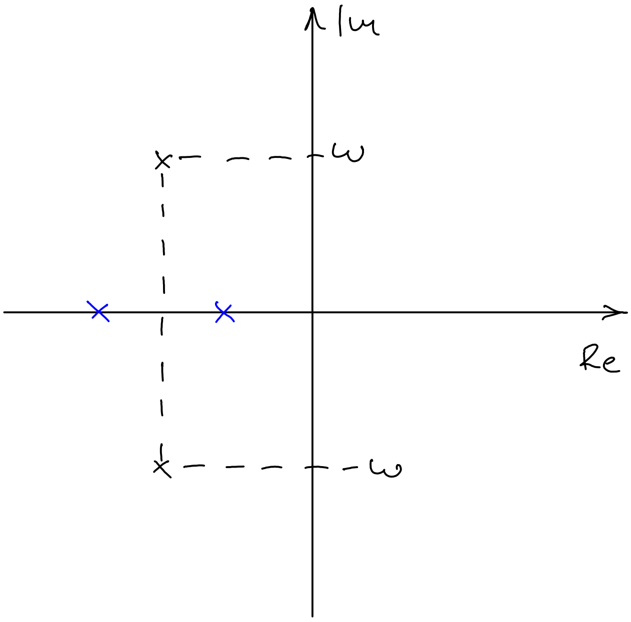
STABILNOST LTI SUSTAVA

3. prosinca 2008
17:06



→ polovi sustava dolaze u kompleksno-konjugiranim parovima

$$s_{p_{1,2}} = \frac{-\sigma \pm j\omega}{2a}$$

↳ rješenja kvadr. jedn.

Primjer:

A) $G(s) = \frac{2}{s+2}$ $\circ\!\!-\!\!\bullet$ $g(t) = 2 \cdot e^{-2t}$

$s_p = -2$

$$\lim_{t \to \infty} g(t) = 0$$

B) $G(s) = \frac{2}{s-2}$ $\circ\!\!-\!\!\bullet$ $g(t) = 2 \cdot e^{2t}$ MOD SUSTAVA

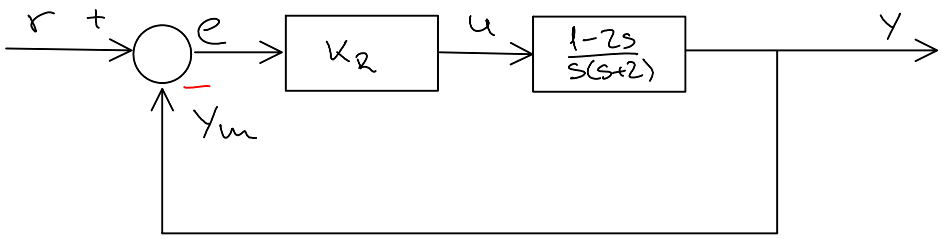
$s_p = 2$

$$\lim_{t \to \infty} g(t) = \infty$$

↳ NESTABILAN

Ako su jednostruki polovi na imaginarnoj osi onda je sustav granično stabilan
Ako su dvostruki ⇒ nestabilan

Primjer: (4. DZ. → Zadatak 1.)



kod hurwitzovog određivanja: napisati f-ciju „cijelog" (zatvorenog) sustava

$$G_0 = k_R \cdot \frac{1-2s}{s(s+2)}$$

$$\hookrightarrow G(s) = \frac{k_R \cdot \frac{1-2s}{s(s+2)}}{1 + k_R \cdot \frac{1-2s}{s(s+2)}} = \frac{k_R \cdot (1-2s)}{s(s+2) + k_R(1-2s)} \, *$$

karakteristična jedn. sustava

jednostavniji način određivanja karakt. jedn.:
kada u povratnoj vezi nema ničega
→ brojnik + nazivnik:
$k_R \cdot (1-2s) + s \cdot (s+2)$ ✓

$$* \quad s^2 + 2s + k_R - 2k_R \cdot s = s^2 + s(2-2k_R) + k_R$$

$$\alpha_{CE} = s^2 + (2-2k_R)s + k_R$$

1. svi članovi moraju biti istog predznaka
⇒ ako je već jedan pozitivan ⇒ moraju biti i ostali

$$\alpha_{CE} = a_n s^n + a_{n-1} s^{n-1} + \dots + a_1 s^1 + a_0$$

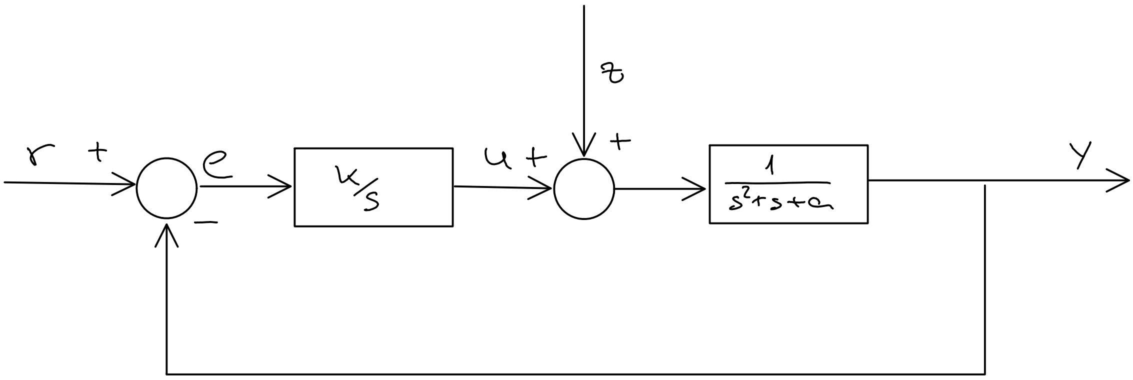
2. sve determinante moraju biti veće od 0!

