Analogna i mješovita obrada signala – FORMULE

Fourierova transformacija

Fourierov red f(t) i težinski faktori F_n

$$f(t) = \sum_{n=-\infty}^{\infty} F_n \cdot e^{jn\omega_0 t}$$

$$F_n = \frac{1}{T_0}\int_{-\frac{T_0}{2}}^{\frac{T_0}{2}} f(t) \cdot e^{-jn\omega_0 t} dt$$

Fourierova transformacija i spektar F(ω)

$$f(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} F(\omega) e^{j\omega t} d\omega$$

$$F(\omega) = \int_{-\infty}^{\infty} f(t) e^{-j\omega t} dt$$

Rastav spektra na realni i imaginarni dio:

$$F_{Re}(\omega) = \int_{-\infty}^{\infty} [f_{Re}(t) cos\omega t + f_{Im}(t) sin\omega t] dt$$

$$F_{Im}(\omega) = \int_{-\infty}^{\infty} [f_{Re}(t) sin\omega t - f_{Im}(t) cos\omega t] dt$$

Inverzna Fourierova transformacija: $F(\omega) = F_{Re}(\omega) - jF_{Im}(\omega)$

$$f_{Re}(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} [F_{Re}(\omega) cos\omega t - F_{Im}(\omega) sin\omega t] d\omega$$

$$f_{Im}(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} [F_{Re}(\omega) sin\omega t + F_{Im}(\omega) cos\omega t] d\omega$$

Za REALNU vremensku funkciju $f(t)=f_{Re}(t)$

$$F_{Re}(\omega) = \int_{-\infty}^{\infty} f(t) cos\omega t dt$$

$$F_{Im}(\omega) = -\int_{-\infty}^{\infty} f(t) sin\omega t dt$$

Uočiti da je F_{Re} parna, a F_{Im} neparna pa vrijedi F(-ω)=F*(ω)
Ako ovo vrijedi, vremenska funkcija je realna.
Vrijedi: $f_{Im}(t)=0$ pa se inverz računa samo preko f_{Re}.

Za IMAGINARNU vremensku funkciju $f(t)=f_{Im}(t)$

$$F_{Re}(\omega) = \int_{-\infty}^{\infty} f(t) sin\omega t dt$$

$$F_{Im}(\omega) = \int_{-\infty}^{\infty} f(t) cos\omega t dt$$

Uočiti da je F_{Re} neparna, a F_{Im} parna, vrijedi F(-ω)=-F*(ω)
Ako ovo vrijedi, vremenska funkcija je imaginarna.
Vrijedi $f_{Re}=0$ pa se inverz računa samo preko $f_{Im}(t)$

Za REALNU I PARNU vremensku funkciju:

$$F_{Re}(\omega) = 2\int_{0}^{\infty} f_P(t) cos\omega t dt$$

$$F_{Im}(\omega) = 0$$

Spektar je čisto realan:

$$f_P(t) = \frac{1}{\pi}\int_{0}^{\infty} F_{Re}(\omega) cos\omega t d\omega$$

Za REALNU I NEPARNU vremensku funkciju:

$$F_{Re}(\omega) = 0$$

$$F_{Im}(\omega) = -2\int_{0}^{\infty} f_N(t) sin\omega t dt$$

Spektar je čisto imaginaran:

$$f_N(t) = -\frac{1}{\pi}\int_{0}^{\infty} F_{Im}(\omega) sin\omega t d\omega$$

Korisne transformacije: $e^{j\omega_0 t} \circ\!\!-\!\!\bullet\ 2\pi\delta(\omega - \omega_0)$

$$\sin(\omega_0 t) \circ\!\!-\!\!\bullet -j\pi(\delta(\omega - \omega_0) - \delta(\omega + \omega_0))$$

$$\cos(\omega_0 t) \circ\!\!-\!\!\bullet\ \pi(\delta(\omega - \omega_0) + \delta(\omega + \omega_0))$$

Svojstva: *Linearnost*: $\mathcal{F}[\alpha f_1(t) + \beta f_2(t)] = \alpha F_1(\omega) + \beta F_2(\omega)$

Simetričnost: $\mathcal{F}[F(t)] = 2\pi \cdot f(-\omega)$

Vremensko skaliranje: $\mathcal{F}[f(at)] = \frac{1}{|a|} F\left(\frac{\omega}{a}\right)$

Vremenski pomak: $\mathcal{F}[f(t - t_0)] = F(\omega) e^{-j\omega t_0}$

Frekvencijski pomak: $\mathcal{F}\left[f(t) \cdot e^{j\omega t_0}\right] = F(\omega - \omega_0)$

