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2. Domaća zadaća – Grupa B

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1. Zadana je nelinearna diferencijalna jednadžba drugog reda:

$$4y''(t) + 4y'(t)\ln y(t) + 3y(t) = 3e^{u(t)}$$

a) Potrebno je linearizirati nelinearnu diferencijalnu jednadžbu u okolini radne točke određene s $u_0 = 2$.

$$3y_0 = 3e^{u_0}\,, \qquad y_0 = e^{u_0} = e^2, \qquad (y_0'' = y_0' = 0)$$

$$y = \Delta y + y_0, \qquad y' = \Delta y', \qquad y'' = \Delta y'', \qquad u = \Delta u + u_0$$

$$y'' = \frac{3}{4}e^u - y'\ln y - \frac{3}{4}y = f(u, y, y')$$

$$\Delta y'' = \underbrace{f(u_0, y_0, 0)}_{0} + \left.\frac{\partial f}{\partial u}\right|_{S.T.} \Delta u + \left.\frac{\partial f}{\partial y}\right|_{S.T.} \Delta y + \left.\frac{\partial f}{\partial y'}\right|_{S.T.} \Delta y'$$

$$\Delta y'' = \frac{3}{4}e^{u_0}\Delta u + \left(-\frac{y_0'}{y_0} - \frac{3}{4}\right)\Delta y + (-\ln y_0)\Delta y'$$

$$\Delta y'' + 2\Delta y' + \frac{3}{4}\Delta y = \frac{3}{4}e^2\Delta u$$

b) Primjeniti Laplaceovu transformaciju na diferencijalnu jednadžbu dobivenu pod a) te odrediti prijenosnu funkciju $G(s) = \frac{Y(s)}{U(s)}$, pri čemu je $Y(s) = \mathcal{L}\{\Delta y(t)\}$ i $U(s) = \mathcal{L}\{\Delta u(t)\}$.

$$\mathcal{L}\{\Delta y(t)'\} = sY(s), \qquad \mathcal{L}\{\Delta y(t)''\} = s^2Y(s)$$

$$s^2Y(s) + 2sY(s) + \frac{3}{4}Y(s) = \frac{3}{4}e^2U(s)$$

$$\frac{Y(s)}{U(s)} = G(s) = \frac{\frac{3}{4}e^2}{s^2 + 2s + \frac{3}{4}}$$

c) Odrediti odziv lineariziranog modela na pobudu Δu prikazanu slikom 2.1. te na temelju aproksimacije linearizacijom skicirati odziv nelinearnog modela na pobudu $u = \Delta u + u_0$.

$$\Delta u(t) = \begin{cases} 0 & t < 0 \\ t & 0 \le t \le 2 \\ 2 & 2 \le t \end{cases}, \qquad \Delta u(t) = t[S(t) - S(t-2)] + 2S(t-2) = tS(t) - (t-2)S(t-2)$$

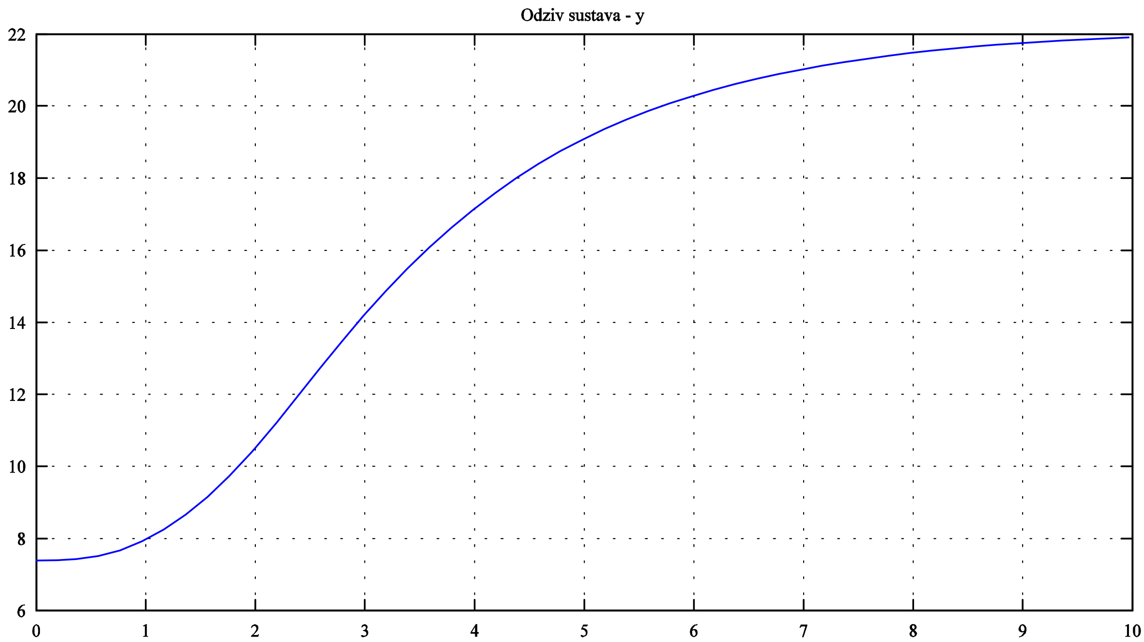
$$\mathcal{L}\{\Delta u(t)\} = U(s) = \frac{1}{s^2} - \frac{1}{s^2}e^{-2s}$$

