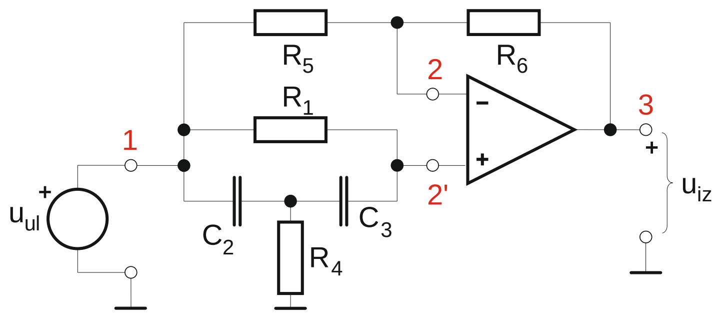
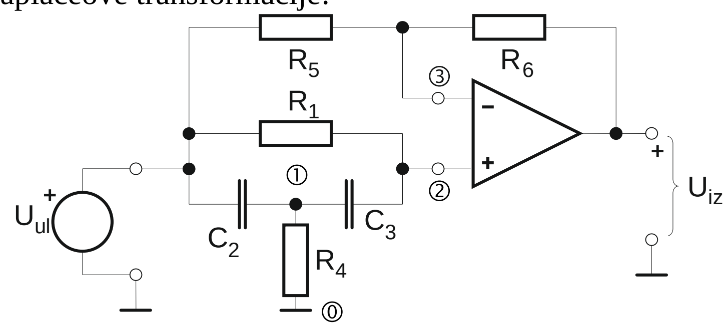
ZAVRŠNI ISPIT IZ ANALOGNE I MJEŠOVITE OBRADE SIGNALA 2016-2017
Rješenja

1. Za aktivni-RC električni filtar prikazan slikom koji realizira pojasnu branu (PB) izračunati metodom napona čvorišta naponsku prijenosnu funkciju $T(s)=U_{iz}(s)/U_{ul}(s)$. Operacijsko pojačalo je idealno ($A\rightarrow\infty$). Usporedbom s općim oblikom PB prijenosne funkcije filtra 2. stupnja odrediti izraze za Q-faktor polova, q_p, frekvenciju polova ω_p, frekvenciju nula ω_z, te pojačanje u području propuštanja k, kao funkcije elemenata. Koliko iznose širina pojasa gušenja B, te gornja i donja granična frekvencija ω_g i ω_d kao funkcije parametara ω_p i q_p ?
(7 bodova)



Opći oblike prijenosne funkcije PB filtra 2. stupnja je: $T_{PB}(s)=\dfrac{U_{iz}(s)}{U_{ul}(s)}=k\cdot\dfrac{s^2+\omega_z^2}{s^2+\dfrac{\omega_p}{q_p}s+\omega_p^2}$

Rješenje: Primjenom Laplaceove transformacije:



Metoda napona čvorišta:

$$(1)\ U_1\left(sC_2+sC_3+\frac{1}{R_4}\right)-U_2sC_3=U_{ul}sC_2\ \Big/\cdot R_4$$

$$(2)\ -U_1sC_3+U_2\left(\frac{1}{R_1}+sC_3\right)=U_{ul}\frac{1}{R_1}\ \Big/: sC_3$$

$$(3)\ U_3\left(\frac{1}{R_5}+\frac{1}{R_6}\right)=U_{ul}\frac{1}{R_5}+U_{iz}\frac{1}{R_6}\ \Big/\cdot R_5R_6$$

$$(4)\ \underline{A(U_2-U_3)=U_{iz}\ \Rightarrow\ U_2=U_3\ \ (A\rightarrow\infty)}$$

Slijedi postepeno računanje korak po korak

$$(3)\Rightarrow\ U_3(R_5+R_6)=U_{ul}R_6+U_{iz}R_5\ \Rightarrow\ U_3=U_{ul}\frac{R_6}{R_5+R_6}+U_{iz}\frac{R_5}{R_5+R_6}$$

