

# FREKVENCIJSKE KARAKTERISTIKE

30. studenog 2008

10:01

$$u(t) = U_m \sin(\omega t + \alpha_0) \rightarrow \boxed{\text{SUSTAV}} \rightarrow y(t) = Y_m \sin(\omega t + \alpha_y)$$

$$G(s) = \frac{B(s)}{N(s)} \quad s = \sigma + j\omega$$

↓  
prigušenje

definiramo:  $\sigma = 0 \rightarrow$  frekv. karakt.  
 $s = j\omega$

Primer: (Zadatak 9.1.)

$$G(s) = \frac{100}{s(1 + s/10)} \rightarrow s = j\omega \Rightarrow G(j\omega) = \frac{100}{j\omega(1 + j\omega/10)}$$

$$\left\{ \begin{aligned} G(j\omega) &= \frac{10}{(2 + j\omega)(10 + j\omega)} \Rightarrow \\ &= \frac{\cancel{10}}{2(1 + j\frac{\omega}{2}) \cdot \cancel{10}(1 + j\frac{\omega}{10})} = \frac{1}{2(1 + j\frac{\omega}{2})(1 + j\frac{\omega}{10})} \end{aligned} \right\}$$

slučajno  
ispireme formu za  
Bodeov dijagram

$$A(\omega)_{[dB]} = 20 \log |G(j\omega)|$$

$$\log(a \cdot b) = \log a + \log b$$

$$\log\left(\frac{a}{b}\right) = \log a - \log b$$

$$z = \frac{z_1}{z_2 \cdot z_3} \rightarrow |z| = \frac{|z_1|}{|z_2| \cdot |z_3|}$$

$$A(\omega) = 20 \log |z_1| - 20 \log |z_2| - 20 \log |z_3|$$

$$\left\{ \begin{aligned} A(\omega) &= \underline{20 \log \frac{1}{2}} + \cancel{20 \log 1} - \underline{20 \log \sqrt{1 + \left(\frac{\omega}{2}\right)^2}} - \underline{20 \log \sqrt{1 + \left(\frac{\omega}{10}\right)^2}} \end{aligned} \right.$$

$$\varphi(\omega) = \cancel{\arctan \frac{0}{\frac{1}{2}}} - \underline{\arctan \frac{\omega}{2}} - \underline{\arctan \frac{\omega}{10}}$$

$10^x = 2$

$$\arctan 0 = 0$$

$$\arctan \infty = \frac{\pi}{2}$$

$$\arctan(-\infty) = -\frac{\pi}{2}$$

$$x = \log 2 = 0,301$$

# Bodeov dijagram

The diagram is a Bode magnitude plot. The vertical axis represents the magnitude  $A(\omega)$  in dB, with major ticks at -20, -10, 0, 10, and 20. The horizontal axis represents the frequency  $\omega$  on a logarithmic scale, with major ticks at  $10^{-1}$ ,  $10^0$ ,  $10^1$ ,  $10^2$ ,  $10^3$ , and  $10^4$ . A horizontal blue line is drawn at  $A(\omega) = -6$  dB. Three asymptotic curves are shown: a red line that is horizontal at -6 dB until  $\omega = 10^0$  and then slopes downward; a green line that is horizontal at 0 dB until  $\omega = 10^1$  and then slopes downward; and a cyan line that is horizontal at 0 dB until  $\omega = 10^1$  and then slopes downward with a steeper negative slope than the green line. A smooth cyan curve represents the actual magnitude response, starting at -6 dB, following the red asymptote closely, and then following the green asymptote. A red label  $A(\omega)$  is placed near the bottom right of the plot.

Hand-drawn Bode phase plot for a system with two poles and one zero. The plot shows the phase angle in degrees versus frequency on a logarithmic scale.

The vertical axis represents the phase angle  $\varphi(\omega)$  in degrees, with markings at  $90^\circ$ ,  $45^\circ$ ,  $0^\circ$ ,  $-45^\circ$ ,  $-90^\circ$ ,  $-135^\circ$ , and  $-180^\circ$ . The horizontal axis represents the frequency  $\omega$  on a logarithmic scale, with markings at  $10^{-1}$ ,  $10^0$ ,  $10^1$ ,  $10^2$ ,  $10^3$ , and  $10^4$ .

The plot shows the asymptotic approximation (cyan line) and the actual phase (red line). The asymptotic approximation starts at  $0^\circ$  for  $\omega < 10^0$ , drops to  $-90^\circ$  at  $\omega = 10^0$ , and drops to  $-180^\circ$  at  $\omega = 10^1$ . The actual phase (red line) starts at  $0^\circ$  for  $\omega < 10^{-1}$ , drops to  $-90^\circ$  at  $\omega = 10^{-1}$ , and drops to  $-180^\circ$  at  $\omega = 10^1$ .

The plot also includes dashed lines for the asymptotes and a dashed line for the actual phase at  $\omega = 10^1$ .

Handwritten notes in red ink include  $-\varphi(\omega)$  and  $-\frac{\pi}{2}$ .

