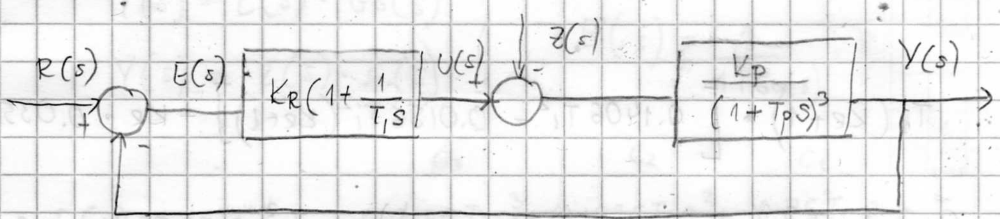
6. DOMAĆA ZADAĆA

(1)



$K_P = 1$

$T_P = 0.25s$

$K_R \geq 0 \quad T_I \geq 0$

a)

$$G_0(s) = K_R\left(1+\frac{1}{T_I s}\right)\cdot\frac{1}{(1+0.25s)^3}$$

$$1+G_0(s) = 1+\frac{K_R\left(1+\frac{1}{T_I s}\right)}{(1+0.25s)^3} = 0 \quad / \cdot (1+0.25s)^3$$

$$(1+0.25s)^3 + K_R\,\frac{T_I s+1}{T_I s} = 0$$

$$T_I\left(s+0.0156s^4+0.75s^2+0.1875s^3\right)+K_R T_I s+K_R=0$$

$$\overset{a_4}{0.0156T_I s^4}+\overset{a_3}{0.1875T_I s^3}+\overset{a_2}{0.75T_I s^2}+\overset{a_1}{s(T_I K_R+T_I)}+\overset{a_0}{K_R}=0$$

(1)

$$T_I>0 \qquad K_R+1>0 \qquad K_R>0$$

$$\boxed{K_R>-1}$$

(2)

$$\begin{vmatrix} a_1 & a_0 \\ a_3 & a_2 \end{vmatrix}>0 \qquad \begin{vmatrix} T_I(K_R+1) & K_R \\ 0.1875T_I & 0.75T_I \end{vmatrix}>0$$

$$0.75T_I^2(K_R+1)-0.1875T_I K_R>0$$

$$0.1875T_I\left(4(K_R+1)T_I-K_R\right)>0$$

$$T_I>0 \qquad \boxed{T_I>\frac{K_R}{4(K_R+1)}}$$

b)

$$D_3 = \begin{vmatrix} a_1 & a_0 & 0 \\ a_3 & a_2 & a_1 \\ 0 & a_4 & a_3 \end{vmatrix} > 0 \qquad \begin{vmatrix} T_i(K_R+1) & K_R & 0 \\ 0.1875\,T_i & 0.75\,T_i & T_i(K_R+1) \\ 0 & 0.0156\,T_i & 0.1875\,T_i \end{vmatrix} > 0$$

$$T_i(K_R+1)\cdot\left[0.1406\,T_i^2 - 0.0156\,T_i^2(K_R+1)\right] - K_R\cdot 0.0352\,T_i^2 > 0$$

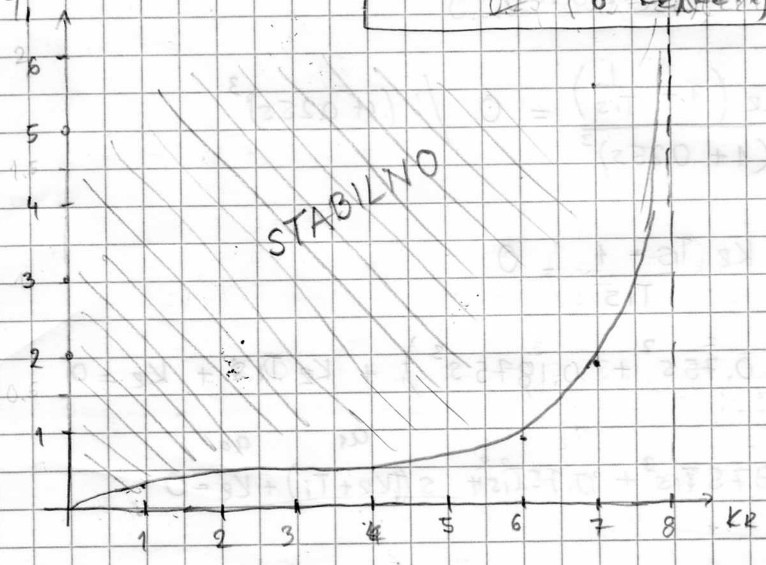
$$(K_R+1)\,T_i^3\left(0.1406 - 0.0156\,(K_R+1)\right) - T_i^2\cdot 0.0352\,K_R > 0$$