Derivacija (dvostrana transformacija):

$$\mathcal{F}\left[\frac{d^n f(t)}{dt^n}\right] = (j\omega)^n F(\omega)$$

Derivacija (jednostrana transformacija):

$$\mathcal{F}\left[\frac{d^n f(t)}{dt^n}\right] = (j\omega)^n F(\omega) - (j\omega)^{n-1} f(0) - \cdots - (j\omega)^0 f^{n-1}(0)$$

Integracija (dvostrana trans.): $\mathcal{F}\left[\int_{-\infty}^{t} f(\tau) d\tau\right] = \frac{F(\omega)}{j\omega}$

Integracija (jednostrana trans.):

$$\mathcal{F}\left[\int_{0}^{t} f(\tau) d\tau\right] = \frac{F(\omega)}{j\omega} + \frac{I(0)}{j\omega}, I(0) = \int_{-\infty}^{0} f(\tau) d\tau$$

Derivacija frekvencijske funkcije:

$$\mathcal{F}^{-1}\left[\frac{d^n F(\omega)}{d\omega^n}\right] = (-jt)^n f(t)$$

Konvolucija vremenskih: $\mathcal{F}[f(t) * g(t)] = F(\omega) \cdot G(\omega)$

Konvolucija frekvencijskih: $\mathcal{F}[f(t) \cdot g(t)] = \frac{1}{2\pi} F(\omega) * G(\omega)$

Spektri nekih singularnih funkcija*:*

$$\mathcal{F}[\delta(t - t_0)] = e^{-j\omega t_0}$$

$$\mathcal{F}[sgn(t)] = -j\frac{2}{\omega}$$

$$\mathcal{F}[\mu(t)] = \pi \cdot \delta(\omega) - j\frac{1}{\omega}$$

Dobivanje izlaza pomoću impulsnog odziva: $G(\omega) = F(\omega) \cdot H(\omega)$

Približno određivanje odziva: $g(t) = f(t) \cdot H(0) = h(t) \cdot F(0)$

Faktor nelinearnog izobličenja: $THD = \frac{\sqrt{A_2^2 + A_3^2 + \cdots + A_n^2}}{A_1}$

Idealni niskopropusni filtar s pravokutnim prozorom:

$$H(\omega) = A_0 e^{-j\omega t_0} \cdot p_{\omega_g}$$

$$h(t) = \frac{A_0 \omega_g}{\pi} \frac{sin\omega_g(t - t_0)}{\omega_g(t - t_0}$$

Idealni visokopropusni filtar s pravokutnim prozorom

$$H(\omega) = A_0(1 - p_{\omega_g}) e^{j\omega t}$$

$$h(t) = A_0 \delta(t - t_0) - \frac{A_0 \omega_g}{\pi} \frac{sin\omega_g(t - t_0)}{\omega_g(t - t_0}$$

Idealni pojasnopropusni filtar:

$$h(t) = 2 \cdot h_{NP}(t) \cdot \cos(\omega_0 t)$$

Hilbertova transformacija:

$$H_I(\omega) = -\frac{1}{\pi}\int_{-\infty}^{\infty} \frac{H_R(y)}{\omega - y} dy \bullet\!\!-\!\!\circ \frac{1}{j} h_N(t)$$

$$H_R(\omega) = \frac{1}{\pi}\int_{-\infty}^{\infty} \frac{H_I(y)}{\omega - y} dy \bullet\!\!-\!\!\circ h_P(t)$$

Za sustave s minimumom faze:

$$H(\omega) = e^{-\alpha(\omega) - j\theta(\omega)} A = e^{-\alpha(\omega)}$$

$$\theta(\omega) = \frac{\omega}{\pi}\int_{-\infty}^{\infty} \frac{\alpha(y)}{y^2 - \omega^2} dy$$

$$\alpha(\omega) = \alpha(0) - \frac{\omega^2}{\pi}\int_{-\infty}^{\infty} \frac{\theta(y)}{y(y^2 - \omega^2)} dy$$

