

# KRATKI SPOJ

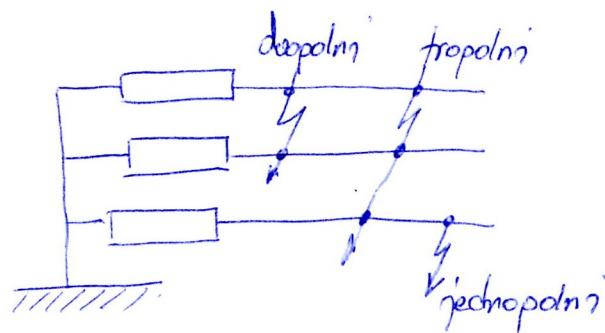
- dvije vrste kvarova:

- 1) uzdužni - prekid vodiča (struje nisu velike)
- 2) poprečni - probor izolacije (ovi kvarovi se nazivaju LS)
  - struja izlazi van (velika opasnost od termičkog razaranja mreže)

uzroci LS-a:

- 1) slom izolacije → zbog povećanja električnog naprezanja
  - zbog smanjenja čvrstode izolacije
  - kombinacijom gore navedenog
  - električna naprezanja
    - pogonski napon
    - povišenje napona
    - unutrašnji prenaponi
    - atmosferski -it
    - utjecaj mreže višeg napona

Zemljospoj - nisu velike struje, nema temičkih naprezanja, ali utječe na kvalitetu napona

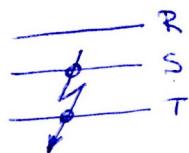


## ZORACUN DVOPOLONOG KS-a

na mjestu lvara:

$$U_s = U_T$$

$$I_s = -I_T$$



$$[U_m^d] = [Z_s^d] \{ [I^d] + [I_m^0] \}$$

$$[U_m^i] = [Z^i] [I_m^i]$$

$$\begin{aligned} U_s &= U^0 + a^2 U^d + a U^i \\ U_T &= U^0 + a U^d + a^2 U^i \end{aligned} \quad \left. \begin{array}{l} U^0 + a^2 U^d + a U^i = U^0 + a U^d + a^2 U^i \\ (a^2 - a) \cdot U^d = (a^2 - a) U^i \end{array} \right.$$

$$U^d = U^i \rightarrow \text{analogno} \quad I^d = -I^i$$

$$U_m^d = Z_{mm}^d + Z_{mn}^d I_m^d$$

$$U_m^i = Z_{mm}^i I_m^i$$

$$Z_{mm}^d + Z_{mn}^d I_m^d = Z_{mm}^i I_m^i = -Z_{mn} I^d$$

$$I_m^d = -\frac{Z_{mm}^d}{Z_{mn}^d + Z_{mm}^i}$$

$$I_m^i = \frac{Z_{mm}^d}{Z_{mn}^d + Z_{mm}^i}$$

## ZORACUN DVOPOLONOG KS-a SA ZEMLJOM

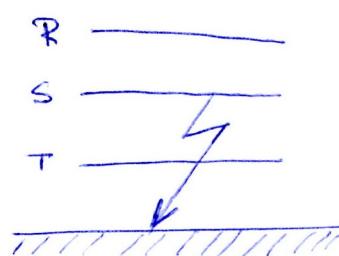
$$U^d = Z_m^d U_m^d + Z_m^d \cdot I_m^d$$

$$U^i = Z^i I_m^i$$

$$U^0 = Z^0 \cdot I_m^0$$

$$I_s = U_T = 0$$

$$Z_2 = 0$$



$$I_m^d = -Z_{mm}^d \frac{Z_m^0 + Z_m^i}{Z_{mn}^d \cdot Z_{mm}^i + Z_{mn}^d Z_{mm}^0 + Z_{mn}^i Z_{mm}^0}$$

$$I_m^i = Z_{mm}^d \frac{Z_m^0}{Z_{mn}^i}$$

$$I_m^0 = Z_{mm}^d \left( \frac{Z_m^i}{Z_{mn}^i} \right)$$

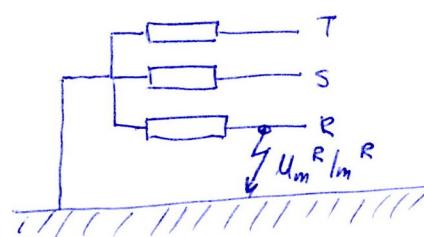
# PRORĀČUN JEDNOSTOPLNOG KRATKOG SPOJA

u bolesnom čvoru postavljamo jednadžba:

$$\sum U_m = 0$$

$$\sum I_m = I_m^d + I_m^i + I_m^o$$

$$T I_m = S I_m = 0$$



$$\begin{vmatrix} R \\ S \\ T \end{vmatrix} \begin{matrix} U_m \\ I_m \\ I_m \end{matrix} = \begin{vmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{vmatrix} \begin{matrix} U_m^o \\ U_m^d \\ U_m^i \end{matrix} \rightarrow \begin{cases} 0 = U_m^o + U_m^d + U_m^i \\ T I_m = 0 \\ S I_m = 0 \end{cases}$$

$$0 = I_m^o + a I_m^d + a^2 I_m^i$$

$$0 = I_m^o + a^2 I_m^d + a I_m^i$$

$$I_m^o = I_m^d = I_m^i$$

$$U_m^d = \sum_{j=1}^n I_j Z_{mj}^d + Z_{mm}^d I_m^o , \quad \sum_{j=1}^n I_j Z_{mj}^d = U_m^z$$

$$U_m^i = Z_{mm}^i \cdot I_m^o$$

$$U_m^o = Z_{mm}^o \cdot I_m^o$$

$$U_m^z + Z_{mm}^d I_m^o + Z_{mm}^i I_m^o + Z_{mm}^o I_m^o = 0$$

$$I_m^o = -\frac{U_m^z}{Z_{mm}^d + Z_{mm}^i + Z_{mm}^o}$$

$$I_{kr} = -3 I_m^o = \frac{3 U_m^z}{Z_{mm}^d + Z_{mm}^i + Z_{mm}^o} = \frac{3 U_m^z}{2 Z_{mm}^d + Z_{mm}^o}$$

