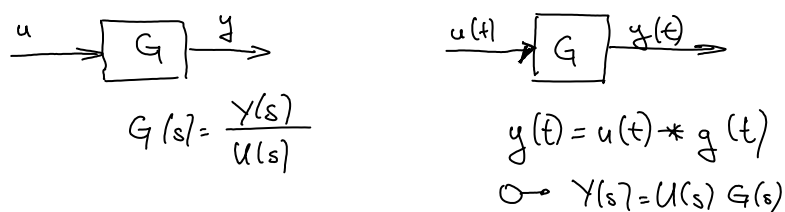
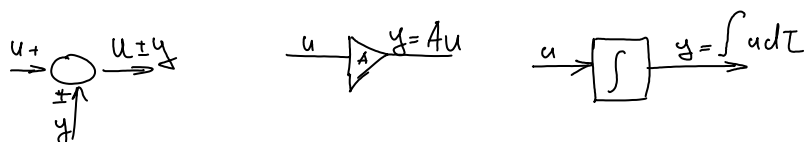


# Automatsko upravljanje

## ④ Formalni prikaz SAU



Impulzni odziv  $\leftarrow g(t) \Leftrightarrow G(s)$

$h(t)$  - prijelazna funkcija  
 $\hookrightarrow u(t) = \mu(t) = \delta(t)$

$$y'' + a_1 y' + a_2 y = b_0 u'' + b_1 u' + b_2 u$$

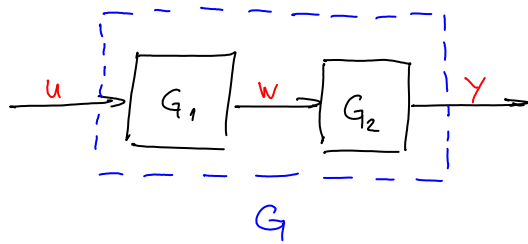
$$Y(s) (s^2 + a_1 s + a_2) = U(s) (b_0 s^2 + b_1 s + b_2)$$

$$\frac{Y(s)}{U(s)} = \frac{b_0 s^2 + b_1 s + b_2}{s^2 + a_1 s + a_2}$$

Karakteristična jedn. ulaza  
→  
karak. jed. izlaza

$$\begin{aligned} y &\Leftrightarrow Y(s) \\ y' &\Leftrightarrow sY(s) \\ y'' &\Leftrightarrow s^2 Y(s) \end{aligned}$$

