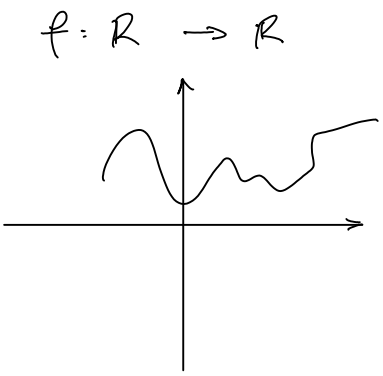
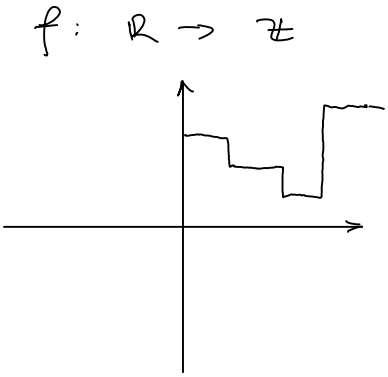
1. UVOD U DIGITALNE SUSTAVE UPRAVLJANJA

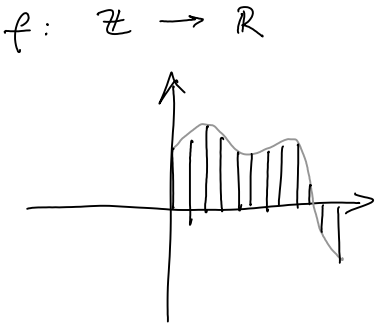
24. siječnja 2009
10:07



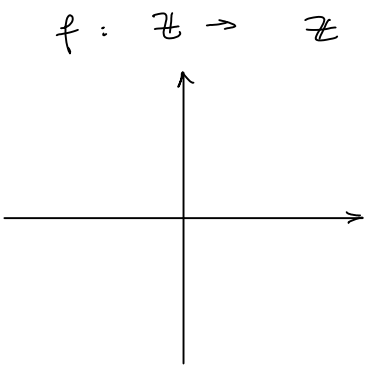
ANALOGNI SIGNAL



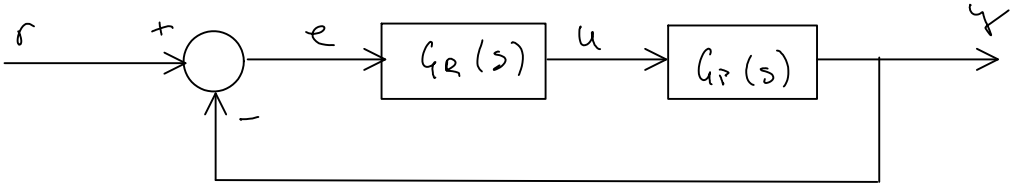
KVANTIZIRANI SIGNAL

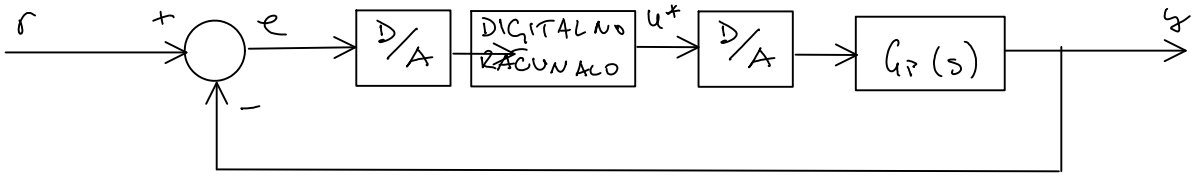


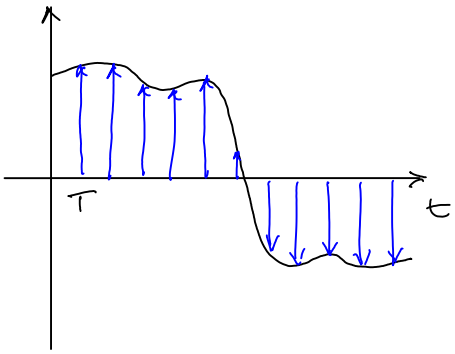
DISKRETNI SIGNAL

