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12. Stabilnost LTI sustava upravljanja

$G(s) = \frac{B(s)}{N(s)}$ $\to$ nule, $\to$ polovi

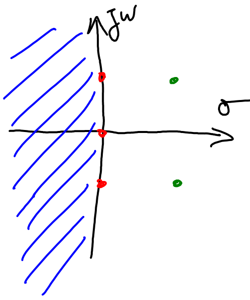
$s = \sigma \pm j\omega$

$s = -2 - j$
$s = -2 + j$

Jednostruki polovi $(s_1 = -1,\ s_2 = -3,\ s_3 = 0)$

$y_h(t) = C_1 e^{s_1 t} + C_2 e^{s_2 t} \dots$

1) $Re\{s_i\} < 0$ $\forall s_i$ stabilan
2) $Re\{s_i\} = 0$ $\exists s_i$ Granično stabilan
3) $Re\{s_i\} > 0$ $\exists s_i$ Nestabilan

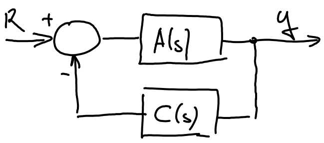


1) $\lim_{s\to\infty} y_h(t) =$
2) $\lim_{s\to\infty} y_h(t) = C_1$ $s=0$
3) $\lim_{s\to\infty} y_h(t) = \infty$ $\exists s_i > 0$

Višestruki $(s_1 = s_2 = -1 \quad s_3 = 4)$

1) $Re\{s_i\} < 0$ $\forall s_i$ stabilan
2) $Re\{s_i\} \geq 0$ $\exists s_i$ nestabilan

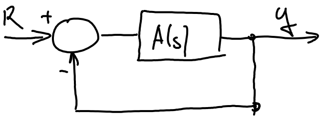
Hurwitzov kriterij stabilnosti



$G(s) = \frac{A(s)}{1 + A(s)C(s)}$

zatvoreni krug

$G_0(s) = A(s)C(s)$ $\Rightarrow$ otvorena grana



$A(s) = G_0(s)$

$$\boxed{G(s) = \frac{G_0(s)}{1 + G_0(s)}}$$

$\to$ Vrijedi samo kada je jedinična povratna veza

– hurwitzov kriterij stabilnosti na temelju $G_0(s)$!

$G(s) = \frac{B(s)}{N(s)} \leftarrow L_{CE} = 1 + G_0(s) = 0$

$a_n s^n + a_{n-1} s^{n-1} \dots + a_1 s^1 + a_0 = 0$

1) Moraju postojati svi koeficijenti

$s^3 + s + 1 = 0$ $\Rightarrow$ nestabilan

- svi koef. moraju biti veći od nule

$$s^3 + 2s^2 - s + 5 = 0 \Rightarrow \text{nestabilan}$$

2) $\underline{n-1}$ Determinanta mora biti pozitivna

$$\begin{array}{c|cccccccc}
D_1 & a_1 & a_0 & 0 & 0 & 0 & 0 & 0 & 0 \\
D_2 & a_3 & a_2 & a_1 & a_0 & 0 & 0 & 0 & 0 \\
D_3 & a_5 & a_4 & a_3 & a_2 & a_1 & a_0 & 0 & 0 \\
D_4 & a_7 & a_6 & a_5 & a_4 & a_3 & a_2 & a_1 & a_0
\end{array}$$

$$D_1 = a_1 > 0$$

$$D_2 = \begin{vmatrix} a_1 & a_0 \\ a_3 & a_2 \end{vmatrix} = a_1 a_2 - a_0 a_3$$

$$D_3 = \begin{vmatrix} a_1 & a_0 & 0 \\ a_3 & a_2 & a_1 \\ a_5 & a_4 & a_3 \end{vmatrix} = a_1 (a_2 a_3 - a_1 a_4) - a_0 (a_3 \cdot a_3 - a_1 \cdot a_5) + 0 (a_3 \cdot a_4 - a_2 a_5)$$

