① Zatvoreni diskretni — ravnina a_1-a_2, Ruth-Hurwitz
↳ stabilnost

$$G_2(z) = \frac{1}{a_2 z^2 + a_1 z + 0.1} \qquad z = \frac{2}{T}\frac{1+w}{1-w} \rightarrow \text{mod. bil. trans.}$$

$$z = \frac{1+w}{1-w} \leftrightarrow \text{bilinear. transf.}$$

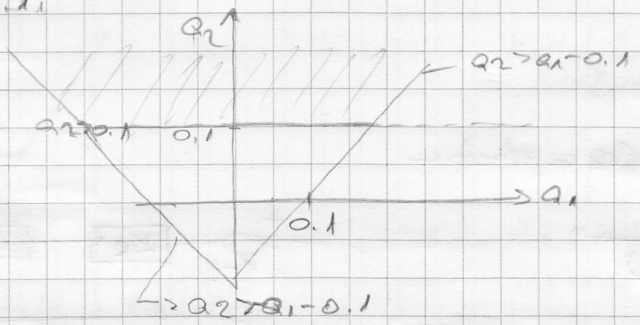
↳ kjer dobimo odzivi frekv. karakteristiku

$$G_2(w) = \frac{1}{a_2\left(\frac{1+w}{1-w}\right)^2 + a_1\frac{1+w}{1-w} + 0.1} = \frac{(1-w)^2}{a_2(1+w)^2 + a_1(1-w^2) + 0.1(1-w)^2}$$

$$= \frac{(1-w)^2}{w^2\underbrace{(a_2 - a_1 + 0.1)}_{g_2} + w\underbrace{(2a_2 - 0.2)}_{g_1} + \underbrace{a_2 + a_1 + 0.1}_{g_0}}$$

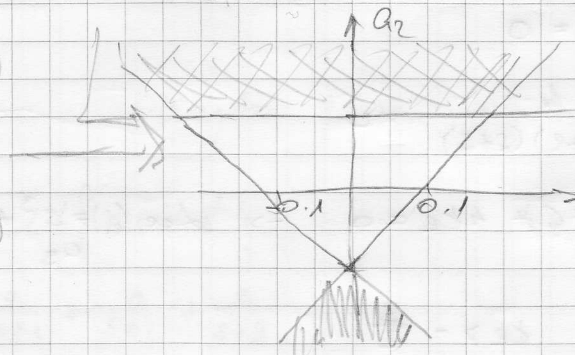
① $g_2 > 0 \Rightarrow a_2 - a_1 > -0.1$
$g_1 > 0 \Rightarrow a_2 > 0.1$
$g_0 > 0 \Rightarrow a_2 + a_1 > 0.1$

$D_{n-1} > 0$, $D_1 > 0$, $g_1 > 0$
↳ $n = 2$



II

$g_2 < 0 \Rightarrow a_2 - a_1 < -0.1$
$g_1 < 0 \Rightarrow a_2 < 0.1$
$g_0 < 0 \Rightarrow a_2 + a_1 < -0.1$
$D_1 = g_1 > 0$



②

$$G_p(s) = \frac{4}{s+2}$$ → upot. bil. transformacije (za promatr. diskretnog sustava u frekv. području)

a) naći ω^* na pojačanju kont. sustava od 0,5

$$G(j\omega^*) = \frac{4}{j\omega^*+2} \qquad |G(j\omega^*)| = \frac{4}{\sqrt{4+\omega^{*2}}} = 0,5 \qquad \omega^* = \sqrt{60} = 7.746 \text{ rad/s}$$

b) naći ω diskr. sust. na kojoj je pojač. 0,5

$$\omega^* = \frac{2}{T}\,tg\,\frac{\omega T}{2} \qquad \omega = \frac{2}{T}\,atang\,\frac{T\omega^*}{2}$$

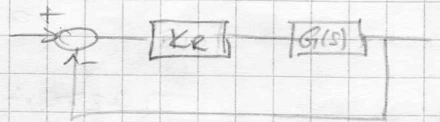
$T = 1 \Rightarrow \omega = 2.636$ rad/s

$T = 0,1 \Rightarrow \omega = 7.39$ rad/s

⑤ zadatak

PID-reg. Ziegler metodom

$$G(s) = \frac{4}{(s+1)(s+2)(s+3)}$$



$$\Delta_{ce} = 1 + G_o(s) = 0$$

$$1 + K_R\,\frac{4}{(s+1)(s+2)(s+3)} = 0$$

$$(s+1)(s^2+5s+6) + 4K_R = 0 \quad \rightarrow \quad \Delta_{ce}(s) = \underset{a_3}{s^3} + \underset{a_2}{6s^2} + \underset{a_1}{11s} + \underbrace{6 + 4K_R}_{a_0}$$