$$T_i^2\left[(0.1406 - 0.0156 - 0.0156\,K_R)(K_R+1) - 0.0352\,K_R > 0\right.$$

$$T_i^2\left[T_i(0.125 - 0.0156\,K_R)(K_R+1) - 0.0352\,K_R\right] > 0$$

$$T_i\,(0.125 - 0.0156\,K_R)(K_R+1) > 0.0352\,K_R$$

$$\boxed{T_i > \frac{2.25\;K_R}{(8-K_R)(K_R+1)}}$$



c)

b)

$$R(s) = \frac{1}{s}$$

$$\left.\begin{aligned} Y(s) &= E(s) \cdot G_o(s) \\ Y(s) &= R(s) \cdot G(s) \end{aligned}\right\} \quad E(s) = \frac{1}{1+G_o(s)} \cdot R(s)$$

$$E(s) = \frac{\overset{c_3}{0.0156 T_1 s^3} + \overset{c_2}{0.1875 T_1 s^2} + \overset{c_1}{0.75 T_1 s} + \overset{c_0}{T_1}}{\underset{d_4}{T_1 0.0156 s^4} + \underset{d_3}{0.1875 T_1 s^3} + \underset{d_2}{0.75 T_1 s^2} + \underset{d_1}{T_1(K_R+1)} s + \underset{d_0}{K_R}}$$

$$I_{3,4} = \frac{c_3^2(-d_0^2 d_3 + d_0 d_1 d_2) + (c_2^2 - 2c_1 c_3) d_0 d_1 d_4 + (c_1^2 - 2c_0 c_2) d_0 d_3 d_4 + c_0^2(-d_1 d_4^2 + d_2 d_3 d_4)}{2 d_0 d_4 (-d_0 d_3 - d_1^2 d_4 + d_1 d_2 d_3)}$$

IZ MATLABA $\Rightarrow$ $\frac{\partial I_{3,4}}{\partial K_R} = 0$ $\frac{\partial I_{3,4}}{\partial T_1} = 0$

$$\boxed{K_R = \frac{11}{4} \qquad T_1 = \frac{11}{8}}$$

c)

$$R(s) = 0 \qquad Z(s) = \frac{1}{s}$$

$$\left.\begin{aligned} Y(s) &= E(s) \cdot G_o(s) - Z(s) \cdot G_p(s) \\ Y(s) &= R(s) - Y(s) \end{aligned}\right\} \quad E(s) = \frac{G_p(s) \cdot Z(s)}{1 + G_o(s)}$$

$$E(s) = \frac{1}{\cancel{s}} \cdot \frac{\overset{c_0}{T_1 \cancel{s}} \cdot (\cancel{1+}0.25s)^3 \cdot \frac{1}{(\cancel{1+}0.25s)^3}}{\underset{d_4}{0.0156 T_1 s^4} + \underset{d_3}{0.1875 T_1 s^3} + \underset{d_2}{0.75 T_1 s^2} + \underset{d_1}{T_1(K_R+1)} s + \underset{d_0}{K_R}}$$

$$\boxed{K_R = 5 \qquad T_1 = \frac{5}{2}}$$

$$D_3 = \begin{vmatrix} a_1 & a_0 & 0 \\ a_3 & a_2 & a_1 \\ 0 & a_4 & a_3 \end{vmatrix} > 0 \qquad \begin{vmatrix} T_i(K_R+1) & K_R & 0 \\ 0.1875\,T_i & 0.75\,T_i & T_i(K_R+1) \\ 0 & 0.0156\,T_i & 0.1875\,T_i \end{vmatrix} > 0$$

$$T_i(K_R+1)\cdot\left[0.1406\,T_i^2 - 0.0156\,T_i^2(K_R+1)\right] - K_R\cdot 0.0352\,T_i^2 > 0$$