$U_m^z \rightarrow$  fazni napon trofazne mreže

## PRORAČUN TOKOVA SNAGA

- zadano → snage ( $P$  i  $Q$ ) u čvoristima tereta
  - konfiguracija mreže
  - iznos napona i djelatne snage u GENERATORSKIM ČVORIŠTIMA
  - napon u regulacijskim elektranama (referentno čvorište)
- cilj proračuna:
  - odrediti napon po iznos i faza u svim čvoristima
- rezultati:
  - vektor napona svih čvorista po iznosu i kutu
  - iznos radne i jačine snage kroz grane mreže
  - injekcije snage u čvoristima
  - gubici snage u mreži
  - snage ref. čvorista

podjela čvorista:

- 1) čvorišta tereta (poznat  $P$  i  $Q$ , traži se  $U$  i  $\varphi$ )
- 2) generatorska čvorišta (poznat  $|V|$ ,  $P$ ,  $Q_{\min}$  i  $Q_{\max}$ , traži se  $Q$  i  $\varphi$ )
- 3) čvorište regulacijske elektrane (poznat  $|V|$ ,  $\varphi=0$ )

## PRORAČUN KRATKOG SPOJA

- zadano → konfiguracija mreže ( $d, i, o$ )
  - uključeni generatori
  - mjesto kvara
- rezultati
  - tropolna struja (snaga), struje 1 KS-a

## METODA OTPORA (relativne veličine)

→ pretpostavljamo da su trofazne snage u starnoj mreži jednake snagama u reduciranoj

$$S = S'$$

$$U' = \frac{U_B}{U_n} \cdot U$$

$$\text{iz } S = S' \Rightarrow \sqrt{3} UI = U'I'$$

$$I' = \frac{\sqrt{3} UI}{U'} = \frac{\sqrt{3} UI}{\frac{U_B}{U_n} U} = \frac{\sqrt{3}}{U_B} U_n I$$

$$Z' = \frac{U'}{I'} = \frac{\frac{U_B}{U_n} U}{\frac{\sqrt{3}}{U_B} U_n I} = \left(\frac{U_B}{U_n}\right)^2 Z \quad \text{ili} \quad Y' = \left(\frac{U_n}{U_B}\right)^2 Y$$

## EDINIČNE VRIJEDNOSTI (per unit)

zadana bazna snaga, bazni napon jednako nazivnom

$$S_{pu} = \frac{S}{S_B} ; \quad U_{pu} = \frac{U}{U_B} ; \quad I_{pu} = \frac{I}{I_B} = \frac{\frac{S}{S_B}}{\frac{\sqrt{3} U}{U_B}} = \frac{S}{S_B} \cdot \frac{U_B}{\sqrt{3} U}$$

$$Y_{pu} = \frac{Y}{Z_B} = Z \cdot \frac{1}{\frac{U_0^2}{S_B}} = Z \frac{S_B}{U_0^2} \quad Y_{[pu]} = Y \frac{U_B^2}{S_B} \quad (U_0 = U_B)$$

## METODA REDUCIRANIH ADMITANCIJA

- proračun LS-a

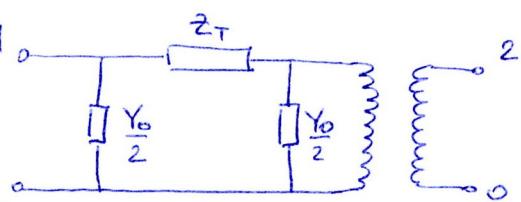
- sve pretvaramo u admittanciju

$$U_B = 1 \text{ kV ili } S_B = 1 \text{ MVA}$$

$$U_n = \frac{U}{U_n} , \quad I_N = \sqrt{3} U_n I , \quad Z_n = \frac{Z}{U_n^2} , \quad Y_n = \frac{U_n^2}{Z} = U_n^2 \cdot Y$$

# TRANSFORMATOR

## IDEALNI TRAFO



$S_n$  (MVA)

$U_n$  (%)

$U_{n1}$  (kV)

$P_k$  (MW) - gubici XS

$i$  (%) - struja mag.

$P_o$  (MW) - gubici PH

$\frac{w_1}{w_2} \rightarrow$  prijenosni omjer

## ZATKI SPOJ

sekundar se kratko spoji, diže se napon primara dok ne potekne nazivna struja

$$U_{k.v.} = \frac{\ln \cdot Z_T}{\frac{U_n}{\sqrt{3}}} \cdot 100\% = \frac{\frac{S_n}{\sqrt{3}U_n} \cdot Z_T}{\frac{U_n}{\sqrt{3}}} \cdot 100\% = Z_T \frac{S_n}{U_n^2} \cdot 100\% \Rightarrow Z_T = \frac{U_{k.v.}}{100} \cdot \frac{U_n^2}{S_n}$$

$$P_k = 3 I_n^2 \cdot R = 3 \left( \frac{S_n}{\sqrt{3}U_n} \right)^2 R = \frac{S_n^2}{U_n^2} \cdot R$$

$$X = \sqrt{Z_T^2 - R^2} = \sqrt{U_n^2 \left( \frac{U_n^2}{S_n} \right)^2 - P_k^2 \left( \frac{U_n^2}{S_n^2} \right)^2} = \frac{U_n^2}{S_n} \sqrt{U_{k.v.}^2 - \left( \frac{P_k}{S_n} \right)^2}$$

$$Z = R + jX = \frac{P_k}{S_n^2} U_n^2 + j \frac{U_n^2}{S_n} \sqrt{U_{k.v.}^2 - \left( \frac{P_k}{S_n} \right)^2}$$

$$Z = \frac{U_n^2}{S_n} \left[ \frac{P_k}{S_n} + j \sqrt{U_{k.v.}^2 - \left( \frac{P_k}{S_n} \right)^2} \right] \quad [\Omega]$$