① $6 + 4K_R > 0 \qquad K_R > -\frac{3}{2}$

② $D = \begin{vmatrix} a_1 & a_0 \\ a_3 & a_2 \end{vmatrix} = \begin{vmatrix} 11 & 6+4K_R \\ 1 & 6 \end{vmatrix} > 0 \qquad 66 - 6 - 4K_R > 0 \qquad K_R < 15$

$K_R \in \langle -\frac{3}{2}, 15 \rangle$

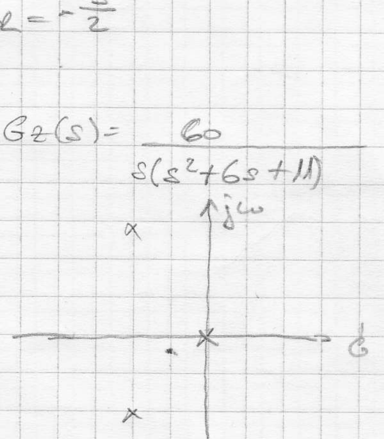
$$G_{ZAT}(s) = \frac{4 \cdot K_R}{s^3 + 6s^2 + 11s + 6 + 4K_R}$$

$K_R = -\frac{3}{2}$

$$G_Z(s) = \frac{60}{s(s^2+6s+11)}$$

$p_1 = 0$

$$p_{2,3} = \frac{-6 \pm \sqrt{36-44}}{2} = \frac{-6 \pm j2\sqrt{2}}{2} = -3 \pm j\sqrt{2}$$

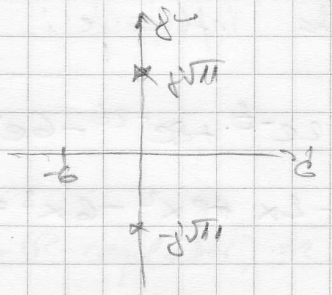


- ravnoteža stabilan
→ nema oscilacija

$k_e = 15$

$$G_z(s) = \frac{60}{s^3+6s^2+11s+66} = \frac{60}{(s+6)(s^2+11)}$$

oscilatoran



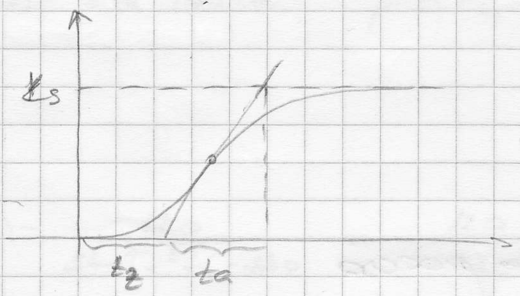
$\rightarrow$ težinska funkci. $g(t)$

$$\frac{A}{s+6} + \frac{Bs+C}{s^2+11} \Rightarrow Ae^{-6t} + B\cos(\sqrt{11}t) + C\cos(\sqrt{11}t)$$

$$\omega = \sqrt{11} \quad T_{kr} = \frac{2\pi}{\sqrt{11}} \quad k_{e,kr} = 15$$

$\rightarrow$ $k_e = 0.6\, k_{e,kr} = 9$ $\qquad T_I = 0.5\, T_{kr} = \frac{\pi}{\sqrt{11}}$ $\qquad T_D = 0.12\, T_{kr} = \frac{0.24\pi}{\sqrt{11}}$

$\rightarrow$ Ziegler 2



$\hookrightarrow$ vrijeme zadržavanja

$$\left.\begin{aligned} k_e &= \frac{1.2\, t_a}{k_s \cdot t_z} \\ T_I &= 2t_z \\ T_D &= 0.5\, t_z \end{aligned}\right\} \text{ šalabahter}$$