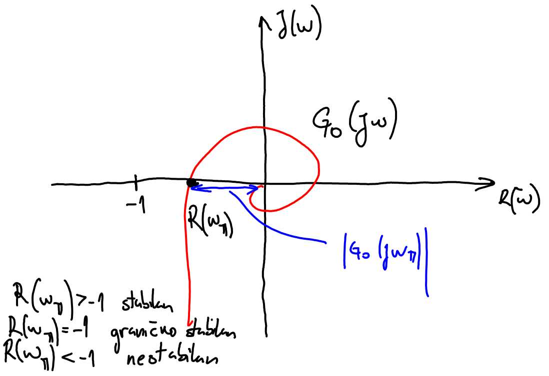
Nyquistov kriterij stabilnosti

ω_π - freq. sustava na kojoj je faza sustava $= \pi$

$$\varphi(\omega_\pi) = \pm \pi$$

$$G_0(j\omega) = R(\omega) + j\, J(\omega)$$

$$J(\omega_\pi) = 0 \qquad R(\omega_\pi) < 0$$

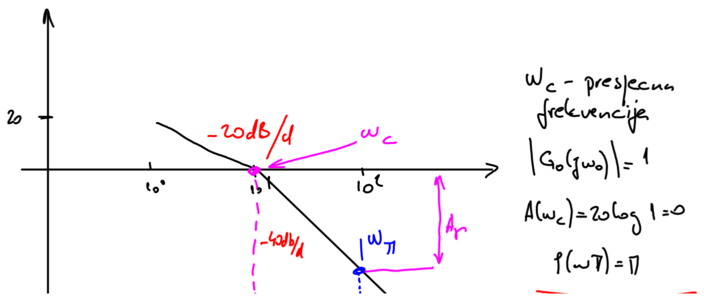


$R(\omega_\pi) > -1$ stabilan

$R(\omega_\pi) = -1$ granično stabilan

$R(\omega_\pi) < -1$ nestabilan

Bodeov kriterij stabilnosti

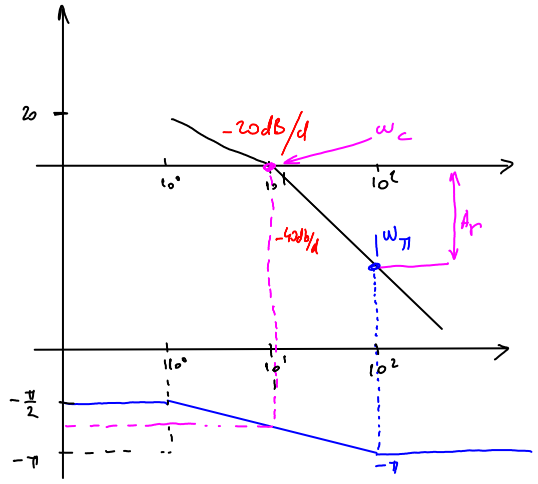


ω_c - presječna frekvencija

$$|G_0(j\omega_c)| = 1$$

$$A(\omega_c) = 20 \log 1 = 0$$

$$\varphi(\omega_\pi) = \pi$$



ω_c - presjecna frekvencija

$|G_o(j\omega_c)| = 1$

$A(\omega_c) = 20\log 1 = 0$

$\varphi(\omega_\pi) = \pi$

$\boxed{A_r = -A(\omega_\pi)}$

$\boxed{\mu = 180° + \varphi(\omega_c)}$

$\boxed{\omega_c < \omega_\pi}$ → stabilan

$\omega_c = \omega_\pi$ granicno

$\omega_c > \omega_\pi$ nestabilno

$$A_r = \underbrace{A(\omega_c)}_{0} - A(\omega_\pi) = -A(\omega_\pi)$$

$$A_r = -A(\omega_\pi) = -20\log\frac{1}{|G_o(j\omega_\pi)|}$$

$$\boxed{A_r = \frac{1}{|G_o(j\omega)|}}$$

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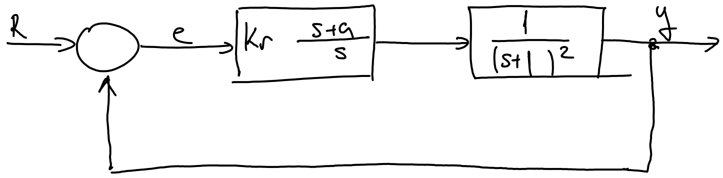
4. Domaća zadaća



