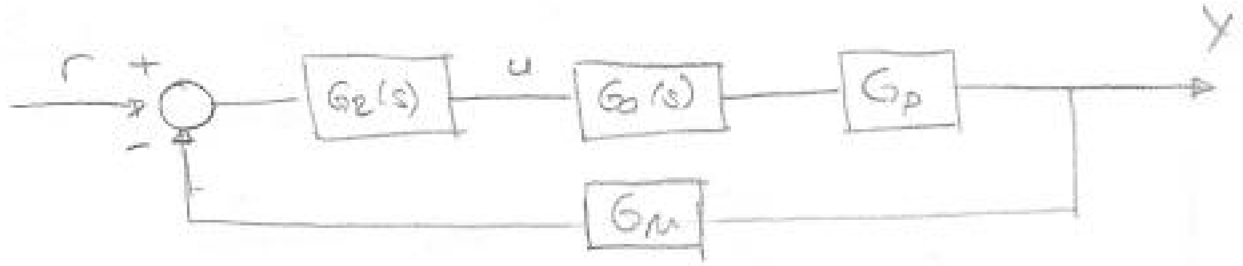
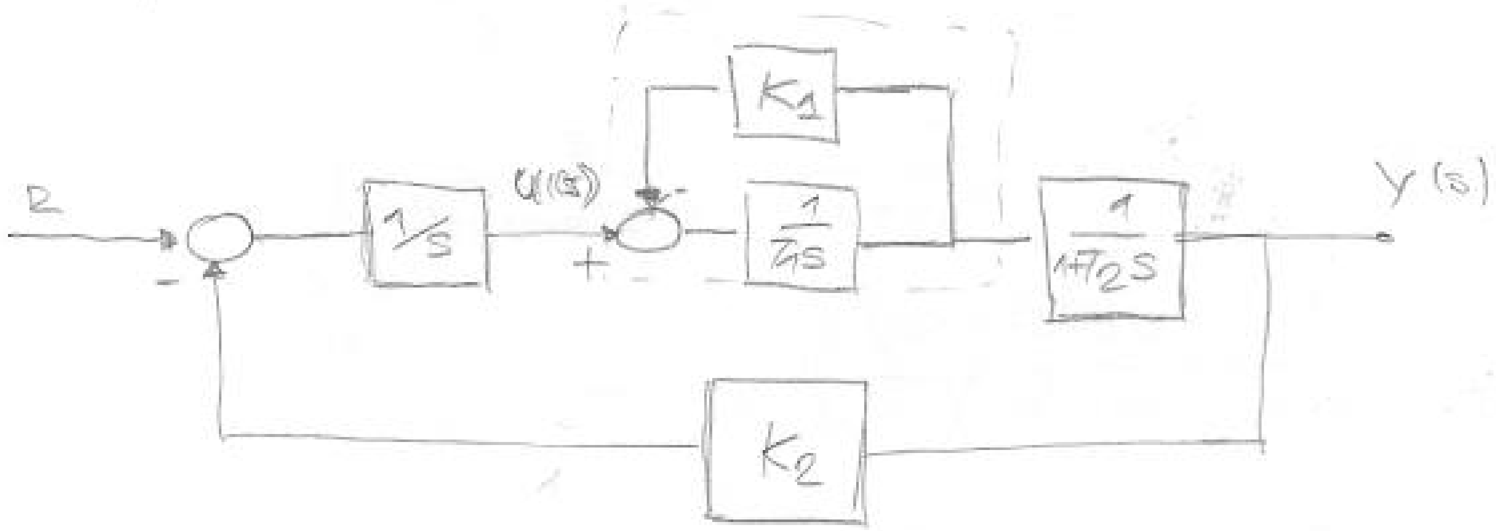
POGLAVLJE 1

1.1. a)



b) $\dfrac{Y}{R} = \dfrac{G_R G_a G_P}{1+G_R G_a G_P}$ $\qquad \dfrac{U}{R} = \dfrac{G_R}{1+G_R G_o G_P}$

1.2



$$G(s) = \frac{U(s)}{R(s)} =$$

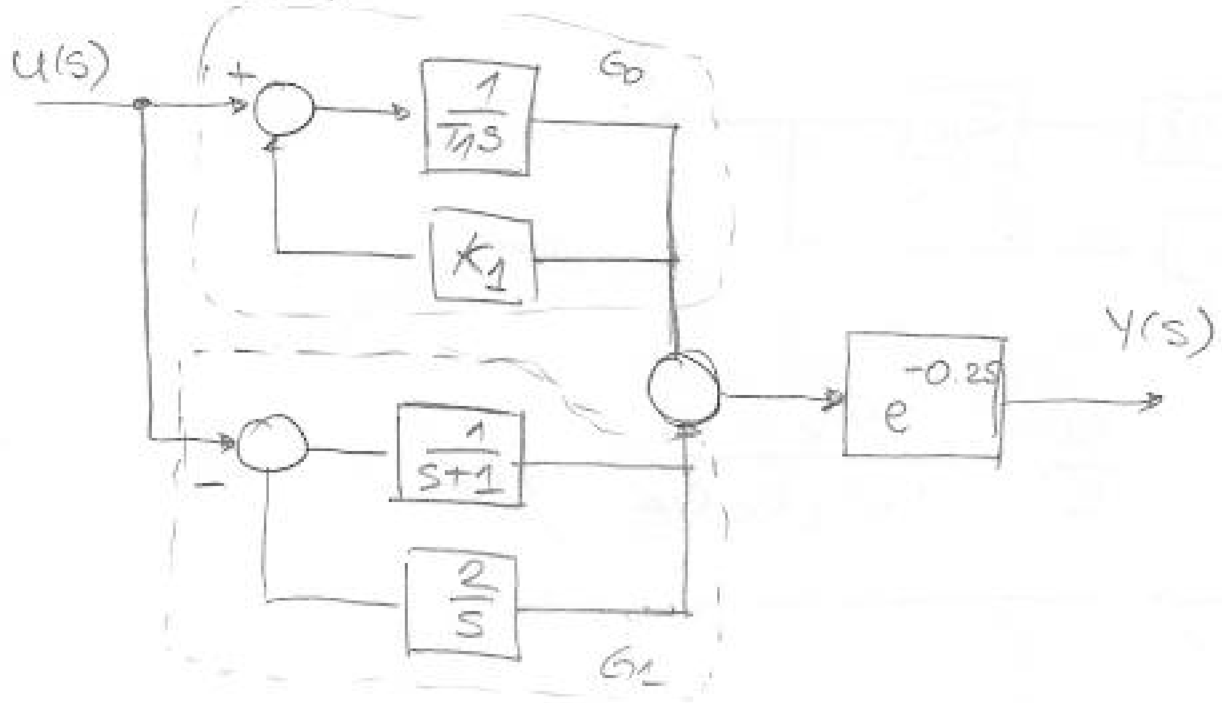
$$G_1(s) = \frac{Y(s)}{R(s)} = \frac{\frac{1}{s}\cdot\frac{1}{T_1 s+k_1}\cdot\frac{1}{1+T_2 s}}{1+\frac{k_2}{s(T_1 s+k_1)(1+T_2 s)}} = \frac{\frac{1}{\cancel{s(T_1 s+k_1)(1+T_2 s)}}}{\frac{s(T_1 s+k_1)(1+T_2 s)+k_2}{\cancel{s(T_1 s+k_1)(1+T_2 s)}}} = \frac{1}{s(T_1 s+k_1)(1+T_2 s)+k_2}$$

$$G_o = \frac{\frac{1}{T_1 s}}{1+\frac{k_1}{T_1 s}} = \frac{\frac{1}{\cancel{T_1 s}}}{\frac{T_1 s+k_1}{\cancel{T_1 s}}} = \frac{1}{T_1 s+k_1}$$

$$G_1(s) = \frac{U(s)\cdot\frac{1}{T_1 s+k_1}\cdot\frac{1}{1+T_2 s}}{R(s)} = \frac{1}{s(T_1 s+k_1)(1+T_2 s)+k_2}$$

$$G(s) = \frac{U(s)}{R(s)} = \frac{(T_1 s+k_1)(1+T_2 s)}{s(T_1 s+k_1)(1+T_2 s)+k_2}$$

3. $G(s) = \frac{Y(s)}{R(s)}$



$$G_0 = \frac{\frac{1}{T_1 s}}{1 + \frac{k_1}{T_1 s}} = \frac{\frac{1}{\cancel{T_1 s}}}{\frac{T_1 \cdot s + k_1}{\cancel{T_1} s}}$$

$$G_0 = \frac{1}{T_1 \cdot s + k_1}$$

$$G_1 = \frac{\frac{1}{s+1}}{1 + \frac{2}{s(s+1)}} = \frac{\frac{1}{\cancel{s+1}}}{\frac{s(s+1)+2}{s\cancel{(s+1)}}}$$

$$G_1 = \frac{s}{s(s+1)+2}$$

$$\frac{Y(s)}{U(s)} = e^{-0.2s}(G_0 + G_1) = e^{-0.2s}\left(\frac{1}{T_1 s + k_1} + \frac{s}{s(s+1)+2}\right)$$

$$= e^{-0.2s}\left(\frac{s(s+1)+2+s(T_1 s + k_1)}{(T_1 s + k_1)(s(s+1)+2)}\right) \ldots$$

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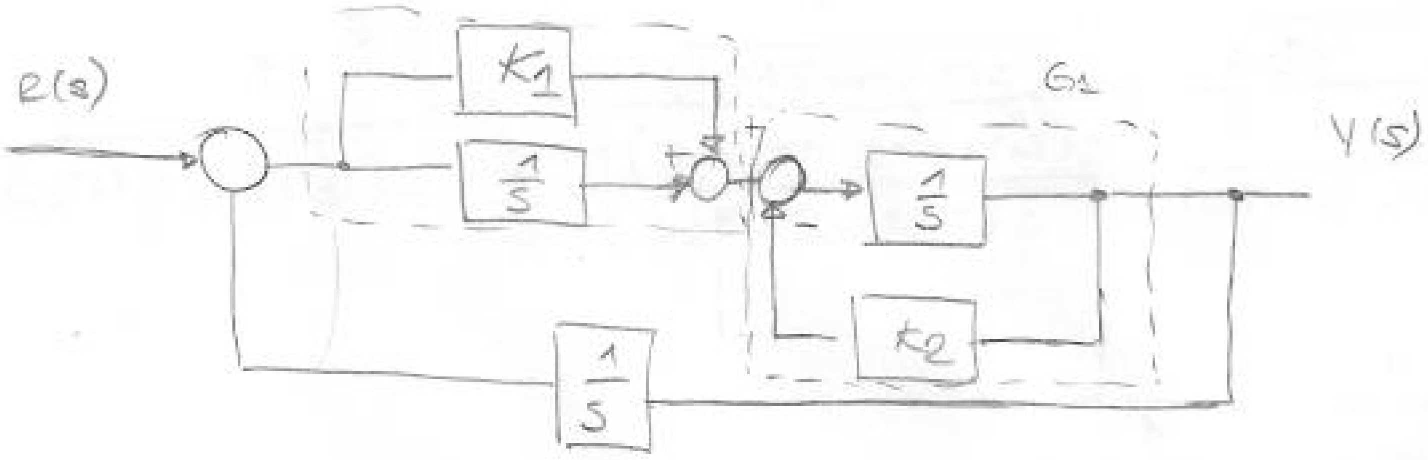
4. $G(s) = \frac{Y(s)}{R(s)}$



$$G_0 = k_1 + \frac{1}{s} = \frac{k_1 s + 1}{s}$$

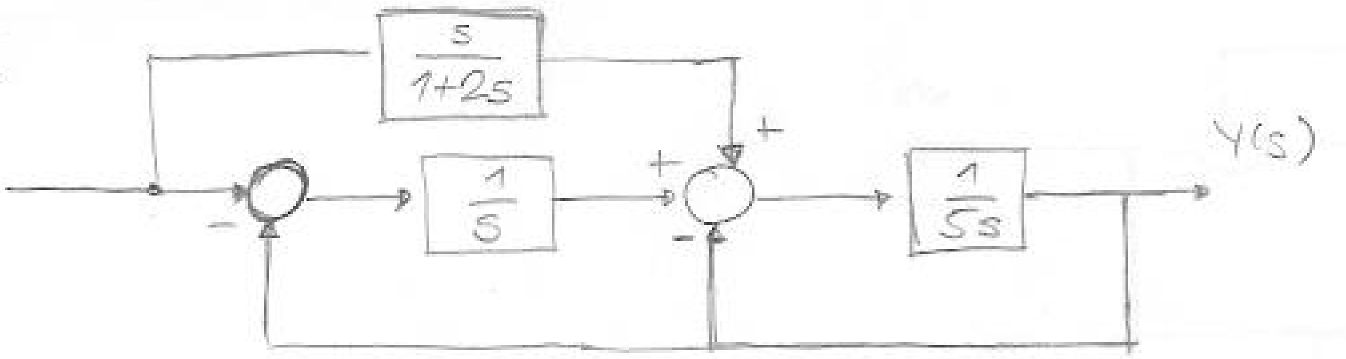
$$G_1 = \frac{\frac{1}{s}}{1 + \frac{k_2}{s}} = \frac{\frac{1}{\cancel{s}}}{\frac{s+k_2}{\cancel{s}}} = \frac{1}{s+k_2}$$

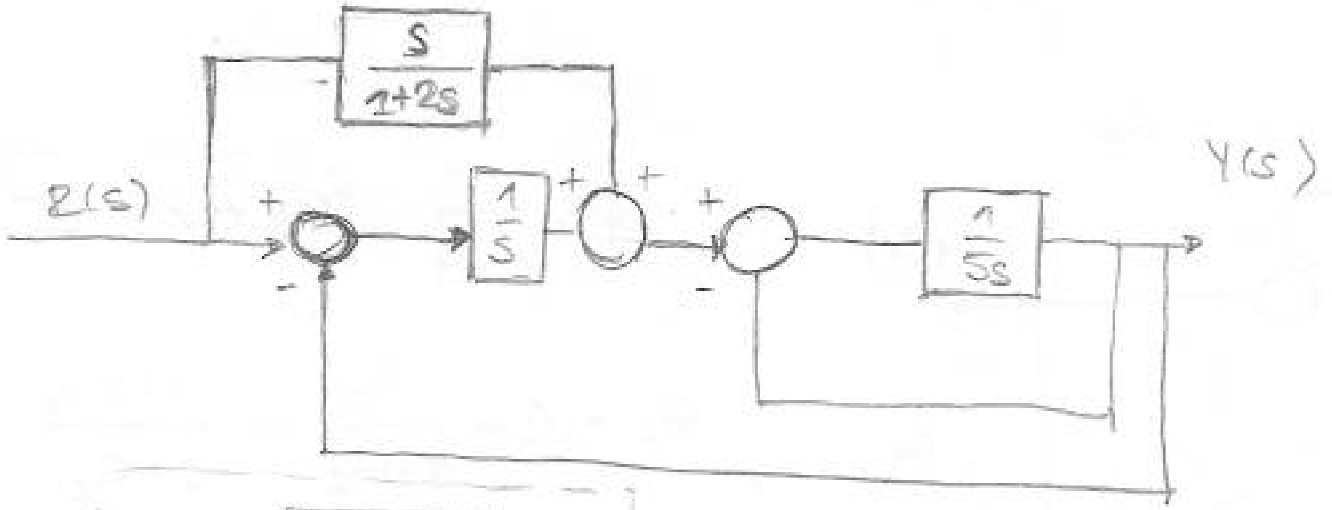
$$\frac{Y}{R} = \frac{G_0 G_1}{1 + \frac{G_0 G_1}{s}} = \frac{\frac{k_1 s + 1}{s(s+k_2)}}{1 + \frac{k_1 s + 1}{s^2(s+k_2)}}$$

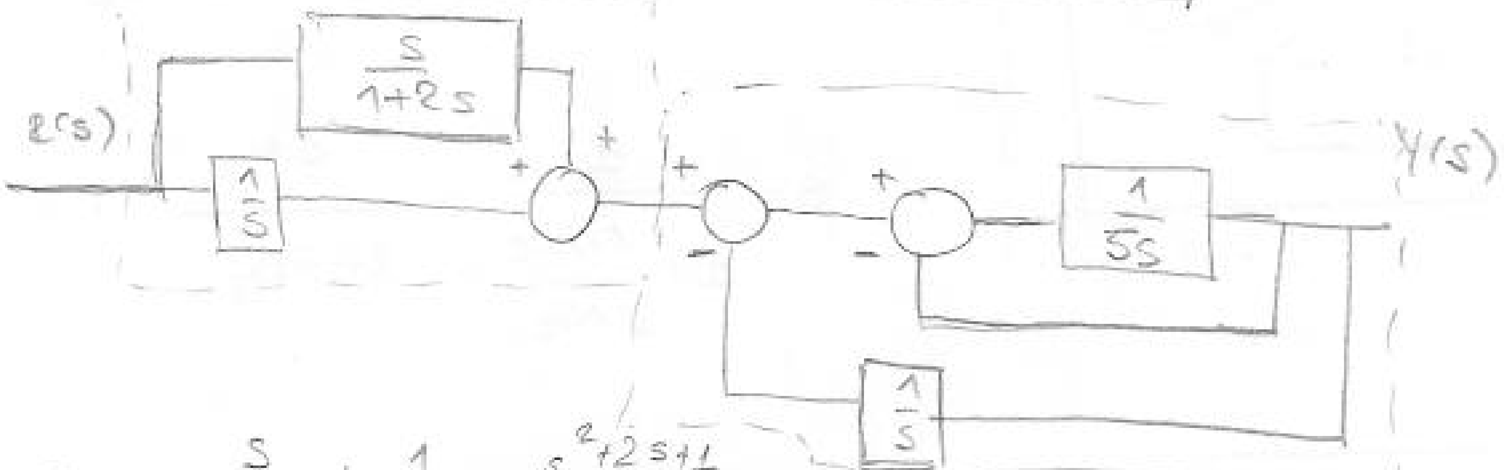
$$\frac{Y}{R} = \frac{\frac{k_1 s + 1}{\cancel{s}(s+k_2)}}{\frac{s^2(s+k_2)+k_1 s + 1}{s^{\cancel{2}}(s+k_2)}} = \frac{s(k_1 s + 1)}{s^2(s+k_2)+k_1 s + 1}$$

5.

$$G(s) = \frac{Y(s)}{R(s)}$$







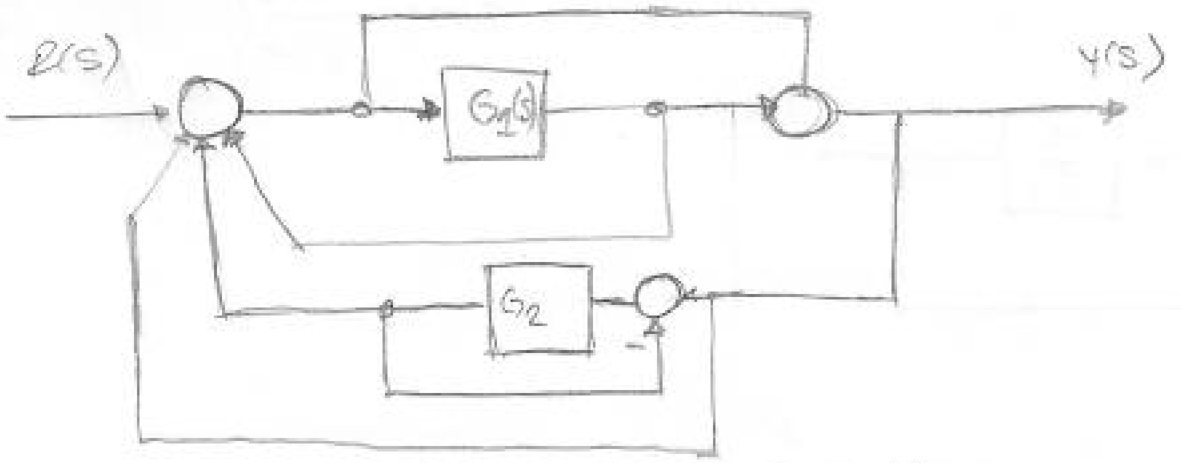
$$G_0 = \frac{s}{1+2s} + \frac{1}{s} = \frac{s^2+2s+1}{s(1+2s)}$$

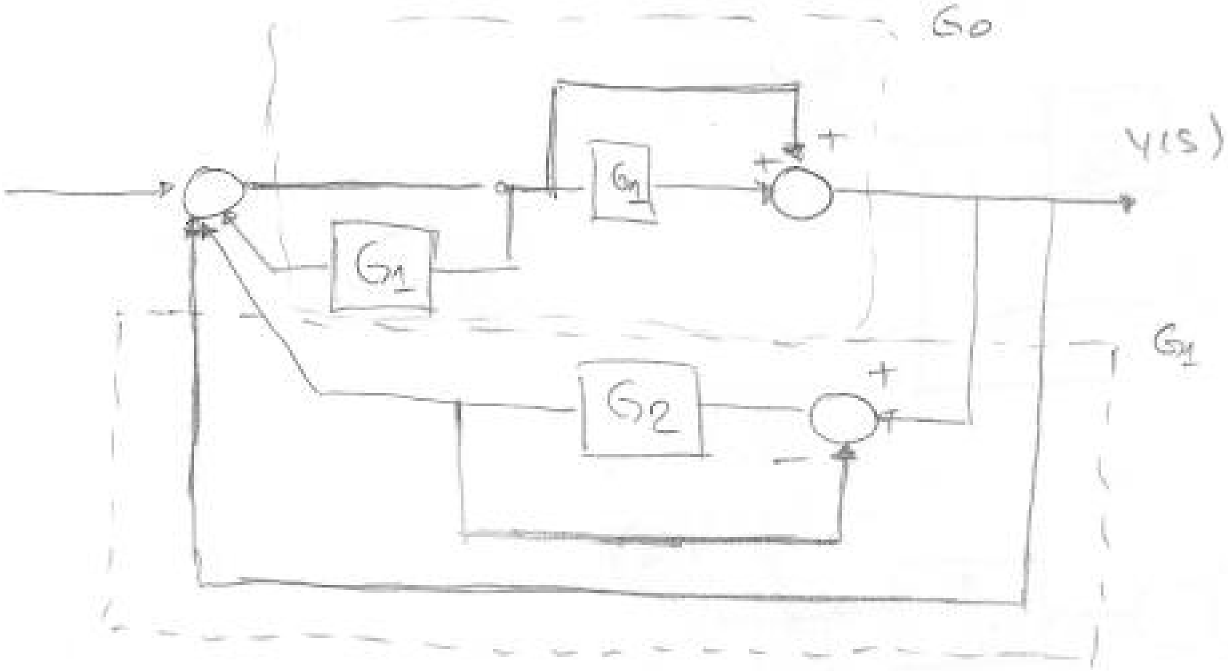
$$G_1 = \frac{\frac{1}{5s}}{1+\frac{1}{5s}\left(\frac{1}{s}+1\right)} = \frac{\frac{1}{5s}}{1+\frac{1}{5s}\,\frac{s+1}{s}} = \frac{\frac{1}{5s}}{1+\frac{s+1}{5s^2}} = \frac{\frac{1}{\cancel{5s}}}{\frac{5s^2+s+1}{\cancel{5}s^{\cancel{2}}}} = \frac{s}{5s^2+s+1}$$

$$G = G_0 \cdot G_1 = \frac{s^2+2s+1}{\cancel{s}(1+2s)} \cdot \frac{\cancel{s}}{5s^2+s+1} = \frac{s^2+2s+1}{(1+2s)(5s^2+s+1)} = \frac{s^2+2s+1}{5s^2+s+1+10s^3+2s^2+2s}$$

$$= \frac{s^2+2s+1}{\ldots}$$

6.



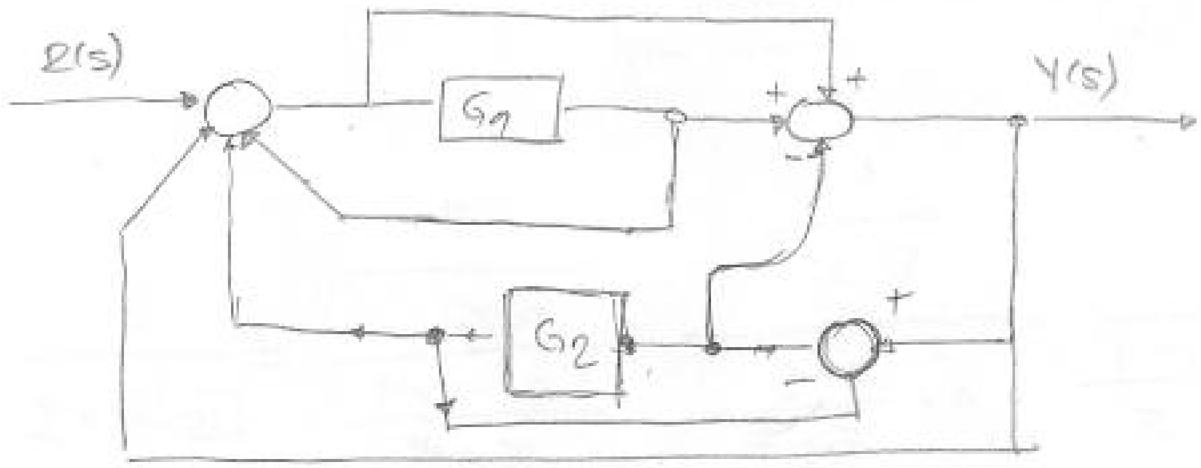


$$G_0 = \frac{1}{1+G_1} \cdot (G_1+1) = 1$$

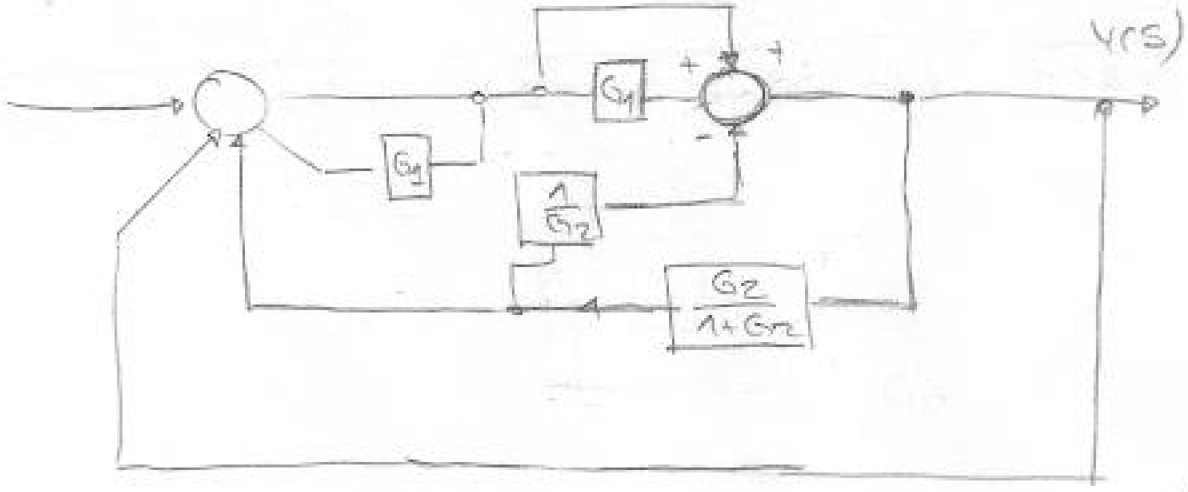
$$G_1 = 1 + \frac{G_2}{1+G_2} = \frac{1+2G_2}{1+G_2}$$

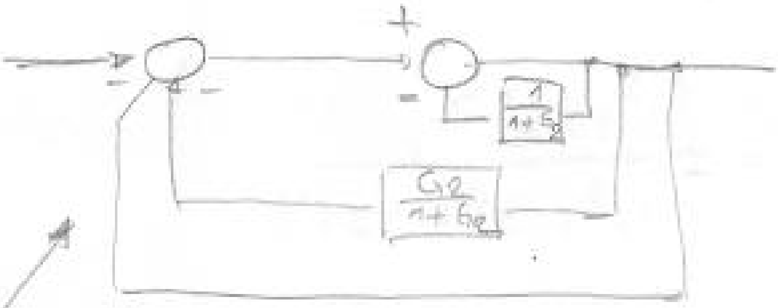
$$G = \frac{1}{1+\frac{1+2G_2}{1+G_2}} = \frac{1+G_2}{2+3G_2}$$

7.