$$Y(s) = U(s)G(s) = \frac{1}{s^2}\frac{\frac{3}{4}e^2}{s^2 + 2s + \frac{3}{4}} - \frac{1}{s^2}\frac{\frac{3}{4}e^2}{s^2 + 2s + \frac{3}{4}}e^{-2s}$$

$$\frac{\frac{3}{4}}{s^2\left(s+\frac{1}{2}\right)\left(s+\frac{3}{2}\right)} = \frac{C_{11}}{s} + \frac{C_{12}}{s^2} + \frac{C_{21}}{s+\frac{1}{2}} + \frac{C_{31}}{s+\frac{3}{2}} = \frac{-\frac{8}{3}}{s} + \frac{1}{s^2} + \frac{3}{s+\frac{1}{2}} + \frac{-\frac{1}{3}}{s+\frac{3}{2}}$$

$$\mathcal{L}^{-1}\left\{\frac{-\frac{8}{3}}{s} + \frac{1}{s^2} + \frac{3}{s+\frac{1}{2}} + \frac{-\frac{1}{3}}{s+\frac{3}{2}}\right\} = \left[-\frac{8}{3} + t + 3e^{-\frac{1}{2}t} - \frac{1}{3}e^{-\frac{3}{2}t}\right]S(t)$$

$$\mathcal{L}^{-1}\{\Delta y(t)\} = e^2\left[-\frac{8}{3} + t + 3e^{-\frac{1}{2}t} - \frac{1}{3}e^{-\frac{3}{2}t}\right]S(t)$$
$$- e^2\left[-\frac{8}{3} + (t-2) + 3e^{-\frac{1}{2}(t-2)} - \frac{1}{3}e^{-\frac{3}{2}(t-2)}\right]S(t-2)$$

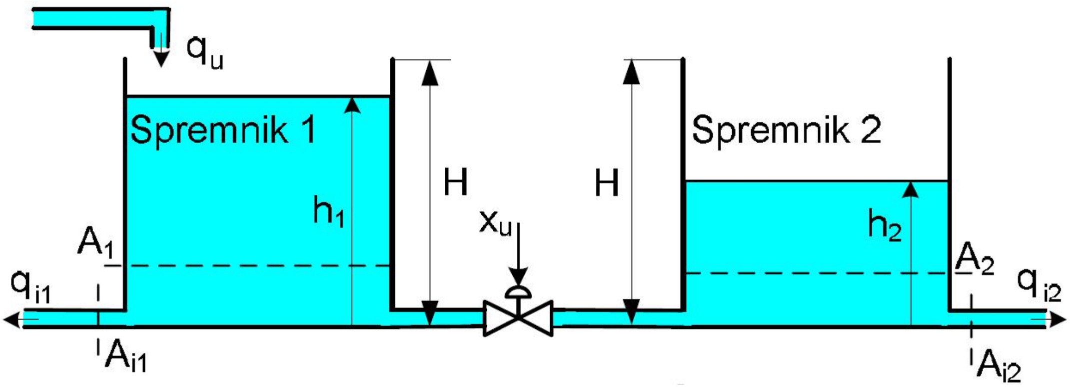


Slika 2.1. Odziv lineariziranog sustava - $y(t) = y_0 + \Delta y(t)$

d) Odrediti stacionarnu vrijednost odziva lineariziranog modela te nagibe tog odziva u trenucima $t = 0\ s$ i $t = 2\ s$.

$$\Delta y_{S.S.} = \lim_{t\to\infty} \Delta y(t) = 2e^2, \qquad \Delta y'(0) = 0, \qquad \Delta y'(2) = 3.4956$$

2. Za sustav skladištenja fluida modeliran nelinearnim modelom u 1. domaćoj zadaći (Slika 2.3) potrebno je:



Slika 2.2. Shema sustava skladištenja fluida

a) Linearizirati nelinearni matematički model u stacionarnoj radnoj točki određenoj otvorenošću ventila na polovici njegovog dozvoljenog radnog područja otvorenosti.