$$G_0 = K_r \frac{s+a}{s(s+1)^2}$$

$$\mathcal{L}_{CE}(s) = 1 + G_0(s) = \frac{s^3+2s^2+s+sK_r+aK_r}{s(s+1)^2} = 0$$

$$s^3+2s^2+s+sK_r+aK_r = 0$$

$a_3 = 1 \quad a_2 = 2 \quad a_1 = 1+K_r \quad a_0 = aK_r$

$a \neq 0 \quad K_r \neq 0 \quad 1,2 > 0$

$K_r \neq -1 \quad K_r + 1 > 0$

$K_r > -1$

$aK_r > 0$

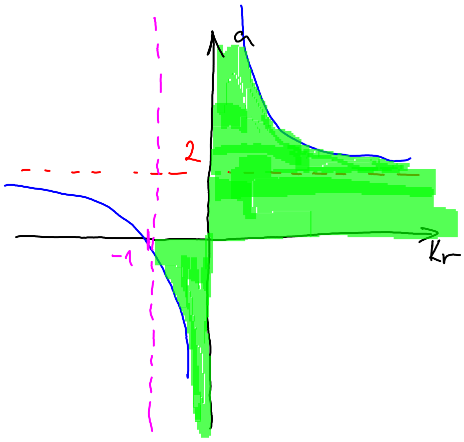
$K_r \in \langle -1, 0 \rangle$, $a < 0$ $\qquad$ $K_r > 0$, $a > 0$

c) $D_1 = a_1 = K_r + 1 > 0 \qquad K_r > 1$

$$D_2 = \begin{vmatrix} a_1 & a_0 \\ a_3 & a_2 \end{vmatrix} = (K_r+1)\cdot 2 - aK_r > 0$$

$a < 2 + \underline{2}$

$\overline{Kr}$



>> syms Kr
>> a = 2 + 2/Kr
ezplot(a, [-1, 10])

$2 + 2K_r > aK_r \;/\; K_r$

1) $K_r > 0$

$2 + \frac{2}{K_r} > a$

2) $K_r < 0$

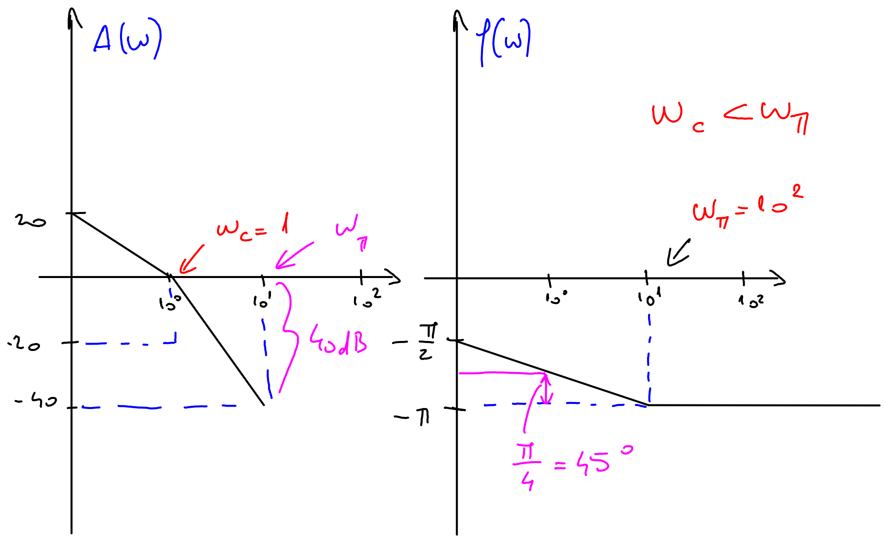
$2 + \frac{2}{K_r} < a$

b) $K_r = 1$
$a = 1$

$$G_o(s) = K_r \frac{s+a}{s} \cdot \frac{1}{(s+1)^2} = \frac{1}{s(s+1)}$$

$$A(\omega) = -20\log(\omega) - 20\log\sqrt{1 + \left(\frac{\omega}{1}\right)^2}$$

$$\varphi(\omega) = -\frac{\pi}{2} + \operatorname{arctg}\frac{\omega}{1}$$



$$G_o(j\omega) = \frac{1}{j\omega(j\omega+1)} = \frac{1}{-\omega^2 + j\omega} = \frac{-1}{\omega^2 - j\omega} = -\frac{1}{\omega}\,\frac{1}{\omega - j}\cdot\frac{\omega + j}{\omega + j} =$$

$$= -\frac{1}{\omega}\,\frac{\omega}{\omega^2+1} + j\left(-\frac{1}{\omega}\right)\frac{1}{\omega^2+1} = \frac{-1}{\omega^2+1} + j\frac{(-1)}{\omega^2+1}$$

