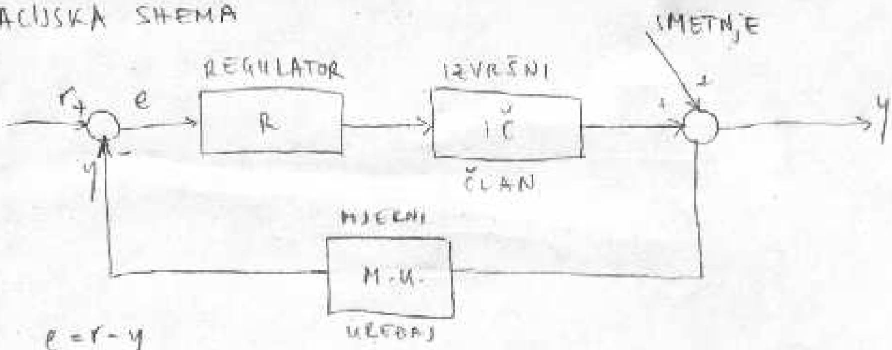
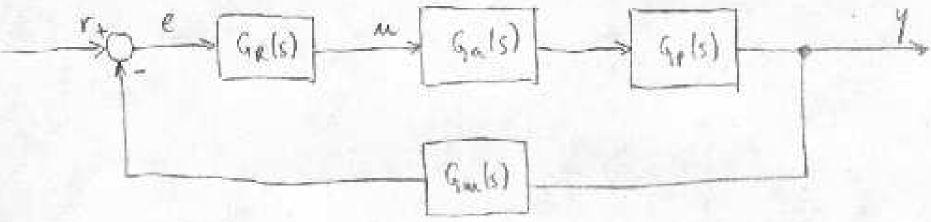
REGULACIJSKA SHEMA



$e = r - y$

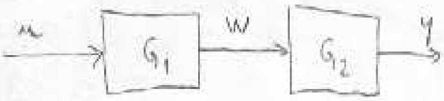
ZADATAK 1. 1. MI. 2007./2008.

zadano je: IZVRŠNI ČLAN $G_a(s)$
REGULATOR $G_R(s)$ u
MJERNI ČLAN $G_m(s)$
OBJEKT UPRAVLJANJA $G_P(s)$ y

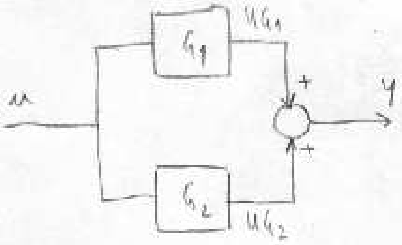


BLOKOVSKI DIJAGRAMI

$Y = U \cdot G$



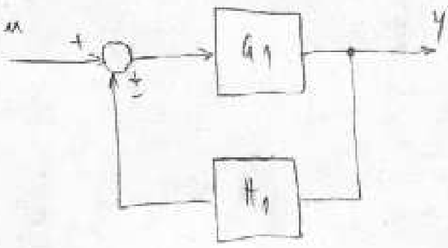
$$\left.\begin{aligned} W &= U \cdot G_1 \\ Y &= W \cdot G_2 \end{aligned}\right\} \begin{aligned} Y &= U \cdot G_1 \cdot G_2 \\ G &= G_1 \cdot G_2 \end{aligned}$$ - kaskadni spoj



$Y = UG_1 + UG_2$

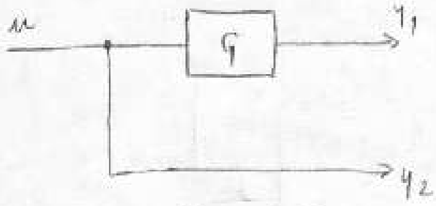
$Y = U(G_1 + G_2)$ - paralelni spoj

$G = G_1 + G_2$

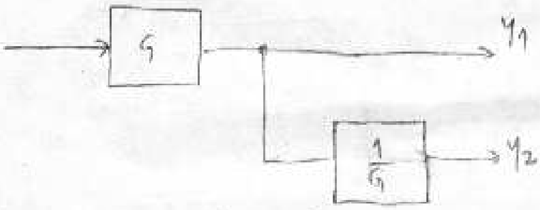


$$G = \frac{G_1}{1 \mp G_1 H_1}$$ - povratna veza

ZADATAK 2.



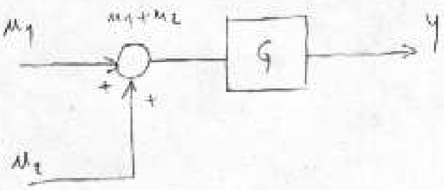
$\Rightarrow$



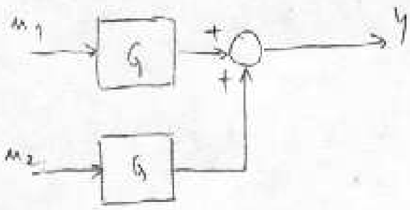
$$Y_1 = U \cdot G$$
$$Y_2 = U$$

$$Y_1 = U \cdot G$$
$$Y_2 = U \cdot G \cdot \frac{1}{G} = U$$

ZADATAK 3.

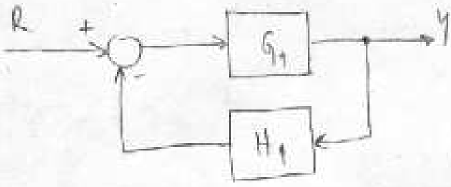


$$Y = (U_1 + U_2)G$$
$$= U_1 G + U_2 G$$

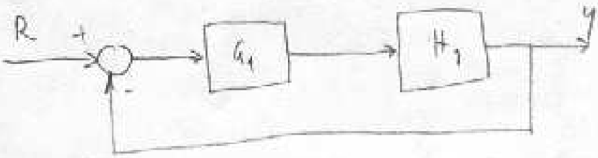


$$Y = U_1 G + U_2 G$$
$$= (U_1 + U_2) G$$

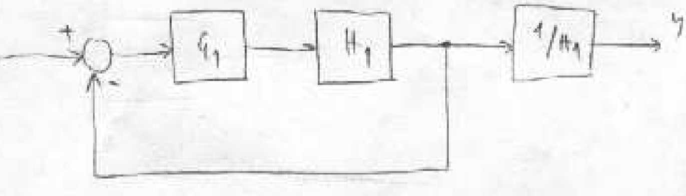
ZADATAK 4.