$$D_1 = a_1 > 0$$

$$D_2 = \begin{vmatrix} a_1 & a_0 \\ a_3 & a_2 \end{vmatrix} \qquad D_3 = \begin{vmatrix} a_1 & a_0 & 0 \\ a_3 & a_2 & a_1 \\ a_5 & a_4 & a_3 \end{vmatrix}$$

$$\left.\begin{array}{l} D_1 > 0: \quad 2 - 2k_R > 0 \Rightarrow k_R < 1 \\ \qquad\quad k_R > 0 \end{array}\right\} k_R \in \langle 0, 1 \rangle \rightarrow \text{sustav je stabilan}$$

Primjer: (2. MI 2007./2008. $\rightarrow$ 3. zadatak)



$$G_o(s) = \frac{k}{s} \cdot \frac{1}{s^2+s+a}$$

$$K_{CE} = k + s^3 + s^2 + as$$
$$= s^3 + s^2 + as + k$$

1) svi koef. > 0

$a > 0$

$k > 0$

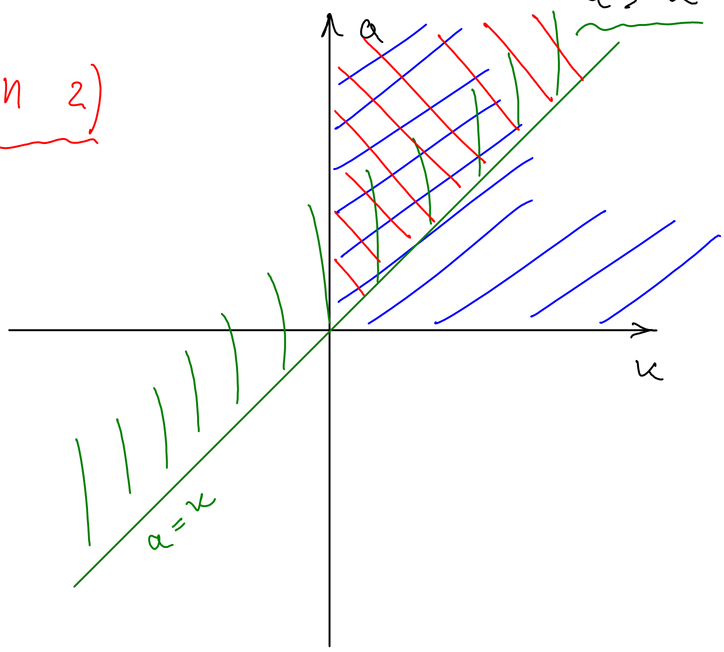
2) determinante > 0 $\quad \forall i \; D_i > 0$

$$D_1 = a_1 = a > 0$$

$$D_2 = \begin{vmatrix} a_1 & a_0 \\ a_3 & a_2 \end{vmatrix} = \begin{vmatrix} a & k \\ 1 & 1 \end{vmatrix} = a - k > 0$$

$a > k$

1) ∩ 2)



„Sve potencije moraju biti zastupljene!

$s^4 + s^3 + s^2 + a_0$

ne smiju biti $\emptyset$!

Period oscilacija na rubu stabilnosti

$s_{p1,2} = -\sigma \pm j\omega_d$

$\omega = 2\pi f = \frac{2\pi}{T}$

uvjet ruba: $k = a$

$\alpha_{CE}(s) = s^3 + s^2 + as + a = s^2(s+1) + a(s+1) = (s^2+a)(s+1)$

$\alpha_{CE}(j\omega) = (j\omega+1)(-\omega^2 + a) = \emptyset \quad \rightarrow \omega = \sqrt{a}$

$T = \frac{2\pi}{\sqrt{a}}$

$s = \pm j\sqrt{a}$ $\quad s = -1$

Primjer: (4. LiV → A1)

$$G_0(s) = \frac{k}{s(s+1)(s+2)}$$

$\alpha_{CE} = k + s(s+1)(s+2)$

$\alpha_{CE}(s) = k + s(s^2+3s+2) = s^3 + 3s^2 + 2s + k$

1) $k > 0$

2) $D_1 = 2 > 0$ ✓

$$D_2 = \begin{vmatrix} a_1 & a_0 \\ a_3 & a_2 \end{vmatrix} = \begin{vmatrix} 2 & k \\ 1 & 3 \end{vmatrix} = 6 - k > 0$$

$k > 6$

1) $\cap$ 2) $k \in \langle 0, 6 \rangle$

rub stabilnosti:

1) $k = 0$ → ne tražimo jer je tada sustav NESTABILAN

2) $s^3 + 3s^2 + 2s + 6 = 0$

$s^2(s+3) + 2(s+3) = (s^2+2)(s+3)$

$s = \pm j\sqrt{2}$ $\quad s_1 = -3$ tu nema oscilacija

$$\omega = 2 \rightarrow T = \frac{2\pi}{\omega} = \frac{2\pi}{\sqrt{2}} = \pi\sqrt{2} \quad [s]$$

