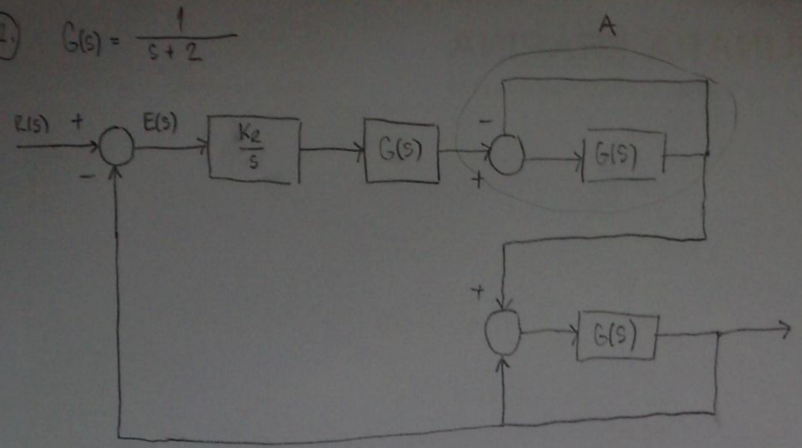
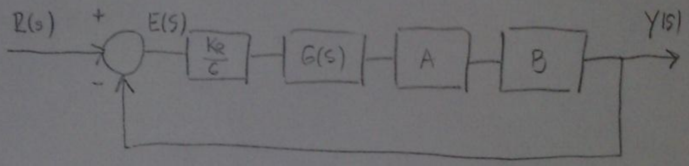
ZI 2009/2010

2. $G(s) = \frac{1}{s+2}$



$$A = \frac{G(s)}{1 + G(s)}$$

$$B = \frac{G(s)}{1 - G(s)}$$



$$G(s) = \frac{\frac{K_R}{s} \cdot \frac{1}{s+2} \cdot \frac{\frac{1}{s+2}}{1+\frac{1}{s+2}} \cdot \frac{\frac{1}{s+2}}{1-\frac{1}{s+2}}}{1 - \frac{K_R}{s} \cdot \frac{1}{s+2} \cdot \frac{\frac{1}{s+2}}{1+\frac{1}{s+2}} \cdot \frac{\frac{1}{s+2}}{1-\frac{1}{s+2}}}$$

$$\frac{K_R}{s} \cdot \frac{1}{s+2} \cdot \frac{\frac{1}{s+2}}{\frac{s+2+1}{s+2}} \cdot \frac{\frac{1}{s+2}}{\frac{s+1}{s+2}} =$$

$$s(s+1)(s+2)(s+3) =$$
$$= (s^2+s)(s+2)(s+3) =$$
$$= (s^3+s^2+2s^2+2s)(s+3) =$$
$$= (s^3+3s^2+2s)(s+3) =$$
$$= s^4 + \cancel{3s^3} + \cancel{3s^3} + \cancel{3s^3} + \cancel{9s^2} + 6s =$$
$$= s^4 + 6s^3 + 11s^2 + 6s$$

$$= \frac{K_R}{s} \cdot \frac{1}{s+2} \cdot \frac{1}{s+3} \cdot \frac{1}{s+1} = K_R \cdot \frac{1}{s(s+2)(s+3)(s+1)}$$

$$Y(s) = K_R \cdot \frac{1}{s(s+1)(s+2)(s+3)}$$

$$R(s) = 1 + \frac{K_R}{s(s+1)(s+2)(s+3)} = \frac{s(s+1)(s+2)(s+3) + K_R}{s(s+1)(s+2)(s+3)}$$

$$\boxed{G(s) = \frac{Y(s)}{R(s)} = \frac{K_R}{s(s+1)(s+2)(s+3) + K_R}} = \frac{K_R}{s^4 + 6s^3 + 11s^2 + 6s + K_R}$$

b) algebarski kriterij stabilnosti

$K_R = ?$

c) Hurwitz = ?

