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Mass 2.1

Bodeov diagram

$$G(s) = 890 \cdot \frac{(s+0.1)(s+2)}{s(s+5)(s+20)}$$

$$G(s) = 890 \cdot \frac{0.1\left(\frac{s}{0.1}+1\right)\cdot 2\left(\frac{s}{2}+1\right)}{s\cdot 5\left(1+\frac{s}{5}\right)\cdot 20\left(\frac{s}{20}+1\right)}$$

$$G(s) = 1.78 \quad \frac{\left(\frac{s}{0.1}+1\right)\left(\frac{s}{2}+1\right)}{s\left(1+\frac{s}{5}\right)\left(1+\frac{s}{20}\right)}$$

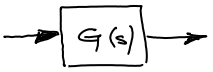
$$G(j\omega) = 1.78 \cdot \frac{\left(\frac{j\omega}{0.1}+1\right)\left(\frac{j\omega}{2}+1\right)}{j\omega\left(1+\frac{j\omega}{5}\right)\left(1+\frac{j\omega}{20}\right)}$$

$$|G(j\omega)| = 1.78 \quad \frac{\sqrt{1^2+\left(\frac{\omega}{0.1}\right)^2}\sqrt{1^2+\left(\frac{\omega}{2}\right)^2}}{\omega\sqrt{1^2+\left(\frac{\omega}{5}\right)^2}\sqrt{1^2+\left(\frac{\omega}{20}\right)^2}}$$

$$A(\omega)_{dB} = 20\log|G(j\omega)| =$$

$$= 20\left(\log(1.78) + \log\sqrt{1+\left(\frac{\omega}{0.1}\right)^2} + \log\sqrt{1+\left(\frac{\omega}{2}\right)^2} - \log\omega - \log\sqrt{1+\left(\frac{\omega}{5}\right)^2} - \log\sqrt{1+\left(\frac{\omega}{20}\right)^2}\right)$$

$u(t) = U\sin(\omega_0 t + \varphi_u)$



$y(t) = Y\sin(\omega_0 t + \varphi_i)$

$y = U \cdot |G(j\omega_0)|$

$G(j\omega) = G(s)\big|_{s=j\omega}$

$\varphi_i = \varphi_u + \measuredangle G(j\omega)$

$z = a + bj = re^{j\varphi}$

$r = \sqrt{Re^2 + Im^2}$

$\varphi = \operatorname{arctg}\frac{Im}{Re}$

$\operatorname{arctg}\frac{-b}{a} = -\operatorname{arctg}\frac{b}{a}$ | četrti kvadrant

$\operatorname{arctg}\frac{b}{-a} = -\operatorname{arctg}\frac{b}{a}$ | drugi kvadrant

$\Downarrow = \pi - \operatorname{arctg}\frac{b}{a}$

$\operatorname{arctg}\frac{-b}{-a} = \pi + \operatorname{arctg}\frac{b}{a}$

$$-\log\omega - \log\sqrt{1+\left(\frac{\omega}{5}\right)^2} - \log\sqrt{1+\left(\frac{\omega}{20}\right)^2}$$

Re (1.78)

$$f(\omega) = 0 + \operatorname{arctg}\frac{\omega}{0.1} + \operatorname{arctg}\frac{\omega}{1} - \operatorname{arctg}\frac{\omega}{0} - \frac{\pi}{2}$$

$$- \operatorname{arctg}\frac{\omega}{5} - \operatorname{arctg}\frac{\omega}{20}$$

$$z_1 \cdot z_2 = r_1 e^{j\varphi_1} \cdot r_2 e^{j\varphi_2} = r_1 \cdot r_2 \, e^{j(\varphi_1+\varphi_2)}$$