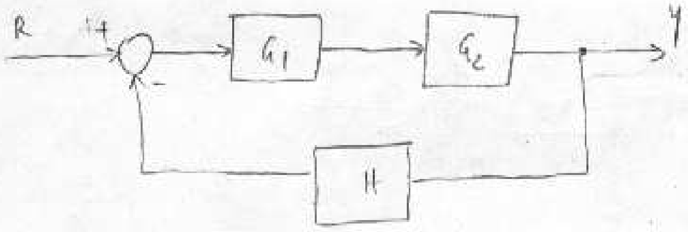


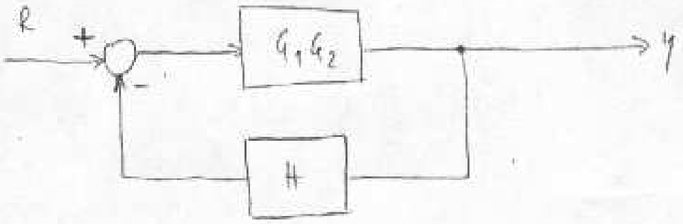
$$G_x = \frac{G_1}{1 + G_1 H_1}$$



$$G_y = \frac{G_1 H_1}{1 + G_1 H_1 \cdot 1} = \frac{G_1 H_1}{1 + G_1 H_1}$$

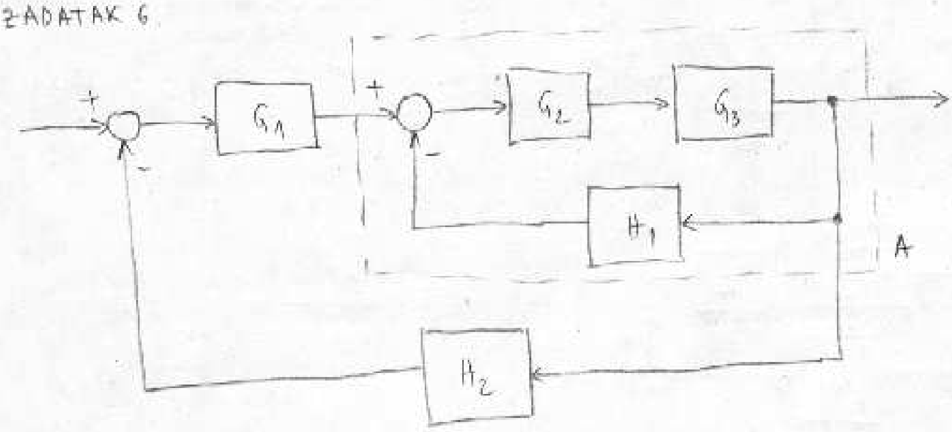




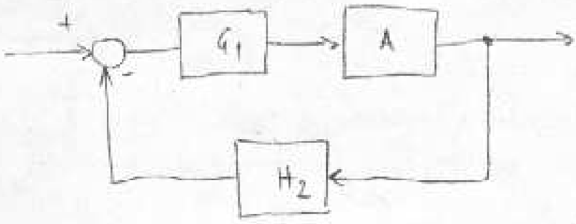


$$G = \frac{G_1 G_2}{1 + G_1 G_2 H}$$

ZADATAK 6.

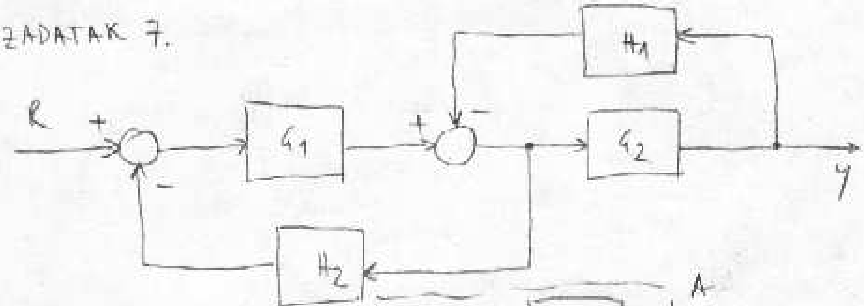


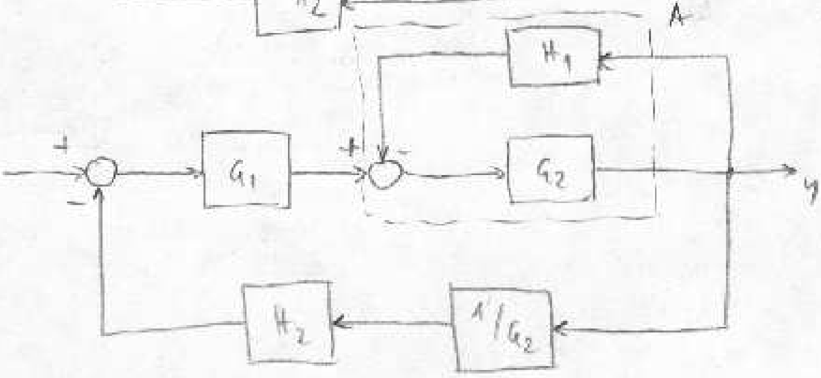
$$A = \frac{G_2 G_3}{1 + G_2 G_3 H_1}$$



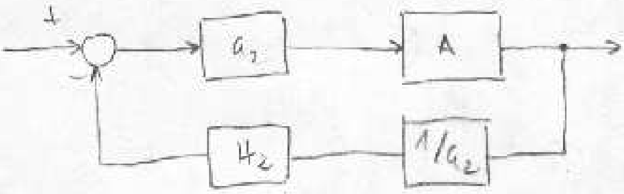
$$G = \frac{G_1 A}{1 + G_2 A H_2} = \frac{G_1 G_2 G_3}{1 + G_2 G_3 H_2 + G_1 G_2 G_3 H_1}$$

ZADATAK 7.