$R(\omega)$ $\quad$ $J(\omega)$

$$\left|G_o(j\omega_o)\right| = \frac{1}{\omega\sqrt{1+\omega^2}} = 1$$

$$\omega^2(1+\omega^2) = 1$$

$$\omega^4 + \omega^2 - 1 = 0$$

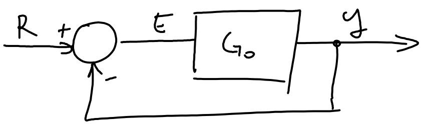
$$\omega^2 = 0.618 \Rightarrow \omega = 0{,}786$$

$$\omega^2 = \cancel{-1{,}618}$$

$A_r = 40\,dB$

$\mu = 45^\circ$

c)



$$e_\infty = \lim_{s\to 0} sE(s)$$

$$R(s) = \frac{2}{s} \qquad K_r = a = 1$$

$$E(s) = R(s) - Y(s)$$

$$Y(s) = E(s)\,G_o(s)$$

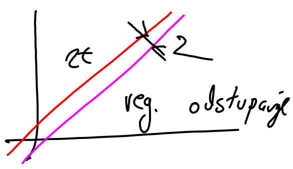
$$E(s) = \frac{R(s)}{1+G_o(s)} \quad , \quad G_o(s) = \frac{1}{s(s+1)}$$

$$E(s) = \frac{2}{s}\,\frac{s(s+1)}{s^2+s+1}$$

$$\lim_{s\to 0} sE(s) = \lim_{s\to 0} 2\,\frac{s(s+1)}{s^2+s+1} = 0$$

(d) $R(s) = \frac{2}{s^2} \;\bullet\!\!-\!\!\circ\; r(t) = 2t\mu(t)$

$$\lim_{s\to 0} s\cdot\frac{2}{s^2}\cdot\frac{s(s+1)}{s^2+s+1} = 2$$



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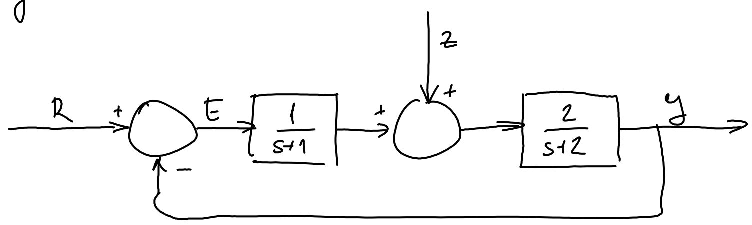
Iz glave



(a) $R(s) = \frac{2}{s}$ $\qquad E(s) = R(s) - Y(s)$

$$E(s) = \frac{R(s)}{1+G_0(s)}$$

$$E(s) = \frac{2}{s} \frac{s^2+2s+1}{s^2+2s+4}$$

$$\lim_{s\to 0} sE(s) = \lim_{s\to 0} \cancel{s} \cdot \frac{2}{\cancel{s}} \frac{s^2+2s+1}{s^2+2s+4} = \frac{1}{2}$$

(b) $z(s) = \frac{1}{s}$, $\Rightarrow R(s) = 0$

$$E(s) = 0 - Y(s)$$

$$Y(s) = \left(Z(s) + E(s)\left(\frac{1}{s+1}\right)\right) \frac{2}{s+2}$$

$$-E(s) = Z(s) \frac{2}{s+2} + E(s) \frac{2}{(s+1)(s+2)}$$

$$-E(s)\left(1 + \frac{2}{(s+1)(s+2)}\right) = Z(s) \frac{2}{(s+2)}$$

$$E(s) \quad \frac{s^2+2s+4}{s+1} = -2 \quad Z(s)$$

$$E(s) = -Z(s) \quad \frac{2(s+2)^2}{s+1}$$

$$e_\infty = \lim_{s\to 0} sE(s) = -\frac{1}{2}$$

c) $\underbrace{Z(s) = \frac{1}{s}}_{/}$, $\underbrace{R(s) = \frac{2}{s}}_{e_{\infty R} = \frac{1}{2}}$