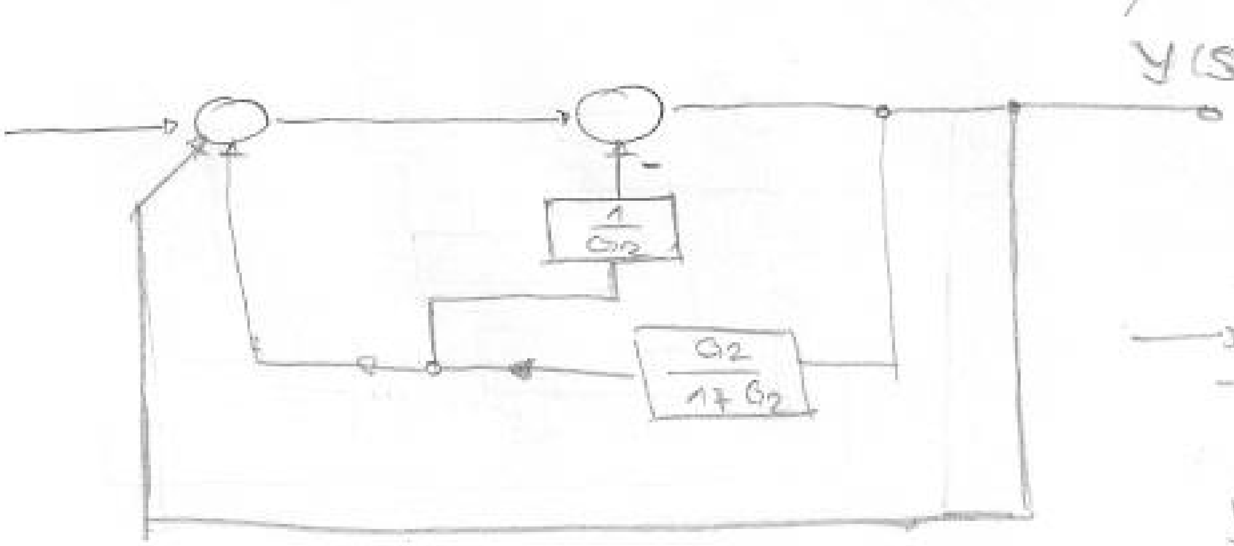
$$G_3 = \frac{G_2}{1+G_2}$$

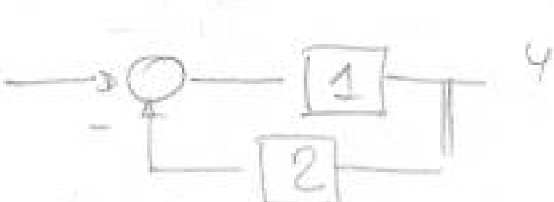




$$\frac{1}{1+G_2} + \frac{G_2}{1+G_2} + 1$$

$$= \frac{1+G_2+1+G_2}{1+G_2} = 2$$

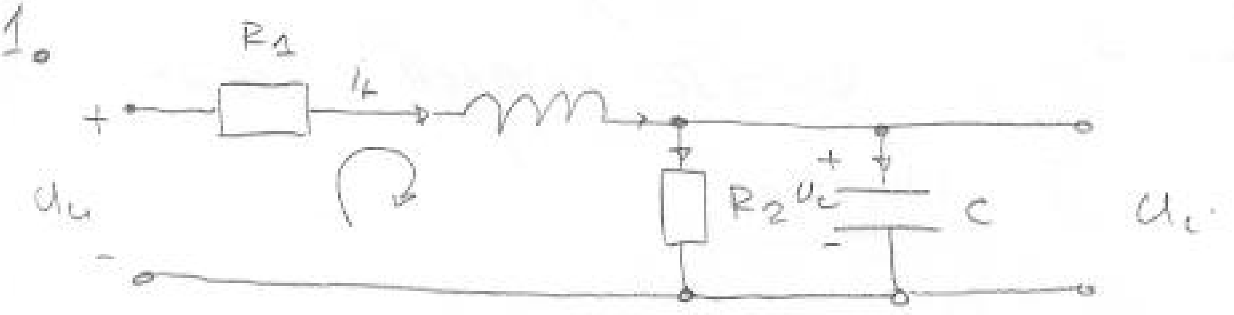




$$\frac{Y}{R} = \frac{1}{1+2} = \frac{1}{3}$$ ✓

POGLAVLJE 2 - matematičko modeliranje procesa

1.



$$I_L = I_R + I_C$$

$$u_u = I_L \cdot R_1 + \dot{I}_L L + u_{iz}$$

$$U_C = \frac{1}{C}\int_0^t I_C(\tau)d\tau \;/^{\circ}$$

$$I_R = u_i / R_2$$

$$C \cdot \dot{U}_C = I_C$$

$$I_L = \frac{1}{R_2} u_{iz} + C \cdot \dot{u}_{iz} \;/^{\circ} \quad (1)$$

$$u_u = \left(\frac{1}{R_2} u_{iz} + C\dot{u}_{iz}\right)R_1 + \left(\frac{1}{R_2}\dot{u}_{iz} + C\ddot{u}_{iz}\right)L + u_{iz}$$

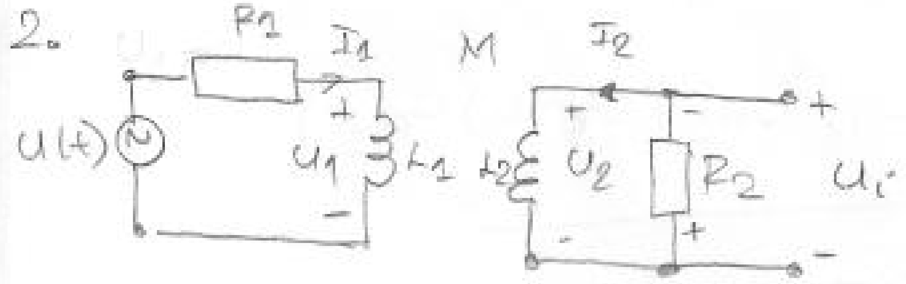
$$\dot{I}_L = \frac{1}{R_2}\dot{u}_{iz} + C \cdot \ddot{u}_{iz} \;/ \quad (2)$$

$$u_u = \frac{R_1}{R_2}u_{iz} + C R_1 \dot{u}_{iz} + \frac{L}{R_2}\dot{u}_{iz} + LC\ddot{u}_{iz} + u_{iz}$$

$$u_u - \dot{u}_{iz}\left(\frac{L - R_1R_2C}{R_2}\right) - LC\ddot{u}_{iz} = u_{iz}\left(\frac{R_1+R_2}{R_2}\right) \;/ \frac{R_2}{R_1+R_2}$$

$$\frac{R_2}{R_1+R_2}u_u - \frac{L - R_1R_2C}{R_1+R_2}\dot{u}_{iz} - \frac{R_2LC}{R_1+R_2}\ddot{u}_{iz} = u_{iz}$$

2.



$$u_1 = L_1 \cdot \dot{I}_1 + M\dot{I}_2 \quad (1)$$

$$u_2 = M\dot{I}_1 + L_2\dot{I}_2 \quad (2)$$

$$u = I_1R_1 + u_1$$

$$u_i = u_{R2} = I_2R_2 \qquad I_2 = \frac{u_i}{R_2} \qquad u_2 = u_i$$

$$u_1 = u - I_1R_1 \;/^{\circ}$$

$$\dot{I}_1 = \frac{u_{iz} - L_2\dot{I}_2}{M} \qquad \ddot{I}_1 = \frac{\dot{u}_{iz} - L_2\ddot{I}_2}{M}$$

$$\dot{u}_1 = \dot{u} - \dot{I}_1R_1$$

(1) derivirano $\dot{u}_1 = L_1\ddot{I}_1 + M\ddot{I}_2$

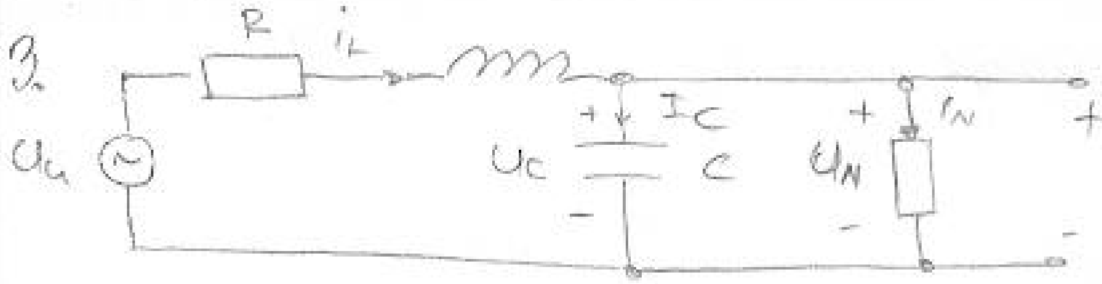
$$\dot{u} - \dot{I}_1R_1 = L_1 \cdot \ddot{I}_1 + M \cdot \ddot{I}_2$$

$$\dot{u} - \frac{u_i - L_2 \cdot \frac{\dot{u}_i}{R_2}}{M}R_1 = L_1 \cdot \frac{\dot{u}_i - L_2\frac{\ddot{u}_i}{R_2}}{M} + M\frac{\ddot{u}_i}{R_2}$$

$$\dot{u} = \frac{L_1}{M}\dot{u}_i - \frac{L_1L_2}{MR_2}\ddot{u}_i + M\frac{\ddot{u}_i}{R_2} + \frac{R_1}{M}u_i - \frac{L_2R_1}{MR_2}\dot{u}_i$$

$$\dot{u} = \ddot{u}_i\left(\frac{M}{R_2} - \frac{L_1L_2}{MR_2}\right) + \dot{u}_i\left(\frac{L_1}{M} - \frac{L_2R_1}{MR_2}\right) + \frac{R_1}{M}u_i$$

3.



$$U_N = k_N \sqrt{i_N}$$

$$k_N = 10 \frac{V}{\sqrt{A}}$$

$R = 10\,\Omega,\ L = 10\,\mu H,\ C = 10\,pF$

$$\dot{I}_L = f_1(I_L, U_C, U_U) \quad (1)$$

$$\dot{U}_C = f_2(I_L, U_C, U_U) \quad (2)$$

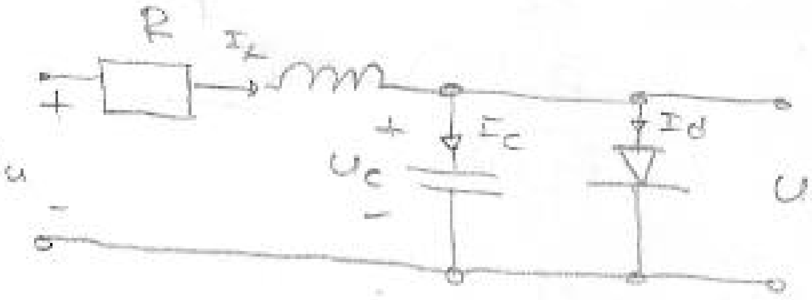
$$\boxed{U_N = U_C}$$

$$U_u = I_L \cdot R + \dot{I}_L \cdot L + U_C \Rightarrow \dot{I}_L = \frac{U_u - U_C - I_L \cdot R}{L} \quad (1)$$

$$U_C = \frac{1}{C} \int_0^t I_C(\tau)\, d\tau \ /^{\circ} \qquad \dot{U}_C = \frac{1}{C} I_C \qquad I_L = I_C + I_N$$

$$\dot{U}_C = \frac{1}{C} (I_L - I_N) = \frac{1}{C} \left(I_L - \left(\frac{U_C}{k_N} \right)^2 \right)$$

4.



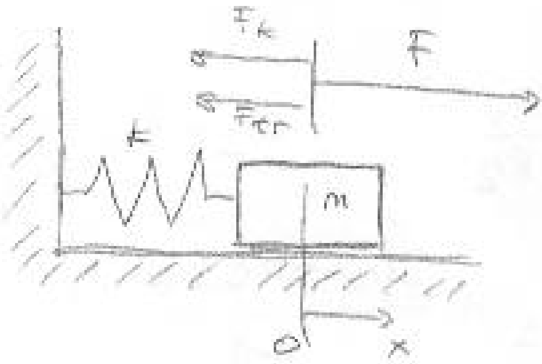
$$I_d = a_1 (U_d - b)^3 - a_2 (U_d - b) + a_3$$

$$\underline{U_C = U_d}$$

$$U_u = I_L \cdot R + \dot{I}_L L + U_C \Rightarrow \dot{I}_L = \frac{U_u - U_C - I_L \cdot R}{L} \quad (1)$$

$$\dot{U}_C = \frac{1}{C} I_C = \frac{1}{C} (I_L - I_d) = \frac{1}{C} \left(I_L - a_1 (U_C - b)^3 + a_2 (U_C - b) - a_3 \right)$$

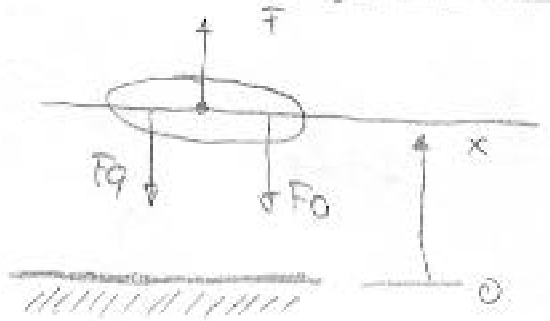
5.



$$m\ddot{x} = F - F_k - F_{tr}$$

$$m\ddot{x} = F - kx^3 - \frac{2 \cdot 10^3}{\pi} \cdot \arctan(\dot{x})$$

6.



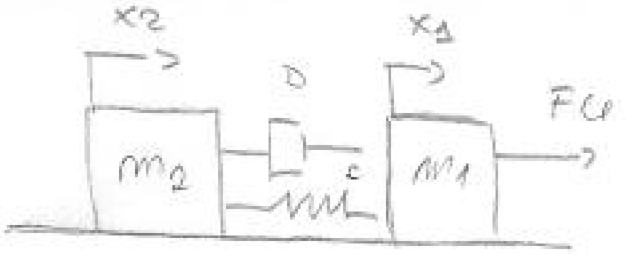
$$m\ddot{x} = F - F_g - F_a$$

$$m\ddot{x} = F - G \frac{Mm}{(R+x)^2} - k_1 (\dot{x})^3 - k_2 \dot{x}$$

$$F_g = G \frac{Mm}{(R+x)^2}$$

$$F_a = k_1 (\dot{x})^3 + k_2 \dot{x}$$

7.



$F_{opruga} = C \cdot \Delta x$

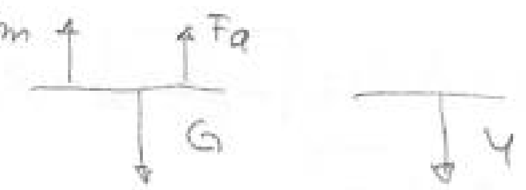
$F_{prigušnica} = D \cdot \Delta \dot{x}$

$$m_1 \ddot{x}_1 = F_u - C(x_1 - x_2) - D(\dot{x}_1 - \dot{x}_2)$$

$$m_2 \ddot{x}_2 = -C(x_2 - x_1) - D(\dot{x}_2 - \dot{x}_1) = C(x_1 - x_2) + D(\dot{x}_1 - \dot{x}_2)$$

8.

$$F_m = \frac{k_1 \cdot i}{(y+k_2)^3}$$

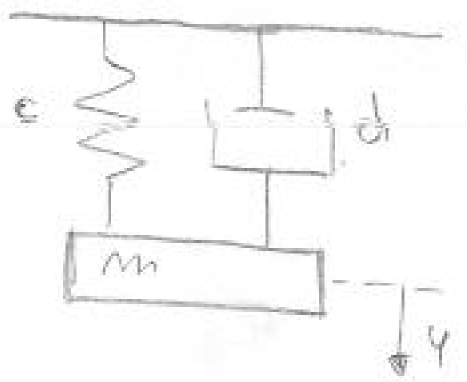


$$F_a = k_3 (\dot{y})^3 + k_4 \dot{y}$$

$$m\ddot{y} = G - F_m - F_a = mg - \frac{k_1 \cdot i}{(y+k_2)^3} - k_3(\dot{y})^3 - k_4 \dot{y} \quad / m$$

$$\ddot{y} = g - \frac{k_1 i}{m(y+k_2)^3} - \frac{k_3}{m}(\dot{y})^3 - \frac{k_4}{m}\dot{y}$$

9.



$$m\ddot{y} = mg - cy - d\dot{y}$$

STACIONARNO STANJE DERIVACIJE = 0

$$mg = cy \qquad y = \frac{mg}{c} = \frac{0.1 \cdot 9,81}{98.1 \, \frac{N}{m}} = 1 \, cm$$

10.

a) $\dot{m}_1 = g_u - g_{12} \qquad \rho A_1 \dot{h}_1 = k_u \cdot \sqrt{x_u} - \rho A_{12} \sqrt{2g(h_1 - h_2)}$

$g_u = k_u \sqrt{x_u}$

$u_i(t) = k_i g_i (t - T_t)$

$$\dot{h}_1 = \frac{k_u \sqrt{x_u}}{\rho A_1} - \frac{A_{12}}{A_1} \sqrt{2g(h_1 - h_2)} \quad (1)$$

$$\dot{m}_2 = g_{12} - g_i \qquad \rho A_2 \dot{h}_2 = \rho A_{12} \sqrt{2g(h_1 - h_2)} - \rho A_i \sqrt{2g h_2} \;\Rightarrow (2)\; \dot{h}_2 = \frac{A_{12}\sqrt{2g(h_1 - h_2)}}{A_2} - \frac{A_i \sqrt{2g h_2}}{A_2}$$

b) $x_{u0} = 0.5$

$$(1) \quad \frac{k_u \sqrt{x_{u0}}}{\rho} = A_{12} \sqrt{2g(h_{10} - h_{20})} \qquad \frac{k_u \sqrt{x_{u0}}}{\rho A_{12}} = \sqrt{2g(h_{10} - h_{20})} = 3.5355$$

$$(2) \quad \cancel{A_{12}} \sqrt{2g(h_{10} - h_{20})} = \cancel{A_i} \sqrt{2g h_{20}} \Rightarrow h_{20} = 0.637 \;, \; h_{10} = 1.2733$$

$$g_i = \rho A_{12} \sqrt{2g h_2} = 35.35 \, m/s \qquad u_i = k_i g_i (t - T_t) = 4.418V$$

11. $g_{u2} = A_v \sqrt{\rho} \sqrt{2\Delta p_1} \cdot x_{u1}$

$g_i = A_v \sqrt{\rho} \sqrt{2\Delta p_2} \cdot x_{u2}$

(1) $\dot{m}_1 = g_{u1} - g_{12}$ $\quad \rho A_1 \dot{h}_1 = 50 - \rho A_{12} \sqrt{2g(h_1-h_2)}$

$$\dot{h}_1 = \frac{50}{\rho A_1} - \frac{A_{12}}{A_1} \sqrt{2g(h_1-h_2)}$$

(2) $\dot{m}_2 = g_{12} + g_{u2} - g_i$ $\quad \rho A_2 \dot{h}_2 = \rho A_{12} \sqrt{2g(h_1-h_2)} + A_v \sqrt{\rho} \sqrt{2\Delta p_2}\, x_{u1} - A_v \rho \sqrt{2gh_2} \cdot x_{u2}$

$$\dot{h}_2 = \frac{A_{12}}{A_2} \sqrt{2g(h_1-h_2)} + \frac{A_v \sqrt{\rho} \sqrt{2\Delta p_1} \cdot x_{u1}}{\rho A_2} - \frac{A_v}{A_2} \sqrt{2gh_2} \cdot x_{u2}$$

12.

$\rho A_1 \cdot \dot{h}_1 = g_u - \rho A_v \sqrt{2g(h_1-h_2)} \cdot x_u$

$\rho A_2 \cdot \dot{h}_2 = \rho A_v \sqrt{2g(h_1-h_2)} \cdot x_u - \rho A_i \sqrt{2gh_2}$

SIGNUM ???
Larbos ???

$$\dot{h}_1 = \frac{g_u}{\rho A_1} - \frac{A_v}{A_1} \sqrt{2g(h_1-h_2)}\, x_u$$

$$\dot{h}_2 = \frac{A_v}{A_2} \sqrt{2g(h_1-h_2)}\, x_u - \frac{A_i}{A_2} \sqrt{2gh_2}$$

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13. $g_i(t) = A_v \sqrt{\rho} \sqrt{2\Delta p} \cdot x_u$

$g_i = A_v \rho \sqrt{2gh} \cdot x_u$

$\dot{m} = \rho \cdot \dot{V}$

$\frac{R_H}{H} = \frac{r}{h} \quad r = \frac{R_H \cdot h}{H}$

$V = \frac{1}{3} B \cdot h = \frac{1}{3} R^2 \pi \cdot h$

$\boxed{\frac{dV}{dt}} = \frac{dV}{dh} \cdot \frac{dh}{dt}$

$\dot{m} = g_u - g_i$

$V = \frac{1}{3} \left(\frac{R_H}{H}\right)^2 \cdot h^3 \pi \;\Big/ \frac{d}{dt}$

$\dot{V} = \left(\frac{R_H}{H}\right)^2 \cdot h^2 \cdot \dot{h}\, \pi$

$\rho \cdot \left(\frac{R_H}{H}\right)^2 \pi \cdot h^2 \cdot \dot{h} = 10 - A_v \rho \sqrt{2gh}\; x_u$

$$\dot{h} = \frac{10}{\rho \left(\frac{R_H}{H}\right)^2 \pi \cdot h^2} - \frac{A_v \sqrt{2gh}\; x_u}{\rho \left(\frac{R_H}{H}\right)^2 \pi \cdot h^2} \quad (1)$$

POGLAVLJE 3 - linearizacija nelinearnih sustava

1. $u(t) \cdot \overset{\circ\circ}{y}(t) + \overset{\circ}{y}(t) + y^2(t) = e^{u-4}$

$y(t) = y_0(t) = 2$

1. Radna točka

$4 = e^{u-2} \;/\ln \quad \ln 4 = u_0 - 2 \quad u_0 = 2 + \ln 4 = 3.38629$

2. $u = u_0 + \Delta u$

$y = y_0 + \Delta y$

3. $u \cdot \overset{\circ\circ}{y} = e^{u-4} - \overset{\circ}{y} - y^2 = F(y, \overset{\circ}{y}, u)$

$\frac{\partial F}{\partial y} = -e^{u_0 - y_0} - 2y_0 = -8 \qquad \frac{\partial F}{\partial \overset{\circ}{y}} = -1 \qquad \frac{\partial F}{\partial u} = e^{u-4} = 4$

a) $3.38629 \cdot \Delta \overset{\circ\circ}{y} = -8\Delta y - \Delta \overset{\circ}{y} + 4\Delta u \Rightarrow \Delta \overset{\circ\circ}{y} = -2.3624\Delta y - 0.295\Delta \overset{\circ}{y} + 1.1812\Delta u$

b) $s^2 \Delta y = -2.3624\Delta y - 0.295 s\Delta y + 1.1812\Delta u$

$\Delta y(s^2 + 2.3624 + 0.295s) = 1.1812\Delta u$

$$\frac{\Delta y}{\Delta u} = \frac{1.1812}{s^2 + 0.295s + 2.3624} = G(s)$$

c) $\Delta u = 0.3$

$$\Delta y(t=\infty) = \lim_{s \to 0} s \cdot G(s) \cdot U(s) = \lim_{s \to 0} \cancel{s} \frac{1.1812}{s^2 + 0.295s + 2.3624} \frac{0.3}{\cancel{s}} = 0.15$$

$y = y_0 + \Delta y = 2 + 0.15 = 2.15$

2. a) $\dot{y} = \frac{u^2}{y} - y^2$

$\frac{\partial f}{\partial y} = u^2\left(-\frac{1}{y^2}\right) - 2y = -\left(\frac{u}{y}\right)^2 - 2y \quad , \frac{\partial f}{\partial u} = \frac{2u}{y}$

$\Delta\dot{y} = \left(-2y - \left(\frac{u}{y}\right)^2\right)\Delta y + \frac{2u}{y}\Delta u$

$\Delta y\left(s + 2y + \left(\frac{u}{y}\right)^2\right) = \frac{2u}{y}\Delta u \Rightarrow \quad \frac{\Delta y}{\Delta u} = \frac{\frac{2u_0}{y_0}}{s + 2y_0 + \left(\frac{u_0}{y_0}\right)^2} = G(s)$

b) $\dot{y} + y^4 = u^2 y$

$\dot{y} = u^2 y - y^4$

$\frac{\partial f}{\partial y} = u^2 - 4y^3 , \quad \frac{\partial f}{\partial u} = 2uy \qquad \Delta\dot{y} = (u^2 - 4y^3)\Delta y + 2uy\Delta u$

$\Delta y(s - (u^2 - 4y^3)) = 2uy\Delta u \quad \Rightarrow \quad \frac{\Delta y}{\Delta u} = \frac{2u_0 y_0}{s - (u_0^2 - 4y_0^3)}$

c) $\ddot{y} + \dot{y} + y^3 = u^3 - \dot{u}y , u_0 = 1$

$\ddot{y} = u^3 - y^3 - \dot{y} - \dot{u}y$

$\frac{\partial f}{\partial y} = -3y^2 - u \quad , \frac{\partial f}{\partial \dot{y}} = -1 , \quad \frac{\partial f}{\partial u} = 3u^2 , \frac{\partial f}{\partial \dot{u}} = -y$

$\Delta\ddot{y} = (-3y^2 - u)\Delta y - \Delta\dot{y} + 3u^2\Delta u - y\Delta\dot{u}$

$\Delta y(s^2 + 3y^2 + u + s) = \Delta u(3u^2 - sy) \quad \Rightarrow \quad \frac{\Delta y}{\Delta u} = \frac{-sy_0 + 3u_0^2}{s^2 + s + 3y_0^2 + u_0}$

d) $\ddot{y} + \dot{y} - \frac{1}{y} = u - y\cdot\dot{u} , y_0 = 1$

$\ddot{y} = u - y\dot{u} + \frac{1}{y} - \dot{y} \qquad \frac{\partial f}{\partial y} = -\frac{1}{y_0^2} = -1 \qquad \frac{\partial f}{\partial u} = 1$

$0 = u_0 + 1 - 0 \quad , u_0 = -1 \qquad \frac{\partial f}{\partial \dot{y}} = -1 \qquad \frac{\partial f}{\partial \dot{u}} = -y_0 = -1$

$\Delta\ddot{y} = -\Delta y - \Delta\dot{y} + \Delta u - \Delta\dot{u}$

$\Delta y(s^2 + s + 1) = \Delta u(-s + 1) \qquad \frac{\Delta y}{\Delta u} = \frac{-s + 1}{s^2 + s + 1}$

3. $R = 10\,\Omega$
$L = 10\,\mu H$
$C = 10\,pF$
$k_N = 10\,\frac{V}{A^{1/2}}$

$I_{L0} = 16\,mA$

$\dot{I}_L = \frac{1}{L}u_u - \frac{R}{L}i_L - \frac{1}{L}u_C$

$\dot{u}_C = \frac{1}{C}I_L - \frac{1}{Ck_N^2}u_C^2 \Rightarrow \frac{1}{C}I_{L0} = \frac{1}{Ck_N^2}u_{C0}^2$

$\sqrt{I_{L0}\,k_N^2} = u_{C0}$

$u_{C0} = \frac{2}{5}V$

$\frac{\partial \dot{I}_L}{\partial u_u} = \frac{1}{L} = \frac{1}{10\cdot 10^{-6}} = 10^5 \quad \frac{\partial \dot{I}_L}{\partial I_L} = -\frac{R}{L} = \frac{10}{10\cdot 10^{-6}} = -10^6 \quad \frac{\partial \dot{I}_L}{\partial u_C} = -\frac{1}{L} = -10^5$

$\Delta\dot{I}_L = 10^5\Delta u_u - 10^6\Delta I_L - 10^5\Delta u_C$

$\frac{\partial \dot{u}_C}{\partial I_L} = \frac{1}{C} = 10^{11} \quad \frac{\partial \dot{u}_C}{\partial u_C} = -\frac{1}{Ck_N^2}2u_{C0} = -8\cdot 10^8 \quad \Delta\dot{u}_C = 10^{11}\Delta I_L - 8\cdot 10^8\Delta u_C$

$$\begin{bmatrix}\Delta\dot{I}_L\\ \Delta\dot{u}_C\end{bmatrix} = \begin{bmatrix}-10^6 & -10^5\\ 10^{11} & -8\cdot 10^8\end{bmatrix}\begin{bmatrix}\Delta I_L\\ \Delta u_C\end{bmatrix} + \begin{bmatrix}10^5\\ 0\end{bmatrix}\Delta u_u$$

$$\Delta u_i = \begin{bmatrix}0 & 1\end{bmatrix}\begin{bmatrix}\Delta I_L\\ \Delta u_C\end{bmatrix} + [0]\,\Delta u_u$$

12.

