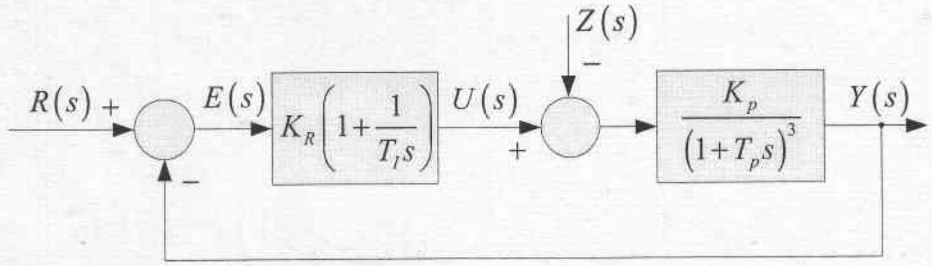
6. domaća zadaća
PID regulator

Priprema za vježbu

Zadatak 1

Zadan je sustav upravljanja s PI regulatorom prikazan blokovskom shemom na Slici 1. Parametri procesa



Slika 1: Sustav upravljanja.

K_p i T_p su:

- Termin A: (08:00-10:00): $K_p = 2$, $T_p = 0.5$ s;
- Termin B: (10:00-12:00): $K_p = 4$, $T_p = 2$ s;
- Termin C: (12:00-14:00): $K_p = 5$, $T_p = 3$ s;
- Termin D: (14:00-16:00): $K_p = 1$, $T_p = 0.25$ s.

Potrebno je:

a) Odrediti područje stabilnosti sustava u ravnini K_R-T_I parametara PI regulatora, uz $K_R \geq 0$ i $T_I \geq 0$;

b) Parametrirati PI regulator minimizacijom ISE kriterija pri odzivu sustava na skokovitu promjenu vodeće veličine;

c) Parametrirati PI regulator minimizacijom ISE kriterija pri odzivu sustava na skokovitu promjenu poremećajne veličine;

d) Parametrirati PI i idealni PID regulator za prikazani proces korištenjem Ziegler-Nicholsove metode ruba stabilnosti;

e) Diskretizirati idealni PID regulator parametriran pod d) korištenjem Tustinova postupka, uz vrijeme uzorkovanja odabrano na temelju frekvencijskih karakteristika otvorenog kontinuiranog regulacijskog kruga. Navedene frekvencijske karakteristike odredite crtanjem Bodeova dijagrama korištenjem aproksimacije pravcima.

f) Odrediti rekurzivnu jednadžbu PID regulatora diskretiziranog pod e).

6. DOMAĆA ZADAĆA
TERMIN B: 10:00 - 12:00

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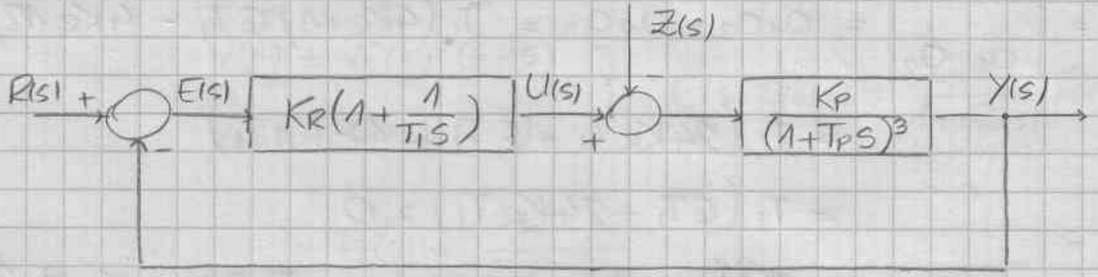


$K_P = 4,\ T_P = 2\,s$

(a) $K_R \geq 0,\ T_I \geq 0$

$$G_0(s) = \frac{K_P K_R (1+T_I s)}{T_I s (1+T_P s)^3}$$

$$G_0(s) = \frac{4K_R(1+T_I s)}{T_I s(1+2s)^3}$$

$$\mathcal{L}_{CE} = 4K_R(1+T_I s) + T_I s(8s^3 + 12s^2 + 6s + 1)$$

$$\mathcal{L}_{CE} = \underbrace{8T_I}_{a_4} s^4 + \underbrace{12T_I}_{a_3} s^3 + \underbrace{6T_I}_{a_2} s^2 + \underbrace{(4K_R T_I + T_I)}_{a_1} s + \underbrace{4K_R}_{a_0}$$