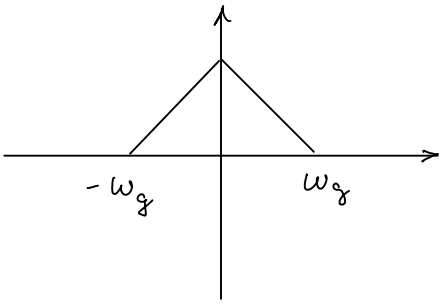


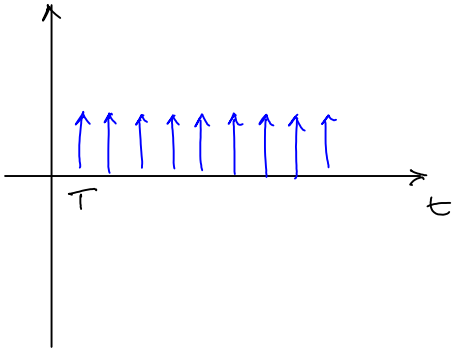
DIGITALNI SIGNAL

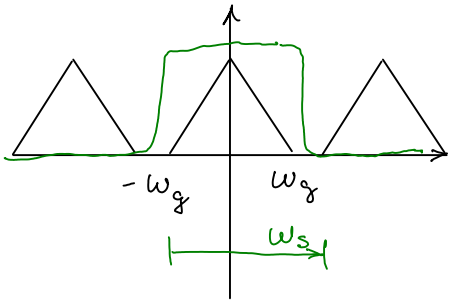


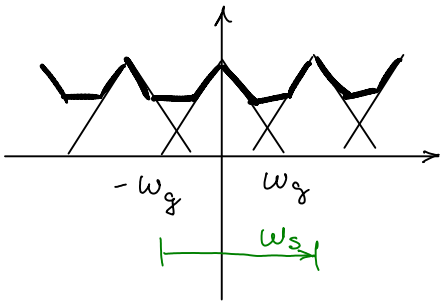






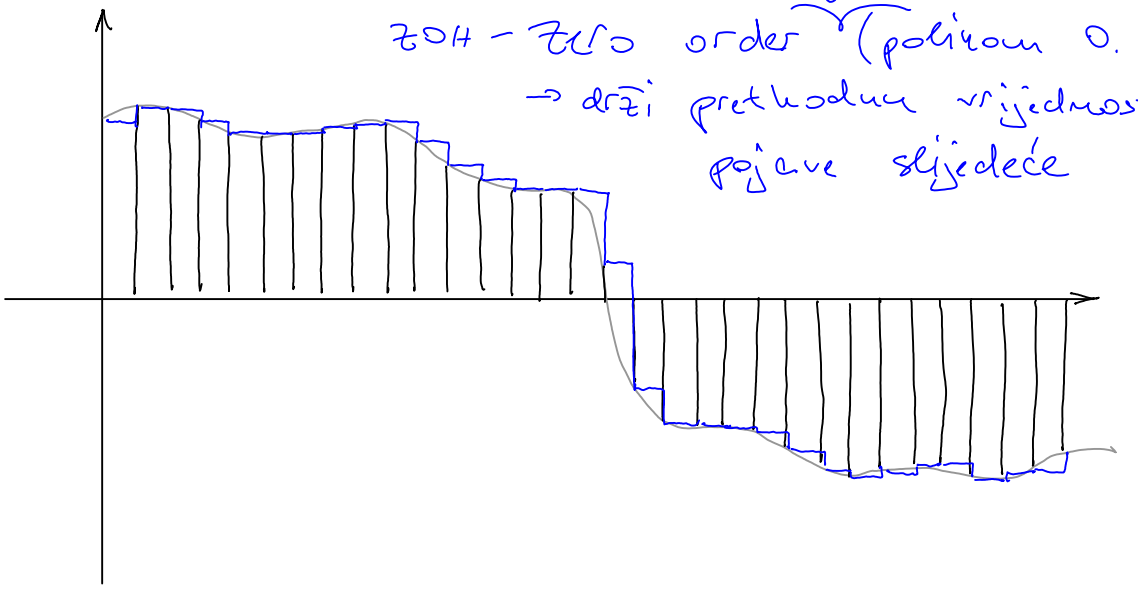




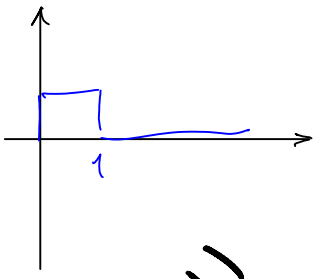


ALIASING

D/A PRETVORBA

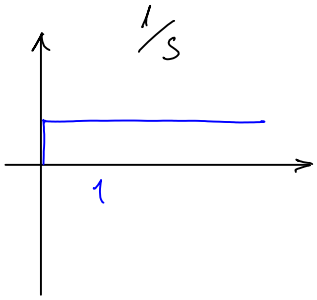


ZOH - Zero order hold (polinom 0. stupnja)
→ drži prethodnu vrijednost do pojave sljedeće