$T_V\cdot\frac{dq_u(t)}{dt} + q_u(t) = k_V\sqrt{x_V}$

$q_{u0} = k_V\sqrt{x_{u0}}$

$G_{EMP} = \frac{x_V}{u} = \frac{1}{T_i s}$

$x_{u0} = \left(\frac{q_{u0}}{k_V}\right)^2$

a) $G_p(s) = \frac{H(s)}{x_V(s)}$

$x_{u0} = 0.25$

$h_0 = 1.25\,m$

$\rho A\dot{h} = q_u - q_i = q_u - \rho A\sqrt{2gh}$

$0 = q_u - q_i \quad q_{u0} = q_{i0} = 0.1\,m^3/s$ VOLUMNI PROTOK

$\dot{m} = \rho\cdot\dot{V} = 1000\cdot 0.1\left[\frac{kg}{m^3}\cdot\frac{m^3}{s}\right] = 100\,\frac{kg}{s} = \rho A_i\sqrt{2gh}$

$T_V\dot{q}_u = k_V\sqrt{x_V} - q_u$

$T_V\Delta\dot{q}_u = \frac{1}{5}\Delta x_V - \Delta q_u$

$\frac{\partial \dot{q}_u}{\partial x_V} = \frac{k_V}{2\sqrt{x_{u0}}} = \frac{0.2}{2\cdot\sqrt{0.25}} = \frac{1}{5}$

$(0.1s + 1)\Delta q_u = \frac{1}{5}\Delta x_V$

$\frac{\Delta q_u}{\Delta x_V} = \frac{1}{0.5s + 5}\,\frac{/:5}{/:5} = \frac{0.2}{1 + 0.1s}$

$\frac{\partial \dot{q}_u}{\partial q_u} = -1$

VOLUMNI PROTOK:

$$A \cdot \dot{h} = q_u - A_v \sqrt{2gh}$$

$$\frac{\partial \dot{h}}{\partial q_u} = 1$$

$$\frac{\partial \dot{h}}{\partial h} = \frac{-A_v \sqrt{2g}}{2\sqrt{h_0}} = 0.04$$

$$A \Delta \dot{h} = \Delta q_u - 0.04 \Delta h$$

$$\Delta \dot{h} = 0.2 \Delta q_u - 8 \cdot 10^{-3} \Delta h \Rightarrow \quad s \Delta h + 8 \cdot 10^{-3} \Delta h = 0.2 \Delta q_u$$

$$\Delta h (s + 8 \cdot 10^{-3}) = 0.2 \Delta q_u \qquad \frac{\Delta h}{\Delta q_u} = \frac{0.2}{s + 8 \cdot 10^{-3}} = \frac{25}{125 s + 1}$$

$$G_p(s) = \frac{H(s)}{X_v(s)} = \frac{\Delta q_u}{\Delta x_v} \cdot \frac{\Delta h}{\Delta q_u} = \frac{0.2}{1 + 0.1 s} \cdot \frac{25}{125 s + 1} = \frac{5}{12.5 x^2 + 125.1 x + 1}$$ W

POGLAVLJE 4 - Laplaceova transformacija, težinska i prijelazna funkcija

1. $G(s)=\dfrac{Y(s)}{U(s)}=\dfrac{2-s}{(s+1)(s+4)}$

$u(t)=10(e^{-t}-e^{-5t})S(t)$

$u(t=0)=\emptyset$

$U(s)=10\left(\dfrac{1}{s+1}-\dfrac{1}{s+5}\right)=10\dfrac{4}{(s+1)(s+5)}$

$U(s)=\dfrac{40}{(s+1)(s+5)}$

a) $Y(s)(s+1)(s+4)=(2-s)U(s)$

$Y(s)(s^2+5s+4)=(2-s)U(s)$

$\ddot{y}+5\dot{y}+4y=2u-\dot{u}$

b) $y(0)=1,\ \dot{y}(0)=-1$

$s^2Y(s)-sy(0)-\dot{y}(0)+5(sY(s)-y(0))+4Y(s)=2U(s)-(sU(s)-u(0))$

$s^2Y(s)-1\cdot s+1+5sY(s)-5+4Y(s)=2U(s)-sU(s)$

$Y(s)(s^2+5s+4)-s-4=2U(s)-sU(s)$

$Y(s)(s^2+5s+4)=+s+4+(2-s)U(s)\qquad Y(s)=\dfrac{(2-s)}{s^2+5s+4}U(s)+\dfrac{s+4}{s^2+5s+4}$

$Y(s)=\dfrac{(2-s)}{(s+1)(s+4)}\dfrac{40}{(s+1)(s+5)}+\dfrac{s+4}{(s+1)\cancel{(s+4)}}$

$Y(s)=40\dfrac{2-s}{(s+1)^2(s+4)(s+5)}+\dfrac{1}{s+1}=\dfrac{40(2-s)+(s+4)(s+1)(s+5)}{(s+1)^2(s+4)(s+5)}$

$=\dfrac{A}{s+1}+\dfrac{B}{(s+1)^2}+\dfrac{C}{s+4}+\dfrac{D}{s+5},\quad A=-\dfrac{49}{6}\quad B=10\quad C=-\dfrac{35}{2}$

$D=\dfrac{80}{3}$

$=-\dfrac{49}{6}\dfrac{1}{s+1}+10\dfrac{1}{(s+1)^2}-\dfrac{35}{2}\dfrac{1}{s+5}+\dfrac{80}{3}\dfrac{1}{s+4}$

$=\left(-\dfrac{49}{6}e^{-t}+10te^{-t}-\dfrac{35}{2}e^{-5t}+\dfrac{80}{3}e^{-4t}\right)S(t)$

---

2. $G(s)=\dfrac{\left(1+\frac{T_1}{K_1}s\right)(1+T_2s)}{s\left(1+\frac{T_1}{K_1}s\right)(1+T_2s)+\frac{K_2}{K_1}}=\dfrac{Y(s)}{U(s)}$

$y(t=0^+)=Y(s=\infty)=\lim_{s\to\infty}s^2Y(s)\cdot U(s)$

$y(t=0^+)=\lim_{s\to\infty}s^2\cdot\dfrac{\left(1+\frac{T_1}{K_1}s\right)(1+T_2s)}{s\left(1+\frac{T_1}{K_1}s\right)(1+T_2s)+\frac{K_2}{K_1}}\quad\dfrac{1}{s}\left(\dfrac{s^3\frac{T_1T_2}{K_1}}{s^3\frac{T_1T_2}{K_1}}\right)=1$

3. $G(s) = \frac{1-s}{(s+2)^2(3s+9)} \; e^{-0.5s}$ kašnjenje

$$\dot{g}(0^+) = \lim_{s\to\infty} s^2 \frac{1-s}{(s+2)^2(3s+9)} \quad \left(\frac{-s^3}{3s^3}\right) = -\frac{1}{3}$$

---

4. $G(s) = \frac{(T_1+1)s^2 + (k_1+1)s + 2}{T_1 s^3 + (k_1+T_1)s^2 + (2T_1+k_1)s + 2k_1} e^{-0.2s}$

$t = 0.2s \qquad \dot{y}(0^+) = 3 \qquad y(s) = 4$

$$3 = \lim_{s\to\infty} s^2 G(s) \cdot \frac{1}{s} \quad \left(\frac{(T_1+1)s^3}{T_1 s^3}\right) = 3 \qquad \frac{T_1+1}{T_1} = 3 \qquad 3T_1 = T_1 + 1 \qquad T_1 = \frac{1}{2}$$

$$4 = \lim_{s\to 0} s G(s) \cdot \frac{1}{s} \quad \left(\frac{2}{2k_1}\right) \qquad \frac{1}{k_1} = 4 \qquad k_1 = \frac{1}{4}$$

---

5. $\frac{y(s)}{u(s)} = \frac{s(k_1 s + 1)}{s^3 + k_2 s^2 + k_1 s + 1} \qquad u(t) = t \cdot S(t) \circ\!\!-\!\!\circ \frac{1}{s^2}$

$$\dot{y}(t=0^+) = \lim_{s\to\infty} s^2 G(s) \cdot \frac{1}{s^2} \quad \left(\frac{k_1 s^2}{s^3}\right) = \emptyset$$

$$y(t=\infty) = \lim_{s\to 0} s G(s) \cdot \frac{1}{s^2} = \frac{1}{1} = 1$$

---

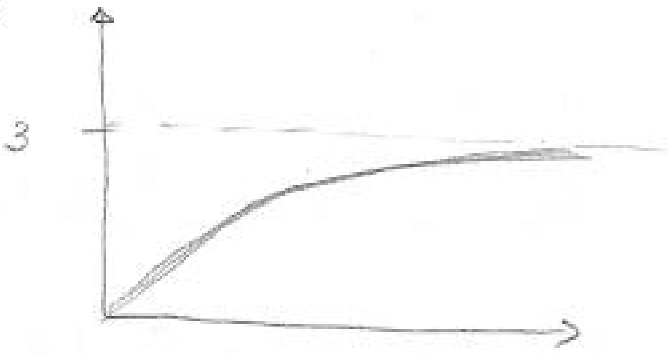
6. $G(s) = \frac{y(s)}{u(s)} = \frac{s^2+2s+1}{(1+2s)(1+s+5s^2)}$

a) $g(t=0^+) = \lim_{s\to\infty} s \frac{s^2+2s+1}{(1+2s)(1+s+5s^2)} \quad \left(\frac{s^3}{10s^3}\right) = \frac{1}{10}$

b) $h(t=\infty) = \lim_{s\to 0} \cancel{s} \frac{s^2+2s+1}{(1+2s)(1+s+5s^2)} \frac{1}{\cancel{s}} = \frac{1}{1} = 1$

7. $G(s) = \dfrac{s^2+2s+3}{s^3+3s^2+3s+1}$

c)



a) $h(t=\infty) = \lim\limits_{s\to 0} s\, G(s)\, \dfrac{1}{s} = 3$

b) $\dot{h}(t=0^+) = \lim\limits_{s\to\infty} s^2 \dfrac{s^2+2s+3}{s^3+3s^2+3s+1} \cdot \dfrac{1}{s} = \underline{1}$

---

8. $G(s) = \dfrac{5}{s^2+5s+6} = \dfrac{5}{(s+2)(s+3)}$

a) $Y(s) = G(s) \cdot U(s) = \dfrac{5}{s(s+2)(s+3)} = \dfrac{A}{s+2} + \dfrac{B}{s+3} + \dfrac{C}{s}$

$= \dfrac{5}{6} \cdot \dfrac{1}{s} - \dfrac{5}{2}\dfrac{1}{s+2} + \dfrac{5}{3}\dfrac{1}{s+3} \quad \bullet\!\!-\!\!\circ \quad \left(\dfrac{5}{6} - \dfrac{5}{2}e^{-2t} + \dfrac{5}{3}e^{-3t}\right) S(t)$

b) točka infleksije (2 derivacija = 0)

$\dot{h}(t) = 5e^{-2t} - 5e^{-3t} \qquad \ddot{h}(t) = -10e^{-2t} + 15e^{-3t} = 0 \; / e^{3t}$

$10 e^{t} = 15 \qquad e^{t} = \dfrac{3}{2} \qquad t = \ln\dfrac{3}{2} = 0.405$

$h(t) = h(t = \ln\dfrac{3}{2}) = 0.216$

---

9. $h(t) = \dfrac{1}{2} e^{-t} (\sin t - \cos t)\, S(t)$

a) $g(t) = ?$ $\quad g(t) = \dfrac{dh}{dt} = -\dfrac{1}{2} e^{-t} (\sin t - \cos t)\, S(t) + \dfrac{1}{2} e^{-t} (\cos t + \sin t) S(t)$

$+ \dfrac{1}{2} e^{-t} (\sin(t) - \cos t) \cdot \delta(t)$

$= e^{-t} \cos t - \dfrac{1}{2} \qquad G(s) = \dfrac{s+1}{(s+1)^2+1} - \dfrac{1}{2} = \dfrac{-\frac{1}{2}s^2}{(s+1)^2+1}$

b) $U(s) = \dfrac{1}{(s+1)^2+1} + \dfrac{2(s+1)}{(s+1)^2+4}$

$Y(s) = G(s)\, U(s)$

$y(t) = \int_0^t y(s)$ KONVOLUCIJA

$= \left(\dfrac{4}{3}\sin 2t - \cos 2t - \dfrac{7}{6}\sin t + \dfrac{1}{2} t \sin t\right) e^{-t}$

10. $h(t) = (2 - e^{-t} - e^{-2t})\, s(t)$

a) $g(t) = \dot{h}$

$g(t) = (e^{-t} + 2e^{-2t})\, s(t) + \underbrace{(2 - e^{-t} - e^{-2t})}^{\emptyset}\, \delta(t)$

$g(t) = (e^{-t} + 2e^{-2t})\, s(t)$

b) $G(s) = \left(\frac{1}{s+1} + \frac{2}{s+2}\right) = \frac{s+2+2s+2}{(s+1)(s+2)} = \frac{3s+4}{(s+1)(s+2)}$

c) $u(t) = 2s(t) - s(t-2)$

$y(t) = h(t) - h(t-2)$
jer je LTI (Vremenski nepromjenjiv)

$u_1(t) = \frac{2}{s}$

$y(s) = \frac{6s+8}{s(s+1)(s+2)} = \frac{4}{s} - \frac{2}{s+1} - \frac{2}{s+2} \circ\!\!-\!\!\circ (4 - 2e^{-t} - 2e^{-2t})\, s(t)$

$u_2(t) = \frac{2}{s} e^{-2s}$ $\quad y(t) = (4 - 2e^{-(t-2)} - 2e^{-2(t-2)})\, s(t-2)$

$y_{uk}(s) = (4 - 2e^{-t} - 2e^{-2t})\, s(t) + (4 - 2e^{-(t-2)} - 2e^{-2(t-2)})\, s(t-2)$

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11. ISTI KAO 10.

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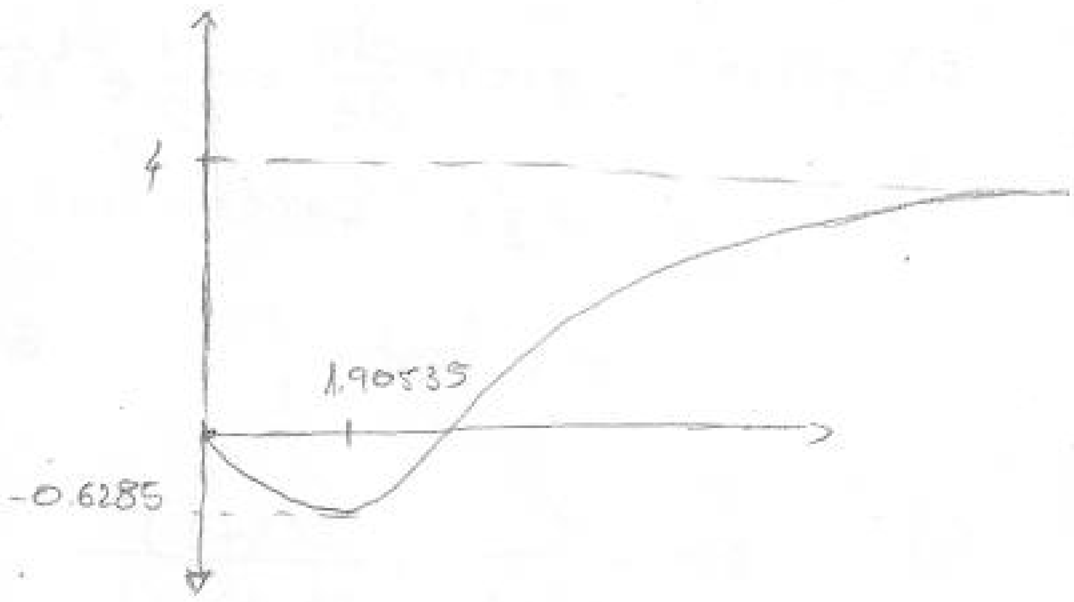
12. $G_p(s) = \frac{4(1-4s)}{(1+10s)(1+2s)}$



$y(t=\infty) = \lim_{s\to 0} s G_p(s) \cdot \frac{1}{s} = 4$

$y(t=0^+) = \lim_{s\to\infty} s \cdot G_p \cdot \frac{1}{s} = 0$

$\dot{y}(t=0^+) = \lim_{s\to\infty} s^2 \cdot G_p \cdot \frac{1}{s} = \left(-\frac{16s^2}{20s^2}\right) = -\frac{4}{5}$

Polovi realni – nema oscilacija

Nula u Desnoj poluravnini – Podbačaj

$H(s) = G(s) \cdot \frac{1}{s} = \ldots$ parcijalno

$= \frac{4}{s} - \frac{7}{s+\frac{1}{10}} + \frac{3}{s+\frac{1}{2}}$

I derivacija $= 0 \Rightarrow$ VRIJEME PODBAČAJA

$h'(t_m) = \frac{7}{10} e^{-t_m/10} - \frac{3}{10} e^{-t_m/2}$

$t_m = 1.90535\,s$

$h(t_m) = -0.6285$

POGLAVLJE 5

1. $\dot{U}_c = -\frac{1}{R_2 C} U_c + \frac{1}{C} I_L$

$\dot{I}_L = -\frac{1}{L} U_c - \frac{R_1}{L} I_L + \frac{1}{L} U_u \quad , \quad u_i = U_c$

a)

$$\begin{bmatrix} \dot{U}_c \\ \dot{I}_L \end{bmatrix} = \overset{A}{\begin{bmatrix} -\frac{1}{R_2 C} & \frac{1}{C} \\ -\frac{1}{L} & -\frac{R_1}{L} \end{bmatrix}} \begin{bmatrix} U_c \\ I_L \end{bmatrix} + \overset{B}{\begin{bmatrix} 0 \\ \frac{1}{L} \end{bmatrix}} [U_u]$$

$$[u_i] = \overset{C}{[1 \quad 0]} [U_c \quad I_L] + \overset{D}{[0]} U_u$$

b) $G(s) = C(sI - A)^{-1} B + D$

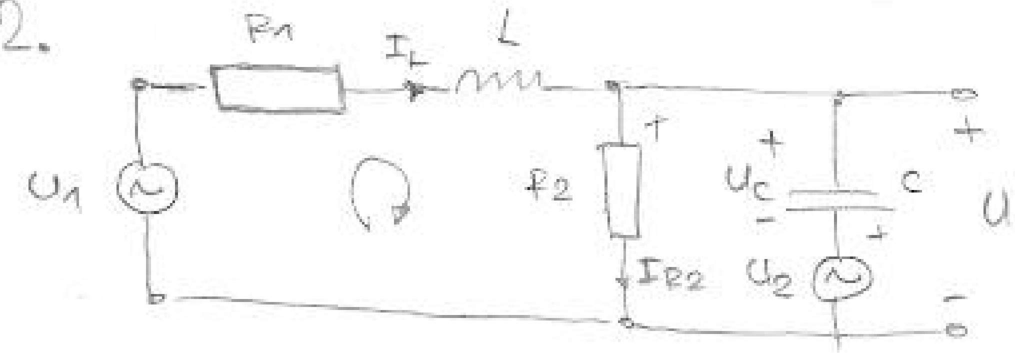
$$= [1 \quad 0] \begin{bmatrix} s + \frac{1}{R_2 C} & -\frac{1}{C} \\ \frac{1}{L} & s + \frac{R_1}{L} \end{bmatrix}^{-1} \begin{bmatrix} 0 \\ \frac{1}{L} \end{bmatrix} + [0]$$

$$= [1 \quad 0] \frac{1}{det} \begin{bmatrix} s + \frac{R_1}{L} & \frac{1}{C} \\ -\frac{1}{L} & s + \frac{1}{R_2 C} \end{bmatrix} \begin{bmatrix} 0 \\ \frac{1}{L} \end{bmatrix} + [0]$$

$$= \frac{1}{s^2 + \frac{s}{R_2 C} + s\frac{R_1}{L} + \frac{R_1}{R_2 L C} + \frac{1}{LC}} \cdot \begin{bmatrix} s + \frac{R_1}{L} & \frac{1}{C} \end{bmatrix} \begin{bmatrix} 0 \\ \frac{1}{L} \end{bmatrix} = \frac{R_2 \cancel{LC}}{R_2 L C s^2 + s(L + R_1 R_2 C) + R_1 + R_2} \cdot \frac{1}{\cancel{LC}}$$

$$= \frac{R_2}{R_2 L C s^2 + s(L + R_1 R_2 C) + R_1 + R_2}$$

2.



$$x = [u_C \; I_L]^T$$
$$u = [u_1 \; u_2]^T$$
$$y = [u_i \; I_{R2}]^T$$

$$\begin{bmatrix} \dot{u}_C \\ \dot{I}_L \end{bmatrix} = \begin{bmatrix} -\frac{1}{R_2C} & \frac{1}{C} \\ -\frac{1}{L} & -\frac{R_1}{L} \end{bmatrix} \begin{bmatrix} u_C \\ I_L \end{bmatrix} + \begin{bmatrix} 0 & -\frac{1}{R_2C} \\ \frac{1}{L} & -\frac{1}{L} \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$$

$$\begin{bmatrix} u_i \\ I_{R2} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ \frac{1}{R_2} & 0 \end{bmatrix} \begin{bmatrix} u_C \\ I_L \end{bmatrix} + \begin{bmatrix} 0 & 1 \\ 0 & \frac{1}{R_2} \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$$

$$u_1 = I_L \cdot R_1 + \dot{I}_L \cdot L + u_i \qquad u_i = u_2 + u_C$$

$$\dot{I}_L = \frac{u_1 - u_i - I_L R_1}{L} = \frac{u_1 - u_2 - u_C - I_L R_1}{L}$$

$$u_C = \frac{1}{C} \int_0^t I_C(\tau) d\tau \Big|^{\circ} \qquad \dot{u}_C = \frac{1}{C} \cdot I_C = \frac{1}{C}(I_L - I_{R2})$$

$$\dot{u}_C = \frac{1}{C}\left(I_L - \frac{u_i}{R_2}\right) = \frac{1}{C}\left(I_L - \frac{u_2 + u_C}{R_2}\right) = \frac{1}{C} I_L - \frac{u_2 + u_C}{R_2 C}$$

$$I_{R2} = \frac{u_i}{R_2} = \frac{u_2 + u_C}{R_2}$$

3. $m_1 \ddot{x}_1(t) = F_u(t) - D(\dot{x}_1 - \dot{x}_2) - c(x_1 - x_2)$

$m_2 \ddot{x}_2 = D(\dot{x}_1 - \dot{x}_2) + c(x_1 - x_2)$

a)

$$\begin{bmatrix} \dot{x}_1 \\ \ddot{x}_1 \\ \dot{x}_2 \\ \ddot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -\frac{c}{m_1} & -\frac{D}{m_1} & \frac{c}{m_1} & \frac{D}{m_1} \\ 0 & 0 & 0 & 1 \\ \frac{c}{m_2} & \frac{D}{m_2} & -\frac{c}{m_2} & -\frac{D}{m_2} \end{bmatrix} \begin{bmatrix} x_1 \\ \dot{x}_1 \\ x_2 \\ \dot{x}_2 \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{1}{m_1} \\ 0 \\ 0 \end{bmatrix} [F_u]$$

$$\begin{bmatrix} x_1 \\ \dot{x}_1 \\ \ddot{x}_1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ -\frac{c}{m_1} & -\frac{D}{m_1} & \frac{c}{m_1} & \frac{D}{m_1} \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ \dot{x}_1 \\ x_2 \\ \dot{x}_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \frac{1}{m_1} \\ 0 \end{bmatrix} [F_u]$$

b) $G(s) = \dfrac{X_1(s)}{F_u(s)}$

$m_1 s^2 x_1 = F_u - D(s x_1 - s x_2) - c(x_1 - x_2)$

$x_1(m_1 s^2 + Ds + c) = F_u + x_2(Ds + c)$

$m_2 s^2 x_2 = D(s x_1 - s x_2) + c(x_1 - x_2)$

$x_2(m_2 s^2 + Ds + c) = x_1(Ds + c) \quad \Rightarrow \quad x_2 = \dfrac{x_1(Ds+c)}{m_2 s^2 + Ds + c}$

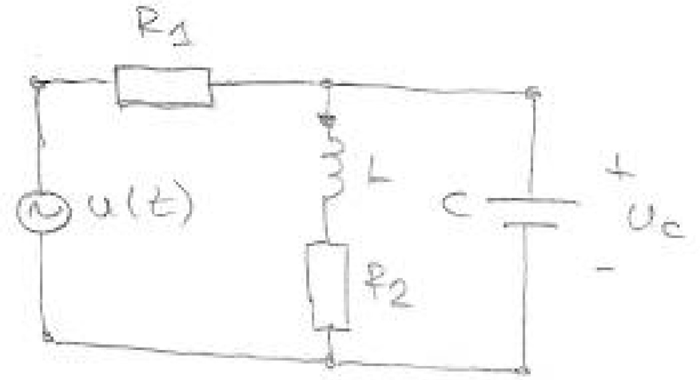
$x_1(m_1 s^2 + Ds + c) = F_u + \dfrac{x_1(Ds+c)}{m_2 s^2 + Ds + c}(Ds + c)$

$x_1\left(\dfrac{(m_1 s^2 + Ds + c)(m_2 s^2 + Ds + c) + (Ds + c)^2}{m_2 s^2 + Ds + c}\right) = F_u$

$$\frac{x_1}{F_u} = \frac{m_2 s^2 + Ds + c}{(m_1 s^2 + Ds + c)(m_2 s^2 + Ds + c) + (Ds + c)^2} = \ldots$$

$$= \frac{m_2 s^2 + Ds + c}{s^2[m_1 m_2 s^2 + (m_1 + m_2) Ds + (m_1 + m_2) c]}$$

4.



I_L, u_c

$$\begin{bmatrix} \dot{u}_c \\ \dot{I}_L \end{bmatrix} = \begin{bmatrix} \frac{-1}{R_1 C} & -\frac{1}{C} \\ \frac{1}{L} & \frac{R_2}{L} \end{bmatrix} \begin{bmatrix} u_c \\ I_L \end{bmatrix} + \begin{bmatrix} \frac{1}{R_1 C} \\ 0 \end{bmatrix} u$$

$$u_c = \dot{I}_L L + I_L \cdot R_2 \qquad \dot{I}_L = \frac{u_c - I_L R_2}{L}$$

$$\dot{u}_c = \frac{1}{C} I_C = \frac{1}{C}(I_1 - I_L) = \frac{1}{C}\left(\frac{u_c - u}{R_1} - u\right)$$

$$u = I_1 R_1 + u_c \qquad I_1 = \frac{u - u_c}{R_1}$$

$$u_c = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} u_c \\ I_L \end{bmatrix} + \begin{bmatrix} 0 \end{bmatrix} u$$

5) ISTO

6.

a) $m\ddot{y} = u - b\dot{y} - cy - d\dot{y}$

b) $m\ddot{y} = u - b\dot{y} - c_1 y - c_2 y$

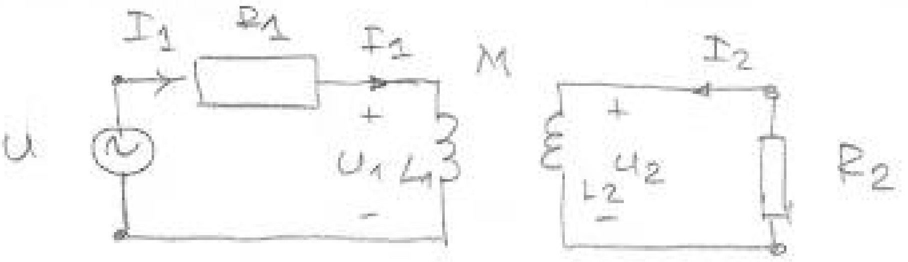
$x = [\, y \quad \dot{y} \,]$

a) $$\begin{bmatrix} \dot{y} \\ \ddot{y} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -\frac{c}{m} & -\frac{b+d}{m} \end{bmatrix} \begin{bmatrix} y \\ \dot{y} \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{1}{m} \end{bmatrix} [\, u \,]$$

b) $$\begin{bmatrix} \dot{y} \\ \ddot{y} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -\frac{c_1+c_2}{m} & -\frac{b}{m} \end{bmatrix} \begin{bmatrix} y \\ \dot{y} \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{1}{m} \end{bmatrix} u$$

5. $u_1 = \dot{I}_1 L_1 + M \dot{I}_2$

$u_2 = M \dot{I}_1 + L_2 \dot{I}_2$



I_1, I_2, u

$u(t) = I_1 R_1 + u_1 \qquad u_1 = u - I_1 R_1 \qquad -I_2 R_2 = M\dot{I}_1 + L_2 \dot{I}_2$

$u_2 = +I_2 R_2 \qquad \dot{u}_2 = \dot{I}_2 R_2 \qquad \dot{I}_2 = \dfrac{+I_2 R_2 - M \dot{I}_1}{L_2}$

$u - I_1 R_1 = \dot{I}_1 L_1 + M \dot{I}_2$

$u - I_1 R_1 = \dot{I}_1 L_1 + M \dfrac{+I_2 R_2 - M \dot{I}_1}{L_2} \quad / \cdot L_2$

$u \cdot L_2 - I_1 R_1 L_2 = \dot{I}_1 L_1 L_2 + M (I_2 R_2 + M \dot{I}_1)$

$u \cdot L_2 - I_1 R_1 L_2 = \dot{I}_1 L_1 L_2 + M I_2 R_2 - M^2 \dot{I}_1 \qquad X = L_1 L_2 - M^2$

$-M R_2 I_2 - R_1 L_2 I_1 + u L_2 = \dot{I}_1 (L_1 L_2 - M^2)$

$$\dot{I}_1 = \frac{-M R_2 I_2 - R_1 L_2 I_1 + u L_2}{X}$$

$$L_2 \cdot \dot{I}_2 = I_2 R_2 - M \left(\frac{-M R_2 I_2 - R_1 L_2 I_1 + u L_2}{X} \right)$$

$$L_2 \dot{I}_2 = I_2 R_2 + \frac{M^2 R_2 I_2}{X} + \frac{M R_1 L_2 I_1}{X} - \frac{M L_2 u}{X}$$

$$\dot{I}_2 = \frac{R_2 L_1 \cancel{L_2} - \cancel{M^2 R_2} + \cancel{M^2 R_2}}{X \cdot \cancel{L_2}} I_2 + \frac{M R_1 \cancel{L_2}}{X \cancel{L_2}} I_1 - \frac{M \cdot \cancel{L_2}}{\cancel{L_2} X} u$$

$$\dot{I}_2 = \frac{1}{X} \left(R_2 L_1 I_2 + M R_1 I_1 - M \cdot u \right)$$

$$\begin{bmatrix} \dot{I}_1 \\ \dot{I}_2 \end{bmatrix} = \frac{1}{L_1 L_2 - M^2} \begin{bmatrix} -R_1 L_2 & -M R_2 \\ M R_1 & R_2 L_1 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} + \begin{bmatrix} L_2 \\ -M \end{bmatrix} u$$

PAŽNJA !!

POGLAVLJE 6 - prikaz pomoću frekvencijske karakteristike

1. $G(s)=\dfrac{32}{s^2+2s+16}=\dfrac{Y(s)}{U(s)}$ $\qquad \omega_r=\omega_n\sqrt{1-2\xi^2}$

$u(t)=\sin(\omega_r t+45^\circ)$ $\qquad \omega_n=4$ $\qquad \omega_r=4\sqrt{1-\frac{1}{8}}$

$G(s)=\dfrac{k\,\omega_n^2}{s^2+2\zeta\omega_n s+\omega_n^2}$ $\qquad 2\zeta\omega_n=2$ $\qquad =4\cdot\sqrt{\frac{7}{8}}$

$\zeta=\frac{1}{4}$ $\qquad \omega_r=\sqrt{14}$

$y(t)=A\sin(\omega_r t+45^\circ+\varphi)$ $\qquad \boxed{G(j\omega)=\dfrac{32}{16-\omega^2+2j\omega}}$

$|G(j\omega_r)|=\dfrac{32}{\sqrt{(16-\omega^2)^2+4\omega^2}}=4.13=A$

$\varphi(j\omega_r)=\operatorname{arctg}\dfrac{0}{32}-\operatorname{arctg}\dfrac{2\omega}{16-\omega^2}=-75.03$

$y(t)=4.13\sin(\sqrt{14}\,t-30.03)$

---

2. 

