Knowledge Representation

Lecture 4: Description Logic Axioms, Reasoning and OWL

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November 6, 2023

The Story so far...

Ontologies:

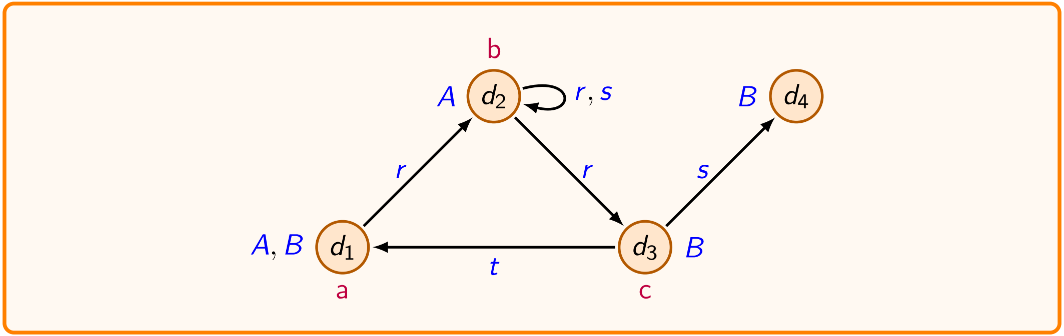
- Formalize conceptualizations
- Fix the meaning of terminology

Description Logics:

- Formalism for specifying ontologies
- The vocabulary consists of concept names, role names and individual names
- Interpretations fix their meaning
- $\mathcal{ALC}$ Concepts can be build using the operators $\top$, $\bot$, $\sqcap$, $\sqcup$, $\neg$, $\exists$, and $\forall$
- A DL ontology is a set of axioms, consisting of to parts:
  - TBox: terminological axioms (GCIs, equivalence axioms)
  - ABox: assertional axioms (concept and role assertions)

Exercise: Axioms

Let's get back to the interpretation $\mathcal{I}$ from last week:



Which of the following axioms does it satisfy:

1. $(a, b) : \; r$
2. $c : \; A$
3. $c : \; \exists s.B$
4. $A \sqsubseteq \exists r.A$
5. $A \sqsubseteq \forall r.(A \sqcup B)$
6. $A \equiv \forall r.(A \sqcup B)$
7. $\exists r.\top \sqsubseteq A$
8. $\exists r.\bot \sqsubseteq B$
9. $\exists r.A \sqsubseteq \forall s.A$

Ontologies

An ontology is a set $\mathcal{O} = \mathcal{A} \cup \mathcal{T}$, where

- $\mathcal{A}$ is an ABox, a finite set of assertions,
- $\mathcal{T}$ is a TBox, a finite set of GCIs,

An interpretation is a model of $\mathcal{O}$ (written $\mathcal{I} \models \mathcal{O}$) if it is a model of all axioms in $\mathcal{O}$.

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- The ABox contains facts about named individuals (data), the TBox contains (terminological) knowledge that applies to all individuals.

Example: An Ontology

$\mathcal{O} = \mathcal{A} \cup \mathcal{T}$ with

$\mathcal{A} = \{\mathit{WN\text{-}KC137} : \ \mathit{Room}, \ (\mathit{WN\text{-}KC137}, \mathit{W\&N}) : \ \mathit{partOf}\}$

$$\mathcal{T} = \{\mathit{Room} \sqsubseteq \neg \mathit{University}, \ \mathit{Room} \sqsubseteq \exists \mathit{partOf}.\mathit{Building}, \\ \mathit{Building} \sqsubseteq \neg \mathit{University}, \ \mathit{Building} \sqsubseteq \neg \mathit{Room}\}$$

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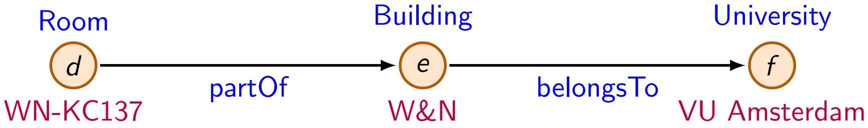
This ontology has many models, for example the following:



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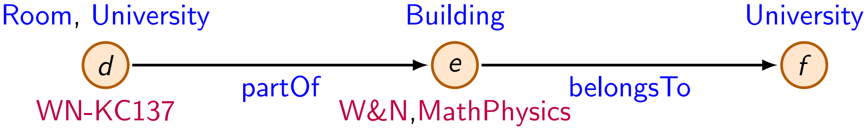
The following interpretation is not a model of the ontology:



Flashback: What is Knowledge Representation?