$$\frac{1}{s} - \frac{1}{s}e^{-s} = \frac{1-e^{-s}}{s}$$

=



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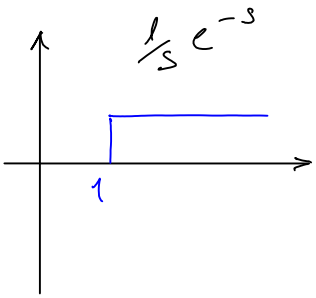
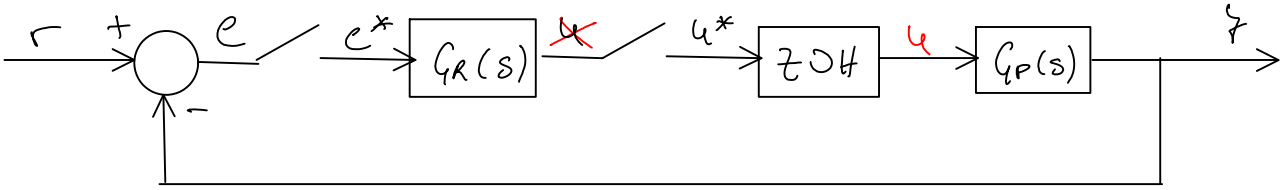
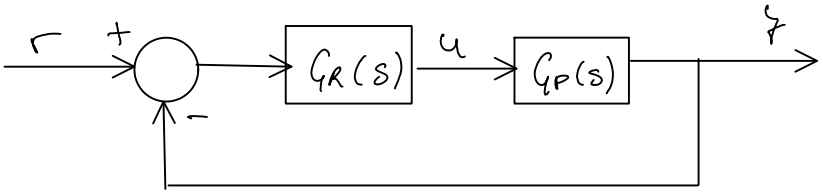






>> proces se v praksi diskretizira ZOH-om

15. PRESLIKAVANJE POLOVA I NULA IZ S U Z RAVNINU

24. siječnja 2009 11:07

$z = e^{sT}$; $T \rightarrow$ period otipkavanja

$s = \sigma + j\omega \rightarrow z = e^{(\sigma + j\omega)T} = e^{\sigma T} \cdot e^{j\omega T}$

$z = |z| e^{j \arg(z)}$

$\Rightarrow |z| = e^{\sigma T} \qquad \arg(z) = \omega T$

Primjer 1.