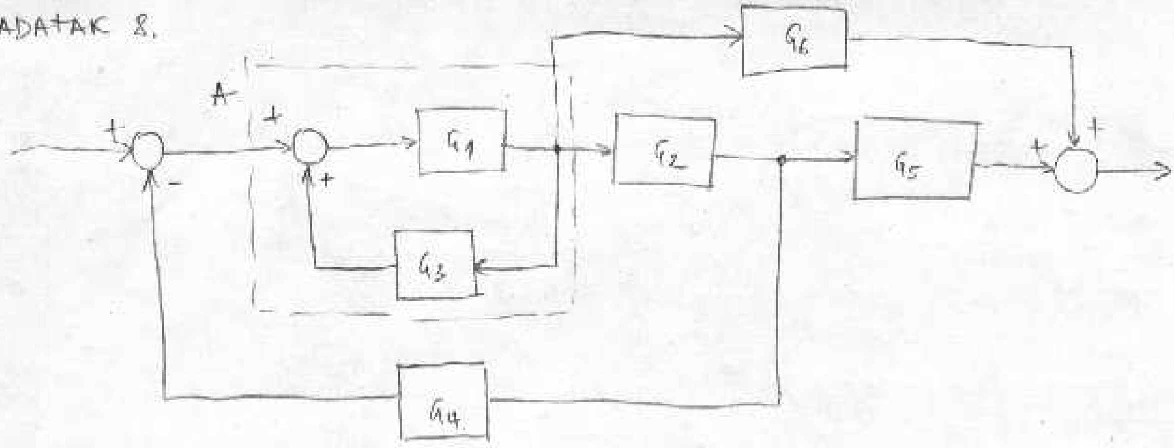


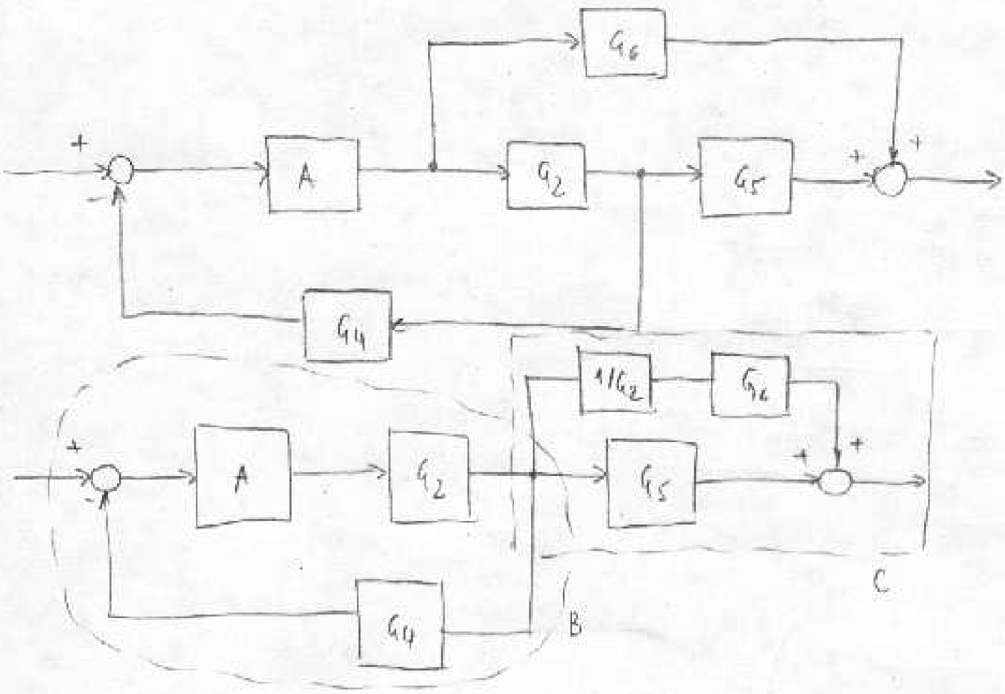
$$A = \frac{G_2}{1 + G_2 H_1}$$



- to riješiti

ZADATAK 8.





$$A = \frac{G_1}{1 - G_1 G_3}$$

$$B = \frac{A}{1 + A G_4}$$

$$C = \frac{G_6}{G_2} + G_5$$

$$G = B \cdot C$$

MASONOV TEOREM

$$G(s) = \frac{1}{\Delta} \sum_i \Delta_i G_i$$ DIREKTNI PUTEVI

P.F. - prijenosne funkcije

DIREKTNI PUTEVI (ZADATAK 8.)

$G_1 G_2 G_5 \qquad G_1 G_6$

PETLJE SUSTAVA (POVRATNE VEZE)

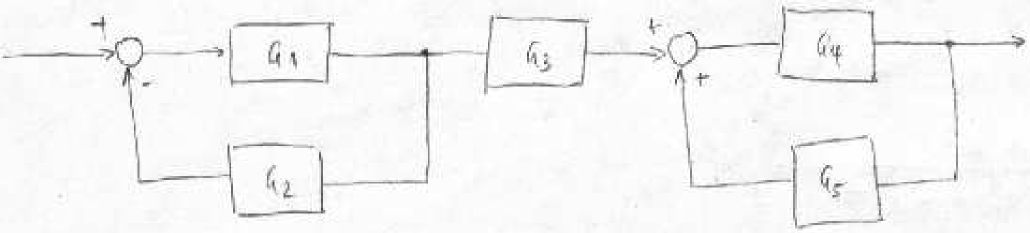
$+G_1 G_3 \qquad -G_1 G_2 G_4$

$\Delta = 1 - \sum$ (P.F. PETLJI) $+ \sum$ (UMNOŽAK P.F. DVIJU PETLJI KOJE SE NE DODIRUJU) $-$

$- \sum$ (—"— TRIJU —"—)

$$\Delta = 1 - (+G_1 G_3 + (-G_1 G_2 G_4)) = 1 - G_1 G_3 + G_1 G_2 G_4$$

ZADATAK 1.