- KR as surrogate
- KR as expression of ontological commitment
- **KR as theory of intelligent reasoning**
  - How to deduce *implicit information*
  - What can be deduced? What should be deduced?
  - Foundation in logics one, but not the only possibility
- KR as medium for efficient computation
- KR as medium of human expression

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- We need to consider what all models have in common.
- This is the same as in propositional and first-order logic.

Example: Entailment

$\mathcal{O} = \mathcal{A} \cup \mathcal{T}$ with

$$\mathcal{A} = \{ \quad \mathit{drKoopmann} : \mathit{AssistantProfessor}, \qquad (\mathit{drKoopmann}, \mathit{KR}) : \mathit{teaches}, \\ (\mathit{mrsAbadi}, \mathit{KR}) : \mathit{attends} \quad \}$$

$$\mathcal{T} = \{ \quad \mathit{AssistantProfessor} \sqsubseteq \mathit{TeachingPersonal}, \qquad \exists \mathit{attends}.\mathit{Course} \sqsubseteq \mathit{Student}, \\ \mathit{TeachingPersonal} \sqsubseteq \forall \mathit{teaches}.\mathit{Course} \quad \}$$

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This ontology entails:

1. $drKoopmann : \ \exists teaches.\top$

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3. $KR : \; Course$

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It does not entail:

1. $\mathit{drKoopmann} : \neg \mathit{Student}$
2. $\mathit{mrsAbadi} : \neg \mathit{TeachingPersonal}$
3. $\mathit{AssistantProfessor} \sqsubseteq \exists \mathit{teaches}.\mathit{Course}$

Because in each case, we can build a model of the ontology in which the axiom is not satisfied.

Entailment is a quite general reasoning task.

With ontologies, we usually have more specific questions we want to answer.

Typical reasoning tasks are the following:

- Subsumption-relationships between concepts
- Individuals in a given concept
- Consistency and coherence of an ontology

Reasoning: Relationships between Concepts

Let C, D be concepts and $a \in \mathbf{I}$.

- If $\mathcal{O} \models C \sqsubseteq D$, we say that C is subsumed by D wrt. $\mathcal{O}$. $C \sqsubseteq_{\mathcal{O}} D$
- If $\mathcal{O} \models C \equiv D$, we say that C is equivalent to D wrt. $\mathcal{O}$. $C \equiv_{\mathcal{O}} D$
- If $C \sqsubseteq_{\mathcal{O}} D$ and $C \not\equiv_{\mathcal{O}} D$, C is strictly subsumed by D wrt. $\mathcal{O}$. $C \sqsubset_{\mathcal{O}} D$
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$$\{\mathit{AILecture} \sqsubseteq \mathit{Lecture},\ \mathit{Lecture} \sqsubseteq \mathit{Course}\} \models \mathit{AILecture} \sqsubseteq \mathit{Course}$$

$$\{\mathit{AILecture} \sqsubseteq \mathit{Lecture},\ \mathit{Lecture} \sqcap \mathit{Room} \sqsubseteq \bot\} \models \mathit{AILecture} \sqcap \mathit{Room} \sqsubseteq \bot$$

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$$\{AILecture \sqsubseteq Lecture,\ Lecture \sqsubseteq Course\} \models AILecture \sqsubseteq Course$$

$$\{AILecture \sqsubseteq Lecture,\ Lecture \sqcap Room \sqsubseteq \bot\} \models AILecture \sqcap Room \sqsubseteq \bot$$

If $C \sqsubset D$, then C is more specific than D (w.r.t. $\mathcal{O}$), and D is more general than C (w.r.t. $\mathcal{O}$).