## ZADNI HOD

- mjerimo na ulazu struju magnetizirajuću

$$I_o = \frac{U_{n1}}{\sqrt{3}} \cdot Y_o ; \quad Y_o = \frac{I_o \sqrt{3}}{U_n} = \frac{i_o I_n \sqrt{3}}{U_n} = \frac{i_o \frac{S_n}{\sqrt{3}U_n} \sqrt{3}}{U_n} = i_o \frac{S}{U_n^2}$$

$$P_o = 3 I_o^2 Q_o = 3 \left( \frac{U_n}{\sqrt{3}} \right)^2 Q_o \Rightarrow Q_o = \frac{P_o}{U_n^2}$$

$$Y_o = \frac{P_o}{U_n^2} - j \frac{S_n}{U_n^2} \sqrt{i_o^2 - \left( \frac{P_o}{S_n} \right)^2} = \frac{S_n}{U_n^2} \left[ \frac{P_o}{S_n} - j \sqrt{i_o^2 - \left( \frac{P_o}{S_n} \right)^2} \right]$$

$$Z_T \text{ p.u.} = \frac{S_B}{S_n} \left[ \frac{P_k}{S_n} + j \sqrt{U_{k.v.}^2 - \left( \frac{P_k}{S_n} \right)^2} \right]$$

$$Y_o \text{ p.u.} = \frac{S_n}{S_B} \left[ \frac{P_o}{S_n} - j \sqrt{i_o^2 - \left( \frac{P_o}{S_n} \right)^2} \right]$$

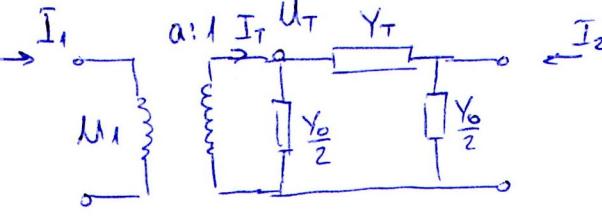
karak. veličine :

$$U_{k.v.} = 4\% \div 15\% U_n$$

$$P_k = 2,5\% \div 0,5\% S_n$$

$$i_o = 2,5\% \div 0,5\% I_n$$

$$P_o = 15\% \div 4\% P_k$$



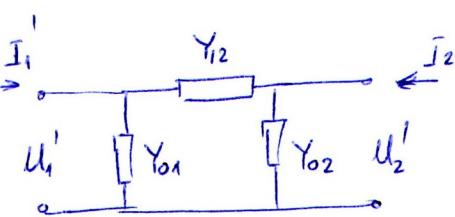
$$I_T = (U_T - U_2) Y_T + U_T \frac{Y_0}{2}$$

$$I_1 = \frac{I_T}{a} = (U_T - U_2) \frac{Y_T}{a} + \frac{U_T}{a} \frac{Y_0}{2}$$

$$\frac{U_1}{U_T} - a = \frac{I_T}{I_1}$$

$$I_1 = \left( \frac{U_1}{a} - U_1 \right) \frac{Y_T}{a} + \frac{U_1}{a^2} \frac{Y_0}{2} = (U_1 - U_{2a}) \frac{Y_T}{a^2} + U_1 \frac{Y_0}{2a^2}$$

$$I_2 = (U_2 - U_T) Y_T + U_2 \frac{Y_0}{2} = (U_2 a - U_1) \frac{Y_T}{a} + U_2 \frac{Y_0}{2}$$



$$I_1' = (U_1' - U_2') Y_{12} + U_1' Y_{01}$$

$$I_2' = (U_2' - U_1') Y_{12} + U_2' Y_{02}$$

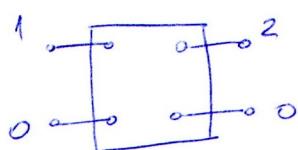
$$Y_{12} = \frac{Y_T}{a}$$

$$a = \frac{\frac{U_1}{U_{1n}}}{\frac{U_2}{U_{2n}}}$$

$$Y_{01} = \frac{Y_T}{a} \left( \frac{1}{a} - 1 \right) + \frac{1}{a^2} \frac{Y_0}{2}$$

$$Y_{02} = Y_T \left( 1 - \frac{1}{a} \right) + \frac{Y_0}{2}$$

## ATRIČNI OBLIK TRANSFORMATORA



$$\begin{vmatrix} I_1 \\ I_2 \end{vmatrix} = \begin{vmatrix} Y_{12} + Y_{01} & -Y_{12} \\ -Y_{12} & Y_{12} + Y_{02} \end{vmatrix} \begin{vmatrix} U_1 \\ U_2 \end{vmatrix}$$

matrica admitancije  $[Y]$   $\rightarrow ij \rightarrow$  zbroj poprečne i uzdужne adm.  
 $ij \rightarrow$  suprotni predznak međuadmitancije

wjet paralelnog rada  $\rightarrow$  ista grupa spoja  
 $\rightarrow$  isti prijenosni omjer  
 $\rightarrow$  snage se razlikuju do 1/3

# METODA GSZ

- mreža od  $n$  čvorova → jedno referentno

$$\vec{U}_i - \vec{U}_{ref} = \sum_{\substack{j=1 \\ j \neq ref}}^n \vec{Z}_{ij} \vec{I}_j$$

matrica  $Y$  samo uzdužni parametri grane  
poprečne admittancije → matrica  $Y'$