### ① Kaskadni spoj



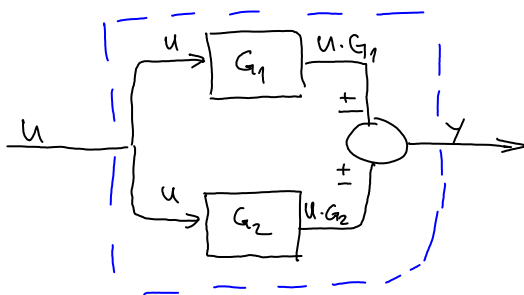
$$w = u \cdot G_1$$

$$y = w \cdot G_2$$

$$y = u \cdot G_1 \cdot G_2$$

$$\frac{y}{u} = G_1 \cdot G_2 = G$$

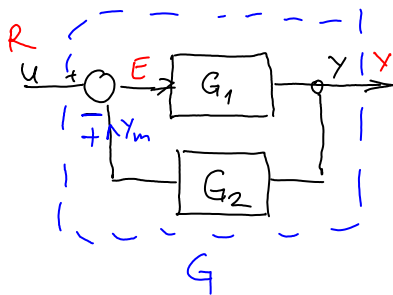
### ② Paralelni spoj



$$y = \pm u G_1 \pm u G_2$$

$$\frac{y}{u} = \pm G_1 \pm G_2$$

### ③ Povratna veza



$$y = E \cdot G_1$$

$$E = R - y_m, \quad y_m = y \cdot G_2$$

$$E = R - y G_2$$

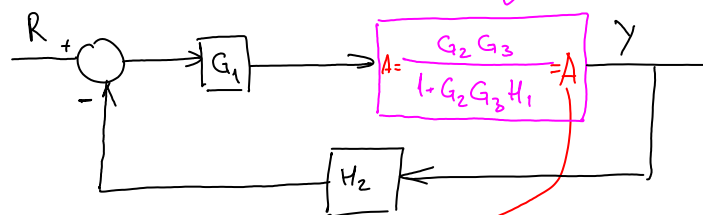
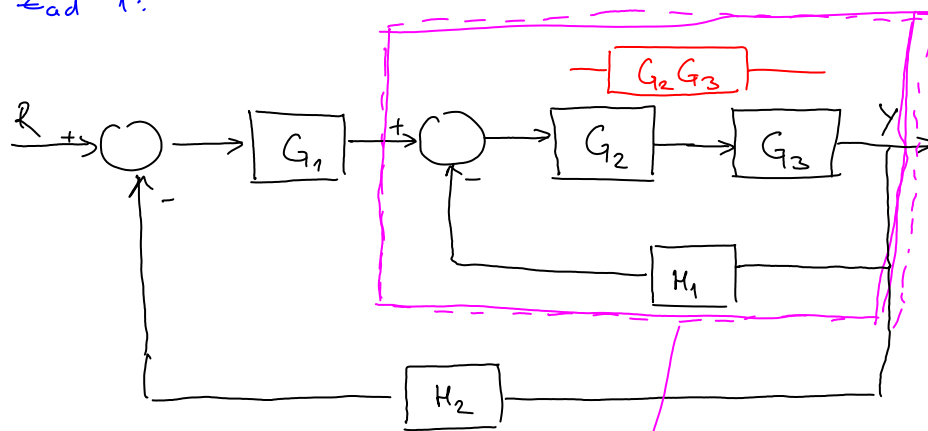
$$y = (R - y G_2) G_1$$

$$\frac{y}{R} = \frac{G_1}{1 \pm G_1 G_2}$$

30.rujan 2009

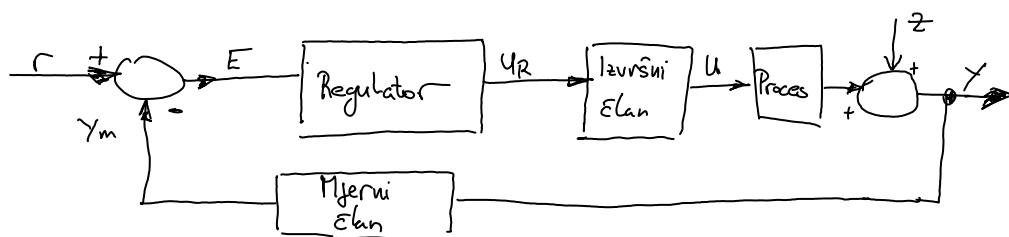
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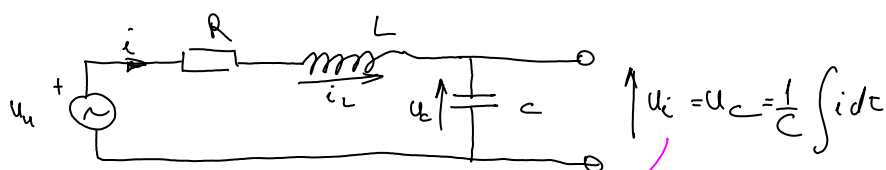


$$G = \frac{A \cdot G_1}{1 + H_2 A G_1}$$

## Osnovna struktura sustava upravljanja



### • Strujni krugovi



$$u_n = u_R + u_L + u_C$$

$$u_L = L \frac{di}{dt}$$

$$u_C = \frac{1}{C} \int i d\tau$$

$$u = iR$$

$$u_C = \frac{1}{C} \int i d\tau$$

$$i = \dot{u}_C$$

$$u_L = L \frac{di}{dt}, i = \dot{u}_C$$

$$u_L = L \frac{d}{dt} \dot{u}_C = LC \ddot{u}_C$$

$$u_R = iR = RC \dot{u}_C$$

$$u_n = RC \dot{u}_C + LC \ddot{u}_C + u_C$$

### Ponašanje

$$\dot{x} = Ax + Bu$$

$$y = Cx + Du$$

$$x \in \mathbb{R}^{n \times 1}$$

$$u \in \mathbb{R}^{m \times 1}$$

$$y \in \mathbb{R}^{p \times 1}$$

variable  
dostupna

ulazi

izlazi

$$\dot{x} = A \cdot x + B \cdot u$$

$$[n \times 1] \quad [n \times n] [n \times 1] \quad [n \times m] [m \times 1]$$

A - ulazna matrica

B - ulazno-izlazna matrica

C - izlazna matrica

$$y = C \cdot x + D \cdot u$$

$$[p \times 1] = [p \times n] [n \times 1] + [p \times m] [m \times 1]$$

$p \times 1$                        $p \times 1$

$D$  - matrica direktnog sustava

Primer

$$\dot{x}_1 = x_1 + u_2$$

$$y_1 = u_1$$

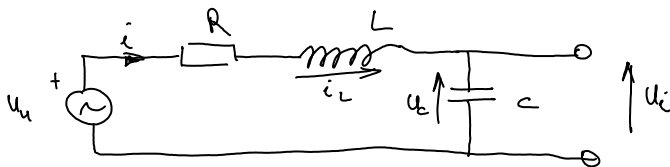
$$\dot{x}_2 = 2x_2$$

$$y_2 = x_2 + x_3$$

$$\dot{x}_3 = x_1 - x_3$$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 1 & 0 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 & 1 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$$

