PRIMJER 12.1

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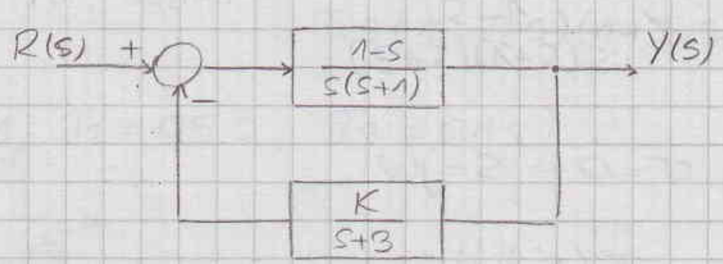
3.AUT1



$$G(s) = \frac{\frac{1-s}{s(s+1)}}{1 + \frac{K}{s+3}\,\frac{1-s}{s(s+1)}} \rightarrow G(s) = \frac{(1-s)(s+3)}{s(s+1)(s+3) + K(1-s)}$$

$$G(s) = \frac{-s^2 - 2s + 3}{s^3 + 4s^2 + s(3-K) + K}$$

$$s^3 + 4s^2 + s(3-K) + K = 0$$

(1) SVI KOEFICIJENTI MORAJU BITI POZITIVNI

$$3 - K > 0 \rightarrow K < 3$$

$$K > 0$$

(2) SVE DETERMINANTE MORAJU BITI POZITIVNE

$$D_1 = a_1 > 0 \rightarrow 3 - K > 0 \rightarrow K < 3$$

$$D_2 = a_1 a_2 - a_0 a_3 > 0 \rightarrow 12 - 7K > 0 \rightarrow K < \frac{12}{7}$$

KONACNO:

$$K \in \langle 0, \frac{12}{5} \rangle$$

PRIMJER 12.2

$$G_0(s) = K\frac{1-s}{s(s+1)(s+3)}$$

ZA $s=\sigma+j\omega$, $\sigma=0 \rightarrow s=j\omega$

$$G_0(j\omega)=\frac{K(1-j\omega)}{j\omega(j\omega+1)(j\omega+3)}$$

ZA PRIMJENU NYQUISTOVOG KRITERIJA, PRIJENOSNU FUNKCIJU OTVORENOG KRUGA POTREBNO JE PRIKAZATI U OBLIKU

$$G_0(j\omega) = Re\{G_0(j\omega)\} + Im\{G_0(j\omega)\}$$

$$Re\{G_0(j\omega)\} := R(\omega)$$

$$Im\{G_0(j\omega)\} := I(\omega)$$

NAKON KRAĆEG RAČUNA DOBIJEMO:

$$R(\omega) = K\frac{\omega^2-7}{\omega^4+10\omega^2+9}, \quad I(\omega) = K\frac{5\omega^2-3}{\omega^5+10\omega^3+9\omega}$$

$R(\omega=\infty)=0 \qquad R(\omega=0) = -K\frac{7}{9}$

$I(\omega=\infty)=0 \qquad I(\omega=0^-) \neq I(\omega=0^+)$

$$I(\omega=0^-)=+\infty, \quad I(\omega=0^+)=-\infty$$

$R(\omega)=0 \rightarrow \omega=\pm\sqrt{7} \qquad I(\omega)=0 \rightarrow \omega=\pm\sqrt{\frac{3}{5}}$

$I(\omega=\sqrt{7}) = \frac{\sqrt{7}}{28}K \qquad R\left(\omega=\sqrt{\frac{3}{5}}\right) = -\frac{5}{12}K$

$I(\omega=-\sqrt{7}) = -\frac{\sqrt{7}}{28}K \qquad R\left(\omega=-\sqrt{\frac{3}{5}}\right) = -\frac{5}{12}K$

$$R\left(\omega=\pm\sqrt{\frac{3}{5}}\right) > -1 \rightarrow -\frac{5}{12}K > -1 \rightarrow K < \frac{12}{5}$$

$$K \in \left\langle 0, \frac{12}{5}\right\rangle$$

PRIMJER 13.3

$$G_0(s) = \frac{K}{T_1 s(1+T_2 s)(1+T_3 s)}$$

$K = 1$, $T_2 = 0{,}5\ s$, $T_3 = 0{,}1\ s$

(a) $\gamma = 45^\circ$

$$G_0(j\omega) = \frac{1}{jT_1\omega(1+j\frac{\omega}{2})(1+j\frac{\omega}{10})}$$