Primer: (Zadatak 9.2.)

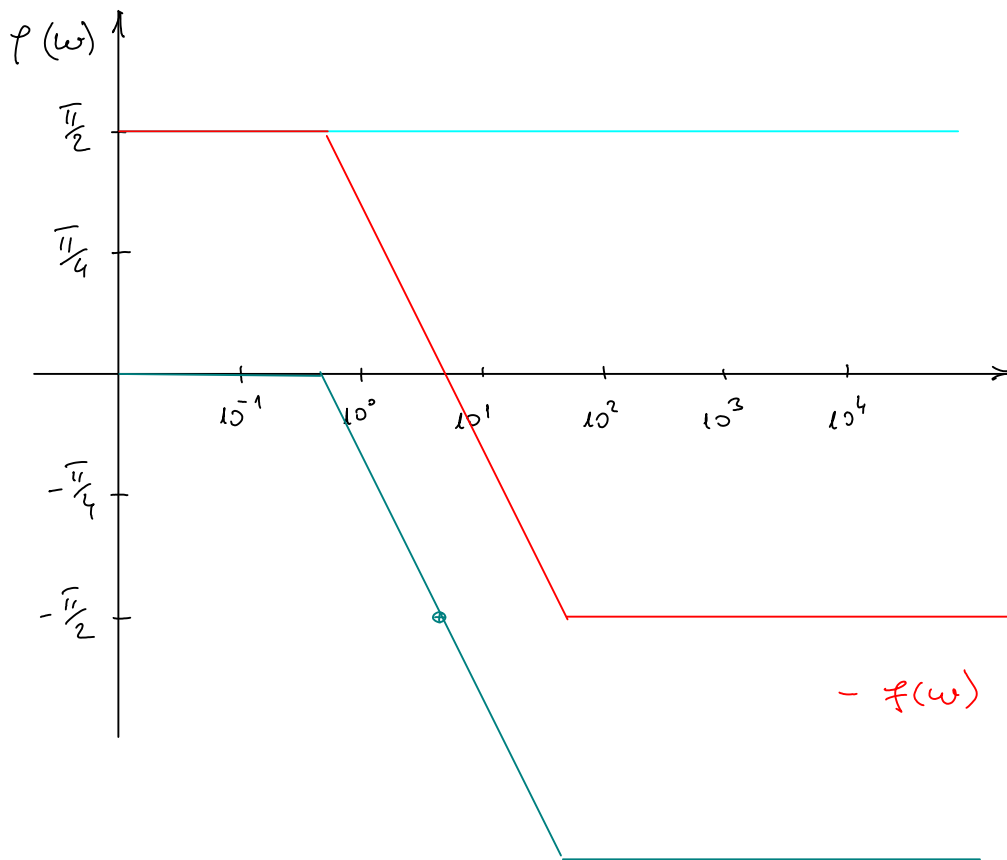
$$G(s) = \frac{k s}{(1 + sT)^2} \quad k = 10, \quad T = 0,2$$

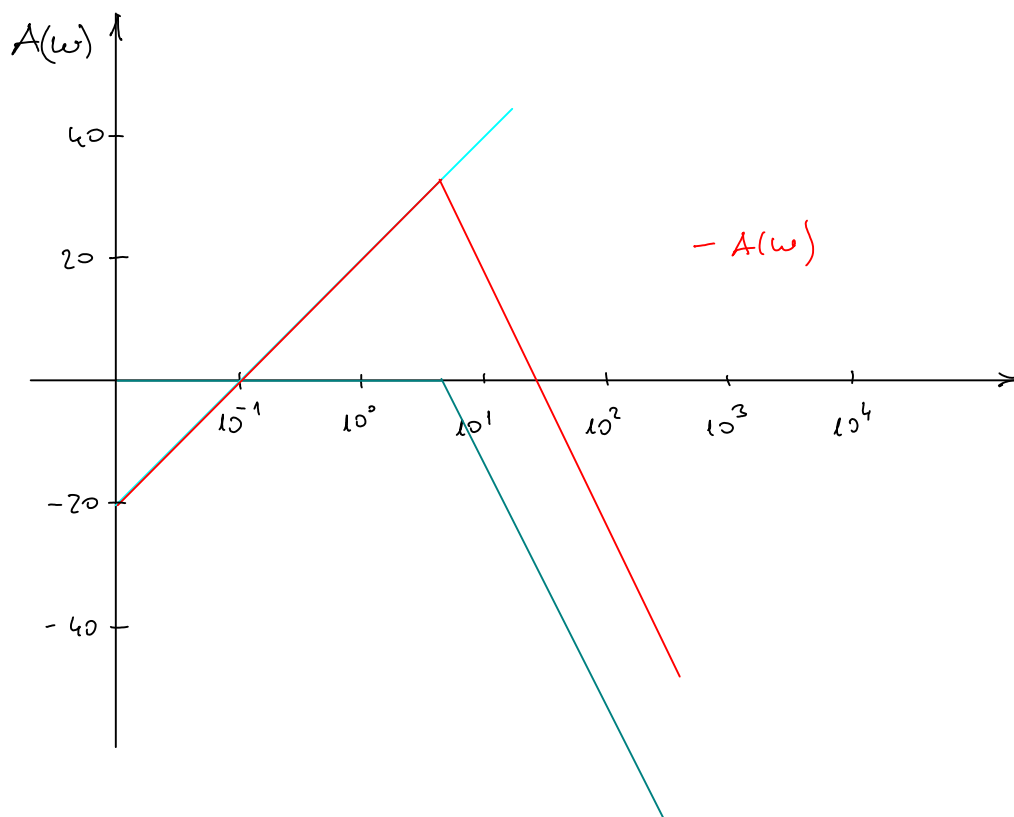
$$G(s) = \frac{10 s}{(1 + 0,2 s)^2} \rightarrow G(j\omega) = \frac{j \frac{\omega}{10}}{\left(1 + j \frac{\omega}{5}\right)^2} = \frac{j \frac{\omega}{10}}{\left(1 + j \frac{\omega}{5}\right)^2}$$

