3. DOMAĆA ZADAĆA

1) a)

$$G(s) = 500\,\frac{s+0{,}1}{(s+5)(s+20)} = 500\cdot\frac{0{,}1\left(\frac{s}{0{,}1}+1\right)}{20\cdot 5\left(\frac{s}{5}+1\right)\left(\frac{s}{20}+1\right)}$$

$$G(s) = \frac{1}{2}\cdot\frac{\left(\frac{s}{0{,}1}+1\right)}{\left(\frac{s}{5}+1\right)\left(\frac{s}{20}+1\right)}$$

$\Rightarrow s = j\omega$

$$G(j\omega) = \frac{1}{2}\cdot\frac{\left(\frac{j\omega}{0{,}1}+1\right)}{\left(\frac{j\omega}{5}+1\right)\left(\frac{j\omega}{20}+1\right)}$$

$$A(\omega) = 20\log|G(j\omega)| = 20\log|G_1(j\omega)|\cdot|G_2(j\omega)|\cdot|G_3^{-1}(j\omega)|\cdot|G_4^{-1}(j\omega)|$$

$$= 20\log\frac{1}{2} + 20\log\sqrt{1+\left(\frac{\omega}{0{,}1}\right)^2} - 20\log\sqrt{1+\left(\frac{\omega}{5}\right)^2} - 20\log\sqrt{1+\left(\frac{\omega}{20}\right)^2}$$

fazna karakteristika

$$\varphi(\omega) = \operatorname{arctg}\frac{\omega}{0{,}1} - \operatorname{arctg}\frac{\omega}{5} - \operatorname{arctg}\frac{\omega}{20}$$

$100-\omega^2 + j25\omega$

$$\varphi(\omega) = \operatorname{arctg}\frac{\omega}{0{,}1} - \operatorname{arctg}\frac{\omega}{5} - \operatorname{arctg}\frac{\omega}{20}$$

$100-\omega^2 + j25\omega$

b) Nyquistov dijagram

$20j\omega + 5j\omega$

$$\Rightarrow G(j\omega) = 500\,\frac{(j\omega+0{,}1)}{(j\omega+5)(j\omega+20)} = 500\cdot\frac{(j\omega+0{,}1)}{(-\omega^2+j25\omega+100)} = 500\cdot\frac{(j\omega+0{,}1)}{(100-\omega^2)+j25\omega}$$

$$= 500\cdot\frac{(j\omega+0{,}1)(100-\omega^2) - (j\omega+0{,}1)\cdot j25\omega}{(100-\omega^2)^2 + 625\omega^2}$$

$$G(j\omega) = 500\cdot\frac{j(100\omega - \omega^3 - 2{,}5\omega) + 10 - 0{,}1\omega^2 + 25\omega^2}{(100-\omega^2)^2 + 625\omega^2}$$

$$G(j\omega) = \underbrace{500\cdot\frac{10+24{,}9\omega^2}{(100-\omega^2)^2+625\omega^2}}_{Re(G)} + \underbrace{j500\,\frac{97{,}5\omega-\omega^3}{(100-\omega^2)^2+625\omega^2}}_{Im(G)} = Re(G)+Im(G)$$