D.P. $G_1 G_3 G_4$

PETLJE $-G_1G_2$, $+G_4G_5$

$$\Delta = 1-\left(-G_1G_2+G_4G_5\right)+\left((-G_1G_2)G_4G_5\right)$$

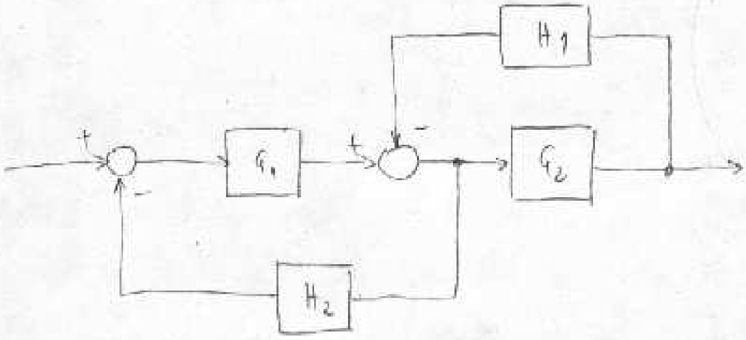
ZADATAK 8 (nastavak)

D.P. $G_1G_2G_5 \longrightarrow \Delta_1 = 1$ - (izbaciti sve petlje koje taj put dodiruje)

$G_1G_6 \longrightarrow \Delta_2 = 1$

$$G(s) = \frac{G_1G_2G_5\cdot 1 + G_1G_6\cdot 1}{1-G_1G_3+G_1G_2G_4}$$

ZADATAK 10.



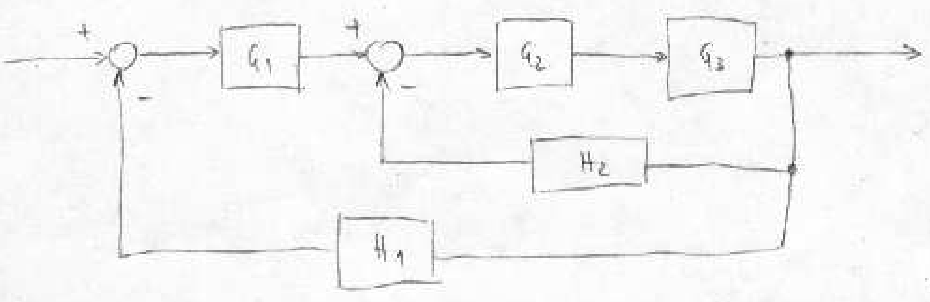
D.P. $G_1G_2 \longrightarrow \Delta_1 = 1$

PETLJE $-G_1H_2$, $-G_2H_1 \longrightarrow \Delta = 1-\left(-G_1H_2-G_2H_1\right)+(0)+\dots$

$\Delta = 1+G_1H_2+G_2H_1$

$$G = \frac{G_1G_2\cdot 1}{1+G_1H_2+G_2H_1}$$

ZADATAK 11.



D.P. $G_1G_2G_3 \rightarrow \Delta_1 = 1$

PETLJE $-G_1G_3H_2$, $-G_1G_2G_3H_1 \rightarrow \Delta = 1 - (\quad)$

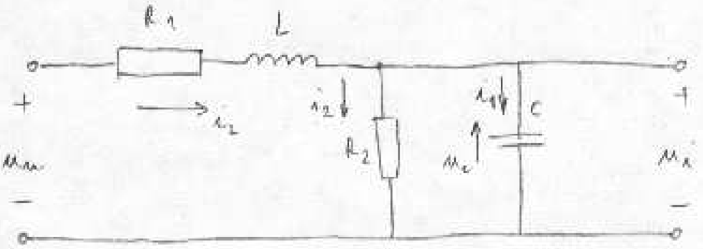
$$\Delta = 1 + G_1G_3H_2 + G_1G_2G_3H_1$$

$$G = \frac{G_1G_2G_3 \cdot 1}{1 + G_1G_3H_2 + G_1G_2G_3H_1}$$

1. DOMAĆA ZADAĆA GRUPA 1

ZADATAK 12.

a)



$u_i = i_2 R_2 \longrightarrow i_2 = \frac{1}{R_2} u_i$

$u_i = \frac{1}{C}\int_0^t i_1(t)\,dt$

$u_u = i_L R_1 + L\frac{di}{dt} + u_i$

$i_L = i_1 + i_2$

$$u_i = \frac{1}{C}\int_{0^-}^{t} i_1(t)\,dt$$

$$\int_{0^-}^{t} i_1(t)\,dt = C u_i \Big/ '$$

$$i_1(t) = C \cdot \dot{u}_i$$

$$i_L = \frac{1}{R_2} u_i + C\dot{u}_i \Big/ '$$

$$\dot{i}_L = \frac{1}{R_2}\dot{u}_i + C\ddot{u}_i$$

$$u_u = \left(\frac{1}{R_2}u_i + C\dot{u}_i\right)R_1 + L\left(\frac{1}{R_2}\dot{u}_i + C\ddot{u}_i\right) + u_i$$

$$CL\ddot{u}_i + \left(R_1C + \frac{L}{R_2}\right)\dot{u}_i + \left(\frac{R_1}{R_2} + 1\right)u_i = u_u \Big/ :CL$$

$$\ddot{u}_i + \frac{R_1R_2C + L}{R_2LC}\dot{u}_i + \frac{R_1 + R_2}{R_2LC}u_i = \frac{1}{LC}u_u$$

DODATAK - MATRICE

$$\dot{x} = Ax + Bu$$
$$y = Cx + Du$$

x - stanje sustava

u - ulaz

y - izlaz

ZADATAK 13.

$$\dot{x}_1 = 2x_1 + x_2 + u_1$$
$$\dot{x}_2 = 5x_1 + x_3 - u_2$$
$$\dot{x}_3 = x_2 + x_3 + 2u_1 - u_2$$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 2 & 1 & 0 \\ 5 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & -1 \\ 2 & -1 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$$

$$y_1 = 3x_1 - 5x_2 + u_1$$
$$y_2 = u_2$$

$$\begin{bmatrix} \dot{y}_1 \\ \dot{y}_2 \end{bmatrix} = \begin{bmatrix} 3 & -5 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$$

