FREKVENCIJSKE KARAKTERISTIKE

30. studenog 2008
10:01

$$u(t) = U_m \sin(\omega t + \alpha_0) \longrightarrow \boxed{\text{SUSTAV}} \longrightarrow y(t) = Y_m \sin(\omega t + \alpha_y)$$

$$G(s) = \frac{B(s)}{N(s)} \qquad s = \sigma + j\omega$$

σ → prigušenje

definiramo: $\sigma = 0 \Rightarrow$ frekv. karakt.

$$s = j\omega$$

Primjer: (Zadatak 9.1.)

$$G(s) = \frac{100}{s(1+s/10)} \rightarrow s = j\omega \Rightarrow G(j\omega) = \frac{100}{j\omega(1+j\omega/10)}$$

slučajno ispravne forme za Bodeov dijagram

$$\left\{ G(j\omega) = \frac{10}{(2+j\omega)(10+j\omega)} \Rightarrow \right.$$

$$\left. = \frac{\cancel{10}}{2(1+j\frac{\omega}{2})\cdot\cancel{10}(1+j\frac{\omega}{10})} = \frac{1}{2}\,\frac{1}{(1+j\frac{\omega}{2})(1+j\frac{\omega}{10})} \right\}$$

$$A(\omega)_{[dB]} = 20 \log |G(j\omega)|$$

$$\log(a \cdot b) = \log a + \log b$$
$$\log\left(\frac{a}{b}\right) = \log a - \log b$$
$$z = \frac{z_1}{z_2 \cdot z_3} \rightarrow |z| = \frac{|z_1|}{|z_2| \cdot |z_3|}$$

$$A(\omega) = 20 \log|z_1| - 20 \log|z_2| - 20 \log|z_3|$$

$$\left\{ A(\omega) = 20 \log \frac{1}{2} + \cancel{20 \log 1} - 20 \log \sqrt{1 + \left(\frac{\omega}{2}\right)^2} - 20 \log \sqrt{1 + \left(\frac{\omega}{10}\right)^2} \right.$$

$\cancel{20\log 1} = 0$

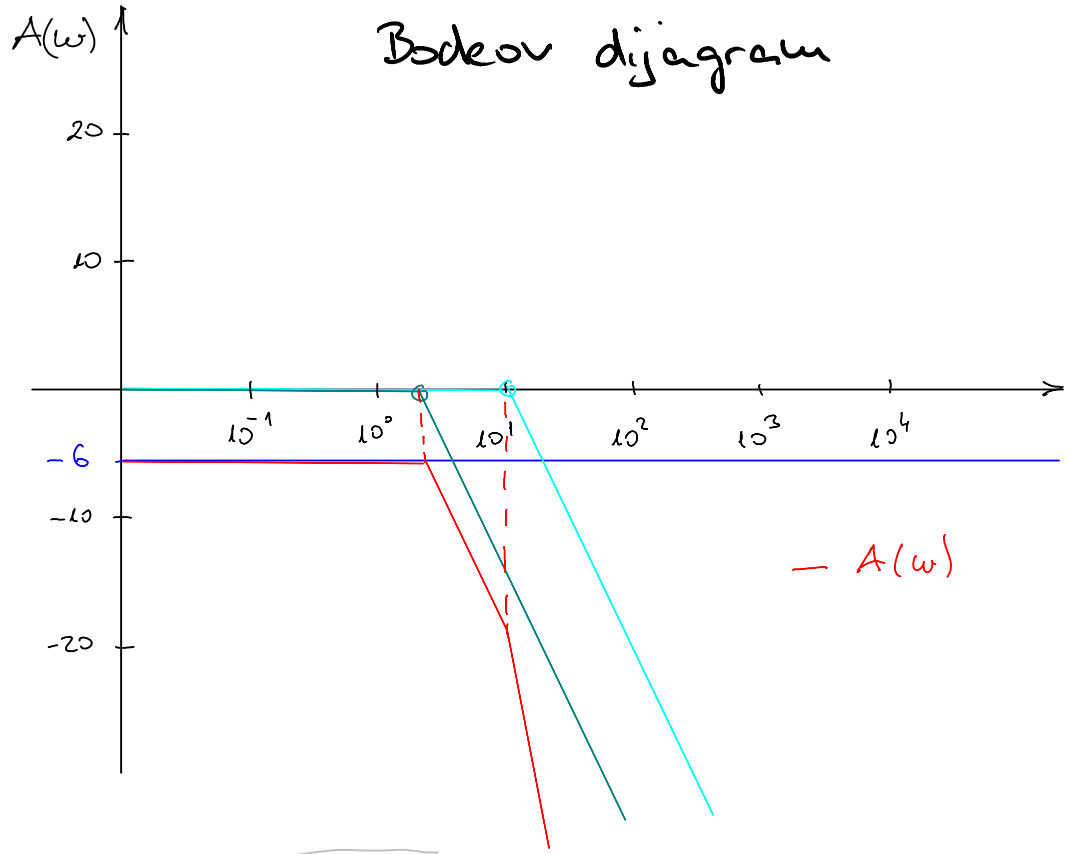
$$\varphi(\omega) = \cancel{\operatorname{arctg} \frac{0}{\frac{1}{2}}} - \operatorname{arctg} \frac{\omega}{2} - \operatorname{arctg} \frac{\omega}{10}$$

$\cancel{\operatorname{arctg}\frac{0}{1/2}} = 0$

$$10^x = 2$$
$$x = \log 2 = 0{,}301$$

$$\operatorname{arctg} 0 = 0$$
$$\operatorname{arctg} \infty = \frac{\pi}{2}$$
$$\operatorname{arctg}(-\infty) = -\frac{\pi}{2}$$

