Zadatak 10.1 Zadan je sustav opisan sljedećom prijenosnom funkcijom:

$$G(s) = \frac{2}{s^2 + as + 4}$$

Potrebno je:

a) Analitički odrediti prijelaznu funkciju sustava (odziv na odskočnu funkciju $S(t)$) kao funkciju parametra a;
b) Odrediti iznos parametra a za koji nadvišenje u prijelaznoj funkciji $h(t)$ iznosi $\sigma_m = 25\%$;
c) Odrediti vrijeme prvog maksimuma t_m prijelazne funkcije sustava?

Zadatak 10.2 Za sustav:

$$G(s) = \frac{1 - 2s}{s^2 + 4s + 20}$$

Potrebno je:

a) Odrediti iznos i trenutak propada u prijelaznoj funkciji zbog postojanja neminimalno fazne nule u sustavu;
b) Odrediti iznos nadvišenja $\sigma_m[\%]$ u prijelaznoj funkciji sustava;
c) Skicirati prijelaznu funkciju sustava.

Zadatak 10.3 Koristeći relacije koje vrijede za sustav $II. reda$ (bez konačnih nula) potrebno je skicirati područje polova u kompleksnoj ravnini da bi se istovremeno zadovoljili sljedeći zahtjevi:

a) Vrijeme prvog maksimuma $t_m < 1s$;
b) Nadvišenje $\sigma_m < 10\%$.

Primjer 10.3 Zadan je sustav opisan sljedećom prijenosnom funkcijom:

$$G(s) = \frac{1}{s^2 + 2s + 2}$$

Potrebno je:

a) Odrediti težinsku funkciju sustava $g(t)$;
b) Odrediti prijelaznu funkciju sustava $h(t)$ tj. odziv sustava na skokovitu pobudu $x(t) = S(t)$;
c) Analitički odrediti vrijeme prvog maksimuma prijelazne funkcije sustava t_m te odgovarajuće nadvišenje u odzivu σ_m;
d) Na temelju rješenja dobivenog pod točkom $a)$ odrediti težinsku funkciju sustava:

$$G(s) = \frac{s + 1}{s^2 + 2s + 2}$$

PRIMJER 10.3

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$$G(s) = \frac{1}{s^2+2s+2}$$

(a) $g(t) \circ\!\!-\!\!\bullet\, G(s)$

$$G(s) = \frac{1}{(s+1)^2+1}$$

$$g(t) = e^{-t}\sin t$$

(b) $H(s) = \frac{1}{s} G(s)$

$$G(s) = \frac{1}{s(s^2+2s+2)} = \frac{C_{11}}{s} + \frac{As+B}{s^2+2s+2}$$

$$C_{11} = G(s)\cdot s \Big|_{s=0} = \frac{1}{2}$$

$$G(s) = \frac{1}{2}\frac{1}{s} + \frac{As+B}{s^2+2s+2} = \frac{\frac{1}{2}s^2+s+1+As^2+Bs}{s(s^2+2s+2)}$$

$$s^2\left(A+\frac{1}{2}\right) + s\left(B+1\right) + 1 = 1$$

$$A = -\frac{1}{2}, \quad B = -1$$

$$G(s) = \frac{1}{2}\frac{1}{s} - \frac{1}{2}\frac{s+2}{(s+1)^2+1}$$

$$G(s) = \frac{1}{2}\frac{1}{s} - \frac{1}{2}\left(\frac{s+1}{(s+1)^2+1} + \frac{1}{(s+1)^2+1}\right)$$