ZADATAK 12. (nastavak)

b) $x = \begin{bmatrix} u_C & i_L \end{bmatrix}^T$

$u = [u_u]$

$y = [u_i]$

$$u_i = u_C$$

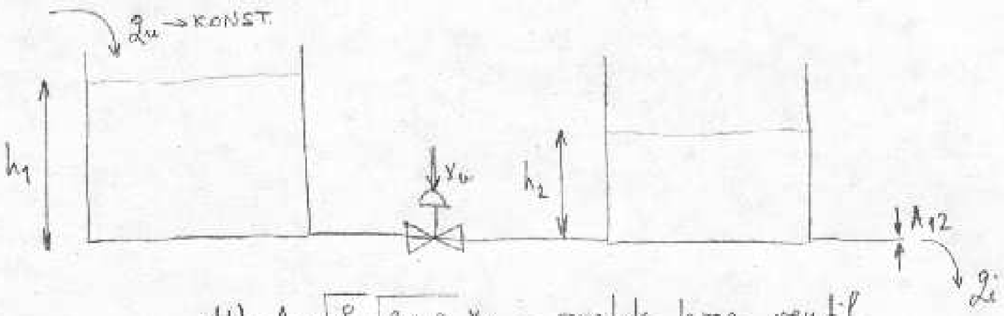
$$y = \underbrace{\begin{bmatrix} 1 & 0 \end{bmatrix}}_{C} \begin{bmatrix} u_C \\ u_L \end{bmatrix} + \underbrace{\begin{bmatrix} 0 \end{bmatrix}}_{D} [u_u]$$

$$i_1 = C\dot{u}_C \rightarrow \dot{u}_C = \frac{1}{C} i_1$$
$$i_L = i_1 + i_2 \rightarrow i_2 = \frac{u_C}{R_2}$$
$$i_1 = i_L - \frac{u_C}{R_2}$$
$$\dot{u}_C = \frac{1}{C}\left(i_L - \frac{u_C}{R_2}\right)$$
$$\dot{u}_C = -\frac{1}{CR_2} u_C + \frac{1}{C} i_L$$
$$u_u = R_1 i_L + L\frac{di}{dt} + u_i$$
$$\dot{i}_L = \frac{1}{L}\left(u_u - i_L R_1 - u_C\right)$$
$$\dot{i}_L = -\frac{1}{L} u_C - \frac{R_1}{L} i_L + \frac{1}{L} u_u$$

$$\begin{bmatrix} \dot{u}_C \\ \dot{i}_L \end{bmatrix} = \underbrace{\begin{bmatrix} -\frac{1}{CR_2} & \frac{1}{C} \\ -\frac{1}{L} & -\frac{R_1}{L} \end{bmatrix}}_{A} \begin{bmatrix} u_C \\ i_L \end{bmatrix} + \underbrace{\begin{bmatrix} 0 \\ \frac{1}{L} \end{bmatrix}}_{B} u_u$$

ZADATAK 14. 2. ZADATAK 1. DZ.

a)



ventil - izvršni član

$q(t) = A_v \sqrt{\rho}\sqrt{2\Delta p}\, x_u$ – protok kroz ventil

$A \frac{dh_1}{dt}\rho = A\frac{dV}{dt}$ → volumni protok (bez ρ)

$q_u - q(t) = A\frac{dV}{dt}$

(1) $q_u - q(t) = A_1 \frac{dh_1}{dt}\rho$

$q_u - A_v\sqrt{\rho}\sqrt{2\Delta p}\, x_u = A_1 \rho \frac{dh_1}{dt}$

$p_1 = \rho g h_1 + p_a$

$p_2 = \rho g h_2 + p_a$

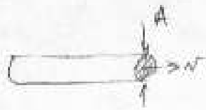
$p_1 - p_2 = \rho g h_1 + \cancel{p_a} - \rho g h_2 - \cancel{p_a}$

$\Delta p = \rho g (h_1 - h_2)$

$q_u - A_v\sqrt{\rho}\sqrt{\rho}\sqrt{2g}\sqrt{h_1-h_2}\, x_u = A_1\rho\frac{dh_1}{dt} \quad / : A_1\rho$

$$\boxed{\frac{dh_1}{dt} = \frac{1}{A_1\rho} q_u - \frac{A_v}{A_1}\sqrt{2g}\sqrt{h_1-h_2}\, x_u}$$

(2) $q(t) - q_i = A_2 \frac{dh_2}{dt}\rho$



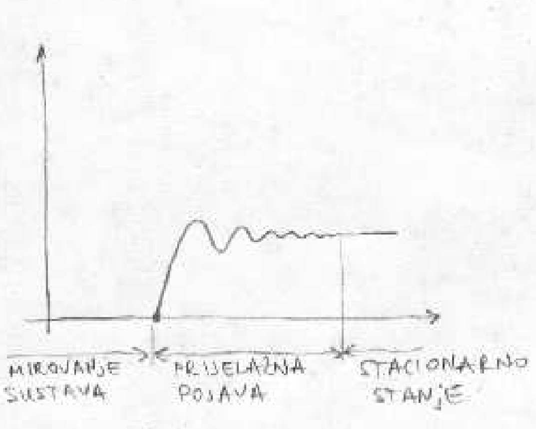
$q = A\rho v$

$q_i = A_{12}\rho\sqrt{2gh_2}$

$A_v\rho\sqrt{2g}\sqrt{h_1-h_2}\, x_u - A_{12}\rho\sqrt{2g}\sqrt{h_2} = A_2\rho\frac{dh_2}{dt} \quad / : A_2\rho$

$$\boxed{\frac{dh_2}{dt} = \frac{A_v}{A_2}\sqrt{2g}\sqrt{h_1-h_2}\, x_u - \frac{A_{12}}{A_2}\sqrt{2g}\sqrt{h_2}}$$

b)



$\frac{dh_1}{dt} = \frac{dh_2}{dt} = 0$ – stacionarno stanje (nema promjene h_1 i h_2)

$$\frac{1}{A_1\rho} q_u - \frac{A_v}{A_1}\sqrt{2g}\sqrt{h_1-h_2}\cdot x_u = 0$$

$$\frac{A_v}{A_2}\sqrt{2g}\sqrt{h_1-h_2}\,x_u - \frac{A_{12}}{A_2}\sqrt{2g}\sqrt{h_2} = 0 \longrightarrow \frac{A_v}{\cancel{A_2}}\sqrt{2g}\sqrt{h_1-h_2}\,x_u = \frac{A_{12}}{\cancel{A_2}}\sqrt{2g}\sqrt{h_2}$$

$$A_v x_u\sqrt{h_1-h_2} = A_{12}\sqrt{h_2}\ \Big|^2$$

$$\left(\frac{A_{12}}{A_v x_u}\right)^2 h_2 = h_1 - h_2$$

$$\boxed{h_1 = h_2\left(\left(\frac{A_{12}}{A_v x_u}\right)^2 + 1\right)}$$

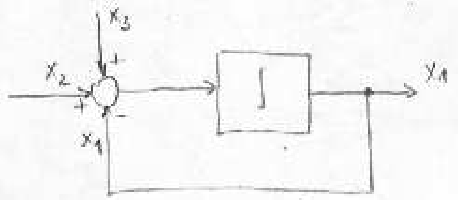
$$\frac{1}{A_1\rho} q_u - \frac{A_{12}}{A_2}\sqrt{2g}\sqrt{h_2}$$

$$\left.\begin{aligned} h_2 &= 0.4507\ m \\ h_1 &= 5\ m \end{aligned}\right\} (3) \rightarrow x_u = 0.397$$