$s = 2 + j1$

$T = 1$

$z = e^2 e^{j1}$

$z = a + bj$ $\qquad \sqrt{a^2+b^2} = e^2 \quad /^2$

$a^2 + b^2 = e^4$

$\left(\frac{\pi^2}{16} + 1\right) a^2 = e^4$

$\arctg \frac{b}{a} = 1$

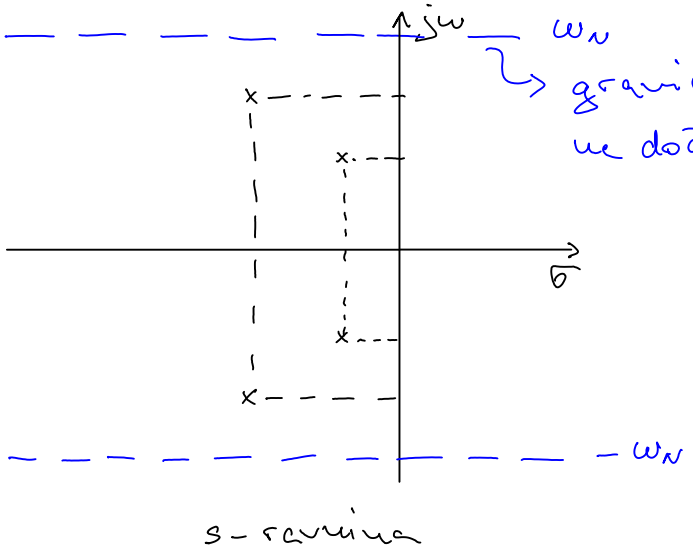
$\frac{b}{a} = \tg 1 = \frac{\pi}{4}$

$b = \frac{\pi}{4} a$

a i b na 5 decimala

$\omega_N = \frac{\omega_s}{2}$ → SHANNON

↳ NIQUIST

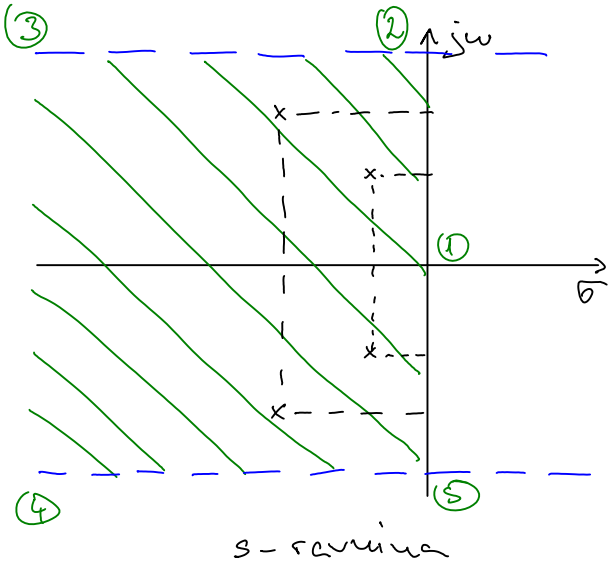


↳ granica da ne dođe do aliasinga

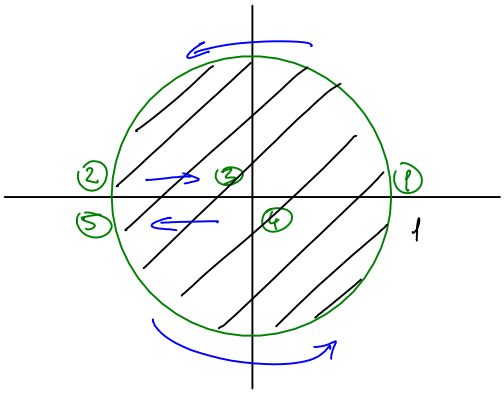
$\omega_s = \frac{2\pi}{T}$

↳ period otipkavanja

s - ravnina



s - ravnina



① $s = 0 + 0j$

$z = e^{0T} e^{j0T} =$

② $s = 0 + \omega_N j$ $\qquad \omega_N \cdot T = \frac{\omega_s}{2} \cdot \frac{2\pi}{\omega_s} = \pi$

$z = \underbrace{e^{0T}}_{1} e^{j\omega_N T}$ $\qquad \omega = \frac{\omega_N}{2} = \pi/2$

③ $s = -\infty + \omega_N j$

$z = \underbrace{e^{-\infty T}}_{0} e^{j\omega_N T} = 0$

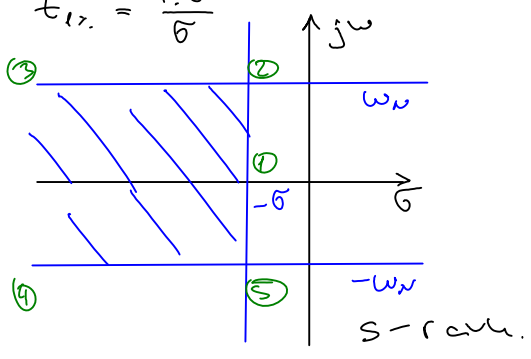
④ $s = -\infty - \omega_N j$

$z = 0$

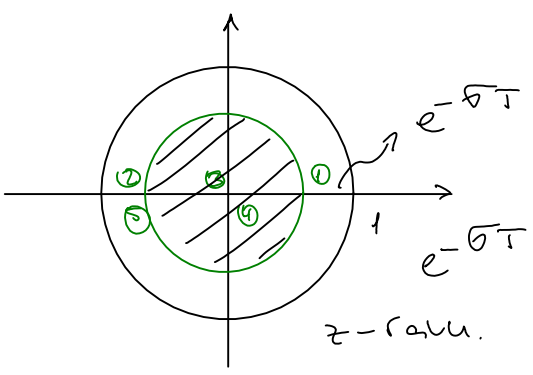
⑤ $s = 0 - \omega_N j$ = ②

- PRESLIKAVANJE DOZVOLJENOG VREMENA USTALJIVANJA

$t_{iz} = \frac{4.6}{\sigma}$



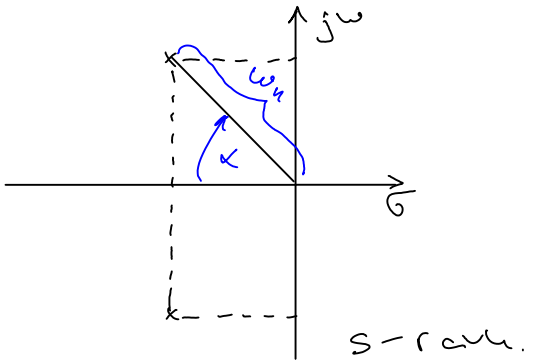
s - ravn.



z - ravn.