Dozvoljeno područje otvorenosti ventila je

$$x_u \in [20.934,100]\ [\%]$$

Prema tome, stacionarne vrijednosti su

$$x_{u0} = 60.467\ [\%], \qquad h_{10} = 4.335\ [m], \qquad h_{20} = 3.015\ [m], \qquad q_{u0} = 40\ [kg/s]$$

Spremnik 1.

$$\frac{dh_1}{dt} = \frac{1}{A_1 \rho} q_u - \frac{A_{i1}\sqrt{2g}}{A_1}\sqrt{h_1} - \frac{A_v\sqrt{2g}}{A_1}\sqrt{(h_1 - h_2)} \cdot \frac{x_u}{100\%}$$

$$\Delta h_1' = \underbrace{\not{f}(h_{10}, h_{20}, x_{u0})}_{0} + \left.\frac{\partial \not{f}}{\partial h_1}\right|_{S.T.} \Delta h_1 + \left.\frac{\partial \not{f}}{\partial h_2}\right|_{S.T.} \Delta h_2 + \left.\frac{\partial \not{f}}{\partial x_u}\right|_{S.T.} \Delta x_u$$

$$\Delta h_1' + x_1 \Delta h_1 - x_2 \Delta h_2 = -y_0 \Delta x_u, \qquad x_1 = \frac{A_{i1}\sqrt{2g}}{A_1}\frac{1}{2\sqrt{h_{10}}} + \frac{A_v\sqrt{2g}}{A_1}\frac{1}{2\sqrt{h_{10} - h_{20}}} \cdot \frac{x_{u0}}{100\%}$$

$$x_2 = \frac{A_v\sqrt{2g}}{A_1}\frac{1}{2\sqrt{h_{10} - h_{20}}} \cdot \frac{x_{u0}}{100\%}, \qquad y_0 = \frac{A_v\sqrt{2g}}{A_1}\sqrt{(h_{10} - h_{20})} \cdot \frac{1}{100\%}$$

Spremnik 2.

$$\frac{dh_2}{dt} = \frac{A_v\sqrt{2g}}{A_2}\sqrt{h_1 - h_2} \cdot \frac{x_u}{100\%} - \frac{A_{i2}\sqrt{2g}}{A_2}\sqrt{h_2}$$

$$\Delta h_2' = \underbrace{f(x_{u0}, h_{10}, h_{20})}_{0} + \frac{\partial f}{\partial h_1}\bigg|_{S.T.} \Delta h_1 + \frac{\partial f}{\partial h_2}\bigg|_{S.T.} \Delta h_2 + \frac{\partial f}{\partial x_u}\bigg|_{S.T.} \Delta x_u$$

$$\Delta h_2' + x_3 \Delta h_2 - x_4 \Delta h_1 = y_1 \Delta x_u, \qquad x_3 = \frac{A_{i2}\sqrt{2g}}{A_2}\frac{1}{2\sqrt{h_{20}}} + \frac{A_v\sqrt{2g}}{A_2}\frac{1}{2\sqrt{h_{10}-h_{20}}} \cdot \frac{x_{u0}}{100\%}$$

$$x_4 = \frac{A_v\sqrt{2g}}{A_2}\frac{1}{2\sqrt{h_{10}-h_{20}}} \cdot \frac{x_{u0}}{100\%}, \qquad y_1 = \frac{A_v\sqrt{2g}}{A_2}\sqrt{(h_{10}-h_{20})} \cdot \frac{1}{100\%}$$

Vrijednosti pojedinih koeficijenata su

$$x_1 = 0.0032, \qquad x_2 = 0.0029, \qquad x_3 = 0.0042, \qquad x_4 = 0.0029, \qquad y_0 = y_1 = 1.2723 \cdot 10^{-4}$$

b) Odrediti matrice A, B, C i D iz zapisa lineariziranog sustava po varijablama stanja:

$$\dot{x} = Ax + Bu$$
$$y = Cx + Du$$

pri čemu su vektori stanja, ulaza i izlaza definirani kao $x = [\Delta h_1 \quad \Delta h_2]^T$, $u = [\Delta x_u]$ i $y = [\Delta h_1]$.