Povratak na struju krug



$$x = [u_c \ i_L]^T \quad u = [u_u] \quad y = [u_i]$$

$$\dot{u}_c \dots u_c, i_L, u_u$$

$$\dot{i}_L \dots u_c, i_L, u_u$$

$$u_i \dots u_c, i_L, u_u$$

$$u_c = \frac{1}{C} \int i dt \quad \left| \quad \frac{d}{dt} \right.$$

$$\dot{u}_c = \frac{1}{C} i, \quad i = i_L$$

$$\dot{u}_c = \frac{i}{C}$$

$$u_L = L \frac{di_L}{dt}$$

$$\frac{di_L}{dt} = \frac{1}{L} u_L = \frac{1}{L} (u_u - u_c - u_R)$$

$$i_R = i_L R$$

$$\frac{di_L}{dt} = \frac{u_L}{L} = \frac{u_u}{L} - \frac{u_c}{L} - \frac{R}{L} i_L$$

$$\begin{bmatrix} \dot{u}_c \\ \frac{d}{dt} i_L \end{bmatrix} = \begin{bmatrix} 0 & \frac{1}{C} \\ -\frac{1}{L} & -\frac{R}{L} \end{bmatrix} \begin{bmatrix} u_c \\ i_L \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{1}{L} \end{bmatrix} [u_u]$$

$$[u_i] = [1 \ 0] \begin{bmatrix} u_c \\ i_L \end{bmatrix} + [0] [u_u]$$

$$\begin{bmatrix} \ddot{u}_c \\ \frac{d}{dt} i_L \end{bmatrix} = \begin{bmatrix} 0 & \frac{1}{C} \\ -\frac{1}{L} & -\frac{R}{C} \end{bmatrix} \begin{bmatrix} u_c \\ i_L \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{1}{L} \end{bmatrix} [u_u]$$

$$[u_i] = [1 \ 0] \begin{bmatrix} u_c \\ i_L \end{bmatrix} + [0] [u_u]$$

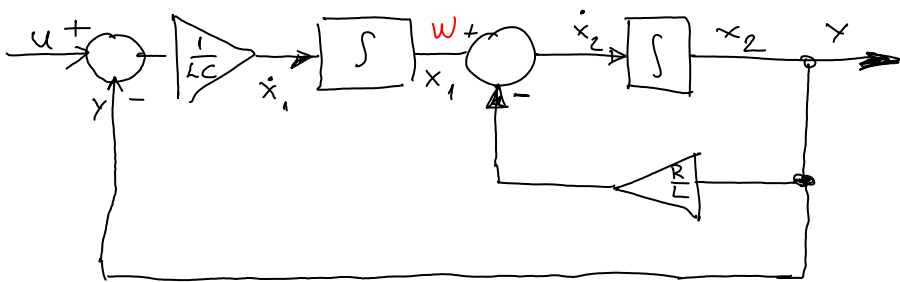
$$u_u = RC\ddot{u}_i + LC\ddot{u}_i + u_i, \quad u_u = u$$

$$u = LCy'' + RCy' + y$$

$$LCy'' = u - RCy' - y \quad / : LC$$

$$y'' = \frac{1}{LC} u - \frac{R}{L} y' - \frac{1}{LC} y \quad / \int \int$$

$$y = \int \left( -\frac{R}{L} y + \underbrace{\int \left( \frac{1}{LC} u - \frac{1}{LC} y \right)}_w \right)$$



$$\dot{x}_1 = \frac{1}{LC} (u - x_2) = -\frac{1}{LC} x_2 + \frac{1}{LC} u$$

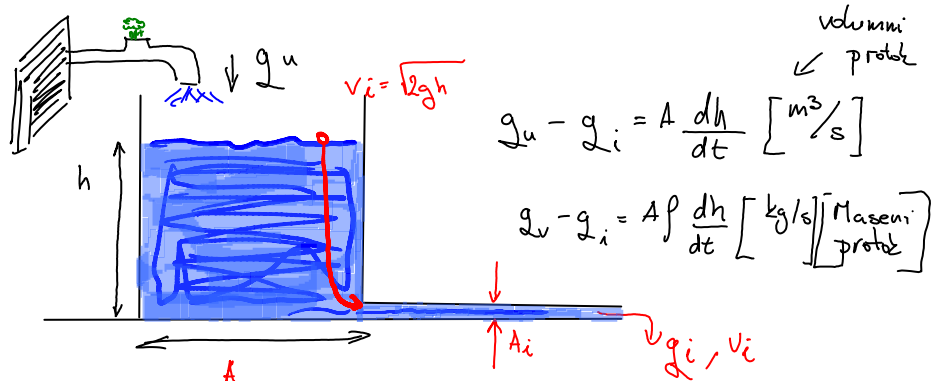
$$\dot{x}_2 = x_1 - \frac{R}{L} x_2 \quad / \quad x_2 = y$$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & -\frac{1}{LC} \\ 1 & -\frac{R}{L} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} \frac{1}{LC} \\ 0 \end{bmatrix} u$$

