Multi-Agent Systems

Homework Assignment 2

MSc AI, VU

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Version: November 15, 2023— Deadline: Wed November 22, 2023 (23h59)

2 Game Theory: Nash equilibrium

2.1 Dining out

Alice and Bob are going to dinner and plan to split the bill evenly no matter who orders what. There are two meals, a cheap one (C) priced at 10 Euro, which gives each of them 12 Euro's worth of pleasure, and an expensive dinner (E) dinner priced at 20 Euro, which gives them each 18 Euro's worth of pleasure.

1. Write down the pay-off matrix.
2. Assuming that they both order simultaneously and without coordinating, what will they order and why?
3. Alice is quite the romantic type and gets an additional s Euro's worth of pleasure if they happen to pick the same meal (either both cheap or both expensive). Bob, on the other hand, is a bit of a contrarian and gets an additional amount of pleasure (also equivalent to s Euro) when they happen to favour different meal choices. Assume that $0 < s \leq 2$. How does this change the pay-off matrix and the Nash equilibrium (or equilibria) of this game?

2.2 Hawk versus Dove

Two animals are in conflict over some resource worth $v > 0$. Simultaneously, they choose whether to behave like hawks (H) or doves (D). Hawks are willing to fight over the good, whereas doves are not. So if one animal chooses hawk and the other dove, the hawk gets everything leaving nothing for the dove. If both behave like doves, they split the resource equally. If however, both adopt an hawk strategy, they fight and on average get half of the food. The fighting however comes at a cost c to both of them.

Questions

- Write down the pay-off matrix for this game.
- Determine the Nash equilibria for this game and discuss how they change as the cost of aggression (c) increases. Do your results make sense?

2.3 Investment in recycling

Two neighbouring countries, $i = 1, 2$, simultaneously choose how many resources r_i (in hours) to spend in recycling activities. The average benefit (for country i) *per hour spent* equals

$$b_i(r_i, r_j) = 10 - r_i + \frac{r_j}{2}$$

Notice that country i's average benefit is increasing in the resources that neighbouring country j spends on his recycling because a clean environment produces positive external effects on other countries. The (opportunity) cost per hour for each country is 4. Hence, the expected utility for country i equals:

$$u_i(r_i, r_j) = b_i(r_i, r_j)r_i - 4r_i.$$

Questions

1. Determine each country's best-response function.
2. Indicate the pure strategy Nash Equilibrium (r_1^*, r_2^*) on the graph;
3. On your previous figure, show how the equilibrium would change if the intercept of one of the countries' average benefit functions b_i fell from 10 to some smaller number. What would this mean for the recycling efforts of both countries?

2.4 Tragedy of the Commons

- n players sharing some common resource (of total size 1)
  - E.g., village green, bandwidth in network, etc.
- Each player i would like to have as big a share $(0 \leq x_i \leq 1)$ as possible!
- However, each player's utility (pay-off) depends on what the others do:

$$u_i(x_i, x_{-i}) = \begin{cases} x_i\,(1 - \sum_{j=1}^{n} x_j) & \text{if } \sum_j x_j < 1 \\ 0 & \text{otherwise} \end{cases}$$

  One way to interpret this utility is that in order to have maximum utility there has to be sufficient (unused) "slack" to accommodate small fluctuations. Think of a highway: initially more cars means more throughput, but at some point the increase in density starts to hamper throughput.

Questions

- Consider the special case where there are only two players (i.e. $n = 2$). Determine the individual shares x_1 and x_2 in the Nash equilibrium for this game.
- Does this Nash equilibrium optimise social welfare which is the aggregated utility of all players (i.e. $u_1(x) + u_2(x)$?
- Can you generalise this result to arbitrary n?