(1) SVI KOEFICIJENTI MORAJU BITI POZITIVNI

$8T_I > 0,\ 12T_I > 0,\ 6T_I > 0 \rightarrow T_I > 0$

$(4K_R + 1)T_I > 0 \rightarrow T_I\left(K_R + \frac{1}{4}\right) > 0$

$\hookrightarrow T_I > 0 \quad K_R > -\frac{1}{4}$

$4K_R > 0 \rightarrow K_R > 0$

PRESJEK: $K_R > -\frac{1}{4},\ T_I > 0$

(2) SVE DETERMINANTE (KOMADA 3) MORAJU BITI VEĆE OD NULA

$D_1 = a_1 = T_I(4K_R + 1) > 0 \rightarrow \quad T_I > 0 \quad K_R > -\frac{1}{4}$

$$D_2 = \begin{vmatrix} a_1 & a_0 \\ a_3 & a_2 \end{vmatrix} = a_1 a_2 - a_0 a_3 = T_1(4K_R+1)\,6T_1 - 4K_R\,12T_1$$

$$= T_1\left(24K_RT_1 + 6T_1 - 48K_RT_1\right)$$

$$= T_1\left(6T_1 - 24K_RT_1\right) > 0$$

$$T_1^2(1-4K_R) > 0 \qquad T_1 \in \mathbb{R},\ K_R < \frac{1}{4}$$

$$D_3 = \begin{vmatrix} a_1 & a_0 & 0 \\ a_3 & a_2 & a_1 \\ 0 & a_4 & a_3 \end{vmatrix} = a_1\left(a_2a_3 - a_1a_4\right) - a_0\left(a_3^2 - 0\right) + 0\cdot\left(a_3a_4 - 0\right)$$

$$= T_1(4K_R+1)\left(72T_1^2 - 8T_1^2(4K_R+1)\right) - 576\,T_1^2K_R$$

$$= T_1^3(4K_R+1)(72 - 32K_R - 8) - 576\,T_1^2K_R$$

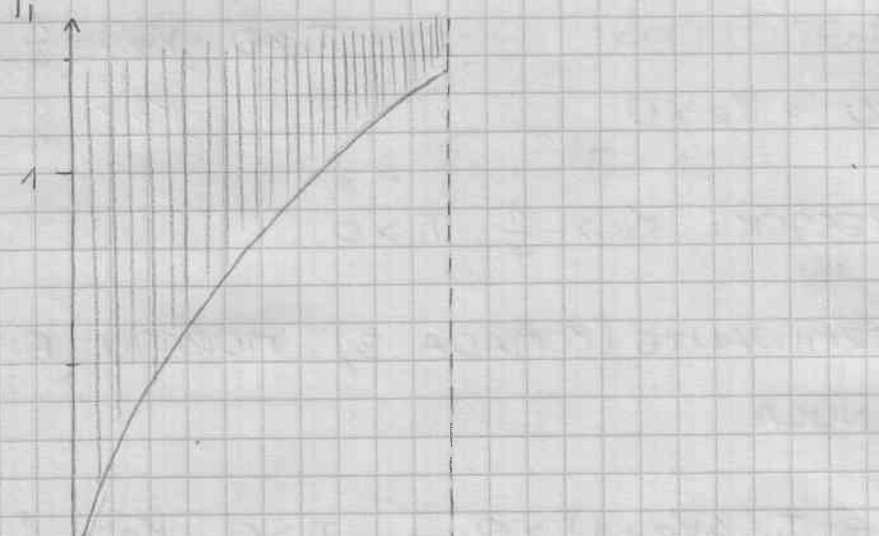
$$= T_1^3\,4\left(K_R+\tfrac{1}{4}\right)(-32)\left(K_R-2\right) - 576\,T_1^2K_R > 0 \quad \Big/ : 128$$

$$T_1^3\left(-K_R^2 + 1.75K_R + 0.5\right) - 4.5\,T_1^2K_R > 0$$

$$T_1^2\left(-T_1K_R^2 + 1.75T_1K_R + 0.5T_1 - 4.5K_R\right) > 0$$

$$T_1\left(-K_R^2 + 1.75K_R + 0.5\right) > 4.5K_R$$

$$T_1 > \frac{4.5K_R}{-K_R^2 + 1.75K_R + 0.5}$$



(b) $R(s) = \frac{1}{s}$

$$\left.\begin{aligned} Y(s) &= E(s)\,G_0(s) \\ Y(s) &= G(s)\,R(s) \end{aligned}\right\} \quad E(s)\,G_0(s) = \frac{G_0(s)}{1+G_0(s)} R(s)$$