$\Rightarrow \omega = 0$

$$Re(G) = \frac{500\cdot 10}{10000} = \frac{1}{2}$$

$$Im(G) = 0$$

$\Rightarrow \omega = \infty$

$$Re(G) = 0$$

$$Im(G) = 0$$

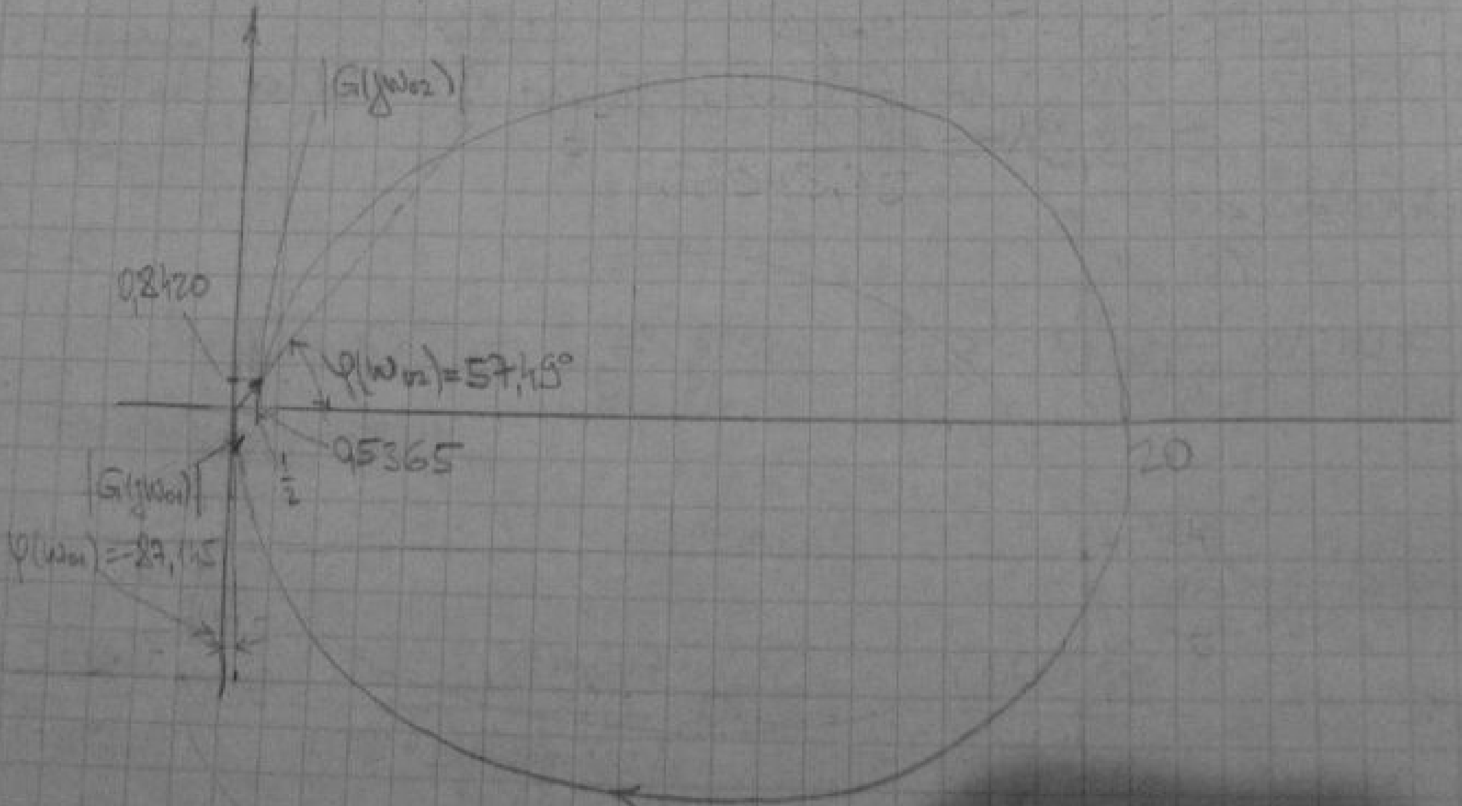
$Re(G) = 0 = 500(10 + 24{,}9\,w^2)$

$\Rightarrow w = \cancel{\frac{1}{\sqrt{}}}\sqrt{\frac{10}{24{,}9}} =$

$Im(G) = 0 \Rightarrow 97{,}5 - w^2 = 9{,}8742$

$\Rightarrow Re(G)\Big|_{w=9{,}8742} = 500 \cdot \frac{10 + 24{,}9 \cdot 97{,}5}{(100 - 97{,}5)^2 + 625 \cdot 97{,}5}$

