

$$\ddot{r} = r\dot{\phi}^2 - \frac{k}{r^2} + U_1$$

$$\ddot{\phi} = -2 \frac{\dot{\phi}\dot{r}}{r} + \frac{U_2}{r}$$

$$(a) \quad x_1 = r, \quad x_2 = \dot{r}, \quad x_3 = \dot{\phi} = \omega$$

$$(1) \quad \dot{x}_1 = x_2$$

$$(2) \quad \dot{x}_2 = x_1 x_3^2 - k \frac{1}{x_1^2} + U_1$$

$$(3) \quad \dot{x}_3 = -2 \frac{x_2 x_3}{x_1} + U_2 \frac{1}{x_1}$$

$$(b) \quad U_1 = U_2 = 0$$

$$\bullet \quad x_{20} = 0$$

$$x_{10} x_{30}^2 - k \frac{1}{x_{10}} = 0 \rightarrow x_{10}^3 x_{30}^2 = k$$

$$\bullet \quad x_{10} = \sqrt{\frac{k}{x_{30}^2}}$$

$$-2 \frac{x_{20} x_{30}}{x_{10}} + 0 = 0 \rightarrow 0 = 0$$

= 0

$$(1) \quad \dot{\Delta x}_1 = \Delta x_2$$

$$(2) \quad \dot{\Delta x}_2 = x_{10} \cdot 2 x_{30} \Delta x_3 + x_{30}^2 \Delta x_1 + 2k \frac{1}{x_{10}^3} \Delta x_1 + \Delta U_1$$

$$\dot{\Delta x}_2 = \left(x_{30}^2 + 2 \frac{k}{x_{10}^3} \right) \Delta x_1 + 2 x_{10} x_{30} \Delta x_3 + \Delta U_1$$

$$(3) \quad \dot{\Delta x}_3 = -2 \frac{x_{30}}{x_{10}} \Delta x_2 - 2 \frac{x_{20}}{x_{10}} \Delta x_3 + 2 \frac{x_{20} x_{30}}{x_{10}^2} \Delta x_1 +$$

$$+ \frac{1}{x_{10}} \Delta U_2 - U_{20} \frac{1}{x_{10}^2} \Delta x_1 = 0$$

$$\dot{\Delta x}_3 = -2 \frac{x_{30}}{x_{10}} \Delta x_2 + \frac{1}{x_{10}} \Delta U_2$$