 4: Reference Tracking for (a) Induction Motor , (b) Series DC Motor , (c) Separately Excited DC Motor , (d) Shunt DC Motor , (e) PM Stepper Motor.

Since $y = h(x) \to 0$ as $t \to \infty$ the above equation is defined follow

$$\dot{\eta}(x) = 0$$

which serve the internal dynamics of the subsystem. If the internal dynamics of the system is zero, it is provided the stability on overall system. The dynamics of a nonlinear system can be classified into two types depends on the partial linearizing approach, one is external dynamics it is necessary to stable and have a good performance and another is called internal dynamics which needs to be stabile only.

5.1 Induction Motor

Let be the considered

$$\eta(x) = \omega \tag{16}$$

Differentiating (16) according to Lie derivative is as follows

$$\dot{\eta}(x) = L_f\eta(x) + L_{g_1}\eta(x)u_{sa} + L_{g_2}\eta(x)u_{sb}$$

$$= \frac{\partial\eta(x)}{\partial x}f + \frac{\partial\eta(x)}{\partial x}g_1u_{sa} + \frac{\partial\eta(x)}{\partial x}g_2u_{sb}$$

$$= \frac{1}{j}(T_e - T_L) = h(x)$$

Since $y = h(x) = 0$ as $t \to 0$, the above equation can be written as

$$\dot{\eta}(x) = 0$$

5.2 Series Motor

From (16)

$$\dot{\eta}(x) = L_f\eta(x) + L_g\eta(x)v_a$$

$$= \frac{\partial\eta(x)}{\partial x}f + \frac{\partial\eta(x)}{\partial x}gv_a$$

$$= \frac{1}{j}(T_e - B\omega - T_L) = h(x)$$

Since $y = h(x) = 0$ as $t \to 0$,the above equation can be written as

$$\dot{\eta}(x) = 0$$

5.3 Separately Excited DC Motor

Let be the considered From (16)

$$\dot{\eta}(x) = L_f\eta(x) + L_{g_1}\eta(x)v_a + L_{g_2}\eta(x)v_f$$

$$= \frac{\partial\eta(x)}{\partial x}f + \frac{\partial\eta(x)}{\partial x}g_1v_a + \frac{\partial\eta(x)}{\partial x}g_2v_f$$

$$= \frac{1}{j}(T_e - B\omega - T_L) + 0.v_a + 0.v_f$$

$$= \frac{1}{j}(T_e - B\omega - T_L) = h(x)$$

Since $y = h(x) = 0$ as $t \to 0$, the above equation can be written as

$$\dot{\eta}(x) = 0$$

Similar results can be obtained for shunt motor and Stepper motor as well.

6 Performance Evalution

The performance of the proposed controller is presented in this section. The simulation results of the proposed controller is presented using Matlab. Matlab simulink model is used to measure the dynamics of both AC and DC machines. The results are presented for individual machine. The values of the parameters for induction motor, series motor, separately excited DC motor and permanent magnet stepper motor used in simulation results are listed in Table 1, Table 2, Table 3 and Table 4 respectively. The results of the closed-loop system using partial feedback linearization controller is presented in Fig. 4. The open-loop provides undesirable nature of speed with overshoot. The magnitude of speed is increased with increasing of time after transient period for induction motor and series motor. This causes poor tracking of the reference voltage and introduces speed variation. This is objectionable in many application, whereas closed-loop responses using proposed controller provides high performance tracking of command signal. The proposed controller reduces steady state error to zero, settling time and rising time for all machines using closed-loop as compared to open-loop system.

The performance of a machine can be varied because of applying load torque. The torque is a breaking force which is proportional to the armature current and inversely proportional to machine speed. The torque creates an acceleration that helps to reduce the speed of the machine. The performance of proposed controller under different load torque for DC machines is presented in Fig. 5. The results show that for all the variation of the load torque the closed-loop system remain stable.