Reasoning: Consistency

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This means that checking consistency is a kind of (non-)entailment test.

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Theorem: If $\mathcal{O}$ is inconsistent, then $\mathcal{O} \models \alpha$ for any axiom α.

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- Some concepts are unsatisfiable w.r.t. any consistent ontology, e.g. $\exists r.\bot$.

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- If all concept names in $\mathcal{O}$ are satisfiable w.r.t. $\mathcal{O}$, then $\mathcal{O}$ is coherent.

- Unsatisfiable concept names indicate an error in the ontology:
  Why use a concept name A if it can only be interpreted as $A^{\mathcal{I}} = \bot^{\mathcal{I}} = \emptyset$?
- Consistency and coherence are basic requirements for any ontology.

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- Materialization is the task of computing all entailments of the form $\mathcal{O} \models a : A$ and $\mathcal{O} \models (a, b) : r$, where $a, b \in \mathbf{I}$, $A \in \mathbf{C}$, and $r \in \mathbf{R}$.

- Classification and materialization make explicit much of the knowledge that is implicitly given by the ontology.

Example: Consistency and Coherence

$$\{\textit{Felix} : \textit{Cat}, \quad \textit{Cat} \sqsubseteq \textit{Animal}, \quad (\textit{Felix}, \textit{Toby}) : \textit{hasFather}, \quad \exists \textit{hasFather}.\top \sqsubseteq \textit{Human}\}$$

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is inconsistent.

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is inconsistent.

$$\{Human \sqcap Animal \sqsubseteq \bot, \quad Werewolf \sqsubseteq Human \sqcap Wolf, \quad Wolf \sqsubseteq Animal\}$$

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$$\{\mathit{Human} \sqcap \mathit{Animal} \sqsubseteq \bot, \quad \mathit{Werewolf} \sqsubseteq \mathit{Human} \sqcap \mathit{Wolf}, \quad \mathit{Wolf} \sqsubseteq \mathit{Animal}\}$$

is consistent, but not coherent, because Werewolf is unsatisfiable.

Example: Consistency and Coherence

$$\{Felix : Cat, \quad Cat \sqsubseteq Animal, \quad (Felix, Toby) : hasFather, \quad \exists hasFather.\top \sqsubseteq Human, \quad Human \sqcap Animal \sqsubseteq \bot\}$$

$$\{Human \sqcap Animal \sqsubseteq \bot, \quad Werewolf \sqsubseteq Human \sqcap Wolf, \quad Wolf \sqsubseteq Animal\}$$

Disjointness axioms ($C \sqsubseteq \neg D$, or equivalently $C \sqcap D \sqsubseteq \bot$) are very useful for debugging ontologies, because they can expose hidden contradictions.

Last Exercise on Reasoning

$$\mathcal{O} = \{ \quad \begin{aligned} \mathit{Alive} &\sqsubseteq \mathit{Animal} \sqcup \mathit{Plant} \\ \mathit{Animal} &\sqsubseteq \exists \mathit{hasParent}.\mathit{Male} \sqcap \exists \mathit{hasParent}.\mathit{Female} \\ \mathit{thomas} &: \mathit{Alive} \\ \mathit{thomas} &: \forall \mathit{hasParent}.\bot \end{aligned} \quad \}$$

What can we say about Thomas?

Reasoning in Practice

- Later in this course, we will learn algorithms to perform these reasoning tasks.
- Until then, we will use special programs (reasoners) for that.
- But first, a few important aspects of DL ontologies.

DL Ontologies are not Programs

- There is no "execution order" of axioms
- There is no "direction" in which axioms are applied:

$$\mathcal{O} = \{ \quad A \sqsubseteq \exists r.B, \quad \exists r.B \sqsubseteq C, \\ a : A, \quad b : \neg C \quad \}$$

We have both $\mathcal{O} \models a : C$ and $\mathcal{O} \models b : \neg A$

Open World and Open Domain

DLs make the open-world assumption, i.e. facts that are not entailed are not necessarily false, but simply unknown.