$$|z_1 \cdot z_2| = r_1 \cdot r_2$$

$$\log(a \cdot b) = \log a + \log b$$

$$\log\frac{a}{b} = \log a - \log b$$

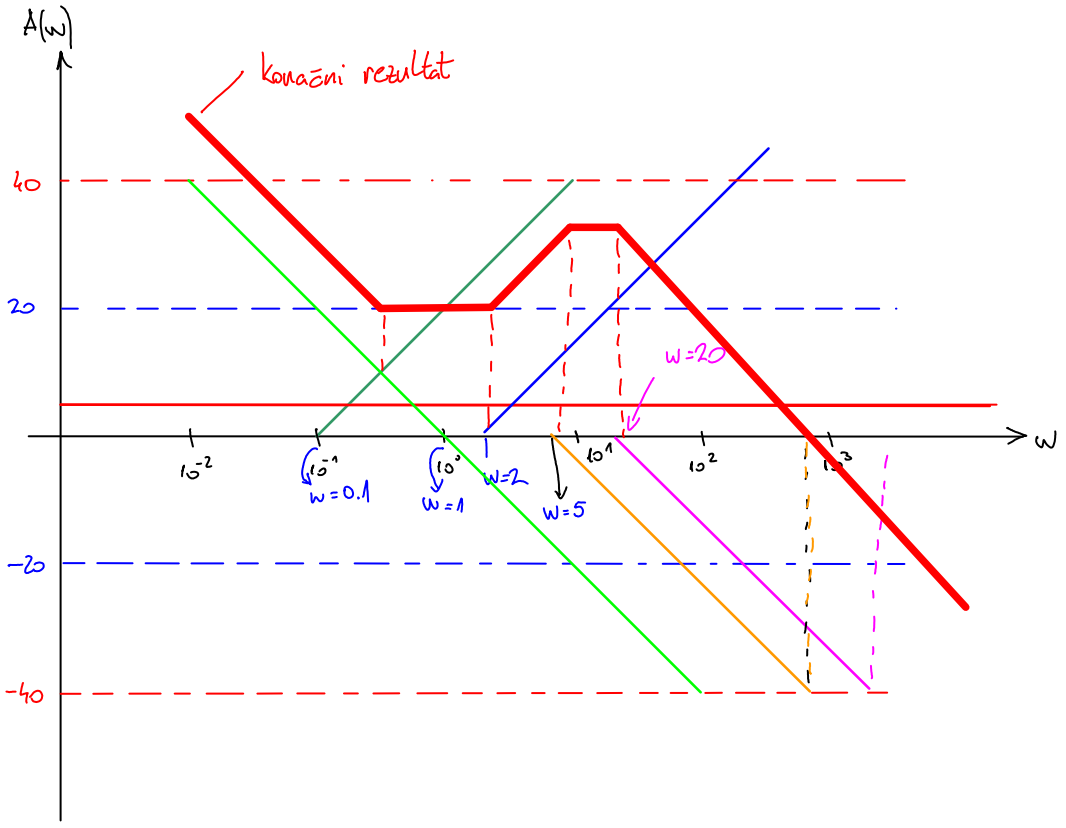
$$\log 1 = 0$$

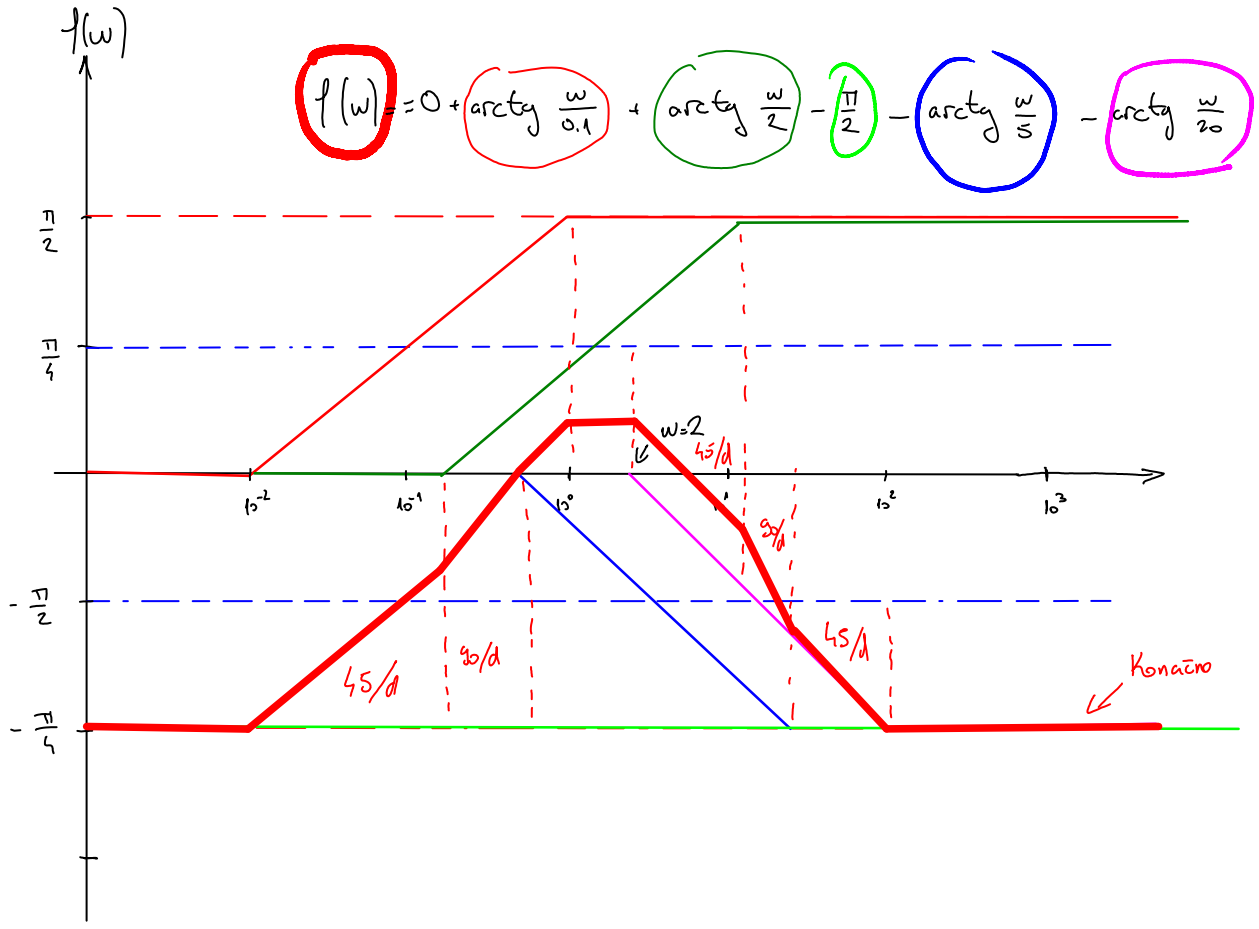
$$10^x = 5 \;/\log$$

$$\log_{10} 10^x = \log_{10} 5$$

$$x \underbrace{\log_{10} 10}_{1} = \log_{10} 5$$

$$x = \log_{10} 5$$




φ(ω)
φ(ω) = 0 + arctg ω/0.1 + arctg ω/2 − π/2 − arctg ω/15 − arctg ω/20
π/2
π/4
−π/2
−π/4
10⁻²
10⁻¹
10⁰
10¹
10²
10³
ω=2
45/d
90/d
45/d
90/d
45/d
Konačno

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Nyquistov diagram

$$G(s) = 500\ \frac{(s+0.1)}{(s+5)(s+20)} = \frac{500(s+0.1)}{s^2+25s+100}$$

$$G(j\omega) = \frac{500(0.1+j\omega)}{(100-\omega^2)+j(25\omega)} \cdot \frac{(100-\omega^2)-j(25\omega)}{(100-\omega^2)-j(25\omega)} =$$

$$= \underbrace{500\ \frac{24.9\omega^2+10}{\omega^4+425\omega^2+10000}} + j\,500\ \frac{-\omega^3+97.5\omega}{\omega^4+425\omega^2+10000}$$

$$\mathrm{Re}\{G(j\omega)\} = R(\omega) \qquad \mathrm{Im}\{G(j\omega)\} = I(\omega)$$