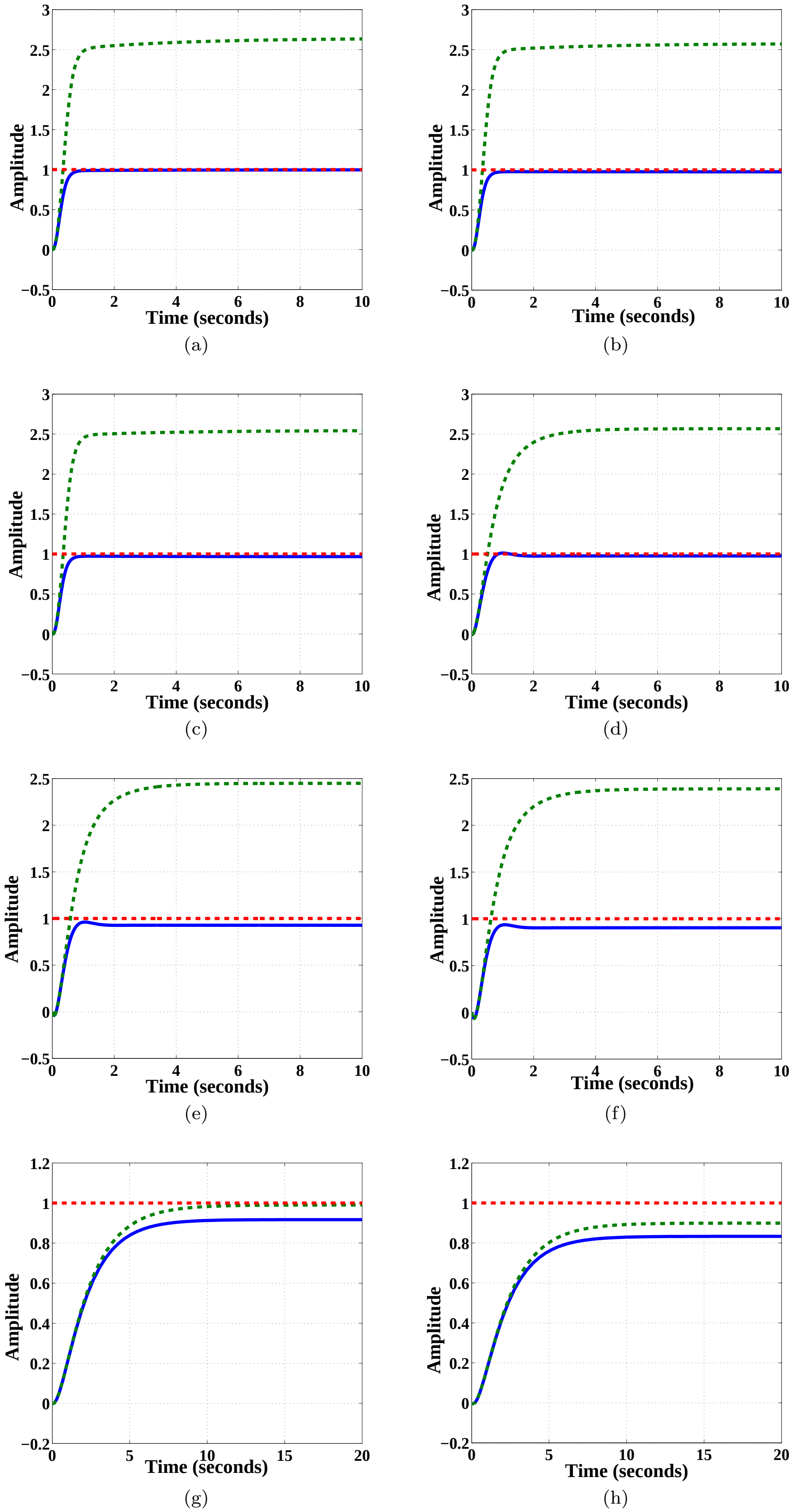

Fig. 5: Reference Tracking for Series DC motor under the load ,(a) 0.01 , (b) 0.03 , (c) 0.04 , Separately Excited DC motor under the load (d) 0.001 , (e) 0.003 , (f) 0.004 , Shunt DC motor under the load (g) 0.001 and (h) 0.002

Table 1: Parameter values for Induction motor

Parameter	Value
L_r	0.0323 H
L_s	0.031747 H
R_r	0.07 Ohm
R_s	0.052 Ohm
M	0.031 H
p	2
j	0.41 Kgm^2

Table 2: Parameter values for Series DC motor

Parameter	Value
L_f	1 H
L_a	0.0014 H
R_a	0.01485 Ohm
R_f	0.07485 Ohm
R_p	0.0796 Ohm
K	0.329 Nm/A^2
B	0.1 Nms/rad
j	0.5 Kgm^2

From Fig. 5 it can be seen that the separately excited DC motor exhibits the negative speed against the load torque. This negative speed is due to the negative or anticlockwise direction of acceleration. The acceleration is produced because of load torque. When the load torque is applied on the Separately excited DC motor , it produces the high armature current as compared to other DC machines. As a result the counter back emf is produced which is equal to the armature voltage. This back emf opposes the armature voltage and produces the negative acceleration that causes the negative speed. when the armature voltage overcomes back emf, the speed becomes positive.

The results given in Fig. 4 and Fig. 5 show the closed loop responses , reference signals and open-loop output. The solid line (-) represents the closed-loop output obtained using the partial feedback linearization controller, the green color dashed line (- -) represents the open-loop output and the red color dashed line (- -) represents the reference input signal.