$\{(\mathit{Bob}, \mathit{Fred}) : \mathit{hasChild}\}$ does not entail $\mathit{Bob} : \mathit{Father}$ or $\mathit{Bob} : \neg\mathit{Father}$.

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$\{(\mathit{Bob}, \mathit{Fred}) : \mathit{hasChild}\}$ does not entail $\mathit{Bob} : \mathit{Father}$ or $\mathit{Bob} : \neg\mathit{Father}$.

They also make the open-domain assumption, i.e. there may be individuals we do not know, and that have no name.

$$\mathcal{O} = \{ \mathit{Human} \sqsubseteq \exists \mathit{hasParent}.\mathit{Mother}, \ \mathit{peter} : \mathit{Human}\}$$
$$\models \mathit{peter} : \exists \mathit{hasParent}.\mathit{Mother}$$
$$\not\models a : \mathit{Mother} \text{ for any } a \in \mathbf{I}$$

Binary vs. *n*-ary Relations

Description logics can only express unary and binary relations.

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could be reformulated as

$Diagnosis(d112)$, $hasDiagnosis(Bob, d112)$,
$associatedDisease(d112, Flu)$, $associatedProbability(d112, High)$

This process is called reification.

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$\mathit{associatedDisease}(d112, \mathit{Flu}), \quad \mathit{associatedProbability}(d112, \mathit{High})$

This process is called reification.

This can lead to more complex axioms, because one has to refer to several roles.

$\exists \mathit{hasDiagnosis}.\exists \mathit{associatedDisease}.\mathit{InfectiousDisease} \sqsubseteq \mathit{Infectious}$

Ontologies in Practice: OWL

Flashback: What is Knowledge Representation?

- KR as surrogate
- KR as expression of ontological commitment
- KR as theory of intelligent reasoning
- KR as medium for efficient computation
- **KR as medium of human expression**
  - humans produce, consume and work with representations
  - representations must be human understandable, but also manageable by humans

Ontologies as Medium of Human Expression

- DL syntax convenient for teaching and research
  - Concise
  - Convenient to write with a pen
  - DLs are *logics*

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  - machine-processable
  - manage large, complex ontologies
  - collaboration

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$\Rightarrow$ Logic is not everything

Requirements for Ontology Languages

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- documentation / comments
- organisation of ontologies into separate modules / files

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  - Cow in Ontology 1 should refer to the same concept as in Ontology 2
- multi-language support
  - Cow means “cow” in English, “Kuh” in German, “vache” in French

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  - Cow in Ontology 1 should refer to the same concept as in Ontology 2
- multi-language support
  - Cow means “cow” in English, “Kuh” in German, “vache” in French

Again others are important for authorship and trust

- Who created/modified an ontology/axiom?
- When was an axiom modified?

OWL 2: Overview

OWL 2 is a standardized ontology language based on XML and RDF that was developed for the Semantic Web, and is used in many applications.

`https://www.w3.org/TR/owl2-overview/`

- **W**eb **O**ntology **L**anguage
- OWL 1 was specified in 2004 by the World Wide Web Consortium (W3C)
- OWL 2 was specified in 2009 by the W3C
- The final standard encompasses different sublanguages
  - more on this later

OWL 2 Terminology

OWL 2 uses different terminology than description logics:

DL	$\rightsquigarrow$	OWL
name	$\rightsquigarrow$	entity
concept name	$\rightsquigarrow$	(named) class
role name	$\rightsquigarrow$	(named) object property
individual name	$\rightsquigarrow$	(named) individual
concept (description)	$\rightsquigarrow$	class expression
axiom	$\rightsquigarrow$	axiom

Syntaxes

OWL 2 ontologies can be written in different formats.

