Službeni šalabahter za kolegij Automatsko upravljanje

1. Tablica Laplaceove transformacije:

$f(t)$	$F(s)$
$t^n e^{\lambda t}$	$\frac{n!}{(s-\lambda)^{n+1}}$
$e^{\sigma t}\sin(\omega t)$	$\frac{\omega}{(s-\sigma)^2+\omega^2}$
$e^{\sigma t}\cos(\omega t)$	$\frac{s-\sigma}{(s-\sigma)^2+\omega^2}$
$t^n f(t)$	$(-1)^n \frac{\mathrm{d}^n F(s)}{\mathrm{d}s^n}$
$\frac{\mathrm{d}^n f(t)}{\mathrm{d}t^n}$	$s^n F(s) - s^{n-1} f(0^-) - s^{n-2} f'(0^-) - \ldots - f^{(n-1)}(0^-)$
$\int_0^t f(\tau) d\tau$	$\frac{F(s)}{s}$
$f(t-a)S(t-a),\ a>0$	$e^{-as}F(s)$
$\frac{1}{a} f(\frac{t}{a}),\ a>0$	$F(as)$

2. Tablica $\mathcal{L}$ i $\mathcal{Z}$-transformacija:

$f(t)$	$F(s)$	$f(kT)$	$F(z)$
$\delta(t)$	1	$1,\ k=0$ $0,\ k\neq 0$	1
1	$\frac{1}{s}$	1	$\frac{1}{1-z^{-1}}$
t	$\frac{1}{s^2}$	kT	$\frac{Tz^{-1}}{(1-z^{-1})^2}$
e^{-at}	$\frac{1}{s+a}$	e^{-akT}	$\frac{1}{1-e^{-aT}z^{-1}}$
te^{-at}	$\frac{1}{(s+a)^2}$	kTe^{-akT}	$\frac{Te^{-aT}z^{-1}}{(1-e^{-aT}z^{-1})^2}$
$1-e^{-at}$	$\frac{a}{s(s+a)}$	$1-e^{-akT}$	$\frac{(1-e^{-aT})z^{-1}}{(1-z^{-1})(1-e^{-aT}z^{-1})}$
$\sin at$	$\frac{a}{s^2+a^2}$	$\sin akT$	$\frac{(\sin aT)z^{-1}}{1-(2\cos aT)z^{-1}+z^{-2}}$
$\cos at$	$\frac{s}{s^2+a^2}$	$\cos akT$	$\frac{1-(\cos aT)z^{-1}}{1-(2\cos aT)z^{-1}+z^{-2}}$
$e^{-at}\sin bt$	$\frac{b}{(s+a)^2+b^2}$	$e^{-akT}\sin bkT$	$\frac{e^{-aT}(\sin bT)z^{-1}}{1-2e^{-aT}(\cos bT)z^{-1}+e^{-2aT}z^{-2}}$
$e^{-at}\cos bt$	$\frac{s+a}{(s+a)^2+b^2}$	$e^{-akT}\cos bkT$	$\frac{1-z^{-1}e^{-aT}\cos bT}{1-2e^{-aT}(\cos bT)z^{-1}+e^{-2aT}z^{-2}}$

3. Hurwitzov kriterij stabilnosti:

$$P(s) = a_n s^n + a_{n-1}s^{n-1} + \ldots + a_1 s + a_0,$$

Uz $a_n > 0$:

- $a_i > 0$, $\forall\ i$;

- sljedećih $n-1$ determinanti su pozitivne:

$$D_1 = a_1 > 0,$$
$$D_2 = \begin{vmatrix} a_1 & a_0 \\ a_3 & a_2 \end{vmatrix} > 0,$$
$$D_3 = \begin{vmatrix} a_1 & a_0 & 0 \\ a_3 & a_2 & a_1 \\ a_5 & a_4 & a_3 \end{vmatrix} > 0,$$
$$\vdots$$
$$D_{n-1} = \begin{vmatrix} a_1 & a_0 & \dots & 0 \\ a_3 & a_2 & \dots & \vdots \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & a_{n-1} \end{vmatrix} > 0.$$

4. Veza karakterističnih veličina u vremenskom području s karakterističnim veličinama u frekvencijskom području:

$t_m = \frac{\pi}{\omega_n\sqrt{1-\zeta^2}}$ $\sigma_m[\%] = 100e^{-\frac{\zeta\pi}{\sqrt{1-\zeta^2}}}$ $t_{1\%} = \frac{4.6}{\zeta\omega_n}$ $t_r = \frac{1.8}{\omega_n}$	$\gamma[^\circ] \approx 70 - \sigma_m[\%]$, za $0.3 < \zeta < 0.8$ $\omega_c \approx \frac{3}{t_m}$, za $0.3 < \zeta < 0.8$ $\omega_r = \omega_n\sqrt{1-2\zeta^2}$