For shunt motor $R_{adj} = 44$ ohm and other parameter values are same as separately excited DC motor

Table 3: Parameter values for separately excited DC motor

Parameter	Value
L_f	60 H
L_a	0.01H
R_a	1 Ohm
R_f	60Ohm
K	0.1238 Nm/A^2
B	0.0011 Nms/rad
j	0.00208Kgm^2

Table 4: Parameter values for PM Stepper DC motor

Parameter	Value
L	0.01H
R	0.6090 Ohm
K	0.507 Nm/A^2
B	0.000170 Nms/rad
j	0.0194Kgm^2

7 Conclusion

In this paper a design of partial feedback linearisation controller is presented for AC and DC machines. To achieve the desirable performance and attenuate the undesirable response the partial feedback linearisation technique is uesd. The performance of proposed controller in DC and AC machines is shown using simulation results. The performance of the controller for a variation of load is presented for different machines as well. The simulation results show that the proposed controller provides reliable and accurate control for both of AC and DC machines.

References

1. S. J. Chapman, *Electric machinery and power system fundamentals.* McGraw-Hill, 2002.
2. S. D. Umans, *Fitzgerald and Kingsley's electric machinery.* McGraw-Hill Higher Education, 2013.
3. S. K. Das, H. R. Pota, and I. R. Petersen, "Damping Controller Design for Nanopositioners: A Mixed Passivity, Negative-Imaginary, and Small-Gain Approach," *IEEE/ASME Transactions on Mechatronics*, vol. 20, no. 1, pp. 416–426, 2015.
4. M. A. Rahman, A. Al Mamun, K. Yao, and S. K. Das, "Design and Implementation of Feedback Resonance Compensator in Hard Disk Drive Servo System: A Mixed Passivity, Negative-Imaginary and Small-Gain Approach in Discrete Time," *Journal of Control, Automation and Electrical Systems*, vol. 26, no. 4, pp. 390–402, 2015.