## POSTUPAK PRORAČUNA

- 1) - učitavanje podataka o mreži (konfiguracija, admittancija grane)
    - učitavanje podataka o injekcijama snaga u čvorovima
  - 2) - formiranje  $Y$  matrice (samo uzdužne admittancije)
    - formiranje  $Y'$  matrice (samo poprečne admittancije)
  - 3) - računanje  $Z$  matrice  $\rightarrow [Z] = [Y]^{-1}$
  - 4) - početne vrijednosti napona čvorova:  $U_i^{(0)} = 1 + j0$  p.u.
  - 5) - računanje stanja u čvorovima:  $I_i^{(0)} = \frac{S_i^*}{U_i^{*(0)}} - Y_i \cdot U_i^{(0)}$ ,  $i = 1, 2, \dots, n-1$
  - 6) - računanje struja i napona u čvorovima:
- $$U_i^{(1)} = U_{ref} + \sum_{j=1}^{i-1} Z_{ij} I_j^{(0)} + \sum_{j=i}^{n-1} Z_{ij} I_j^{(0)}$$
- ) da li je uvjet zadovoljen:  
 $|U_i^{(n)} - U_i^{(n-1)}| < \epsilon$ ,  $\epsilon = 0,001 \div 0,0001$

Općenito:

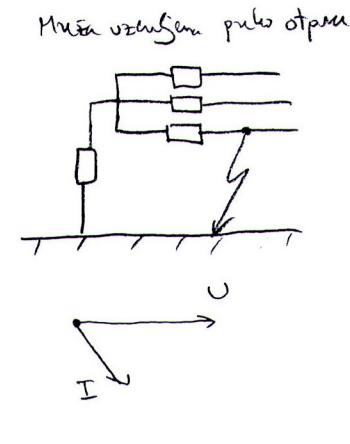
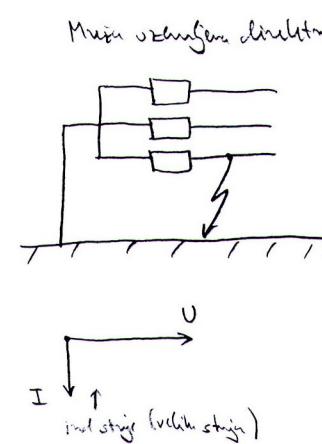
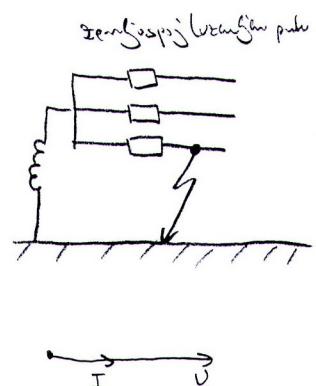
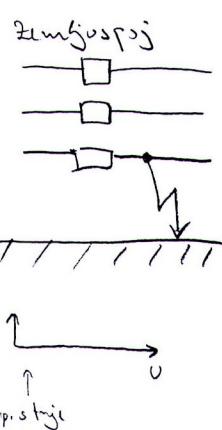
$$U_i^{(n+1)} = U_{ref} + \sum_{j=1}^{i-1} Z_{ij} I_j^{(n+1)} + \sum_{j=i}^{n-1} Z_{ij} I_j^{(n)}$$

$$I_i^{(n+1)} = \frac{S_i^*}{U_i^{*(n+1)}} - Y_i U_i^{(n+1)}, i = 1, 2, \dots, n-1$$

- ne ulaze generatori u proračun

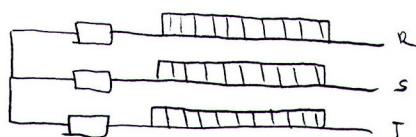
metoda povoljna za mali broj čvorova (20) jer je konvergentna  
dobra je za oko 100 čvorova

## Vektorske slike

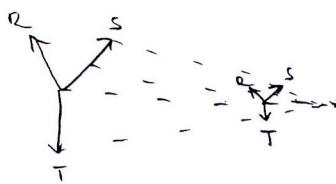
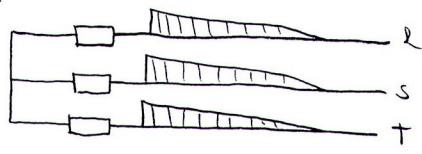


## JEDNOPOLNI KRATKI SPOJ

strukcija:

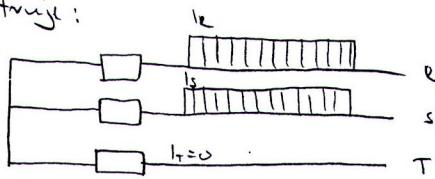


naponi:

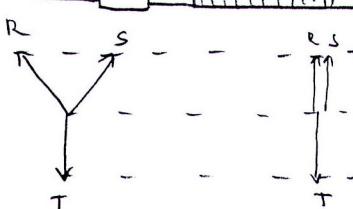
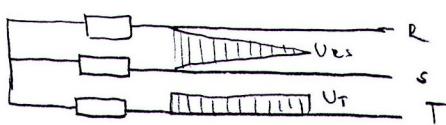


## DVOPOLNI KRATKI SPOJ

strukcija:

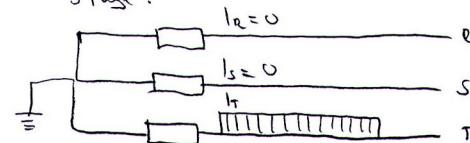


naponi:

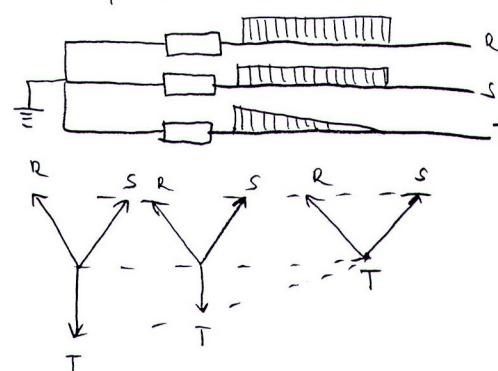


## JEDNOPOLNI KRATKI SPOJ

strukcija:



magneti:



## Simetrične komponente

$${}^R\!U_i = {}^0\!U_i + {}^d\!U_i + {}^i\!U_i$$

$${}^S\!U_i = {}^0\!U_i + \alpha {}^d\!U_i + \alpha^{-1} {}^i\!U_i$$

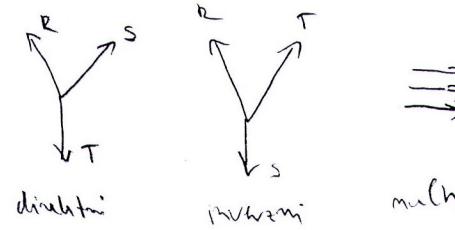
$${}^T\!U_i = {}^0\!U_i + \alpha^2 {}^d\!U_i + \alpha^1 {}^i\!U_i$$

$$\begin{vmatrix} {}^R\!U_i \\ {}^S\!U_i \\ {}^T\!U_i \end{vmatrix} = \begin{vmatrix} 1 & 1 & 1 \\ 1 & \alpha^2 & \alpha \\ 1 & \alpha & \alpha^2 \end{vmatrix} \begin{vmatrix} {}^0\!U_i \\ {}^d\!U_i \\ {}^i\!U_i \end{vmatrix}$$

$$\alpha = -\frac{1}{2} + j\frac{\sqrt{3}}{2}$$

$$\alpha^2 = -\frac{1}{2} - j\frac{\sqrt{3}}{2}$$

$$\begin{vmatrix} {}^0\!U_i \\ {}^d\!U_i \\ {}^i\!U_i \end{vmatrix} = \frac{1}{3} \begin{vmatrix} 1 & 1 & 1 \\ 1 & \alpha & \alpha^2 \\ 1 & \alpha^2 & \alpha \end{vmatrix} \begin{vmatrix} {}^R\!U_i \\ {}^S\!U_i \\ {}^T\!U_i \end{vmatrix}$$



## SNAGE U ČVORIŠTIMA

$$\vec{s}_i = \vec{v}_i \cdot \vec{i}_i^* = P_i + j Q_i$$

$$\vec{s}_i = |\vec{v}_i| \cdot \sum_{j=1}^m |v_j| |Y_{ij}| (\cos(\delta_i - \delta_j - \Theta_{ij}) + j \sin(\delta_i - \delta_j - \Theta_{ij}))$$

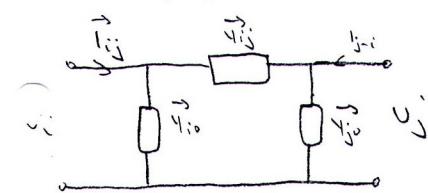
-djelatna snaga u čvoru i:

$$P = |\vec{v}_i| \sum_{j=1}^m v_j |Y_{ij}| \cos(\delta_i - \delta_j - \Theta_{ij})$$

-javna snaga u čvoru i:

$$Q = |\vec{v}_i| \sum_{j=1}^m v_j |Y_{ij}| \sin(\delta_i - \delta_j - \Theta_{ij})$$

## SNAGE U GRANAMA



$$\vec{I}_{i-j} = (v_i - v_j) Y_{ij} + v_i \vec{y}_{io}$$

$$\vec{s}_{i-j} = v_i \cdot \vec{I}_{i-j}^* = v_i [(v_i - v_j) Y_{ij} + v_i \vec{y}_{io}]^*$$

|  $y_{ij} \rightarrow$  uzdužna admittancija  
grane

$$\vec{s}_{j-i} = v_j \cdot \vec{I}_{j-i}^* = v_j [(v_j - v_i) Y_{ij} + v_j \vec{y}_{jo}]^*$$

gubici snage  $\Rightarrow \Delta S = \vec{s}_{i-j} + \vec{s}_{j-i}$

$$\boxed{\Delta S = (v_i^* - v_j^*) Y_{ij}^* (v_i - v_j) - (|v_i|^2 - |v_j|^2) \frac{B_{ij}}{2}}$$

## VRĆUNI TOLKUĆE SNAGE

-osnovni metodi ponašanja tolkova snage:

- metoda Gauss-Seidel pomoću 2 matrice
- metoda Gauss-Seidel pomoću 4 matrice
- metoda Newton Raphson

# STOSMUJERNI MODEL TOKOVA SNAGE

$$P_i = \sum_{j=1}^n U_i U_j Y_{ij} \cos(\delta_i - \delta_j - \theta_{ij})$$

$$Q_i = \sum_{j=1}^n U_i U_j Y_{ij} \sin(\delta_i - \delta_j - \theta_{ij})$$

- pretpostavimo:  $U_i \approx U_j \approx 1$

$$Y_{ij} = B_{ij} e^{j\varphi_{ij}}$$

$$Y_{ii} = B_{ii} e^{j\varphi_{ii}}$$

$$P_i = B_{ii} \cos(\varphi_{ii}) + \sum_{\substack{j=1 \\ j \neq i}}^n B_{ij} \sin(\delta_i - \delta_j)$$

$$Q_i = B_{ii} \sin(\varphi_{ii}) - \sum_{\substack{j=1 \\ j \neq i}}^n B_{ij} \cos(\delta_i - \delta_j)$$