1. Početna vrijednost

$$\lim_{\omega\to\infty} R(\omega) = 0$$

$$\lim_{\omega\to\infty} I(\omega) = 0$$

2. Konačna vrijednost

$$\lim_{\omega\to 0} R(\omega) = 0.5$$

$$\lim_{\omega\to 0} I(\omega) = 0$$

3. Nultočke

$R(\omega)=0 \Rightarrow 24.9\omega^2 = -10 \Rightarrow \omega \notin \mathbb{R}$

$I(\omega)=0 \Rightarrow \omega_1 = 0$, $\omega_2 = \sqrt{97.5} = 9.874$

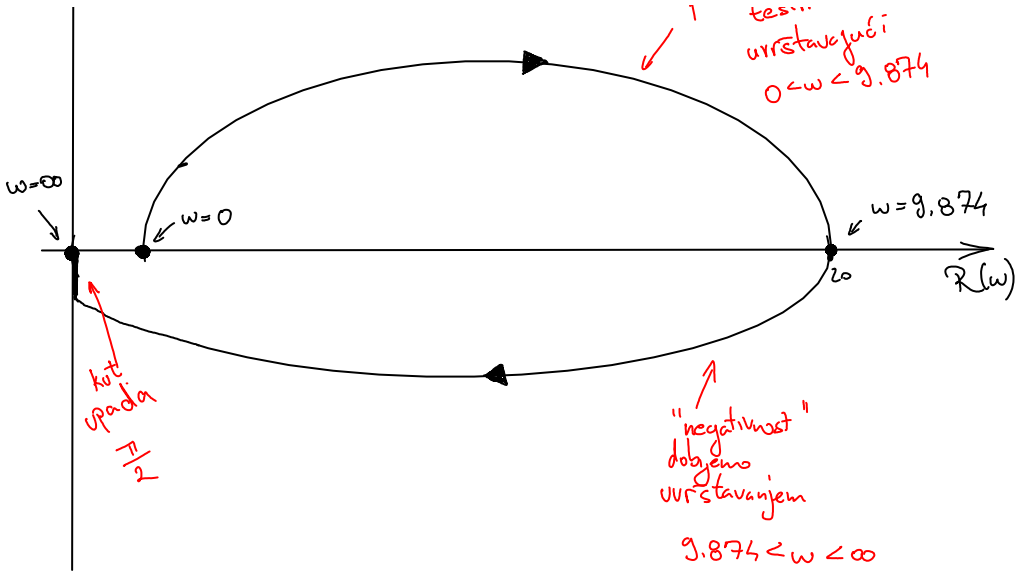
$R(\omega_1)=0$
$R(\omega_2)=20$

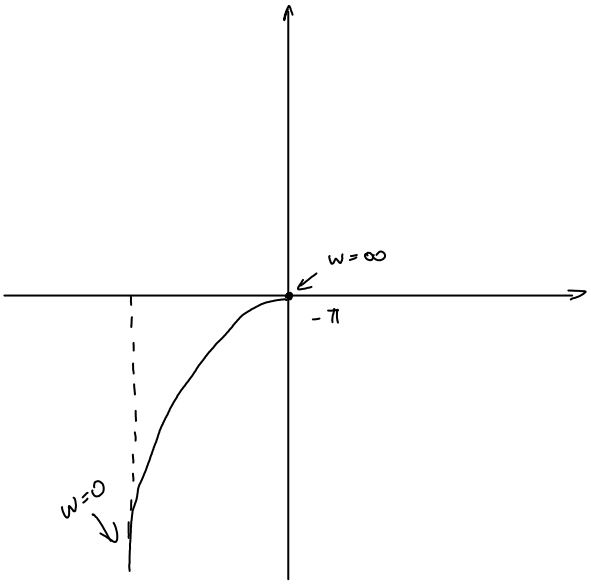
4. Kut upada

