VRIJE UNIVERSITEIT AMSTERDAM

Knowledge Representation

2023/2024

Exercise Sheet 1 – Classical and Description Logics

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Exercise 1.1 Consider an interpretation I with

$$I(a) = \textbf{false} \qquad I(b) = \textbf{false} \qquad I(c) = \textbf{false} \qquad I(d) = \textbf{false}$$

Which of the following propositional formulas are satisfied by this interpretation:

(a) $(a \land b) \lor \neg c \lor \neg d$

(b) $(a \land b) \lor (\neg c \land d)$

(c) $(a \rightarrow \neg b) \lor (\neg c \rightarrow d)$

(d) $(\neg a \rightarrow b) \land (c \rightarrow \neg d)$

Exercise 1.2 Consider the two formulas

$$F = \neg p \land \neg q \qquad G = p \rightarrow q$$

(a) Which truth assignments to p and q give models of F and G?

(b) Does F entail G?

(c) Does G entail F?

Exercise 1.3 Assume that we use propositional logic for knowledge representation and that the reasoning problem we have to solve is the following: given a formula F, decide whether F is satisfiable. Which of the following algorithms is sound/complete/terminating?

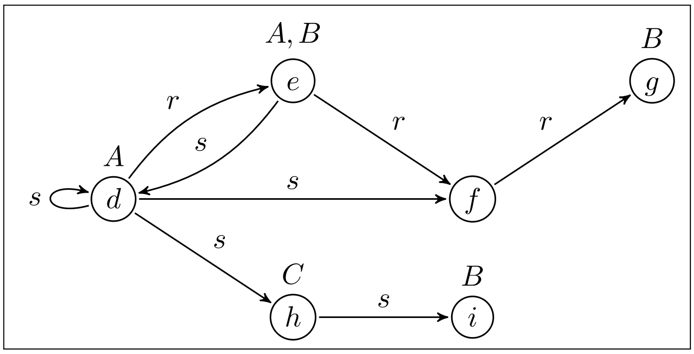
(a) Always return "yes".

(b) Always return "no".

(c) Enter an infinite loop, never return.

(d) Go through all truth assignments for the variables in F one after the other. For each truth assignment, check whether it gives a model for F. If a satisfying truth assignment is found, return "yes". Otherwise return "no".

Exercise 1.4 Consider the interpretation $\mathcal{I}$ represented as the following graph:



For each of the following concepts D, write down the elements in their interpretation $D^{\mathcal{I}}$:

(a) $\neg A$

(b) $A \sqcap \neg B$

(c) $\exists r.(A \sqcup B)$

(d) $\forall r.(A \sqcup B)$

(e) $\exists s.\exists s.\neg A$

(f) $\exists r.\forall r.\neg A$

Exercise 1.5

(a) Express the following phrases using $\mathcal{ALC}$ concepts:

(i) "persons that do not have a friendly neighbour,"

(ii) "persons that have a neighbour that is not friendly."

(b) Construct an interpretation where an element satisfies the concept for (i), but not the concept for (ii).

(c) Construct an interpretation where an element satisfies the concept for (ii), but not the concept for (i).

Exercise 1.6 Which of the following $\mathcal{ALC}$ axioms corresponds to the following English statement:

- "A Mule is an animal that has a horse and a donkey as a parent."

(a) $\mathrm{Mule} \equiv \mathrm{Animal} \sqcap \exists \mathrm{hasParent}.(\mathrm{Horse} \sqcap \mathrm{Donkey})$

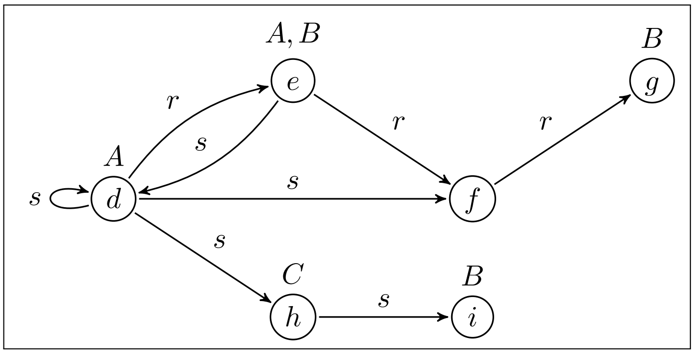
(b) $\mathrm{Mule} \equiv \mathrm{Animal} \sqcap \exists \mathrm{hasParent}.\mathrm{Horse} \sqcap \exists \mathrm{hasParent}.\mathrm{Donkey}$

(c) $\mathrm{Mule} \equiv \mathrm{Animal} \sqcap (\exists \mathrm{hasParent}.\mathrm{Horse} \sqcup \exists \mathrm{hasParent}.\mathrm{Donkey})$

Exercise 1.7 Translate the following axiom into English:

$$\mathrm{KRTeacher} \equiv \exists \mathrm{teaches}.(\mathrm{Course} \sqcap \forall \mathrm{hasTopic}.(\mathrm{DL} \sqcup \mathrm{Arg} \sqcup \mathrm{PGM}))$$

Exercise 1.8 We consider the same graph from Exercise 1.4, but with a different reading: This time, we see it as an *ABox* $\mathcal{A}$, where the nodes are individual names, i.e. $d, e, f, g, h, i \in \mathbf{I}$, and the labels correspond to concept and role assertions, e.g. $e : A$, $g : B$, $(d, e) : r$, etc.



We also consider again the following set of concepts:

$$S \quad = \quad \{ \quad \neg A, \quad A \sqcap \neg B, \quad \exists r.(A \sqcup B), \quad \forall r.(A \sqcup B), \quad \exists s.\exists s.\neg A, \quad \exists r.\forall r.\neg A \quad \}.$$

(a) For each concept $D \in S$, what are the instances of D w.r.t. $\mathcal{A}$, i.e. for which individual names $x \in \{d, e, f, g, h, i\}$ does $\mathcal{A} \models x : D$ hold?

(b) For each concept $D \in S$, what are the instances of D w.r.t. the ontology $\mathcal{O} = \mathcal{A} \cup \mathcal{T}$, where $\mathcal{T} = \{C \sqsubseteq \neg A,\ B \sqsubseteq \forall r.C\}$?