$$h(t) = \frac{1}{2}\left(1 - e^{-t}\left(\cos(t) + \sin(t)\right)\right)$$

(c) $\dot{h}(t) = 0 \rightarrow g(t) = 0$

$$e^{-t}\sin(t) = 0, \quad e^{-t} \neq 0 \qquad \sin(t + k\pi) = 0$$

$$t + k\pi = 0, \quad k = \dots, -2, -1, 0, 1, 2, \dots$$

$$t = -k\pi$$

$t = 0, \ k = 0$

$t = \pi, \ k = -1$

$t = 2\pi, \ k = -2$

$$\ddot{h}(t) = \dot{g}(t) = e^{-t}\left(\cos(t) - \sin(t)\right)$$

ZA $t = 0$

$$\ddot{h}(t) = e^{-0}\left(\cos(0) - \sin(0)\right) > 0$$

ZA $t = \pi$

$$\ddot{h}(t) = e^{-\pi}\left(\cos(\pi) - \sin(\pi)\right) < 0$$

$t = \pi$ – VRIJEME PRVOG MAKSIMUMA

$$h(\pi) = \frac{1}{2}\left(1 - e^{-\pi}\left(\cos(\pi) - \sin(\pi)\right)\right)$$

$$h(\pi) = 0{,}522 := y_m$$

$$h(\infty) = 0{,}5 := y_{ss}$$

$$\sigma_m\,[\%] = 100\,\frac{y_m - y_{ss}}{y_{ss}}$$

- $\sigma_m = 4.4\,\%$

(d) $G(s) = \frac{s+1}{s^2+2s+2} = \frac{1}{s^2+2s+2} + s\,\frac{1}{s^2+2s+1}$

$y(t) \circ\!\!-\!\!\bullet\ Y(s)$

$\dot{y}(t) \circ\!\!-\!\!\bullet\ sY(s)$

$g(t) = g(t)_{(a)} + \dot{g}(t)_{(a)}$

$g(t) = e^{-t}\sin(t) + e^{-t}(\cos(t) - \sin(t))$

- $g(t) = e^{-t}\cos(t)$

ZADATAK 10.1

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OPĆENITO, PRIJENOSNA FUNKCIJA PT2-ČLANA JE

$$G(s) = K \frac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2} \qquad (1)$$

PREMA TOME, DIFERENCIJALNA JEDNADŽBA KOJA OPISUJE PT2-ČLAN GLASI

$$\ddot{y} + 2\xi\omega_n\dot{y} + \omega_n^2 y = \omega_n^2 u(t)$$

KORJENI SUSTAVA SU:

$$s_{P1,2} = \frac{-2\xi\omega_n \pm \sqrt{4\xi^2\omega_n^2 - 4\omega_n^2}}{2}$$

$$s_{P1,2} = -\xi\omega_n \pm \omega_n\sqrt{\xi^2 - 1}$$

$$s_{P1,2} = -\xi\omega_n \pm j\omega_n\sqrt{\xi^2 - 1}$$

OPĆENITO, PIŠEMO

$$s_{P1,2} = -\sigma \pm j\omega_d$$

PRI ČEMU VRIJEDI

$$\sigma = \xi\omega_n$$

$$\omega_d = \omega_n\sqrt{1-\xi^2}$$

HOMOGENO RJEŠENJE JE

$$y_h(t) = C_1 e^{-s_{P1}t} + C_2 e^{-s_{P2}t}$$

$$y_h(t) = e^{-\sigma t}\left(C_1 e^{j\omega_d} + C_2 e^{-j\omega_d}\right)$$

$$y_h(t) = e^{-\sigma t}\left(A\cos(\omega_d t) + B\sin(\omega_d t)\right)$$

TRAŽIMO PRIJELAZNU FUNKCIJU, TJ. ODZIV NA STEP FUNKCIJU

$$U(t) = S(t) \rightarrow y_P(t) = C$$
$$\dot{y}_P(t) = \ddot{y}_P(t) = 0$$

$$0 + 2\xi\omega_n \cdot 0 + C\omega_n^2 = K\omega_n^2 \rightarrow C = K$$

$$y_P(t) = K, \quad \forall t \geq 0$$

TOTALNI ODZIV KAO ZBROJ PARTIKULARNOG I HOMOGENOG RJEŠENJA

$$y(t) = e^{-\sigma t}\left(A\cos(\omega_d t) + B\sin(\omega_d t)\right) + K$$

UVRŠTAVANJE POČETNIH UVJETA

PRETPOSTAVKA: $y(0^-) = y(0^+) = 0$
$\dot{y}(0^-) = \dot{y}(0^+) = 0$

$$y(0) = A + K = 0 \rightarrow A = \underline{\underline{-K}}$$

$$\dot{y}(t) = (-\sigma)\, e^{-\sigma t}\left(A\cos(\omega_d t) + B\sin(\omega_d t)\right)$$
$$+ e^{-\sigma t}\left((-A\omega_d)\sin(\omega_d t) + (B\omega_d)\cos(\omega_d t)\right)$$

$$\dot{y}(t) = e^{-\sigma t}\left[(-A\sigma + B\omega_d)\cos(\omega_d t) + \right.$$
$$\left. + (-B\sigma - A\omega_d)\sin(\omega_d t)\right]$$

$$\dot{y}(0) = -A\sigma + B\omega_d = 0 \rightarrow B\omega_d = -K\sigma$$

$$B = \frac{-K\xi}{\sqrt{1-\xi^2}}$$

KONAČNO RJEŠENJE GLASI:

$$y(t) = K\left(1 + e^{-\sigma t}\left[-\cos(w_d t) - \frac{\xi}{\sqrt{1-\xi^2}}\sin(w_d t)\right]\right)$$

KAKO JE OVO ODZIV NA STEP

$$y(t) \mapsto h(t)$$

- $h(t) = K\left(1 - e^{-\sigma t}\left[\cos(w_d t) + \frac{\xi}{\sqrt{1-\xi^2}}\sin(w_d t)\right]\right)$ (2)

$$\cos(x) = \sin\left(x + \frac{\pi}{2}\right)$$

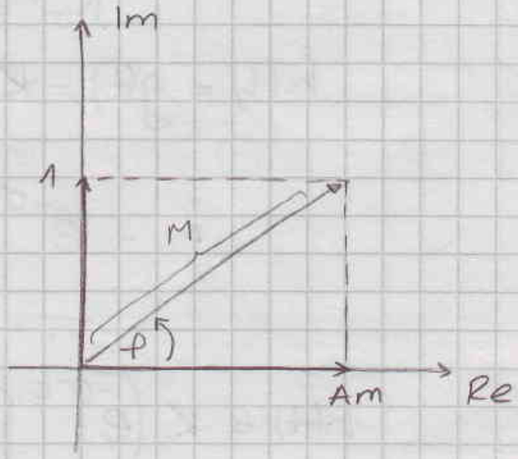
$\rightarrow \sin\left(w_d t + \frac{\pi}{2}\right) + A_m \sin(w_d t)$, $A_m = \frac{\xi}{\sqrt{1-\xi^2}}$

$$\sin\left(w_d t + \frac{\pi}{2}\right) \longmapsto 0 + j$$

$$A_m \sin(w_d t) \longmapsto A_m + 0j$$

$$= 0 + j + A_m + 0j$$

$$= A_m + j$$



$$M = \sqrt{A_m^2 + 1}$$

$$M = \frac{1}{\sqrt{1-\xi^2}} \qquad \varphi = \arccos\frac{A_m}{M}$$

$$\varphi = \arccos\xi$$

KONAČNO RJEŠENJE:

- $h(t) = K\left[1 - \frac{1}{\sqrt{1-\xi^2}}e^{-w_n \xi t}\sin\left(w_n\sqrt{1-\xi^2}\,t + \arccos\xi\right)\right]$ (3)

(a)
$$G(s) = \frac{2}{s^2 + as + 4} \qquad (4)$$

USPOREDIMO IZRAZ (4) SA IZRAZOM (1)

$$\left.\begin{aligned} &\cdot\ K\omega_n^2 = 2 \\ &\cdot\ 2\xi\omega_n = a \\ &\cdot\ \omega_n^2 = 4 \end{aligned}\right\} \ \omega_n = 2,\ \xi = \frac{a}{4},\ K = \frac{1}{2}$$