Primjer: (4. LiV → A2)

$$G_o(s) = \frac{1}{ks(s+0,5)(s+2)}$$

$$\alpha_{CE} = ks(s^2+2,5s+1)+1$$
$$= ks^3 + 2,5ks^2 + ks + 1$$

1) $\left.\begin{array}{l} k>0 \\ 2,5k>0 \\ k>0 \end{array}\right\} \Rightarrow k>0$

2) $D_1 = k > 0$

$$D_2 = \begin{vmatrix} a_1 & a_0 \\ a_3 & a_2 \end{vmatrix} = \begin{vmatrix} k & 1 \\ k & 2,5k \end{vmatrix} = 2,5k^2 - k > 0$$

$(2,5k-1)\cdot k > 0$

(iz 1. uvjeta) $\hookrightarrow > 0 \Rightarrow 2,5k > 1$

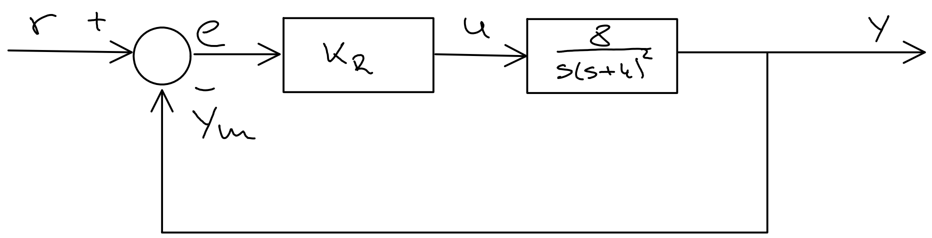
$k > 0,4$ ✓

$k > 0,4 \Rightarrow k \in \langle 0,4 ; \infty \rangle$

Period oscilacija na rubu stabilnosti?

$\Rightarrow k = 0,4$ rub stabilnosti

Primjer: (4. DZ. → 2. zadatak)



$K_R = 1,664$

$$G_o(s) = \frac{13,312}{s(s+4)^2}$$

$$G_o(j\omega) = \frac{13,312}{j\omega(j\omega+4)^2} = \frac{13,312}{16j\omega(1+j\frac{\omega}{4})(1+j\frac{\omega}{4})} = \frac{0,832}{j\omega(1+\ldots)(\ldots)}$$

$$A(\omega) = 20\log(0{,}832) - 20\log(\omega) - 40\log\sqrt{1+(\omega/4)^2}$$

$$\varphi(\omega) = \operatorname{arctg}\frac{0}{0{,}832} - \operatorname{arctg}\frac{\omega}{0} - 2\operatorname{arctg}\frac{\omega}{4}$$

$$= -\frac{\pi}{2} - 2\operatorname{arctg}\frac{\omega}{4}$$

$5+3j \to \operatorname{arctg}\frac{3}{5} = 30^\circ$

$5-3j \to \operatorname{arctg} -\frac{3}{5} = -\operatorname{arctg}\frac{3}{5} = -30^\circ$ ✓

$-5+3j \to \operatorname{arctg}\frac{3}{-5} = \cancel{-\operatorname{arctg}\frac{3}{5} = -30^\circ}$

$\left.\begin{array}{l} \mathrm{Im} = 3 > 0 \\ \mathrm{Re} = -5 < 0 \end{array}\right\} \Rightarrow$ II. kvadrant $\Rightarrow$

$\to \operatorname{arctg}\frac{3}{-5} = -\operatorname{arctg}\frac{3}{5} + \pi = 150^\circ$ ✓

(180°)

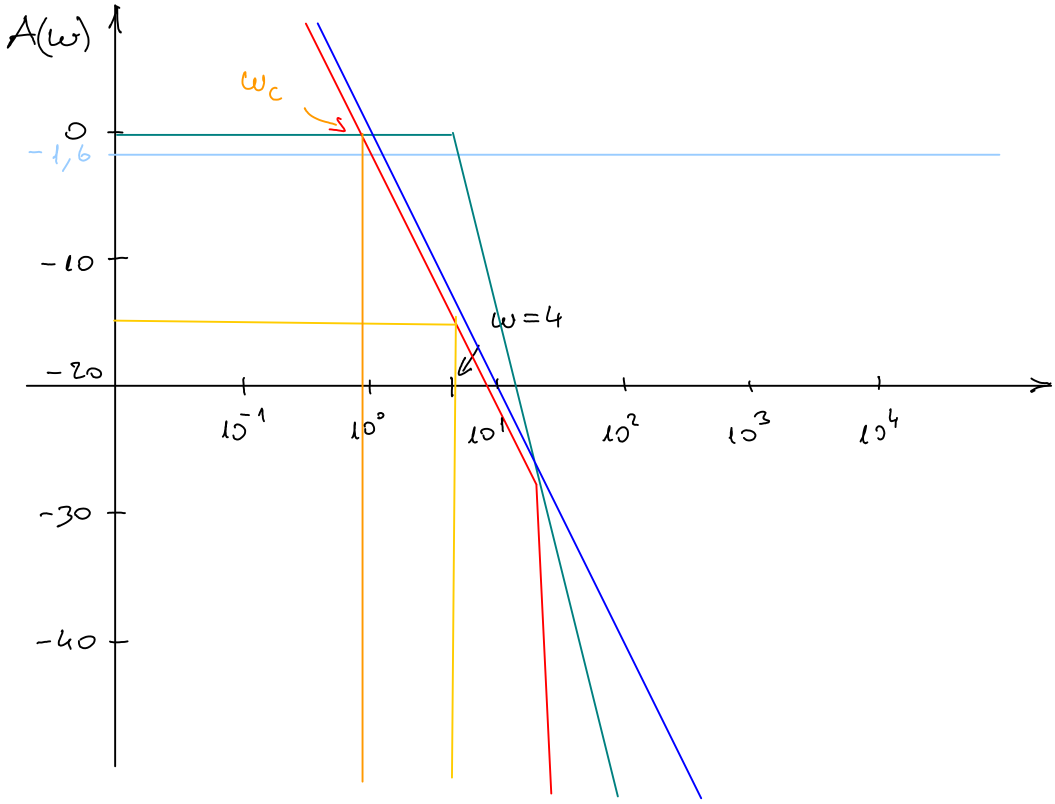
$-5-3j \to \cancel{\operatorname{arctg}\frac{+3}{+5} = \operatorname{arctg}\frac{3}{5} = 30^\circ}$