$$E(s) = \frac{R(s)}{1+G_0(s)}$$

NAKON KRAĆEG UVRŠTAVANJA DOBIJEMO

$$E(s) = \frac{8T_1 s^3 + 12 T_1 s^2 + 6T_1 s + T_1}{8T_1 s^4 + 12 T_1 s^3 + 6T_1 s^2 + T_1(4K_R+1)s + 4K_R}$$

$c_0 = T_1$, $c_1 = 6T_1$, $c_2 = 12T_1$, $c_3 = 8T_1$

$d_0 = 4K_R$, $d_1 = T_1(4K_R+1)$, $d_2 = 6T_1$, $d_3 = 12T_1$, $d_4 = 8T_1$

$$I_{3,4} = \frac{c_3^2(-d_0^2 d_3 + d_0 d_1 d_2) + (c_2^2 - 2c_1c_3)d_0d_1d_4 + (c_1^2 - 2c_0c_2)d_0d_3d_4 + \ldots}{2d_0d_4(-d_0d_3^2 - d_1^2 d_4 + d_1d_2d_3)}$$

NAKON DULJEG RAČUNA DOBIJEMO KOBASU (UVJETNO REČENO). NAKON ŠTO DOBIJEMO IZRAZ $I_{3,4}(K_R, T_1)$ RAČUNAMO NOVE UVJETE

$$\frac{\partial I_{3,4}}{\partial K_R} = 0, \quad \frac{\partial I_{3,4}}{\partial T_1} = 0$$

DOBIJEMO DVIJE JEDNADŽBE S DVIJE NEPOZNANICE ČIJA SU REALNA RJEŠENJA

$$K_R = \frac{11}{6}, \quad T_1 = 11$$

(C) $Z(s) = \frac{1}{s}$

$$Y(s) = E(s)G_0(s) - Z(s)G_P(s)$$
$$Y(s) = -E(s)$$

$$Z(s)G_P(s) = E(s)\left(G_0(s) + 1\right)$$

$$E(s) = \frac{G_P(s)}{G_0(s) + 1} Z(s)$$

NAKON KRAĆEG UVRŠTAVANJA DOBIJEMO

$$E(s) = \frac{4T_1}{8T_1 s^4 + 12T_1 s^3 + 6T_1 s^2 + (T_1 + 4K_R T_1)s + 4K_R}$$

$c_0 = 4T_1$

$d_0 = 4K_R$, $d_1 = T_1(1 + 4K_R)$, $d_2 = 6T_1$, $d_3 = 12T_1$, $d_4 = 8T_1$

NAKON ŠTO DOBIJEMO IZRAZ ZA $I_{3,4}$, UVRSTIMO GA U UVJETE

$$\frac{\partial I_{3,4}}{\partial K_R} = 0 \qquad \frac{\partial I_{3,4}}{\partial T_1} = 0$$

IZ SUSTAVA OD DVIJE JEDNADŽBE S DVIJE NEPOZNANICE DOBIJEMO TRAŽENE KOEFICIJENTE

$$K_R = \frac{5}{4}, \quad T_1 = 10$$

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(d) ZIEGLER-NICHOLSOVA METODA RUBA STABILNOSTI

$$G_o(s) = K_{RP}\frac{4}{(1+2s)^3} \rightarrow G_o(j\omega) = K_{RP}\frac{4}{(1+2j\omega)^3}$$

$$G_o(j\omega) = \frac{4K_{RP}}{(1-12\omega^2)+j(6\omega-8\omega^3)}$$

ODNOSNO

$$G_o(j\omega) = \frac{4K_{RP}}{(1-12\omega^2)^2+(6\omega-8\omega^3)}\left[(1-12\omega^2)+j(8\omega^3-6\omega)\right]$$

$$Im\{G_o(j\omega)\} = 0 \quad ZA \quad \omega_\pi = \frac{\sqrt{3}}{2}\ s^{-1}$$