Bodeov dijagram

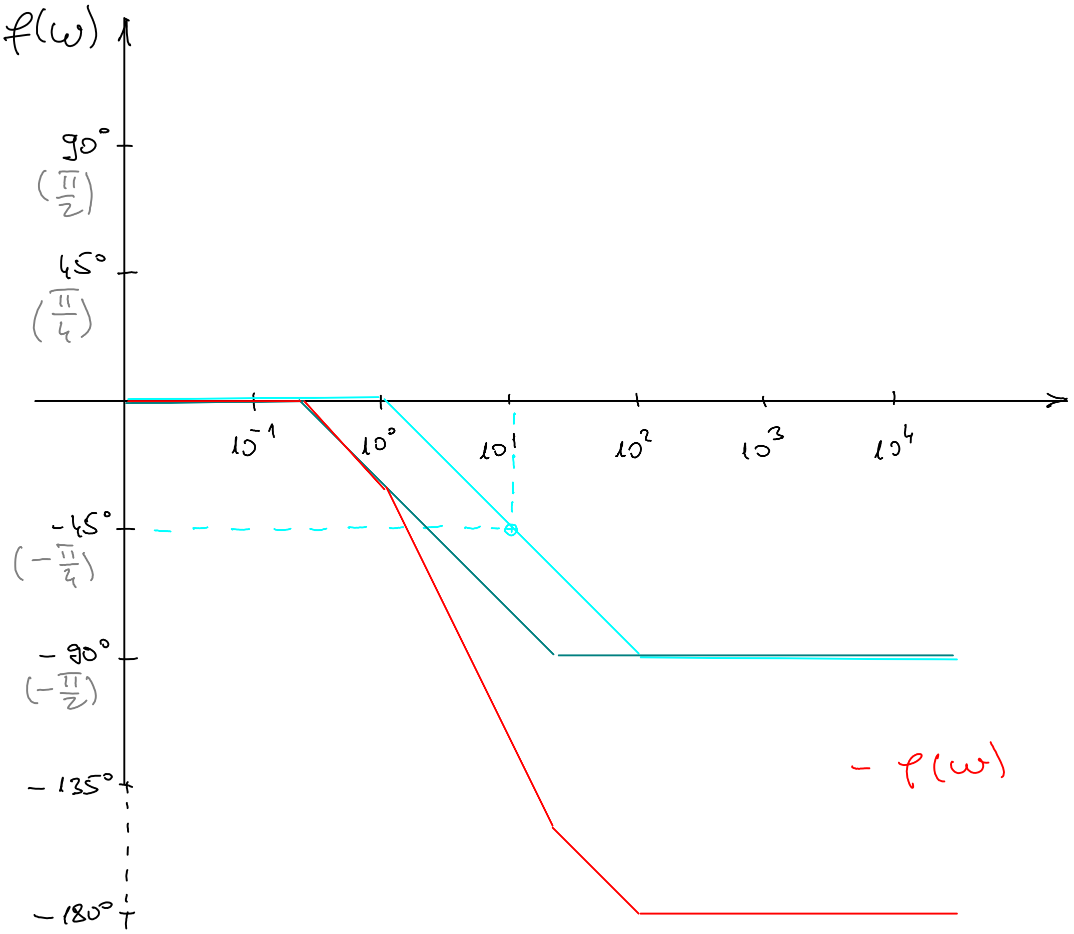


$$-20\log\sqrt{1+\left(\frac{\omega}{2}\right)^2}$$

− pada
+ raste

20 → nagib

2 → lomna frekvencija



Primjer: (Zadatak 9.2.)

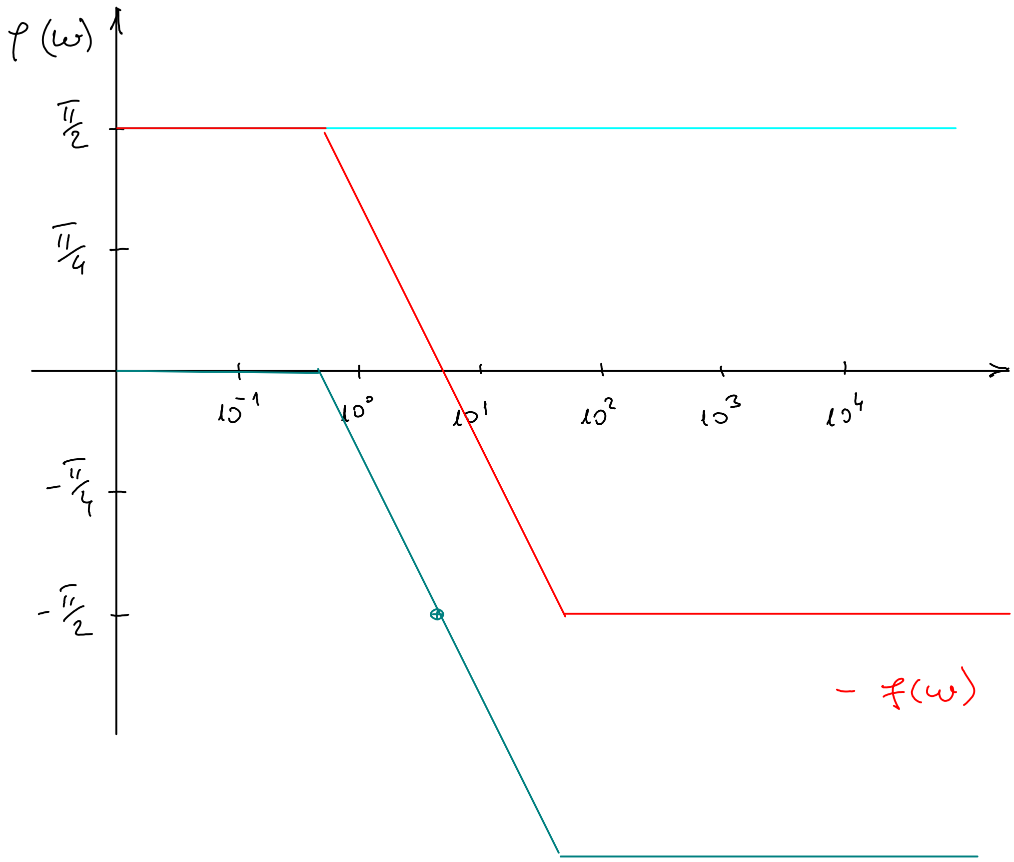
$$G(s) = \frac{ks}{(1+sT)^2} \qquad k = 10, \quad T = 0{,}2$$