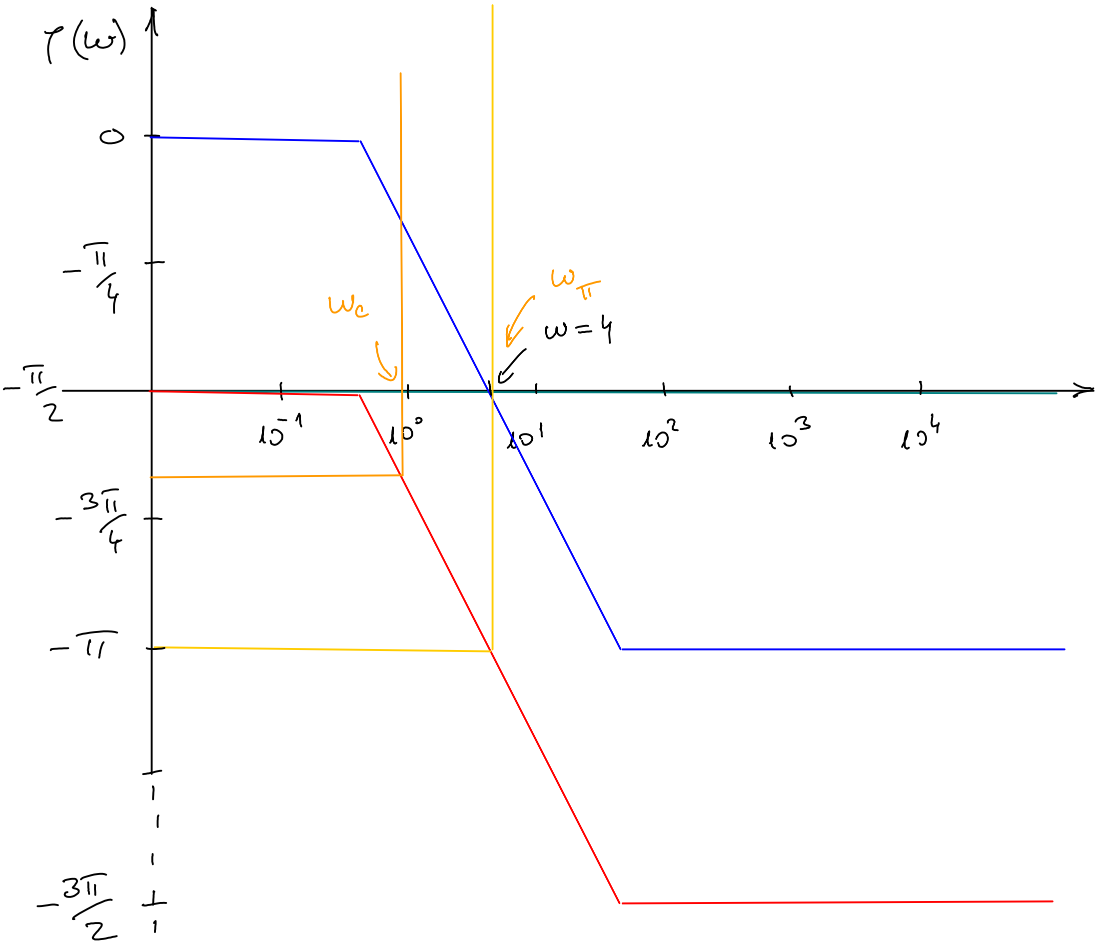
$\left.\begin{array}{l} \mathrm{Im} = -3 < 0 \\ \mathrm{Re} = -5 < 0 \end{array}\right\}$ III. kvadrant $\Rightarrow$

$\to \operatorname{arctg}\frac{-3}{-5} = \operatorname{arctg}\frac{3}{5} + \pi = 210^\circ$ ✓

(180°)