$$A(\omega) = 20 \log \frac{\omega}{10} - 40 \log \sqrt{1 + \left(\frac{\omega}{5}\right)^2}$$

$$\varphi(\omega) = \arctg \frac{\omega}{10} - 2 \arctg \frac{\omega}{5} = \frac{\pi}{2} - 2 \arctg \frac{\omega}{5}$$

$$\log 5 = 0,69$$





Primer: (Zadatak 3.4.)

$$G(s) = \frac{20(s-2)}{(s+1)(s+10)}$$

$$G(j\omega) = \frac{\cancel{20}(1-j\frac{\omega}{2})}{(1+j\frac{\omega}{1})(1+j\frac{\omega}{10}) \cdot \cancel{10}} = -2 \cdot \frac{(1-j\frac{\omega}{2})}{(1+j\frac{\omega}{1})(1+j\frac{\omega}{10})}$$

$$A(\omega) = 20 \log 2 + \cancel{20 \log \sqrt{1+(\frac{\omega}{1})^2}} - \cancel{20 \log \sqrt{1+(\frac{\omega}{1})^2}} - 20 \log \sqrt{1+(\frac{\omega}{10})^2}$$

$| -2 | = 2!$

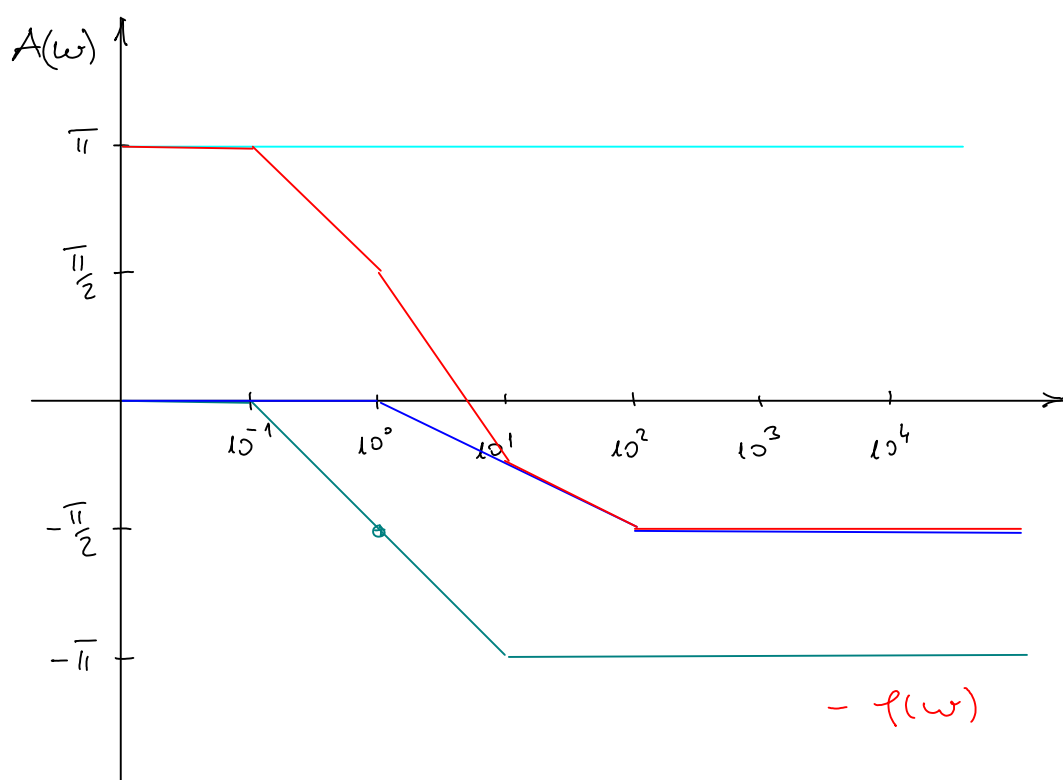
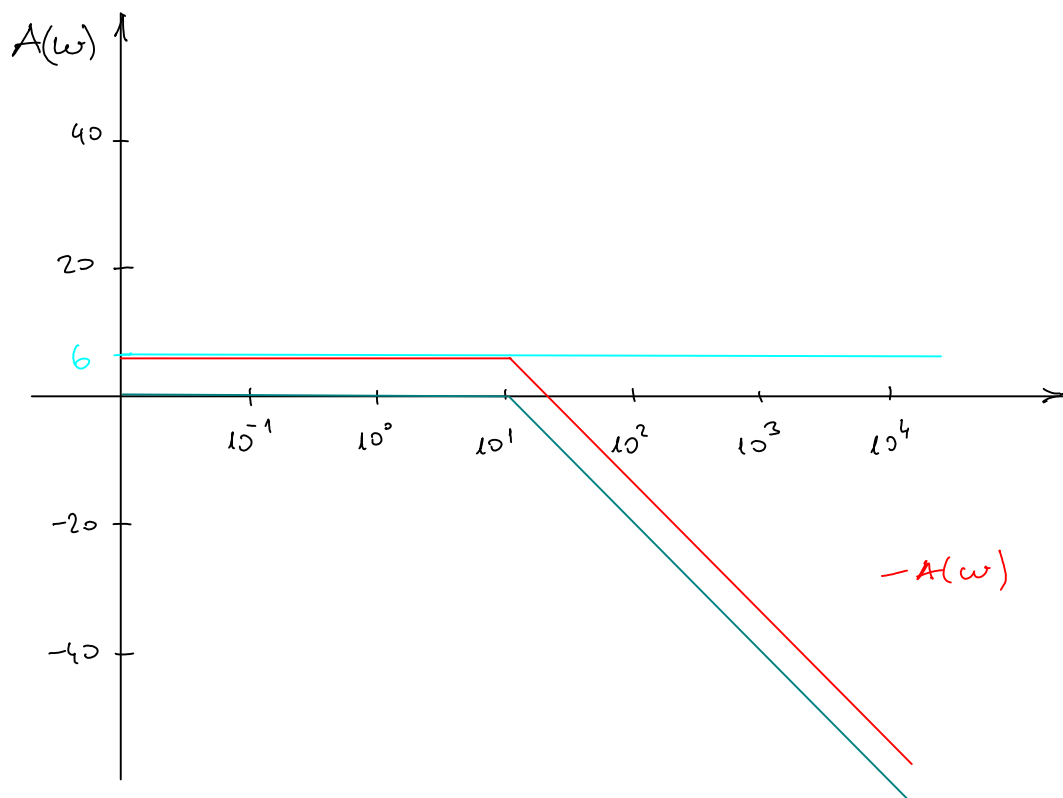
$$A(\omega) = \underline{20 \log 2} - \underline{20 \log \sqrt{1+(\frac{\omega}{10})^2}}^*$$

$$\varphi(\omega) = \arctg \frac{0}{-2} + \arctg \left( \frac{-1}{1} \right)^* - \arctg \left( \frac{\omega}{1} \right) - \arctg \left( \frac{\omega}{10} \right)$$

$$\varphi(\omega) = \underline{\pi} - \underline{2 \cdot \arctg \left( \frac{\omega}{1} \right)} - \underline{\arctg \left( \frac{\omega}{10} \right)}$$