Uz oznaku $\alpha=\dfrac{R_5}{R_5+R_6}\ \Rightarrow\ U_3=U_{ul}(1-\alpha)+U_{iz}\alpha$, i zajedno sa (4) $\Rightarrow$

$$U_2 = U_3 = U_{ul}(1-\alpha) + U_{iz}\alpha$$

$$(2) \Rightarrow U_1 = U_2\left(\frac{1}{sC_3R_1}+1\right) - U_{ul}\frac{1}{sC_3R_1}$$

$$(1) \Rightarrow U_1(sR_4C_2 + sR_4C_3 + 1) - U_2R_4sC_3 = U_{ul}R_4sC_2$$

(2) → (1) (rješavamo se U_1) ⇒

$$\left[U_2\left(\frac{1}{sC_3R_1}+1\right) - U_{ul}\frac{1}{sC_3R_1}\right](sR_4C_2 + sR_4C_3 + 1) - U_2R_4sC_3 = U_{ul}R_4sC_2$$

$$U_2\left(\frac{1}{sC_3R_1}+1\right)(sR_4C_2 + sR_4C_3 + 1) - U_2R_4sC_3 = U_{ul}\frac{1}{sC_3R_1}(sR_4C_2 + sR_4C_3 + 1) + U_{ul}R_4sC_2 \Big/ \cdot sC_3R_1$$

$$U_2(sC_3R_1 + 1)(sR_4C_2 + sR_4C_3 + 1) - U_2R_1R_4s^2C_3^2 = U_{ul}(s^2R_1C_2C_3R_4 + sR_4C_2 + sR_4C_3 + 1)$$

$$U_2(s^2R_1C_2C_3R_4 + sC_3R_1 + sR_4C_2 + sR_4C_3 + 1) = U_{ul}(s^2R_1C_2C_3R_4 + sR_4C_2 + sR_4C_3 + 1)$$

$$[U_{ul}(1-\alpha) + U_{iz}\alpha](s^2R_1C_2C_3R_4 + sC_3R_1 + sR_4C_2 + sR_4C_3 + 1) = U_{ul}(s^2R_1C_2C_3R_4 + sR_4C_2 + sR_4C_3 + 1)$$

$$U_{iz}\alpha(s^2R_1C_2C_3R_4 + sC_3R_1 + sR_4C_2 + sR_4C_3 + 1) =$$
$$U_{ul}(s^2R_1C_2C_3R_4 + sR_4C_2 + sR_4C_3 + 1) - U_{ul}(1-\alpha)(s^2R_1C_2C_3R_4 + sC_3R_1 + sR_4C_2 + sR_4C_3 + 1)$$

$$U_{iz}\alpha(s^2R_1C_2C_3R_4 + sC_3R_1 + sR_4C_2 + sR_4C_3 + 1) =$$
$$U_{ul}(s^2R_1C_2C_3R_4 + sR_4C_2 + sR_4C_3 + 1) - U_{ul}(s^2R_1C_2C_3R_4 + sC_3R_1 + sR_4C_2 + sR_4C_3 + 1) +$$
$$+ U_{ul}\alpha(s^2R_1C_2C_3R_4 + sC_3R_1 + sR_4C_2 + sR_4C_3 + 1)$$

$$T(s) = \frac{U_{iz}(s)}{U_{ul}(s)} = \frac{s^2R_1C_2C_3R_4 + s\left[C_3R_1\left(1-\frac{1}{\alpha}\right) + R_4C_2 + R_4C_3\right] + 1}{s^2R_1C_2C_3R_4 + s(C_3R_1 + R_4C_2 + R_4C_3) + 1}$$

Konačno je:

$$T(s) = \frac{U_{iz}(s)}{U_{ul}(s)} = \frac{s^2 + s\dfrac{C_3R_1(1-1/\alpha) + R_4C_2 + R_4C_3}{R_1C_2C_3R_4} + \dfrac{1}{R_1C_2C_3R_4}}{s^2 + s\dfrac{C_3R_1 + R_4C_2 + R_4C_3}{R_1C_2C_3R_4} + \dfrac{1}{R_1C_2C_3R_4}}$$ (3 boda)