$r(t)=2\sin(3t+\frac{\pi}{6})$

$y_0(t)=\sin(3t-\frac{2\pi}{3})$

$\gamma=\frac{\pi}{6}$ $\qquad \frac{\pi}{6}+\varphi=-\frac{2\pi}{3}$ $\qquad \varphi=-\frac{2\pi}{3}-\frac{\pi}{6}=-\frac{5\pi}{6}$

$|G_0(j\omega)|=1=|G_0(j\cdot 3)|$ $\qquad G_0(j\cdot 3)=\frac{1}{2}\angle-\frac{5\pi}{6}=-\frac{\sqrt{3}}{4}-\frac{1}{4}i$

$G_z=\dfrac{G_0(j\cdot 3)}{1+G_0(j\cdot 3)}=-0.4766-0.65108j=0.80689\angle\overset{\varphi}{-2.2027}$

$|G_z|=0.806$ $\qquad A=|G_z|\ 2$ ← amplituda od $r(t)$

$y_z=A\sin(3\cdot t+\frac{\pi}{6}+\varphi)=1.61379\sin(3\cdot t-1.679)$

3. $s_{1,2} = 2.3(-1 \pm j\sqrt{3})$

$A(0) = 1$

$u(t) = \sin(\omega_e t - 45°)$, ω_e = lowna frekvencija

$$G(s) = \frac{K}{\frac{1}{\omega_n^2}s^2 + \frac{23}{\omega_n}s + 1}$$

$$P = (s - 2.3(-1+j\sqrt{3}))(s - 2.3(-1-j\sqrt{3}))$$

$$P = (s+2.3-2.3j\sqrt{3})(s+2.3+2.3\sqrt{3})$$

$$P = s^2 + \frac{23}{5}s + \frac{529}{25} = 0 \,/\cdot \frac{25}{529}$$

$$G(s) = \frac{K}{\frac{25}{529}s^2 + \frac{5}{23}s + 1}$$

$$P = \frac{25}{529}s^2 + \frac{5}{23}s + 1$$

$$G(s=0) = 1 = \lim_{s \to 0} G(s) = 1 \qquad k = 1 \qquad G_z(s) = \frac{1}{\frac{25}{529}s^2 + \frac{5}{23}s + 1}$$

$$G_z = \frac{G_o}{1+G_o} \Rightarrow G_o = \frac{G_z}{1-G_z} = \frac{529}{25s^2+115s} = \frac{529}{25s(s+4.6)} = \frac{21.16}{s(s+4.6)}$$

$$= \frac{4.6}{s\cdot\left(\frac{1}{4.6}s+1\right)}$$

$\omega_e = 4.6$ rad/s ?!

$$|G_z(j\omega_e)| = 1$$

$$\varphi(G_2(j\omega_e)) = -\frac{\pi}{2}$$

$$y(t) = 1\cdot 1 \sin\left(4.6t - \frac{\pi}{4} - \frac{\pi}{2}\right) = 1\sin(4.6t - 135)$$

4. $s_{p1,2} = \left(-\frac{1}{2} \pm \frac{\sqrt{3}}{2} j\right)$

$s_{p1,2} = -\zeta\omega_n \pm j\,\omega_n\sqrt{1-\zeta^2}$

a) $\omega_r, \omega_b = ?$

$\omega_r = \omega_n\sqrt{1-2\zeta^2}$

$\omega_n = \sqrt{Re^2 + Im\,g^2} = \sqrt{\frac{1}{4} + \frac{3}{4}} = 1 \qquad \zeta\omega_n = \frac{1}{2} \quad \zeta = \frac{1}{2}$

$\omega_r = 1 \cdot \sqrt{1 - \frac{1}{2}} = \sqrt{\frac{1}{2}} = \frac{\sqrt{2}}{2} = 0.707$

$G(s) = \frac{1}{s^2 + s + 1} \Rightarrow G(j\omega) = \frac{1}{1-\omega^2 + j\omega} \quad |G(j\omega)| = \frac{1}{\sqrt{(1-\omega^2)^2 + \omega^2}}$

$20 \log |G(j\omega_b)| = -3dB \qquad 20 \log \frac{1}{\sqrt{(1-\omega^2)^2 + \omega^2}} = -3dB$

$20[\log 1 - (\log\sqrt{(1-\omega)^2 + \omega^2}\,] = -3dB$

$-20 \log((1-\omega^2)^2 + \omega^2)^{\frac{1}{2}} = -3dB \qquad 10 \log((1-\omega^2)^2 + \omega^2) = 3$

$10^{0.3} = 1 - 2\omega^2 + \omega^4 + \omega^2 \qquad 1\,99526 = \omega^4 - \omega^2 + 1 \qquad \omega^4 - \omega^2 - 0\,99526 = 0$

$\omega^2 = t \qquad \omega^2 = 1.6159 \qquad \omega = 1{,}27$ rad/s

b) $\gamma = ?$

$G_o = \frac{G_z}{1-G_z} = \frac{\frac{1}{s^2+s+1}}{1 - \frac{1}{s^2+s+1}} = \frac{\frac{1}{\cancel{s^2+s+1}}}{\frac{s^2+s+1-1}{\cancel{s^2+s+1}}} = \frac{1}{s^2+s} = \frac{1}{s(s+1)}$

$G_o(j\omega) = \frac{1}{j\omega(j\omega+1)} \qquad |G_o(j\omega_c)| = 1 = \frac{1}{\omega_c\sqrt{1+\omega_c^2}}$

$\omega_c\sqrt{1+\omega_c^2} = 1 \,/^2 \qquad \omega_c^2(1+\omega_c^2) - 1 = 0 \qquad \omega_c^4 + \omega_c^2 - 1 = 0$

$t = \omega_c^2 \qquad t = 0.618 \Rightarrow \omega_c = 0.786$

$\varphi(\omega_c) = -128.16 \qquad \gamma = 51.84$

c) $\omega_c \cdot T = 10^\circ$

$T = \frac{\frac{10}{180}\cdot\pi}{0.786} = 0.222$

5. $G_0 = \dfrac{1}{\frac{1}{1.41^2}s^2 + \frac{2\cdot 0.71}{1.41}s + 1} = \dfrac{1}{0.50s^2 + s + 1}$

$G_0(j\omega) = \dfrac{1}{1 - 0.5\omega^2 + j\omega}$ $\qquad |G_0(j\omega_c)| = 1 = \dfrac{1}{\sqrt{(1-0.5\omega^2)^2 + \omega^2}}$

$1 = 1 - \omega_c^2 + \omega_c^2 + \omega_c^4 \qquad \omega_c = \phi$

$\varphi(\omega_c) = -\mathrm{arctg}\dfrac{\omega}{1-0.5\omega^2} = \phi$

$\gamma = 180 + \varphi_0(\omega_c) = 180^\circ$ FAZNO OSIGURANJE $T_t \in [0, \infty\rangle s$

$\omega_c \cdot T_t = 55^\circ \qquad T_t = \dfrac{\frac{55}{180}\cdot\pi}{0} = \infty$!! VRIJEDI ZA BILO KOJI KUT!

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6. a)

$G_0 = \dfrac{400\, K_R}{s(s+8)(s+50)}$

a) Bode

b) $(\log 0.8, -90)$

$y + 90 = -45(x - \log 0.8) \qquad y(x = \log 5) = -45 \log\frac{5}{0.8} - 90 = -125.8146$

$(\log 5, -125.8146)$

$y + 125.8146 = -90(x - \log 5) \qquad , y = -180$

$x = 1.300 \qquad x = \log\omega \qquad \omega = 10^x = 19.98\, rad/s \approx 20\, rad/s$

$(\log 8, 20\log 8)$

$y + 20\log 8 = -40(\log 20 - \log 8) \qquad y = -33.97\, dB$

$A(\omega_\pi) = 33.97 dB = 20\log K_R \Rightarrow K_R = 50$ [na rubu stabilnosti]

$K_R \in \langle 0, 50\rangle$

7. a) potrebno znati napomet članove

$$G_p(s) = \frac{s+10}{s(s+1)}$$

b) Bode $G(s) = G_p \cdot G_R = \frac{s+10}{s(s+1)} \quad 200\,\frac{s+10}{s+1}$

c) ω_c, $\gamma = ?$

sve je podignuto za 6dB

$(\log 10, 6)$ $\quad y-6 = -20(x-\log 10)$ $\quad y=0$

$\frac{6}{20} + \log 10 = x$ $\quad x = \frac{13}{10}$ $\quad \omega_c = 10^x = 19.95\ rad/s$

Fazno: $(\log 10, 180)$

$y+180 = +90(x-\log 10)$ $\quad y = -153.005 = \varphi_o(\omega_c)$

$\gamma = 180 - 153 = 27°$

8. $K_R = 24.5$ $\quad G_o = \frac{24.5 \cdot 2}{(1+0.1s)(1+0.002s)} = \frac{49}{(0.1s+1)(0.002s+1)}$

$K_{KR,krit} = ?$

$(\log 10, 33.8)$

$y - 33.8 = -20(x - \log 10) \Rightarrow y = 0$ $\quad x = 2.69$ $\quad \omega_c = 489.778\ rad/s$

$\omega_\pi = 5000\ rad/s$

$(2.69, 0)$

$y = -20(x-2.69) = -20(\log 500 - 2.69) = -0.1794$

$(\log 500, -0.1794)$

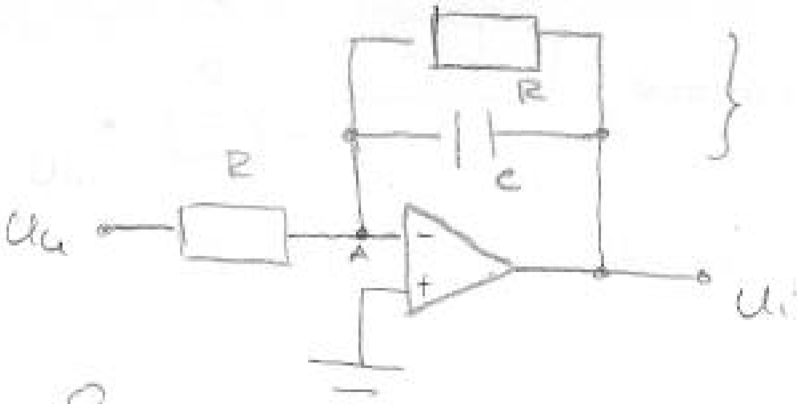
$y + 0.1794 = -40(x - \log 500)$ $\quad y = -40 \log \frac{5000}{500} - 0.1794$

$y = -40.1794$ $\quad A(\omega_\pi) = 40.1794\,dB$

$20 \log K_R = 40.1794$ $\quad K_R = 10^{40.1794/20} = 102.0868$

$K_{Rkrit} = 2501.1289$

9. $R = 100\,k\Omega$
$C = 1\,\mu F$



$$\frac{R\cdot\frac{1}{sC}}{R+\frac{1}{sC}} = \frac{\frac{R}{sC}}{\frac{RsC+1}{sC}} = \frac{R}{RsC+1} = Z$$

a)

$$U_A\left(\frac{1}{R} + \cancel{\frac{RsC+1}{R}}^{\,0}\right) = U_u\frac{1}{R} + U_i\frac{sRC+1}{R} \qquad -U_u\frac{1}{\cancel{R}} = U_i\frac{sRC+1}{\cancel{R}}$$

$$\frac{U_i}{U_u} = \frac{-1}{sRC+1} = \frac{-1}{0.1s+1}$$

b) BODE

c) $u_u(t) = 2\sin(100t)$

$\omega = 100$

$y - 0 = -20(x - \log 10) \qquad y = -20\log 10 = -20$

$20\log A = -20 \qquad A = 10^{-1} = 0.1$ na $\omega = 100$

$U_i = 0.1\cdot 2 = 0.2\,V$

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10. $G_0(s) = \frac{K_0}{s(1+10s)(1+2s)} \qquad G_0(j\omega) = \frac{K_0}{(j\omega)(1+10j\omega)(1+2j\omega)}$

$\varphi_0(\omega_c) = \frac{Im(\omega_c)}{Re(\omega_c)}$

a) BODE

$\gamma = 50^\circ \qquad \varphi_0(\omega_c) = -130^\circ$

$$G_0(j\omega) = \frac{-K_0 j}{\omega(1+10j\omega)(1+2j\omega)}\cdot\frac{(1-10j\omega)}{(1-10j\omega)}\cdot\frac{(1-2j\omega)}{(1-2j\omega)} = \frac{-K_0 j(1-10j\omega)(1-2j\omega)}{\omega(1+100\omega^2)(1+4\omega^2)}$$

$$= \frac{-K_0 j(1-2j\omega-10j\omega+20j^2\omega^2)}{A} = \frac{-K_0 j(1-20\omega^2-12j\omega)}{A}$$

$$= \frac{K_0(-j+20j\omega^2+12j^2\omega)}{A} = \frac{K_0(-12\omega-j+20j\omega^2)}{A}$$

$$tg(-130) = \frac{-1+20\omega_c^2}{-12\omega_c} \qquad 14.30\,\omega_c = 20\omega_c^2 - 1 \qquad 20\omega_c^2 + 14{,}30\,\omega_c - 1 = 0$$

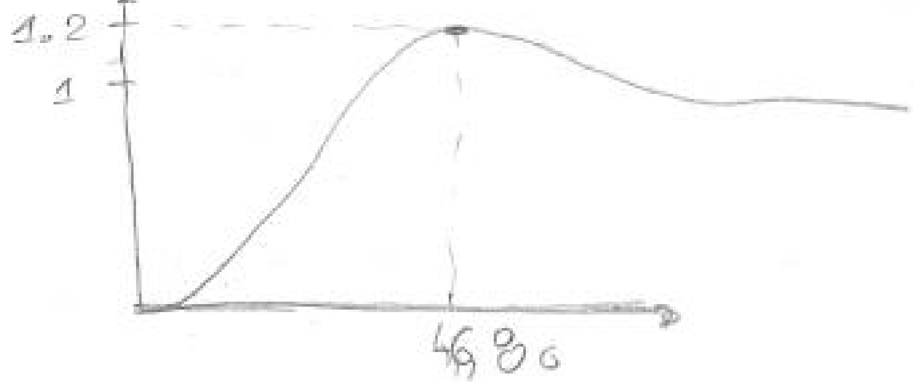
$\omega_c = 0.0641$ rad/s

$$1 = \frac{K_0}{0.0641\sqrt{1+100\omega_c^2}\sqrt{1+4\omega_c^2}} \qquad K_0 = 0.07676$$

Razlika jer su iščitavali iz Bode-a.

b) $\gamma \approx 70 - \sigma_m \qquad \sigma_m = 20\%$

$t_m \approx \dfrac{3}{\omega_c} = 46{,}80\,s$



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11. $G_o(s) = \dfrac{1}{sT_1(s+1)(s+5)}$

a) BODE UZ $T_1 = 0.5$

$$G_o(s) = \frac{1}{s\,0.5(s+1)\cdot 5(\frac{1}{5}s+1)} = \frac{0.4}{s(s+1)(\frac{1}{5}s+1)}$$

b) IZ BODEA ω_c i γ

$|G_o(j\omega_c)| = 1$

za $\omega = 1 \quad A = -7.9588$

$(\log 1, -7.9588)$, $k = -20$

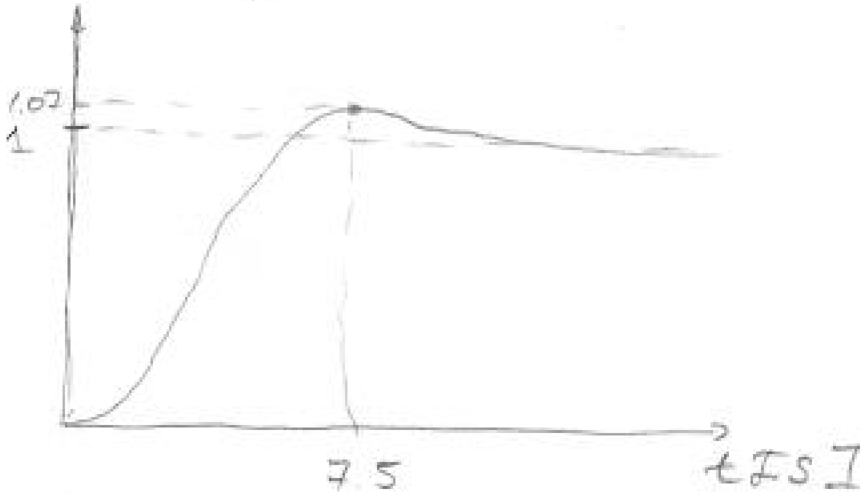
$y + 7.95 = +20(x - \log 1) \qquad x = -0.3975 \qquad \omega = 10^x = 0.400\,rad/s$

$(\log 0{,}1, -90)$

$y + 90 = -45(x - \log 0.1)$, za $x = 0.4 \qquad y = -117.1125 = \varphi_o(\omega_c)$

$\gamma = 62.8875 \qquad \sigma_m = 70 - \gamma = 7.11\%$

$t_m = \dfrac{3}{\omega_c} = 7.5\,s$



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12. $G_p(s) = \dfrac{k_P}{(1+T_1 s)(1+T_2 s)(1+T_3 s)}$, $T_1 = 2s$, $T_2 = 3s$, $T_3 = 4s$, $k_P = 1$

PID regulator: $G_R(s) = K_R\left(1 + \dfrac{1}{T_i s} + T_D s\right) = \left(\dfrac{T_i s + 1 + T_D T_i s^2}{T_i s}\right) \cdot k_R$

$$= \frac{K_R}{T_i}\left(\frac{T_i T_D s^2 + T_i s + 1}{s}\right)$$

$$G_0 = G_P \cdot G_R = \frac{1}{(1+2s)(1+3s)(1+4s)} \cdot 1.02 \cdot \frac{6.31 s^2 + 5.13025 s + 1}{s}$$

$$\frac{1.02}{(1+2s)(1+3s)(1+4s)} \cdot \frac{(s+0.324)(s+0.487)}{s}$$

$$= \frac{1.02}{s(1+\cancel{2s})(1+\cancel{3s})(1+4s)} \quad 0.324\,(\overbrace{\cancel{\tfrac{1}{0.324}s+1}}^{3s+1})\,(\overbrace{\cancel{\tfrac{1}{0.487}s+1}}^{2s+1}) \cdot 0.487$$

$$G_0 \approx \frac{0.1609}{s(1+4s)}$$

13. $G_o(s) = K_R\left(1 + \frac{1}{T_i s}\right) \cdot \frac{1-s}{(s+1)(s+10)}$

PI regulator

a) PI regulator $= \frac{T_i s + 1}{T_i s}$ ⟶ mora se pokratiti sa $(s+1)$

$T_i = 1s$

b) $K_R = 1$, $T_i = 1s$

$$G_o(s) = 1 \cdot \left(\frac{s+1}{s}\right) \cdot \frac{1-s}{\cancel{(s+1)}(s+10)} = \frac{1-s}{s(s+10)} = 0.1 \frac{1-s}{s(\frac{1}{10}s+1)}$$

BODE

c) $A(\omega_\pi) = 20dB$, sustav treba podići za $10 dB$

$20 \log K_R = 10$ $\qquad K_R = 3.16$ $\quad t_1 \quad G_o = 0.316 \frac{1-s}{s(\frac{1}{10}s+1)}$

d) $(\log 1, -10)$

$y + 10 = -20(x - \overset{0}{\widehat{\log 1}})$ $\qquad 0 + 10 = -20x \qquad x = -0.5 \qquad \omega = 10^x \quad \omega_c = 0.316$

$(\log 0.1, -90)$

$y + 90 = -45(x - \log 0.1)$, $x = -0.5 \implies y = -112.5 = \varphi_o(\omega_c)$

$\gamma = 180 + \varphi_o(\omega_c) = 67.5°$

14. $G_0 = k\frac{(s-1)}{(s+1)(s+10)}$

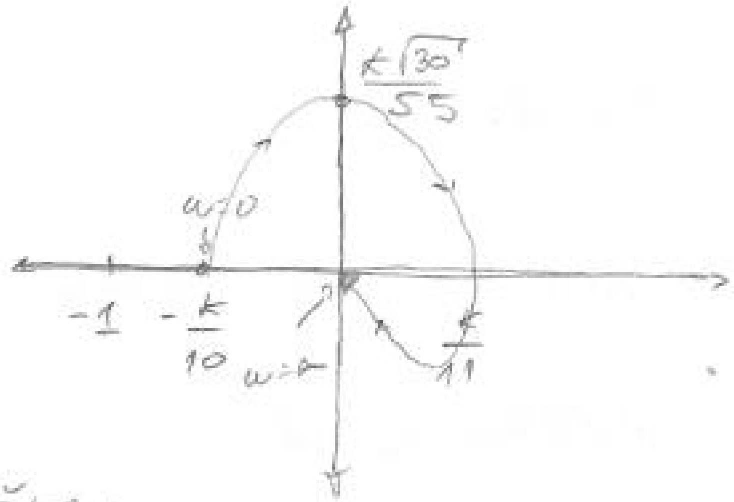
$$G_0(j\omega) = k\frac{(j\omega-1)}{(j\omega+1)(j\omega+10)} \cdot \frac{(1-j\omega)(10-j\omega)}{(1-j\omega)(10-j\omega)} = k\frac{(j\omega-1)(1-j\omega)(10-j\omega)}{(1+\omega^2)(\omega^2+100)}$$

$$= \frac{k}{A}\cdot(j\omega - j^2\omega^2 - 1 + j\omega)(10-j\omega) = \frac{k}{A}(\omega^2-1+2j\omega)(10-j\omega)$$

$$= \frac{k}{A}(10\omega^2 - j\omega^3 - 10 + j\omega + 20j\omega - 2j^2\omega^2) = \frac{k}{A}(12\omega^2-10+21j\omega - j\omega^3)$$

$A = (1+\omega^2)(\omega^2+100)$

$k>0$



$Re = \frac{k}{A}(12\omega^2-10)$ $\quad Im = \frac{k}{A}\omega(21-\omega^2)$

$\omega = 0 \quad Re = \frac{-k}{10} \quad Im = 0$

$\omega = \infty \quad Re = 0 \quad Im = 0$ (ulazi iz 4. kvadranta)

$Im = 0 \quad \omega = \sqrt{21} \quad Re = \frac{k}{11}$

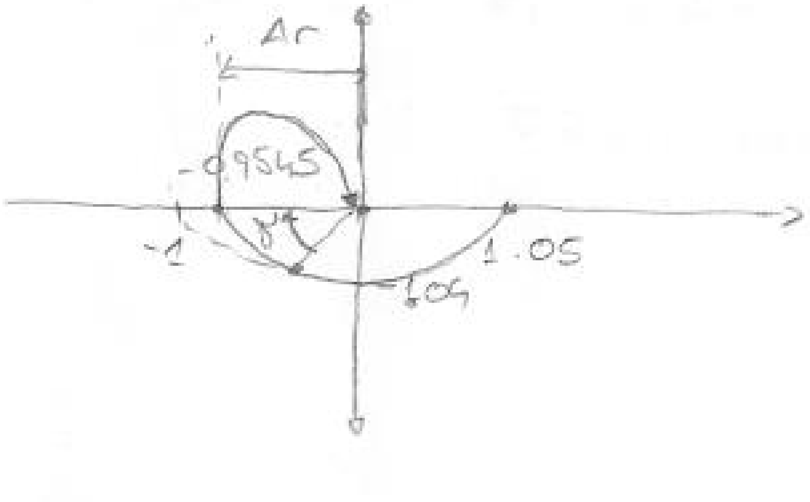
$Re = 0 \quad \omega = \sqrt{\frac{5}{6}} \quad Im = \frac{k\sqrt{30}}{55}$

KRITIČNA TOČKA: $(-1, 0j)$ $\quad -\frac{k}{10} > -1 \;/(\cdot 10) \quad k < 10$

za $K<0$ je isti Nyquist samo ga treba preslikati

$\frac{k}{11} > -1 \quad k > -11 \quad k \in \langle -11, 10 \rangle$

b) $k = -10.5$



$Re = 1.05 \; (\omega = 0), \; Im = 0$

$Im = 0 \quad Re = -0.9545$

$Re = 0 \quad Im = -1.04$

$A_r = \frac{1}{0.9545} = 1.0476$

$A(\omega_c) = 1 = -10.5\frac{\sqrt{\omega^2+1}}{\sqrt{\omega^2+1}\sqrt{\omega^2+100}}$ $\quad \omega^2 = 10.25 \quad \omega_c = 3.201$ rad

$\varphi = \mathrm{arctg}\frac{\omega(21-\omega^2)}{12\omega^2-10} = 16.93°$

15. $G_0(s) = k_0 \cdot \dfrac{1-0.1s}{(1+2s)(1+0.02s)}$, $G_0(j\omega) = k_0 \dfrac{(1-0.1j\omega)}{(1+2j\omega)(1+0.02j\omega)}$. Racionalizacija

$$G_0(j\omega) = k_0 \frac{(1-0.1j\omega)(1-2j\omega)(1-0.02j\omega)}{(1+4\omega^2)(1+0.0004\omega^2)} = \frac{K_0}{A}\left(1 \overbrace{-2j\omega - 0.1j\omega}^{-2.1j\omega} - 0.2\omega^2\right)(1-0.02j\omega)$$

$$= \frac{K_0}{A}\left(1 - 0.02j\omega - 2.1j\omega - 0.042\omega^2 - 0.2\omega^2 + 0.004j\omega^3\right)$$

$$= \frac{K_0}{A}\left(1 - 0.242\omega^2 + j(-2.12\omega + 0.004\omega^3)\right)$$

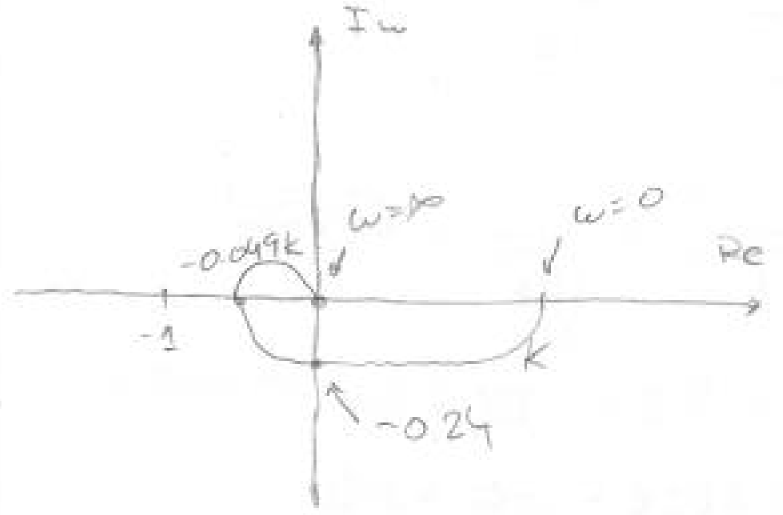
$Re = \dfrac{K_0}{A}(1-0.242\omega^2)$ $\qquad I\omega = \dfrac{0.004\omega^3 - 2.12\omega}{A} K$

$\omega = 0 \quad Re = K \quad I\omega = 0$

$\omega = \infty \quad Re = 0 \quad I\omega = 0$ (ulazi iz 2 kvadranta)

$Re = 0 \quad 1 - 0.242\omega^2 = 0 \quad \omega^2 = 4.13 \quad \omega = 2.03 \quad I\omega = -0.24353K$

$I\omega = 0 \quad \omega^2 = 530 \quad \omega = 23.02 \quad Re = -0.049505k$



$-0.049505k > -1$

$k < 20.19 \qquad k \in \langle 0, 20.19 \rangle$

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16. $G(s) = \dfrac{Y(s)}{U(s)}$

$\sigma_m[\%] = 100\, e^{-\pi\zeta/\sqrt{1-\zeta^2}} = 5\% \qquad \dfrac{-\pi\zeta}{\sqrt{1-\zeta^2}} = \ln\dfrac{5}{100} = -2.99 \,|^2$

$\pi^2\zeta^2 = 8.974(1-\zeta^2) \qquad \zeta^2 = \dfrac{8.974}{18.84} = 0.476 \qquad \zeta = 0.69$

$t_m - T_t = 0.5s = \dfrac{\pi}{\omega_n\sqrt{1-\zeta^2}}$, $T_t \to$ mrtvo vrijeme (kašnjenje)

$\omega_n = 8.68\,rad/s \qquad G(s) = \dfrac{K}{\frac{1}{75.34}s^2 + \frac{69}{434}s + 1} e^{-s} \qquad Y(s) = \lim_{s\to 0} s \cdot G(s) \cdot \frac{1}{s} = 1$

$k = 1$

$$G(s) = \frac{e^{-s}}{\frac{1}{75.34}s^2 + \frac{69}{434}s + 1}$$

a) nastavak

zbog kašnjenja od 1s u $t=0^+$ nema odziva, pa tako ni nagiba $\dot{y}=0$

b) $$G_0 = \frac{G}{1-G} = \frac{\frac{75.34}{s^2+11.98s+75.34}}{1-\frac{75.34}{s^2+11.98s+75.34}} = \frac{75.34}{s^2+11.98s} = \frac{75.34}{s(s+11.98)}$$

$$G_0(j\omega) = \frac{75.34}{j\omega(j\omega+11.98)}$$

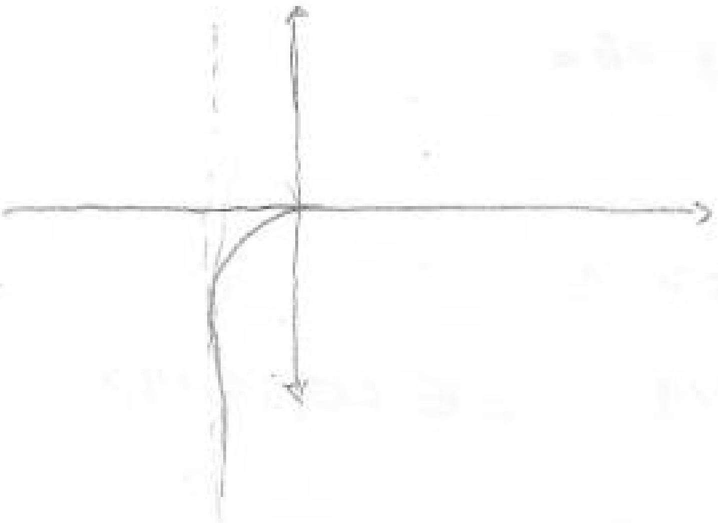
$$G_0(j\omega) = \frac{-75.34j}{(j\omega+11.98)}\,\frac{(11.98-j\omega)}{11.98-j\omega}\,\frac{1}{\omega}$$

$$G_0(j\omega) = \frac{-75.34j(11.98-j\omega)}{(11.98^2+\omega^2)\,\omega}$$

$$Re = \frac{-75.34\omega}{\omega(\omega^2+143.52)}, \quad I\omega = \frac{-902.5732}{\omega(\omega^2+143.52)}$$

$\omega = 0 \quad Re = -0.524 \quad I\omega = -\infty$ * sustav 2 reda (2 kvadranta)

$\omega = \infty \quad Re = 0 \quad I\omega = 0$



c) $G_0(s) = \frac{1}{s}$, $r(t) = 2\sin(3t + \frac{\pi}{6})$

$$G_2 = \frac{\frac{1}{s}}{1+\frac{1}{s}} = \frac{\frac{1}{s}}{\frac{s+1}{s}} = \frac{1}{s+1}$$

$$|Y(j\omega)| = 2\cdot\frac{1}{\sqrt{\omega^2+1}} = \frac{2}{\sqrt{10}} = 0.632$$

$\varphi = -1.249°$ $x(t) = 0.632\sin(3t - 0.725)$

$-\omega T_t = \varphi \quad -3\cdot 1 = \varphi \quad \varphi = -3\,\text{rad} \quad y(t) = 0.632(3t-3-0.725)$

6.17

$$G_0(s) = \frac{K}{a_3s^3+a_2s^2+a_1s+1}$$ ← nema konačnih nula; ← 3 red (3 kvadranta)

$$G_0(j\omega) = \frac{K}{-a_3j\omega^3 - a_2\omega^2 + a_1j\omega + 1}$$

$$G_0(j\omega) = \frac{K}{1-a_2\omega^2 + j(\omega a_1 - a_3\omega^3)}$$

$$G_0(j\cdot 1) = -j = \frac{K}{1-a_2+j(a_1-a_3)}$$

$1 - a_2 = 0 \quad a_2 = 1$

$$-j = \frac{K}{j(a_1-a_3)} \quad a_1 - a_3 = K$$

$G_0(j\cdot 3) = -0.25$

$$-0.25 = \frac{K}{1-9a_2+j(3a_1-27a_3)}$$

$$-0.25 = \frac{K}{1-9a_2} \quad -0.25(1-9) = K \Rightarrow K = 2$$

$3a_1 - 27a_3 = 0 \quad a_1 = 9a_3$

$8a_3 = 2 \quad a_3 = \frac{1}{4} \quad a_1 = \frac{9}{4}$

a) $G(s) = \dfrac{2}{0.25s^3 + s^2 + 2.25s + 1}$

b) $u(t) = 2 + 3\sin t - 4\cos 3t$

$\underbrace{2}_{\omega=0} \quad \underbrace{3\sin t}_{\omega=1} \quad \underbrace{4\cos 3t}_{\omega=3}$

$x = -j$

$\operatorname{arctg} = -\dfrac{\pi}{2}$

$|x| = 1$

$y(s) = 2 \cdot K + 3 \cdot |-j| \sin t - 4 \cdot |-0.25| \cos 3t$

$= 4 + 3\sin\left(t - \frac{\pi}{2}\right) - \cos 3t$

---

18. $K > 0, T_1 > 0, T_2 > 0$

$G_0 = \dfrac{K}{(1+sT_1)(1+sT_2)}$

$G_0(j\omega) = \dfrac{K}{(1+j\omega T_1)(1+j\omega T_2)} \cdot \dfrac{(1-j\omega T_1)(1-j\omega T_2)}{(\quad - \quad // \quad - \quad)}$

$G_0(j\omega) = \dfrac{K \cdot (1-j\omega T_1)(1-j\omega T_2)}{(1+(\omega T_1)^2)(1+(\omega T_2)^2)} = \dfrac{K}{A}(1 - j\omega T_2 - j\omega T_1 - \omega^2 T_1 T_2)$

$= \dfrac{K}{A}(1 - \omega^2 T_1 T_2 - j(\omega T_2 + \omega T_1))$

$Re = K \cdot \dfrac{1 - \omega^2 T_1 T_2}{(1+(\omega T_1)^2)(1+(\omega T_2)^2)}$

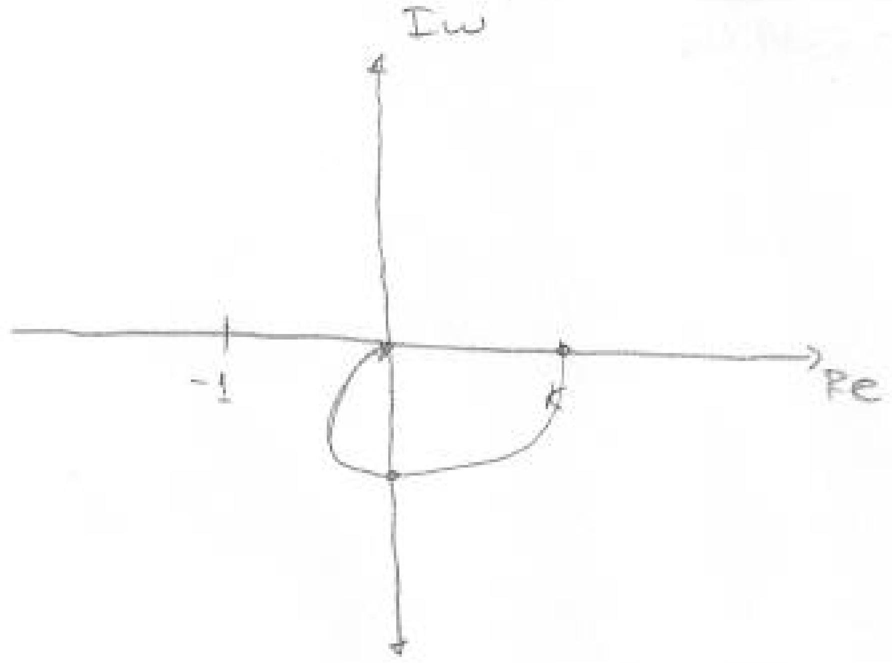
$I\omega = \dfrac{-\omega(T_1 + T_2)}{(1+(\omega T_1)^2)(1+(\omega T_2)^2)} \cdot K$

$\omega = 0$

$Re = K \qquad I\omega = 0$

$\omega = \infty \qquad Re = 0 \qquad I\omega = 0$

$Re = 0 \qquad \omega = \dfrac{1}{\sqrt{T_1 T_2}} \qquad I\omega = \dfrac{-K\sqrt{T_1 T_2}}{T_1 + T_2}$



Nikad ne ulazi u 2kvadrant pa je nemoguće da bude NESTABILAN.

