Zadatak 2.

Zadana je prijenosna funkcija otvorenog kruga

$$G(s) = 4\frac{1-s}{(1+10s)(1+0.1s)}$$

Nacrtaj Nyquistov dijagram, odredi amplitudno osiguranje (A_r),frekvencija na kojoj je fazna karakteristika 180°(ω_π),preskočnu frekvenciju (ω_c) i fazno osiguranje (γ)

$$G(j\omega) = 4\frac{1-j\omega}{(1+j10\omega)(1+j0.1\omega)} = \cdots (nakon\ sređivanja) \ldots =$$
$$= 4\frac{1-11.1\omega^2}{(1-\omega^2)^2+(10.1\omega)^2} + j4\frac{\omega^3-11.1\omega}{(1-\omega^2)^2+(10.1\omega)^2} = Re(\omega) + jIm(\omega)$$

Početne vrijednosti su:

$$\omega = 0 \qquad Re(\omega=0) = 4 \qquad Im(\omega=0) = 0$$

Konačne vrijednosti su:

$$\omega = \infty \quad \lim_{\omega\to\infty} Re(\omega) = 0 \quad \left(približava\ se\ s\ lijeve\ strane\ jer\ je\ Re\sim\frac{-\omega^2}{\omega^4}\right)$$
$$\lim_{\omega\to\infty} Im(\omega) = 0 \quad \left(približava\ se\ odozgo\ jer\ je\ Im\sim\frac{\omega^3}{\omega^4}\right)$$

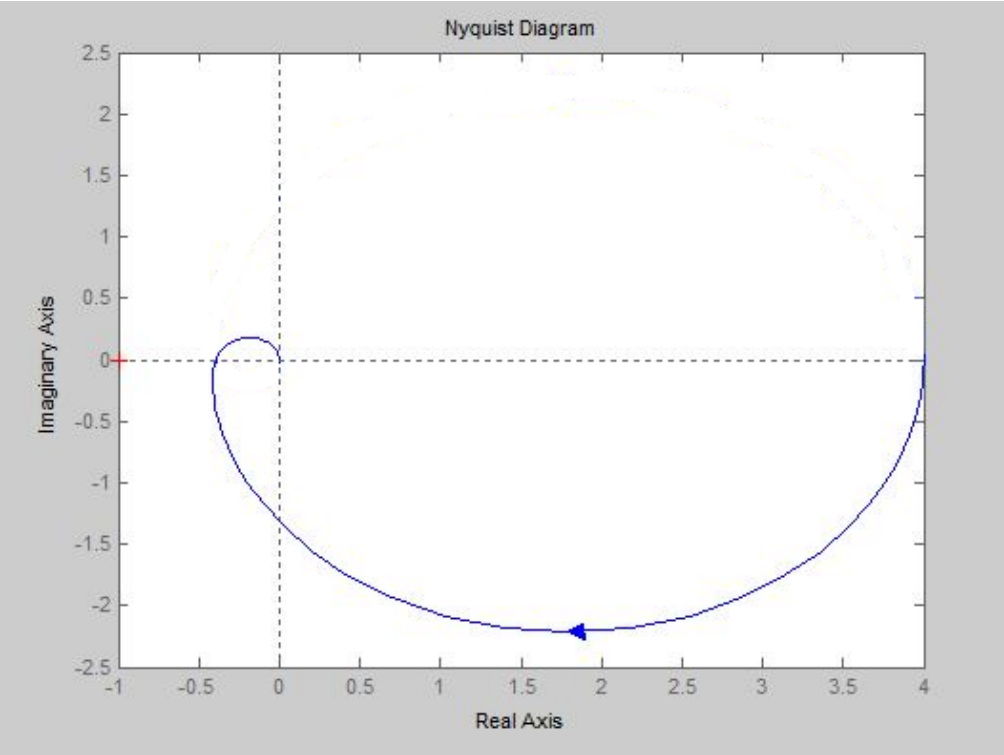
Frekvencije na kojima si realni i imaginarni dijelovi jednaki nuli

$$Re(\omega) = 0 \quad \rightarrow \quad 1-11.1\omega^2 = 0 \quad \rightarrow \omega = 0.3$$

$$Im\ (\omega = 0.3) = -1.32$$

$$Im(\omega) = 0 \quad \rightarrow \omega^3 - 11.1\omega = 0 \quad \rightarrow \quad \omega_1 = 0 \ i\ \omega_2 = 3.33$$

$$Re(\omega = 0) = 4 \quad ; \quad Re(\omega = 3.33) = -0.3961$$



$\omega_\pi = 3.33 \quad \rightarrow kut\ Nyquistovog\ dijagrama\ je - 180°$

Amplitudno osiguranje je max. pojačanje koje mogu staviti, a da sustav bude stabilan

$$\frac{1}{A_r} = 0.3961 \quad \rightarrow A_r = 2.5246 \quad \rightarrow A_{rdB} = 20 \log A_r = 8.04\ dB$$

Presječna frekvencija ω_c

$$|G(j\omega_c)| = 1$$

$$|G(j\omega)| = 4\frac{\sqrt{1+\omega^2}}{\sqrt{1+100\omega^2}\sqrt{1+0.01\omega^2}} = 1$$

$$\omega^4 + 84\omega^2 - 15 = 0$$

~~$\omega^2 = -84{,}88$~~ $\quad \omega^2 = 0.17817$

$\omega = \omega_{c=} 0.4221\ rad/s$

Fazno osiguranje γ

$$\gamma = 180° + \varphi_0(\omega_c)$$

I. način

$$\gamma = arctg\frac{Im(\omega_c)}{Re(\omega_c)} = arctg\frac{\omega_c(\omega_c^2 - 11.1)}{1 - 11.1\omega_c^2} = 78°$$

II. način

$$\varphi_{0(\omega_c)} = arctg\ \frac{Im}{Re} = arctg2(\ 4.7152\ ) = -180° + arctg(4.7152) = -102°$$

$$\gamma = 180° - 102° = 78°$$