$$[y] = [0 \ 1] \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + [0] u$$

$$G = C(sI - A)^{-1} B$$

# Fluidi



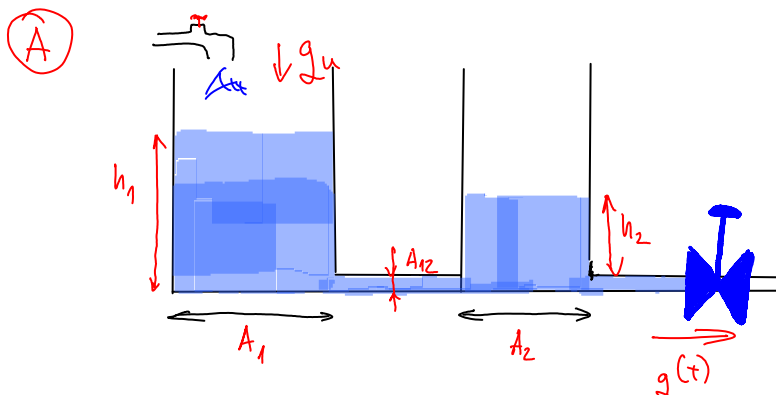
$$q_u - q_i = A \frac{dh}{dt} \left[ \frac{m^3}{s} \right]$$

$$q_u - q_i = A \rho \frac{dh}{dt} \left[ \frac{kg}{s} \right] \left[ \frac{Masseni}{protek} \right]$$

$$q_i = A_i v_i \left[ \frac{m^3}{s} \right]$$

$$q_i = A_i \rho v_i \left[ \frac{kg}{s} \right]$$

$$P = \frac{F}{S} = \frac{\rho g h S}{S} = \rho g h$$



$$q(t) = A_0 \sqrt{\rho} \sqrt{2 \Delta P} \frac{\times u}{100\%}$$

$$\textcircled{1} \quad \textcircled{q_u} - q_{12} = A_1 \rho \frac{dh_1}{dt}$$

$$q_{12} = A_{12} \rho \sqrt{2g(h_1 - h_2)}$$

$$\frac{dh_1}{dt} = \frac{1}{A_1 \rho} q_u - \frac{A_{12}}{A_1} \sqrt{2g} \sqrt{h_1 - h_2}$$

$$\Delta P = P_1 - P_2$$

$$P_1 = P_a + \rho g h_2$$

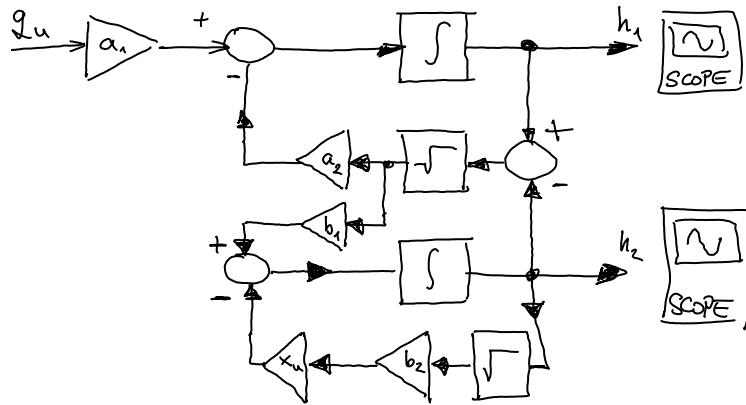
$$P_2 = P_a$$

$$\Delta P = \rho g h_2$$

$$\textcircled{2} \quad q_{12} - q(t) = A_2 \rho \frac{dh_2}{dt}$$

$$q(t) = A_0 \sqrt{\rho} \sqrt{2 \Delta P} \frac{\times u}{100\%}$$

$$\bullet \quad \frac{dh_2}{dt} = \frac{A_{12}}{A_2} \sqrt{2g} \sqrt{h_1 - h_2} - \frac{A_v}{A_2} \sqrt{2g} \sqrt{h_2} \quad \frac{x_u}{100\%}$$



b) Stacionarno stanje

$h_{10}$  — visina  $h_1$  u stacionarnom stanju

$$h_{10} = f(x_{u0})$$