19. $G_z(s) = \dfrac{1}{0.5539s^2+0.9326s+1}$

$$G_0 = \frac{G_z}{1-G_z} = \frac{1}{0.5539s^2+0.9326s} \qquad G_0(j\omega) = \frac{1}{0.9326j\omega - 0.5539\omega^2}$$

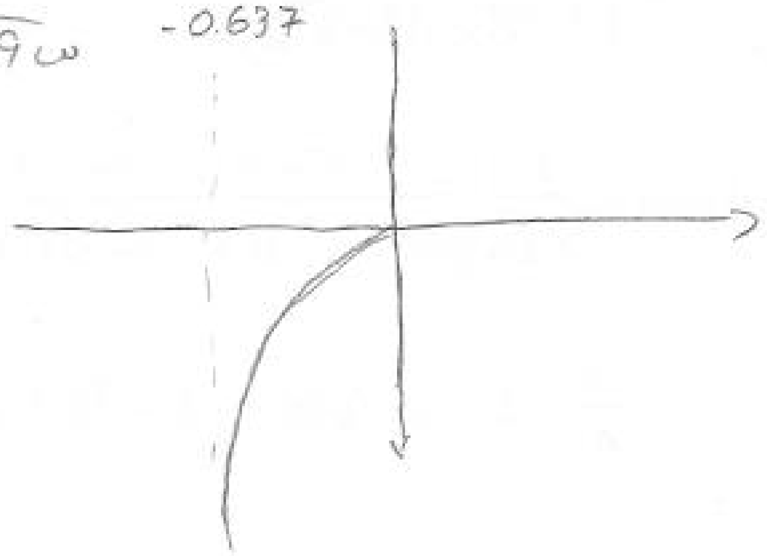
$$G_0(j\omega) = \frac{-1}{0.5539\omega^2 - 0.9326j\omega} \cdot \frac{0.5539\omega^2+0.9326j\omega}{-\,\text{''}\,-} = \frac{-(0.5539\omega^2+0.9326j\omega)}{0.306\omega^4+0.869\omega^2}$$

$$Re = \frac{-0.5539\omega^2}{0.306\omega^4+0.869\omega^2} \qquad I\omega = \frac{-0.9326\omega}{0.306\omega^4+0.869\omega^2}$$

$$= \frac{-0.5539}{0.306\omega^2+0.869} \qquad = \frac{-0.9326}{0.306\omega^3+0.869\omega}$$

$\omega = 0$ $\quad Re = -0.637$ $\quad I\omega = -\infty$

$\omega = \infty$ $\quad Re = 0$ $\quad I\omega = 0$



$$G_0'(s) = G_0 e^{-sT_t} = \frac{1}{0.5539s^2+0.9326s} e^{-sT_t}$$

$$|G_0(j\omega_c)| = 1 = \frac{1}{\sqrt{(0.5539\omega^2)^2+(0.9326\omega)^2}} \qquad 0.306\omega^4+0.869\omega^2-1=0$$

$$\omega_c = \sqrt{0.878\ rad/s} = 0.937\ rad/s$$

$$\varphi(\omega_c) = \operatorname{arctg} \frac{-0.9326\omega}{-0.5539\omega^2} = \operatorname{arctg} \frac{0.9326}{0.5539\cdot\omega_c} = -119.09^\circ$$

$$\gamma = 180 + \varphi_0(\omega_c) = 60.9035^\circ$$

$\omega_c \cdot T_t < \gamma \qquad T_t < 1.1345s$

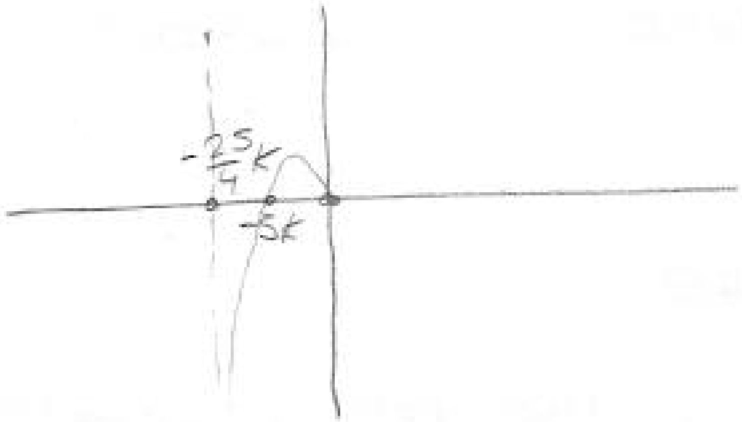
20. $G_0(s) = 20K\dfrac{(1-s)}{s(s+4)}$, $G_0(j\omega) = 20K\dfrac{(1-j\omega)}{j\omega(4+j\omega)} \cdot \dfrac{j(4-j\omega)}{j(4-j\omega)}$

$$G_0(j\omega) = \frac{20Kj(1-j\omega)(4-j\omega)}{j^2\omega(16+\omega^2)} = \frac{-20Kj(1-j\omega)(4-j\omega)}{\omega(16+\omega^2)} = \frac{-20Kj(4-j\omega-4j\omega-\omega^2)}{\omega(16+\omega^2)}$$

$$= \frac{-20Kj(4-5j\omega-\omega^2)}{\omega(16+\omega^2)} = \frac{20K(-4j-5\omega+j\omega^2)}{\omega(16+\omega^2)} = \frac{20K(-5\omega+j(\omega^2-4))}{\omega(16+\omega^2)}$$

$Re = \dfrac{-100K\omega}{\omega(16+\omega^2)}$ $\quad Im = \dfrac{\omega^2-4}{\omega(16+\omega^2)}$ $\quad Im = 0 \quad \omega = \pm 2$

$\omega = 0 \quad Re = -\dfrac{25}{4}K \quad Im = -\infty$



$\omega = \infty \quad Re = 0 \quad Im = 0$

$\omega = 2 \quad Re = -5K \quad Im = 0$

$-5K > -1 \; /\cdot(-5) \qquad K < 0.2 \qquad K \in \langle 0, 0.2 \rangle$

b) BODE za $K = 0.1$ i OČITATI A, γ

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21. $G_0 = \dfrac{K}{T_1 s(1+T_2 s)(1+T_3 s)}$, $K = 1$, $T_2 = 0.5s$, $T_3 = 0.1s$

$\gamma = 60° = 180 + \varphi_0(\omega_c) \Rightarrow \varphi_0(\omega_c) = -120°$

$G_0(j\omega) = \dfrac{K}{T_1(j\omega)(1+T_2 j\omega)(1+T_3 j\omega)}$ $\Rightarrow$ Racionalizacija ...

$$tg(-120) = \frac{\omega_c^2 T_2 T_3 + 1}{\omega_c(T_2+T_3)} = \frac{0.05\omega_c^2 + 1}{0.6\omega_c} \qquad 0.05\omega_c^2 - 0.6\sqrt{3}\,\omega_c + 1 = 0$$

$\omega_{c1} = 1.01157, \quad \omega_{c2} = 19.7712$

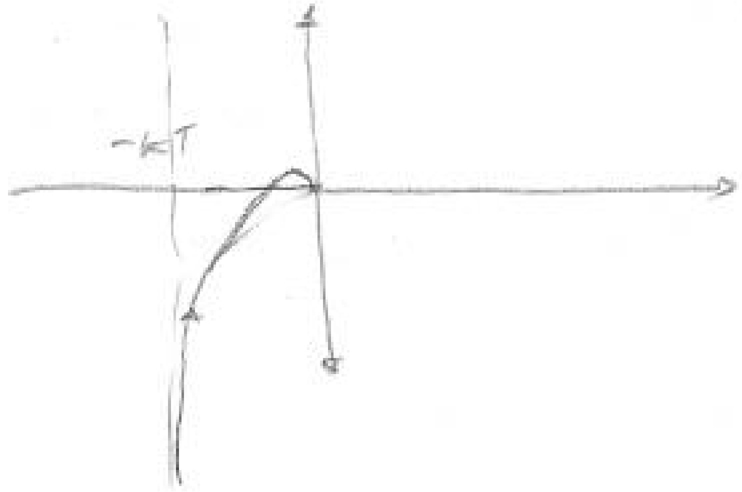
22. $G_0(s)=\dfrac{K}{s(1+Ts)}$, $G_0(j\omega)=\dfrac{K}{j\omega(1+jT\omega)}\;\dfrac{j}{j}$

$$= \frac{-Kj}{\omega}\cdot\frac{1-jT\omega}{1+T^2\omega^2}=\frac{K(-j-T\omega)}{\omega(1+T^2\omega^2)}$$

$$Re=\frac{-KT\omega}{\omega(1+T^2\omega^2)}=\frac{-KT}{(1+T^2\omega^2)}\quad,\quad Im=\frac{-K}{\omega(1+T^2\omega^2)}$$

$\omega=0 \quad Re=-KT \qquad Im=-\infty$

$\omega=\infty \quad Re=0 \qquad Im=0$



b) BODE

c) faza je 180 tek u $\omega=\infty=\omega_\pi \Rightarrow A(\omega_\pi)=\infty \;, K\to\infty$

POGLAVLJE 7

1. $G_o = \dfrac{K}{(s+a)(s+1)(s+2)}$, $a = -\dfrac{2}{3}$, $k = \dfrac{4}{3}$ $\quad G_o = \dfrac{\frac{4}{3}}{(s-\frac{2}{3})(s+1)(s+2)}$

$$G_z = \frac{G_o}{1+G_o} = \frac{\dfrac{\frac{4}{3}}{(s-\frac{2}{3})(s+1)(s+2)}}{1+\dfrac{\frac{4}{3}}{(s-\frac{2}{3})(s+1)(s+2)}} = \frac{\dfrac{\frac{4}{3}}{\cancel{A}}}{\dfrac{A+\frac{4}{3}}{\cancel{A}}} = \frac{\frac{4}{3}}{A+\frac{4}{3}}$$

$$= \frac{\frac{4}{3}}{(s-\frac{2}{3})(s^2+3s+2)+\frac{4}{3}} = \frac{\frac{4}{3}}{s^3+3s^2+\cancel{2s}-\frac{2}{3}s^2-\cancel{2s}-\cancel{\frac{4}{3}}+\cancel{\frac{4}{3}}} = \frac{\frac{4}{3}}{s^3+\frac{7}{3}s^2} = \frac{\frac{4}{3}}{s^2(s+\frac{7}{3})}$$

$s_{1,2} = 0 \qquad s_3 = -\dfrac{7}{3}$

2.

$G_p(s) = K\dfrac{s-1}{(s+15)(s+1)s}$ $\qquad \dfrac{1}{5} = \lim\limits_{s\to 0} s^{\cancel{2}} \cdot G_p \cdot \dfrac{1}{\cancel{s}}$

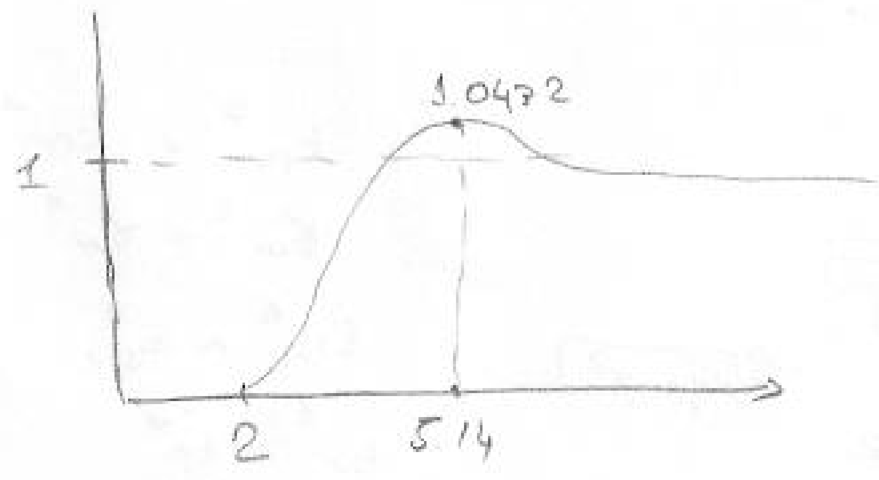
$\dfrac{1}{5} = k \cdot \dfrac{-1}{15}$ $\quad k = -\dfrac{15}{5} = -3$ $\qquad G_p(s) = -3 \cdot \dfrac{s-1}{s(s+15)(s+1)}$

3. $s_{p1,2} = -1 \pm j$ $\qquad p = (s-(1-j)(s-(-1+j)) = (s+1+j)(s+1-j)$

$T_t = 2 \quad k = 1$ $\qquad = (s+1)^2 + 1 = s^2+2s+1+1 = s^2+2s+2$

$\omega_n^2 = 2 \Rightarrow \omega_n = \sqrt{2}$ $\qquad 2\zeta\omega_n = 2 \qquad \zeta = \dfrac{1}{\omega_n} = \dfrac{1}{\sqrt{2}} = \dfrac{\sqrt{2}}{2}$

$t_m = \dfrac{\pi}{\omega_n\sqrt{1-\zeta^2}} + 2 = 5.14s$



$\sigma_m[\%] = 4.32$

$t_{1\%} = \dfrac{4.6}{3\omega_n} + 2 = 6.6s$

$t_r = 1.27s$ $\qquad G(s) = \dfrac{\sqrt{2}}{s^2+2s+2}$ $\qquad y(\infty) = 1$

4. $G_0 = \frac{2(1-s)}{s(s+4)}$ $\qquad G_2 = \frac{G_0}{1+G_0} = \frac{\frac{2(1-s)}{s(s+4)}}{1+\frac{2(1-s)}{s(s+4)}} = \frac{\frac{2(1-s)}{\cancel{s(s+4)}}}{\frac{s^2+4s+2-2s}{\cancel{s(s+4)}}} = \frac{2(1-s)}{s^2+2s+2}$

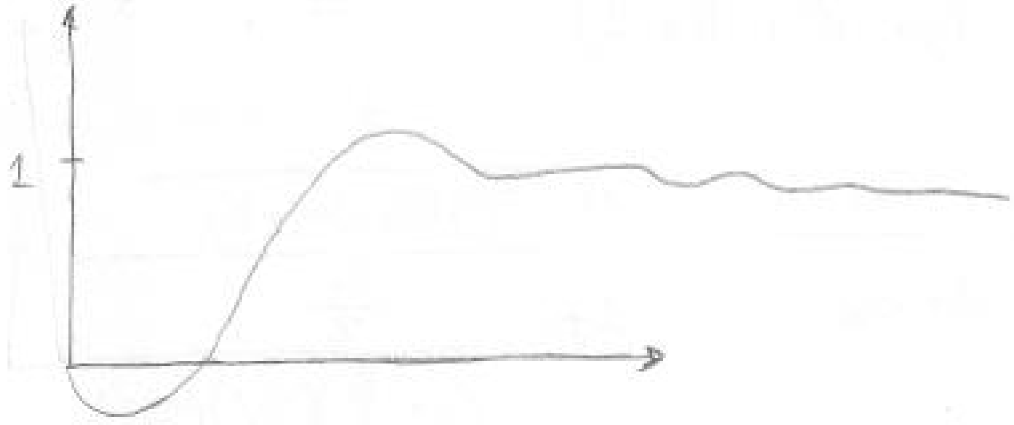
$s_{P1,2} = -1 \pm j \rightarrow$ OSCILACIJE

$s_N = 1 \rightarrow$ PODBAČAJ

$y(\infty) = \lim_{s\to 0} s \cdot G_2 \frac{1}{s} = 1$

$y(0) = \lim_{s\to\infty} s \cdot G_2 \frac{1}{s} = \emptyset$

$\dot{y}(0^+) = \lim_{s\to\infty} s^2 G_2 \frac{1}{s} = -2$



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5. $G_0(s) = k_0 \cdot \frac{1-0.1s}{(1+2s)(1+0.02s)}$ $\qquad G_2(s) = \frac{G_0}{1+G_0}$

$G_2(s) = \frac{k_0 \frac{1-0.1s}{(1+2s)(1+0.02s)}}{1+k_0\frac{1-0.1s}{(1+2s)(1+0.02s)}} = \frac{k_0 \frac{1-0.1s}{\cancel{A}}}{\frac{(1+2s)(1+0.02s)+k_0(1-0.1s)}{\cancel{A}}}$

$G_2(s) = k_0 \frac{1-0.1s}{(1+2s)(1+0.02s)+k_0(1-0.1s)}$

$k_0 = 5$

$G_{R1} = \frac{5}{6} \cdot \frac{1-0.1s}{(1+0.22351s)\cdot(1+0.0298s)}$

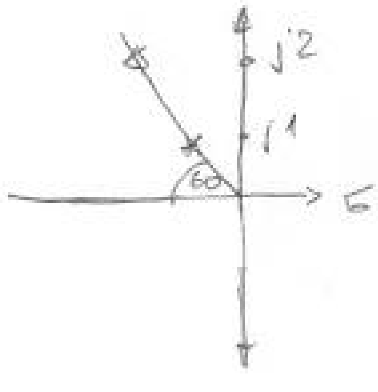
Podbačaj, nema oscilacija

$k_0 = 15$

$G_{R2} = \frac{15}{16} \frac{1-0.1s}{(1+(0.01+0.04j)\cdot s)(1+(0.01-0.04j)s)}$

Podbačaj i oscilacije

6.



$s_{P1,2} = -\zeta\omega_n + j\omega_n\sqrt{1-\zeta^2}$

$\cos 60 = \zeta \qquad \zeta = \frac{1}{2}$

$\mathrm{tg}60^\circ = \frac{1}{\zeta\omega_n} \qquad \zeta\omega_{n1} = \frac{\sqrt{3}}{3} \qquad \zeta\omega_{n2} = \frac{2\sqrt{3}}{3}$

$t_m = \frac{\pi}{\omega_n\sqrt{1-\zeta^2}}$

$\sigma_m[\%] = 100 e^{-\zeta\pi/\sqrt{1-\zeta^2}}$

$t_{1\%} = \frac{4.6}{\zeta\omega_n}$

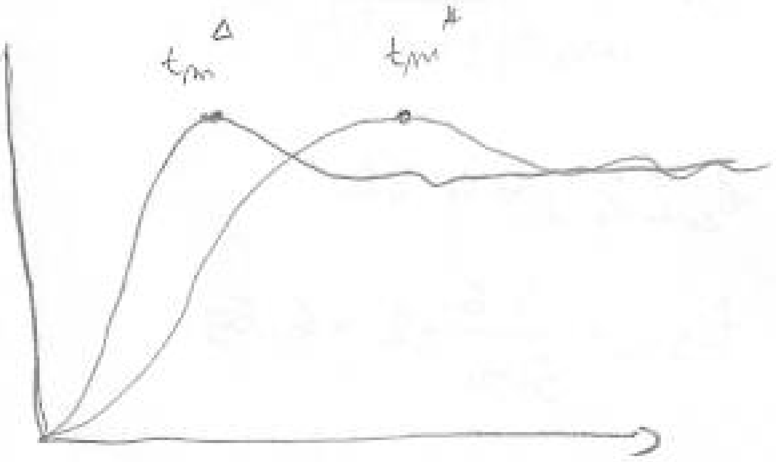
$t_r = 1.8/\omega_n$

$t_m^\Delta < t_m^*$

$\sigma_m^\Delta = \sigma_m^*$

$t_{1\%}^\Delta < t_{1\%}^*$

$t_r^\Delta < t_r^*$



Slika može biti i preciznija

7. $G(s)=\dfrac{32}{s^2+2s+16} = \dfrac{K\cdot \omega_n^2}{s^2+2\zeta\omega_n+\omega_n^2}$

$\omega_n=4 \qquad 2\zeta\omega_n=2 \qquad \zeta=\frac{1}{4}$

$t_m=\dfrac{\pi}{\omega_n\sqrt{1-\zeta^2}}=0.811\,s \qquad t_{1\%}=\dfrac{4.6}{\zeta\omega_n}=4.6\,s$

$\sigma_w=100\,e^{-\pi\zeta/\sqrt{1-\zeta^2}}=44.43\% \qquad t_r=\dfrac{1.8}{\omega_n}=0.45\,s$

8.
a) $G_0=K_R\dfrac{2}{(1+0.1s)(1+0.002s)} \qquad G_z=\dfrac{G_0}{1+G_0}$

$G_z=\dfrac{K_R\frac{2}{A}}{\frac{A+2K_R}{A}}=\dfrac{2K_R}{(1+0.1s)(1+0.002s)+2K_R}$

$0.98=\lim_{s\to 0} s\cdot G_z\cdot\frac{1}{s}=\dfrac{2K_R}{1+2K_R} \qquad 1+2K_R=2.04K_R \Rightarrow K_R=24.5$

b) $G_z=\dfrac{49}{1+0.002s+0.1s+2\cdot10^{-4}s^2+49}=\dfrac{49}{2\cdot10^{-4}s^2+0.102s+50}=\dfrac{\frac{49}{50}}{4\cdot10^{-6}s^2+2{,}04\cdot10^{-3}s+1}$

$\dfrac{1}{\omega_n^2}=\dfrac{4}{10^6} \qquad \omega_n=\sqrt{\dfrac{10^6}{4}}=500 \qquad 2{,}04\cdot10^{-3}=\dfrac{2\zeta}{\omega_n}\Rightarrow\zeta=0{,}51$

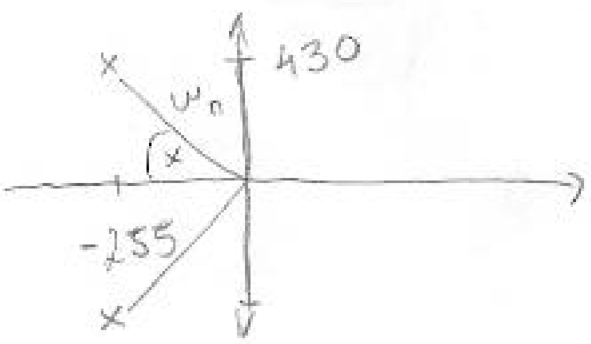
$t_w=\dfrac{\pi}{\omega_n\sqrt{1-\zeta^2}}=7{,}3\cdot10^{-3}\,s$

$\sigma_w[\%]=100\,e^{-\pi\zeta/\sqrt{1-\zeta^2}}=15.525$

$h(t_w)=0{,}98+0{,}98\cdot 0{,}155=1{,}1321$

c) $s_{p1,2}=-255\pm430.08j$

$\cos\alpha=\zeta$



9. $h_r(\infty) = 1$ $G_o = \dfrac{k \cdot \omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$

$\sigma_w[\%] = 8\%$

$t_w = 3s$

$\dfrac{8}{100} = e^{-\pi\zeta/\sqrt{1-\zeta^2}}$ / $\ln\dfrac{8}{100} \cdot \sqrt{1-\zeta^2} = -\pi\zeta$ $/^2$ $6.3793(1-\zeta^2) = \pi^2\zeta^2$

$\zeta^2 = 0.3925 \Rightarrow \zeta = 0.626$

$t_w = 3s = \dfrac{\pi}{\sqrt{1-\zeta^2} \cdot \omega_n} \Rightarrow \omega_n = \dfrac{\pi}{3\sqrt{1-0.3925}} = 1.3435$

$s_{p12} = -\zeta\omega_n \pm j\,\omega_n\sqrt{1-\zeta^2} = -0.841 \pm j\,1.04$

$G_o = \dfrac{1.804k}{s^2 + 1.68s + 1.804}$ $h = \lim\limits_{s\to 0} s \cdot G_o \cdot \dfrac{1}{s} = 1 \Rightarrow k = 1$

$G_o(s) = \dfrac{1.804}{s^2 + 1.68s + 1.804} = \dfrac{1}{0.554s^2 + 0.931s + 1}$

---

10. $G_o = \dfrac{k}{s(1+Ts)}$, $T = 10$ $G_o = \dfrac{k}{s(1+10s)}$ $\sigma_w = 10\%$

$G_z = \dfrac{G_o}{1+G_o} = \dfrac{\frac{k}{s(1+10s)}}{1 + \frac{k}{s(1+10s)}} = \dfrac{\frac{k}{\cancel{s(1+10s)}}}{\frac{10s^2+s+k}{\cancel{s(1+10s)}}} = \dfrac{k}{10s^2+s+k} = \dfrac{1}{\frac{10}{k}s^2 + \frac{1}{k}s + 1}$

$10\% = 100e^{\frac{-\pi\zeta}{\sqrt{1-\zeta^2}}}$ $\ln 0.1\,\sqrt{1-\zeta^2} = -\pi\zeta$ $5.30(1-\zeta^2) = \pi^2\zeta^2$

$\zeta = 0.591$

$\dfrac{10}{k} = \dfrac{1}{\omega_n^2}$ $\omega_n^2 = \dfrac{k}{10} \Rightarrow \omega_n = \sqrt{\dfrac{k}{10}}$

$\dfrac{1}{k} = \dfrac{2\zeta}{\omega_n}$ $\dfrac{\omega_n}{k} = 2\zeta = 1.18$ $\dfrac{\sqrt{\frac{k}{10}}}{k} = 1.18$, $\sqrt{\dfrac{\frac{k}{10}}{\frac{k^2}{1}}}$

$\sqrt{\dfrac{1}{10k}} = 1.18$ $/^2$ $\dfrac{1}{10k} = 1.39 \Rightarrow \underline{\underline{k = 0.0715}}$

11. $G_0 = \frac{10k_R}{(s+2)(s+5)}$

$$G_z = \frac{G_0}{1+G_0} = \frac{\frac{10k_R}{(s+2)\cancel{(s+5)}}}{\frac{(s+2)(s+5)+10k_R}{(s+2)\cancel{(s+5)}}} = \frac{10k_R}{(s+2)(s+5)+10k_R}$$