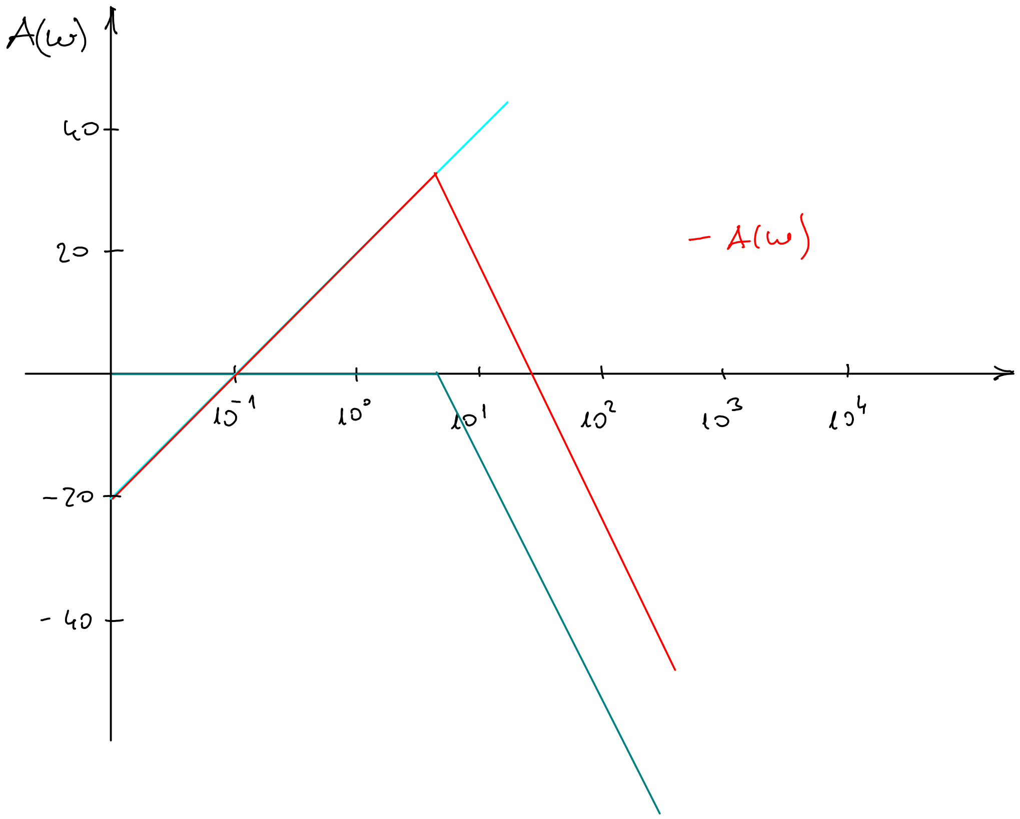
$$G(s) = \frac{10s}{(1+0{,}2s)^2} \rightarrow G(j\omega) = \frac{j\,\frac{\omega}{1/10}}{\left(1+j\,\frac{\omega}{1/0{,}2}\right)^2} = \frac{j\,\frac{\omega}{10^{-1}}}{\left(1+j\,\frac{\omega}{5}\right)^2}$$

$$A(\omega) = 20\log\frac{\omega}{10^{-1}} - 40\log\sqrt{1+\left(\frac{\omega}{5}\right)^2}$$

$$\varphi(\omega) = \operatorname{arctg}\frac{\frac{\omega}{10^{-1}}}{0} - 2\operatorname{arctg}\frac{\omega}{5} = \frac{\pi}{2} - 2\operatorname{arctg}\frac{\omega}{5}$$

$\log 5 = 0{,}69$





Primjer: (Zadatak 9.4.)

$$G(s) = \frac{20\,(s-1)}{(s+1)(s+10)}$$

$$G(j\omega) = \frac{\cancel{-20}\,2\,(1-j\,\frac{\omega}{1})}{(1+j\,\frac{\omega}{1})(1+j\,\frac{\omega}{10})\cdot\cancel{10}} = -2\cdot\frac{(1-j\,\omega/1)}{(1+j\,\omega/1)(1+j\,\omega/10)}$$

$$A(\omega) = 20\log 2 + \cancel{20\log\sqrt{1+(\omega/1)^2}} - \cancel{20\log\sqrt{1+(\omega/1)^2}} - 20\log\sqrt{1+(\omega/10)^2}$$

$|-2| = 2!$

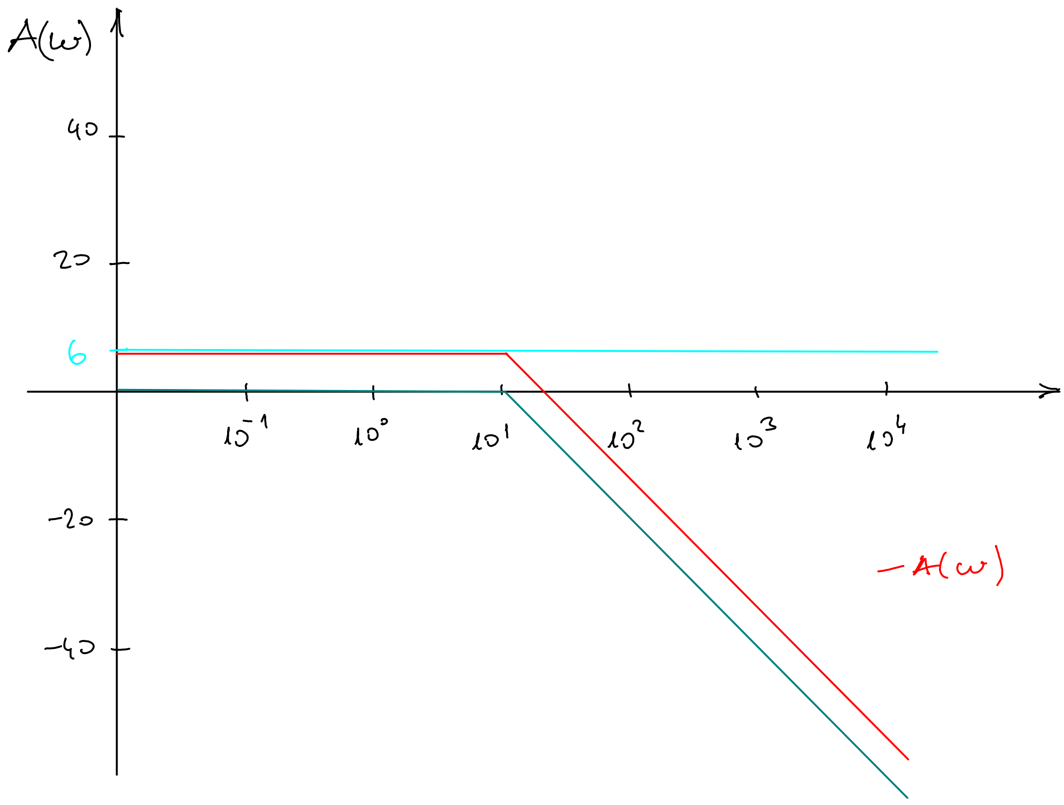
$$A(\omega) = 20\log 2 - 20\log\sqrt{1+(\omega/10)^2} \;*$$