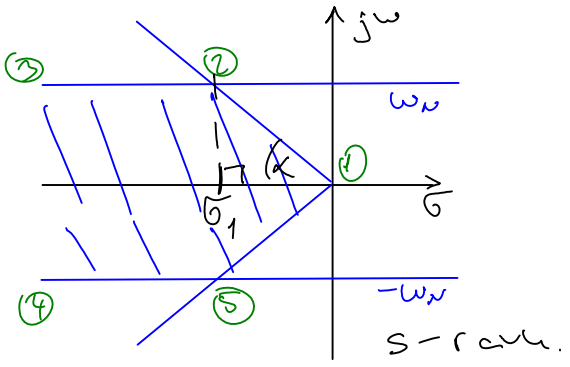
- PRESLIKAVANJE DOZVOLJENOG IZNOSA PRIGUŠENJA

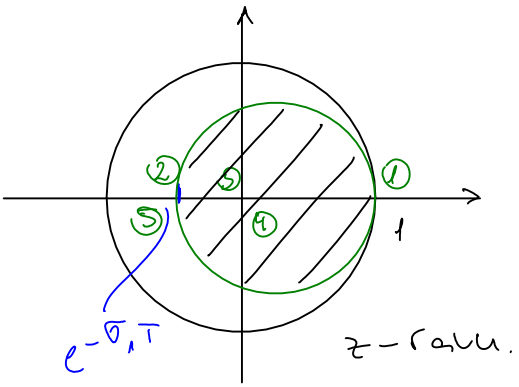
$$\sigma = e^{-\frac{\pi \xi}{\sqrt{1-\xi^2}}} \cdot 100 \quad [\%]$$



$$s = -\underbrace{\xi \omega_n}_{\sigma} \pm j \underbrace{\omega_n \sqrt{1-\xi^2}}_{\omega_d}$$

$$\alpha = \arccos \xi$$

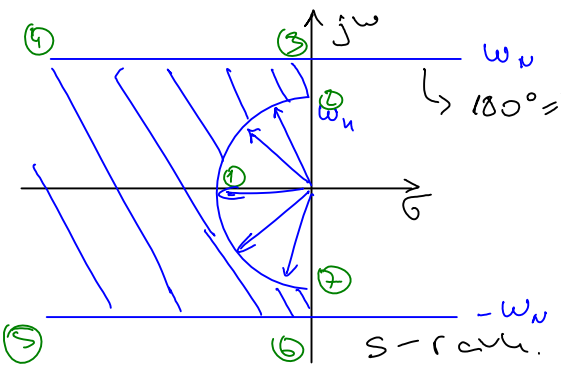




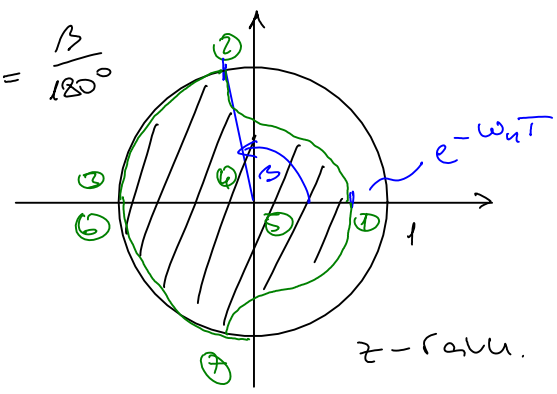
$$tg\alpha = \frac{\omega_N}{\sigma_1}$$

$\sigma_1 = \dots$
↳ $e^{-\sigma_1 T}$

- PRESLIKAVANJE PODRUČJA DOZVOLJENOG VREMENA PORASTA $t_r = \frac{1,8}{\omega_n}$



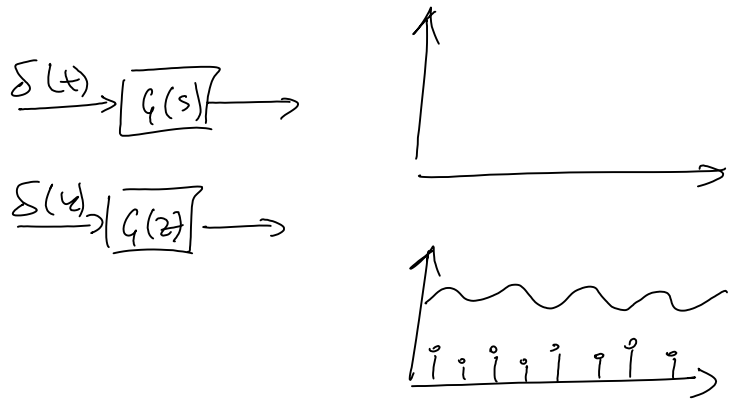
↳ $180° \Rightarrow \frac{\omega_n}{\omega_N} = \frac{\beta}{180°}$