\*  $\begin{cases} \text{Re} > 0 \\ \text{Im} < 0 \end{cases} \Rightarrow \text{IV. kvadr.} \Rightarrow \text{"-"} \text{ se može izvesti van}$

$$20 \log 2 = 6$$



Pri mjeri :

$$G(s) = \frac{20(1-s)}{(s+1)(s+10)}$$

$$G(j\omega) = \frac{2(1-j\frac{\omega}{1})}{(1+j\frac{\omega}{1})(1+j\frac{\omega}{10})}$$

$$\varphi(\omega) = \cancel{\arctan \frac{0}{2}} + \arctan \left( \frac{-\frac{\omega}{1}}{1} \right) - \arctan \left( \frac{\omega}{1} \right) - \arctan \frac{\omega}{10}$$

„Problem“ sa dodatnim  $\pi$  : kada je  $\operatorname{Re} < 0$ !

$$G(s) = \frac{20(1-s)}{-10(1+s)(1+s)}$$

$$G(j\omega) = \frac{-20(1-j\frac{\omega}{1})}{-10(1+j\frac{\omega}{1})(1-j\frac{\omega}{10})}$$

$$\varphi(\omega) = \arctan \frac{0}{2} + \arctan \left( \frac{-\frac{\omega}{1}}{1} \right) - \arctan \left( \frac{\omega}{1} \right) - \arctan \left( \frac{-\frac{\omega}{10}}{1} \right)$$

## Nyquistov dijagram

Realna forma prijenosne funkcije za crtanje Nyquist

$$G(j\omega) = \operatorname{Re} \{ G(j\omega) \} + j \operatorname{Im} \{ G(j\omega) \}$$