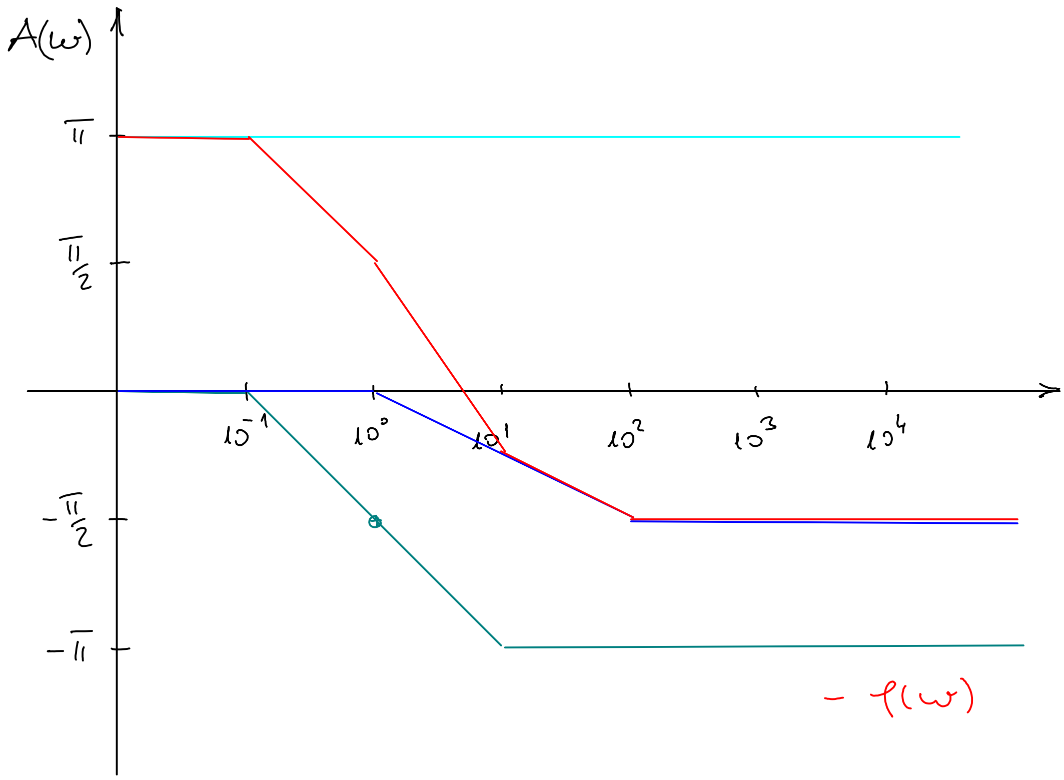
$$\varphi(\omega) = \text{arctg}\,\frac{0}{-2} + \text{arctg}\left(\frac{-\omega/1}{1}\right) - \text{arctg}\left(\frac{\omega}{1}\right) - \text{arctg}\left(\frac{\omega}{10}\right)$$

$$\varphi(\omega) = \pi - 2\cdot\text{arctg}\left(\frac{\omega}{1}\right) - \text{arctg}\left(\frac{\omega}{10}\right)$$

$*$ $\left.\begin{array}{l} Re > 0 \\ Im < 0 \end{array}\right\} \Rightarrow$ IV. kvadr. $\Rightarrow$ „−" se može izvaditi van

$20\log 2 = 6$


A(ω)
40
20
6
-20
-40
10⁻¹
10⁰
10¹
10²
10³
10⁴
− A(ω)


A(ω)
π
π/2
-π/2
-π
10⁻¹
10⁰
10¹
10²
10³
10⁴
− φ(ω)

Primjeri :

$$G(s) = \frac{20(1-s)}{(s+1)(s+10)}$$

$$G(j\omega) = \frac{2(1-j\omega/1)}{(1+j\omega/1)(1+j\omega/10)}$$

$$\varphi(\omega) = \cancel{\mathrm{arctg}\,\frac{0}{2}}\ \emptyset + \mathrm{arctg}\left(\frac{-\omega/1}{1}\right) - \mathrm{arctg}\left(\frac{\omega}{1}\right) - \mathrm{arctg}\,\frac{\omega}{10}$$

„Problem" sa dodavanjem π : kada je $Re < 0$!

$$G(s) = \frac{20(1-s)}{-10(1+s)(1+s)}$$

$$G(j\omega) = \frac{-20(1-j\omega/1)}{-10(1+j\omega/1)(1-j\omega/10)}$$

$$\varphi(\omega) = \mathrm{arctg}\,\frac{0}{2} + \mathrm{arctg}\left(\frac{-\omega/1}{1}\right) - \mathrm{arctg}\left(\frac{\omega}{1}\right) - \mathrm{arctg}\left(\frac{-\omega/10}{1}\right)$$

Nyquistov dijagram

Željena forma prijenosne fcije za crtanje Nyquista

$$G(j\omega) = \underbrace{Re\{G(j\omega)\}}_{R(\omega)} + \underbrace{Im\{G(j\omega)\}}_{I(\omega)}$$

$R(\omega=0)\ldots \rightarrow \lim\limits_{\omega\to 0^+}$ (zdesna jer crtamo samo za pozitivne frekvencije)

$I(\omega=0)\ldots$

$R(\omega=\infty)\ldots$

$I(\omega=\infty)\ldots$

$R(\omega=?) = 0$

$I(\omega=?) = 0$

$$z = \frac{a+bj}{c-dj}\cdot\frac{c+dj}{c+dj} = \frac{(a+bj)(c+dj)}{c^2+d^2}$$

Primjer: (Zadatak 9.1.)

$$G(s) = \frac{100}{s\left(1+\frac{s}{10}\right)}$$

$$G(j\omega) = \frac{100}{j\omega\cdot\frac{1}{10}(10+j\omega)} = \frac{1000}{j\omega(10+j\omega)} = \frac{1000}{-\omega^2+10j\omega} \cdot \frac{-\omega^2-10j\omega}{-\omega^2-10j\omega}$$

$$= \frac{-1000\omega^2-10000j\omega}{\omega^4+100\omega^2}$$

$$G(j\omega) = \underbrace{\frac{-1000}{\omega^2+100}}_{R(\omega)} + j\cdot\underbrace{\frac{-10\,000}{\omega^3+100\omega}}_{I(\omega)}$$

$$R(\omega) = \frac{-1000}{\omega^2+100}$$

$$I(\omega) = \frac{-10000}{\omega^3+100\omega}$$

$$R(\omega=0) = -10$$

$$I(\omega=0) = -\infty$$

$$R(\omega=\infty) = 0$$

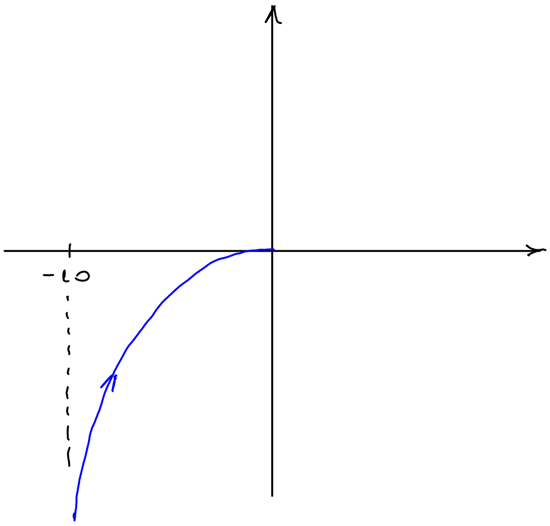
$$I(\omega=\infty) = 0$$

($R(\omega) = \frac{\omega^2+1}{\dots}$ $\quad \omega^2+1=0 \Rightarrow \cancel{\omega=\pm j}$

imaginarne zanemarujemo

$= \frac{\omega^2-1}{\dots}$ $\quad \omega^2=1 \Rightarrow \omega_1=1 \quad \cancel{\omega_2=-1}$

negativne zanemarujemo)