Parametri (usporedba s općim oblikom): (2 boda)

$$T_{PB}(s) = \frac{U_{iz}(s)}{U_{ul}(s)} = k\cdot\frac{s^2 + \omega_z^2}{s^2 + \dfrac{\omega_p}{q_p}s + \omega_p^2} \Rightarrow k = 1;\ \omega_p^2 = \omega_z^2 = \frac{1}{R_1C_2C_3R_4} \Rightarrow \omega_p = \omega_z = \frac{1}{\sqrt{R_1C_2C_3R_4}}$$

$$\frac{\omega_p}{q_p} = \frac{C_3R_1 + R_4C_2 + R_4C_3}{R_1C_2C_3R_4} \Rightarrow q_p = \omega_p\frac{R_1C_2C_3R_4}{C_3R_1 + R_4C_2 + R_4C_3} = \frac{\sqrt{R_1C_2C_3R_4}}{C_3R_1 + R_4C_2 + R_4C_3}$$

$q_z = \infty \Rightarrow$ uvjet za PB glasi: $C_3R_1(1/\alpha - 1) = R_4C_2 + R_4C_3$

Širina pojasa gušenja $B = \dfrac{\omega_p}{q_p}$ [rad/s] (1 bod)

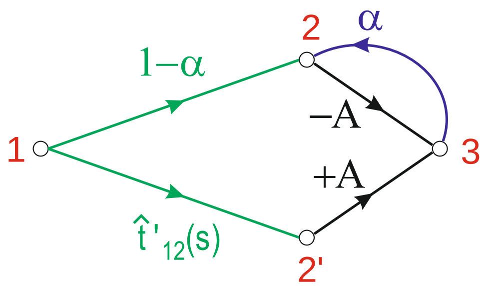
Gornja i donja granična frekvencija pojasa gušenja su (isti izrazi kao u slučaju pojasnog propusta):

$$\omega_{g,d} = \omega_p\sqrt{1 + \frac{1}{4q_p^2}} \pm \frac{\omega_p}{2q_p}$$ [rad/s] $\Rightarrow B = \omega_g - \omega_d$ [rad/s] (1 bod)

2. Za Bikvadratnu sekciju prikazanu u prethodnom zadatku nacrtati dijagram toka signala (DTS). Izračunati naponsku prijenosnu funkciju $T(s)=U_{iz}(s)/U_{ul}(s)$ koristeći DTS. Za izračun prijenosne funkcije DTS-a koristiti Masonovo pravilo. Pritom odrediti sve prijenosne funkcije (transmisije) u dijagramu toka signala, uvrstiti ih u DTS, te provjeriti je li prijenosna funkcija u prvom zadatku točna. Koje sve prijenosne funkcije i pod kojim uvjetima se mogu realizirati? Napisati moguće prijenosne funkcije i potrebne uvjete. Operacijsko pojačalo je idealno ($A\rightarrow\infty$).

(6 bodova)

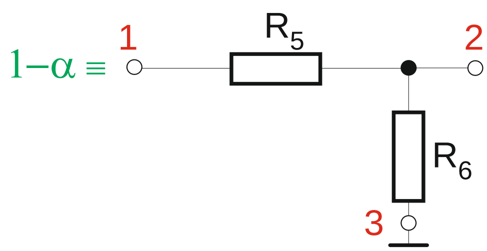
Rješenje:

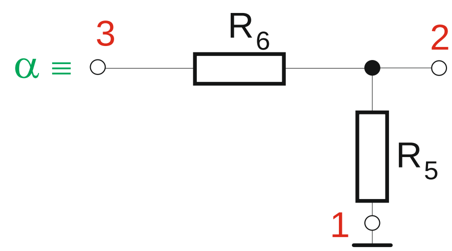


$$T(s)=\frac{U_{iz}(s)}{U_{ul}(s)}=\frac{U_3(s)}{U_1(s)}=\frac{\hat{t}'_{12}A-A(1-\alpha)}{1+A\alpha}=\left.\frac{\hat{t}'_{12}-(1-\alpha)}{\frac{1}{A}+\alpha}\right|_{A\to\infty}=\frac{\hat{t}'_{12}-(1-\alpha)}{\alpha}=\frac{\hat{t}'_{12}}{\alpha}+\left(1-\frac{1}{\alpha}\right)$$