OPĆENITO

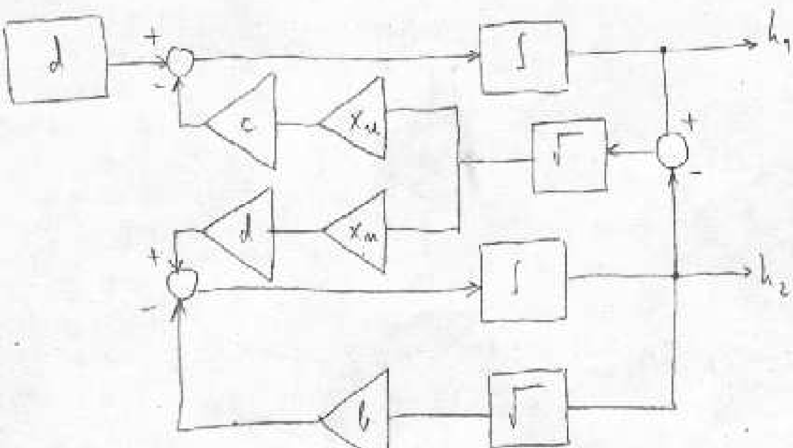
$$\frac{dx_1}{dt} = x_2 - x_1 + x_3 \ \Big|\int$$

$$x_1 = \int(-x_1 + x_2 + x_3)\,dt$$



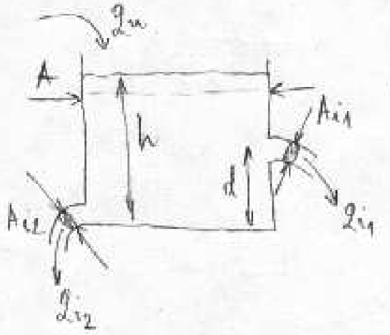
c) $$\frac{dh_2}{dt} = \underbrace{\boxed{\frac{A_v}{A_2}\sqrt{2g}}}_{=a}\sqrt{h_1-h_2}\,x_u - \underbrace{\boxed{\frac{A_{12}}{A_2}\sqrt{2g}}}_{=b} h_2$$

$$\frac{dh_1}{dt} = \underbrace{\boxed{\frac{1}{A_1\rho}}}_{=d} q_u - \underbrace{\boxed{\frac{A_v}{A_1}\sqrt{2g}}}_{=c}\sqrt{h_1-h_2}\,x_u$$



ZADATAK 15.

1. LIV. IZLAZNI TEST



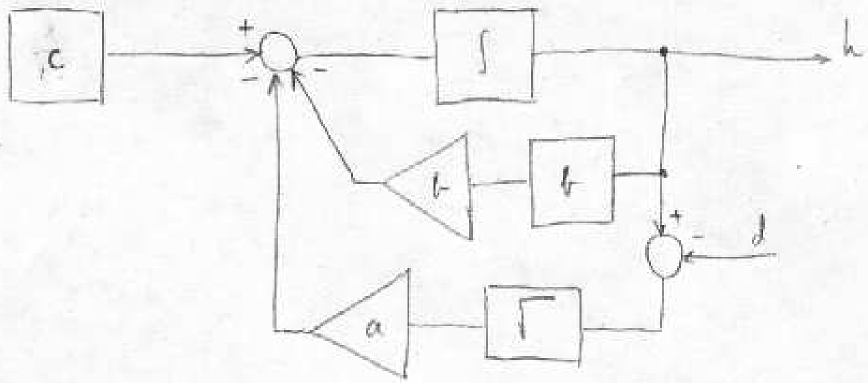
$$q_u - (q_{i1} + q_{i2}) = A \frac{dh}{dt} \rho$$

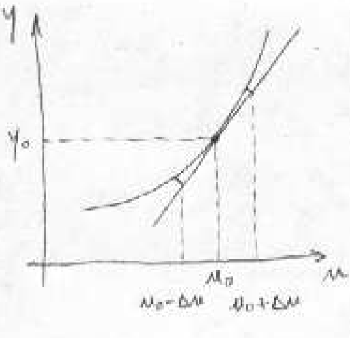
$$q_{i1} = A_{i1} \rho \sqrt{2g(h-d)}$$

$$q_{i2} = A_{i2} \rho \sqrt{2g h}$$

$$q_u - A_{i1} \rho \sqrt{2g} \sqrt{h-d} - A_{i2} \rho \sqrt{2g} \sqrt{h} = A\rho \frac{dh}{dt} \quad / : A\rho$$

$$\frac{dh}{dt} = \underbrace{\boxed{\frac{1}{A\rho} q_u}}_{c} - \underbrace{\boxed{\frac{A_{i1}}{A} \sqrt{2g}}}_{a} \sqrt{h-d} - \underbrace{\boxed{\frac{A_{i2}}{A} \sqrt{2g}}}_{b} \sqrt{h}$$

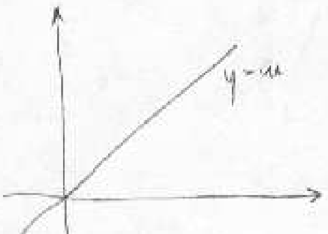


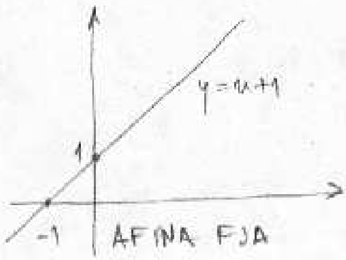


$$y = f(u)$$

$$y \sim f(u_0) + \frac{df}{du}\bigg|_{\substack{y=y_0\\u=u_0}}(u-u_0) + \underbrace{\frac{1}{2!}\frac{d^2f}{du^2}\bigg|_{\substack{y=y_0\\u=u_0}}(u-u_0)^2 + \dots}_{0}$$

$$y \sim f(u_0) + \frac{df}{du}\bigg|_{\substack{y=y_0\\u=u_0}}\Delta u \quad \text{– TAYLOROV RED}$$





$$y_1 = u_1 + 1$$
$$y_2 = u_2 + 1$$
$$u = \alpha u_1 + \beta u_2$$
$$y = u + 1$$
$$y = \alpha u_1 + \beta u_2 + 1$$
$$y = \alpha y_1 + \beta y_2$$
$$y = \alpha u_1 + \alpha + \beta u_2 + \beta$$
$$y = \alpha u_1 + \beta u_2 + \alpha + \beta$$

ZADATAK 16.

