VU VRIJE UNIVERSITEIT AMSTERDAM

Knowledge Representation

2023/2024

Exercise Sheet 2 – Description Logics and Non-Monotonic Reasoning

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Dr. Patrick Koopmann, Dr. Atefeh Keshavarzi Zafarghandi

Exercise 2.1 ($\mathcal{EL}$ Completion) Use the completion algorithm to compute the materialization of the following ontology, that is, to compute all entailments of the form $a : A$, where A is a concept name.

$$\begin{aligned}
&\mathcal{O} = \mathcal{T} \cup \mathcal{A} \\
&\mathcal{T} = \{\ \exists \mathrm{hasParent}.\mathrm{Person} \sqsubseteq \mathrm{Child}, && \exists \mathrm{hasParent}.\mathrm{Child} \sqsubseteq \mathrm{GrandChild}, \\
&\qquad \mathrm{Child} \sqsubseteq \mathrm{Person}, && \mathrm{Person} \sqsubseteq \exists \mathrm{hasParent}.\mathrm{Person} \quad \} \\
&\mathcal{A} = \{\ (\mathrm{peter}, \mathrm{mary}) : \mathrm{hasParent}, && (\mathrm{lilian}, \mathrm{mary}) : \mathrm{hasParent}, \\
&\qquad \mathrm{mary} : \mathrm{Person} \quad \}
\end{aligned}$$

Note: the concepts assigned to the interpretations of named individuals are *not marked* as initial concepts!

Exercise 2.2 (Tableaux Procedure) We want to use the tableaux procedure to decide concept subsumption. Assume we have the following TBox:

$$\mathcal{T} = \{ \quad \forall r.\neg A \sqsubseteq \exists r.B, \qquad \neg C \sqsubseteq D \quad \}$$

We want to know whether $\mathcal{T} \models \forall r.A \sqsubseteq \exists r.C$

(a) Reduce the subsumption problem to a concept unsatisfiability problem.

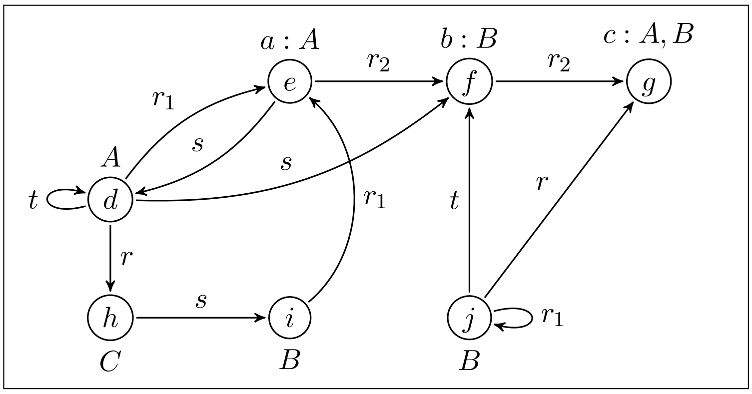
(b) Normalize the TBox and the concept whose satisfiability you would check.

(c) If possible, find a branch that leads to a clash.

(d) If possible, find a complete branch without clash.

(e) Does $\mathcal{T} \models \forall r.A \sqsubseteq \exists r.C$?

Exercise 2.3 ($\mathcal{SROIQ}(D)$) Consider the RBox $\mathcal{R} = \{$ Fun(t), $\quad r_1 \circ r_2 \sqsubseteq r_1, \quad r \sqsubseteq s, \quad t \sqsubseteq s \}$ and the interpretation illustrated as follows:



(a) Complete the interpretation to a model of $\mathcal{R}$.

(b) Find the elements from the interpretation domain that belong to the following concepts:

$$\forall r_1.B \qquad \exists r_1.\{c\} \qquad \forall r_1^{-}.B \qquad \geq 2t.\top \qquad \geq 2s.(A \sqcup B)$$

(c) Which of the following axioms are entailed by $\mathcal{R}$?

$$\text{Fun}(s) \qquad \text{Fun}(r) \qquad \text{Fun}(t^{-}) \qquad \text{Dom}(t) \sqsubseteq \leq 1t.\top$$

Exercise 2.4 Which of the following sentences contains a defeasible rule?

(a) The meeting will be held in the conference room, unless there is a scheduling conflict.

(b) Water boils at 100 degrees Celsius at standard atmospheric pressure.

(c) The warranty covers all damages, unless it is deliberately.

(d) A square has four equal sides and four right angles.

(e) The bus departs at 9 AM.

(f) She typically wake up early morning.

(g) AI students usually like logics.

Exercise 2.5 How do you interpret the following default rules? Which of them are considered normal default rules?

(a) $\frac{\text{Robot: Work}}{\text{Work}}$

(b) $\frac{\text{Robot: Work} \wedge \text{HandsnotBroken}}{\text{Work}}$

(c) $\frac{\text{Tomato:} \neg \text{Ripe}}{\text{Green}}$

(d) $\frac{\text{Suspect(x):} \neg \text{Guilty(x)}}{\text{Innocent(x)}}$

Exercise 2.6 Given default theory $T = (W, D)$ s.t $W = \{$Dutch(bart), Logician(bart)$\}$, $\delta_1 = \frac{\text{Dutch}(x)\text{:Sporty}(x)}{\text{IceScater}(x)}$, $\delta_2 : \frac{\text{Logician}(x)\text{:Philosopher}(x)}{\text{Philosopher}(x)}$ $\delta_3 : \frac{\text{Philosopher}(x)\text{:}\neg\text{Sporty}(x)}{\neg\text{Sporty}(x)}$. Draw the associated process tree.

Exercise 2.7 Given default theory $T = (W, D)$ with $W = \{a, d\}$, and $D = \{\delta_1, \delta_2, \delta_3\}$ such that $\delta_1 = \frac{a:b}{b}$, $\delta_2 = \frac{b:c}{c}$, $\delta_3 = \frac{d:\neg c}{\neg c}$. Draw the process tree of T.

Exercise 2.8 Given AF $F = (\{a,b,c,d,e\}, \{(a,b),(b,a),(b,c),(c,d),(d,e),(e,c)\})$. Draw the associated graph and indicate the sets of semantics for admissible, preferred, grounded, complete.

Exercise 2.9 Given AF $F = (\{a,b,c,d\}, \{(a,b),(b,a),(a,c),(b,c),(c,d)\})$. An AF is called **coherent** if the set of preferred extensions is equal to the set of stable extensions. Check if F is coherent.