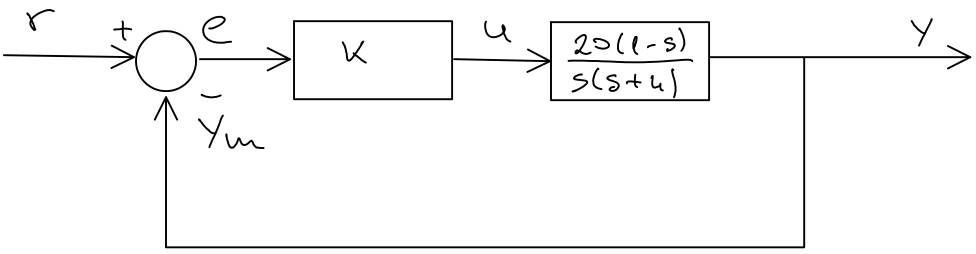
$20\log 0{,}832 = -1{,}6\ \mathrm{dB}$





ω_π desno od ω_c => sustav je stabilan

Primjer: (2. MI 2007./2008. → 1. b) zadatak)



b) $K = 0{,}1$

$$G_o(s) = \frac{1}{10}\,\frac{20(1-s)}{s(s+4)} = 2\,\frac{1-s}{s(s+4)}$$

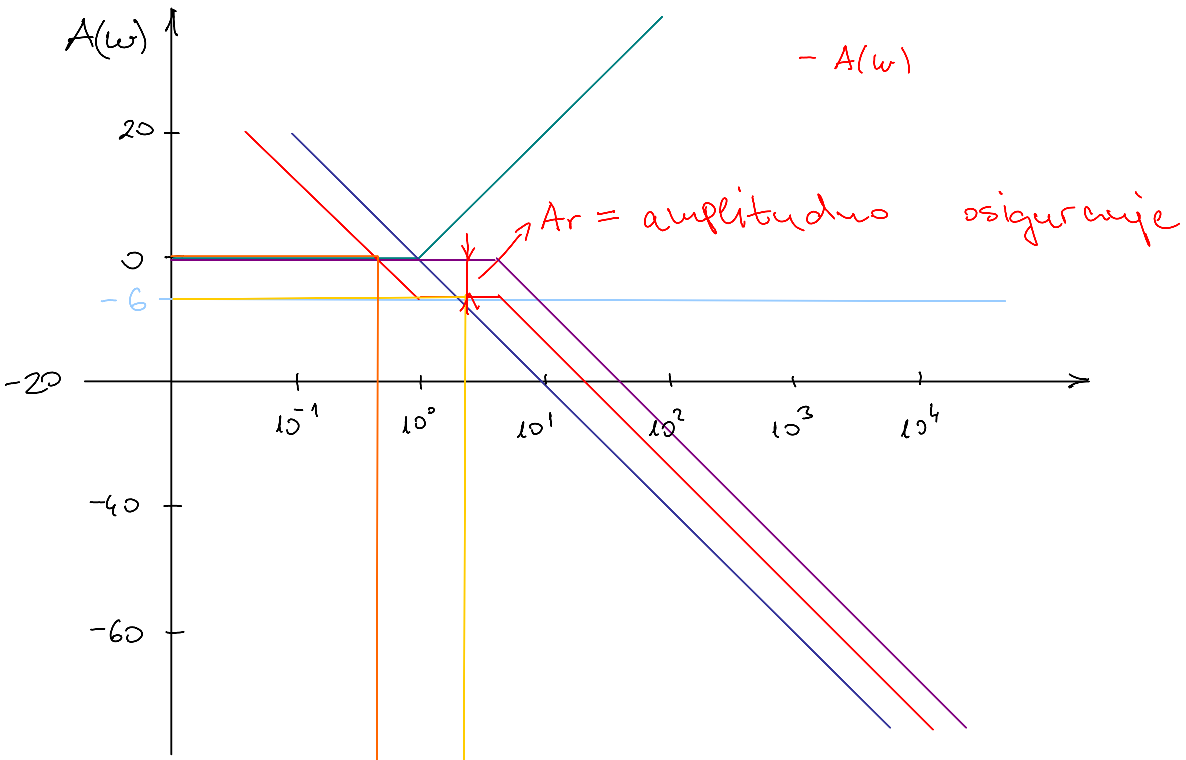
$$G_o(j\omega) = 2\cdot\frac{1-j\omega}{j\omega(j\omega+4)} = \frac{1}{2}\cdot\frac{1-j\,\omega/1}{j\omega(1+j\,\omega/4)}$$