UVRSTIMO DOBIVENE IZRAZE U IZRAZ (3)

$$h(t) = \frac{1}{2}\left(1 - \frac{1}{\sqrt{1-\frac{a^2}{16}}} e^{-\frac{a}{2}t} \sin\left(2\sqrt{1-\frac{a^2}{16}}\, t + \arccos\frac{a}{4}\right)\right)$$

(b) $\dot{h}(t) = 0$

DERIVIRAMO IZRAZ (2)

$$\dot{h}(t) = g(t) = K\left(+\sigma e^{-\sigma t}\left(\cos(\omega_d t) + \frac{\xi}{\sqrt{1-\xi^2}}\sin(\omega_d t)\right)\right.$$
$$\left. - e^{-\sigma t}\left(\left(-\omega_d\right)\sin(\omega_d t) + \frac{\xi\omega_d}{\sqrt{1-\xi^2}}\cos(\omega_d t)\right)\right)$$

$$\dot{h}(t) = K\left(e^{-\sigma t}\left[\left(\sigma - \frac{\xi\omega_d}{\sqrt{1-\xi^2}}\right)\cos(\omega_d t) + \left(\frac{\sigma\xi}{\sqrt{1-\xi^2}} + \omega_d\right)\sin(\omega_d t)\right]\right)$$

$$\dot{h}(t) = K\left(e^{-\sigma t}\left[\frac{\xi\omega_n\sqrt{1-\xi^2} - \xi\omega_n\sqrt{1-\xi^2}}{\sqrt{1-\xi^2}}\cos(\omega_d t)\right.\right.$$
$$\left.\left. + \frac{\xi^2\omega_n + \omega_n(1-\xi^2)}{\sqrt{1-\xi^2}}\sin(\omega_d t)\right]\right)$$

$$\cdot\ \dot{h}(t) = g(t) = K\left(\frac{\omega_n}{\sqrt{1-\xi^2}} e^{-\xi\omega_n t}\sin(\omega_d t)\right) \qquad (5)$$

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$\dot{h}(t) = 0$ – TRAŽIMO VRIJEME EKSTREMA

$$\underbrace{\frac{K\omega_n}{\sqrt{1-\xi^2}} e^{-\xi\omega_n t}}_{\text{UVIJEK RAZLIČITO OD NULE, OSIM U } t=+\infty} \sin(\omega_d t) = 0$$

$$\sin(\omega_d t - k\pi) = 0 \rightarrow \omega_d t_m = k\pi$$

VRIJEME PRVOG EKSTREMA (MAKSIMUMA), $k=1$

$$t_m = \frac{\pi}{\omega_d} \qquad (6)$$

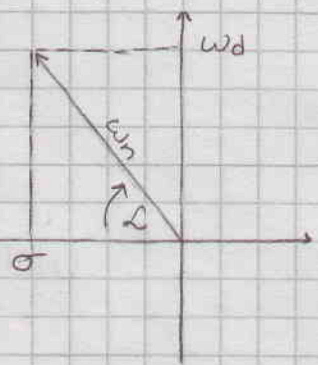
IZNOS MAKSIMUMA:

$$h(t_m) = K\left(1 - \frac{1}{\sqrt{1-\xi^2}} e^{-\omega_n \xi \frac{\pi}{\omega_n\sqrt{1-\xi^2}}} \sin\left(\omega_d \cdot \frac{\pi}{\omega_d} + \arccos\xi\right)\right)$$

$$h(t_m) = K\left(1 - \frac{1}{\sqrt{1-\xi^2}} e^{\frac{-\xi\pi}{\sqrt{1-\xi^2}}} \underbrace{\sin(\arccos\xi + \pi)}_{-\sin(\arccos\xi)}\right)$$