NA RUBU STABILNOSTI VRIJEDI $Re\{G_o(j\omega_\pi)\} = -1$

$$K_{RP} = 2$$

KRITIČNI IZNOS PERIODA JE

$$T_{kr} = \frac{2\pi}{\omega_\pi} = \frac{4\sqrt{3}}{3}\pi\ s^{-1}$$

· PI REGULATOR

$K_R = 0.45\ K_{RP} = 0.9$

$T_I = 0.85\ T_{kr} = 6.17\ s$

$$G_R(s) = 0.9\left(1+\frac{1}{6.17s}\right)$$

· PID REGULATOR

$K_R = 0.6\ K_{RP} = 1.2$

$T_I = 0.5\ T_{kr} = 3.63\ s$

$T_D = 0.12\ T_{kr} = 0.87\ s$

$$G_R(s) = 1.2\left(1+\frac{1}{3.63s}+0.87s\right)$$

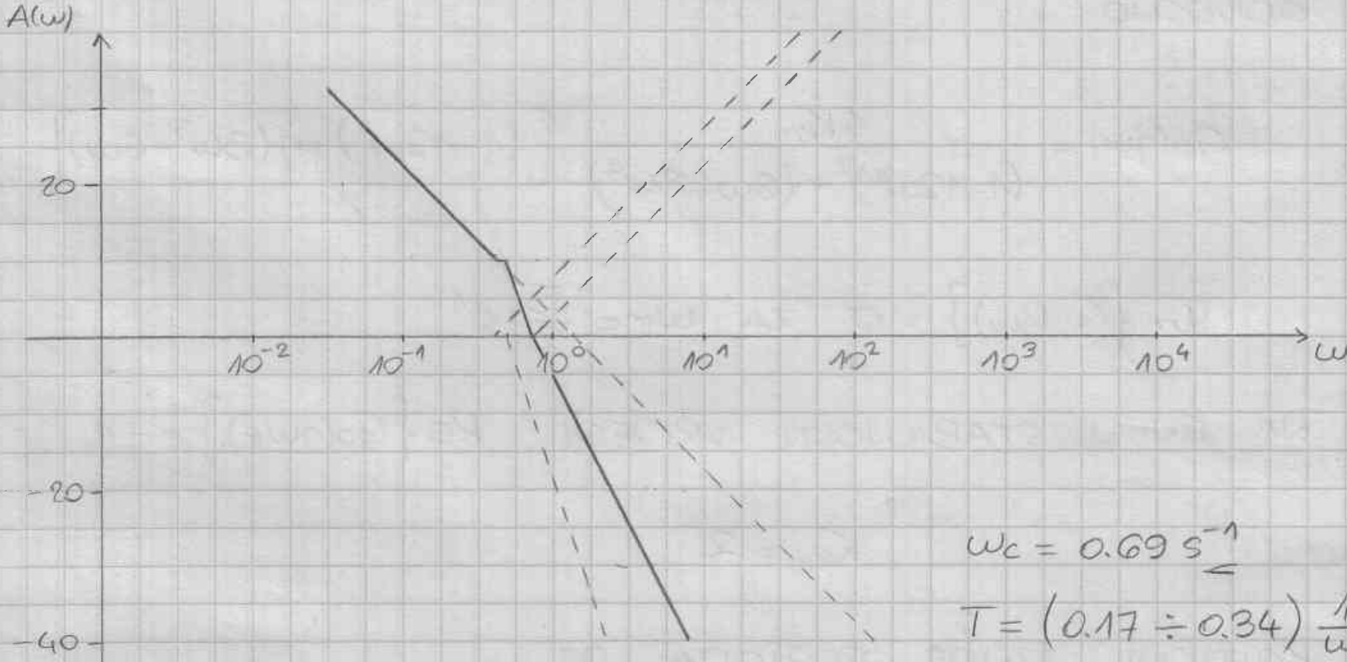
(e) $G_R(s) = 1.2\left(1 + \frac{1}{3.63s} + 0.87s\right)$

$$G_0(s) = 1.2\ \frac{3.63s + 1 + 3.16s^2}{3.63s} \cdot \frac{4}{(1+2s)^3}$$

$$G(s) = \frac{\left(1+\frac{s}{0.46}\right)\left(1+\frac{s}{0.69}\right)}{\frac{s}{1.33}\left(1+\frac{s}{0.5}\right)^3}$$

$$A(\omega) = 20\log\sqrt{1+\left(\frac{\omega}{0.46}\right)^2} + 20\log\sqrt{1+\left(\frac{\omega}{0.69}\right)^2} - 20\log\frac{\omega}{1.33} - 60\log\sqrt{1+\left(\frac{\omega}{0.5}\right)^2}$$