- zanemarimo  $Q$  ( $Q_i = 0$ )  $\rightarrow$  kut  $(\delta_i - \delta_j)$  jeck mali pun:

$$P_i = \sum_{\substack{j=1 \\ j \neq i}}^n B_{ij} (\delta_i - \delta_j)$$

$$P_i = \sum_{j=1}^{n-1} B_{ij} \delta_j, \quad \delta_n = \delta_{n+1} = 0$$

$$|P| = |B| |\delta|$$

$\downarrow$   
 $(n-1) \times (n-1)$

$$\delta_i = (U_i - U_{n+1})$$

- kol matrica  $B$  su dijagonalki (-), a vandijagonalki (+)

$$|P| = |B| |\delta|$$

$$P_{i-j} = \frac{j(\delta U_i - \delta U_j)}{j X_{ij}}$$

$$P_{i-j} = -P_{j-i} \quad (j \neq j \Rightarrow 0, \text{ mimo dijelne gubitke } P_{loss} = I^2 R)$$

v svim mrežama mno gubitku  $t_j$  zanemri se (u izvjesnim mrežama gubici idu u refleksu električne snage)

## • MATEMATIČKI MODEL KRATKOG SPOTA

- stanje zdrave mreže:

$$\begin{vmatrix} U_1 \\ U_2 \\ \vdots \\ U_m \end{vmatrix} = \begin{vmatrix} z_{11} & \cdots & z_{1m} \\ \vdots & \ddots & \vdots \\ z_{m1} & \cdots & z_{mm} \end{vmatrix} \begin{vmatrix} l_1 \\ l_2 \\ \vdots \\ l_m \end{vmatrix}$$

↑   ↑  
veliki svi odgovarajuće  
(velovi, gaji, trosk., poljopr.)

struktu riječi ulaze u čvorište

- stanje bolesne mreže:

$$\begin{vmatrix} U_1 \\ \vdots \\ U_m \end{vmatrix}^B = |z| \cdot \left\{ \begin{vmatrix} l_1 \\ l_2 \\ \vdots \\ l_m \end{vmatrix} + \begin{vmatrix} 0 \\ \vdots \\ 0 \\ 0 \end{vmatrix} \right\} \quad l_k = -l_m$$

$$[U_m]^B = [U_m]^2 + [z][l_m]$$

- tropolski VS  $\Rightarrow$

$$dU_m = 0$$

$$0 = U_m^2 + dZ_{mm} l_m \Rightarrow l_m = -\frac{U_m^2}{dZ_{mm}}$$

- ne simetrični VS  $\Rightarrow$   $[dU_m]^S = [dZ_{mm}] \cdot \{ |d|l_m| + |l_m| \}$

$$[iU_m]^S = [iZ_{mm}] [il_m]$$

$$[\circ U_m]^S = [\circ Z_{mm}] [\circ l_m]$$

# METODA GSY

mocia od m. vlastita - jedno referenčno

$$|\vec{v}| = |\vec{y}| |\vec{v}|$$

$$\vec{v}_{(m-1)} = \vec{y}_{(m-1)1} \vec{v}_1 + \vec{y}_{(m-1)2} \vec{v}_2 + \dots + \vec{y}_{(m-1)m} \vec{v}_m$$

$$\vec{v}_m = \frac{1}{y_{mm}} [\vec{v} - \vec{y}_{m1} \vec{v}_1 - \dots - \vec{y}_{m(m-1)} \vec{v}_{m-1}]$$

$$\vec{v}_m = \frac{\vec{s}_m^*}{\vec{v}^*}$$

$$v_m^{(n)} = \frac{1}{y_{mn}} \left[ \frac{\vec{s}_m^*}{\vec{v}^*} - \vec{y}_{m1} v_n^{(n)} - \dots - \vec{y}_{m(n-1)} v_n^{(n)} \right]$$

⇒ općinito:

$$k L_i = \frac{s_i^*}{y_{ii}} \quad , \quad k L_{ij} = \frac{y_{ij}}{y_{ii}}$$

$$v_i^{(n+1)} = \frac{k L_i}{v_i^{(n)*}} - \sum_{j=1}^{i-1} k L_{ij} v_j^{(n+1)} - \sum_{j=i+1}^m k L_{ij} v_j^{(n)}$$

→ ujet točnosti

$$|v_i^{(n+1)} - v_i^{(n)}| < \varepsilon$$

→ ve funkciju obrazuju  $\Rightarrow \Delta v_2^{(n)} = v_2^{(n)} - v_2^{(n-1)}$   $\rightarrow n_2 \div 2$

$$v_2^{(n)}_{\text{obrazci}} = v_2^{(n)} + \Delta v_2^{(n)}$$

→ prednost metoda → Y matrica ima manje elmenta

→ redoslijed mnogo istanja

# METODA NEWTON-RAPSON

mjefiksnijer, majkovanogenerativni matrica (mane  $\rightarrow$  u svakoj iteraciji se rješava sustav linearnih jednica) 10 iteracija (15,ako je sustav na nizu stabilitetu)

mreža od n čvorova - jedna čvorova referenca

poznate snage u čvorovima:

- generatorsko i čvorovačka  $\rightarrow P_i = U_i \cdot \sum_{j=1}^m U_j Y_{i-j} \cos(\delta_i - \delta_j - \omega_{ij})$ ,  $i = 1, \dots, m-1$

- čvorovačka tenzor  $\rightarrow Q_i = U_i \sum_{j=1}^m U_j Y_{i-j} \sin(\delta_i - \delta_j - \omega_{ij})$ ,  $i = 1, \dots, m-q-1$

- potrebno izmjeriti:  $U_i$   $i = 1, \dots, m-q-1$

$$\delta_i, i = 1, \dots, m-1$$

## POSTUPAK PRORAKUNA

1) - učitavanje podataka o mrezi (konfiguracija, električni grani)