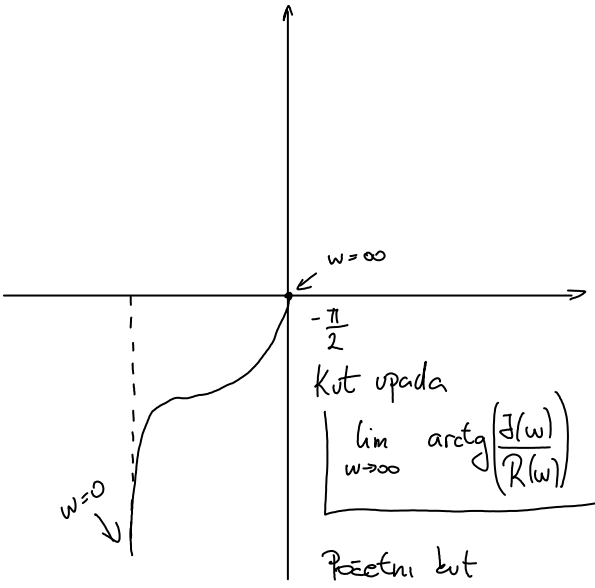
$$\lim_{\omega\to\infty} f(\omega) = -\frac{\pi}{2}$$

$I(\omega)$ ↑

"pozitivnost" testirat





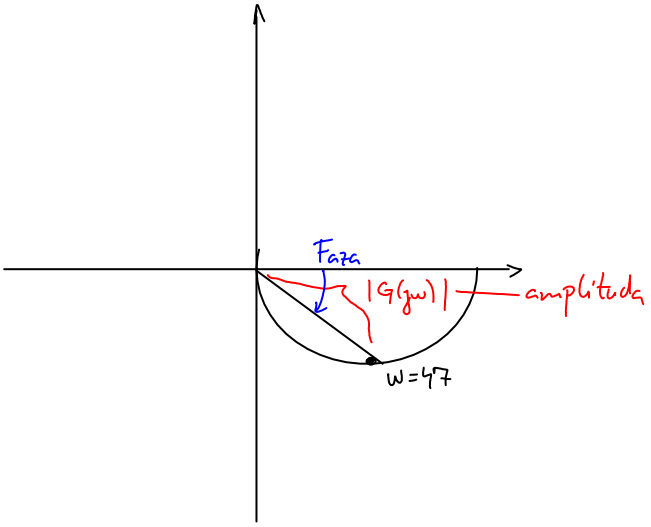


Kut upada

$$\lim_{w \to \infty} \operatorname{arctg}\left(\frac{I(w)}{R(w)}\right)$$

Početni kut

$$\lim_{w \to 0} \left(\operatorname{arctg}\left(\frac{I(w)}{R(w)}\right) \right)$$



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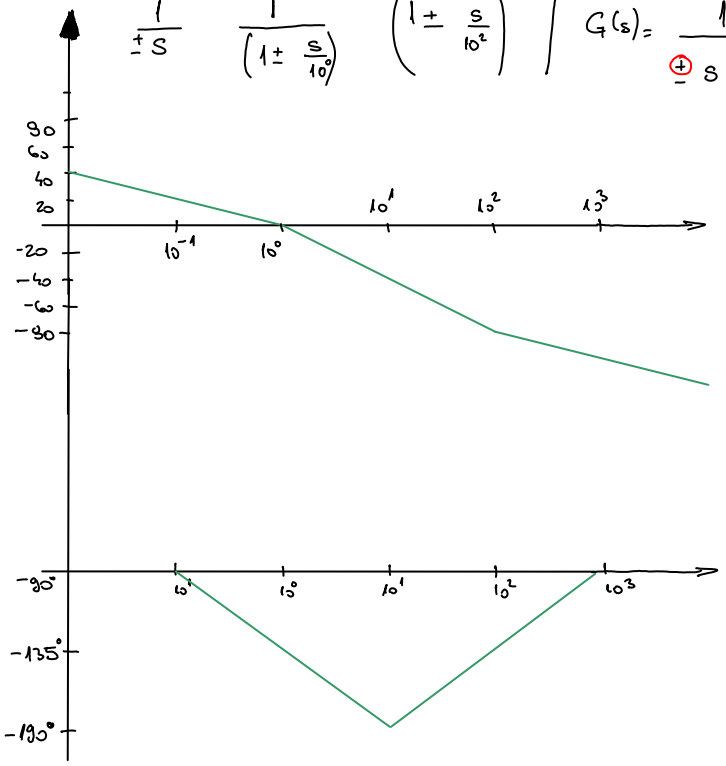
$-20\log(\omega) \Leftarrow \frac{1}{j\omega}$