TRAŽIMO ω_c:

$$G_0(j\omega) = \frac{20}{jT_1\omega(2+j\omega)(10+j\omega)}$$

$$G_0(j\omega) = \frac{20}{jT_1\omega(20-\omega^2+12j\omega)}$$

$$f_0(\omega) = -\frac{\pi}{2} - \operatorname{arctg}\frac{12\omega}{20-\omega^2}$$

$$\gamma = \pi + f_0(\omega_c)$$

$$\frac{\pi}{4} = \pi - \frac{\pi}{2} - \operatorname{arctg}\frac{12\omega_c}{20-\omega_c^2}$$

$$\frac{3\pi}{4} = -\operatorname{arctg}\frac{12\omega_c}{20-\omega_c^2} \rightarrow 20-\omega_c^2 = 12\omega$$

$$\omega^2 + 12\omega_c - 20 = 0 \rightarrow \omega_c = 1.48\ rads^{-1}, \quad \cancel{\omega_c = -13.48\ rads^{-1}}$$

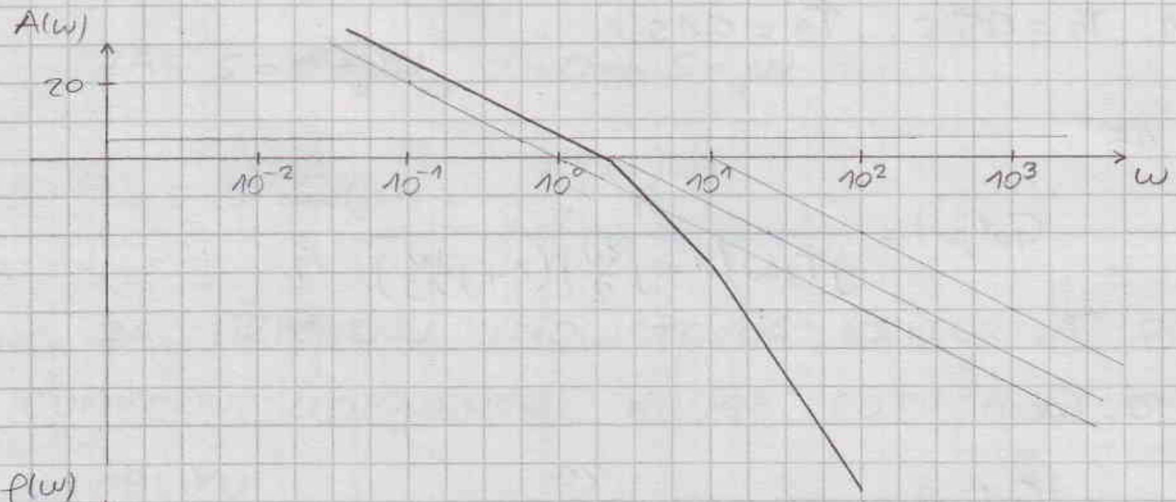
$$|A(\omega_c)| = 1$$

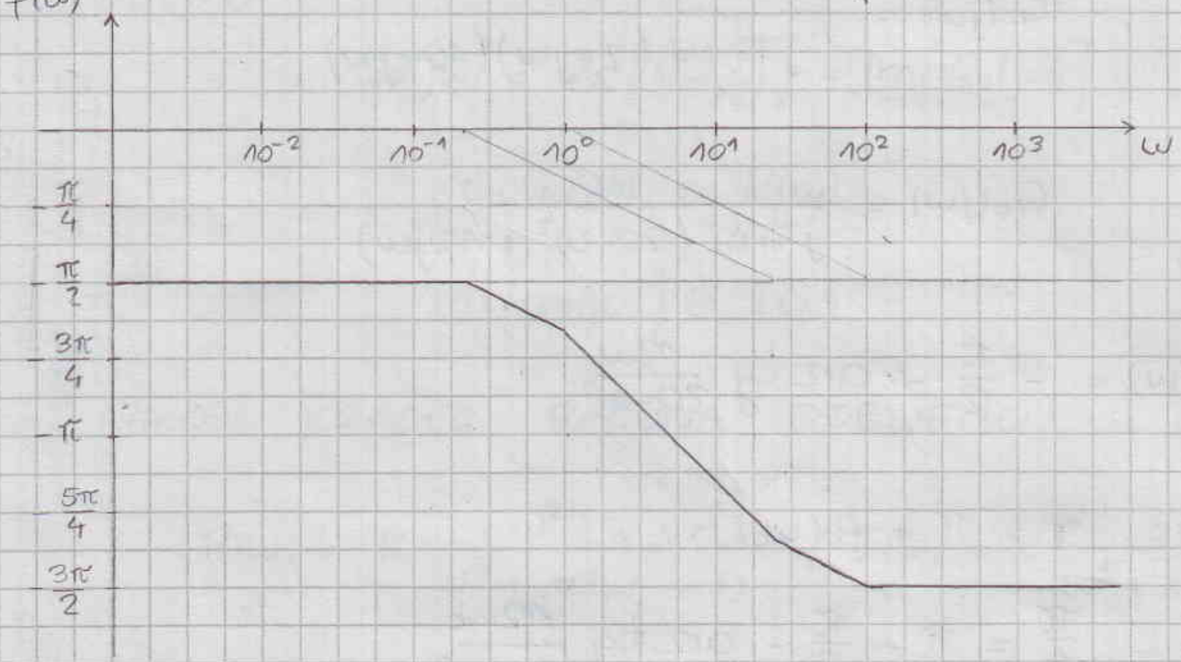
$$\frac{20}{T_1\omega_c}\frac{1}{\sqrt{(20-\omega_c^2)^2+(12\omega_c)^2}} = 1$$