$$\alpha(s) = s^4 + 6s^3 + 11s^2 + 6s + K_R$$

$a_0 = +K_R \quad a_1 = 6 \quad a_2 = 11 \quad a_3 = 6 \quad a_4 = 1$

$$\boxed{K_R > 0}$$

II. UVJET

$D_1 > 0 \Rightarrow a_1 > 0 \quad 6 > 0$ ✓

$D_2 > 0 \quad D_2 = \begin{vmatrix} a_1 & a_0 \\ a_3 & a_2 \end{vmatrix} > 0$

$$D_2 = \begin{vmatrix} 6 & K_R \\ 6 & 11 \end{vmatrix} > 0$$

$$6 \cdot 11 - 6K_R > 0$$

$$66 - 6K_R > 0$$

$$-6K_R > -66$$

$$6K_R < 66$$

$$\boxed{K_R < 11}$$

$D_3 > 0 \quad \begin{vmatrix} a_1 & a_0 & 0 \\ a_3 & a_2 & a_1 \\ a_5 & a_4 & a_3 \end{vmatrix} > 0 \quad \begin{vmatrix} 6 & K_R & 0 \\ 6 & 11 & 6 \\ 0 & 1 & 6 \end{vmatrix} > 0$

$$6 \cdot (66 - 6) - K_R(36) + 0 > 0$$

$$360 - 36K_R > 0$$

$$-36K_R > -360$$

$$\boxed{K_R < 10}$$

$K_R \in \langle 0, 10 \rangle$ ✓

c) $r(t) = S(t) \Rightarrow R(s) = \frac{1}{s}$

$K_R = 5$

$e_\infty = ?$

$E(s) = ?$

$$E(s) = R(s) \cdot \frac{1}{1 + G_0(s)} = \frac{1}{s} \cdot \frac{\frac{1}{1}}{\frac{s(s+1)(s+2)(s+3) + K_R}{s(s+1)(s+2)(s+3)}} =$$

$$= \frac{1}{s} \cdot \frac{s(s+1)(s+2)(s+3)}{s(s+1)(s+2)(s+3) + K_R} = \frac{\cancel{s}(s+1)(s+2)(s+3)}{\cancel{s} \cdot (s(s+1)(s+2)(s+3) + K_R)}$$

$$= \frac{(s+1)(s+2)(s+3)}{s(s+1)(s+2)(s+3) + K_R}$$

$$e_\infty = \lim_{s \to 0} s \cdot E(s) = \lim_{s \to 0} \cancel{s} \cdot \frac{1}{\cancel{s}} \cdot \frac{s(s+1)(s+2)(s+3)}{s(s+1)(s+2)(s+3) + K_R} = \emptyset$$

d) $r(t) = t \cdot S(t)$

$K_R = ?$

$e_\infty = 1$

$$\lim_{s \to 0} s \cdot E(s) = 1$$

$$\lim_{s \to 0} s \cdot R(s) \cdot \frac{1}{1 + G_0(s)} = 1$$

$(s+1)(s+2)(s+3) =$

$= (s^2 + 3s + 2)(s+3) =$

$= (s^3 + 3s^2 + 2s + 3s^2 + 9s + 6) =$

$= \underline{(s^3 + 6s^2 + 11s + 6)}$

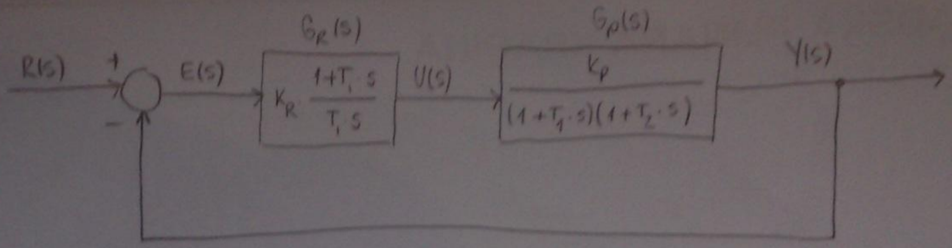
$$\lim_{s \to 0} \cancel{s} \cdot \frac{1}{\cancel{s^2}} \cdot \frac{\cancel{s}(s+1)(s+2)(s+3)}{s(s+1)(s+2)(s+3) + K_R} = 1$$