16. POSTUPCI DISKRETIZACIJE KONT. SUSTAVA

24. siječnja 2009
12:23

(1.1) ... TEŽINSKE FUNKCIJE (očuvane svojstva)



$$G(z) = \mathcal{Z}\{G(s)\}$$

$$G(s) = \frac{K}{s+a}$$

$$G(z) = K\frac{z}{z-e^{-aT}}$$

(1.2) ... PRIJELAZNE FUNKCIJE (očuvane svojstva)

(ZOH - DISKRETIZACIJA)

$$G(z) = (1-z^{-1})\,\mathcal{Z}\left\{\frac{G(s)}{s}\right\}$$

Primjer 2.

$$G(s) = \frac{2}{s+2} \quad ; \quad T = 1s$$

$$\mathcal{Z}\left\{\frac{2}{s(s+2)}\right\} \qquad \frac{C_{11}}{s} + \frac{C_{21}}{s+2} \quad ; \quad C_{11} = 1 \qquad C_{21} = -1$$

$$= \mathcal{Z}\left\{\frac{1}{s+0} - \frac{1}{s+2}\right\} = \frac{z}{z-1} - \frac{z}{z-e^{-2}}$$

$$G(z) = (1-z^{-1})\left(\frac{z}{z-1} - \frac{z}{z-e^{-2}}\right)$$

(2.1) POSTUPAK USKLAĐENIH POLOVA I NULA

$$G(s) = \frac{\prod_{i=1}^{m} (s - s_{Ni})}{\prod_{i=1}^{n} (s - s_{Pi})}$$

m – broj nula

n – broj polova

$n \geq m \Rightarrow$ nekauzalan sustav

1) $z_{Pi} = e^{s_{Pi} T}$; $z_{Ni} = e^{s_{Ni} T}$

2) $n - m > 1 \Rightarrow$ postavljamo $n - m - 1$ nula u $z = -1$

$$G(z) = K^* \frac{(z+1)^{\max(n-m-1)} \prod_{i=1}^{m} (z - z_{Ni})}{\prod_{i=1}^{n} (z - z_{Pi})}$$

$\lim_{s \to 0} |G(s)| = \lim_{z \to 1} |G(z)| \Rightarrow$ iz ovoga dobijemo K^*

Primjer 3.

$$G(s) = 5 \frac{s+3}{(s+1)(s+2)} \quad ; \quad T = 1s$$

$n = 1$; $n = 2$

1) $z_{P1} = e^{-1}$

$z_{P2} = e^{-2}$

$z_{N1} = e^{-3}$

2) $n - m = 1 \not> 1$

$$G(z) = K^* \frac{z - e^{-3}}{(z - e^{-1})(z - e^{-2})}$$

Primjer 4.

$$G(s) = \frac{(s+5)}{(s+1)(s+2)(s+3)(s+4)}$$

$n = 1$

$n = 4$

$n - m = 4 - 1 = 3 > 1$ ✓

$n - m - 1 = 2$

$$G(z) = K^* \frac{(z+1)^2 (z - e^{-5})}{(z - e^{-1})(z - e^{-2})(z - e^{-3})(z - e^{-4})}$$

(3.1) DISKRETIZACIJA TUSTINOVIM POSTUPKOM

$$s = \frac{2}{T}\,\frac{z-1}{z+1}$$

(3.2) UNAPRIJEDNA DIFERENCIJA

$$s = \frac{z-1}{T}$$

(3.3) UNAZADNA DIFERENCIJA

$$s = \frac{z-1}{zT}$$

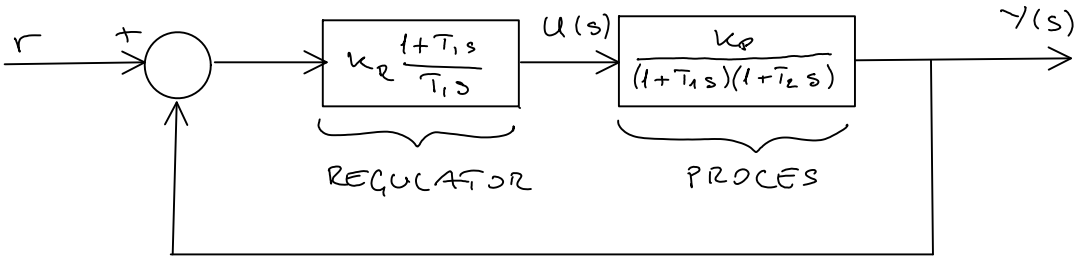
BILINEARNE TRANSFORMACIJE:

$$z = \frac{1 + \Omega\frac{T}{2}}{1 - \Omega\frac{T}{2}}$$

Pri crtanju Bodea: umjesto $\Omega \rightarrow s$

5. DOMAĆA ZADAĆA

24. siječnja 2009
13:25



$K_P = 3$

$T_1 = T_1 = 1\,s$

$T_2 = 0{,}3\,s$

a) $\gamma = 60°$ (fazno osiguranje)