Primjer: (Zadatak 9.4.)

$$G(s) = \frac{20(s-1)}{(s+1)(s+10)}$$

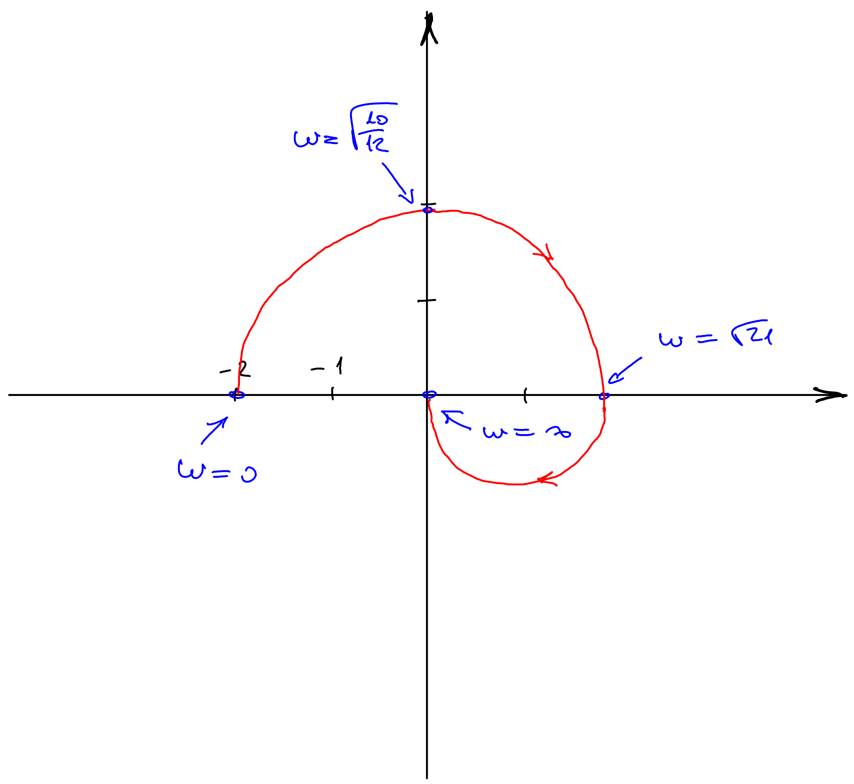
$$R(\omega) = \frac{20(12\omega^2-10)}{(10-\omega^2)^2+(11\omega)^2}, \quad I(\omega) = \frac{20(-\omega^3+21\omega)}{(10-\omega^2)^2+(11\omega)^2}$$

$$R(\omega=0) = -2$$
$$I(\omega=0) = 0$$

$$R(\omega=\infty) = 0$$
$$I(\omega=\infty) = 0$$

$$R(\omega_1) = 0 \Rightarrow \cancel{20}(12\omega_1^2-10) = 0$$
$$12\omega_1^2 = 10$$
$$\omega_1 = \sqrt{\tfrac{10}{12}} \Rightarrow I(\omega_1) = 1,99$$

$$I(\omega_2) = 0 \Rightarrow -\omega_2^3 + 21\omega_2 = 0$$
$$\omega_2^2 = 21$$
$$\omega_2 = \sqrt{21} \Rightarrow R(\omega_2) = 1,818$$



Polovi, nule i vremenski odziv

30. studenog 2008
13:23

PT_2 član : $$G(s) = \frac{K}{\frac{1}{\omega_n^2}s^2 + \frac{2\xi}{\omega_n}s + 1}$$

K – statičko pojačanje

$$s_{p_{1,2}} = \frac{-\frac{2\xi}{\omega_n} \pm \sqrt{\frac{4\xi^2}{\omega_n^2} - \frac{4}{\omega_n^2}}}{\frac{2}{\omega_n^2}} = \frac{-\frac{\cancel{2}\xi}{\cancel{\omega_n}} \pm \frac{\cancel{2}}{\cancel{\omega_n}}\sqrt{\xi^2-1}}{\frac{\cancel{2}}{\omega_n^{\cancel{2}}}} = -\xi\omega_n \pm \omega_n\sqrt{\xi^2-1}$$

$$s = -\sigma \pm j\omega_d$$
$$s = -\xi\omega_n \pm \omega_n\sqrt{\xi^2-1}$$
$$s = -\xi\omega_n \pm \underbrace{j\omega_n\sqrt{1-\xi^2}}_{\omega_d}$$

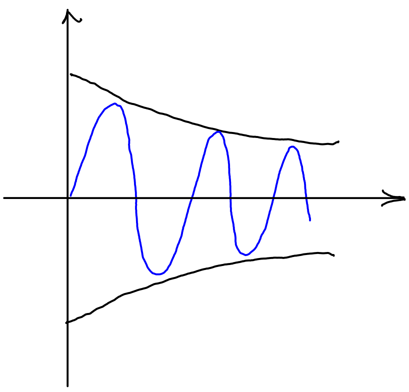
kompleksni polovi uvijek dolaze u kompleksno – konjugiranim parovima