$= \frac{1218875}{60943{,}75} = 20$



kut ulaza u ishodište

za $w=\infty \Rightarrow$

za $w=\infty$ $G(jw)$ ima oblik

$$G(jw) = \frac{jw}{jw \cdot jw} = \frac{1}{jw} = -j\frac{1}{w} \Rightarrow \operatorname{arctg}(G(jw)) = -\frac{\pi}{2}$$

c) $u(t) = 2\sin(\omega_0 t + \phi) = 2e^{j\phi}e^{j\omega_0 t}$

$y(t) = 2\sin(\omega_0 t) = 2e^{j\phi}e^{j\omega_0 t}$

$$G(j\omega) = \frac{2e^{j\phi}e^{j\omega_0 t}}{2e^{j\omega_0 t}} = e^{j\phi}$$

općenito vrijedi

$$G(j\omega) = A(\omega) \cdot e^{j\phi}$$

$$\Rightarrow \boxed{A(\omega) = 1 = |G(j\omega)|}$$

$$G(j\omega) = \cos\phi + j\sin\phi$$

$$|G(j\omega)| = 1 = \frac{500 \cdot \sqrt{\omega^2 + (0{,}1)^2}}{\sqrt{(\omega^2+5^2)(\omega^2+20^2)}}$$

$$\left(\sqrt{(\omega^2+5^2)(\omega^2+20^2)} = 500\sqrt{\omega^2+(0{,}1)^2}\right)^2$$

$$\omega^4 + 400\omega^2 + 25\omega^2 + 10000 = 250000\omega^2 + 2500$$

$$\omega^4 - 249575\omega^2 + 7500 = 0$$

$$\omega_{1,2}^2 = \frac{249575 \pm \sqrt{249575^2 - 4 \cdot 7500}}{2} = \frac{249575 \pm 249574{,}9399}{2}$$

$$\omega^2_{1,2} = \frac{249575 \pm \sqrt{249575^2 - 4\cdot 7500}}{2} = \frac{249575 \pm 249574,9399}{2}$$

$$\omega_1 = \pm 499,574 \;/\!/ \quad \Rightarrow \quad \boxed{\omega_{01} = 499,574 \ \frac{rad}{s}}$$

$$\omega_2 = \pm 0,173 \;/\!/ \quad \quad \boxed{\omega_{02} = 0,173 \ \frac{rad}{s}}$$

1°
$$\varphi(\omega_{01}) = \operatorname{arctg} \frac{\operatorname{Im}(G(j\omega))}{\operatorname{Re}(G(j\omega))} = \operatorname{arctg} \frac{(97,5 - 499,574^2)\cdot 499,574}{10 + 24,9\cdot 499,574^2} = \operatorname{arctg} \frac{-124639063,7}{6214407,119}$$

$$\underline{\varphi(\omega_{01}) = \operatorname{arctg} -20,055 = -87,145°}$$

2°
$$\varphi(\omega_{02}) = \operatorname{arctg} \frac{(97,5 - 0,173^2)\cdot 0,173}{10 + 24,9\cdot 0,173^2} = \operatorname{arctg} \frac{16,8623}{10,7452} = 57,49°$$

d)

c – ima dva rješenja za ω_0 i dva za $\varphi(\omega_0)$

e) ⇒ na nacrtanom dijagramu pod b)

$$G(j\omega_{01}) = 500\left(\frac{6214407{,}119}{(100-499{,}574^2)^2+625\cdot 499{,}57^2} - j\frac{124632063{,}7}{(100-499{,}574^2)^2+625\cdot 499{,}57^2}\right)$$

$$G(j\omega_{01}) = 0{,}0498 - j0{,}9987$$

$|G(j\omega_{01})| \approx 1$ – malo veće od 1

$$G(j\omega_{02}) = 500\left(\frac{10{,}7452 + j16{,}8623}{(100-0{,}173^2)^2+625\cdot 0{,}173^2}\right) = 0{,}5365 + j0{,}8420$$

$|G(j\omega_{02})| \approx 1$ – malo manje od 1

$\arg(G(j\omega_{01})) = -87{,}145°$

$\arg(G(j\omega_{02})) = 57{,}49°$

2) $G(s) = 500 \dfrac{as+0{,}1}{\ldots}$

2) $G(s) = 500 \frac{as + 0{,}1}{(s+5)(s+20)}$

a)
$$\frac{500(as + 0{,}1)}{(s+5)(s+20)} = \frac{A}{(s+5)} + \frac{B}{(s+20)}$$

$$As + 20A + Bs + 5B = 500a \cdot s + 50$$

1° $A + B = 500a \Rightarrow B = 500a - A$

2° $20A + 5B = 50$

$$20A + 5(500a - A) = 50$$

$$15A = 50 - 2500a \quad /:15$$

$$A = \frac{10}{3} - \frac{500a}{3}$$

$$B = 500a - A$$

$$B = 500a - \frac{10}{3} + \frac{500a}{3}$$

$$B = \frac{2000a}{3} - \frac{10}{3}$$

$$G(s) = \frac{1}{s+5}\left(\frac{10}{3} - \frac{500a}{3}\right) + \frac{1}{s+20}\left(\frac{2000a}{3} - \frac{10}{3}\right)$$