$$\arccos\xi = \alpha \rightarrow \sin\alpha = \frac{\omega_d}{\omega_n}$$

$$\hookrightarrow \alpha = \arcsin\frac{\omega_d}{\omega_n}$$



$$h(t_m) = K\left(1 + e^{\frac{-\xi\pi}{\sqrt{1-\xi^2}}}\right), \quad \lim_{t\to\infty} h(t) = K$$

$$\sigma = \frac{h(t_m) - h(\infty)}{h(\infty)} \cdot 100 \; [\%]$$

$$\sigma = 100\, e^{\frac{-\xi\pi}{\sqrt{1-\xi^2}}} \; [\%] \qquad (7)$$

$$25 = 100\, e^{\frac{-\xi\pi}{\sqrt{1-\xi^2}}} \rightarrow \xi = \pm 0{,}403$$

$$\xi = \frac{a}{4} \rightarrow a = \pm 1.61485$$

ZA $a<0$ → NESTABILAN SUSTAV:

$$\sigma = \xi \omega_n \rightarrow \sigma = \frac{a}{4}\omega_n$$

KONAČNO RJEŠENJE:

$$\underline{\underline{a = 1.61485}}$$

(c) $t_m = \frac{\pi}{\omega_d} \rightarrow t_m = 1.717\ s$ ZA $a = 1.61485$

ZADATAK 10.2

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$$G(s) = \frac{1-2s}{s^2+4s+20}$$

(a) $g(t) \circ\!\!-\!\bullet\ G(s)$

$$G(s) = \frac{1-2s}{(s+2)^2+16} = -2\,\frac{s-\frac{1}{2}}{(s+2)^2+16}$$

$$G(s) = -2\left(\frac{s+2}{(s+2)^2+16} - \frac{5}{2}\cdot\frac{1}{4}\,\frac{4}{(s+2)^2+16}\right)$$

$$g(t) = -2\left(e^{-2t}\cos(4t) - \frac{5}{8}\,e^{-2t}\sin(4t)\right)$$

$$g(t) = \frac{5}{4}\,e^{-2t}\sin(4t) - 2e^{-2t}\cos(4t)$$

$$g(t) = e^{-2t}\left(\frac{5}{4}\sin(4t) - 2\cos(4t)\right)$$

$\dot{h}(t) = 0$ - TRAŽIMO VRIJEME EKSTREMA

$$g(t) = \dot{h}(t) = 0$$

$$e^{-2t}\left(\frac{5}{4}\sin(4t) - 2\cos(4t)\right) = 0$$

$$e^{-2t} \neq 0\ ,\quad \frac{5}{4}\sin(4t) = 2\cos(4t)$$

$$tg(4t + k\pi) = \frac{8}{5}$$

$$k = \ldots, -2, -1, 0, 1, 2, \ldots$$

$$4t + k\pi = 1.01$$

$t = 0,2525\ s,\quad k = 0$

$t = 1,0379\ s,\quad k = -1$

$t = 1,8233\ s,\quad k = -2$

$\ddot{h}(t) > 0$ - MINIMUM

$\ddot{h}(t) < 0$ - MAXIMUM

$\ddot{h}(t) = 0$ - TOČKA INFLEKSIJE

$$\ddot{h}(t) = \dot{g}(t)$$

$$\dot{g}(t) = -2e^{-2t}\left(\frac{5}{4}\sin(4t) - 2\cos(4t)\right)$$
$$+ e^{-2t}\left(5\cos(4t) + 8\sin(4t)\right)$$

$$\dot{g}(t) = e^{-2t}\left(-\frac{5}{2}\sin(4t) + \frac{16}{2}\sin(4t) + 4\cos(4t) + 5\cos(4t)\right)$$

$$\dot{g}(t) = e^{-2t}\left(\frac{11}{2}\sin(4t) + 9\cos(4t)\right)$$

ZA $t = 0{,}2525$

$$\dot{g}(t) = e^{-0.505}\left(\frac{11}{2}\cdot 0{,}846 + 9\cdot 0{,}540\right) > 0$$