$$s_{p_{1,2}} = -\xi\omega_n \pm j\omega_n\sqrt{1-\xi^2}$$

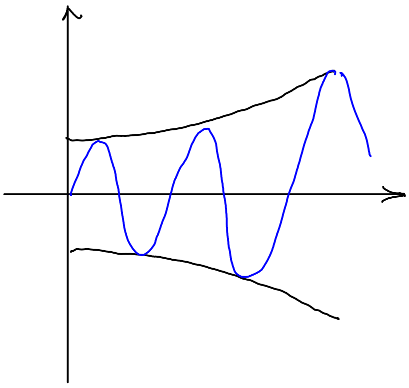
$\omega_n > 0$

$0 < \xi < 1 \rightarrow$

sustav je oscilatoran i prigušen

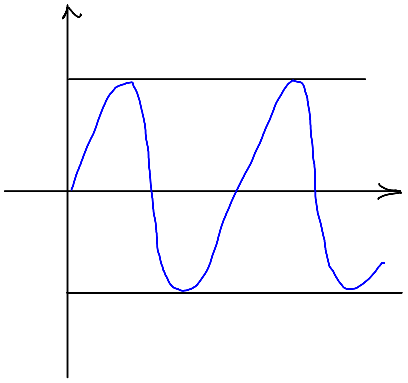


$-1 < \xi < 0$

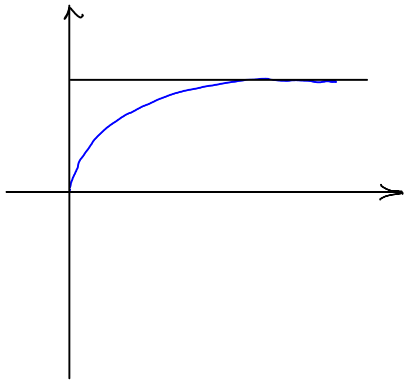


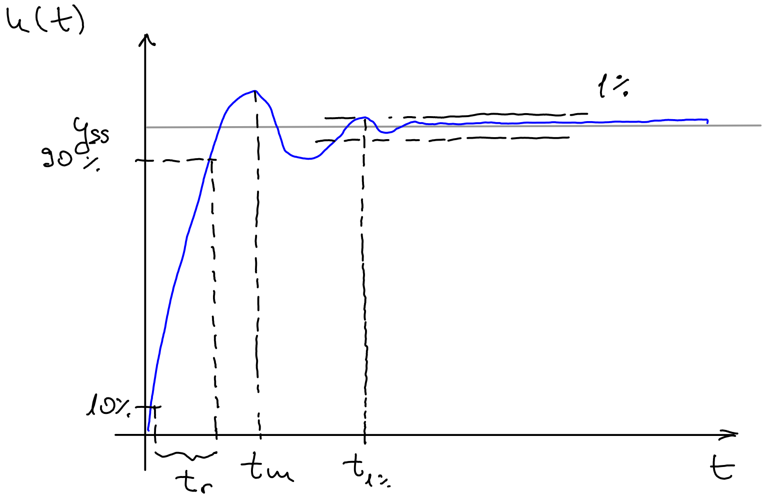
→ sustav je oscilatoran i raspirujuć ?

$\xi = 0$



$\xi = 1$



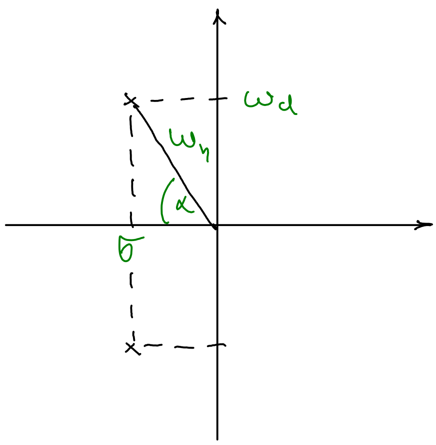


$$\sigma_u[\%] = 100\, e^{\frac{-\pi \xi}{\sqrt{1-\xi^2}}}$$

$$t_{1\%} = \frac{4,6}{\xi \omega_n}$$

$$t_r = \frac{1,8}{\omega_n}$$

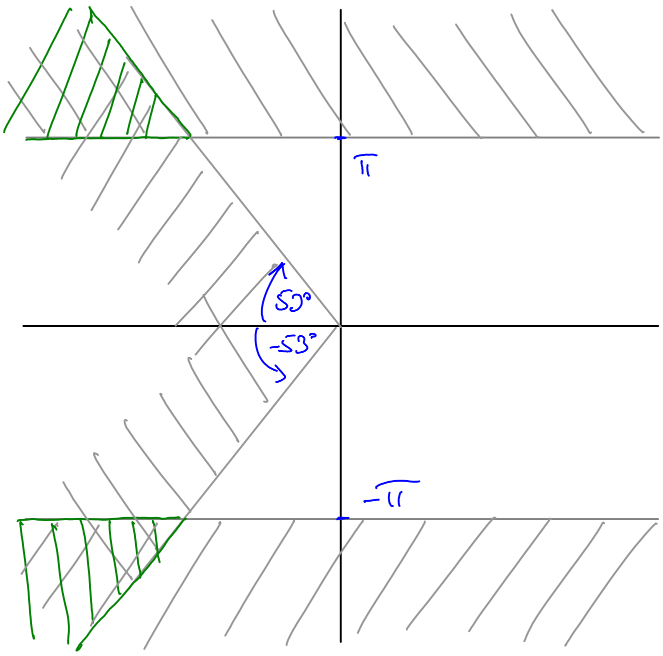
$$t_m = \frac{\pi}{\underbrace{\omega_n \sqrt{1-\xi^2}}_{\omega_d}}$$



Primjer: (Zadatak s 2. MI)

$$t_m < 1$$

$$\frac{\pi}{\omega_d} < 1 \rightarrow \frac{\omega_d}{\pi} > 1 \Rightarrow \omega_d > \pi$$



$$\sigma_m < 0{,}1$$

$$e^{\frac{-\pi \xi}{\sqrt{1-\xi^2}}} < 0{,}1 \quad / \ln$$

$$\frac{-\pi \xi}{\sqrt{1-\xi^2}} < -2{,}3$$

$$\xi = \pm 0{,}5912$$

$\hookrightarrow \alpha = \pm 53°$

Primjer: (Zadatak iz DZ)

$$u(t) = 2 \sin(\omega_0 t + \varphi) \rightarrow A = 2$$

$$y(t) = 2 \sin(\omega_0 t) \rightarrow A = 2$$

$\Rightarrow |G(j\omega)|_{\omega_0} = 1 \Rightarrow \omega_0$

Amplitudna frekv. karakt.