$$g(t) = \left(\frac{10}{3} - \frac{500a}{3}\right)e^{-5t} + \left(\frac{2000a}{3} - \frac{10}{3}\right)e^{-20t}$$

$$h'(t) = g(t) \Rightarrow H(s) = \frac{1}{s} G(s)$$

$$H(s) = \frac{500\,(as+0{,}1)}{s(s+5)(s+20)}$$

$$\frac{(as+0{,}1)\cdot 500}{s(s+5)(s+20)} = \frac{A}{s} + \frac{B}{s+5} + \frac{C}{s+20}$$

$$A(s^2+25s+100) + Bs^2 + 20Bs + Cs^2 + 5Cs = (as+0{,}1)\cdot 500$$

1° $A+B+C=0$

2° $25A+20B+5C = 500a$

3° $100A = 50$

$$\boxed{A = 0{,}5}$$

1° $B+C = -0{,}5 \quad / \cdot (-20)$

3° $100A = 50$

$\boxed{A = 0{,}5}$

1° $B + C = -0{,}5 \quad \cdot(-20)$

2° $12{,}5 + 20B + 5C = 500a$

1° $-20B - 20C = 10$

2° $12{,}5 + 20B + 5C = 500a$ +

$12{,}5 - 15C = 10 + 500a$

$2{,}5 = -500a + 15C$

$$\boxed{C = -\frac{100}{3}a + \frac{0{,}5}{3}}$$

$$B = -0{,}5 - C = -0{,}5 + \frac{100}{3}a - \frac{0{,}5}{3}$$

$$B = \frac{100}{3}a - \frac{2}{3} \quad //$$

$$H(s) = \frac{1}{2} + \left(\frac{100}{3}a - \frac{2}{3}\right)\frac{1}{s+5} - \left(\frac{100}{3}a - \frac{0{,}5}{3}\right)\frac{1}{s+20}$$

$$h(t) = \frac{1}{2} + \left(\frac{100}{3}a - \frac{2}{3}\right)e^{-5t} - \left(\frac{100}{3}a - \frac{0{,}5}{3}\right)e^{-20t}$$

b) kad se neki od prirodnih modova ne bi vidjeli znači da bi koeficijenti uz e^{-5t} ili e^{-20t} bili jednaki nuli

$\Rightarrow$ 1°

$$\frac{100}{3}a - \frac{2}{3} = 0$$

$$100a = 2$$

$$a_1 = \frac{1}{50}$$

2°

$$\frac{100}{3}a - \frac{0{,}5}{3} = 0$$

$$100a = 0{,}5$$

$$a = \frac{1}{2} \cdot \frac{1}{100}$$

$$a_2 = \frac{1}{200}$$

$$a_2 = \frac{1}{200}$$

c) nadvišenje ⇒ vrijednost funkcije veća od vrijednosti u ustaljenom stanju

⇒ da bi f-ja imala nadvišenje, $h(t) - h(\infty)$ izraz mora imati pozitivne nultočke

$$h(\infty) = \lim_{t\to\infty} h(t) = \frac{1}{2} + \frac{1}{e^{\infty}}\left(\frac{100}{3}a - \frac{2}{3}\right) - \frac{1}{e^{\infty}}\left(\frac{100}{3}a - \frac{0{,}5}{3}\right)$$

$$h(\infty) = \frac{1}{2}$$

$$\Rightarrow \quad h(t) - h(\infty) = \left(\frac{100}{3}a - \frac{2}{3}\right)e^{-5t} - \left(\frac{100}{3}a - \frac{0{,}5}{3}\right)e^{-20t} = 0$$