$e_{\infty_z} = -\frac{1}{2}$

$e_{\infty} = e_{\infty_R} + e_{\infty_z}$

$e_{\infty} = \frac{1}{2} + \left(-\frac{1}{2}\right) = 0$

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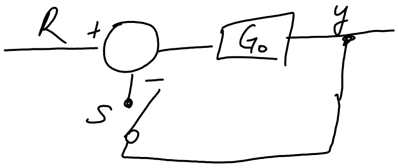
4 Ak. god 2008/2009



$$r(t) = 2\sin\left(3t + \frac{\pi}{6}\right)$$

$$y_0(t) = \sin\left(3t - \frac{2\pi}{3}\right)$$

$$|y| = |U| \cdot |G_0|$$

$$|G_0| = \frac{1}{2}, \quad \measuredangle G_0 = -\frac{5\pi}{6}$$

$$\varphi_{ul} + \measuredangle G_0 = \varphi_{iz}$$

$$G_0 = |G_0| \measuredangle G_0 = \frac{1}{2} \measuredangle -\frac{5\pi}{6} \Big|_{\omega=3}$$

$$y = U \cdot G$$

$$G = \frac{G_0}{1+G_0} = \frac{\frac{1}{2} \measuredangle -\frac{5\pi}{6}}{1 + \frac{1}{2} \measuredangle -\frac{5\pi}{6}} = 0.806 \angle -126^\circ \Big|_{\omega=3}$$