$t_m = 0,7s = \frac{\pi}{\omega_n\sqrt{1-\zeta^2}}$

$\omega_n\sqrt{1-\zeta^2} = \frac{\pi}{0,7} = 4,48$

$s_{1,2} = \frac{-7 \pm \sqrt{49 - 4\cdot 1(10+10k_R)}}{2}$

$G_z = \frac{10k_R}{s^2+7s+10+10k_R}$

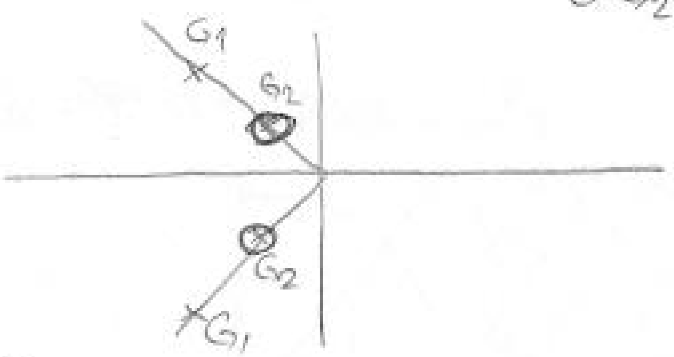
$j\frac{\sqrt{49-40(1+k_R)}}{2} = 4,48$

$\frac{\sqrt{40(1+k_R)-49}}{2} = 4,48$ $\quad 40(1+k_R)-49 = 80,2816$ $\quad \underline{k_R = 2,23}$ ✓

$G_z = \frac{22,3k_R}{s^2+7s+32,3204} \rightarrow s_{p1,2} = -\frac{7}{2} \pm \frac{112}{25}j$ $\quad \omega_n\sqrt{1-\zeta^2}$ $\quad t_w = \frac{\pi}{\omega_n\sqrt{1-\zeta^2}} = 0,7$

---

12. $s_{1,2} = -\sigma_1 \pm j\omega_1, \; \sigma_1 > 0$ $\quad$ o G_2 polovi



$\sigma_w[G_1] = \sigma_w[G_2]$ $\quad \zeta(G_1) = \zeta(G_2)$

$2t_r(G_1) = t_r(G_2)$ $\quad t_r = \frac{1.8}{\omega_n}$

$2\cdot\frac{\cancel{1,8}}{\omega_{n1}} = \frac{\cancel{1,8}}{\omega_{n2}}$ $\quad \omega_{n1} = 2\omega_{n2} \Rightarrow \omega_{n2} = \frac{\omega_{n1}}{2}$

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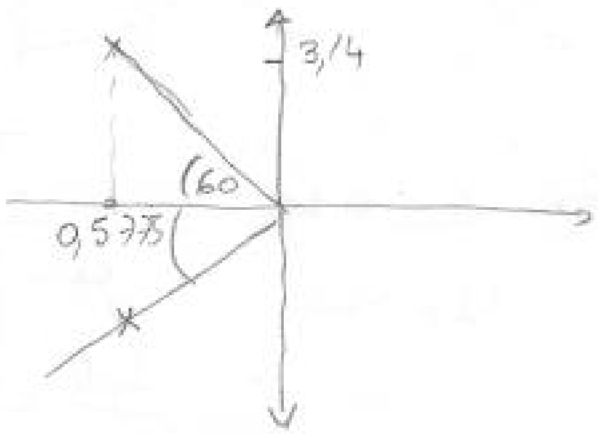
13. $t_m = 3,14s$

$\sigma_w = 16,3$



a)

$t_w = 3,14 = \frac{\pi}{\omega_n\sqrt{1-\zeta^2}}$ $\quad \omega_n = \frac{\pi}{3,14\sqrt{1-\zeta^2}} \Rightarrow \omega_n = 1.155$

$16,3 = 100\, e^{-\pi\zeta/\sqrt{1-\zeta^2}} \Rightarrow \zeta = 0,5$

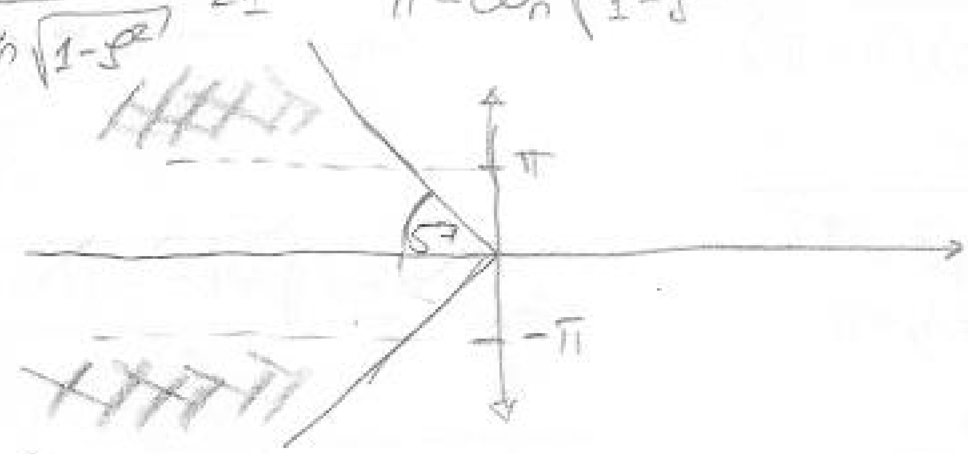
$G_z = \frac{1}{0,7496s^2+0,865s+1}$

b) $G_0 = \frac{G_z}{1-G_z} = \frac{\frac{1}{A}}{\frac{0,7496s^2+0,865s+1-1}{A}} = \frac{1}{0,7496s^2+0,865s} = \frac{1}{s(0,865+0,7496s)}$

Sustav je na rubu stabilnosti, jer imamo pol $s=0$

14. $t_w < 1s \longrightarrow \frac{\pi}{\omega_n\sqrt{1-\zeta^2}} < 1 \quad \pi < \omega_n\sqrt{1-\zeta^2}$

$\sigma_w < 10\%$



$100 e^{-\pi\zeta/\sqrt{1-\zeta^2}} < 10$

$-\pi\zeta < \sqrt{1-\zeta^2}\ln 0.1$

$\pi^2\zeta^2 < (1-\zeta^2)\,5.30 \qquad \zeta^2 < 0.349 \qquad -0.591 < \zeta < 0.591$

$\alpha = a\cos\zeta = 53.77$

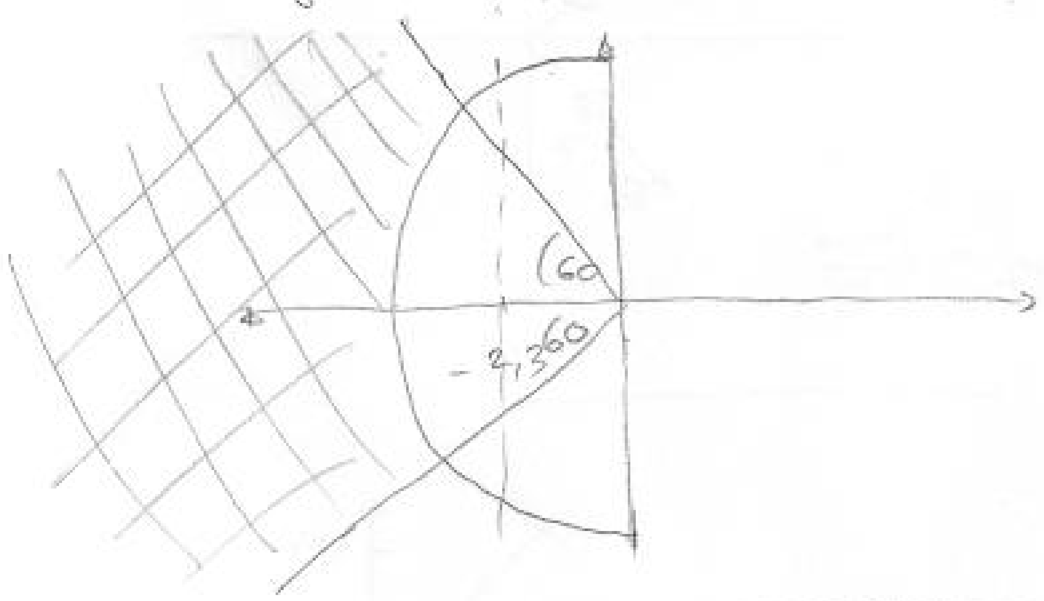
15. $t_{1\%} < 2s \qquad \frac{4.6}{\zeta\omega_n} < 2 \qquad 2.3 < \zeta\omega_n$

$t_r < 0.5s \qquad \frac{1{,}8}{\omega_n} < 0{,}5 \qquad 3{,}6 < \omega_n$

$\sigma_w < 16.3\%$

$100 e^{-\pi\zeta/\sqrt{1-\zeta^2}} < 16.3 \qquad -\pi\zeta < \sqrt{1-\zeta^2}\ln\frac{16.3}{100} \qquad \pi^2\zeta^2 < (1-\zeta^2)\,3.290$

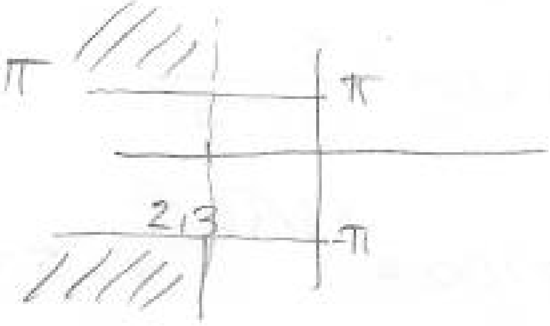
$\zeta^2 < 0.2499 \quad -0.499 < \zeta < 0.499 \qquad \alpha = a\cos(0{,}499) \approx 60°$



16. $t_w < 1s \qquad \frac{\pi}{\omega_n\sqrt{1-\zeta^2}} < 1 \qquad \omega_n\sqrt{1-\zeta^2} > \pi$

$t_{1\%} < 2s$

$\frac{4{,}6}{\zeta\omega_n} < 2 \qquad \zeta\omega_n > 2{,}3$



17. $\omega_n > 4$

$t_r = \frac{1.8}{\omega_n} \qquad \omega_n > \frac{1{,}8}{t_r} = 4 \quad t_r = 0.45s \Rightarrow t_r > 0.45s$

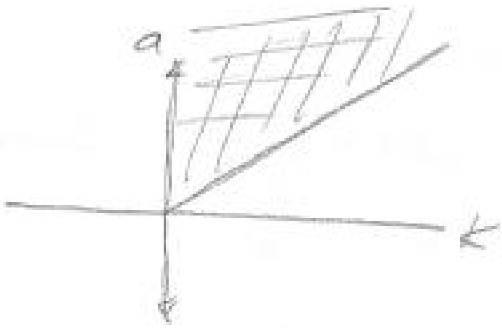
$\alpha = \frac{3\pi}{4} = 45° \Rightarrow \zeta = \cos\alpha = \frac{\sqrt{2}}{2} \qquad \frac{\sqrt{2}}{2} < \zeta < \frac{\sqrt{2}}{2}$ (σ_w ?!)

$\sigma_w = 100 e^{-\pi\zeta/\sqrt{1-\zeta^2}} = 4{,}32 \Rightarrow \sigma_w < 4{,}32\%$

POGLAVLJE 8

1. $G_0=\frac{K}{s(s^2+s+a)}$ $\quad G_z=\frac{G_0}{1+G_0}=\frac{\frac{K}{s(s^2+s+a)}}{1+\frac{K}{s(s^2+s+a)}}=\frac{\frac{K}{\cancel{A}}}{\frac{s(s^2+s+a)+K}{\cancel{A}}}$

$G_z=\frac{K}{s^3+s^2+as+K}$ $\quad P=s^3+s^2+as+K$



I $a_3,a_2,a_1,a_0>0$ $\quad a>0 \quad K>0$

II $a_1>0 \quad a>0$

$a_1a_2-a_0a_3>0 \quad a-K>0 \quad a>K$

b) $a=K$

$P=s^3+s^2+as+a=s^2(s+1)+a(s+1)=(s+1)(s^2+a)$

$s^2=-a \quad s=j\sqrt{a} \quad j\omega=j\sqrt{a} \Rightarrow \omega=\sqrt{a}$

2. $P=1+G_0=1+K_R\frac{s-1}{(s+2)(s+3)(s+4)}=\frac{(s+2)(s+3)(s+4)+K_R(s-1)}{(s+2)(s+3)(s+4)}=0$

$(s+2)(s+3)(s+4)+K_R(s-1)=0 \quad (s^2+5s+6)(s+4)+K_R(s-1)=0$

$s^3+4s^2+5s^2+20s+6s+24+K_Rs-K_R=0 \quad s^3+9s^2+26s+K_Rs+24-K_R=0$

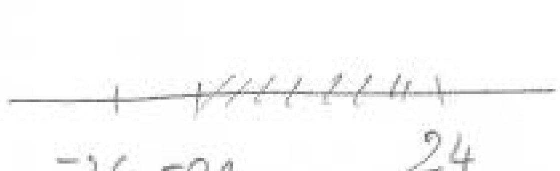
$s^3+9s^2+s(26+K_R)+(24-K_R)=0$

I $a_3,a_2,a_1,a_0>0 \quad 26+K_R>0 \quad K_R>-26\ , \quad 24-K_R>0 \quad K_R<24$

II $a_1>0 \quad 26+K_R>0 \quad K_R>-24$

$a_1a_2-a_0a_3>0 \quad 9(26+K_R)-24+K_R>0 \quad 234+9K_R-24+K_R>0$

$K_R>-21$



$K_R\in\langle-21,24\rangle$

3.

$$P = 1 + G_0 = 1 + \frac{k}{(s+a)(s+1)(s+2)} = \frac{(s+a)(s+1)(s+2)+k}{(s+a)(s+1)(s+2)} = \emptyset$$

$$(s+a)(s^2+3s+2) + k = 0 \qquad s^3 + 3s^2 + 2s + as^2 + 3as + 2a + k = 0$$

$$s^3 + s^2(3+a) + s(2+3a) + 2a + k = \emptyset$$

I $a_3, a_2, a_1, a_0 > 0$

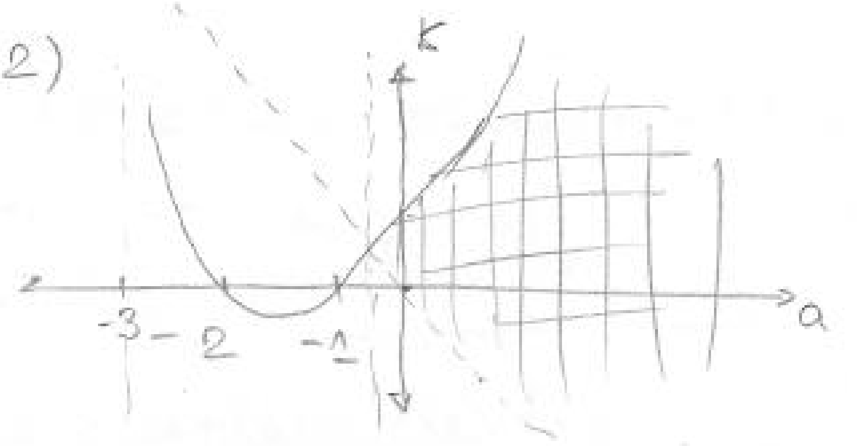
$3+a>0 \quad a>-3 \qquad 2a+k>0 \quad k>-2a$

$2+3a>0 \quad a>-\frac{2}{3}$

II $2+3a>0 \quad a>-\frac{2}{3}$

$(2+3a)(3+a) - 2a - k > 0 \qquad 6 + 2a + 9a + 3a^2 - 2a - k > 0 \qquad 6 + 9a + 3a^2 > k$

$k < 3(a+1)(a+2)$



4.

$$P = 1 + G_0(s) = \frac{s(1+10s)(1+2s)+k}{s(1+10s)(1+2s)} = \frac{s(1+2s+10s+20s^2)+k}{s(1+10s)(1+2s)} = \frac{20s^3+12s^2+s+k}{s(1+10s)(1+2s)} = 0$$

$$20s^3 + 12s^2 + s + k = 0$$

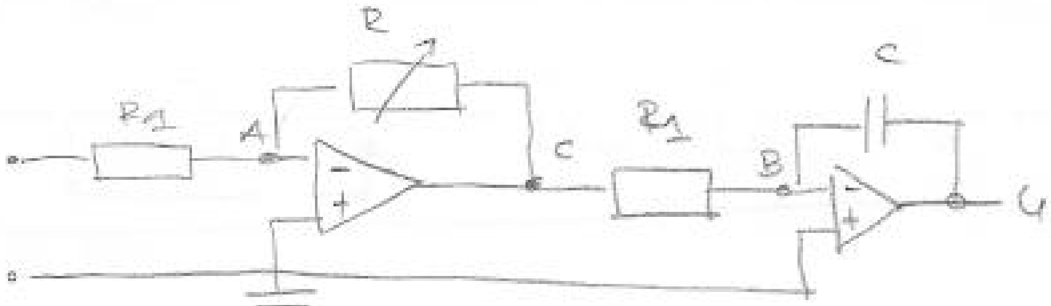
I $a_3, a_2, a_1, a_0 \qquad k>0$

$k \in \langle 0, 0.6 \rangle$

II $a_1 > 0$

$a_1 a_2 - a_0 a_3 > 0 \qquad 12 - 20k > 0 \qquad k < \frac{12}{20} \qquad k < 0{,}6$

5. $G_0 = G_R \cdot \frac{10}{(s+2)(s+5)}$



$R_1 = 100\,k\Omega$

$U_A\left(\cancel{\frac{1}{R_1}+\frac{1}{R}}\right)^0 = e\frac{1}{R_1} + U_C\frac{1}{R}$ $U_C\frac{1}{R} = -e\frac{1}{R_1}$ $U_C = -e\frac{R}{R_1}$

$U_B\left(\cancel{\frac{1}{R_1}+sC}\right)^0 = U sC + U_C\frac{1}{R_1}$ $-UsC = U_C\frac{1}{R_1}$ $-UsC = -e\frac{R}{R_1}\cdot\frac{1}{R_1}$

$UsC = +e\frac{R}{R_1^2}$ $\frac{U}{e} = \frac{R}{R_1^2 sC}$

$G_0 = \frac{R}{sCR_1^2}\cdot\frac{10}{(s+2)(s+5)}$ $P = 1+G_0 = 1+\frac{10R}{sCR_1^2(s+2)(s+5)}$

$P = \frac{sCR_1^2(s+2)(s+5)+10R}{sCR_1^2(s+2)(s+5)} = \emptyset$ $sCR_1^2(s+2)(s+5)+10R=\emptyset$

$sCR_1^2(s^2+7s+10)+10R=0$ $s^3CR_1^2+7s^2CR_1^2+10sCR_1^2+10R=0$

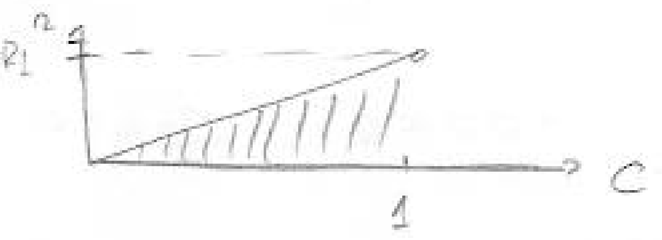
I $a_3, a_2, a_1, a_0 > 0$ $R_1^2C>0$ $C>0$, $R>0$

II $a_1>0$ $C>0$

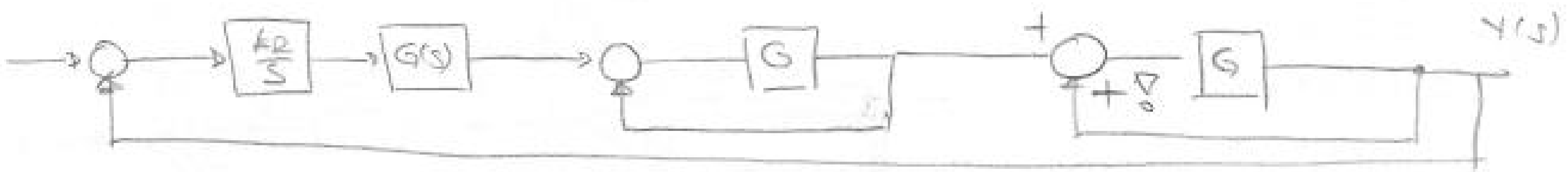
$a_1a_2-a_0a_3>0$ $10CR_1^2\cdot 7CR_1^2 - 10R\,CR_1^2>0$ $70C^2R_1^4-10RR_1^2C>0 \,/:R_1^2$

$70C^2R_1^2-10RC>0 \,/:10$ $7C^2R_1^2-RC>0 \,/:C$ $7CR_1^2-R>0$

$R<7CR_1^2$



6.



$$G_0 = \frac{k_R}{s} \cdot G \cdot \frac{G}{1+G} \; \frac{G}{1-G} = \frac{k_R}{S} \cdot \frac{1}{s+2} \cdot \frac{\frac{1}{s+2}}{1+\frac{1}{s+2}} \cdot \frac{\frac{1}{s+2}}{1-\frac{1}{s+2}}$$

$$= \frac{k_R}{s} \cdot \frac{1}{s+2} \cdot \frac{\frac{1}{\cancel{s+2}}}{\frac{s+3}{\cancel{s+2}}} \cdot \frac{\frac{1}{\cancel{s+2}}}{\frac{s+1}{\cancel{s+2}}} = \frac{k_R}{s} \cdot \frac{1}{s+2} \cdot \frac{1}{s+3} \cdot \frac{1}{s+1}$$

$$P = 1 + G_0 = 1 + \frac{k_R}{s(s+1)(s+2)(s+3)} = \frac{s(s+1)(s+2)(s+3) + k_R}{s(s+1)(s+2)(s+3)} = \emptyset$$

$$(s^2+s)(s^2+5s+6) + k_R = 0 \qquad s^4 + 5s^3 + 6s^2 + s^3 + 5s^2 + 6s + k_R = 0$$

$$\underset{a_4}{s^4} + \underset{a_3}{6s^3} + \underset{a_2}{11s^2} + \underset{a_1}{6s} + \underset{a_0}{k_R} = \emptyset$$

I $\quad a_4, a_3, a_2, a_1, a_0 \qquad k_R > 0$

II $\quad a_1 > 0 \qquad 6 > 0$ ✓

$a_1 a_2 - a_0 a_3 > 0 \qquad 66 - 6k_R > 0 \qquad 66 > 6k_R \qquad k_R < 11$

$$D_3 = \begin{vmatrix} a_1 & a_0 & 0 \\ a_3 & a_2 & a_1 \\ a_5 & a_4 & a_3 \end{vmatrix} > 0 = \begin{vmatrix} 6 & k_R & 0 \\ 6 & 11 & 6 \\ 0 & 1 & 6 \end{vmatrix} \begin{matrix} 6 & k_R \\ 6 & 11 \\ 0 & 1 \end{matrix} = 6^2 \cdot 11 - 6^2 - 6^2 k_R > 0 \; / : 6^2$$

$11 - 1 - k_R > 0 \qquad 10 > k_R \qquad k_R < 10$

$k_R \; \langle 0, 10 \rangle$

POGLAVLJE 9

1. $G_0(s)=\dfrac{1}{sT(s+1)(s+5)}$, $r(t)=2t\,S(t)$, $e_\infty=5$

$r(s)=\dfrac{2}{s^2}$

$e_\infty=5=\lim\limits_{s\to 0} s\cdot E(s)$

$E(s)=\dfrac{1}{1+G_0}\,(R(s)-\overset{0}{Z(s)})=\dfrac{1}{1+G_0}\,R(s)$

$E(s)=\dfrac{1}{1+\dfrac{1}{sT(s+1)(s+5)}}\cdot\dfrac{2}{s^2}=\dfrac{sT(s+1)(s+5)}{sT(s+1)(s+5)+1}\cdot\dfrac{2}{s^2}$

$E(s)=\dfrac{2T(s+1)(s+5)}{s^2T(s+1)(s+5)+1}$

$e_\infty=5=\lim\limits_{s\to 0} s\cdot E(s)=10T=5 \quad T=0.5s$

2. $e_\infty=?$ $\quad z(t)=0.5t\,S(t)$ $\quad z(s)=\dfrac{1}{2}\cdot\dfrac{1}{s^2}$

$E(s)=R(s)-Y(s)$

$Y(s)=\left(E(s)\cdot\dfrac{K}{s}+Z(s)\right)\dfrac{1}{s^2+s+a}$

$E(s)=0-\left(E(s)\dfrac{K}{s}+Z(s)\right)\dfrac{1}{s^2+s+a}$

$E(s)+E(s)\cdot\dfrac{K}{s}\cdot\dfrac{1}{s^2+s+a}=-Z(s)\dfrac{1}{s^2+s+a}$

$E(s)\left[1+\dfrac{K}{s(s^2+s+a)}\right]=-Z\,\dfrac{1}{s^2+s+a}$

$E(s)\left[\dfrac{s^3+s^2+as+K}{s(\cancel{s^2+s+a})}\right]=-Z\,\dfrac{1}{\cancel{s^2+s+a}}$ $\qquad E(s)=\dfrac{-s}{s^3+s^2+as+K}\,Z$

$e_\infty=\lim\limits_{s\to 0} s\cdot E(s)=\lim\limits_{s\to 0}\cancel{s}\cdot\dfrac{-\cancel{s}}{s^3+s^2+as+K}\cdot\dfrac{1}{2}\cdot\dfrac{1}{\cancel{s^2}}=\dfrac{-1}{2K}$

3. $G_o = \dfrac{K}{(1+sT_1)(1+sT_2)}$

$r(t) = (1-t)S(t-1) = -(t-1)S(t-1) = -\dfrac{1}{s^2} e^{-s}$

$E(s) = \dfrac{1}{1+G_o} R(s) = \dfrac{1}{1+\dfrac{K}{(1+sT_1)(1+sT_2)}} = \dfrac{(1+sT_1)(1+sT_2)}{(1+sT_1)(1+sT_2)+K} \cdot \left(-\dfrac{1}{s^2} e^{-s}\right)$

$e_\infty(t) = \lim_{s \to 0} s \dfrac{(1+sT_1)(1+sT_2)}{(1+sT_1)(1+sT_2)+K}\left(-\dfrac{1}{s^2} e^{-s}\right) = -\infty$

4. $G_p = \dfrac{1}{s(s+1)(s+2)}$, $r(t) = tS(t)$ $R(s) = \dfrac{1}{s^2}$

$G_R = PI$ struktura $= K_R\left(1+\dfrac{1}{T_i s}\right) = K_R \dfrac{T_i s+1}{T_i s}$

$1+G_o = 1 + K_R \dfrac{T_i s+1}{T_i s} \dfrac{1}{s(s+1)(s+2)} = \dfrac{T_i s^2(s+1)(s+2)s + K_R(T_i s+1)}{s^2(s+1)(s+2)\,T_i s} \quad \dfrac{1}{s^2}$

$E(s) = \dfrac{1}{1+G_o} \cdot R(s) = \dfrac{\cancel{s^2}(s+1)(s+2)\cdot T_i}{T_i s^2(s+1)(s+2) + K_R(T_i s+1)} \cdot \dfrac{1}{\cancel{s^2}}$

$e_\infty = \lim_{s \to 0} s \cdot E(s) = \dfrac{0}{K_R} = \emptyset$

5. $G_o = \dfrac{K\omega_n^2}{s^2+2\xi\omega_n s+\omega_n^2}$ $r(t) = S(t)$ $R(s) = \dfrac{1}{s}$

$G_z = \dfrac{G_o}{1+G_o e^{-sT_t}} = \dfrac{\dfrac{K\omega_n^2}{s^2+2\xi\omega_n s+\omega_n^2}}{1+\dfrac{K\omega_n^2}{s^2+2\xi\omega_n s+\omega_n^2} e^{-sT_t}} = \dfrac{K\omega_n^2}{s^2+2\xi\omega_n s+\omega_n^2+K\omega_n^2 e^{-sT_t}}$

$y(\infty) = 0.5 = \lim_{s \to 0} \cancel{s} \cdot \dfrac{K\omega_n^2}{s^2+2\xi\omega_n s+\omega_n^2+K\omega_n^2 e^{-sT_t}} \cdot \dfrac{1}{\cancel{s}} = \dfrac{K\omega_n^2}{\omega_n^2+K\omega_n^2}$

$0.5 = \dfrac{1.9881\,K}{1.9881+1.9881\,K}$ $1.9881 \cdot 0,5(1+K) = K \cdot \cancel{1,9881}$

$0,5+0,5K = K$ $0,5 = 0,5K$

$\underline{K=1}$

6. Poglavlje 8, 6 zadatak (stranica 47)

$$G_0(s) = \frac{K_R}{s(s+1)(s+2)(s+3)} \quad , \quad P = 1 + G_0 = \frac{s(s+1)(s+2)(s+3)+K_R}{s(s+1)(s+2)(s+3)}$$

a) $K_R = 5$

$$e_\infty = \lim_{s \to 0} s \cdot \frac{1}{1+G_0} \cdot R(s) = \lim_{s \to 0} \cancel{s} \frac{s(s+1)(s+2)(s+3)}{s(s+1)(s+2)(s+3)+K_R} \cdot \frac{1}{\cancel{s}} = \frac{0}{K_R} = \phi$$

b)

$$1 = \lim_{s \to 0} \cancel{s} \frac{\cancel{s}(s+1)(s+2)(s+3)}{s(s+1)(s+2)(s+3)+K_R} \cdot \frac{1}{\cancel{s^2}} = \frac{6}{K_R} = 1 \Rightarrow \quad K_R = 6$$

7.

$$G_R = K_R\left(1 + \frac{1}{T_i s}\right) = K_R \frac{T_i s+1}{T_i s} = 0{,}5 \, \frac{15s+1}{15s} = \frac{15s+1}{30s}$$

$$G_0 = G_R \cdot G_P = \frac{15s+1}{30s} \cdot \frac{4(1-4s)}{(1+10s)(1+2s)} = \frac{2}{15s} \frac{(15s+1)(1-4s)}{(1+10s)(1+2s)}.$$

$$E(s) = \frac{15s(1+10s)(1+2s)}{15s(1+10s)(1+2s)+2(15s+1)(1-4s)} \quad R(s)$$

$$e_\infty = \lim_{s \to 0} \cancel{s} E(s) \cdot \frac{1}{\cancel{s}} = \lim_{s \to 0} E(s) = 0$$

$$e_\infty = \lim_{s \to 0} s \cdot E(s) \cdot \frac{1}{s^2} = \frac{15}{2} = 7.5$$