$y = u^4$ oko radne točke $u_0 = 1$

S.T. – statička točka

$$y_0 = u_0^4 = 1$$
$$y = \Delta y + y_0$$
$$u = \Delta u + u_0$$
$$y \sim f(u_0) + \frac{df}{du}\bigg|_{S.T.}\Delta u$$
$$\Delta y + \cancel{y_0} = \cancel{\underset{y_0}{f(u_0)}} + 4u_0^3\,\Delta u$$
$$\Delta y = 4\Delta u$$

ZADATAK 17.

$$\dot{y}(t) + y(t)u(t) = u^2(t)$$
$$u_0 = \frac{1}{2}$$
$$\dot{y}_0 = 0 \rightarrow 0 + y_0 u_0 = u_0^2$$
$$y_0 = \frac{1}{2}$$
$$y = \Delta y + y_0 \rightarrow \dot{y} = \Delta\dot{y}$$
$$u = \Delta u + u_0$$
$$\dot{y}(t) = u^2(t) - y(t)u(t) = f(u, y)$$
$$\dot{y}(t) \sim \underset{0}{f(u_0, y_0)} + \frac{\partial f}{\partial u}\bigg|_{S.T.}\Delta u + \frac{\partial f}{\partial y}\bigg|_{S.T.}\Delta y$$
$$\Delta\dot{y} = (2u - y_0)\Delta u + (-u_0)\Delta y$$
$$\Delta\dot{y} = \frac{1}{2}\Delta u - \frac{1}{2}\Delta y$$
$$2\Delta\dot{y} + \Delta y = \Delta u$$

ZADATAK 18.

$$\ddot{y}(t)+5\dot{y}(t)+y(t)+y^3(t)=\ln(u(t))$$

$$y_0=1$$

$$\ddot{y}_0=\dot{y}_0=0$$

$$0+5\cdot 0+y_0+y_0^3=\ln(u_0)$$

$$2\ \ln(u_0) \Rightarrow u_0=e^2$$

$$y=y_0+\Delta y \quad \rightarrow \dot{y}=\Delta\dot{y}$$
$$\ddot{y}=\Delta\ddot{y}$$

$$u=u_0+\Delta u$$

$$\ddot{y}=\ln(u)-5\dot{y}-y-y^3=f(u,y,\dot{y})$$

$$\ddot{y}\approx \underset{0}{f(u_0,y_0,0)}+\frac{\partial f}{\partial u}\bigg|_{S.T.}\Delta u+\frac{\partial f}{\partial y}\bigg|_{S.T.}\Delta y+\frac{\partial f}{\partial \dot{y}}\bigg|_{S.T.}\Delta\dot{y}$$

$$\Delta\ddot{y}=\frac{1}{u_0}\Delta u+(-1-3y_0^2)\Delta y+(-5)\Delta\dot{y}$$

$$\Delta\ddot{y}+5\Delta\dot{y}+4\Delta y=e^{-2}\Delta u$$

ZADATAK 19.

$$\ddot{y}+\dot{y}-\frac{1}{y}=u-ye^{\dot{u}}$$

$$y_0=1$$

$$\ddot{y}=\dot{y}=\dot{u}=0$$

$$-\frac{1}{y_0}=u_0-y_0 \rightarrow u_0=y_0-\frac{1}{y_0}$$
$$u_0=0$$

$$y=y_0+\Delta y \rightarrow \dot{y}=\Delta\dot{y}$$
$$\ddot{y}=\Delta\ddot{y}$$

$$u=u_0+\Delta u \rightarrow \dot{u}=\Delta\dot{u}$$

$$\ddot{y}=u-ye^{\dot{u}}-\dot{y}+\frac{1}{y}=f(u,\dot{u},y,\dot{y})$$

$$\ddot{y}\approx \underset{0}{f(u_0,0,y_0,u)}+\frac{\partial f}{\partial u}\bigg|_{S.T.}\Delta u+\frac{\partial f}{\partial \dot{u}}\bigg|_{S.T.}\Delta\dot{u}+\frac{\partial f}{\partial y}\bigg|_{S.T.}\Delta y+\frac{\partial f}{\partial \dot{y}}\bigg|_{S.T.}\Delta\dot{y}$$

$$\Delta\ddot{y}=\Delta u+(-y_0\ 1\cdot\Delta\dot{u})+(-1-\frac{1}{y_0^2})\Delta y+(-1)\Delta\dot{y}$$

$$\Delta\ddot{y}+\Delta\dot{y}+2\Delta y=\Delta u-\Delta\dot{u}$$

LAPLACEOVA TRANSFORMACIJA

$$F(s) = \int_0^{\infty} e^{-st} f(t)\,dt$$

$$f(t) \circ\!\!-\!\!\circ F(s)$$

$$\mathcal{L}\{f(t)\} = F(s)$$

VREMENSKA DOMENA

$$1 \circ\!\!-\!\!\circ \frac{1}{s}$$

$$2 \circ\!\!-\!\!\circ \frac{2}{s}$$

$$t \circ\!\!-\!\!\circ \frac{1}{s^2}$$

$$t^n \circ\!\!-\!\!\circ \frac{n!}{s^{n+1}}$$

$$\sin(\omega t) \circ\!\!-\!\!\circ \frac{\omega}{s^2+\omega^2}$$

$$\cos(\omega t) \circ\!\!-\!\!\circ \frac{s}{s^2+\omega^2}$$

$$\delta(t) \circ\!\!-\!\!\circ 1$$

$$f(t) \circ\!\!-\!\!\circ F(s)$$

$$e^{at} f(t) \circ\!\!-\!\!\circ F(s-a)$$

$$e^{t} t \circ\!\!-\!\!\circ \frac{1}{(s-1)^2}$$

$$\cos(2t)\, e^{t} \circ\!\!-\!\!\circ \frac{s-2}{(s-2)^2+4}$$

$$f(t-a) \circ\!\!-\!\!\circ F(s) \cdot e^{-as}$$

$$\cos(3t) \circ\!\!-\!\!\circ \frac{s}{s^2+9}$$

$$\cos(3(t-2)) \circ\!\!-\!\!\circ \frac{s}{s^2+9} e^{-2s}$$

$$t^n f(t) \circ\!\!-\!\!\circ (-1)^n \frac{d^n}{ds^n}(F(s))$$

$$t\sin(t) \circ\!\!-\!\!\circ (-1)^1 \frac{d}{ds}\left(\frac{1}{s^2+1}\right) = (-1)\frac{-2s}{(s^2+1)^2} = \frac{2s}{(s^2+1)^2}$$