$$T_1 = 0.537\ s$$

$$A(\omega) = 20\log 1{,}862 - 20\log\omega - 20\log\sqrt{1+\left(\frac{\omega}{2}\right)^2} - 20\log\sqrt{1+\left(\frac{\omega}{10}\right)^2}$$

$$\varphi(\omega) = -\frac{\pi}{2} - \operatorname{arctg}\frac{\omega}{2} - \operatorname{arctg}\frac{\omega}{10}$$





$$G_0(j\omega) = 37{,}24 \cdot \frac{1}{j\omega} \cdot \frac{1}{(20-\omega^2)+12j\omega}$$

$$G_0(j\omega) = 37{,}24 \cdot \frac{(-j)}{\omega} \cdot \frac{20-\omega^2-12j\omega}{(20-\omega^2)^2+(12\omega)^2}$$

$$G_0(j\omega) = \underbrace{\frac{-446.88}{(20-\omega^2)^2+(12\omega)^2}}_{R(\omega)} + j\underbrace{\frac{37{,}24\,(\omega^2-20)}{\omega\left[(20-\omega^2)^2+(12\omega)^2\right]}}_{I(\omega)}$$

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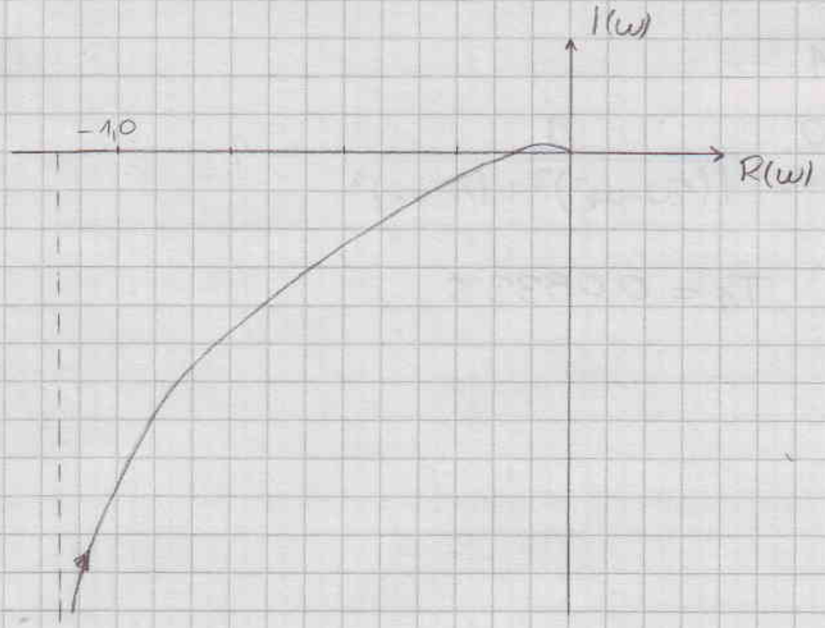
$R(\omega=\infty) = 0 \qquad R(\omega=0) = -1.117$

$I(\omega=\infty) = 0 \qquad I(\omega=0^+) = -\infty$

$R(\omega) = 0 \rightarrow \omega = \infty$

$I(\omega) = 0 \rightarrow \omega = \sqrt{20} \rightarrow R(\omega=\sqrt{20}) = -0.155$



(b)

$$G(s) = \frac{G_o(s)}{1 + G_o(s)}$$

$$G(s) = \frac{1}{T_1 T_2 T_3\, s^3 + T_1(T_2+T_3)\, s^2 + T_1 s + 1}$$

$$0.0268\, s^3 + 0.3216\, s^2 + 0.536\, s + 1 = 0$$

„JEDNOSTAVNIM" POSTUPKOM (= MATLAB) DOBIJEMO:

$$s_{p1} = -10.42$$

$$s_{p2,3} = -0.787 \pm j\, 1.72$$

$$\xi \omega_n = 0{,}787 \quad , \quad \omega_n \sqrt{1-\xi^2} = 1{,}72$$

$$\xi = 0.42 \, , \, \omega_n = 1.873$$

(c) GRANICA STABILNOSTI : $\varphi_0(\omega_c) = -\pi \rightarrow \gamma = 0°$

$$\varphi_0(\omega_c) = -\frac{\pi}{2} - \operatorname{arctg} \frac{12\omega_c}{20-\omega_c^2} = \pi$$

$$\operatorname{arctg} \frac{12\omega_c}{20-\omega_c^2} = \frac{\pi}{2}$$

$$\frac{12\omega_c}{20-\omega_c^2} = \infty \rightarrow 20-\omega_c^2 = 0 \rightarrow \omega_c = \sqrt{20}$$

$$|A(\omega_c)| = 1$$

$$\frac{20}{T_1\omega_c} \frac{1}{\sqrt{(20-\omega_c^2)^2 + (12\omega_c)^2}} = 1$$

$$T_1 = 0.0833\ s$$

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ZADATAK 12.1

$$G_0(s) = \frac{10}{(1+s)(1+0.4s)(1+0.1s)}$$

$$G_0(s) = \frac{1000}{(1+s)(10+4s)(10+s)}$$

$$\mathcal{L}_{CE} = 1000 + (1+s)(10+4s)(10+s)$$

$$\mathcal{L}_{CE} = 1000 + (s+1)(4s^2+50s+100)$$

$$\mathcal{L}_{CE} = 1000 + (4s^3+50s^2+100s+4s^2+50s+100)$$

$$\mathcal{L}_{CE} = 4s^3+54s^2+150s+1100$$

(1) $4>0,\ 5>0,\ 150>0,\ 1100>0$

(2) $D_1 = 150 > 0$

$$D_2 = \begin{vmatrix} 150 & 1100 \\ 4 & 54 \end{vmatrix} = 150\cdot 54 - 1100\cdot 4 = 3700 > 0$$

ZADATAK 12.2

$$G_0(j\omega) = \frac{1000}{-4j\omega^3 - 54\omega^2 + 150j\omega + 100}$$

$$G_0(j\omega) = \frac{1000\left((100-54\omega^2) - j(150\omega - 4\omega^3)\right)}{(100-54\omega^2)^2 + (150\omega - 4\omega^3)^2}$$

$$I(\omega) = 0 \rightarrow \omega(150-4\omega^2) = 0, \quad \omega_1 = 0\ s^{-1}$$

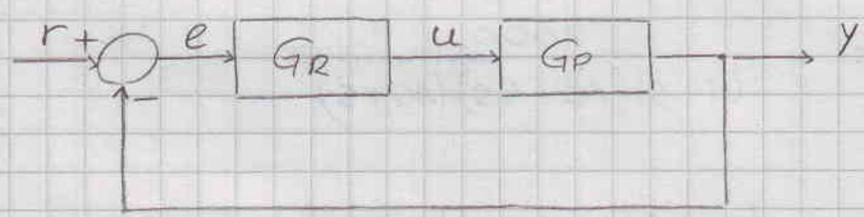
$$\omega_2 = 6.12\ s^{-1} = \omega_\pi$$

UVJET STABILNOSTI: $R(\omega_\pi) > -1$

$$R(\omega_\pi) = -0{,}544 > \underline{-1}$$

ZADATAK 12.3

$$G_P(s) = \frac{1}{s(s^2+s+1)(s+2)}, \quad G_R(s) = K_R$$



$$G_O(s) = G_P(s)\,G_R(s)$$

$$G_O(s) = \frac{K_R}{s(s^2+s+1)(s+2)}$$

$$\mathcal{L}_{CE} = s(s^2+s+1)(s+2) + K_R$$

$$\mathcal{L}_{CE} = s(s^3+2s^2+s^2+2s+s+2) + K_R$$

$$\mathcal{L}_{CE} = s(s^3+3s^2+3s+2) + K_R$$

$$\mathcal{L}_{CE} = s^4+3s^3+3s^2+2s+K_R$$

(1) $K_R > 0$

(2) $D_1 = 2 > 0$

$$D_2 = \begin{vmatrix} 2 & K_R \\ 3 & 3 \end{vmatrix} = 6 - 3K_R > 0 \rightarrow K_R < 2$$

$$D_3 = \begin{vmatrix} 2 & K_R & 0 \\ 3 & 3 & 2 \\ 0 & 1 & 3 \end{vmatrix} = 2\begin{vmatrix} 3 & 2 \\ 1 & 3 \end{vmatrix} - K_R\begin{vmatrix} 3 & 2 \\ 0 & 3 \end{vmatrix}$$

$$= 2(9-2) - K_R(9-0)$$

$$= 14 - 9K_R > 0 \rightarrow K_R < \frac{14}{9}$$

$$K \in \langle 0, \frac{14}{9} \rangle$$