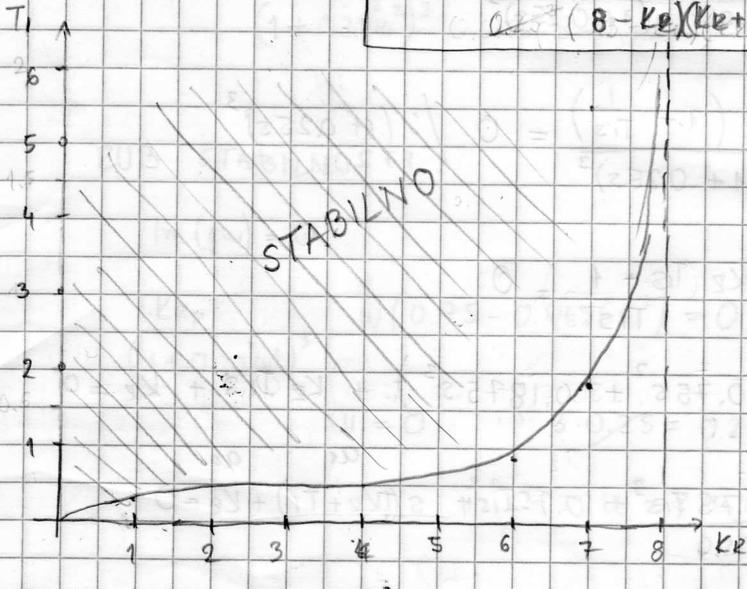
$$(K_R+1)\,T_i^3\left(0.1406 - 0.0156(K_R+1)\right) - T_i^2\cdot 0.0352\,K_R > 0$$

$$T_i^2\left[\left(0.1406 - 0.0156 - 0.0156K_R\right)(K_R+1) - 0.0352K_R > 0\right.$$

$$T_i^2\left[T_i(0.125 - 0.0156K_R)(K_R+1) - 0.0352K_R\right] > 0$$

$$T_i(0.125 - 0.0156K_R)(K_R+1) > 0.0352\,K_R$$

$$\boxed{T_i > \frac{2.25\,K_R}{(8-K_R)(K_R+1)}}$$



$K_{Rkp} = 8 \quad T_{kr} = \frac{\sqrt{3}}{6}\pi$

PI - regulator

$K_R = 0.45 \cdot K_{Rp} = 3.6$

$T_I = 0.85 \cdot \frac{\sqrt{3}}{6} \cdot \pi = 0.771$

PID - regulator

$K_R = 0.6 \cdot K_{Rp} = 4.8$

$T_I = 0.5\, T_{kr} = 0.453\, s$

$T_D = 0.12\, T_{kr} = 0.109\, s$

PI $\boxed{G_R(s) = 3.6 \cdot \left(1 + \frac{1}{0.771 s}\right)}$

$\boxed{G_R(s) = 4.8 \cdot \left(1 + \frac{1}{0.453 s} + 0.109 s\right)}$

PID

e)

$$G_R(s) = 4.8 \cdot \left(1 + \frac{1}{0.453 s} + 0.109 s\right)$$

$$G_O(s) = G_R(s) \cdot G_P(s) = \frac{4.8 \cdot \left(1 + \frac{1}{0.453 s} + 0.105 s\right)}{s(1+0.25s)^3}$$

$$= \frac{10.6 \left(0.453 s + 1 + 0.05 s^2\right)}{s\left(1 + \frac{s}{4}\right)^3} = \frac{0.53 \cdot \left(s^2 + 9.06 s + 20\right)}{s\left(1 + \frac{s}{4}\right)^3}$$

$$= 0.53 \frac{(s + 3.81)(s + 5.25)}{s\left(1 + \frac{s}{4}\right)^3} = 10.6 \frac{\left(\frac{s}{3.81} + 1\right)\left(\frac{s}{5.25} + 1\right)}{s\left(1 + \frac{s}{4}\right)^2}$$