$$\lim_{s \to 0} \frac{s^3 + 6s^2 + 11s + 6}{s^4 + 6s^3 + 11s^2 + 6s + K_R} = 1$$

$$\frac{6}{K_R} = 1 \Rightarrow$$

$$\boxed{K_R = 6}$$

3



$G_p \quad K_p = 3 \quad T_1 = 1s \quad T_2 = 0.3s$

a/ $G_0(s) = G_R(s) \cdot G_p(s)$

$T_I = T_1 = 1s$

$$G_0(s) = K_R \cdot \frac{\cancel{1+s}}{s} \cdot \frac{3}{(\cancel{1+s})(1+0{,}3s)} =$$

$$= K_R \cdot \frac{3}{s(1+0.3s)}$$

$$G_0(j\omega) = K_R \cdot \frac{3}{j\omega(1+0.3j\omega)} = K_R \cdot \frac{3}{j\omega\left(1+j\,\frac{\omega}{3.33}\right)}$$

$\gamma = 60°$

$\gamma = 180° + \varphi(\omega_c) \Rightarrow \varphi(\omega_c) = 60° - 180°$

$$\operatorname{arctg}\frac{0}{3K_R} - \operatorname{arctg}\frac{\omega}{\varnothing} - \operatorname{arctg}\frac{\frac{\omega}{3.33}}{1} = -120°$$

$$-\frac{\pi}{2} - \operatorname{arctg}\frac{\omega}{3.33} = -120°$$

$$-\operatorname{arctg}\frac{\omega}{3.33} = -30$$

$$-\frac{\omega}{3.33} = \frac{-\sqrt{3}}{3} \Rightarrow \boxed{\omega_c = 1.92257 \text{ rad}}$$

$$|G_0(j\omega_c)| = 1$$

$$\left| \frac{3 \cdot K_R}{j\omega_c\,(1+0.3j\omega_c)} \right| = 1$$

$$\frac{3K_R}{\omega_c \sqrt{1+\left(\frac{\omega_c}{3.33}\right)^2}} = 1$$

$$3K_R = \omega_c \cdot \sqrt{1+\left(\frac{\omega_c}{3.33}\right)^2} \Big/^2$$

$$9K_R^2 = \omega_c^2 \cdot \left(1+\left(\frac{\omega_c}{3.33}\right)^2\right)$$

$$9K_R^2 = 1.92257^2 \cdot \left(1+\left(\frac{1.92257}{3.33}\right)^2\right)$$

$$9K_R^2 = 4.95086 \Rightarrow \boxed{K_R = 0.74168} \Rightarrow$$

$$\boxed{G_R(s) = 0.74168 \cdot \frac{1+s}{s}}$$

b) $T = (0.17 \div 0.34)\,\frac{1}{\omega_c}$, $\omega_c = 1.92257$

$T = 88.42\,ms \div 176.84\,ms \Rightarrow$

$$\boxed{T = 100\,ms}$$

c) $G_R(s) = 0.74168 \dfrac{1+s}{s}$

TUSTIN: $s = \dfrac{2}{T} \cdot \dfrac{z-1}{z+1}$, $T = 100\,ms$

$$G_R(z) = 0.74168 \cdot \frac{1 + 20\frac{z-1}{z+1}}{20 \cdot \frac{z-1}{z+1}} =$$

