ZADATAK 1) ⇒ A

$$\ddot{y}(t)\frac{1}{y(t)} + 6\dot{y}(t) + 4y(t) = 2e^{2u(t)}$$

a) $u_0 = 0$

$$4y_0 = 2e^{2u_0} \Rightarrow 4y_0 = 2 \Rightarrow y_0 = \frac{1}{2}$$

$$y = y_0 + \Delta y \Rightarrow \dot{y} = \Delta\dot{y} \Rightarrow \ddot{y} = \Delta\ddot{y}$$

$$u = u_0 + \Delta u$$

$$\ddot{y} = y \cdot 2e^{2u} - 6y\dot{y} - 4y^2 = f(u, y, \dot{y})$$

$$\Delta\ddot{y} = \underbrace{f(u_0, y_0, \dot{y}_0)}_{0} + \frac{\partial f}{\partial u}\Big|_{RT} \cdot \Delta u + \frac{\partial f}{\partial y}\Big|_{RT} \cdot \Delta y + \frac{\partial f}{\partial \dot{y}}\Big|_{RT} \cdot \Delta\dot{y}$$

$$\Delta\ddot{y} = 4 \cdot y e^{2u}\Big|_{RT} \cdot \Delta u + (2e^{2u} - 6\dot{y} - 8y)\Big|_{RT} \cdot \Delta y - 6y\Big|_{RT} \cdot \Delta\dot{y}$$

$$\Delta\ddot{y} = 2\Delta u - 2\Delta y - 3\Delta\dot{y}$$

$$\Rightarrow \Delta\ddot{y} + 3\Delta\dot{y} + 2\Delta y = 2\Delta u$$

b) $G(s) = \frac{Y(s)}{U(s)}$

$$Y(s)(s^2 + 3s + 2) = 2U(s)$$

$$\Rightarrow G(s) = \frac{2}{s^2 + 3s + 2}$$

c) $u(t) = t[S(t) - S(t-1)] + S(t-1) = tS(t) - tS(t-1) - S(t-1)$

$$u(t) = tS(t) - (t-1)S(t-1)$$

$$U(s) = \frac{1}{s^2} - \frac{1}{s^2}e^{-s}$$

$$Y(s) = G(s) \cdot U(s) = \frac{2}{s^2 + 3s + 2} \cdot \frac{1}{s^2}(1 - e^{-s})$$

$$s^2 + 3s + 2 = 0$$

$$s_1 = -1, \quad s_2 = -2$$

$$\frac{2}{s^2(s+1)(s+2)} = \frac{A}{s} + \frac{B}{s^2} + \frac{C}{(s+1)} + \frac{D}{(s+2)}$$

$$C = 2 \quad , \quad D = -\frac{1}{2}$$

$$As(s+1)(s+2) + B(s+1)(s+2) + 2s^2(s+2) - \frac{1}{2}s^2(s+1) = 2$$

$$As(s^2+3s+2) + B(s^2+3s+2) + 2(s^3+2s^2) - \frac{1}{2}(s^3+s^2) = 2$$

$$As^3 + 3As^2 + 2As + Bs^2 + 3Bs + 2B + 2s^3 + 4s^2 - \frac{1}{2}s^3 - \frac{1}{2}s^2 = 2$$

$$A + 2 - \frac{1}{2} = 0 \Rightarrow A = -\frac{3}{2}$$

$$3A + B + 4 - \frac{1}{2} = 0 \Rightarrow -\frac{9}{2} + B + 4 - \frac{1}{2} = 0 \Rightarrow B = 1$$

$$Y(s) = \left(-\frac{3}{2}\cdot\frac{1}{s} + \frac{1}{s^2} + \frac{2}{s+1} - \frac{1}{2}\cdot\frac{1}{s+2}\right)(1-e^{-s})$$

$$\Rightarrow \Delta y(t) = \left(-\frac{3}{2} + t + 2e^{-t} - \frac{1}{2}e^{-2t}\right)S(t) - \left(-\frac{3}{2} + (t-1) + 2e^{-(t-1)} - \frac{1}{2}e^{-2(t-1)}\right)S(t-1)$$

d) $\Rightarrow \Delta y(\infty) = \lim\limits_{s\to 0} s\cdot Y(s) = \lim\limits_{s\to 0} \frac{2}{s(s+1)(s+2)}(1-e^{-s}) =$

$$= \lim_{s\to 0} \frac{2(1+e^{-s})}{(s+1)(s+2)+s(s+2)+s(s+1)} = 1$$

$$\Rightarrow \Delta \dot{y}(0) = \lim_{s\to\infty} s^2\cdot Y(s) = \lim_{s\to\infty} \frac{2}{(s+1)(s+2)}(1-e^{-s}) =$$

$$= \lim_{s\to\infty} \frac{\frac{2}{s^2}}{(1+\frac{1}{s})(1+\frac{2}{s})}(1-e^{-s}) = 0$$

$$\Rightarrow \Delta \dot{y}(1) = (0+1-2e^{-1}+e^{-2}) - (0+1-2e^{-0}+e^{0}) = 0{,}4$$