$$|y| = |U| \cdot |G| \Rightarrow |y| = 2 \cdot 0.806 = 1.612$$

$$\measuredangle y = -96^\circ$$

$$y(t) = 1.612 \sin(3t - 96^\circ)$$

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(4) 2007/2008

$t_m < 1$

$\sigma_m < 10\%$

$t_r < 5.4s$

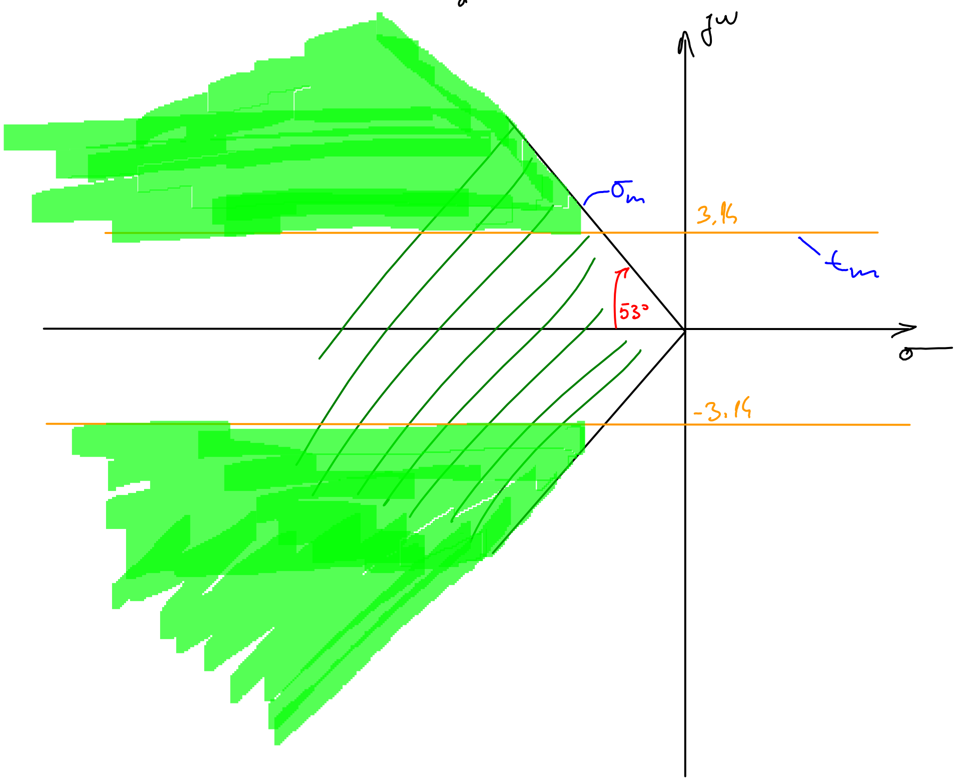
$t_{1\%} < 18.4s$

$\sigma_m = 100 e^{-\frac{\pi \xi}{\sqrt{1-\xi}}} \Rightarrow \xi = 0.592$

$\alpha = \arccos \xi = 53°$

$t_m = \frac{\pi}{\omega_d} \Rightarrow \omega_d = 3.14 s^{-1}$

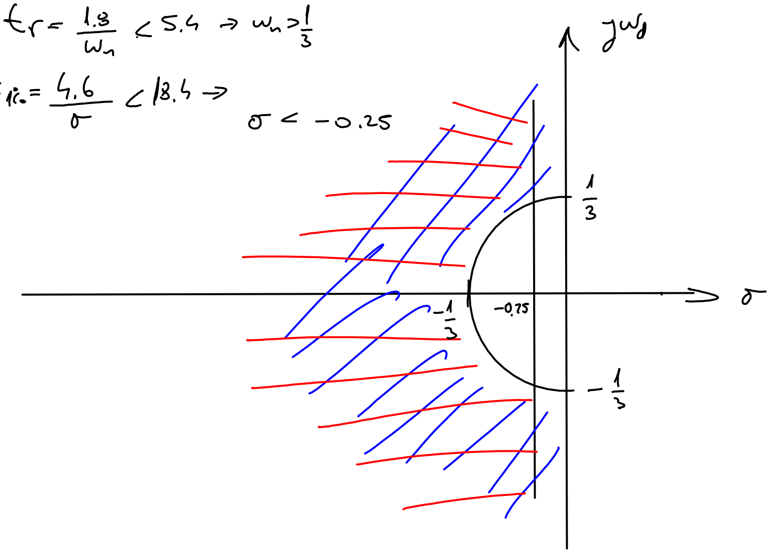
$\frac{\pi}{\omega_d} < 1 \Rightarrow \omega_d > \pi$



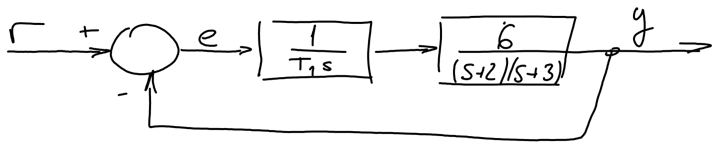
$t_r = \frac{1.8}{\omega_n} < 5.4 \Rightarrow \omega_n > \frac{1}{3}$

$t_{1\%} = \frac{4.6}{\sigma} < 18.4 \Rightarrow$

$\sigma < -0.25$



(2) 4 Dz 2009/2010



(a) $t_m \approx 3s$

$$\boxed{t_m \approx \frac{3}{\omega_c}\,[s]}$$

presjecna freq otvorenog kruga

$\Downarrow$

$$\omega_c \approx 1 s^{-1}$$

$$G_0(s) = \frac{6}{T_1 s (s+2)(s+3)}$$

$$G_0(s) = \frac{6}{T_1 j\omega (j\omega+2)(j\omega+3)}$$

$$|G_0(j\omega_c)| = \frac{6}{\omega_c T_1 \sqrt{4+\omega_c^2}\,\sqrt{9+\omega_c^2}} = 1$$

$\Leftarrow \omega_c = 1$

$\Downarrow$

$$T_1 = 0{,}8485\,s$$

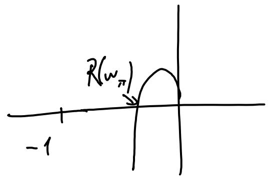
b)

c) $A_r = \dfrac{1}{|G(j\omega_\pi)|}$

$$G_0(j\omega) = \underbrace{\frac{-35{,}3565}{\omega^4+13\omega^2+36}}_{R(\omega)} + j\underbrace{\frac{7.0713(\omega^2-6)}{\omega(\omega^4+13\omega^2+36)}}_{J(\omega)}$$

$J(\omega_\pi) = 0 \quad R(\omega_\pi) = \sqrt{6}$

$R(\omega_\pi) = -0.23571$



Ar

$K \cdot R(\omega_\pi) = -1 \Rightarrow Ar = 4.2425 \Rightarrow 12.5\,dB$

$\mu = 180 + \varphi(\omega_c)$

$\omega_c = 1\,s^{-1}$

$$\varphi(\omega_c) = \operatorname{arctg} = \frac{\frac{7.071\,(\omega_c^2 - 6)}{\omega_c}}{-35.3565} \Rightarrow \varphi(\omega_c) = -135^\circ$$

$\mu = 45^\circ \quad \boxed{\sigma_m = 70 - \mu \; [\%]}$

$\sigma_m = 25\,\%$

d) $\sigma_m = 25\%$
$t_m = 3s$

$$\sigma = 100\, e^{-\frac{\pi \xi}{\sqrt{1-\xi^2}}} \Rightarrow \xi = 0.4037$$

$$t_m = \frac{\pi}{\omega_n \sqrt{1-\xi^2}} \Rightarrow \omega_n = 1.1446\,s^{-1}$$

$$s_{p1,2} = -\sigma \pm j\omega_d$$

$$s_{p1,2} = -\xi\omega_n \pm j\omega_n\sqrt{1-\xi^2}$$

$$s_{p1,2} = -0.46 \pm j\,1.04$$