5. Z. Z. Liu, F. L. Luo, and M. H. Rashid, “Speed Nonlinear control of DC Motor Drive With Field Weakening,” *IEEE Transactions on Industry Applications*, vol. 39, no. 2, pp. 417–423, 2003.
6. M. George *et al.*, “Speed Control of separately Excited DC Motor,” *American journal of applied sciences*, vol. 5, no. 3, pp. 227–233, 2008.
7. P. Olejnik and J. Awrejcewicz, “Low-Speed Voltage-Input Tracking Control of a DC-Motor Numerically Modelled by a Dynamical System with Stick-Slip Friction,” *Differential Equations and Dynamical Systems*, pp. 1–11, 2013.
8. B. Allaoua, B. Gasbaoui, and B. Mebarki, “Setting Up PID DC Motor Speed Control Alteration Parameters Using Particle Swarm Optimization Strategy,” *Leonardo Electronic Journal of Practices and Technologies*, vol. 14, pp. 19–32, 2009.
9. A. T. Azar and F. E. Serrano, “Robust IMC–PID Tuning for Cascade Control Systems with Gain and Phase Margin Specifications,” *Neural Computing and Applications*, vol. 25, no. 5, pp. 983–995, 2014.
10. S. Das, N. Mondol, M. Rana, and P. Das, “Genetic algorithm based optimal PI controller for position control of Maxon S-DC motor with dSPACE,” in *Informatics, Electronics & Vision (ICIEV), 2012 International Conference on.* IEEE, 2012, pp. 184–189.
11. A. T. Azar and F. E. Serrano, “Design and Modeling of Anti Wind up PID Controllers,” in *Complex System Modelling and Control Through Intelligent Soft Computations.* Springer, 2015, pp. 1–44.
12. T. Furuhashi, S. Sangwongwanich, and S. Okuma, “A Position-and-Velocity Sensorless Control for Brushless DC Motors Using an Adaptive Sliding Mode Observer,” *IEEE Transactions on Industrial electronics*, vol. 39, no. 2, pp. 89–95, 1992.
13. Q. Z. e. Ahmad Taher Azar, *Advances and Applications in Sliding Mode Control systems*, 1st ed., ser. Studies in Computational Intelligence 576. Springer International Publishing, 2015.
14. B. Meghni, D. Dib, and A. T. Azar, “A Second-Order Sliding Mode and Fuzzy Logic Control to Optimal Energy Management in Wind Turbine with Battery Storage,” *Neural Computing and Applications*, pp. 1–18, 2016.
15. T.-C. Chen and T.-T. Sheu, “Model Reference Neural Network Controller for Induction Motor Speed Control,” *IEEE Transactions on Energy Conversion*, vol. 17, no. 2, pp. 157–163, 2002.
16. M. A. Rahman and M. A. Hoque, “Online Self-Tuning ANN-Based Speed Control of a PM DC Motor,” *IEEE/ASME Transactions On Mechatronics*, vol. 2, no. 3, pp. 169–178, 1997.
17. A. T. Azar and S. Vaidyanathan, *Handbook of Research on Advanced Intelligent Control Engineering and Automation.* IGI Global, 2014.
18. A. H. O. Ahmed, “Optimal Speed Control for Direct Current Motors Using Linear Quadratic Regulator,” *Journal of Science and Technology*, vol. 13, pp. 1605–427, 2012.
19. M. N. Uddin and M. A. Rahman, “High-Speed Control of IPMSM Drives Using Improved Fuzzy Logic Algorithms,” *IEEE transactions on industrial electronics*, vol. 54, no. 1, pp. 190–199, 2007.
20. S. Bolognani, S. Bolognani, L. Peretti, and M. Zigliotto, “Design and Implementation of Model Predictive Control for Electrical motor Drives,” *IEEE Transactions on industrial electronics*, vol. 56, no. 6, pp. 1925–1936, 2009.
21. H. Grabner, W. Amrhein, S. Silber, and W. Gruber, “Nonlinear Feedback Control of a Bearingless Brushless DC Motor,” *IEEE/ASME transactions on mechatronics*, vol. 15, no. 1, pp. 40–47, 2010.
22. A. T. Azar and S. Vaidyanathan, *Chaos Modeling and Control Systems Design.* Springer, 2015, vol. 581.
23. S. Das, N. Mondol, and N. Sultana, “Design And Implement Of A State Feedback Position Output Controller For A Maxon S-Dc Motor With Dspace,” 2011.
24. M. Mahmud, H. Pota, M. Aldeen, and M. Hossain, “Partial Feedback Linearizing Excitation Controller for Multimachine Power Systems to Improve Transient Stability,” *IEEE Transactions on Power Systems*, vol. 29, no. 2, pp. 561–571, 2014.
25. M. Mahmud, H. Pota, and M. Hossain, “Full-Order Nonlinear Observer-Based Excitation Controller Design for Interconnected Power Systems via Exact Linearization Approach,” *International Journal of Electrical Power & Energy Systems*, vol. 41, no. 1, pp. 54–62, 2012.
26. H. K. Khalil and J. Grizzle, “Nonlinear systems,” 1996.
27. J.-J. E. Slotine, W. Li *et al.*, *Applied nonlinear control.* prentice-Hall Englewood Cliffs, NJ, 1991, vol. 199, no. 1.
28. A. Isidori, *Nonlinear control systems.* Springer Science & Business Media, 2013.
29. M. Mahmud, M. Hossain, and H. Pota, “Nonlinear Excitation Controller for Power Systems Using Zero Dynamic Design Approach,” in *North American Power Symposium (NAPS), 2011.* IEEE, 2011, pp. 1–6.
30. T. Von Raumer, J. M. Dion, L. Dugard, and J.-L. Thomas, “Applied Nonlinear Control of an Induction Motor Using Digital Signal Processing,” *IEEE Transactions on Control Systems Technology*, vol. 2, no. 4, pp. 327–335, 1994.
31. J. H. Chang and B. K. Kim, “Minimum-time Minimum-Loss Speed Control of Induction Motors Under Field-Oriented Control,” *IEEE transactions on industrial electronics*, vol. 44, no. 6, pp. 809–815, 1997.
32. S. Mehta and J. Chiasson, “Nonlinear Control of a Series DC Motor: Theory and Experiment,” *IEEE Transactions on industrial electronics*, vol. 45, no. 1, pp. 134–141, 1998.
33. J. Chiasson, “Nonlinear Differential-Geometric Techniques for Control of a Series DC Motor,” *IEEE Transactions on Control Systems Technology*, vol. 2, no. 1, pp. 35–42, 1994.
34. A. P. Memon, W. A. Khan, R. H. Memon, and A. A. Akhund, “Laboratory Studies of Speed Control of DC Shunt Motor and the Analysis of Parameters Estimation,” *International Journal of Emerging Trends in Electrical and Electronics (IJETEE–ISSN: 2320-9569)*, vol. 9, no. 1, 2013.
35. J. Chiasson and M. Bodson, “Nonlinear Control of a Shunt DC Motor,” *IEEE Transactions on Automatic Control*, vol. 38, no. 11, pp. 1662–1666, 1993.
36. M. Bodson and J. Chiasson, “Nonlinear and Adaptive Control of a Shunt DC Motor,” in *Systems Engineering, 1991., IEEE International Conference on.* IEEE, 1993, pp. 73–76.
37. M. Zribi and J. Chiasson, “Position Control of a PM Stepper Motor by Exact Linearization,” *IEEE Transactions on Automatic Control*, vol. 36, no. 5, pp. 620–625, 1991.
38. A. T. Azar and F. E. Serrano, “Stabilization of Mechanical Systems with Backlash by PI Loop Shaping,” *International Journal of System Dynamics Applications (IJSDA)*, vol. 5, no. 3, pp. 21–46, 2016.