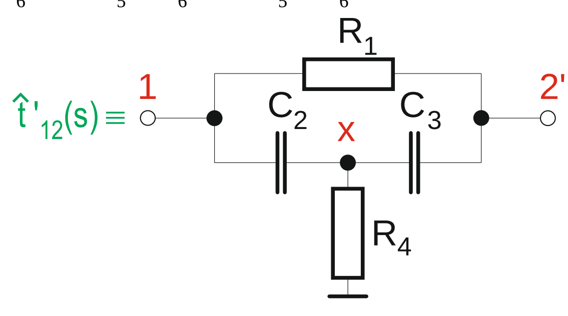
(2 boda)

Pojedine transmisije:





$$1-\alpha=1-\frac{R_6}{R_5+R_6}=\frac{R_5+R_6-R_6}{R_5+R_6}=\frac{R_5}{R_5+R_6};\qquad \alpha=\frac{R_5}{R_5+R_6}$$



Metoda napona čvorišta:

$$(1)\ U_x\left(sC_2+sC_3+\frac{1}{R_4}\right)-U_2sC_3=U_1sC_2\ \Big/\cdot R_4$$

$$(2)\ -U_xsC_3+U_2\left(\frac{1}{R_1}+sC_3\right)=U_1\frac{1}{R_1}\ \Big/:sC_3$$

$$(2)\Rightarrow U_x=U_2\left(\frac{1}{sC_3R_1}+1\right)-U_1\frac{1}{sC_3R_1}$$

$$(1) \Rightarrow U_x(sR_4C_2 + sR_4C_3 + 1) - U_2R_4sC_3 = U_1R_4sC_2$$

(2) $\rightarrow$ (1) (rješavamo se U_x) $\Rightarrow$

$$\left[U_2\left(\frac{1}{sC_3R_1}+1\right) - U_1\frac{1}{sC_3R_1}\right](sR_4C_2 + sR_4C_3 + 1) - U_2R_4sC_3 = U_1R_4sC_2$$

$$U_2\left(\frac{1}{sC_3R_1}+1\right)(sR_4C_2 + sR_4C_3 + 1) - U_2R_4sC_3 = U_1\frac{1}{sC_3R_1}(sR_4C_2 + sR_4C_3 + 1) + U_1R_4sC_2 \Big/ \cdot sC_3R_1$$

$$U_2(sC_3R_1 + 1)(sR_4C_2 + sR_4C_3 + 1) - U_2R_1R_4s^2C_3^2 = U_1(s^2R_1C_2C_3R_4 + sR_4C_2 + sR_4C_3 + 1)$$

$$U_2(s^2R_1C_2C_3R_4 + sC_3R_1 + sR_4C_2 + sR_4C_3 + 1) = U_1(s^2R_1C_2C_3R_4 + sR_4C_2 + sR_4C_3 + 1)$$

$$\hat{t}'_{12}(s) = \frac{U_2(s)}{U_1(s)} = \frac{s^2R_1C_2C_3R_4 + sR_4C_2 + sR_4C_3 + 1}{s^2R_1C_2C_3R_4 + sC_3R_1 + sR_4C_2 + sR_4C_3 + 1} = \frac{s^2 + s\dfrac{R_4C_2 + R_4C_3}{R_1C_2C_3R_4} + \dfrac{1}{R_1C_2C_3R_4}}{s^2 + s\dfrac{C_3R_1 + R_4C_2 + R_4C_3}{R_1C_2C_3R_4} + \dfrac{1}{R_1C_2C_3R_4}}$$

Koristili smo jedan dio istih izraza kao u prvom zadatku! (2 boda)