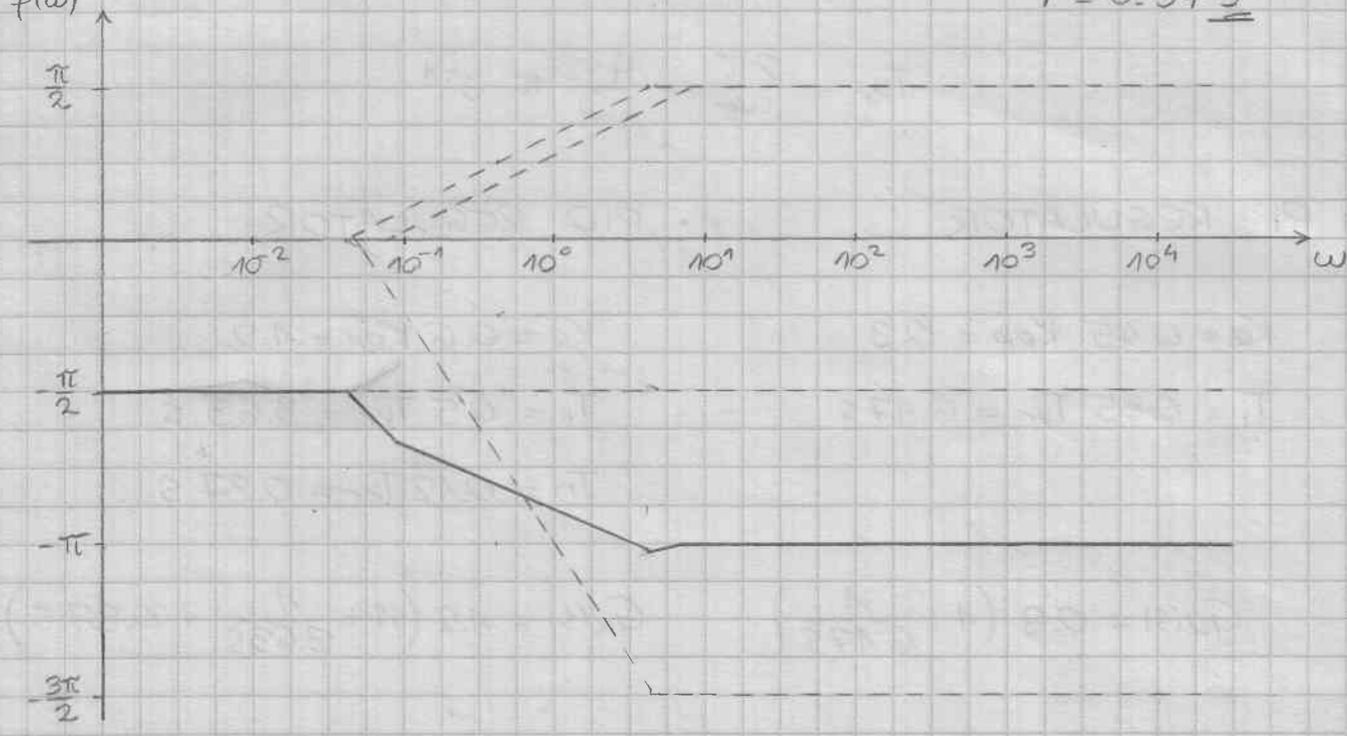
$$\varphi(\omega) = -\frac{\pi}{2} + \operatorname{arctg}\frac{\omega}{0.46} + \operatorname{arctg}\frac{\omega}{0.69} - 3\operatorname{arctg}\frac{\omega}{0.5}$$



$\omega_c = 0.69\ s^{-1}$

$T = (0.17 \div 0.34)\ \frac{1}{\omega_c}$

$T = 0.37\ s$



TUSTINOV POSTUPAK: $s = \frac{2}{T} \frac{z-1}{z+1}$, $T = 0.37\,s$

$$G_R(z) = G_R(s)\Big|_{s = 5.41 \frac{z-1}{z+1}}$$

$$G_R(z) = \frac{6.91 - 11.18 z^{-1} + 4.52 z^{-2}}{1 - z^{-2}}$$

(f) REKURZIVNA RELACIJA

$$u(k) = u(k-2) + 6.91\, e(k) - 11.18\, e(k-1) + 4.52\, e(k-2)$$

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