Izlazni

$\frac{1}{\pm s}$ $\frac{1}{\left(1 \pm \frac{s}{10^0}\right)}$ $\left(1 \pm \frac{s}{10^2}\right)$ $G(s) = \frac{1 \pm \frac{s}{10^2}}{\pm s\left(1 \pm \frac{s}{1}\right)} = \frac{0.01(s+100)}{s(s+1)}$



$-\operatorname{arctg}\left(\frac{\omega}{0}\right) = -\frac{\pi}{2}$

$-\operatorname{arctg}\left(\frac{-\omega}{0}\right) = \operatorname{arctg}\frac{\omega}{0} = \frac{\pi}{2}$

$-\operatorname{arctg}\left(\frac{\omega}{1}\right)$ = faza pada

$-\operatorname{arctg}\left(\frac{-\omega}{1}\right) = \operatorname{arctg}\left(\frac{\omega}{1}\right)$ = faza raste

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Polovi, nule i vremenski odziv

- PT2 - S - članovi bez nula

$$G(s) = \frac{b_0}{a_2 s^2 + a_1 s + a_0} \Rightarrow G(s) = K \frac{\omega_n^2}{s^2 + 2\xi\omega_n + \omega_n^2}$$

K - pojačanje sustava
ω_n - frek. neprigušenih oscilacija
ξ - relativno prigušenje

$$s_p = -\sigma \pm j\omega_d$$

σ - apsolutno prigušenje
ω_d - frekv. prigušenih oscilacija

$$s^2 + 2\xi\omega_n + \omega_n^2 = 0 \Rightarrow s_{1,2} = \frac{-2\xi\omega_n \pm \sqrt{4\omega_n^2\xi^2 - 4\omega_n^2}}{2}$$

$$s_{1,2} = -\xi\omega_n \pm \omega_n\sqrt{\xi^2 - 1}$$

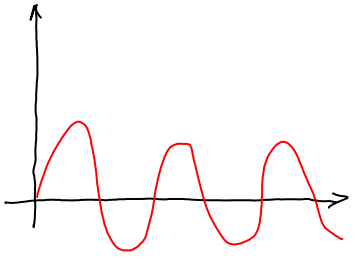
$$s_{1,2} = -\xi\omega_n \pm j\omega_n\sqrt{1 - \xi^2}$$