$$\left(\frac{100}{3}a - \frac{2}{3}\right)e^{-5t} = \left(\frac{100}{3}a - \frac{0{,}5}{3}\right)e^{-20t} \quad \Big| \cdot e^{5t}$$

$$\frac{100a-2}{3} = e^{-15t}\left(\frac{100a-0{,}5}{3}\right)$$

$$\frac{100a-0{,}5}{100a-2} = e^{15t} \Rightarrow$$

$$15t = \ln \frac{100a - 0{,}5}{100a - 2}$$

$$t = \frac{1}{15} \ln \frac{100a - 0{,}5}{100a - 2}$$

ukoliko nadvišenje postoji $t > 0 \Rightarrow$

$$\Rightarrow \ln \frac{100a - 0{,}5}{100a - 2} > 0$$

$$\Rightarrow \frac{100a - 0{,}5}{100a - 2} > 1$$

$$\frac{\cancel{100a} - 0{,}5 - \cancel{100a} + 2}{100a - 2} > 0$$

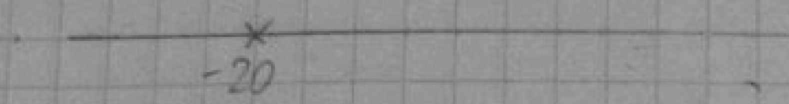
$$\frac{1{,}5}{100a - 2} > 0$$

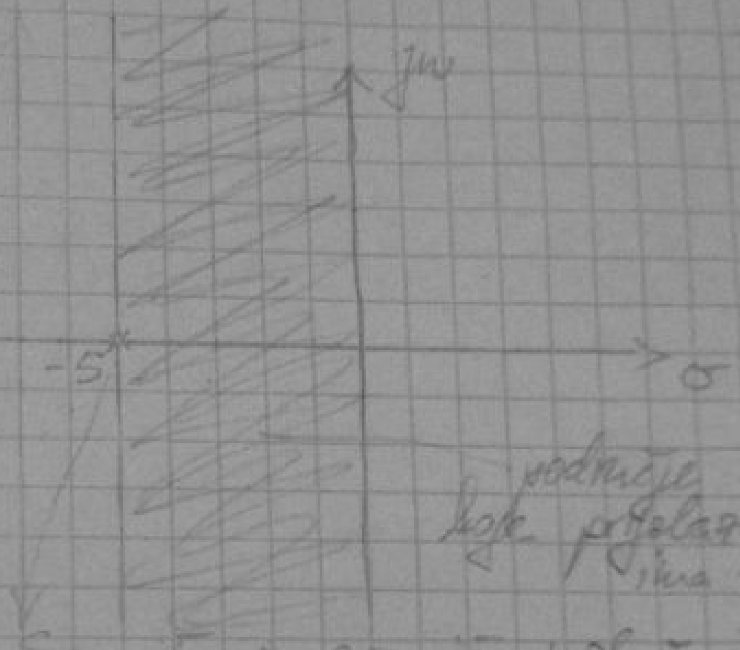
$$\Rightarrow 100a - 2 > 0 \Rightarrow a > \frac{2}{100}$$