Uvrstimo sve u gornji izraz za prijenosnu funkciju:

$$T(s) = \frac{U_{iz}(s)}{U_{ul}(s)} = \frac{\hat{t}'_{12}}{\alpha} + \left(1 - \frac{1}{\alpha}\right) = \frac{\dfrac{1}{\alpha}\left(s^2 + s\dfrac{R_4C_2 + R_4C_3}{R_1C_2C_3R_4} + \dfrac{1}{R_1C_2C_3R_4}\right)}{s^2 + s\dfrac{C_3R_1 + R_4C_2 + R_4C_3}{R_1C_2C_3R_4} + \dfrac{1}{R_1C_2C_3R_4}} + \left(1 - \frac{1}{\alpha}\right)$$

$$T(s) = \frac{U_{iz}(s)}{U_{ul}(s)} \frac{\dfrac{1}{\alpha}\left(s^2 + s\dfrac{R_4C_2 + R_4C_3}{R_1C_2C_3R_4} + \dfrac{1}{R_1C_2C_3R_4}\right) + \left(1 - \dfrac{1}{\alpha}\right)\left(s^2 + s\dfrac{C_3R_1 + R_4C_2 + R_4C_3}{R_1C_2C_3R_4} + \dfrac{1}{R_1C_2C_3R_4}\right)}{s^2 + s\dfrac{C_3R_1 + R_4C_2 + R_4C_3}{R_1C_2C_3R_4} + \dfrac{1}{R_1C_2C_3R_4}}$$

$$T(s) = \frac{U_{iz}(s)}{U_{ul}(s)} \frac{s^2 + s\dfrac{C_3R_1 + R_4C_2 + R_4C_3}{R_1C_2C_3R_4} + \dfrac{1}{R_1C_2C_3R_4} - \dfrac{1}{\alpha}s\dfrac{C_3R_1}{R_1C_2C_3R_4}}{s^2 + s\dfrac{C_3R_1 + R_4C_2 + R_4C_3}{R_1C_2C_3R_4} + \dfrac{1}{R_1C_2C_3R_4}}$$

$$T(s) = \frac{U_{iz}(s)}{U_{ul}(s)} = \frac{s^2 + s\dfrac{C_3R_1(1 - 1/\alpha) + R_4C_2 + R_4C_3}{R_1C_2C_3R_4} + \dfrac{1}{R_1C_2C_3R_4}}{s^2 + s\dfrac{C_3R_1 + R_4C_2 + R_4C_3}{R_1C_2C_3R_4} + \dfrac{1}{R_1C_2C_3R_4}}$$

Dobili smo istu prijenosnu funkciju kao i u prvom zadatku. (1 bod)

Možemo realizirati (1 bod)

1. pojasnu branu: $T_{PB}(s) = \dfrac{U_{iz}(s)}{U_{ul}(s)} = \dfrac{s^2 + \omega_p^2}{s^2 + s(\omega_p / q_p) + \omega_p^2}$; Uvjet: $C_3R_1(1 - 1/\alpha) + R_4C_2 + R_4C_3 \Rightarrow \alpha$

2. svepropusnu prijenosnu funkciju: $T_{SP}(s) = \dfrac{U_{iz}(s)}{U_{ul}(s)} = \dfrac{s^2 - s(\omega_p / q_p) + \omega_p^2}{s^2 + s(\omega_p / q_p) + \omega_p^2}$

Uvjet: $C_3R_1(1 - 1/\alpha) + R_4C_2 + R_4C_3 = -(C_3R_1 + R_4C_2 + R_4C_3) \Rightarrow \alpha$

Ovo pitanje nije postavljeno: Kojoj filtarskoj klasi pripada navedena Bikvadratna sekcija?

Ne pripada niti jednoj klasi: nema povratnu vezu $t_{32}(s)$ za realizaciju konjugirano-kompleksnih polova.