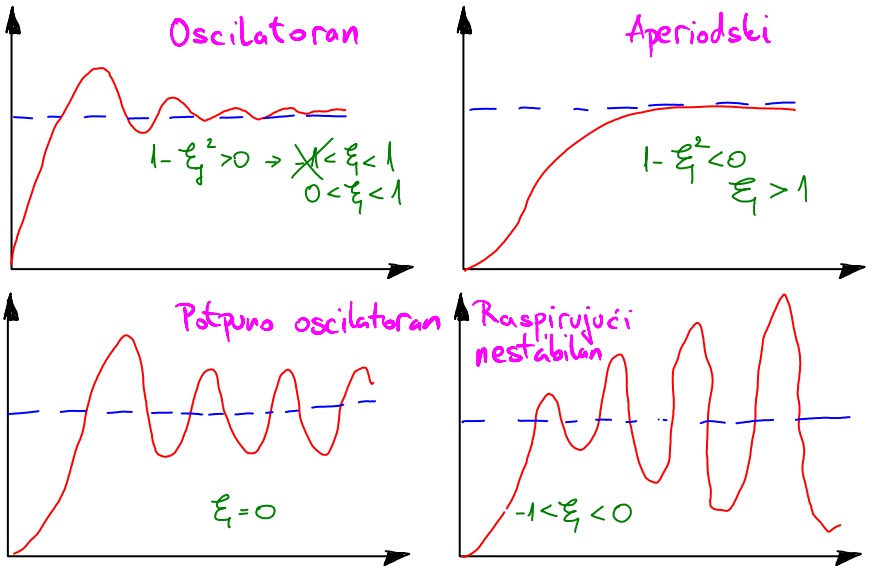
$\sigma = \xi\omega_n$

$\omega_d = \omega_n\sqrt{1 - \xi^2}$

$$\frac{1}{(s+1)(s+2)} = \frac{1}{s+1} + \frac{-1}{(s+2)} \circ\!\!-\!\!\bullet\; e^{-t} - e^{-2t}$$

$$\frac{1}{s^2+1} \circ\!\!-\!\!\bullet\; \sin(t)$$




Oscilatoran
1-ξ²>0 → ξ<1
0<ξ<1
Aperiodski
1-ξ²<0
ξ>1
Potpuno oscilatoran
ξ=0
Raspirujući
nestabilan
-1<ξ<0

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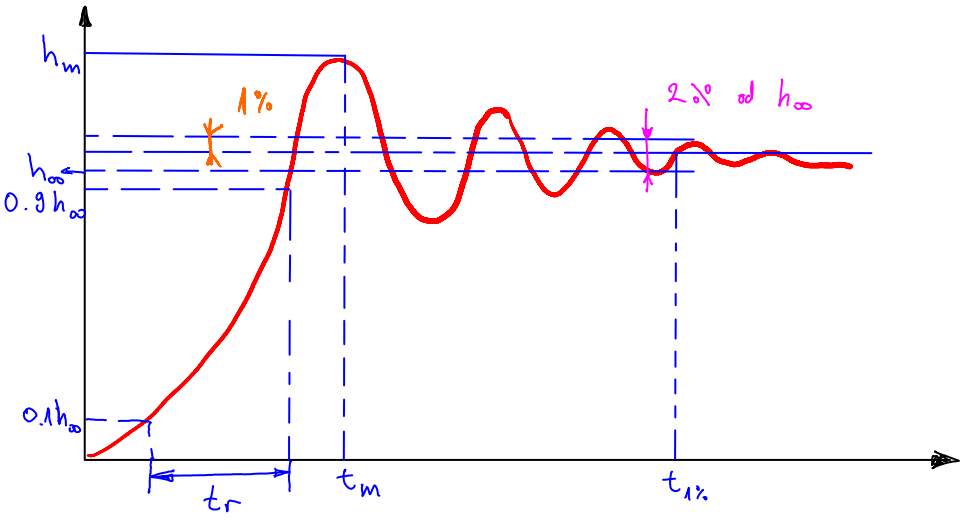


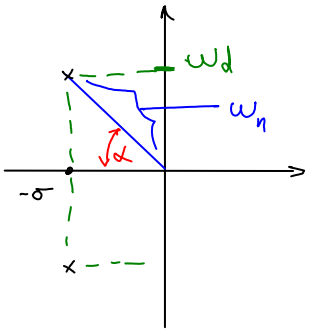
$$\sigma_m = \frac{h_m - h_\infty}{h_\infty} \cdot 100\%$$

$$t_{1\%} = \frac{4.6}{\sigma} = \frac{4.6}{\xi \omega_n}$$

$$t_r = \frac{1.8}{\omega_n}$$

$$\sigma_m = 100\, e^{-\frac{\pi \xi}{\sqrt{1-\xi^2}}}$$

$$t_m \simeq \frac{\pi}{\omega_d} = \frac{\pi}{\omega_n \sqrt{1-\xi^2}}$$



$\alpha = \arccos \xi$

Formule vrijede samo za PT2-S članove
Bez nule

2. $G(s) = 500\ \dfrac{as+0.1}{(s+5)(s+20)}$

a) $h(t),\ g(t) = ?$

$$H(s) = 500\ \frac{as+0.1}{s(s+5)(s+20)} = \frac{C_{11}}{s} + \frac{C_{21}}{s+5} + \frac{C_{31}}{s+20}$$