$\omega_0 = +30{,}93$ rad/s

$\omega_0 = \cancel{-30{,}93}$ rad/s

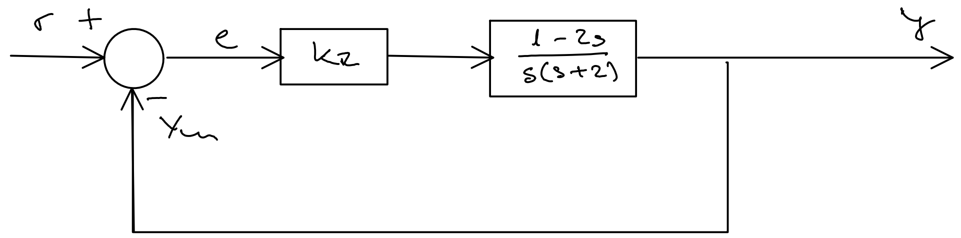
$$G(s) = 32 \frac{s+1}{(s+2)(s+8)}$$

$$Y_m = U_m \cdot |G(j\omega)|_{\omega=\omega_0}$$

$$\measuredangle y = \measuredangle u + \measuredangle G(j\omega)|_{\omega=\omega_0}$$

$\hookrightarrow$ ovo se može odrediti i iz Bode-a

Primjer: (1. zadatak iz 4. D.Z.)



$$k_r = \frac{1}{2}$$

$$G(s) = \frac{\frac{1-2s}{s(s+2)}}{\frac{2s(s+2)+1-2s}{2s(s+2)}} = \frac{1-2s}{2s^2+2s+1}$$

$$H(s) = \frac{1}{s}G(s) = \frac{1-2s}{s(2s^2+2s+1)} = \frac{C_{11}}{s} + \frac{As+B}{2s^2+2s+1}$$

$$C_{11} = H(s)\cdot s\big|_{s=0} = 1$$

$$H(s) = \frac{1}{s} + \frac{As+B}{2s^2+2s+1} = \frac{1-2s}{s(2s^2+2s+1)}$$

$$= \frac{2s^2+2s+1+As^2+Bs}{s(2s^2+2s+1)} = \frac{s^2(2+A)+s(2+B)+1}{s(2s^2+2s+1)}$$

$$2+A=0 \Rightarrow A=-2$$

$$2+B=-2 \Rightarrow B=-4$$

$$H(s) = \frac{1}{s} + \frac{-2s-4}{2s^2+2s+1} = \frac{1}{s} - \frac{s+2}{s^2+s+\frac{1}{2}}$$

$$\frac{s+2}{s^2+s+\frac{1}{2}} = \frac{s+\frac{1}{2}}{\left(s+\frac{1}{2}\right)^2+\frac{1}{4}} + \frac{3}{\cancel{2}}\cdot\cancel{2}\,\frac{\frac{1}{2}}{\left(s+\frac{1}{2}\right)^2+\frac{1}{4}} \;\circ\!\!-\!\!\bullet$$

$$e^{-\frac{1}{2}t}\left(\cos\left(\tfrac{1}{2}t\right)+3\sin\left(\tfrac{1}{2}t\right)\right)$$

$$\frac{s}{s^2+\omega^2} \;\circ\!\!-\!\!\bullet\; \cos(\omega t)$$

$$\frac{\omega}{s^2+\omega^2} \;\circ\!\!-\!\!\bullet\; \sin(\omega t)$$

$$\frac{s+a}{(s+a)^2+\omega^2} \;\circ\!\!-\!\!\bullet\; \cos(\omega t)\,e^{-at}$$

$$h(t) = 1 - e^{-\frac{1}{2}t}\left(\cos\left(\tfrac{1}{2}t\right)+3\sin\left(\tfrac{1}{2}t\right)\right)$$

$$g(t) = \dot{h}(t) = \left(2\sin\left(\tfrac{1}{2}t\right)-\cos\left(\tfrac{1}{2}t\right)\right)e^{-\frac{1}{2}t}$$

$$u(t) = 1 - e^{-\frac{1}{2}t}\left(\cos\left(\tfrac{1}{2}t\right) + 3\sin\left(\tfrac{1}{2}t\right)\right)$$

$$g(t) = \dot{u}(t) = e^{-\frac{1}{2}t}\left(2\sin\left(\tfrac{1}{2}t\right) - \cos\left(\tfrac{1}{2}t\right)\right) = 0$$

$$g(t) = 0 \longrightarrow$$

$$2\sin\left(\tfrac{1}{2}t\right) - \cos\left(\tfrac{1}{2}t\right) = 0$$

$$2\sin\left(\tfrac{1}{2}t\right) = \cos\left(\tfrac{1}{2}t\right) \quad / : 2\cos\left(\tfrac{1}{2}t\right)$$

$$\operatorname{tg}\left(\tfrac{1}{2}t\right) = \tfrac{1}{2}$$

PERIODIČNOST: $\operatorname{tg}\left(\tfrac{1}{2}t + k\pi\right) = \tfrac{1}{2}$, $k \in \mathbb{Z}$