Functional Syntax (used in the OWL 2 specification and the OWL API)

```
SubClassOf( Lecture  Course )
```

RDF/XML Syntax (main interchange format)

```
<owl:Class rdf:about="Lecture">
  <rdfs:subClassOf rdf:resource="Course">
</owl:Class>
```

Syntaxes II

OWL/XML Syntax

```
<SubClassOf>
  <Class IRI="Lecture"><Class IRI="Course">
</SubClassOf>
```

Turtle Syntax

```
Lecture rdfs:subClassOf Course
```

Manchester Syntax (used by Protégé and in this lecture)

```
Class: Lecture
  SubClassOf: Course
```

IRIs

In OWL 2, the ontology and every entity (class, property, individual, datatype) is uniquely identified by an Internationalized Resource Identifier (IRI).

> http://kai.vu.nl/university-ontology#Lecture
> http://kai.vu.nl/university-ontology#Course

Ideally, IRIs should be dereferenceable, e.g. lead to a web page about the entity.

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Prefix definitions are used to abbreviate IRIs.

`Prefix:` uo: http://kai.vu.nl/university-ontology#

Complex Expressions in OWL Manchester Syntax

Let C, D, r, a be DL names or OWL entities, depending on the context.

DL syntax	Manchester syntax	Remark
$\top$	owl:Thing	(a special named class)
$\bot$	owl:Nothing	(a special named class)
$C \sqcap D$	C `and` D	
$C \sqcup D$	C `or` D	
$\neg C$	`not` C	
$\exists r.C$	r `some` C	
$\forall r.C$	r `only` C	
$C \sqsubseteq D$	C `SubClassOf` D	

Protégé

Protégé is a freely available IDE for developing OWL ontologies

The structure of an ontology, as it is displayed in Protégé, is a bit different to what we have seen so far:

- Usually, you do not directly enter axioms
- Rather, you add information to classes (concept names):
  - what are they equivalent to
  - what are they subsumed by
- Also, you have to explicitly specify your vocabulary before you can use it
  - every class, object property and individual has to be added before you can use it

Little Tour of Protégé

We look at some basics of Protégé:

- Adding classes and properties
- Adding logical information on classes
- Adding general class axioms
- Opening and saving OWL files
- Loading example ontologies
- Using a reasoner
- Querying complex concepts
- Explaining reasoning results

Accessing OWL from Software

There are various libraries to access and work with OWL ontologies and reasoners:

- OWL API: the most comprehensive available library, available for Java
  - Use of *software design patterns* might make this harder for less experienced software developers
- OwlReady2: a library for Python that allows to integrate OWL classes with Python classes
  - Optimized for specific use cases (querying data, reasoning, ...)
- DeepOnto: a Python library for ontology engineering with Deep learning
  - quite recent
- dl4python: Description Logic view on OWL
  - special development for this course
  - can be used from Python, Scala and Java
  - example file illustrates relevant functionalities

Some Advice on Creating Ontologies

Class Hierarchy

- Before adding complex axioms, first define the class hierarchy ($\sqsubseteq$-axioms).
- Flesh out the hierarchy with common superconcepts, missing siblings.

Organism

- Animal
  - Mammal
    - Cat
    - ...
  - Fish
    - Trout
    - ...
  - Carnivore
  - Herbivore
  - Omnivore
- Plant
  - Tree
  - Grass
  - Wheat

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- Ideally, much of this information was already elicited, otherwise we have to ask the domain experts again.
- Once the class hierarchy is fixed, we can add definitions.

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- Animal
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Definitions

Identify which terms should be defined:

- Depends on the goals of the ontology
- General terms like "Organism" probably don't need a definition
- Some terms are easier to define than others, e.g. "Cat" vs. "Carnivore".
- For many terms, the information about their place in the class hierarchy is enough.

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- For many terms, the information about their place in the class hierarchy is enough.

Intensional definitions consist of the superclass(es) and any distinguishing characteristics.

> A cat is a mammal that has paws, legs, and a tail.
>
> A carnivore is an animal that eats only meat.
>
> A pet is a domesticated animal that lives with humans.

Definitions (II)

Distinguish between full definitions ($\equiv$) and partial definitions ($\sqsubseteq$)!