$t = 0{,}2525$ - VRIJEME PROPADA

$$H(s) = \frac{1-2s}{s(s^2+4s+20)}$$

$$H(s) = \frac{C_{11}}{s} + \frac{As+B}{s^2+4s+20}$$

$$C_{11} = H(s)\cdot s\Big|_{s=0} = \frac{1}{20}$$

$$\frac{As+B}{s^2+4s+20} = \frac{1-2s}{s(s^2+4s+20)} - \frac{1}{20}\cdot\frac{1}{s}$$

$$\frac{(As+B)s}{s(s^2+4s+20)} = \frac{\cancel{1} - 2s - \frac{1}{20}s^2 - \frac{1}{5}s - \cancel{1}}{s(s^2+4s+20)}$$

$$As^2 + Bs = -\frac{1}{20}s^2 - \frac{11}{5}s, \quad A = -\frac{1}{20}, \quad B = -\frac{11}{5}$$

$$H(s) = \frac{1}{20}\frac{1}{s} - \frac{1}{20} \cdot \frac{s+44}{(s+2)^2+16}$$

$$H(s) = \frac{1}{20}\frac{1}{s} - \frac{1}{20}\left(\frac{s+2}{(s+2)^2+16} + \frac{21}{2}\frac{4}{(s+2)^2+16}\right)$$

$$h(t) = \frac{1}{20}\left(1 - e^{-2t}\cos(4t) - \frac{21}{2}e^{-2t}\sin(4t)\right)$$

$$h(t) = \frac{1}{20}\left(1 - e^{-2t}\left(\cos(4t) + \frac{21}{2}\sin(4t)\right)\right)$$

ZA $t = 0.2525\,s$

- $h(t) = -0.234$

(b) ODREĐUJEMO VRIJEME U KOJEM FUNKCIJA DOSEGNE SVOJ PRVI MAKSIMUM

ZA $t = 1.0379\,s$

$$\ddot{h}(t) = \dot{g}(t) = e^{-2.0758}\left(\frac{11}{2}\cdot(-0{,}847) + 9(-0{,}532)\right) < 0$$