$\downarrow$   
 $R(\omega)$

$\downarrow$   
 $I(\omega)$

$R(\omega=0) \dots \rightarrow \lim_{\omega \rightarrow 0^+}$  (zdeske još crtamo samo za pozitivne frekvencije)

$I(\omega=0) \dots$

$R(\omega=\infty) \dots$

$I(\omega=\infty) \dots$

$R(\omega=?) = 0$

$I(\omega=?) = 0$

$$z = \frac{a+bj}{c-dj} \cdot \frac{c+dj}{c+dj} = \frac{(a+bj)(c+dj)}{c^2 + d^2}$$

Primer: (Zadatak 9.1.)

$$G(s) = \frac{100}{s(1 + \frac{s}{10})}$$

$$G(j\omega) = \frac{100}{j\omega \cdot \frac{1}{10}(10 + j\omega)} = \frac{1000}{j\omega(10 + j\omega)} = \frac{1000}{- \omega^2 + 10j\omega} \quad \text{[ } -\omega^2 - 10j\omega \text{]}$$

$$= \frac{-1000\omega^2 - 10000j\omega}{\omega^4 + 100\omega^2}$$

$$G(j\omega) = \frac{-1000}{\omega^2 + 100} + j \frac{-10000}{\omega^3 + 100\omega}$$

$\downarrow$   $\downarrow$   
 $R(\omega)$   $I(\omega)$

$$R(\omega) = \frac{-1000}{\omega^2 + 100}$$

$$I(\omega) = \frac{-10000}{\omega^3 + 100\omega}$$

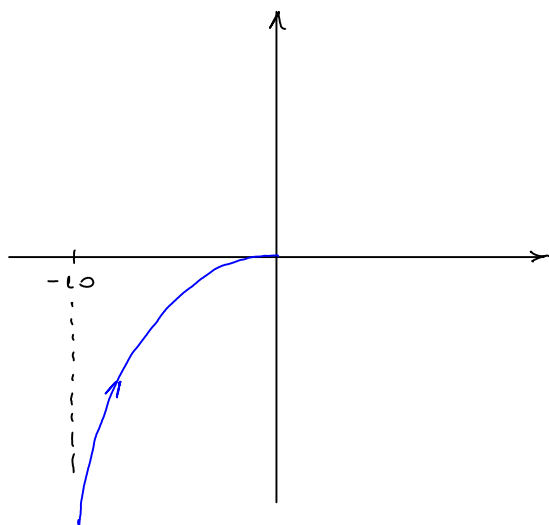
$$R(\omega=0) = -10$$

$$I(\omega=0) = -\infty$$

$$R(\omega=\infty) = 0$$

$$I(\omega=\infty) = 0$$

$( R(\omega) = \frac{\omega^2 + 1}{\dots} \quad \omega^2 + 1 = 0 \Rightarrow \omega = \pm j$   
 imaginarna zeneracija  
 $= \frac{\omega^2 - 1}{\dots} \quad \omega^2 - 1 = 0 \Rightarrow \omega_1 = 1 \quad \omega_2 = -1$   
 negativna zeneracija



Primer: (zadatak 2.4.)

$$G(s) = \frac{20(s-1)}{(s+1)(s+10)}$$

$$R(\omega) = \frac{20(12\omega^2 - 10)}{(10 - \omega^2)^2 + (11\omega)^2}, \quad I(\omega) = \frac{20(-\omega^3 + 21\omega)}{(10 - \omega^2)^2 + (11\omega)^2}$$