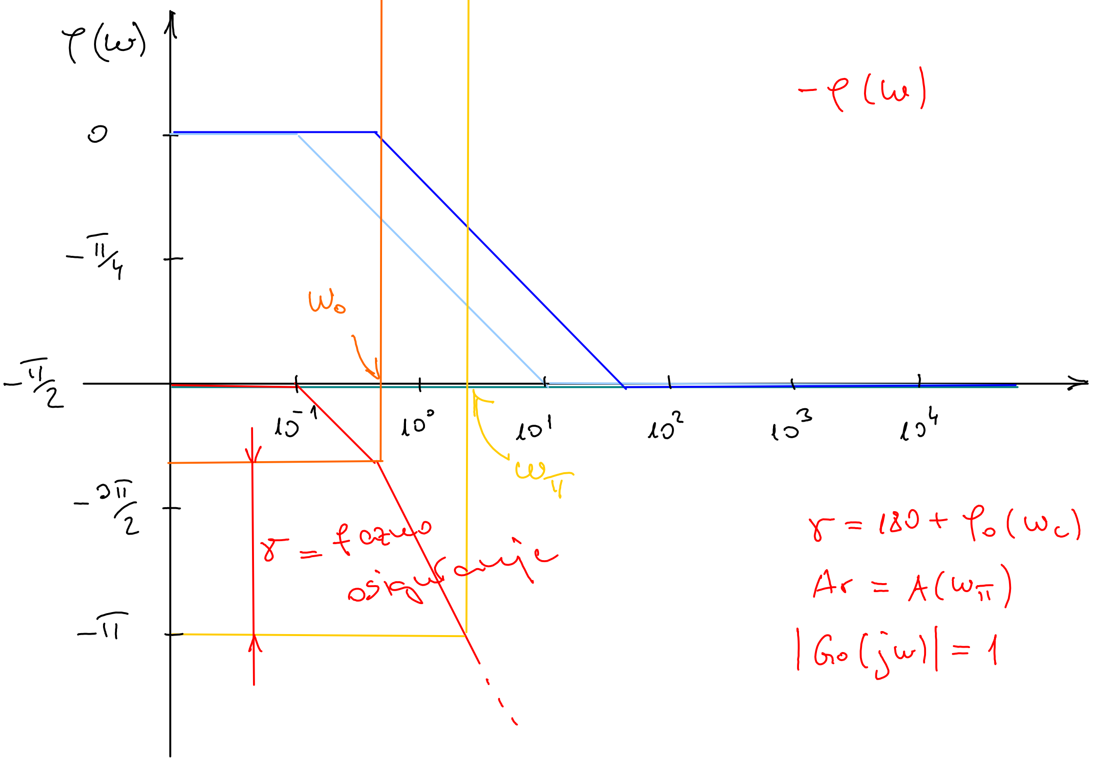
$$A(\omega)_{[dB]} = 20\log A(\omega)$$

$$A(\omega) = \underbrace{20\log\frac{1}{2}} + \underbrace{20\log\sqrt{1+\left(\frac{\omega}{1}\right)^2}} - \underbrace{20\log\omega} - \underbrace{20\log\sqrt{1+\left(\frac{\omega}{4}\right)^2}}$$

$$\varphi(\omega) = \cancel{\operatorname{arctg}\frac{0}{0,5}} + \operatorname{arctg}\frac{-\omega}{1} - \operatorname{arctg}\frac{\omega}{0} - \operatorname{arctg}\frac{\omega}{4}$$

$$\varphi(\omega) = \underbrace{-\operatorname{arctg}\frac{\omega}{1}} - \underbrace{\frac{\pi}{2}} - \underbrace{\operatorname{arctg}\frac{\omega}{4}}$$





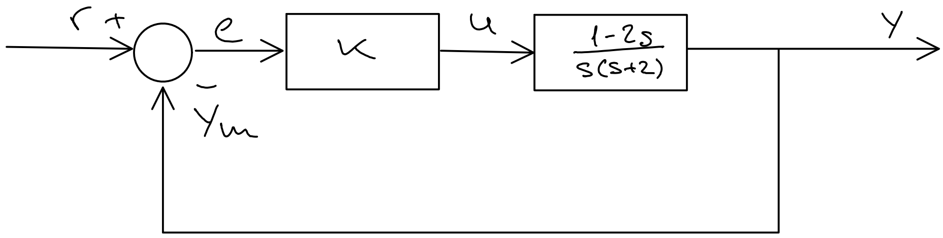
$\gamma = 180 + \varphi_0(\omega_c)$

$Ar = A(\omega_\pi)$

$|G_0(j\omega)| = 1$

REGULACIJSKO ODSTUPANJE

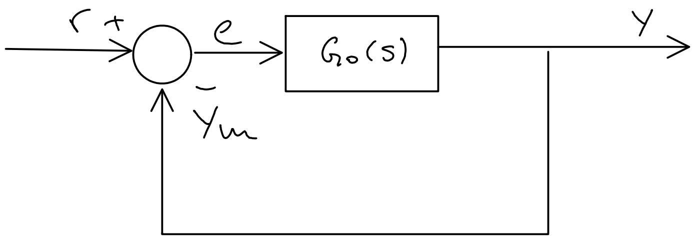
3. prosinca 2008
20:24



c) Odrediti regulacijsko odstupanje u ustaljenom stanju za:

$$R(s) = \frac{5}{s} \; ; \quad K_R = 0{,}5$$

$$e_\infty = \lim_{t\to\infty} e(t) = \lim_{s\to 0} sE(s)$$



$$E(s) = R(s) - Y(s) \qquad\qquad Y(s) = E(s)\cdot G_0(s)$$

$$Y(s) = R(s)\cdot G(s) \qquad\qquad E(s) = R(s) - E(s)\cdot G_0(s)$$

$$G(s) = \frac{G_0(s)}{1+G_0(s)} \qquad\qquad E(s) = \frac{R(s)}{1+G_0(s)}$$

$$G_0(s) = \frac{1-2s}{2s(s+2)}$$

$$E(s) = \frac{\frac{5}{s}}{1+\frac{1-2s}{2s(s+2)}} = \frac{10\cancel{s}(s+2)}{\cancel{s}(2s(s+2)+1-2s)}$$

$$= \frac{10s+20s}{2s^2+2s+1} \rightarrow e_\infty = 0$$

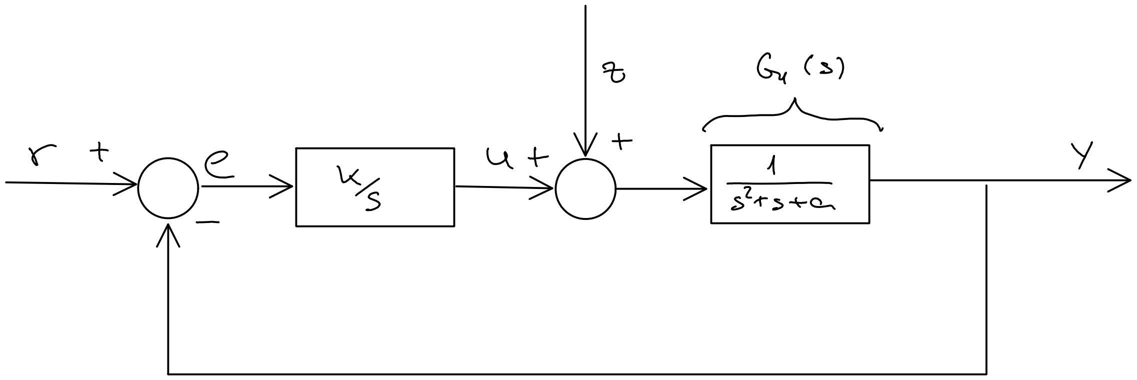
$$e_\infty = \lim_{s\to 0}\frac{10s+20s}{2s^2+2s+1} = 0$$