$$\frac{1}{a} f\left(\frac{t}{a}\right) \circ\!\!-\!\!\circ F(as)$$

$$f(t) = \cos(2t) \circ\!\!-\!\!\circ \frac{s}{s^2+4}$$

$$\frac{1}{5}\cos\left(2\frac{t}{5}\right) \circ\!\!-\!\!\circ \frac{5s}{(5s)^2+4}$$

$$\boxed{\begin{aligned} y &\circ\!\!-\!\!\circ Y(s) \\ y' &\circ\!\!-\!\!\circ sY(s) - y(0) \\ y'' &\circ\!\!-\!\!\circ s^2Y(s) - sy(0) - y'(0) \end{aligned}}$$

ZADATAK 20.

$$\ddot{y} + 5\dot{y} + 6y = u$$

$u = t \qquad y(0) = 1$

$\qquad\qquad y'(0) = 2$

$y \circ\!\!-\!\!\bullet\, Y(s) \qquad\qquad u \circ\!\!-\!\!\bullet\, U(s) = \frac{1}{s^2}$

$y' \circ\!\!-\!\!\bullet\, sY(s) - y(0) = sY(s) - 1$

$y'' \circ\!\!-\!\!\bullet\, s^2Y(s) - sy(0) - y'(0) = s^2Y(s) - s - 2$

$$s^2Y(s) - s - 2 + 5(sY(s) - 1) + 6Y(s) = U(s)$$

$$Y(s)\left[s^2+5s+6\right] = U(s) + s + 2 + 5$$

$$Y(s)\left[s^2+5s+6\right] = U(s) + s + 7$$

$$Y(s) = \underbrace{\frac{U(s)}{s^2+5s+6}}_{y_r(s)} + \underbrace{\frac{s+7}{s^2+5s+6}}_{y_s(s)}$$

$$U(s) = \frac{1}{s^2}$$

$$s^2+5s+6 \longrightarrow \frac{-5 \pm \sqrt{25-24}}{2} = \frac{-5 \pm 1}{2}$$

$$-2, -3 \Rightarrow (s+2)(s-3)$$

$$Y(s) = \underbrace{\frac{1}{s^2(s+2)(s+3)}}_{A(s)} + \underbrace{\frac{s+7}{(s+2)(s+3)}}_{G(s)}$$

$$G(s) = \frac{s+7}{(s+2)(s+3)} = \frac{C_{11}}{s+2} + \frac{C_{21}}{s+3} = \frac{5}{s+2} - \frac{4}{s+3} \;\bullet\!\!-\!\!\circ\; 5e^{-2t} - 4e^{-3t}$$

$$C_{11} = G(s)(s+2)\Big|_{s=-2} = 5 \qquad\qquad C_{ij} = \frac{1}{(r_i-j)!}\left\{\frac{ds^{r_i-j}}{ds^{r_i-j}}\left[A(s)(s-s_i)^{r_i}\right]\right\}\Bigg|_{s=s_i}$$

$$C_{21} = G(s)(s+3)\Big|_{s=-3} = -4$$

$$\frac{C_{11}}{s} + \frac{C_{12}}{s^2} + \frac{C_{21}}{s+2} + \frac{C_{31}}{s+3} = \frac{-5}{36}\frac{1}{s} + \frac{1}{6}\frac{1}{s} + \frac{1}{4}\frac{1}{s+2} - \frac{1}{9}\frac{1}{s+3}$$

$$C_{21} = A(s)(s+2)\Big|_{s=-2} = \frac{1}{4}$$

$$C_{31} = A(s)(s+3)\Big|_{s=-3} = -\frac{1}{9}$$

$r = 2$

$$C_{11} = \frac{1}{1!}\left(\frac{d}{ds}\left(\frac{1}{(s+2)(s+3)}\right)\right)\Bigg|_{s=0} = \frac{-2s-5}{((s+2)(s+3))^2}\Bigg|_{s=0} = \frac{-5}{36}$$

$$C_{12} = \frac{1}{0!}\left[\frac{d^0}{ds^0}\,\frac{1}{(s+2)(s+3)}\right]\Bigg|_{s=0} = \frac{1}{6}$$

PRIJENOSNA FUNKCIJA

$$G(s) = \frac{U(s)}{Y(s)}$$

$$y'' - 5y' + 6y = u' + 2u$$

$$G(s) = \frac{s+2}{s^2 + 5s + 6}$$

$Y(s) = G(s)$ – težinska fja

$u(t) = \delta(t)$

$U(s) = 1$

$H(s) = \frac{1}{s} G(s)$ – prijelazna fja

$u(t) = \mu(t)$

$U(s) = \frac{1}{s}$

$$G(s) = \frac{s+2}{s^2+5s+6}$$

$$u(t) = 3\delta(t) \rightarrow U(s) = \frac{3}{s}$$

$$Y(s) = U(s)\,G(s) = \frac{3}{s} \frac{s+2}{s^2+5s+6} = \frac{3s+6}{s^3+5s^2+6s}$$

$$y(0^+) = \lim_{s\to\infty} sY(s) = \lim_{s\to\infty} \frac{3s^2+6s}{s^3+5s^2+6s} = \lim_{s\to\infty} s^2 Y(s)$$

$$\dot{y}(0^+) = \lim_{s\to\infty} s \cdot sY(s) = \lim_{s\to\infty} s^2 Y(s) = \frac{3s^3+6s^2}{s^3+5s^2+6s} = 3$$

$$y(\infty) = \lim_{s\to 0} sY(s) = \lim \frac{3s^2+6s}{s^3+5s^2+6s} = \frac{3s+6}{s^2+5s+6} = 1$$