$$R(\omega=0) = -2$$

$$I(\omega=0) = 0$$

$$R(\omega=\infty) = 0$$

$$I(\omega=\infty) = 0$$

$$R(\omega_1) = 0 \Rightarrow 20(12\omega_1^2 - 10) = 0$$

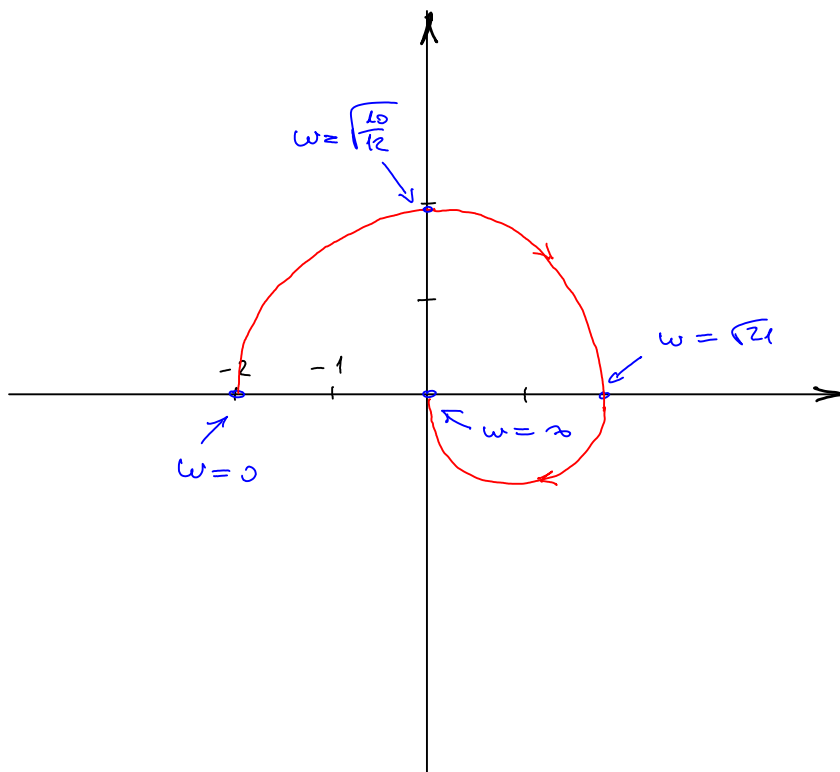
$$12\omega_1^2 = 10$$

$$\omega_1 = \sqrt{\frac{10}{12}} \rightarrow I(\omega_1) = 1,99$$

$$I(\omega_2) = 0 \Rightarrow -\omega_2^3 + 21\omega_2 = 0$$

$$\omega_2^2 = 21$$

$$\omega_2 = \sqrt{21} \Rightarrow R(\omega_2) = 1,818$$





# Polovi, nule i vremenski odziv

30. studenog 2008

13:23

P12 član :  $G(s) = \frac{K}{\frac{1}{\omega_n^2} s^2 + \frac{2\zeta}{\omega_n} s + 1}$

$K$  - statičko pojačanje

$$s_{p1,2} = \frac{-\frac{2\zeta}{\omega_n} \pm \sqrt{\frac{4\zeta^2}{\omega_n^2} - \frac{4}{\omega_n^2}}}{\frac{2}{\omega_n^2}} = \frac{-\frac{2\zeta}{\omega_n} \pm \frac{2}{\omega_n} \sqrt{\zeta^2 - 1}}{\frac{2}{\omega_n^2}} = -\zeta \omega_n \pm \omega_n \sqrt{\zeta^2 - 1}$$

$$s = -\sigma \pm j\omega_d$$

$$s = -\zeta \omega_n \pm \omega_n \sqrt{\zeta^2 - 1}$$

$$s = -\zeta \omega_n \pm j \underbrace{\omega_n \sqrt{1 - \zeta^2}}_{\omega_d}$$

kompleksni polovi uvijek dolaze u kompleksno-konjugiranim parovima

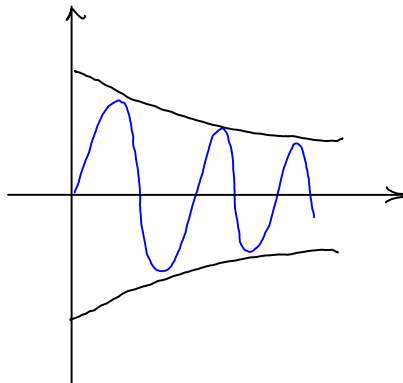
$$s_{p1,2} = -\zeta \omega_n \pm j \omega_n \sqrt{1 - \zeta^2}$$