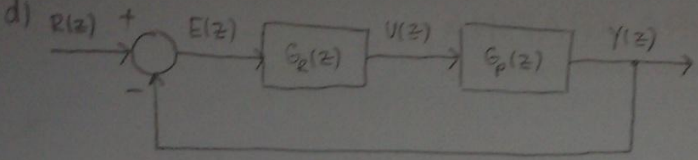
$$= 0.74168 \frac{\frac{z+1+20(z-1)}{z+1}}{\frac{20(z-1)}{z+1}} =$$

$$= 0.74168 \cdot \frac{z+1+20z-20}{20(z-1)} = 0.037084 \cdot \frac{21z-19}{z-1} =$$

$$= \frac{0.778764z - 0.704596 \;/\cdot z^{-1}}{z-1 \;/\cdot z^{-1}} = \frac{0.7787 - 0.7046z^{-1}}{1-z^{-1}} = \frac{U(z)}{E(z)}$$

$$u(k) - u(k-1) = 0.7787\,e(k) - 0.7046\,e(k-1)$$

d)



$$G_p(z) = (1 - z^{-1}) \cdot Z\left\{ \frac{G_p(s)}{s} \right\}$$

$$\frac{G_p(s)}{s} = \frac{\frac{3}{(1+s)(1+0.3s)}}{s} = \frac{3}{s\,(1+s)(1+0.3s)}$$

$$\frac{c_{11}}{s} + \frac{c_{21}}{1+s} + \frac{c_{31}}{1+0.3s} = \frac{3}{s} + \frac{-4.2857}{1+s} + \frac{0.3857}{1+0.3s}$$

$$c_{11}(s=0) = 3$$

$$c_{21}(s=-1) = -\frac{30}{7} = -4.2857$$

$$c_{31}\left(s=-\frac{10}{3}\right) = \frac{27}{70} = 0.3857$$

$$Z\left\{ \frac{G_p(s)}{s} \right\} = 3 \cdot \frac{1}{1-z^{-1}} + \frac{-4.2857}{1-e^{-1 \cdot 100ms}\, z^{-1}} + \frac{1.2857}{1-e^{-3.33 \cdot 100ms}\, z^{-1}} =$$

$$= 3 \cdot \frac{z}{z-1} - 4.2857 \cdot \frac{z}{z-0.9048} + 1.2857 \cdot \frac{z}{z-0.7167} =$$

$$\frac{3 \cdot z \cdot (z-0.9048) \cdot (z-0.7167) - 4.2857 z \cdot (z-1)(z-0.7167) + 1.2857 z (z-1)(z-0.9048)}{(z-1) \cdot (z-0.9048)(z-0.7167)} =$$

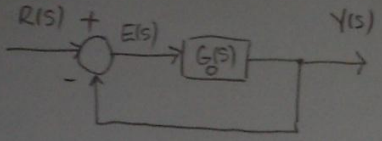
$$= \frac{0.0437 z^2 + 0.037 z}{(z-1)(z^2 - 1.6215 z + 0.6484)}$$

$$G_p(z) = \frac{\cancel{z-1}}{\cancel{z}} \cdot \frac{\cancel{z}(0.0437z + 0.037)}{\cancel{(z-1)}(z^2 - 1.6215z + 0.6484)} =$$

$$\boxed{= \frac{0.0437z + 0.037}{z^2 - 1.6215z + 0.6484}}$$

e)

$$\Delta\varphi = -\omega_c \cdot \frac{T}{2} = -1.92257 \cdot \frac{100ms}{2} = -0.09612 \, rad \cdot \frac{180}{\pi} = -5.507^\circ$$



$$E(s) = R(s) - E(s)\,G_0(s)$$

$$E(s) + E(s)G_0(s) = R(s)$$

$$E(s)\,(1 + G_0(s)) = R(s)$$

$$E(s) = \frac{R(s)}{1 + G_0(s)}$$

recimo + u povratnoj:

$$E(s) = R(s) + E(s) \cdot G_0(s)$$

$$E(s) - E(s)G_0(s) = R(s)$$

$$E(s)\,(1 - G_0(s)) = R(s)$$

$$E(s) = R(s) \cdot \frac{1}{1 - G_0(s)}$$