POGLAVLJE 10

1. $s_{p1,2} = -1 \pm j$ $\qquad z = e^{sT} = e^{(-\sigma + j\omega)T} = e^{-\sigma T} e^{j\omega T}$

a) $T = T_1 = 0{,}1\,s$ $\qquad |z| = e^{-\sigma T}$

$T = T_2 = 1\,s$ $\qquad \arg z = \omega T$

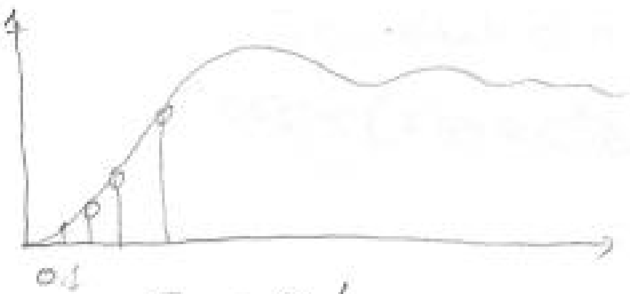
$T = T_1 = 0.1s$ $\qquad T = T_2 = 1s$

$|z| = 0.904$ $\qquad |z| = 0{,}367$

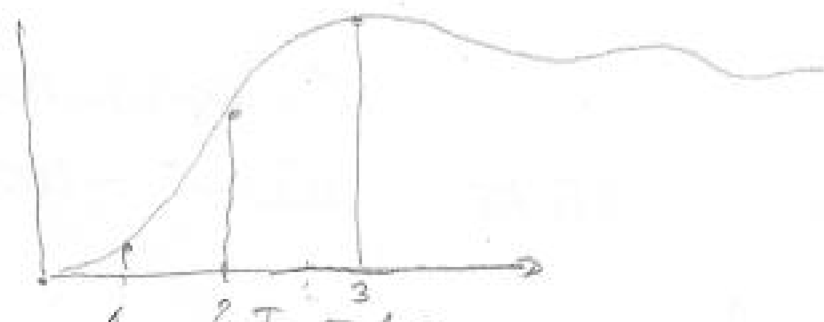
$\arg(z) = 0.1 rad = 5.7°$ $\qquad \arg(z) = 57{,}29°$

b) Prijelazne će biti iste i u S i u Z. Samo u Z treba pripaziti na periode uzorkovanja

$$G(s) = \frac{1}{s^2 + 2s + 2}$$



$T_1 = 0.1$



$T_1 = 1s$

2. $$G(s) = \frac{\frac{4}{3}}{s^2 + \frac{2\sqrt{3}}{3}s + \frac{4}{3}}$$

TUSTIN ?? $\qquad z = \dfrac{1 + s\frac{T}{2}}{1 - s\frac{T}{2}}$

$s^2 + \frac{2\sqrt{3}}{3}s + \frac{4}{3} = 0$ $\qquad s = \frac{2}{T}\frac{z-1}{z+1}$

(1) $T = T_1 = 0.1$

(2) $T = T_2 = 1s$

$s_{p1,2} = -\frac{\sqrt{3}}{3} \pm j$

uz $T = T_1$ i Tustina

$$G(z) = \frac{\frac{4}{3}\cdot(z^2 + 2z + 1)}{424.427z^2 - 797.33z + 378.239} \rightarrow = 0 \qquad z_{1,2} = 0{,}939 \pm 0{,}09j$$

$T = T_2$

$$G(z) = \frac{3\sqrt{3}(z+1)^2}{4((3+4\sqrt{3})z^2 - 4\sqrt{3}z + 4\sqrt{3} - 3)} \rightarrow = 0 \qquad z_{1,2} = \frac{2 \pm 3j}{4 + \sqrt{3}}$$

Puno jednostavnije bez Tustina ! $z = e^{s_{pi}T}$

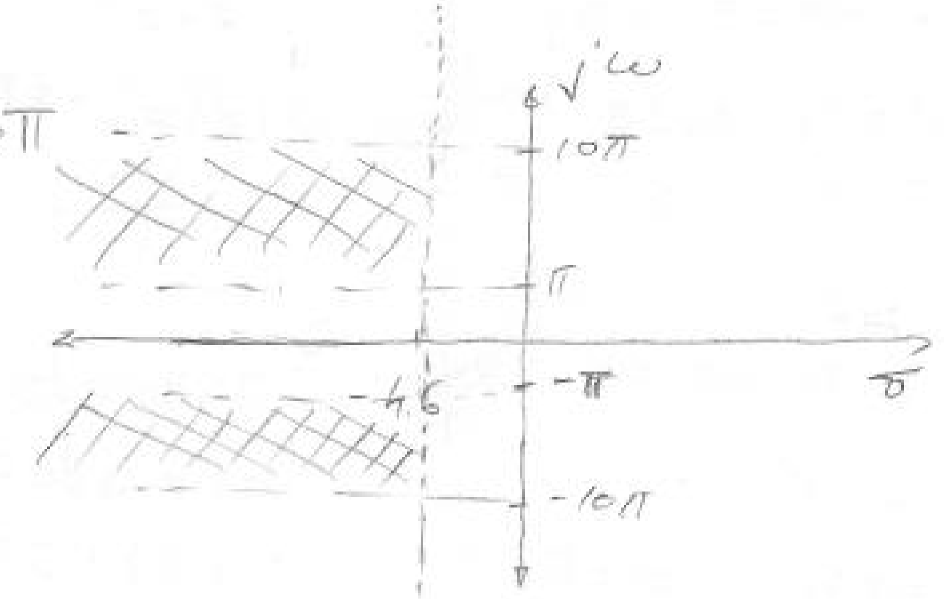
3. $t_m < 1s$

$t_{1\%} < 1s$

$T = 0,1s$

a) $\frac{\pi}{\omega_n\sqrt{1-\zeta^2}} < 1$ $\omega_n\sqrt{1-\zeta^2} > \pi$

$\omega > \pi$



$t_{1\%} < 1s$

$\frac{4.6}{\zeta\omega_n} < 1s$ $\zeta\omega_n > 4.6$

$\sigma > 4.6$

$\omega_s = \frac{2\pi}{T} = 2\omega_N$ $\omega_N = \frac{\pi}{T} = 10\pi$

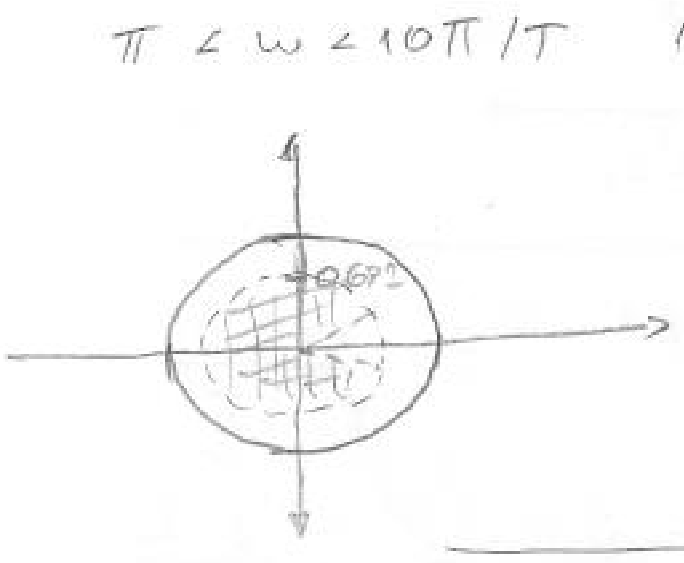
b) $z = e^{-\sigma T} e^{j\omega T}$

$\sigma > 4.6 \;/(-1)$ $-\sigma < -4.6 \Rightarrow |z| = e^{-\sigma T} = e^{-\sigma T} < e^{-4.6 \cdot 0.1}$

$|z| < 0.631$

$18^\circ < arg(z) < 180^\circ$ $-\pi > \omega > -10\pi$

$\pi < \omega < 10\pi / T$ $18^\circ < \omega T < 180^\circ$ $-18^\circ > arg(z) > -180^\circ$



4. $z = \frac{1}{4} + \frac{\sqrt{3}}{4} j$, $T = 10s$

$$z = \frac{1}{2} e^{j\frac{\pi}{3}} = e^{s_{pi} T} = e^{-\sigma T} e^{j\omega T}$$

$$\frac{1}{2} = e^{-\sigma T} \Rightarrow \sigma = 0{,}0693147$$

$$\frac{\pi}{3} = \omega T \Rightarrow \omega = \frac{\pi}{30}$$

$$s = 0{,}0693147 \pm j\,0.10472$$

$$\frac{\pi}{3} = \omega T + 2k\pi$$

$$\omega = \frac{\pi}{30} - 0{,}628k$$

$\omega = 0{,}10472 - 0{,}628k \Rightarrow$ ALIAS POLOVI

$$s = 0{,}0693147 \pm j\,(0{,}10472 - 0{,}628k)$$

POGLAVLJE 11 - postupci diskretizacije kontinuiranih sustava

1. $G_R(s)=K_R\frac{1+T_is}{T_is}$, $T=0.1$ $K_R=2.7$ $T_i=3.78$

$$s=\frac{2}{T}\frac{z-1}{z+1} \qquad G_R(s)=K_R\frac{1+T_i\frac{2}{T}\frac{z-1}{z+1}}{T_i\frac{2}{T}\frac{z-1}{z+1}}=K_R\frac{\frac{T(z+1)+2T_i(z-1)}{T\cancel{(z+1)}}}{\frac{2T_i(z-1)}{T\cancel{(z+1)}}}$$

$$G_R(s)=K_R\frac{T(z+1)+2T_i(z-1)}{2T_i(z-1)}=2.7\frac{0.1(z+1)+2\cdot 3.78(z-1)}{2\cdot 3.78(z-1)}=2.7\frac{0.1z+0.1+7.56z-7.56}{7.56(z-1)}$$

$$G_R(z)=2.7\frac{7.66z-7.44z}{7.56(z-1)}=2.7\cdot\frac{7.66}{7.56}\frac{(z-0.971)}{z-1}=2.7357\frac{z-0.971}{z-1}$$

---

2. PI regulator $=K_R\left(1+\frac{1}{T_is}\right)=K_R\frac{T_is+1}{T_is}$

a) $s=\frac{2}{T}\frac{z-1}{z+1}$

$$G_R(z)=K_R\frac{\frac{2T_i}{T}\frac{z-1}{z+1}+1}{\frac{2T_i}{T}\frac{z-1}{z+1}}=K_R\frac{\frac{2T_i(z-1)+T(z+1)}{\cancel{T(z+1)}}}{\frac{2T_i}{\cancel{T}}\frac{z-1}{\cancel{z+1}}}=K_R\frac{T(z+1)+2T_i(z-1)}{2T_i(z-1)}$$

$$G_R(z)=K_R\frac{\left(\frac{T}{2T_i}+1\right)z+\frac{T}{2T_i}-1}{z-1}=\frac{U(z)}{E(z)}\quad /:z$$

$$\frac{U(z)}{E(z)}=K_R\frac{\left(\frac{T}{2T_i}+1\right)+\left(\frac{T}{2T_i}-1\right)z^{-1}}{1-z^{-1}}$$

$$U(z)(1-z^{-1})=K_R\left(\frac{T}{2T_i}+1\right)E(z)+\left(\frac{T}{2T_i}-1\right)z^{-1}E(z)$$

$$U(k)-U(k-1)=K_R\left(\frac{T}{2T_i}+1\right)E(k)+\left(\frac{T}{2T_i}-1\right)E(k-1)$$

b) $G_R = K_R \frac{T_i s + 1}{T_i s}$ $\Rightarrow$ EULEROVOM UNAZADNOM $z = 1/1 - sT$

$s = \frac{z-1}{zT}$

$$G_R = K_R \frac{T_i \frac{z-1}{zT} + 1}{T_i \frac{z-1}{zT}} = K_R \frac{\frac{T_i(z-1) + zT}{\cancel{zT}}}{T_i \frac{z-1}{\cancel{zT}}} = K_R \frac{T_i(z-1) + zT}{T_i(z-1)} = \frac{U(z)}{E(z)}$$

$$\frac{U(z)}{E(z)} = \frac{z(T_i + T) - T_i}{T_i(z-1)} = K_R \frac{(T_i + T) - T_i z^{-1}}{T_i(1 - z^{-1})}$$

$$T_i U(k) - T_i U(k-1) = (T_i + T) E(k) - T_i E(k-1) \, / : T_i$$

$$U(k) = U(k-1) + K_R \left(1 + \frac{T}{T_i}\right) E(k) - K_R \, E(k-1)$$

3. PI regulator $= K_R \left(1 + \frac{1}{T_i s}\right) = K_R \frac{T_i s + 1}{T_i s}$ $\quad s = \frac{z-1}{zT}$

ISTI KAO PRETHODNI (B)

4. $G_p(s) = \frac{2}{s+2}$ $\quad T = 0.1s$

$$G_p(z) = (1 - z^{-1}) \mathcal{Z}\left\{\frac{G_p(s)}{s}\right\} = \text{ZOH diskretizacija}$$

$$= (1 - z^{-1}) \mathcal{Z}\left\{\frac{2}{s(s+2)}\right\} = (\text{službeni šalabahter}) = \frac{0.181 z^{-1}}{\cancel{(1-z^{-1})}(1 - 0.181 z^{-1})} \cdot \cancel{(1-z^{-1})}$$

$$G_p(z) = \frac{0.181 z^{-1}}{1 - 0.181 z^{-1}}$$

5. $\omega_c = 30\,s^{-1}$

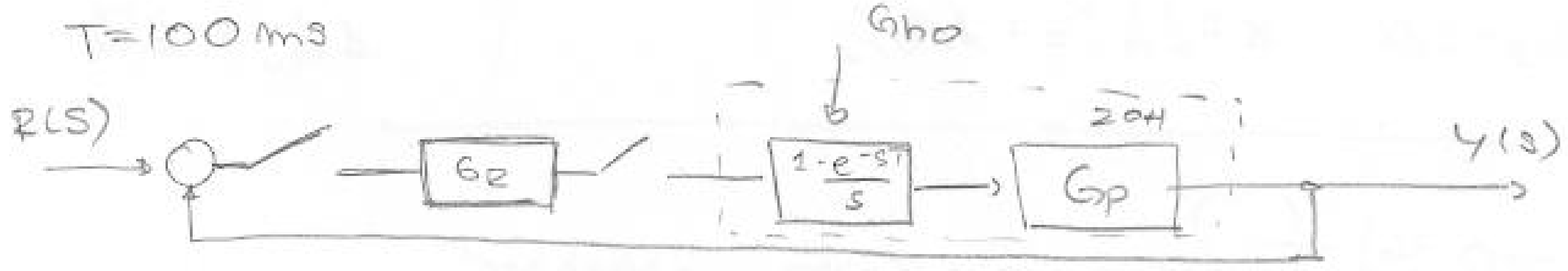
$T = 0.01\,s$

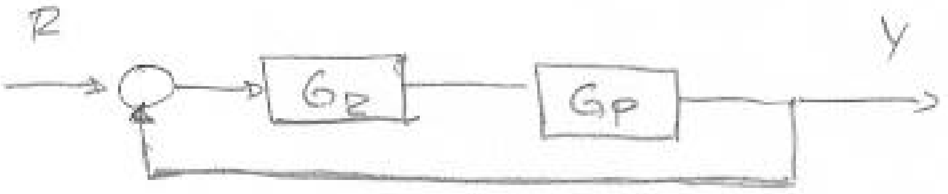
BILINEARNA TRANSFORMACIJA ⇒ TUSTIN

$$\omega^* = \frac{2}{T}\,tg\,\frac{\omega T}{2} = \frac{2}{0.01}\,arctg\,\frac{30\cdot 0.01}{2} = 30.22\,s^{-1}$$

6. $G_R(s) = 0.7\,\frac{1+s}{s}$, $G_p(s) = \frac{5}{(1+0.3s)(1+s)}$

T=100 ms





G_R - Z transformacija

G_p - ZOH diskretizacija

$$G_R(s) = 0.7\,\frac{s+1}{s} = 0.7\left(1+\frac{1}{s}\right) \xrightarrow{s\to z} 0.7\left(1+\frac{z}{z+1}\right)$$

$$G_R(z) = 0.7\,\frac{z+1+z}{z+1} = 0.7\,\frac{2z+1}{z+1}$$

$$G_p(s) = \frac{5}{(1+0.3s)(1+s)}$$

$$G_p(z) = (1-z^{-1})\,\mathcal{Z}\left(\frac{G_p(s)}{s}\right) = (1-z^{-1})\,\mathcal{Z}\left\{\frac{5}{s(1+0.3s)(1+s)}\right\}$$

$$\frac{1}{s}G_p(s) = \frac{A}{s} + \frac{B}{1+0.3s} + \frac{C}{1+s} = \frac{5}{s} + \frac{9}{14}\cdot\frac{1}{1+0.3s} - \frac{50}{7}\,\frac{1}{s+1}$$

$$A = \cancel{s}\,\frac{5}{\cancel{s}(1+0.3s)(1+s)} = 5 \qquad B = \frac{5}{s(1+s)}\Big|_{s=-\frac{10}{3}} = \frac{9}{14} \qquad C = -\frac{50}{7}$$

$$\mathcal{Z}\left(\frac{5}{s} + \frac{9}{14}\,\frac{1}{1+0.3s} - \frac{50}{7}\,\frac{1}{s+1}\right) = 5\,\frac{z}{z-1} + \frac{15}{7}\,\frac{z}{z-0.716} - \frac{50}{7}\,\frac{z}{z-0.904}$$

$$G_p(z) = \frac{z-1}{z}\left(A\frac{z}{z-1} + B\,\frac{z}{z-0.716} + C\,\frac{z}{z-0.904}\right) = A + B\,\frac{z-1}{z-0.716} + C\,\frac{z-1}{z-0.904}$$

$$= 5 + \frac{15}{7}\,\frac{z-1}{z-0.716} - \frac{50}{7}\,\frac{z-1}{z-0.904} \quad \_\,56\,\_ \quad = \frac{0.07z+0.059}{z^2-1.62z+0.64}$$

7. $\omega_c = 0.1\,rad/s$

$T = 1s$

$$\Delta\varphi = \frac{\omega T}{2} = \frac{0.1 \cdot 1}{2} = \frac{1}{20}\,rad = 2.86^\circ$$

$$\gamma[^\circ] = 70 - \sigma_m[\%] \Rightarrow \sigma_m = 70 - \gamma[\%]$$

DISKRETIZACIJA: $\sigma_{m1}[\%] = 70 - (70 - \Delta\sigma_{m2} + 2.86)$

$\sigma_{m1} = \sigma_{m2} - 2.86 \quad \Delta\sigma[\%] = +2.86\%$

FAZNO KAŠNJENJE SMANJUJE FAZNO OSIGURANJE

$\gamma_2 = \gamma_1 - 2.86$

$70 - \sigma_{m2} = 70 - \sigma_{m1} - 2.86$

$-(\sigma_{m2} - \sigma_{m1}) = -2.86$

$\Delta\sigma_m = -2.86$

---

8. $T = (0.17 \div 0.34)\frac{1}{\omega_c}$

$T = (\frac{1}{10} \div \frac{1}{4})t_r$

Formule $\Rightarrow$ službeni šalabahter

$\omega_c = 10^{-2} \Rightarrow T = (0.17 \div 0.34) \cdot 10^2 = (17 \div 34)\,s \qquad T_s = \frac{17+34}{2} = 25.55$

ZOH unosi kašnjenje $\frac{T}{2}$ (MRTVO VRIJEME)

$$\omega_c \frac{T}{2} = \Delta\gamma = \frac{25.55 \cdot 10^{-2}}{2} = 0.1275\,\frac{rad}{s} = 7.30^\circ$$ (točnije bi bilo napisano -7.3)

$\gamma_2 = \gamma_1 - 7.3$

$70 - \sigma_{m2} = 70 - \sigma_{m1} - 7.3 \qquad \Delta\sigma_m = 7.3\%$

$$\varphi(\omega_c) = -\frac{T}{2}\omega_c = -\frac{(17, 34)}{2} \cdot 10^{-2} = -(0{,}085, 0{,}17)\,rad = -(4.87, 9.74)^\circ$$

$\Delta\sigma_m = (4.87, 9.74)\%$

9.

$$G_0(s) = \frac{1}{4T_i s(1+2T_i s)}$$

$$G_0(j\omega_c) = \frac{1}{4T_i j\omega_c} \cdot \frac{1}{(1+2T_i j\omega_c)}$$

$$|G_0(j\omega_c)| = \frac{1}{4T_i\omega_c} \frac{1}{\sqrt{1+4T_i^2\omega_c^2}} = 1$$

$$4T_i\omega_c \cdot \sqrt{1+4T_i^2\omega_c^2} = 1 \quad |^2$$

$$16T_i^2\omega_c^2(1+4T_i^2\omega_c^2) = 1$$

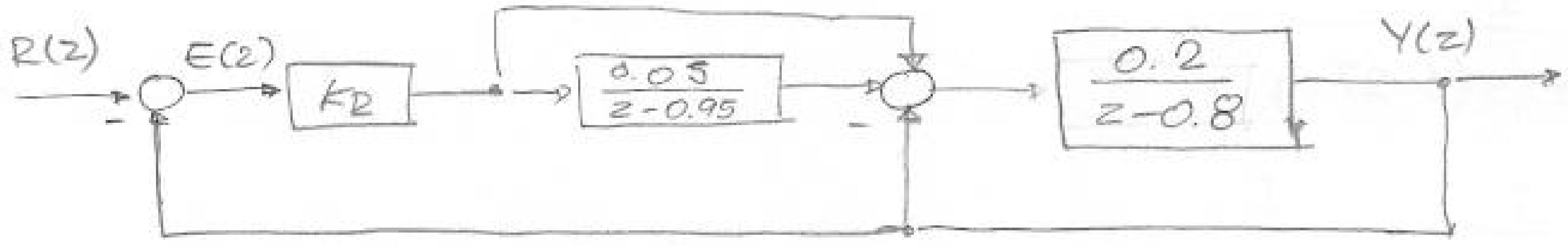
$$64T_i^4\omega_c^4 + 16T_i^2\omega_c^2 - 1 = 0 \qquad T_i^2\omega_c^2 = x$$

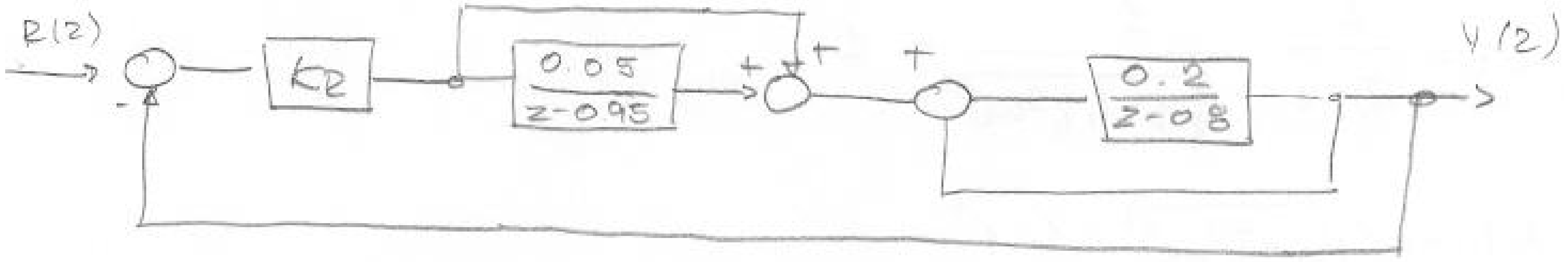
$$64 \cdot x^2 + 16x - 1 = 0 \qquad x_1 = \frac{-1+\sqrt{2}}{8} \qquad \cancel{x_2 = -0{,}304}$$

$$T_i^2\omega_c^2 = \frac{-1+\sqrt{2}}{8} \Rightarrow \omega_c = \sqrt{\frac{\sqrt{2}-1}{8T_i^2}} = \frac{1}{2T_i}\sqrt{\frac{\sqrt{2}-1}{2}} = \frac{0.22754}{T_i}$$

$$T = (0.17 \div 0.34)\frac{1}{\frac{0.22754}{T_i}} = (0.17 \div 0.34)\frac{T_i}{0.22754}$$

10. $r(k) = S(k)$





$E(z) = R(z) - Y(z)$

$$Y(z) = E(z) \cdot K_R \left(1 + \frac{0.05}{z-0.95}\right) \cdot \frac{\frac{0.2}{z-0.8}}{1 + \frac{0.2}{z-0.8}}$$

$r(k) = S(k)$

$r(z) = \frac{z}{z-1}$

$$Y(z) = E \cdot K_R \frac{z-0.9}{z-0.95} \cdot \frac{0.2}{z-0.6}$$

$$E\left(1 + K_R \frac{z-0.9}{z-0.95} \cdot \frac{0.2}{z-0.6}\right) = \frac{z}{z-1}$$

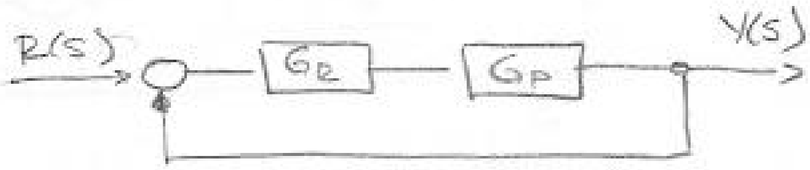
$$E(z) = \frac{1}{1 + K_R \frac{(z-0.9) \cdot 0.2}{(z-0.95)(z-0.6)}} \cdot \frac{z}{z-1} = \frac{z(z-0.95)(z-0.6)}{(z-0.95)(z-0.6) + 0.2K_R(z-0.9)} \cdot \frac{1}{z-1}$$

$$e(\infty) = \lim_{z \to 1} \frac{\cancel{z-1}}{\cancel{z}} \cdot \frac{\cancel{z}(z-0.95)(z-0.6)}{(z-0.95)(z-0.6) + 0.2K_R(z-0.9)} \cdot \frac{1}{\cancel{z-1}}$$

$$e(t=\infty) = \frac{0.02}{0.02 + 0.2K_R \cdot 0.1} = \frac{0.02}{0.02 + 0.02K_R} = \frac{1}{1+K_R}$$

POGLAVLJE 12 - stabilnost linearnih diskretnih sustava upravljanja

1. $G_R(z) = K_R \dfrac{z}{0.5z - 0.4}$, $G_P(z) = \dfrac{0.1813 z^{-1}}{1 - 0.8187 z^{-1}}$



$$G_P(z) = \frac{0.1813 \frac{1}{z}}{1 - 0.8187 \frac{1}{z}} = \frac{0.1813 \frac{1}{z}}{\frac{z - 0.8187}{z}} = \frac{0.1813}{z - 0.8187}$$

$$G_0 = G_R \cdot G_P = K_R \frac{0.1813 z}{(0.5z - 0.4)(z - 0.8187)} = \frac{0.1813 z \cdot K_R}{0.5z^2 - 0.80935z + 0.32748} \begin{matrix} /2 \\ /2 \end{matrix}$$

$$= \frac{0.3626 z K_R}{z^2 - 1.618704z + 0.65496}$$

$$P = 1 + G_0 = z^2 - 1.618704z + 0.65496 + 0.3626 z K_R$$

$$= \underbrace{z^2}_{a_2} + \underbrace{(0.3626 K_R - 1.618704)}_{a_1} z + \underbrace{0.65496}_{a_0} = \phi$$

a) $F(1) > 0$

$1 + 0.3626 K_R - 1.618704 + 0.65496 > 0$ $\qquad 0.3626 K_R > -0.036226$

$K_R > -0.0999 \Rightarrow K_R > -0.1$

b) $n = 2$

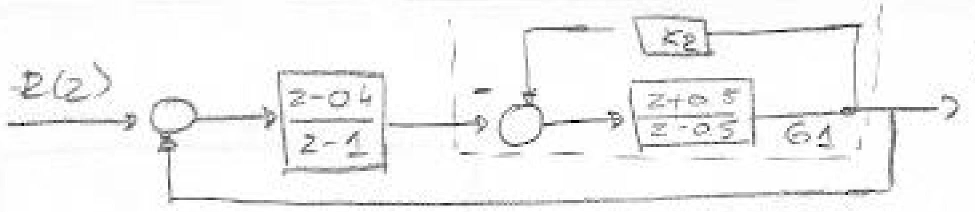
$(-1)^2 F(-1) > 0 \qquad F(-1) > 0 \qquad 1 - 0.3626 K_R + 1.618704 + 0.65496 > 0$

$-0.3626 K_R > -3.273664 \qquad K_R < 9.0283$

provjeravamo matricu do $2n - 3 = 2 \cdot 2 - 3 = 1$

$|a_0| < |a_n|$ ✓ $\qquad K_R \in \langle -0.1, 9.0283 \rangle$

2. $G(z) = \frac{Y(z)}{R(z)} = ?$ Jury ?



$$G_1 = \frac{\frac{z+0.5}{z-0.5}}{1+k_2\frac{z+0.5}{z-0.5}} = \frac{z+0.5}{z-0.5+zk_2+0.5k_2} = \frac{z+0.5}{(1+k_2)z+0.5(k_2-1)}$$

$$G_2(z) = \frac{\frac{z-0.4}{z-1}\cdot\frac{z+0.5}{(1+k_2)z+0.5(k_2-1)}}{1+\frac{z-0.4}{z-1}\,\frac{z+0.5}{(1+k_2)z+0.5(k_2-1)}} = \frac{\frac{z-0.4}{z-1}\cdot\frac{z+0.5}{\cancel{(1+k_2)z+0.5(k_2-1)}}}{(z-1)[(1+k_2)z+0.5(k_2-1)]+(z-0.4)(z+0.5)}$$

$$G_2(z) = \frac{(z-0.4)(z+0.5)}{(1+k_2)z^2+0.5z(k_2-1)-z(1+k_2)-0.5(k_2-1)+z^2+0.1z-0.2}$$

$$G_2(z) = \frac{z^2+0.1z-0.2}{z^2\underbrace{(2+k)}_{a_2}+z\underbrace{(-1.4-0.5k)}_{a_1}\underbrace{+0.3-0.5k}_{a_0}}$$

$P = z^2(2+k)+z(-1.4-0.5k)+0.3-0.5k$

a) $f(1)>0$ $2+\cancel{k}-1.4-\cancel{0.5k}-\cancel{0.5k}+0.3$ $0.9>0\ \forall k$

$(-1)^2 f(-1)>0$ $+2+k+1.4+0.5k+0.3-0.5k>0$

$k>-3.7$ (1)