3. Pomoću Bikvadratne sekcije prikazane u 1. zadatku realizirati (širokopojasni) PB filtar 2. reda s faktorom dobrote polova q_p=1/4 i normaliziranom centralnom frekvencijom ω_0=1. Izračunati sve normalizirane elemente filtra tako da se u proračunu izaberu kapaciteti C_2=C_3=1. Izračunati pojačanje u području propuštanja *k*, širinu pojasa gušenja *B*, te gornju i donju graničnu frekvenciju područja gušenja (normalizirane vrijednosti)? (7 bodova)

Rješenje:
Najprije napišimo normaliziranu PB prijenosnu funkciju.

$$H_{PB}(s) = k \cdot \frac{s^2 + \omega_z^2}{s^2 + (\omega_p / q_p)s + \omega_p^2} = \frac{s^2 + 1}{s^2 + 4s + 1}$$

U realizaciji koristimo bikvadratnu sekciju s karakteristikom pojasne brane s jednim pojačalom koja je prikazana na slici u zadatku 1. U proračunu ćemo pretpostaviti C_2=C_3=C=1 pa će izrazi za ω_p, q_p i k iz zadatka 1 poprimiti jednostavniji oblik:

k=1;

$$\omega_p = \omega_z = \frac{1}{\sqrt{R_1 C_2 C_3 R_4}} \Rightarrow \omega_p = \omega_z = \frac{1}{C\sqrt{R_1 R_4}}\,;\; q_p = \frac{\sqrt{R_1 C_2 C_3 R_4}}{C_3 R_1 + R_4 C_2 + R_4 C_3} \Rightarrow q_p = \frac{\sqrt{R_1 R_4}}{R_1 + 2R_4}\,;$$

$$q_z = \infty \Rightarrow \text{uvjet za PB glasi: } C_3 R_1 (1/\alpha - 1) = R_4 C_2 + R_4 C_3 \Rightarrow \alpha = \frac{1}{2R_4 / R_1 + 1}$$

Izračun jednadžbi iz uvjeta za ω_p i q_p:

Iz $q_p = \frac{1}{4}$ slijedi $\frac{1}{4} = \frac{\sqrt{R_1 R_4}}{R_1 + 2R_4} \Rightarrow R_1 + 2R_4 = 4\sqrt{R_1 R_4}$ Iz ω_p=1 slijedi $1 = \frac{1}{C\sqrt{R_1 R_4}} \Rightarrow C\sqrt{R_1 R_4} = 1$

(2 boda)

Proračun elemenata: Uz odabir C=1 je C_2=1, C_3=1 i računamo: $\sqrt{R_1 R_4} = 1 \Rightarrow R_1 = \frac{1}{R_4}$

Odnosno: $R_1 + 2R_4 = 4 \Rightarrow R_1 + \frac{2}{R_1} = 4\,;\; R_1^2 - 4R_1 + 2 = 0 \Rightarrow (R_1)_{1,2} = 2 \pm \sqrt{2^2 - 2} = 2 \pm \sqrt{2}$

Imamo dva rješenja: $(R_1)_1 = 2 + \sqrt{2} = 3{,}41421; (R_4)_1 = 1/(R_1)_1 = 0{,}292839$

$(R_1)_2 = 2 - \sqrt{2} = 0{,}585786; (R_4)_2 = 1/(R_1)_2 = 1{,}70711$

Pojačanje iznosi: $k = 1$

Konačno svi normalizirani elementi glase: C_1=1, C_2=1, $R_1 = 0{,}585786$, $R_4 = 1{,}70711$. (2 boda)

Širina pojasa gušenja (normalizirana) je : $B = \frac{\omega_p}{q_p} = 4$ (1 bod)

Gornja i donja granična frekvencija područja gušenja:

$$\omega_{g,d} = \omega_p \sqrt{1 + \frac{1}{4q_p^2}} \pm \frac{\omega_p}{2q_p} = \sqrt{1 + \frac{16}{4}} \pm \frac{4}{2} = \sqrt{5} \pm 2$$

$\omega_g = 4{,}23607$; $\omega_d = 0{,}23607$; $B = 4{,}23607 - 0{,}23607 = 4$ (1 bod)

Uvjet za PB $\Rightarrow \alpha = \frac{1}{2R_4 / R_1 + 1} = \frac{1}{2R_4^2 + 1} = 0{,}146447$

Uz oznaku $\alpha = \frac{R_5}{R_5 + R_6}$ i odabir $R_5 = 1 \Rightarrow R_6 = R_5\left(\frac{1}{\alpha} - 1\right) = 5{,}82843$. (1 bod)