Pri rješavanju su korisni integrali:

$$\int \frac{dy}{y(y^2 - \omega^2)} = \frac{1}{2\omega^2} ln\left|1 - \frac{\omega^2}{y^2}\right|$$

$$\int \frac{dy}{y^2 - \omega^2} = \frac{1}{2\omega} ln\left|\frac{y - \omega}{y + \omega}\right|$$

$$\cos(x \pm y) = cosxcosy \mp sinxsiny$$

$$2sin^2 x = 1 - \cos(2x)$$

$$2cos^2 x = 1 + \cos(2x)$$

$$4sin^3 x = 3sinx - \sin(3x)$$

$$4cos^3 x = 3cosx - \cos(3x)$$

$$8\sin^4 x = 3 - \cos(2x) + \cos(4x)$$

$$8cos^4 x = 3 + \cos(2x) + \cos(4x)$$

Pojačala

Neinvertirajući spoj:

$$A_{REAL}=\frac{u_{iz}}{u_{ul}}=\left(1+\frac{R_2}{R_1}\right)\frac{1}{1+\frac{1+\frac{R_2}{R_1}}{a}}$$

$$A_{IDEAL}=1+\frac{R_2}{R_1}$$

Kad ne zanemarujemo ulazni otpor r_D i izlazni otpor r_O:

$$A=\frac{\left(1+\frac{R_2}{R_1}\right)a+\frac{r_O}{r_D}}{1+a+\frac{R_2}{R_1}+\frac{R_2+r_O}{r_D}+\frac{r_O}{R_1}}\approx\left(1+\frac{R_2}{R_1}\right)\frac{1}{1+\frac{1}{T}}$$

$$T=a\beta$$

$$\beta=\frac{R_1}{R_1+R_2}$$

$$R_{ul}=\left(1+\frac{a}{1+\frac{R_2+r_O}{R_1}}\right)r_D+R_1||(R_2+r_O)\approx r_D(1+T)$$

$$R_{iz}=\frac{r_O}{1+\frac{a+\frac{r_O}{R_1}+\frac{r_O}{r_D}}{1+\frac{R_2}{R_1}+\frac{R_2}{r_D}}}\approx r_O\frac{1}{1+T}$$

Invertirajući spoj:

$$A_{REAL}=\frac{u_{iz}}{u_{ul}}=-\frac{R_2}{R_1}\frac{1}{1+\frac{1+\frac{R_2}{R_1}}{a}}$$

$$A_{IDEAL}=-\frac{R_2}{R_1}$$

Kad ne zanemarujemo ulazni otpor r_D i izlazni otpor r_O:

$$A=-\frac{aR_2-r_O}{(1+a)R_1+(R_2+r_O)\left(1+\frac{R_1}{r_D}\right)}\approx-\frac{R_2}{R_1}\frac{1}{1+\frac{1}{T}}$$

$$R_{ul}=R_1+\frac{R_2+r_O}{1+a\frac{R_2+r_O}{r_D}}$$

$$R_{iz}=r_O\frac{1}{1+T}$$

Invertirajuće zbrajalo (R_F u povratnoj vezi, R_B od + stezaljke prema masi): $U_{iz}=-\left(\frac{U_1}{R_1}+\frac{U_2}{R_2}+\frac{U_3}{R_3}\right)\cdot R_F$

$$R_B\approx R_1||R_2||R_3||R_F$$

Neinvertirajuće zbrajalo:

$$U_{iz}=\left(\frac{U_1}{R_1}+\frac{U_2}{R_2}+\frac{U_3}{R_3}\right)\cdot R_1||R_2||R_3\cdot(1+\frac{R_F}{R_B})$$

Zbrajalo za diferencijalne signale:

$$U_{iz}=(U_{1H}-U_{1L})\frac{R}{R_1}+(U_{2H}-U_{2L})\frac{R}{R_2}+(U_{3H}-U_{3L})\frac{R}{R_3}$$

Diferencijsko pojačalo:

$$R_{ulD}=2R_1, R_{ulZ}=\frac{R_1+R_2}{2}$$

$$u_{iz}=A_Du_D\pm A_Zu_Z$$

$$A_Z=\frac{1-\frac{R_2R_3}{R_1R_4}}{1+\frac{R_3}{R_4}}$$

$$A_D=\frac{1}{2}\cdot\frac{1+2\frac{R_2}{R_1}+\frac{R_2R_3}{R_1R_4}}{1+\frac{R_3}{R_4}}$$

Za $\frac{R_3}{R_4}=\frac{R_1}{R_2}$ vrijedi: $A_Z=0, A_D=\frac{R_2}{R_1}$

Faktor potiskivanja (**CMRR**): $CMRR_{dB}=20log_{10}\left|\frac{A_D}{A_Z}\right|$