d) $R(s) = \frac{2}{s^2}$, $K = 0{,}5$

$$E(s) = \frac{R(s)}{1+G_0(s)} = \frac{\frac{2}{s^2}}{1+\frac{1-2s}{2s(s+2)}} = \frac{\frac{2}{s^{\cancel{2}}}}{\frac{4s^2+4s+1-2s}{2\cancel{s}(s+2)}} = \frac{4s+8}{s(4s^2+2s+1)}$$

$$e_\infty = \lim_{s\to 0} \frac{\cancel{s}(4s+8)}{\cancel{s}(4s^2+2s+1)} = \frac{8}{1} = 8$$

Primjer: (2. MI 2007./2008.)



$$z(t) = 0{,}5t\,S(t) \quad \circ\!-\!\bullet \quad Z(s) = 0{,}5\cdot\frac{1}{s^2} = \frac{1}{2s^2}$$

$$Y(s) = E(s)\cdot G_0(s) + Z(s)\cdot G_1(s)$$

$$E(s) = \cancel{R(s)} - Y(s) = -Y(s)$$

0

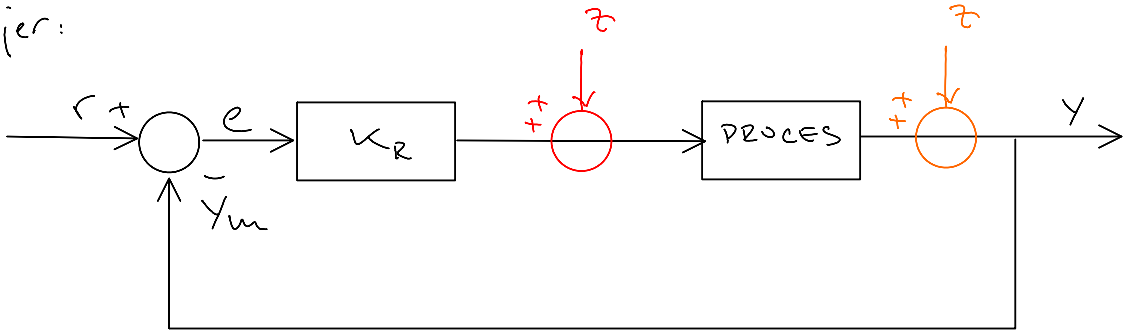
$$Es(-1 - G_0(s)) = G_1(s)\cdot Z(s)$$

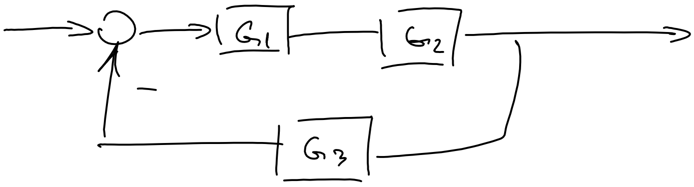
$$E(s) = Z(s)\cdot\frac{-G_1(s)}{1+G_0(s)} = \frac{1}{2s^2}\cdot\frac{-\frac{1}{s^2+s+a}}{1+\frac{K}{s(s^2+s+a)}} = \frac{1}{2s^{\cancel{2}}}\cdot\frac{\frac{-1}{\cancel{s^2+s+a}}}{\frac{K+s(s^2+s+a)}{\cancel{s(s^2+s+a)}}}$$

$$= \frac{-1}{2s(s^3+s^2+as+K)}$$

$$e_\infty = \lim_{s\to 0} sE(s) = \frac{-\cancel{s}}{2\cancel{s}(s^3+s^2+as+K)} = \frac{-1}{2K} \checkmark$$

Primjer:





$$G_0 = G_1 G_2 G_3$$

Primjer: (2. MI 2007./2008. → 2. Zadatak)

$$G(s) = \frac{32}{s^2+2s+16}$$

a) 4 boda   Odziv u ustaljenom stanju

$$u(t) = \sin(\omega_r t + 45^\circ)$$

$$u(t) = U_m \sin(\omega_0 t + \alpha_u)$$
$$y(t) = Y_m \sin(\omega_0 t + \alpha_y)$$
} moraju biti iste faze

$$Y_m = U_m |G(j\omega_0)|$$
$$\alpha_y = \alpha_u + \angle G(j\omega_0)$$

$$y(t) = Y_m \sin(\omega_r t + \alpha_y)$$

$$\omega_r = \omega_n \sqrt{1-2\xi^2}$$

$$G(s) = \frac{K}{\frac{1}{\omega_n^2}s^2 + \frac{2\xi}{\omega_n}s + 1}$$

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nadvišenje u [%] : $\sigma_m = \frac{Y_m - Y_{ss}}{Y_{ss}} \cdot 100\%$