5. Ziegler-Nichols:
ZN1 - metoda ruba stabilnosti
ZN2 - metoda prijelazne funkcije

varijanta	tip regulatora	K_R	T_I	T_D
ZN1	P	$0.5K_{Rkr}$	$-$	$-$
ZN1	PI	$0.45K_{Rkr}$	$0.85T_{kr}$	$-$
ZN1	PID	$0.6K_{Rkr}$	$0.5T_{kr}$	$0.12T_{kr}$
ZN2	P	$\frac{t_a}{t_z K_s}$	$-$	$-$
ZN2	PI	$0.9\frac{t_a}{t_z K_s}$	$3.33t_z$	$-$
ZN2	PID	$1.2\frac{t_a}{t_z K_s}$	$2t_z$	$0.5t_z$

6. Preporuke za odabir perioda uzorkovanja:

$$T = (0.17 \div 0.34)\frac{1}{\omega_c}$$
$$T = (\frac{1}{10} \div \frac{1}{4})t_r$$

7. Postupci diskretizacije:

Postupak diskretizacije	Provedba
Očuvanje svojstava kontinuirane težinske funkcije	$G(z) = \mathcal{Z}\{G(s)\}$
ZOH	$G(z) = (1 - z^{-1})\mathcal{Z}\{\frac{G(s)}{s}\}$
Tustinov postupak	$s = \frac{2}{T}\frac{z-1}{z+1}$
Eulerova unazadna diferencija	$s = \frac{z-1}{Tz}$
Eulerova unaprijedna diferencija	$s = \frac{z-1}{T}$

8. Juryjev kriterij stabilnosti:

$$f(z) = a_0 + a_1 z + a_2 z^2 + ... + a_n z^n$$

Redak	z^0	z^1	z^2	...	z^{n-k}	...	z^{n-2}	z^{n-1}	z^n
1	a_0	a_1	a_2	...	a_{n-k}	...	a_{n-2}	a_{n-1}	a_n
2	a_n	a_{n-1}	a_{n-2}	...	a_k	...	a_2	a_1	a_0
3	b_0	b_1	b_2	...	b_{n-k}	...	b_{n-2}	b_{n-1}	
4	b_{n-1}	b_{n-2}	b_{n-3}	...	b_{k-1}	...	b_1	b_0	
5	c_0	c_1	c_2	...	c_{n-k}	...	c_{n-2}		
6	c_{n-2}	c_{n-3}	c_{n-4}	...	c_{k-2}	...	c_0		
$\vdots$				$\vdots$		$\vdots$			
$2n-5$	p_0	p_1	p_2	p_3					
$2n-4$	p_3	p_2	p_1	p_0					
$2n-3$	q_0	q_1	q_2						

$$b_k = \begin{vmatrix} a_0 & a_{n-k} \\ a_n & a_k \end{vmatrix}, \quad c_k = \begin{vmatrix} b_0 & b_{n-k-1} \\ b_{n-1} & b_k \end{vmatrix}, \quad d_k = \begin{vmatrix} c_0 & c_{n-k-2} \\ c_{n-2} & c_k \end{vmatrix},$$

$$\vdots$$

$$q_0 = \begin{vmatrix} p_0 & p_3 \\ p_3 & p_0 \end{vmatrix}, \qquad q_1 = \begin{vmatrix} p_0 & p_2 \\ p_3 & p_1 \end{vmatrix}, \qquad q_2 = \begin{vmatrix} p_0 & p_1 \\ p_3 & p_2 \end{vmatrix}$$

Nužni i dovoljni uvjeti stabilnosti:

- Uvjet a) : $f(1) > 0, \ (-1)^n f(-1) > 0$
- Uvjet b) : $|a_0| < |a_n|, \ |b_0| > |b_{n-1}|, \ |c_0| > |c_{n-2}|, \ |d_0| > |d_{n-3}|, ... \ |q_0| > |q_2|$

9. Bilinearna transformacija: $z = \frac{1+w}{1-w}$

10. Modificirana bilinearna transformacija: $z = \dfrac{1 + \Omega\frac{T}{2}}{1 - \Omega\frac{T}{2}}$