$$\boxed{a > \frac{1}{50}}$$

h(t) ima nadvišenje

h(t) ima nadvišenje za $a \in \langle \frac{1}{50}, \infty \rangle$





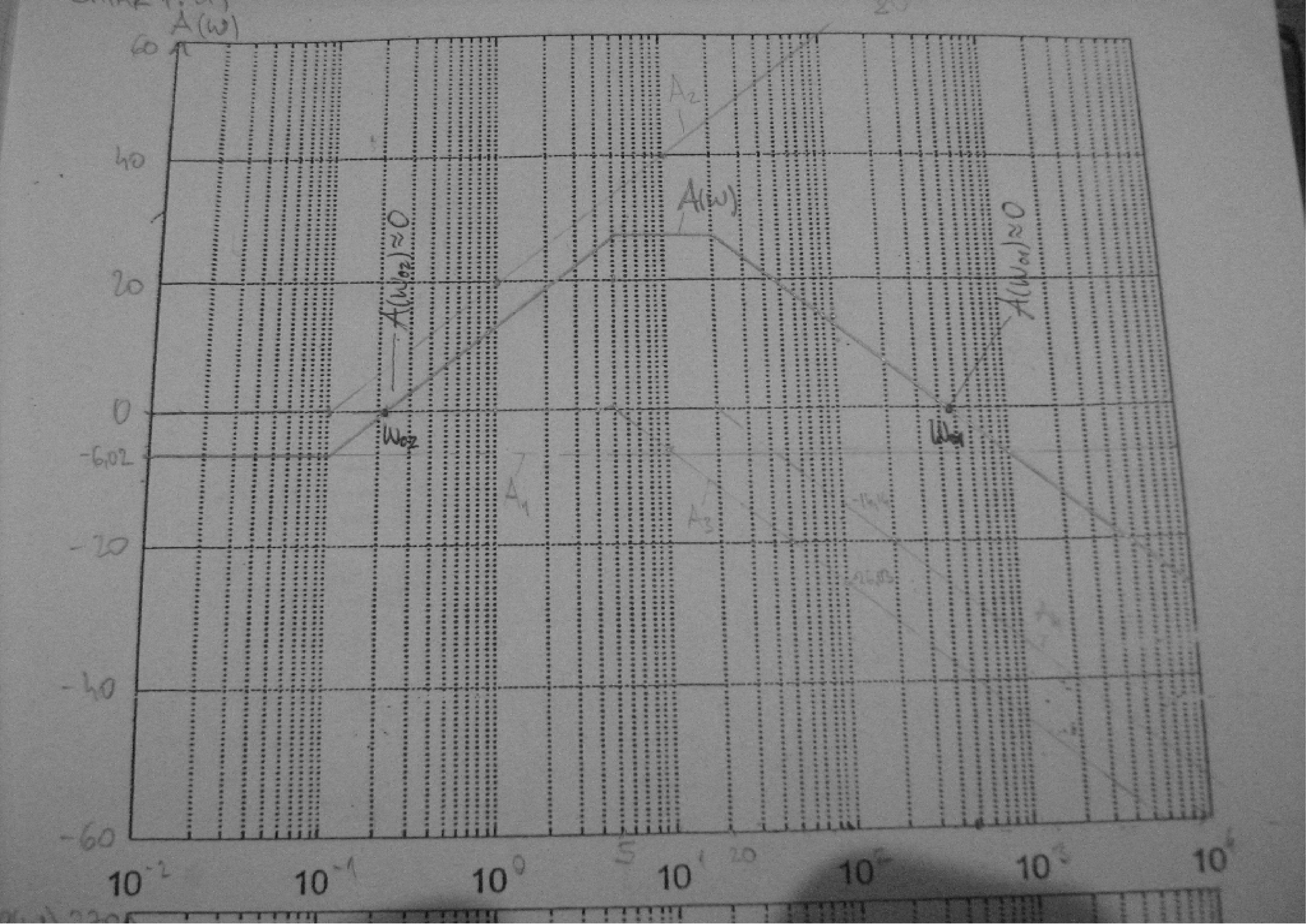
d) da bi h(t) imala podbačaj mora vrijediti:

$$h'(0^+) < 0$$

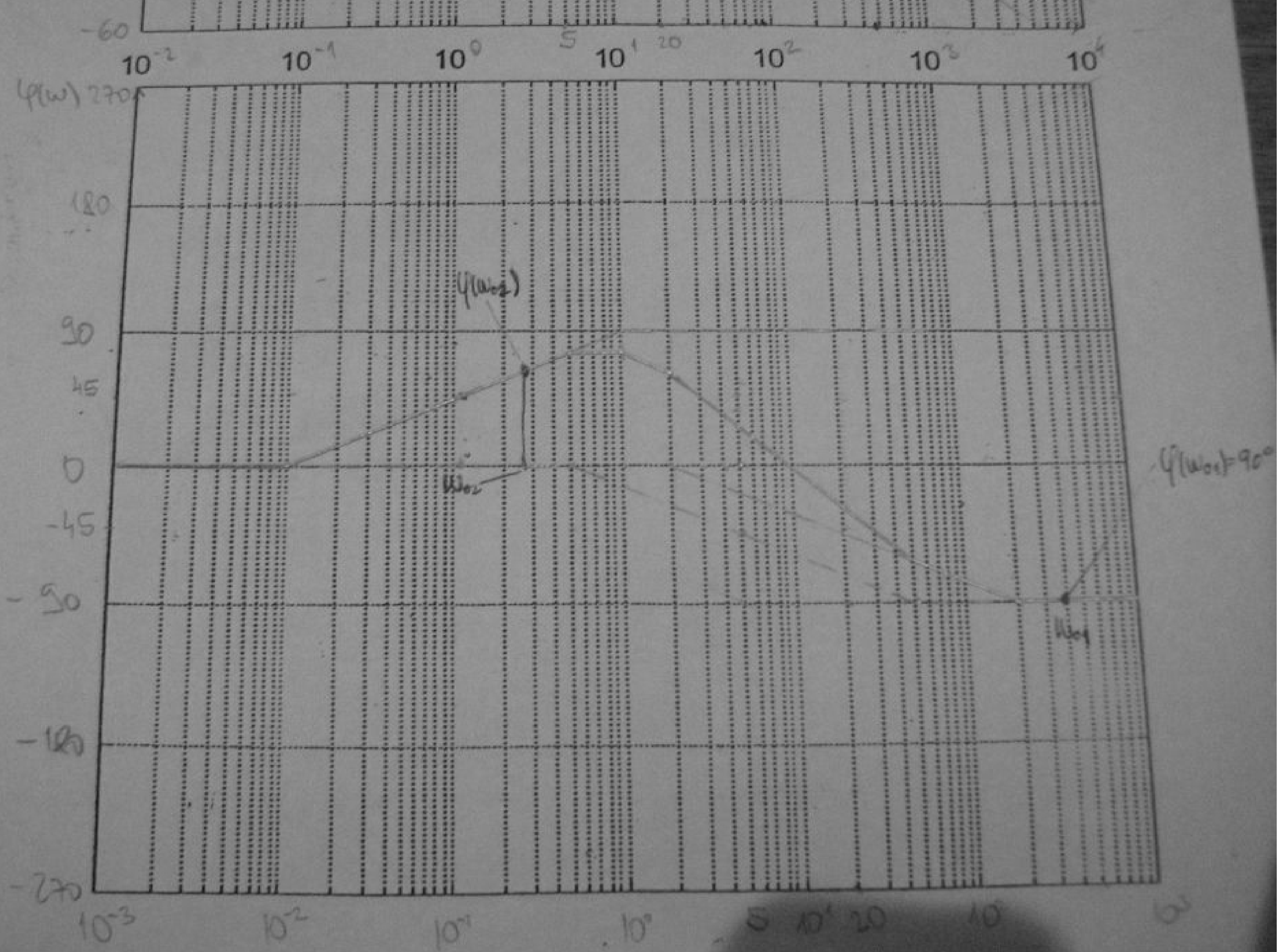
$$h'(0^+) = \left(\frac{100}{3}a - \frac{2}{3}\right)\cdot(-5)e^{0} - \left(\frac{100}{3}a - \frac{0{,}5}{3}\right)\cdot(-20)e^{-20t} < 0$$

$$-\frac{500a}{3} + \frac{10}{3} < \frac{10}{3} - \frac{2000}{3}a$$

⇒ $a < 0$ ⇒ h(t) ima podbačaj za $a < 0$!



A(w)
60
40
20
0
-6,02
-20
-40
-60
A2
A(w)
A(w02)≈0
A(w01)≈0
w02
w01
A1
A3
-14,14
-26,03
5
20
10^-2
10^-1
10^0
10^1
10^2
10^3
10^4


-60
5
20
10^{-2}
10^{-1}
10^{0}
10^{1}
10^{2}
10^{3}
10^{4}
$\varphi(\omega)$ 270
180
$\varphi(\omega_{02})$
90
45
0
ω_{02}
$\varphi(\omega_{01})$=90°
-45
-90
ω_{01}
-180
-270
10^{-3}
10^{-2}
10^{-1}
10^{0}
5
10^{1}
20
10^{2}
10^{3}