$$\omega_n > 0$$

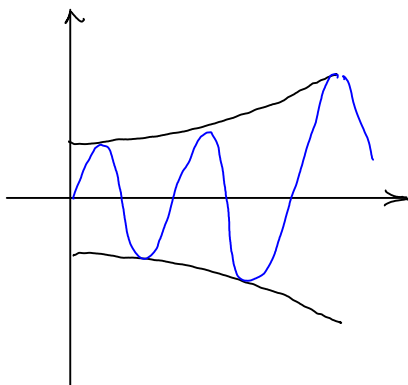
$$0 < \zeta < 1 \rightarrow$$

sustav je oscilatoran

i prigušen



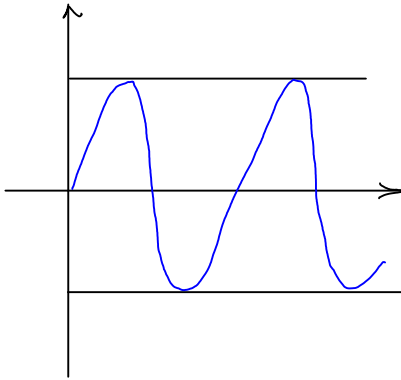
$$-1 < \zeta < 0$$



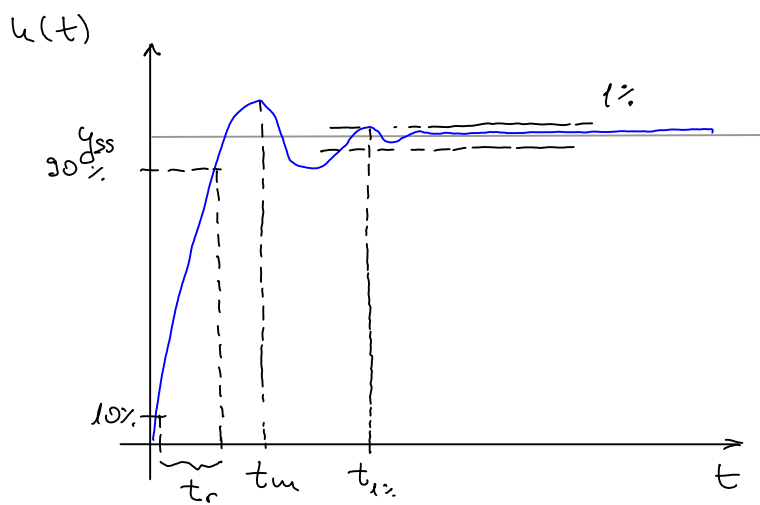
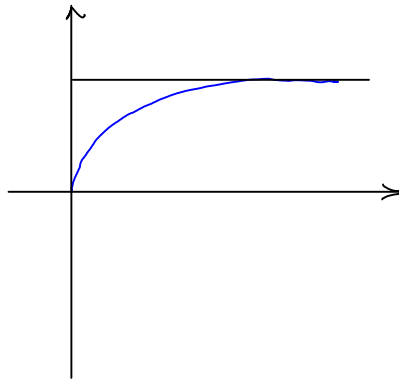
$\rightarrow$  sustav je oscilatoran

i raspirujući?

$$\xi = 0$$



$$\xi = 1$$

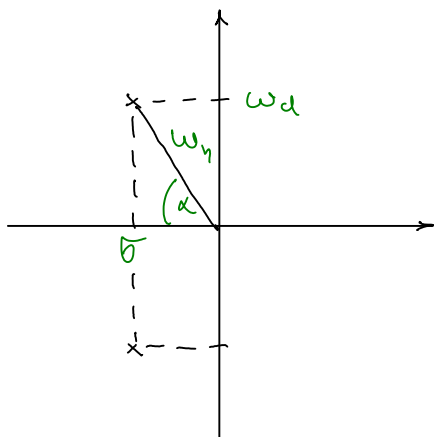


$$\sigma_u [\%] = 100 e^{\frac{-\pi \xi}{\sqrt{1-\xi^2}}}$$

$$t_{w.} = \frac{4,6}{\xi \omega_n}$$

$$t_r = \frac{1,8}{\omega_n}$$

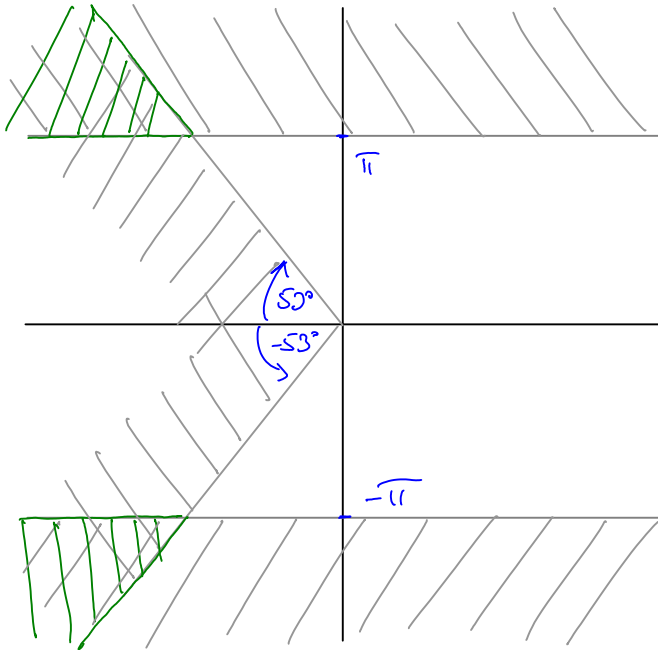
$$t_m = \frac{\pi}{\underbrace{\omega_n \sqrt{1-\xi^2}}_{\omega_d}}$$



Primer: (Zadatek s 2. MI)

$$t_m < 1$$

$$\frac{\pi}{\omega_d} < 1 \rightarrow \frac{\omega_d}{\pi} > 1 \Rightarrow \omega_d > \pi$$



$$\sigma_m < 0,1$$

$$e^{\frac{-\pi \xi}{\sqrt{1-\xi^2}}} < 0,1 \quad | \text{ln}$$

$$\frac{-\pi \xi}{\sqrt{1-\xi^2}} < -2,3$$

$$\xi = \pm 0,5512$$

$$\hookrightarrow \alpha = \pm 53^\circ$$

Primer: (Zadatek iz DZ)

$$u(t) = 2 \sin(\omega_0 t + \varphi)$$

$$y(t) = 2 \sin(\omega_0 t)$$

$$G(s) = 32 \frac{s+1}{(s+2)(s+8)}$$

$$Y_m = U_m \cdot |G(j\omega)|_{\omega=\omega_0}$$

$$\angle y = \angle u + \angle G(j\omega)|_{\omega=\omega_0}$$

$\hookrightarrow$  ovo se može odrediti i iz Bode-a

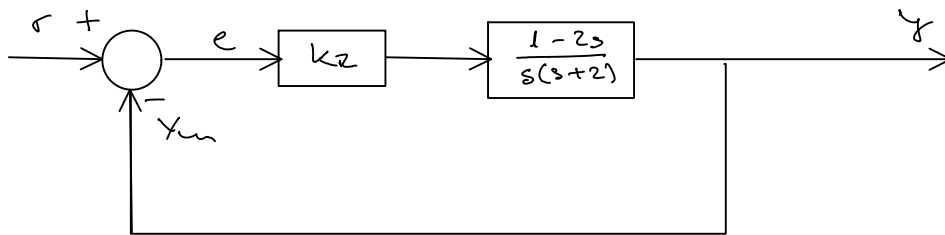
Amplitudna frekv.  
karakter.