$\gamma = \pi + \arg(G_0(j\omega_c))$

$G_0(s) = K_R \frac{s+1}{s} \cdot \frac{3}{(s+1)(1+0{,}3s)}$

$= \frac{3K_R}{(1+0{,}3s)s}$

$|G_0(j\omega_c)| = 1$

$G_0(j\omega_c) = \frac{3K_R}{j\omega_c(1+j0{,}3\omega_c)}$

$\varphi_0 = \underbrace{\arctg\frac{0}{3K_R}}_{0} - \underbrace{\arctg\frac{\omega_c}{0}}_{\pi/2} - \arctg(0{,}3\omega_c)$

$\varphi_0(\omega_c) = \arg(G_0(j\omega_c)) = \frac{\pi}{2} - \arctg(0{,}3\omega_c) = -120$

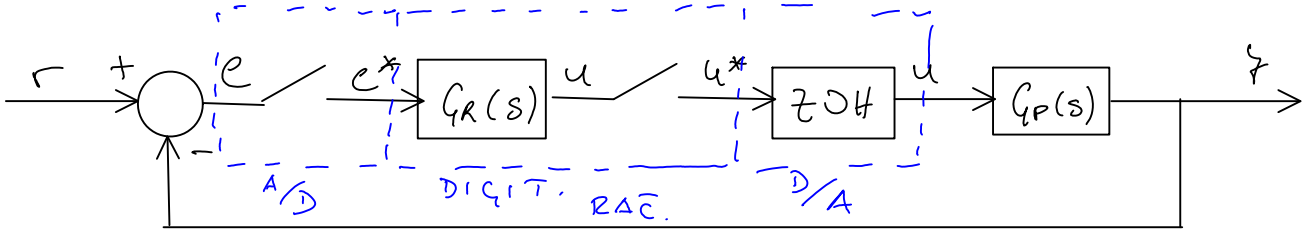
$\omega_c = 1{,}9245\ s^{-1}$

$|G_0(j\omega_c)| = \frac{3K_R}{\omega_c\sqrt{1+(0{,}3\omega_c)^2}} = 1$

$K_R = 0{,}74074$

$G_R(s) = 0{,}74074\ \frac{s+1}{s}$

vremenski kontinirana domena



$T = (0{,}17 \sim 0{,}34)\ 1/\omega_c$

$T = 88\,ms \sim 170\,ms \rightarrow T_f = 100\,ms = 0{,}1s$

$G_R(s) = 0{,}74074\ \frac{s+1}{s}$

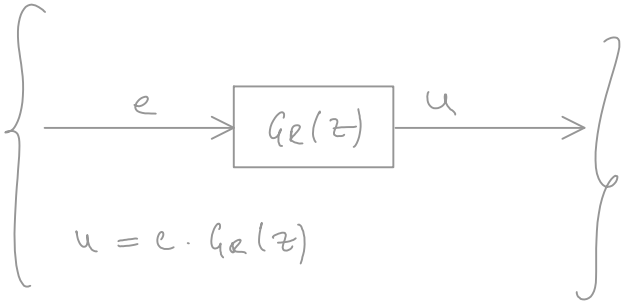
a) Tustinova relacija

$$s = \frac{2}{T}\ \frac{z-1}{z+1}$$

$$G_R(z) = 0{,}74074\ \frac{\frac{2}{0{,}1}\cdot\frac{z-1}{z+1}+1}{\frac{2}{0{,}1}\cdot\frac{z-1}{z+1}} = 0{,}77749\ \frac{1-0{,}90436\,z^{-1}}{1-z^{-1}}$$