- učitavanje podataka o inicijalnim snage u čvorovima

2) - postavljanje inicijalne napona čvorova (prvopostavljanje inicijalne)

$$U_i^{(0)} = 1 + j0 \text{ p.u.}$$

3) - formiranje  $Y$  matrice

4) - računanje snaga u čvorovima:

$$P_{ini}^{(0)} = \sum_{j=1}^m U_i^{(0)} \cdot U_j^{(0)} Y_{i-j} \cos(\delta_i - \delta_j - \omega_{ij}) \quad i = 1, \dots, m-1$$

$$Q_{ini}^{(0)} = \sum_{j=1}^m U_i^{(0)} U_j^{(0)} Y_{i-j} \sin(\delta_i - \delta_j - \omega_{ij}) \quad i = 1, \dots, m-1-q$$

5)  $\Delta P_i^{(0)} = P_{zad} - P_{ini}^{(0)}, \quad i = 1, \dots, m-1$

$$\Delta Q_i^{(0)} = Q_{zad} - Q_{ini}^{(0)}, \quad i = 1, \dots, m-1-q$$

6) - provjera kriterija točnosti

$$\Delta P_i^{(0)} < \epsilon$$

$$\Delta Q_i^{(0)} < \epsilon$$

7) Računanje  $\Delta \delta_i^{(0)}, \Delta U_i^{(0)}$  pomoću  $\Delta P_i^{(0)}$  i  $\Delta Q_i^{(0)}$ ; Jakobijske matrice

$$\begin{vmatrix} \Delta P \\ \Delta Q \end{vmatrix} = |J| \begin{vmatrix} \Delta \delta \\ \Delta U \end{vmatrix} = \begin{vmatrix} J_1 & J_2 \\ J_3 & J_n \end{vmatrix} \begin{vmatrix} \Delta \delta \\ \Delta U \end{vmatrix} = \begin{vmatrix} \frac{\partial P}{\partial \delta} & \frac{\partial P}{\partial U} \\ \frac{\partial Q}{\partial \delta} & \frac{\partial Q}{\partial U} \end{vmatrix} \begin{vmatrix} \Delta \delta \\ \Delta U \end{vmatrix}$$

↓  
Jacobijev  
matrica

↓  
podmatrica

$$J_1 = \frac{\partial P}{\partial \delta}$$

$$\frac{\partial P_i}{\partial \delta_i} = -U_i \sum_{\substack{j=1 \\ j \neq i}}^n U_j Y_{ij} \sin(\delta_i - \delta_j - \Theta_{ij})$$

$$\frac{\partial P}{\partial \delta_j} = U_i U_j Y_{ij} \sin(\delta_i - \delta_j - \Theta_{ij})$$

$$J_2 = \frac{\partial P}{\partial U}$$

$$\frac{\partial P_i}{\partial U_i} = 2U_i Y_{ii} \cos(-\Theta_{ii}) + \sum_{\substack{j=1 \\ j \neq i}}^n U_j Y_{ij} \cos(\delta_i - \delta_j - \Theta_{ij})$$

$$\frac{\partial P}{\partial U_j} = U_i Y_{ij} \cos(\delta_i - \delta_j - \Theta_{ij})$$

$$J_3 = \frac{\partial Q}{\partial \delta}$$

$$\frac{\partial Q_i}{\partial \delta_i} = U_i \sum_{\substack{j=1 \\ j \neq i}}^n U_j Y_{ij} \cos(\delta_i - \delta_j - \Theta_{ij})$$

$$\frac{\partial Q_j}{\partial \delta_i} = -U_i U_j Y_{ij} \cos(\delta_i - \delta_j - \Theta_{ij})$$

$$J_n = \frac{\partial Q}{\partial U}$$

$$\frac{\partial Q_i}{\partial U_i} = 2U_i Y_{ii} \sin(-\Theta_{ii}) + \sum_{\substack{j=1 \\ j \neq i}}^n U_j Y_{ij} \sin(\delta_i - \delta_j - \Theta_{ij})$$

$$\frac{\partial Q_j}{\partial U_i} = U_i Y_{ij} \sin(\delta_i - \delta_j - \Theta_{ij})$$

- dimenzija Jacobijev matrica  $\rightarrow$

$(m-n) \times (m-n)$	$(m-1) \times (m-1-\delta)$
$(m-n-\delta) \times (m-n-\delta)$	$(m-1-\delta) \times (m-1-\delta)$

$$= (2n-g-2) \times (2n-g-2)$$

→ v2 zámenovanie 32; 33 miest:

$$\begin{vmatrix} \Delta P \\ \Delta Q \end{vmatrix} = \begin{vmatrix} J_1 & 0 \\ 0 & J_2 \end{vmatrix} \begin{vmatrix} \Delta \delta \\ \Delta U \end{vmatrix}$$

$$|\Delta S| = |J_1|^{-1} |\Delta P|$$

$$|\Delta U| = |J_2|^{-1} |\Delta Q|$$

→ opčinito miest:

$$P_{ini}^{(u)} = \sum_{j=1}^m V_i^{(u)} V_j^{(u)} Y_{ij} \cos(\delta_i^{(u)} - \delta_j^{(u)} - \Theta_{ij}^{(u)}), \quad i = 1, \dots, m-1$$

$$Q_{ini}^{(u)} = \sum_{j=1}^m V_i^{(u)} V_j^{(u)} Y_{ij} \sin(\delta_i^{(u)} - \delta_j^{(u)} - \Theta_{ij}^{(u)}), \quad i = 1, \dots, m-1$$

$$\Delta P_i^{(u)} = P_{izad} - P_{ini}^{(u)}$$

$$\Delta Q_i^{(u)} = Q_{izad} - Q_{ini}^{(u)}$$

Schubijevé počiatky:

$$\rightarrow \left( \frac{\partial P_i}{\partial \delta_i} \right)^{(u)} = -V_i^{(u)} \sum_{\substack{j=1 \\ j \neq i}}^m V_j^{(u)} Y_{ij} \sin(\delta_i^{(u)} - \delta_j^{(u)} - \Theta_{ij}^{(u)}), \quad j \neq i$$

$$\left( \frac{\partial P_i}{\partial \delta_j} \right)^{(u)} = V_i^{(u)} V_j^{(u)} Y_{ij} \sin(\delta_i^{(u)} - \delta_j^{(u)} - \Theta_{ij}^{(u)}), \quad i \neq j, j \neq i$$

$$\rightarrow \left( \frac{\partial Q_i}{\partial U_i} \right)^{(u)} = 2V_i^{(u)} Y_{ii} \sin(-\Theta_{ii}) + \sum_{\substack{j=1 \\ j \neq i}}^m V_j^{(u)} Y_{ij} \sin(\delta_i^{(u)} - \delta_j^{(u)} - \Theta_{ij}^{(u)}), \quad j \neq i$$

$$\left( \frac{\partial Q_i}{\partial U_j} \right)^{(u)} = V_i^{(u)} Y_{ij} \sin(\delta_i^{(u)} - \delta_j^{(u)} - \Theta_{ij}^{(u)}), \quad i \neq j, j \neq i$$

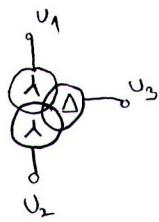
$$|\Delta \delta|^u = |J_1^{(u)}|^{-1} |\Delta P|^{(u)}$$

$$|\Delta U|^u = |J_2^{(u)}|^{-1} |\Delta Q|^{(u)}$$

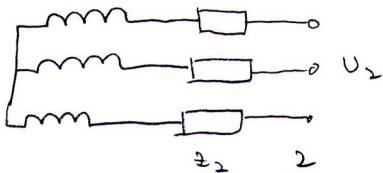
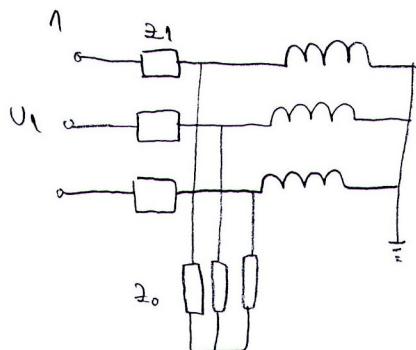
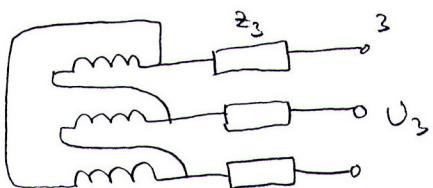
$$V_i^{(u+1)} = V_i^{(u)} + \Delta V_i^{(u)}$$

$$\delta_i^{(u+1)} = \delta_i^{(u)} + \Delta \delta_i^{(u)}$$

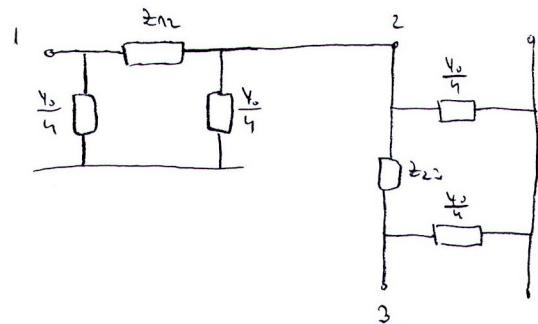
nonanutni transformator



- svaki trafo transformator, ali se raznjava da nema potrošnja



$$\begin{aligned} Z_{12} &= Z_1 + Z_2 \\ Z_0 &= Z_1 + Z_3 \\ Z_{23} &= Z_2 + Z_3 \end{aligned} \quad \left. \begin{array}{l} Z_1 = \frac{1}{2} (Z_{12} + Z_{13} - Z_{23}) \\ Z_2 = \frac{1}{2} (Z_{12} + Z_{23} - Z_{13}) \\ Z_3 = \frac{1}{2} (Z_{13} + Z_{23} - Z_{12}) \end{array} \right\}$$



- neka imamo mrežu od 100 čvorišta → Gauss-Seidel pomoću  $Z \leftrightarrow 5$  iteracija
- Gauss-Seidel pomoću  $Y \leftrightarrow 300$  iteracija

GSZ- Najbolje radi za manji broj čvorišta (idealno je do 20 čvorišta), ali može se primjenjivati do 100 čvorišta (za 100 čvorišta matrica  $Z$  ima 10000 članova, pa se preko toga ne ide), no pozitivna strana jest da je metoda konvergentna i obično završi u 5-10 iteracija.

GSY- Nije konvergentna pa ima hrpu iteracija, ne znam koja je pozitivna strana, ali se sjećam da  $Y$  matrica nešto ne ovisi o broju čvorišta ovo-ono.

Mislim da je stvar u tome da u GSZ nemres u  $Y$  matricu staviti generatorska cvorista, dok kod GSY mozes i zato je bolji GSY kaj se tog tice... jedino kaj je manje konvergentan...

mislim da jedino u newton raphson idu generatorska cvorista a GSY je bolji od GSZ za veliki broj cvorista jer  $Y$  matrica ima masu nula, pa je proracun jedne iteracije dosta brzi, a  $Z$  matrica ima sve pozitivne vrijednosti i treba dosta da se izracuna inverz, npr  $100 \times 100$

U GSZ reduciraš  $Y$  matricu (križaš redak i stupac referentnog čvorišta), u GSY to ne radiš zato jer si potrp'o i generatorska čvorišta u  $Y$  matricu, tu ti se to vidi.

dobro za zapantiti je da je matrica impedancije cvorisata singularna sto znaci da je determinanta nula i nema inverza, zato micemo jedan referentni redak kada racunamo  $Z$  matricu.