$$\Delta h_1' = -x_1 \Delta h_1 + x_2 \Delta h_2 - y_0 \Delta x_u$$

$$\Delta h_2' = x_4 \Delta h_1 - x_3 \Delta h_2 + y_1 \Delta x_u$$

$$y = \Delta h_1$$

Prema prikazanim jednadžbama slijedi zapis sustava u matričnom obliku

$$\begin{bmatrix} \Delta h_1' \\ \Delta h_2' \end{bmatrix} = \begin{bmatrix} -x_1 & x_2 \\ x_4 & -x_3 \end{bmatrix} \begin{bmatrix} \Delta h_1 \\ \Delta h_2 \end{bmatrix} + \begin{bmatrix} -y_0 \\ y_1 \end{bmatrix} \Delta x_u$$

$$[y] = [1 \quad 0] \begin{bmatrix} \Delta h_1 \\ \Delta h_2 \end{bmatrix} + [0] \Delta x_u$$

c) Odrediti prijenosnu funkciju $G(s) = \frac{H_1(s)}{X_u(s)}$, uz $H_1(s) = \mathcal{L}\{\Delta h_1\}$ i $X_u(s) = \mathcal{L}\{\Delta x_u\}$.

$$sH_1(s) + x_1 H_1(s) + b_0 X_u(s) = x_2 H_2(s), \qquad H_2(s) = \frac{s + x_1}{x_2} H_1(s) + \frac{y_0}{x_2} X_u(s)$$

$$sH_2(s) + x_3 H_2(s) - x_4 H_1(s) = y_1 X_u(s), \qquad H_2(s)(s + x_3) - x_4 H_1(s) = y_1 X_u(s)$$

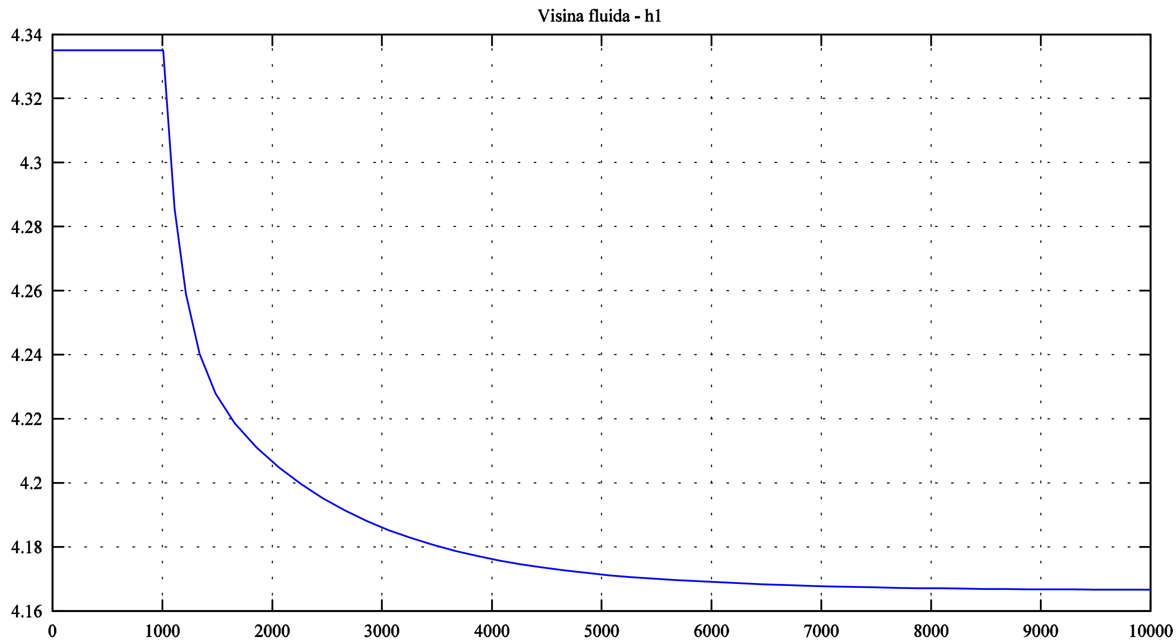
$$H_1(s)[s^2 + s(x_1 + x_3) + (x_1 x_3 - x_2 x_4)] = X_u(s)[-y_0 s + x_2 y_1 - x_3 y_0]$$

$$G(s) = \frac{H_1(s)}{X_u(s)} = \frac{-y_0 s + x_2 y_1 - x_3 y_0}{s^2 + s(x_1 + x_3) + (x_1 x_3 - x_2 x_4)}$$

d) Odrediti odziv razine fluida u prvom spremniku lineariziranog modela na skokovitu promjenu otvorenosti ulaznog ventila $\Delta x_u = 5$ [%] te ga skicirati.