karakteristični polinom regulacijskog odstupanja e

u

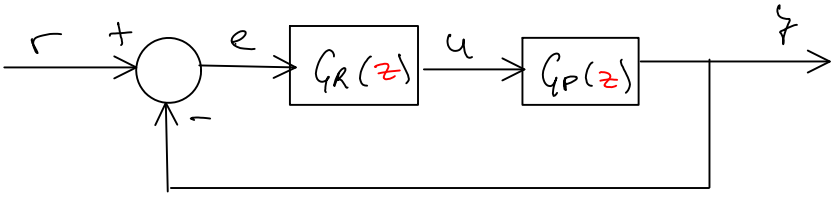


$z^{-1} \rightarrow y(k-1)$

$u(k) - u(k-1)$

$$u(k) - u(k-1) = 0{,}77749\ e(k) - 0{,}70344\ e(k-1) + u(k-1)$$

e)



$$G_P(s) = \frac{3}{(s+1)(1+0{,}3s)} = \frac{3}{(s+1)\frac{3}{10}(s+\frac{10}{3})} = \frac{10}{(s+1)(s+\frac{10}{3})}$$

$$\frac{G_P(s)}{s} = \frac{10}{s(s+1)(s+10/3)} = \frac{C_{11}}{s} + \frac{C_{21}}{s+1} + \frac{C_{31}}{s+10/3}$$

$C_{11} = 3 \qquad C_{21} = -4,28571 \qquad C_{31} = 1,28571$

$$\frac{G_P(s)}{s} = \frac{3}{s+0} - \frac{4,28571}{s+1} + \frac{1,28571}{s+10/3}$$

$$\mathcal{Z}\left\{\frac{3}{s+0}\right\} = \frac{3z}{z-1}$$

$$\mathcal{Z}\left\{\frac{-4,\dots}{s+1}\right\} = \frac{-4,\dots z}{z-e^{-0,1}} = 0,90$$

$$\mathcal{Z}\left\{\frac{1,28\dots}{s+10/3}\right\} = \frac{1,28\dots z}{1-e^{-\frac{10}{3}\cdot\frac{1}{10}}} = \dots$$

$G_P(z) = (1-z^{-1})(\dots)$

OBAVEZNO POKRATITI NULTOČKE AKO JE MOGUĆE

f) Jurgev = Jusin kriterij !!!

$$G_P(z) = \frac{0,77749\ (z-0,90476)}{(z-1)} \cdot \frac{0,04320(z+0,87245)}{(z-0,90476)(z-0,71651)}$$

$$G_0(z) = \frac{0,003359\,z + 0,02930}{z^2 - 1,71651\,z + 0,71651}$$

$f(z) =$ brojnik $(G_0(z))$ + nazivnik $(G_0(z))$

$$f(z) = z^2 - 1,68292\,z + 0,74581$$

$a_2 \qquad a_1 \qquad a_0$

$u = 2$

1) $f(1) = 0,06289 > 0$ ✓

$(-1)^u f(-1) = 3,42873 > 0$ ✓

2)

REDAK	z^0	z^1	z^2	...	z^{n-2}	z^{n-1}	z^n
1	a_0	a_1	a_2	...	a_{n-2}	a_{n-1}	a_n
2	a_n	a_{n-1}	...		a_2	a_1	a_0
3	b_0	b_1	b_2	...	b_{n-2}	b_{n-1}	X
4	b_{n-1}	b_{n-2}	b_{n-3}	...	b_1	b_0	X
5							
6							
⋮							

REDAK	z^0	z^1	z^2
1	a_0	a_1	a_2

$|a_0| < |a_2|$ ✓

SUSTAV JE STABILAN ✓

(g) $z = \frac{1+\Omega\frac{T}{2}}{1-\Omega\frac{T}{2}}$

$$G_0(z) = \frac{0{,}03359\,(z+0{,}87228)}{(z-0{,}71651)(z-1)}$$

$$G_0(\Omega) = 2{,}21842\,\frac{\left(1+\frac{\Omega}{293{,}00156}\right)\left(1-\frac{\Omega}{20}\right)}{\Omega\left(1+\frac{\Omega}{3{,}30292}\right)}$$

↳ modificirana bilinearna transformacija

potrebno za npr. Bodeov dijagram

$\rightarrow s = j\omega$

$\Omega = j\omega^*$

$$G_0(j\omega^*) = 2{,}21842\,\frac{\left(1+j\frac{\omega^*}{293{,}00156}\right)\left(1-j\frac{\omega^*}{20}\right)}{j\omega^*\left(1+j\frac{\omega^*}{3{,}30292}\right)}$$

$|G_0(j\omega_c^*)| = 1$

$\gamma = \pi + \arg\left(G_0(j\omega_c^*)\right) = 55°$

u slučaju:

$$\lim_{s\to 0} \frac{5}{s(s+1)} = \lim_{z\to 1} \frac{K * z}{(z-1)(z-2)}$$

dalje normalno...