$$3e^{-2t}\left(3\cos(\omega t)+\sin(\omega t)\right)$$

$$3\sin\left(\omega t+\frac{\pi}{2}\right)$$

$$\cos(\omega t)=\sin\left(\omega t+\frac{\pi}{2}\right)$$

$$3\angle \pi/2$$

$$U_m\sin(\omega t+\alpha)\to U_{ef}\angle\alpha$$

$$a+bj$$

$$\sqrt{a^2+b^2}=3 \qquad \operatorname{arctg}\frac{b}{a}=\frac{\pi}{2}$$

$$a^2+b^2=9 \qquad \frac{b}{a}=\operatorname{tg}\frac{\pi}{2}=+\infty \Rightarrow a=0$$

$$b=3$$

$$1\angle 0$$

$$a+bj$$

$$+$$

$$\sqrt{a^2+b^2}=1 \qquad \operatorname{arctg}\frac{b}{a}=0$$

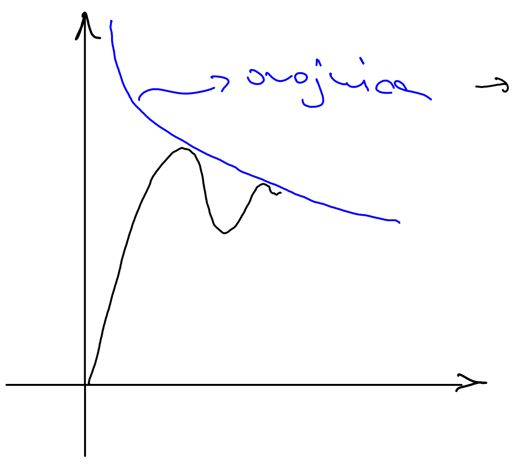
$$a^2+b^2=1 \qquad \frac{b}{a}=\operatorname{tg}0=0 \Rightarrow b=0$$

$$a=1$$

$$1+3j \Rightarrow \sqrt{1^2+3^2}\angle\operatorname{arctg}\frac{3}{1}=\sqrt{10}\angle 71^\circ$$

$$=\sqrt{10}\sin(\omega t+71^\circ)$$

$3\sqrt{10}\,e^{-2t}\sin(\omega t+71^\circ)\to$ ovojnica!



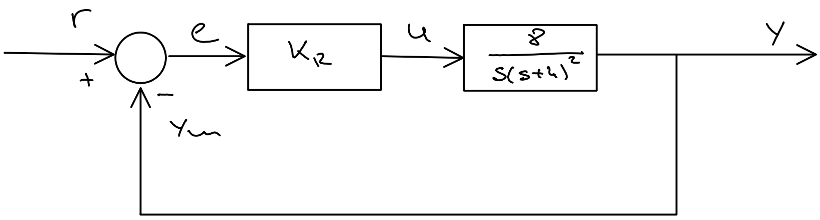
→ nju izjednačimo s 1,01 u(∞)

tj. tražimo vrijeme ustaljivanja: $t_{1\%}$

NYQUIST

3. prosinca 2008
22:05

Primjer : (4. DZ. → 2. Zadatak)



$K_R = 1{,}664$

$$G_o(j\omega) = \underbrace{Re\{G_o(j\omega)\}}_{R(\omega)} + \underbrace{Im\{G_o(j\omega)\}}_{I(\omega)}$$

$$G_o(j\omega) = \underbrace{\frac{-64\,k_R}{(\omega^2+16)^2}}_{R(\omega)} + j\,\underbrace{\frac{8\,k_R(\omega^2-16)}{\omega^5+32\omega^3+256\omega}}_{I(\omega)}$$

1) $\omega = 0^+$

$R(\omega = 0^+) = -0{,}416$

$I(\omega = 0^+) = -\infty$

2) $\omega = +\infty$

$R(\omega = +\infty) = 0$

$I(\omega = +\infty) = 0$

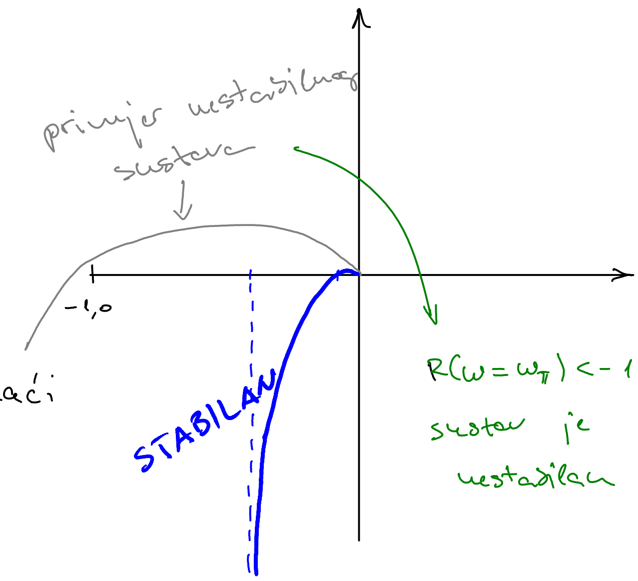
3) $R(\omega_1) = 0$

$\omega_1 \to$ ne može se naći

4) $I(\omega_2) = 0$

$\to \omega_2 = 4$

$R(\omega_2 = 4) = -0{,}10418$



neminimalnofazne nula → nula u desnoj
poluravnini PODBAČAJ

minimalnofazne nula →
NEMA PODBAČAJ