$$X_u(s) = \frac{5}{s}, \qquad H_1(s) = X_u(s)G(s) = \frac{5}{s}\frac{-y_0 s + x_2 y_1 - x_3 y_0}{s^2 + s(x_1 + x_3) + (x_1 x_3 - x_2 x_4)}$$

$$\mathcal{L}^{-1}\{H_1(s)\} = \Delta h_1(t) = \left(-0.1644 + 0.0775 e^{-7.5721\cdot 10^{-4} t} + 0.0869 e^{-6.6428\cdot 10^{-3} t}\right) S(t)$$



Slika 2.3. Visina razine fluida u prvom spremniku $h_1(t)$

e) Odrediti stacionarnu vrijednost odziva lineariziranog modela za pobudu pod d), kao i nagib odziva u trenutku $t = 0$ $[s]$. Odredite razliku u stacionarnom stanju između odziva dobivenim aproksimacijom sustava linearnim modelom u okolini radne točke i odziva stvarno modela.

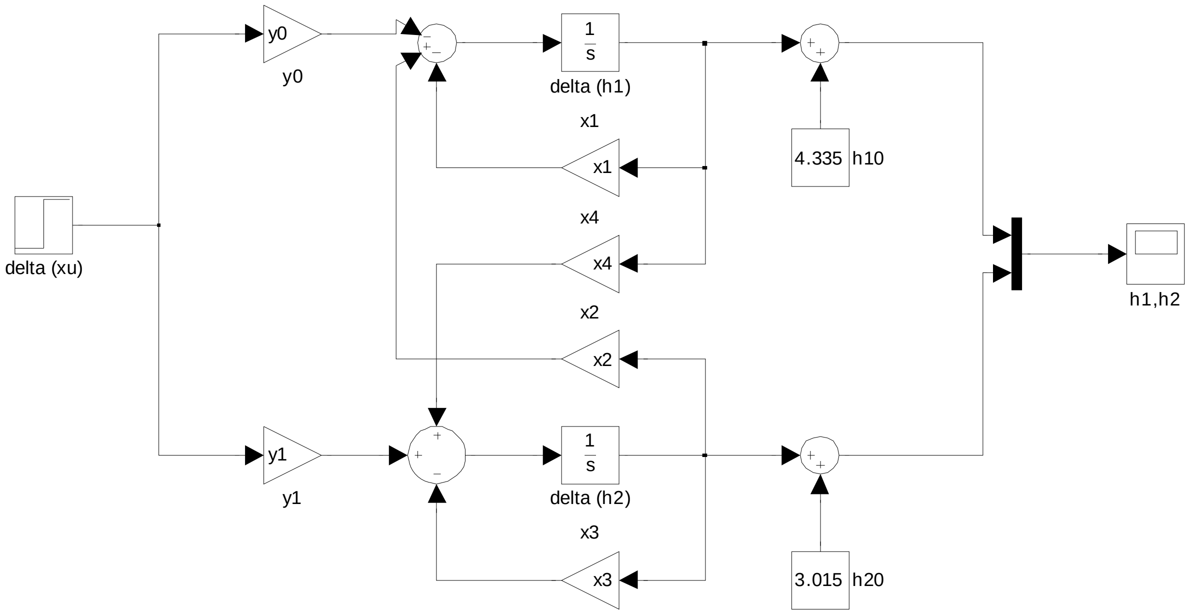
$$\Delta h_{1_{SS}} = \lim_{s\to 0} s H_1(s) = \frac{5(x_2 y_1 - x_3 y_0)}{x_1 x_3 - x_2 x_4}, \qquad \Delta h_{1_{SS}} = -0.1644\ [m]$$

$$\Delta h_1'(0) = \lim_{s\to\infty} s^2 H_1(s) = \lim_{s\to\infty} \frac{-5 y_0 s^2 + 5(x_2 y_1 - x_3 y_0)s}{s^2 + s(x_1 + x_3) + (x_1 x_3 - x_2 x_4)},$$
$$\Delta h_1'(0) = -5 y_0 = -6.36 \cdot 10^{-4}$$

$$h_{1_{[N.M.]}} = 4.185\ [m], \qquad h_{1_{[L.M.]}} = h_{10} + \Delta h_1 = 4.171\ [m]$$

$$err. = h_{1_{[N.M.]}} - h_{1_{[L.M.]}} = 1.4\ [cm]$$

f) Nacrtajte simulacijsku shemu lineariziranog modela sustava skladištenja fluida.



Slika 2.4. Simulacijska shema lineariziranog modela