$C_{11} = 0.5$, $C_{21} = \frac{100}{3}(a-0.02)$, $C_{31} = \frac{100}{3}(a-0.005)$

$$h(t) = \left(0.5 + \frac{100}{3}(a-0.02)e^{-5t} - \frac{100}{3}(a-0.005)e^{-20t}\right)\mu(t)$$

$$g(t) = h'(t) = \left(-\frac{500}{3}(a-0.02)e^{-5t} - \frac{2000}{3}(a-0.005)e^{-20t}\right)\mu(t)$$

b) $y_h(t) = C_1 e^{s_1 t} + C_2 e^{s_2 t} + C_3 e^{s_3 t} + \dots + C_n e^{s_n t}$

prirodni modovi sustava

$a = 0.02 \quad s = -5$

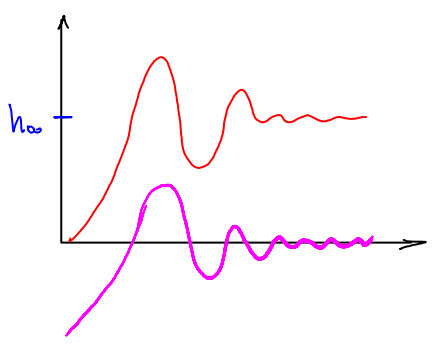
$a = 0.005 \quad s = -20$

$$0.02s + 0.1 = 0.02\left(s + \frac{0.1}{0.02}\right) = 0.02(s+5)$$

$$\underbrace{\frac{(s-1)}{(s-1)(s+2)}}_{\text{Nestabilan sustav}} = \frac{1}{s+2}$$

→ Ne smijemo kratiti nestabilne polove

c)



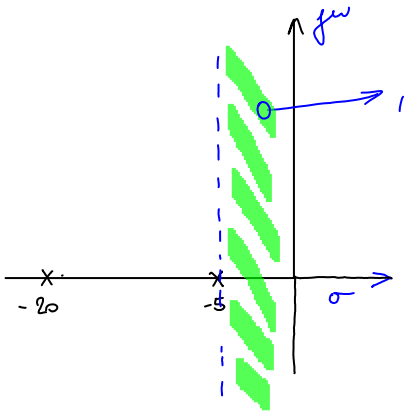
$h_\infty = \lim_{t\to\infty} h(t) = 0.5$

$$h(t) - h_\infty = \frac{100}{3}(a-0.02)e^{-5t} - \frac{100}{3}(a-0.005)e^{-20t} = 0$$

$$e^{15t} = \frac{a-0.005}{a-0.02}$$

$$t = \frac{1}{15}\ln\frac{a-0.005}{a-0.02}$$

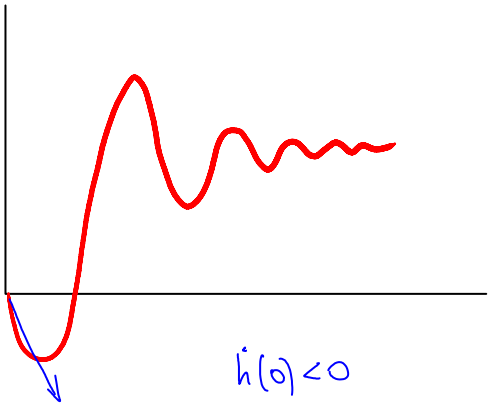
$$\frac{a-0.005}{a-0.02} > 1 \Rightarrow a > 0.02$$



$a \in \langle 0.02, +\infty \rangle$

$0.02s + 0.1 = 0.02(s+5)$

$\infty \cdot s + 0.1 = \infty(s+0)$



$\dot{h}(0) < 0$

$\dot{h}(0) = g(0) < 0 \Rightarrow -\frac{500}{3}(a-0.02) + \frac{2000}{3}(a-0.005) < 0$

$a < 0$

$a < 0$