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$$Carnivore \equiv Animal \sqcap \forall eats.Meat$$
$$Herbivore \equiv Animal \sqcap \forall eats.Plant$$
$$Pet \equiv Animal \sqcap \exists livesWith.Human$$
$$Animal \equiv Organism \sqcap \exists eats.Organism$$

$$Cat \sqsubseteq Mammal \sqcap \exists bodyPart.Paw \sqcap \exists bodyPart.Leg \sqcap \exists bodyPart.Tail$$
$$Cow \sqsubseteq Mammal \sqcap \forall eats.Grass$$

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Distinguish between full definitions ($\equiv$) and partial definitions ($\sqsubseteq$)!

$$\mathit{Carnivore} \equiv \mathit{Animal} \sqcap \forall \mathit{eats}.\mathit{Meat}$$
$$\mathit{Herbivore} \equiv \mathit{Animal} \sqcap \forall \mathit{eats}.\mathit{Plant}$$
$$\mathit{Pet} \equiv \mathit{Animal} \sqcap \exists \mathit{livesWith}.\mathit{Human}$$
$$\mathit{Animal} \equiv \mathit{Organism} \sqcap \exists \mathit{eats}.\mathit{Organism}$$

$$\mathit{Cat} \sqsubseteq \mathit{Mammal} \sqcap \exists \mathit{bodyPart}.\mathit{Paw} \sqcap \exists \mathit{bodyPart}.\mathit{Leg} \sqcap \exists \mathit{bodyPart}.\mathit{Tail}$$
$$\mathit{Cow} \sqsubseteq \mathit{Mammal} \sqcap \forall \mathit{eats}.\mathit{Grass}$$

- Often, not everything can be fully defined, due to the restrictions of the ontology language.
- We will later see a more expressive DL, but this one will be restricted too.

Class Hierarchy (II)

In general, the class hierarchy is not a tree, but a directed acyclic graph with multiple inheritance.

$$Cow \sqsubseteq Mammal \qquad Cow \sqsubseteq Herbivore$$

(*Mammal* and *Herbivore* are unrelated)

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(Mammal and $\mathit{Herbivore}$ are unrelated)

Instead of specifying all subclass-superclass relationships, it is easier to specify only a tree and let the reasoner infer the implicit ones.

$$\mathit{Grass} \sqsubseteq \mathit{Plant} \qquad \mathit{Mammal} \sqsubseteq \mathit{Animal}$$

$$\mathit{Herbivore} \equiv \mathit{Animal} \sqcap \forall \mathit{eats}.\mathit{Plant}$$

$$\mathit{Cow} \sqsubseteq \mathit{Mammal} \sqcap \forall \mathit{eats}.\mathit{Grass}$$

This entails $\mathit{Cow} \sqsubseteq \mathit{Herbivore}$, so we do not have to explicitly add this axiom to the ontology.

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Instead of specifying all subclass-superclass relationships, it is easier to specify only a tree and let the reasoner infer the implicit ones.

Definitions can affect the (inferred) class hierarchy.

Note: “Some” Does Not Mean “Only”

When writing definitions, it is not trivial to find the right one.

A common modeling error is to swap $\forall$ and $\exists$:

$$\mathit{Grass} \sqsubseteq \mathit{Plant}$$

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$$Grass \sqsubseteq Plant$$

$$Herbivore \equiv Animal \sqcap \forall eats.Plant$$

$$Cow \sqsubseteq Mammal \sqcap \exists eats.Grass$$

Cow is not subsumed by Herbivore!

(A cow must eat "at least 1 Grass", but could eat other things.)

Note: "Only" Does Not Mean "Some"

$$Cow \sqsubseteq \forall eats.Grass$$

Cow is not subsumed by $\exists eats.Grass$, not even $\exists eats.\top$.

(A cow can eat only Grass, but does not have to eat anything.)

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Cow is not subsumed by $\exists eats.Grass$, not even $\exists eats.\top$.

(A cow can eat only Grass, but does not have to eat anything.)

$Animal \equiv Organism \sqcap \exists eats.Organism$

$Mammal \sqsubseteq Animal$

$Cow \sqsubseteq Mammal \sqcap \forall eats.Grass$

entails $Cow \sqsubseteq \exists eats.Grass$.