$$\tfrac{1}{2}t + k\pi = \operatorname{arctg}\tfrac{1}{2}$$

$$t = 0{,}927 + 2k\pi$$

$k = 0 \rightarrow t = 0{,}927$

$k = 1 \rightarrow t_1 =$

$k = 2 \rightarrow t_2 =$

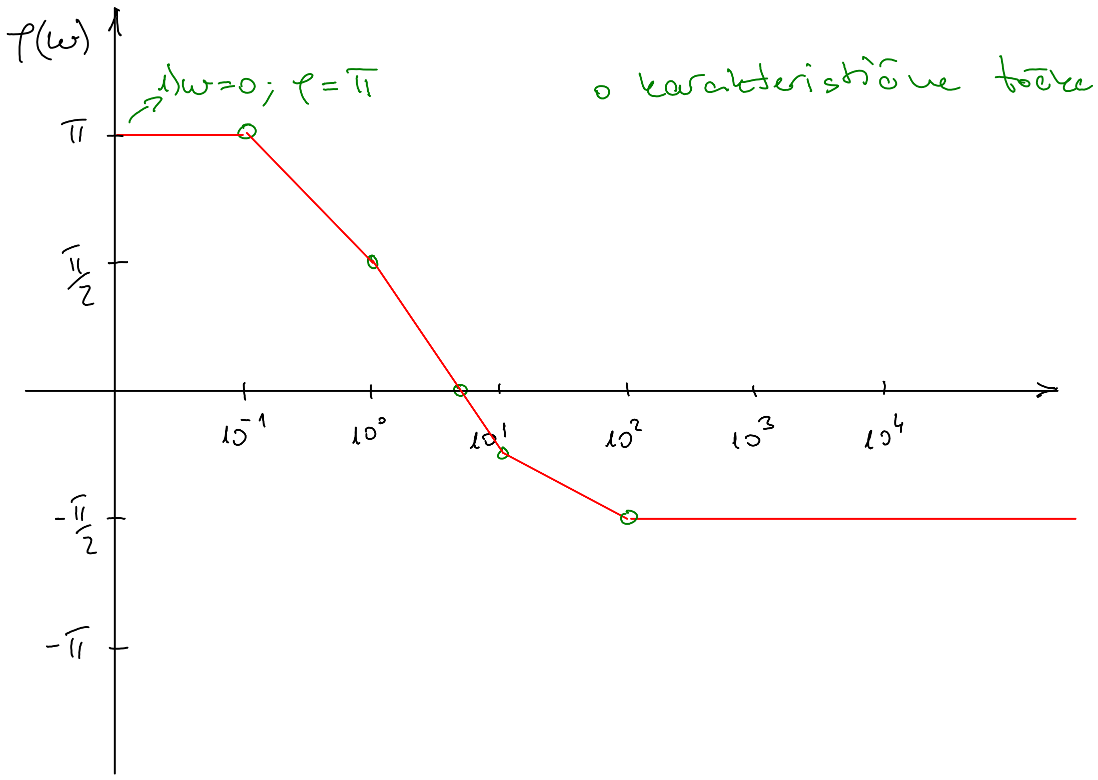
$$\ddot{u}(t) = \dot{g}(t)$$

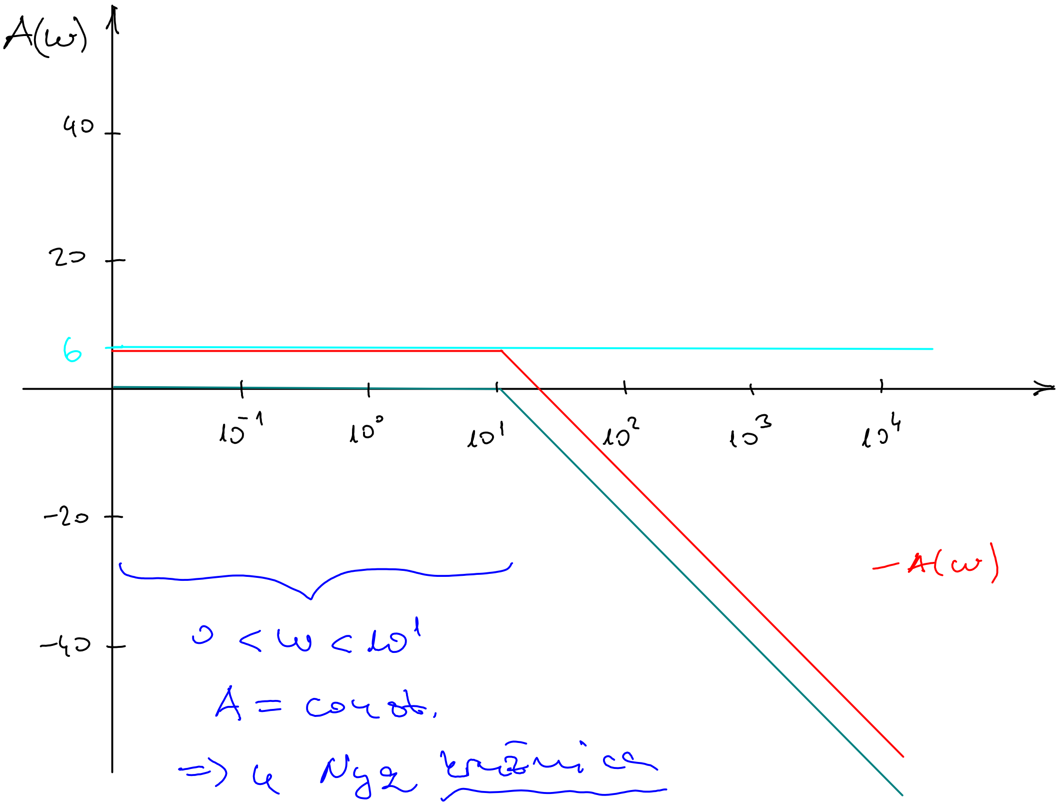
$\ddot{u}(t) < 0 \Rightarrow$ MAX

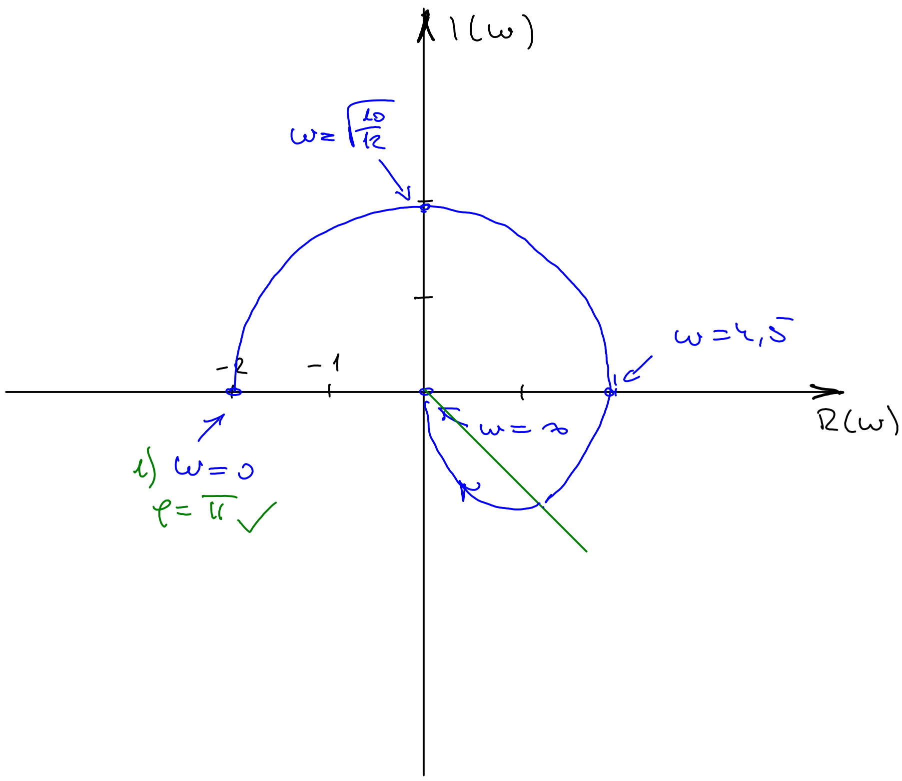
$\ddot{u}(t) > 0 \Rightarrow$ MIN

$\ddot{u}(t) = 0 \Rightarrow$ PREGIB

AD. NYQUIST: KUTEVI UPADA




A(ω)
40
20
6
-20
-40
10^{-1}
10^0
10^1
10^2
10^3
10^4
— A(ω)
0 < ω < 10^1
A = const.
=> u Nyq kružnica


I(ω)
R(ω)
ω = $\sqrt{\frac{10}{12}}$
ω = 4,5
ω = ∞
-2
-1
1) ω = 0
φ = π