$$\Rightarrow |G(j\omega)|_{\omega_0} = 1 \Rightarrow \omega_0$$

$$\omega_0 = +30,93 \text{ rad/s}$$

$$\omega_0 = -30,93 \text{ rad/s}$$

Primer: (1. zadatak iz 4. D.Z.)



$$k_2 = \frac{1}{2}$$

$$G(s) = \frac{\frac{1-2s}{s(s+2)}}{\frac{2s(s+2)+1-2s}{2s(s+2)}} = \frac{1-2s}{2s^2+2s+1}$$

$$H(s) = \frac{1}{s} G(s) = \frac{1-2s}{s(2s^2+2s+1)} = \frac{C_1}{s} + \frac{As+B}{2s^2+2s+1}$$

$$C_1 = H(s) \cdot s \Big|_{s=0} = 1$$

$$H(s) = \frac{1}{s} + \frac{As+B}{2s^2+2s+1} = \frac{1-2s}{s(2s^2+2s+1)}$$

$$= \frac{2s^2+2s+1+As^2+Bs}{s(2s^2+2s+1)} = \frac{s^2(2+A) + s(2+B) + 1}{s(2s^2+2s+1)}$$

$$2+A=0 \Rightarrow A=-2$$

$$2+B=-2 \Rightarrow B=-4$$

$$H(s) = \frac{1}{s} + \frac{-2s-4}{2s^2+2s+1} = \frac{1}{s} - \frac{s+2}{s^2+s+\frac{1}{2}}$$

$$\frac{s+2}{s^2+s+\frac{1}{2}} = \frac{s+\frac{1}{2}}{(s+\frac{1}{2})^2+\frac{1}{4}} + \frac{3}{2} \cdot \frac{\frac{1}{2}}{(s+\frac{1}{2})^2+\frac{1}{4}} \rightarrow 0$$

$$\frac{3}{s^2+\omega^2} \rightarrow 3 \cos(\omega t)$$

$$\frac{\omega}{s^2+\omega^2} \rightarrow \sin(\omega t)$$

$$\frac{s+a}{(s+a)^2+\omega^2} \rightarrow \cos(\omega t) e^{-at}$$

$$e^{-\frac{1}{2}t} \left( \cos\left(\frac{1}{2}t\right) + 3 \sin\left(\frac{1}{2}t\right) \right)$$

$$u(t) = 1 - e^{-\frac{1}{2}t} \left( \cos\left(\frac{1}{2}t\right) + 3 \sin\left(\frac{1}{2}t\right) \right)$$

$$g(t) = \dot{u}(t) = \left( 2 \sin\left(\frac{1}{2}t\right) - \cos\left(\frac{1}{2}t\right) \right) e^{-\frac{1}{2}t}$$

$$u(t) = 1 - e^{-\frac{1}{2}t} (\cos(\frac{1}{2}t) + 3 \sin(\frac{1}{2}t))$$

$$\dot{g}(t) = \dot{u}(t) = e^{-\frac{1}{2}t} (2 \sin(\frac{1}{2}t) - \cos(\frac{1}{2}t)) = 0$$

$$g(t) = 0 \rightarrow$$

$$2 \sin(\frac{1}{2}t) - \cos(\frac{1}{2}t) = 0$$

$$2 \sin(\frac{1}{2}t) = \cos(\frac{1}{2}t) \quad / : 2 \cos(\frac{1}{2}t)$$

$$\tan(\frac{1}{2}t) = \frac{1}{2}$$

$$\text{PERIODIČNOST: } \tan(\frac{1}{2}t + k\pi) = \frac{1}{2}, \quad k \in \mathbb{Z}$$

$$\frac{1}{2}t + k\pi = \arctan \frac{1}{2}$$

$$t = 0,927 + 2k\pi$$

$$k=0 \rightarrow t = 0,927$$

$$k=1 \rightarrow t_1 =$$

$$k=2 \rightarrow t_2 =$$

$$\ddot{u}(t) = \ddot{g}(t)$$

$$\ddot{u}(t) < 0 \Rightarrow \text{MAX}$$

$$\ddot{u}(t) > 0 \Rightarrow \text{MIN}$$

$$\ddot{u}(t) = 0 \Rightarrow \text{PREGIŠ}$$

## AD. NYQUIST : KUTEVI UPADA