Note: “And” Does Not Mean “Or”

“Cows eat grass and grain.”

$$Cow \sqsubseteq \forall eats.(Grass \sqcap Grain) \qquad Grass \sqsubseteq \neg Grain$$

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Cow and $\exists eats.\top$ are disjoint!

(A cow can eat only things that are at the same time Grass and Grain, which do not exist.)

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$$Cow \sqsubseteq \forall eats.(Grass \sqcup Grain) \qquad Grass \sqsubseteq \neg Grain$$

- Axioms like $Cow \sqsubseteq \forall eats.(Grass \sqcup Grain)$ are called closure axioms, because they define the range of $eats$ for all cows.

Closure Axioms

Often, closure axioms are used in combination with existential restrictions:

$$\begin{aligned} \mathit{Book} &\sqsubseteq \exists \mathit{hasPart}.\mathit{Cover} \\ \mathit{Book} &\sqsubseteq \exists \mathit{hasPart}.\mathit{Page} \\ \mathit{Book} &\sqsubseteq \forall \mathit{hasPart}.(\mathit{Cover} \sqcup \mathit{Page}) \end{aligned}$$

This is like saying *“A book has those parts and no other.”*

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$$
\begin{aligned}
\mathit{Cell} &\sqsubseteq \exists \mathit{hasPart}.\mathit{PlasmaMembrane}\\
\mathit{Cell} &\sqsubseteq \exists \mathit{hasPart}.\mathit{Mitochondrion}\\
\mathit{Cell} &\sqsubseteq \exists \mathit{hasPart}.\mathit{EndoPlasmicReticulum}\\
\mathit{Cell} &\sqsubseteq \exists \mathit{hasPart}.\mathit{Nucleus}\\
\mathit{Cell} &\sqsubseteq \forall \mathit{hasPart}.(\mathit{PlasmaMembrane}\\
&\qquad\qquad\qquad \sqcup \mathit{Mitochondrion}\\
&\qquad\qquad\qquad \sqcup \mathit{EndoPlasmicReticulum}\\
&\qquad\qquad\qquad \sqcup \mathit{Nucleus})
\end{aligned}
$$

This is like saying *"A cell has those parts and no other."*

Covering Axioms

- Closure axioms are one way of *closing* descriptions under the open world assumption
- Another such technique is using covering axioms
- Covering axioms work well in combination with disjointness axioms

$$\begin{aligned} Herbivore &\equiv Animal \sqcap \forall eats.Plant \\ Carnivore &\equiv Animal \sqcap \forall eats.Animal \\ Animal &\equiv Herbivore \sqcup Carnivore \sqcup Omnivore \\ Animal &\sqsubseteq \neg Plant \end{aligned}$$

- These axioms entail:

$$Animal \sqcap \exists eats.Animal \sqcap \exists eats.Plant \sqsubseteq Omnivore$$

Disjointness Axioms

- Covering axioms work well in combination with disjointness axioms
- Disjointness axioms furthermore help finding bugs via incoherence

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- Covering axioms work well in combination with disjointness axioms
- Disjointness axioms furthermore help finding bugs via incoherence
- OWL (and Protégé) offers convenient syntactic sugar:
  - Make a set of class expressions pair-wise disjoint
  - Define a class as disjoint union

Note: Value Restrictions on the Left-Hand Side

Value restrictions can behave strangely on the left-hand side of GCIs (and therefore also in full definitions).

$$\forall eats.Grass \sqsubseteq Cow$$

$$Cow \equiv \forall eats.Grass$$

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With one of these axioms, anything that does not eat is a cow.

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Value restrictions can behave strangely on the left-hand side of GCIs (and therefore also in full definitions).

$$\forall eats.Grass \sqsubseteq Cow$$
$$Cow \equiv \forall eats.Grass$$

With one of these axioms, anything that does not eat is a cow.

$$Tornado \sqsubseteq \neg Organism$$
$$\exists eats.\top \sqsubseteq Organism$$
$$\forall eats.Grass \sqsubseteq Cow$$

entail $Tornado \sqsubseteq \forall eats.\bot \sqsubseteq Cow$.