$t = 1.0379$ – VRIJEME PRVOG MAKSIMUMA

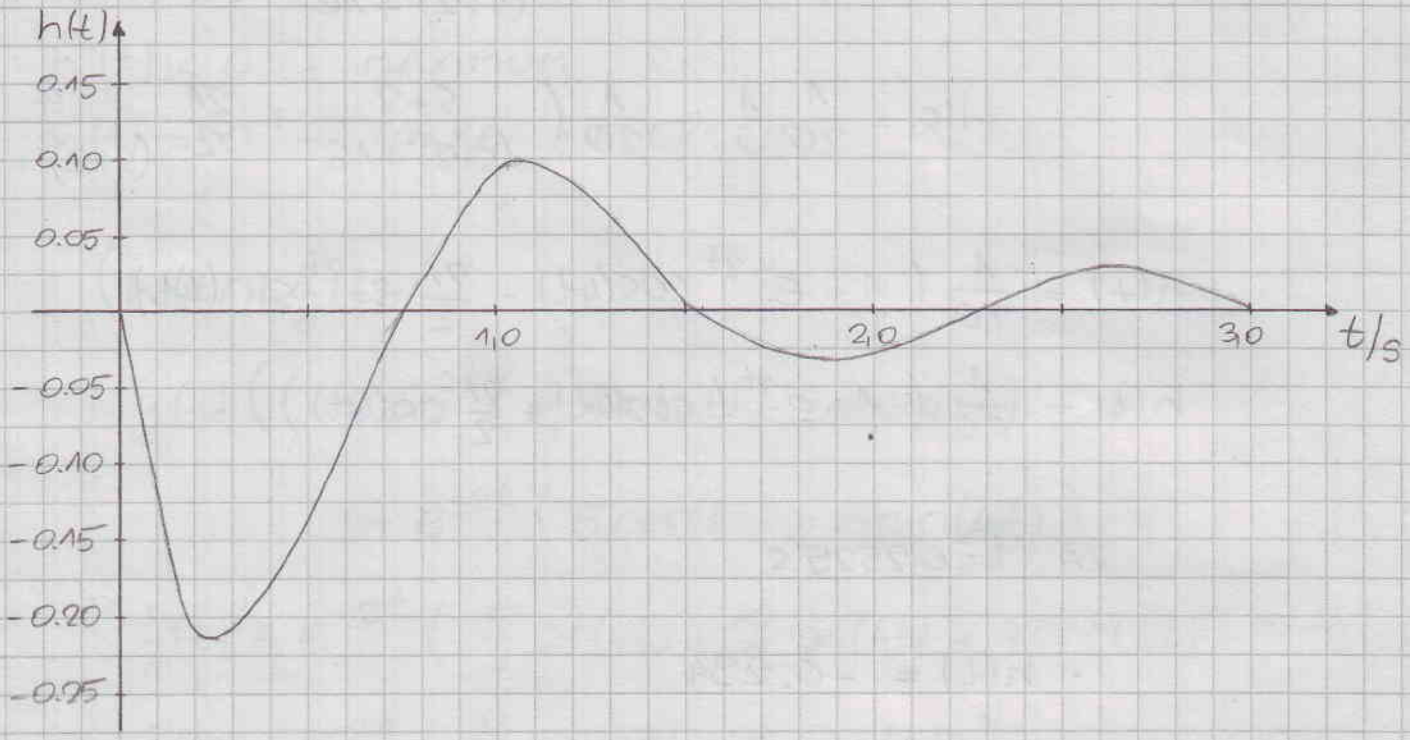
$h(t) = 0.109 := y_m$

$h(\infty) = 0.05 := y_{ss}$

$$\sigma_m\,[\%] = \frac{y_m - y_{ss}}{y_{ss}} \cdot 100$$

- $\sigma_m = 118\,\%$

(c) SKICA PRIJELAZNE FUNKCIJE

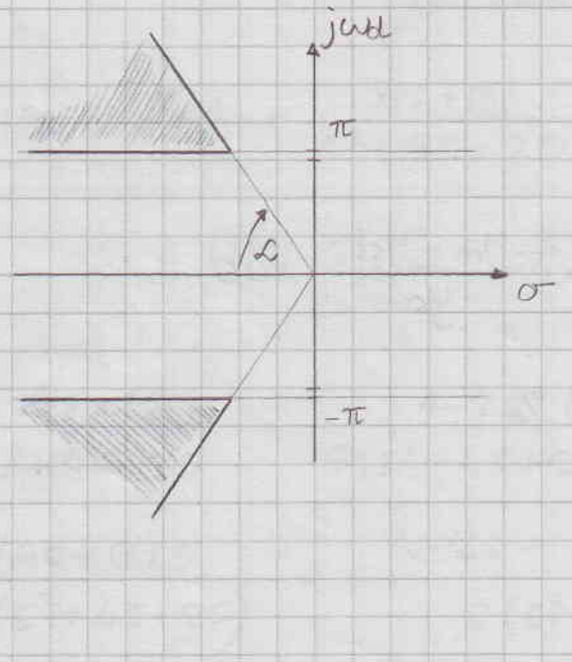


ZADATAK 10.3

$t_m < 1s \qquad t_m = \frac{\pi}{\omega_d} \rightarrow \omega_d = \frac{\pi}{t_m}, \quad \omega_d = \pm\pi$

$\sigma_m = 10\% \qquad \sigma_m = 100\, e^{-\frac{\xi\pi}{\sqrt{1-\xi^2}}} \rightarrow \xi = \pm 0{,}591$

$\measuredangle = \arccos \xi \rightarrow \measuredangle = \pm 53.76^\circ$



DOPUŠTENO PODRUČJE POLOVA!