$\boxed{\text{Zadatak}}$

$$G_p(s) = \frac{4}{(s+1)(s+2)(s+3)}$$ Ziegler 2 na procesu

prijelazna funkcija: $h(t) = \mathcal{L}^{-1}\left\{G(s)\cdot\frac{1}{s}\right\}$

$$\frac{4}{(s+1)(s+2)(s+3)}\cdot\frac{1}{s} = \frac{A}{s} + \frac{B}{s+1} + \frac{C}{s+2} + \frac{D}{s+3}$$

$$A = \frac{4}{(0+1)(0+2)(0+3)} = \frac{2}{3} \qquad B = \left.\frac{4}{(s+2)(s+3)}\frac{1}{s}\right|_{s=-1} = \frac{4}{1(2)(-1)} = -2$$

$$C = \frac{4}{-1\cdot 1\cdot(-2)} = 2 \qquad D = \frac{4}{(-2)(-1)}\,\frac{1}{-3} = -\frac{2}{3}$$

$$h(t) = \frac{2}{3} - 2e^{-t} + 2e^{-2t} - \frac{2}{3}e^{-3t}, \quad t \geq 0$$

$$h'(t) = 2e^{-t} - 4e^{-2t} + 2e^{-3t}$$

$$h''(t) = -2e^{-t} + 8e^{-2t} - 6e^{-3t}$$

točka infleksije : $h''(t) = 0$

$-2e^{-t} + 8e^{-2t} - 6e^{-3t} = 0 \qquad x = e^{-t}$

$-2x + 8x^2 - 6x^3 = 0 \quad /:x \qquad -6x^2 + 8x - 2 = 0$

$x = \frac{-8 \pm \sqrt{64-48}}{-12} = \frac{-8 \pm 4}{-12} \Rightarrow x_1 = 1$

$x_2 = \frac{1}{3}$

$x = 1 \qquad \ln 1 = -t$

$0 = -t$

$x = \frac{1}{3} \quad \ln\frac{1}{3} = -t$

$\ln 3 = t$

$h(t = x = \frac{1}{3}) = \frac{2}{3} - 2 \cdot \frac{1}{3} + 2 \cdot \frac{1}{9} - \frac{2}{3}\frac{1}{27} = \frac{16}{81} \qquad \left(\ln 3, \frac{16}{81}\right)$

$h'(t) = 2\frac{1}{3} - 4\frac{1}{9} + 2\frac{1}{27} = \frac{8}{27}$

$y - y_1 = k(x - x_1)$

$y - \frac{16}{81} = \frac{8}{27}(x - \ln 3)$

$(tz = y = 0) \qquad tz = \ln 3 \cdot \frac{2}{3}$

$ta + tz \Rightarrow y = ks \qquad ks \Rightarrow$ pojačanje procesa

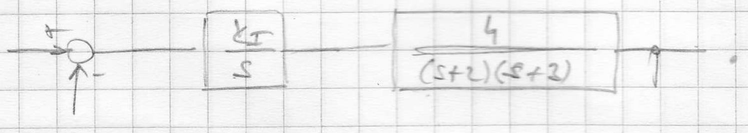
$ks = \lim_{s \to 0} G(s) = \frac{4}{(0+1)(0+2)(0+3)} = \frac{2}{3}$

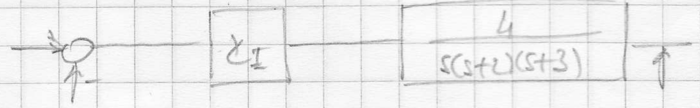
$ks - \frac{16}{81} = \frac{8}{27}(ta + tz - \ln 3) \Rightarrow \qquad ta = \frac{9}{4}$

$k_e = \frac{1.2\, ta}{ks \cdot tz}$

Zadatak 4.

naći I regul. → Z-N 1





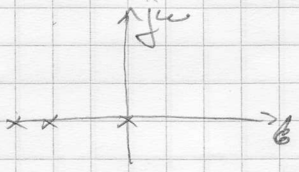
$\Rightarrow K_I$ (P-regulator) =?

$$G_z(s) = \frac{4K_R}{s^3+5s^2+6s+K_R} \qquad K_R>0$$

$$D = \begin{vmatrix} 6 & K_R \\ 1 & 5 \end{vmatrix} > 0 \Rightarrow K_R \in (0, 30)$$

$K_R = 0 \dots$

$\rightarrow \Delta_e(s) = s(s^2+5s+6) \qquad p = 0, -2, -3$



$K_R = 30 \dots$

$$\rightarrow G_z(s) = \frac{120}{s^3+5s^2+6s+30} = \frac{120}{(s+5)(s^2+6)} \Rightarrow \quad \omega = \sqrt{6} \quad T_{Kr} = \frac{2\pi}{\sqrt{6}}$$

$K_R = 0.5\,K_{Rkr} = 15$