$s^2 + 9.06 s + 20 = 0$

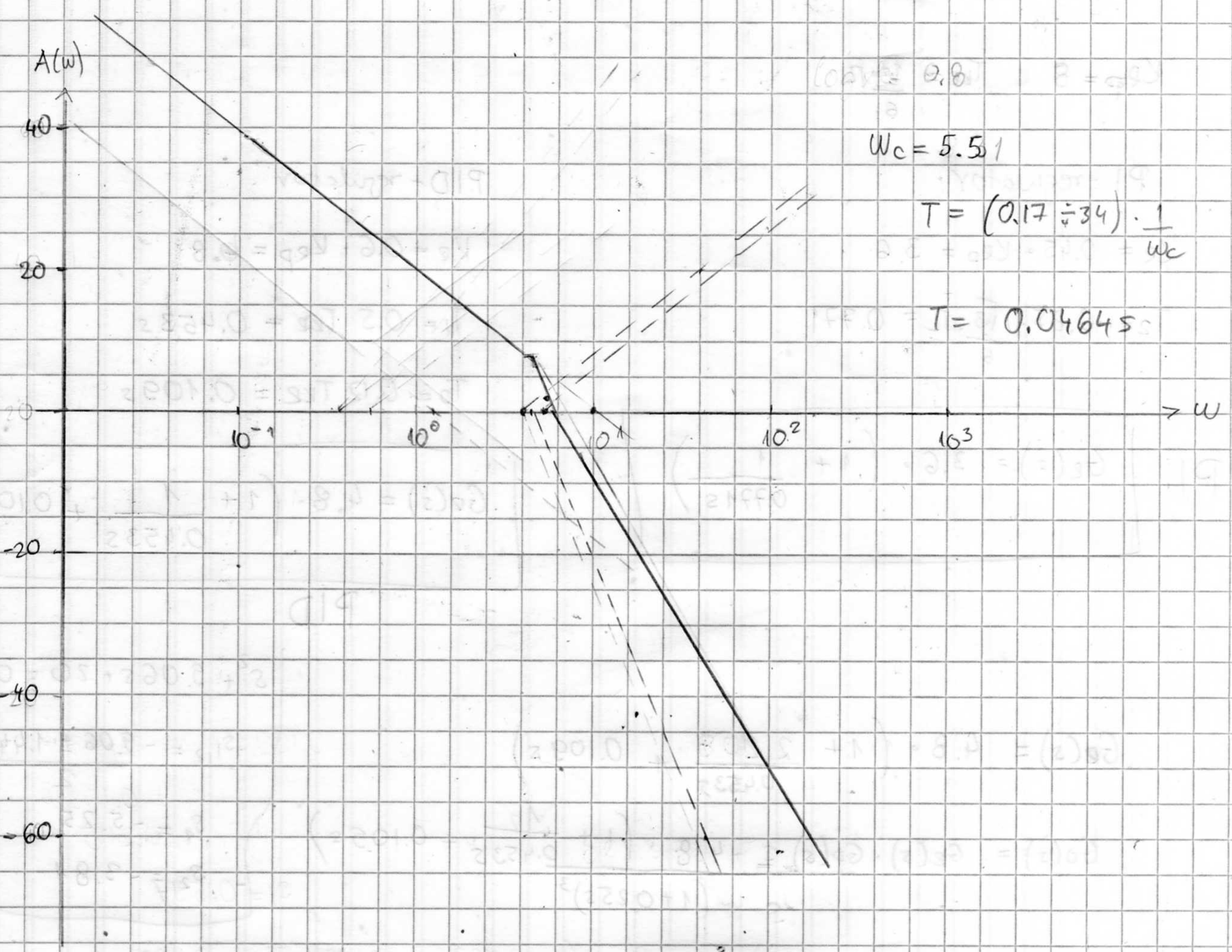
$s_{1,2} = -\frac{9.06 \pm 1.44346}{2}$

$s_1 = -5.25$

$s_2 = -3.81$

$$A(\omega)|_{dB} = 20 \log \sqrt{1 + \left(\frac{\omega}{3.81}\right)^2} + 20 \log \sqrt{1 + \frac{\omega}{5.25}} - 20 \log \frac{\omega}{10.6} - 60 \log \sqrt{1 + \left(\frac{\omega}{4}\right)^2}$$

$$f(\omega) = -\frac{\pi}{2} + \operatorname{arctg} \frac{\omega}{3.81} + \operatorname{arctg} \frac{\omega}{5.25} - 3 \operatorname{arctg} \frac{\omega}{4}$$



$W_c = 5.51$

$T = (0.17 \div 34) \cdot \frac{1}{W_c}$

$T = 0.0464 s$

f) $s = \frac{2}{T} \frac{z-1}{z+1}$ $T = 0.0464$

$$G_R(z) = G_R(s)\Big|_{s = 43.1 \frac{z-1}{z+1}}$$

$$G_R(z) = 4.8 \left(1 + \frac{1}{0.453 \cdot 43.1 \frac{z-1}{z+1}} + 0{,}106 \cdot 43.1 \cdot \frac{z-1}{z+1}\right)$$

$$= 4.8 + 0{,}246 \cdot \frac{z+1}{z-1} + 4.7 \frac{z-1}{z+1}$$

$$= \frac{4.8z^2 - 4.8 + 0.246z^2 + 0.492z + 0.246 + 4.7z^2 - 9.4z + 4.7}{z^2 - 1}$$

$$= \frac{9.746z^2 - 8.08z - 0.1}{z^2 - 1} = \frac{9.764 - 8.08z^{-1} - 0.1z^{-2}}{1 - z^{-2}}$$

$$u(k) = u(k-2) + 9.746\, e(k) - 8.082\, e(k-1) + 90.16\, e(k-2)$$