T mreža u povratnoj vezi:

$$A=-\frac{R_2+R_3+\frac{R_2R_3}{R_4}}{R_1}$$

Statičke nesavršenosti: ulazna struja pomaka (I_{pom}), ulazna struja (I_b), ulazno napon pomaka (U_{pom}):

$$U_{iz,pom}=\left\{|U_{pom}|+|I_{pom}|\frac{R_3+(R_1||R_2)}{2}+|I_b|(R_3-R_1||R_2)\right\}\left(1+\frac{R_2}{R_1}\right)$$

On se minimizira ako vrijedi $R_3=R_1||R_2$

Dinamičke nesavršenosti: frekvencijska ovisnost pojačanja operacijskog pojačala:

$$A(jf)=\frac{1}{1+j\frac{f}{f_t}}$$

$$u_{iz}=U_m\left(1-e^{-\frac{t}{\tau}}\right), \tau=\frac{1}{2\pi f_t}, t_r=\frac{0.35}{f_t}$$

Brzina porasta izlaznog napona:

$$BP=\frac{du_0}{dt}|max=(za\ sinus)U_0\omega_0=(za\ trokut)\frac{4U_0\omega_0}{2\pi}$$

Frekvencijska ovisnost pojačanja operacijskog pojačala:

$$a(jf)=\frac{a_0}{1+j\frac{f}{f_g}}$$

$$a_0f_g=a'\cdot f'$$

Utjecaj izlazne impedancije: $Z_{iz}=r_O\frac{1+j\frac{f}{f_b}}{1+j\frac{f}{f_B}}$

Na f_b $R_{iz}=\frac{r_O}{1+a_0\beta}$, a ,a f_B $R_{iz}=r_O$.

Utjecaj ulazne impedancije: $Z_{ul}=R_{ul}\frac{1+j\frac{f}{f_b}}{1+j\frac{f}{f_t}}$

Na f_b $R_{ul}=\frac{R}{1+a_0\beta}$, a ,a f_t $R_{ul}=R$;($R_{ul}=\frac{R}{1+a_0}$)

Izmjenična pojačala:

Pojačalo u invertirajućem spoju

Donju graničnu frekvenciju određuje serijski spojeni kondenzator C_1: $f_{DG}=\frac{1}{2\pi R_1C_1}$, $\beta=\frac{Z_1}{Z_1+Z_2}$

$$A=-\frac{R_2}{R_1}\cdot\frac{jf}{f_1+jf}\cdot\frac{1}{1+\frac{1}{a\beta}}$$

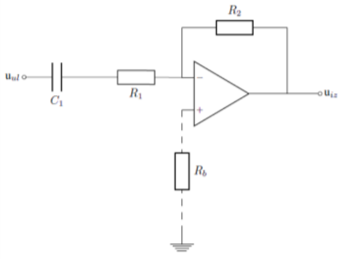
$$|Z_{ul}|=\left|R_1+\frac{1}{j\omega C_1}\right|$$

Derivator

$$A=-\frac{R_2}{Z_1}\frac{1}{1+\frac{1}{a\beta}}, \beta=\frac{Z_1}{Z_1+R_2}$$

$$A=\frac{-R_2C_1s}{1+s\left(R_1C_1+\frac{1}{a_0 2\pi f_g}\right)+s^2\frac{C_1(R_1+R_2)}{a_0 2\pi f_g}}$$

$$u_{iz}(t)=-R_2C_1\frac{du_{ul}}{dt}==-\tau\frac{du_{ul}}{dt}$$



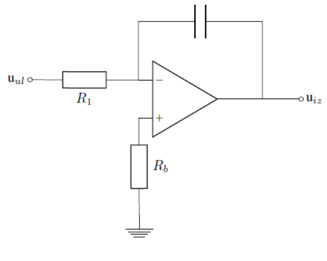
Integrator

$$A=-\frac{1}{R_1C_2s}\cdot\frac{1}{1+\frac{s+2\pi f_g}{a_0 2\pi f_g}\frac{1+R_1C_2s}{R_1C_2s}}$$

$$\omega_i=\frac{1}{\tau_i}=\frac{1}{R_1C_2}$$

f_g je u oba slučaja granična frekvencija operacijskog pojačala

$$u_{iz}=U_0-\frac{1}{\tau}\int_{t_0}^{t_0+\tau}u_{ul}(t)dt$$



Pomak istosmjerne razine:

Invertirajući spoj:

$$u_{iz}=-\frac{R_2}{R_1}u_{ul}+U_p(1+\frac{R_2}{R_1})$$

Neinvertirajući spoj:

$$u_{iz}=u_{ul}\left(1+\frac{R_2}{R_1+R_0}\right)-U_p\cdot\frac{R_2}{R_1}$$