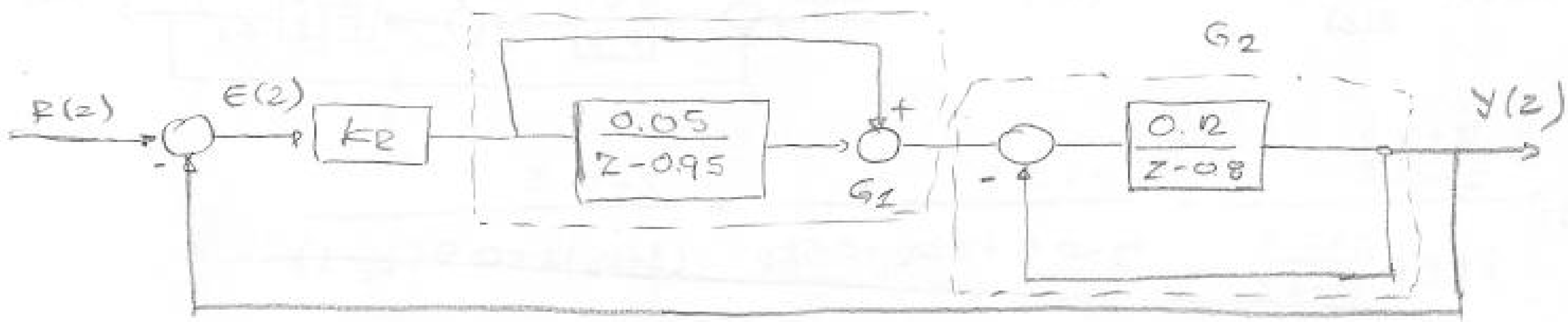
$2n-3 = 2\cdot 2-3 = 1$

$|a_0|<|a_2|$

$|0.3-0.5k|<|2+k|$ (2) $k\in\langle-\infty,-4.6)\cup\langle-1.13,\infty\rangle$

(1) $\cap$ (2) $k\in\langle-1.13, \inf\rangle$

3.



$$G_1 = 1 + \frac{0.05}{z-0.95} = \frac{z-0.95+0.05}{z-0.95} = \frac{z-0.9}{z-0.95}$$

$$G_2 = \frac{\frac{0.2}{z-0.8}}{1+\frac{0.2}{z-0.8}} = \frac{\frac{0.2}{\cancel{z-0.8}}}{\frac{z-0.8+0.2}{\cancel{z-0.8}}} = \frac{0.2}{z-0.6}$$

$$G_z = \frac{K_R \frac{z-0.9}{z-0.95}\cdot\frac{0.2}{z-0.6}}{1+\frac{K_R(z-0.9)\cdot 0.2}{(z-0.95)(z-0.6)}} = \frac{0.2K_R(z-0.9)}{(z-0.95)(z-0.6)+0.2K_R(z-0.9)}$$

$$G_z = \frac{0.2K_R(z-0.9)}{z^2-1.55z+0.57+0.2K_Rz-0.18K_R} = \frac{0.2K_R(z-0.9)}{\underbrace{z^2}_{a_2}+z\underbrace{(0.2K_R-1.55)}_{a_1}\underbrace{+0.57-0.18K_R}_{a_0}}$$

Jury a) $n=2$ $F(1)>0$ $1+0.2K_R-1.55+0.57-0.18K_R>0$

$0.02+0.02K_R>0$ $0.02K_R>-0.02$ $K_R>-1$ (1)

$F(-1)>0$ $1-0.2K_R+1.55+0.57-0.18K_R>0$

$3.12-0.38K_R>0$ $3.12>0.38K_R$ $K_R<8.210$ (2)

$|a_0|<|a_n|$

$|0.57-0.18k|<1$

$k=\frac{19}{6}$ (granica)

I $k<\frac{19}{6}$

$0.57-0.18k<1$

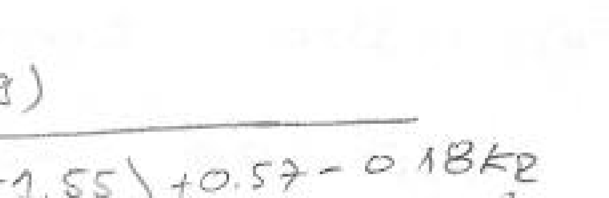
$k>-2.38$ (3)

II $k\geq\frac{19}{6}$

$-0.57+0.18k<1$

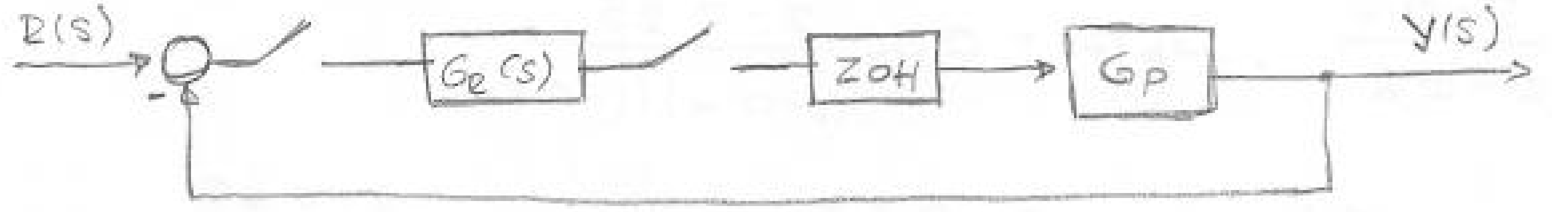
$k<8.72$ (4)

Ukupno rješenje: presjek svega (1,2,3,4)



$K_R \in \langle -1, 8.21 \rangle$

4.



$G_R = \frac{2}{s}$, $G_P(s) = \frac{2}{s+1}$ $\qquad G_P(z) = (1-z^{-1})\, \mathcal{Z}\left\{\frac{2}{s(s+1)}\right\}$

$\mathcal{Z}\{G_R\} = 2\frac{z}{z-1}$ $\qquad G_P(z) = 2(1-z^{-1})\, \mathcal{Z}\left\{\frac{1}{s} - \frac{1}{s+1}\right\}$

$$G_P(z) = 2(1-z^{-1})\cdot\left(\frac{1}{1-z^{-1}} - \frac{1}{1-e^{-T}z^{-1}}\right) = 2\left(1 - \frac{1-z^{-1}}{1-e^{-T}z^{-1}}\right) = 2\frac{1-e^{-T}}{z-e^{-T}}$$

$$G_0 = 2\frac{z}{z-1}\cdot 2\frac{1-e^{-T}}{z-e^{-T}} = \frac{4z(1-e^{-T})}{(z-1)(z-e^{-T})}$$

$$P = 1+G_0 = (z-1)(z-e^{-T}) + 4z(1-e^{-T}) = z^2 - ze^{-T} - z + e^{-T} + 4z - 4ze^{-T}$$

$$= z^2 + z(-e^{-T} - 1 + 4 - 4e^{-T}) + e^{-T} = z^2 + (3-5e^{-T})z + e^{-T}$$

HURWITZ $\qquad z = \frac{1+w}{1-w}$

$$\left(\frac{1+w}{1-w}\right)^2 + (3-5e^{-T})\frac{1+w}{1-w} + e^{-T} = \emptyset \qquad (1+w)^2 + (3-5e^{-T})(1+w)(1-w) + e^{-T}(1-w)^2 = 0$$

$$1 + 2w + w^2 + (3-5e^{-T})(1-w^2) + e^{-T}(1-2w+w^2) = 0$$

$$1 + 2w + w^2 + 3 - 3w^2 - 5e^{-T} + 5w^2e^{-T} + e^{-T} - 2we^{-T} + w^2e^{-T} = 0$$

$$w^2(1 - 3 + 5e^{-T} + e^{-T}) + w(2-2e^{-T}) + 1 + 3 - 5e^{-T} + e^{-T} = 0$$

$$\underbrace{(6e^{-T}-2)}_{a_2}w^2 + \underbrace{(2-2e^{-T})}_{a_1}w + \underbrace{4-4e^{-T}}_{a_0} = \emptyset$$

a) $a_2, a_1, a_0 > 0$ $\qquad 6e^{-T} - 2 > 0 \quad e^{-T} > \frac{1}{3}$, $\quad 2-2e^{-T} > 0 \quad 2 > 2e^{-T} \quad 1 > e^{-T}$

$\qquad 4 - 4e^{-T} > 0 \quad 4 > 4e^{-T} \quad 1 > e^{-T}$

$e^{-T} > \frac{1}{3}$ / ln $\quad -T > \ln\frac{1}{3} \quad T < -\ln\frac{1}{3} \quad T < 1.09$

$e^{-T} < 1$ / ln $\quad -T < \ln 1 \quad T > -\ln e^0 \quad T > 0$

$T \in \langle 0, 1.09 \rangle$

b) $2 - 2e^{-T} > 0$

5. $G_R(z) = 10\,\frac{z-0.4}{z-0.6}$, $G_P(z) = 0.04\,\frac{z-0.25}{(z-0.4)(z-0.6)}$

$$P = 1 + G_0 = 1 + 10\,\frac{\cancel{z-0.4}}{z-0.6}\cdot 0.04\,\frac{z-0.25}{(\cancel{z-0.4})(z-0.6)} = 1 + 0.4\,\frac{z-0.25}{(z-0.6)^2}$$

$$= \frac{(z-0.6)^2 + 0.4z - 0.1}{(z-0.6)^2} = \frac{z^2 - 1.2z + 0.36 + 0.4z - 0.1}{(z-0.6)^2} = \frac{z^2 - 0.8z + 0.26}{(z-0.6)^2} = 0$$

$$z^2 - 0.8z + 0.26 = 0 \qquad \left(\frac{1+w}{1-w}\right)^2 - 0.8\left(\frac{1+w}{1-w}\right) + 0.26 = 0$$

$$(1+w)^2 - 0.8(1-w^2) + 0.26(1-w)^2 = 0 \qquad 1 + 2w + w^2 - 0.8 + 0.8w^2 + 0.26 - 0.52w + 0.2\ldots$$

$$= \underbrace{2.06}_{a_2}w^2 + \underbrace{1.48}_{a_1}w\ \underbrace{+0.46}_{a_0} = 0$$

I $a_2, a_1, a_0 > 0$ ✓

II $a_1 > 0$

} SUSTAV JE STABILAN

POGLAVLJE 13

1. $G_p(s) = \dfrac{4(1-4s)}{(1+10s)(1+2s)}$, $G_R = K_R\left(1+\dfrac{1}{Ts}\right)$

$G_o = \dfrac{4k_R(1-4s)}{(1+10s)(1+2s)}$, $G_o(j\omega) = 4K_R\dfrac{(1-4j\omega)}{(1+10j\omega)(1+2j\omega)} \cdot \dfrac{(1-10j\omega)(1-2j\omega)}{(1-10j\omega)(1-2j\omega)}$

$= 4K_R\dfrac{(1-4j\omega)(1-10j\omega)(1-2j\omega)}{(1+100\omega^2)(1+4\omega^2)} = 4K_R\dfrac{(1-14j\omega-40\omega^2)(1-2j\omega)}{(1+100\omega^2)(1+4\omega^2)}$

$= 4k_R\dfrac{(1-2j\omega-14j\omega-28\omega^2-40\omega^2+80j\omega^3)}{(1+100\omega^2)(1+4\omega^2)} = \dfrac{4K_R}{\longrightarrow}\left((1-68\omega^2)+j(80\omega^3-16\omega)\right)$

$I\omega g = 0 \Rightarrow 80\omega^3-16\omega=0 \quad \omega(80\omega^2-16)=0$

$\omega = \pm\sqrt{\dfrac{16}{80}} = \pm\dfrac{\sqrt{5}}{5} \approx 0.447$

$|G_o(j\omega_c)| = 1 = 4k_R\dfrac{\sqrt{1+16\omega^2}}{\sqrt{(1+100\omega^2)(1+4\omega^2)}} \Rightarrow K_{R_{krit}} = \dfrac{3}{4} = 0.75$

PI $\quad k_R = 0{,}45 \cdot k_{Rkr} = 0.3375$

$\omega_\pi = \dfrac{\sqrt{5}}{5} = \dfrac{2\pi}{T_{kr}} \Rightarrow T_{kR} = 14.0496\,s \qquad T_i = 0{,}85 \cdot T_{kR} = 11.94\,s$

$G_R = 0.3375 \cdot \left(1+\dfrac{1}{11.94\,s}\right)$

2. $G_p(s) = \dfrac{k_p}{(1+T_1 s)(1+T_2 s)(1+T_3 s)}$, $k_p = 1$ $T_1 = 2$, $T_2 = 3$, $T_3 = 4$

$$G_p(s) = \frac{1}{(1+2s)(1+3s)(1+4s)}$$

$$G_o = \frac{k_{Rkri}}{(1+2s)(1+3s)(1+4s)}$$

$$P = 1+G_o = (1+2s)(1+3s)(1+4s) + k_{R\,kri} = 24s^3 + 26s^2 + 9s + 1 + k_R$$

Hurwitz

I $a_3, a_2, a_1, a_0 > 0$ $\qquad k_R > 0$

II $a_1 > 0$ $\qquad 9 > 0$

$a_1 a_2 - a_0 a_3 > 0$ $\qquad 26 \cdot 9 - 24(1+k_R) > 0$ $\qquad 234 - 24(1+k_R) > 0$

$234 > 24(1+k_R)$ $\qquad 9.75 > 1+k_R$ $\qquad 8.75 > k_R$ $\qquad K_{RKRI} = 8.75$

$$P = 24s^3 + 26s^2 + 9s + 9.75 = \phi$$

$s_1 = -1.083$

$s_2 = 6.5 \cdot 10^{-13} + 0.6123 j$ $\qquad s = j\omega \qquad 0.6123 = \omega = \dfrac{2\pi}{T}$

$s_3 = 6.5 \cdot 10^{-13} - 0.6123 j$ $\qquad T_{KRI} = \dfrac{2\pi}{0.6123} = 10.2616$

$K_R = 0.6 K_{RKR} = 5.25$ $\qquad T_I = 0.5 \cdot T_{KRI} = 5.1308$ $\qquad T_D = 1.2313$

$$G_R(s) = 5.25\left(1 + \frac{1}{5.1308\, s} + 1.2313\, s\right)$$

3. $G_R = PI$ struktura $\qquad G_p(s) = \dfrac{1}{s(s+1)(s+2)}$

$$G_o'(s) = K_R \frac{1}{s(s+1)(s+2)} \qquad G_o(j\omega) = K_R \frac{1}{j\omega(j\omega+1)(j\omega+2)}$$

$$G_o(j\omega) = K_R \frac{1}{j\omega(j\omega+1)(j\omega+2)} \cdot \frac{j(1-j\omega)(2-j\omega)}{j(1-j\omega)(2-j\omega)} = -K_R j \frac{(1-j\omega)(2-j\omega)}{\omega(1+\omega^2)(4+\omega^2)}$$

$$= -K_R \cdot j \frac{2-3j\omega-\omega^2}{\omega(1+\omega^2)(4+\omega^2)} = -K_R \frac{(2-\omega^2)j+3\omega}{\omega(1+\omega^2)(4+\omega^2)}$$

$$Img = 0 \qquad 2-\omega^2 = 0 \qquad \omega^2 = 2 \qquad \omega = \pm\sqrt{2} \qquad \omega_\pi = +\sqrt{2}$$

$$1 = K_R \frac{1}{\omega\sqrt{1+\omega_\pi^2}\sqrt{4+\omega_\pi^2}} \Rightarrow K_{R_{KRI}} = 6 \qquad \omega_\pi = +\sqrt{2} = \frac{2\pi}{T_{KRI}}$$

$$T_{KRI} = \frac{2\pi}{\sqrt{2}} = 4{,}4428s \qquad \boxed{G_R = 2.7\left(1 + \frac{1}{3.77s}\right)}$$

$$K_R = 0{,}45 \cdot K_{RKRI} = 2.7 \;,\; T_I = 0.85 T_{KR} = 3.77638\,s$$

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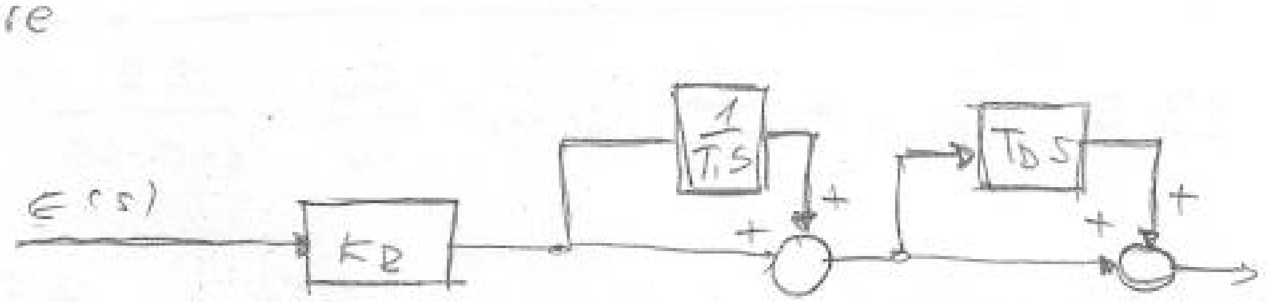
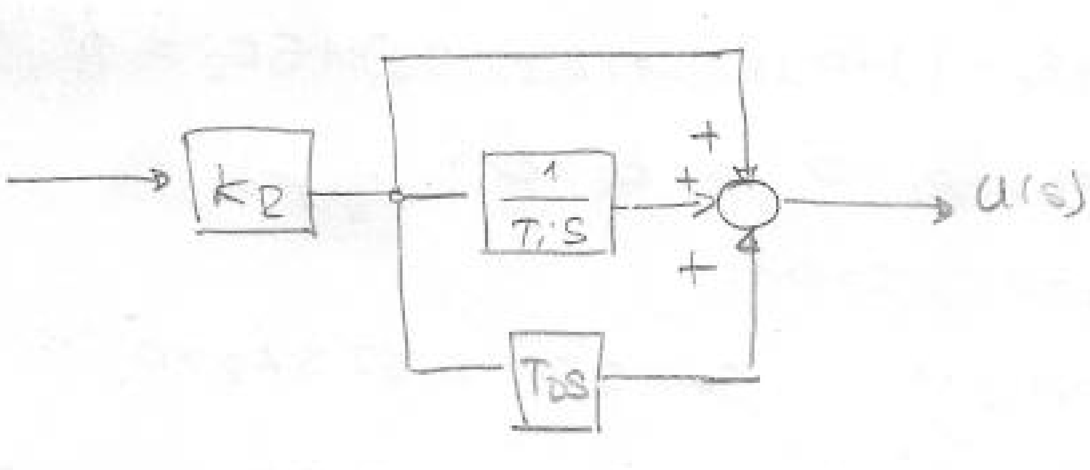
4. $G_R(s) = PID$ paralelne strukture

$$G_p(s) = \frac{5}{(s+2)(s+1)^2}$$



SERIJSKI

$$G_R = K_R\left(1 + \frac{1}{T_I s} + T_D s\right)$$



4. nastawek

$$G_P = \frac{5}{(s+2)(s+1)^2} \qquad G_0'(s) = \frac{5K_R}{(s+2)(s+1)^2}$$

$$G_0(j\omega) = \frac{5K_R}{(j\omega+2)(j\omega+1)^2} = \frac{5K_R}{(2+j\omega)(j^2\omega^2+2j\omega+1)} = \frac{5K_R}{(2+j\omega)(1-\omega^2+2j\omega)}$$

$$= \frac{5K_{Rkrit}}{(2-2\omega^2+4j\omega+j\omega-j\omega^3-2\omega^2)} = \frac{5K_{Rkrit}}{(2-4\omega^2+j(5\omega-\omega^3))} \cdot \frac{2-4\omega^2-j(5\omega-\omega^3)}{-\text{||}-}$$

$$= \frac{5K_{Rkrit}}{(2-4\omega^2)^2+(5\omega-\omega^3)^2} \cdot \frac{2-4\omega^2-j(5\omega-\omega^3)}{-\text{||}-}$$

Nyquist $Im_g = 0$ $\quad 5\omega-\omega^3=0 \quad \omega(5-\omega^2)=\phi \quad \omega_\pi=\sqrt{5}$

$$1 = \frac{5K_{Rkrit}}{\sqrt{(2-4\omega_\pi^2)^2+(5\omega_\pi-\omega_\pi^3)^2}} \Rightarrow K_{Rkrit} = \frac{18}{5} = 3.6$$

$$\omega_\pi = \frac{2\pi}{T_{kri}} \Rightarrow T_{kR} = \frac{2\pi}{\sqrt{5}} = 2.8099 \qquad G_R = 2.16\left(1+\frac{1}{1.4T_i}+0.337T_D\right)$$

$K_R = 0.6K_{Rkri} = 2.16 \qquad T_i = 0.5T_{kR} = 1.40 \qquad T_D = 0.12\ T_{kR} = 0.337$

13.5 $\quad G_{EMP} = \frac{1}{5s}, \quad G_V = \frac{Q_u}{X_V} = \frac{0.2}{1+0.1s}, \quad G_S = \frac{H}{Q_u} = \frac{25}{1+125s}$

$$G_0'(s) = K_R \cdot G_V \cdot G_S \cdot G_{EMP} = K_R\ \frac{0.2}{1+0.1s}\ \frac{25}{1+125s} \cdot \frac{1}{5s}$$

$p = 1+G_0 = (1+0.1s)(1+125s)(5s)+5K_R = \phi \qquad 62.5s^3+625.5s^2+5s+5K_R=\phi$

I $\quad a_3, a_2, a_1, a_0 > 0 \Rightarrow K_R > 0$

II $\quad a_1 > 0 \qquad 5 > 0$

$a_1a_2 - a_0a_3 > 0 \qquad 5 * 625.5 - 5 \cdot 62.5K_R > 0 \qquad 10.008 - K_R > 0 \qquad 10.008 < K_R$

$K_{Rkrit} = 10 \qquad K_R = 0.5K_{Rkrit} = 5$

6. $G_0(s) = \dfrac{0.6}{(1+40s)(1+25s)}$

$$Y(s) = G_0(s)\cdot U(s) = G_0(s)\cdot\frac{1}{s} = \frac{0.6}{(1+40s)(1+25s)\cdot s} = \frac{A}{s} + \frac{B}{1+40s} + \frac{C}{1+25s}$$

$$A = s\cdot\frac{0.6}{s(1+40s)(1+25s)} = 0.6$$

$$B = \cancel{(1+40s)}\cdot\frac{0.6}{\cancel{(1+40s)}(1+25s)s}\Bigg|_{s=-\frac{1}{40}} = -64$$

$$C = \cancel{(1+25s)}\,\frac{0.6}{(1+40s)\cancel{(1+25s)}s} = 25$$

$$y(s) = \frac{0.6}{s} - \frac{64}{1+40s} + \frac{25}{1+25s} = \frac{0.6}{s} - 1.6\,\frac{1}{\frac{1}{40}+s} + 1\,\frac{1}{\frac{1}{25}+s}$$

$$y(t) = \left(0.6 - 1.6\,e^{-t/40} + 1\cdot e^{-\frac{t}{25}}\right)S(t),\ t>0$$

TOČKA INFLEKSIJE: $\ddot{y} = \phi$

$$\dot{y} = \left(-1.6\cdot\left(-\frac{1}{40}\right)e^{-\frac{t}{40}} + \left(-\frac{1}{25}\right)e^{-\frac{t}{25}}\right) + \underbrace{\left(0.6 - 1.6e^{-\frac{t}{40}} + e^{-\frac{t}{25}}\right)\delta(t)}_{=0,\ \text{uvrh}\cdot t=0}$$

$$\dot{y} = \left(+\frac{1}{25}e^{-\frac{t}{40}} - \frac{1}{25}e^{-\frac{t}{25}}\right)$$

$$\ddot{y} = \left(\frac{+1}{25}\cdot\left(-\frac{1}{40}\right)e^{-\frac{t}{40}} - \frac{1}{25}\cdot\left(-\frac{1}{25}\right)e^{-\frac{t}{25}}\right) = \phi$$

$$-\frac{1}{1000}e^{-\frac{t}{40}} + \frac{1}{625}e^{-\frac{t}{25}} = \phi \qquad \frac{1}{1000}e^{-\frac{t}{40}} = \frac{1}{625}e^{-\frac{t}{25}}$$

$$\frac{625}{1000} = e^{-\frac{t}{25}+\frac{t}{40}} \rightarrow \cancel{t} = 31.333\,s = t_i$$

$$y(t_i) = \left(0.6 - 1.6e^{-31.333/40} + e^{-\frac{31.333}{25}}\right) = 0.1545$$

$$k = \text{nagib} = \dot{y}(t_i) = 6.85\cdot10^{-3} = 0.00685$$

Tangenta: $y - y_i = k_i(t - t_i)$ $\quad y - 0.1545 = 0.00685(t - 31.333)$

$$y(t) = 0.00685t - 0.06013105$$

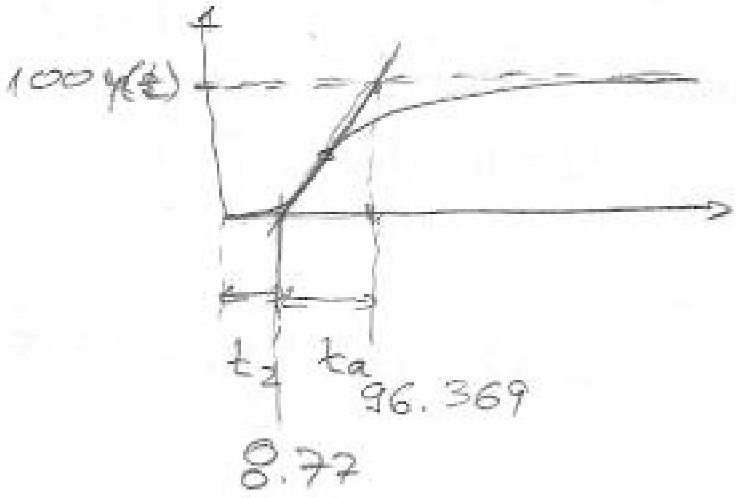
$\longrightarrow$

6. nastavak

$y = 0.00685t - 0.06013105$

$y = 0 \Rightarrow 0.00685t = 0.06013105$

$t = 8.77825s = t_z$



$y(t \to \infty) = 0.6 = k_s$

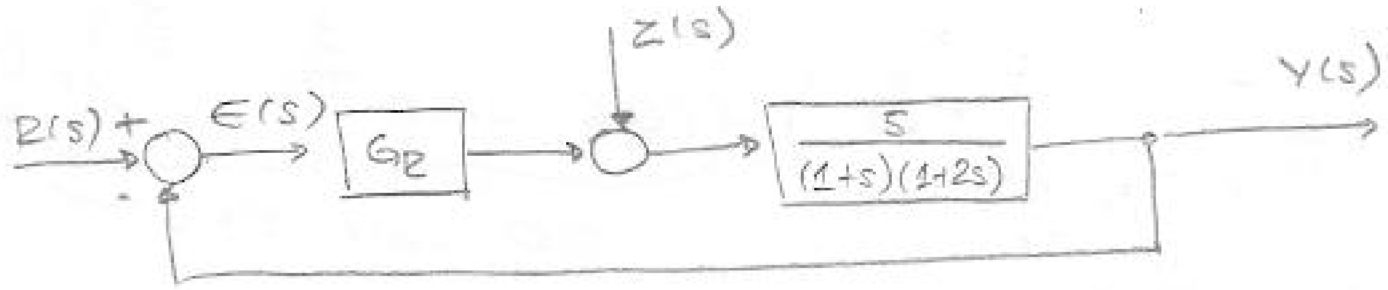
$0.6 = 0.00685t - 0.06013105 \qquad t = 96.36949$

$t_a = 96.369 - 8.77 = 87.591s$

PI $\qquad k_R = 0.9 \dfrac{t_a}{t_z \cdot k_s} \qquad T_I = 3.33 t_z$

$k_R = 0.9 \cdot \dfrac{87.591}{8.77 \cdot 0.6} = 14.98 \qquad T_I = 29.23s$

7.



a) $Y = G_P(s) \cdot \frac{1}{s} = \dfrac{5}{s(1+s)(1+2s)} \qquad Y(s) = \dfrac{A}{s} + \dfrac{B}{1+s} + \dfrac{C}{1+2s}$

$A = 5 \qquad B = 5 \qquad C = -20$

$Y(s) = \dfrac{5}{s} + \dfrac{5}{1+s} - \dfrac{20}{1+2s} \quad \circ\!\!-\!\!\circ \quad y(t) = \left(5 + 5e^{-t} - 10e^{-\frac{t}{2}}\right)s(t)$

b) $G_R(s) = k_R\left(1 + \frac{1}{T_I s}\right)$

$\dot{y}(t) = \left(-5e^{-t} + 5e^{-\frac{t}{2}}\right)s(t) + \underbrace{\left(5 + 5e^{-t} - 10e^{-\frac{t}{2}}\right)\delta(t)}_{\phi} \quad t=0$

$\ddot{y}(t) = 5e^{-t} - \frac{5}{2}e^{-t/2} = \phi$

$5e^{-t} = \frac{5}{2}e^{-t/2}$

$2 = e^{\frac{t}{2}} \qquad t_i = 1.386 \qquad y(t_i) = \dfrac{5}{4} \qquad \dot{y}(t_i) = 1.2499 \qquad \rightarrow$

7. nastavak

$$t_i = 1.386 \qquad y(t_i) = \frac{5}{4} \qquad \dot{y}(t_i) = k = 1.2499$$

$$y - \frac{5}{4} = 1.2499(t - 1.386) \qquad \text{za } y = 0 \quad t = t_z$$

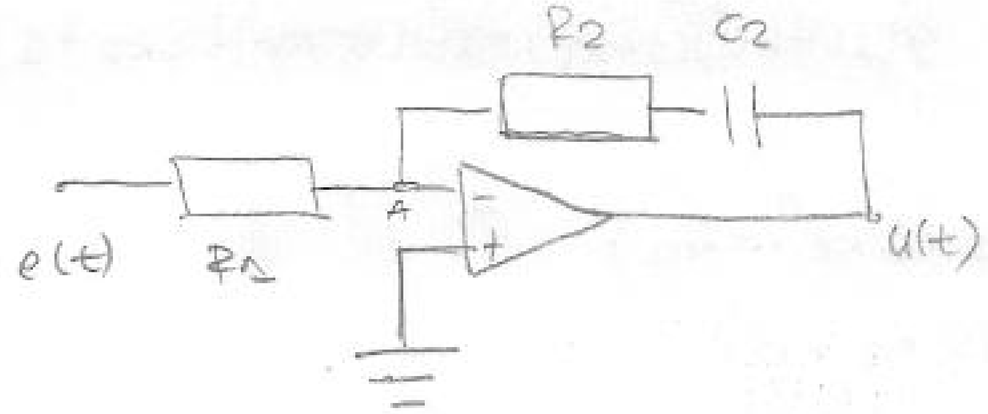
$$t_z = 0.3859$$

$$y(t=\infty) = 5 = k_s \qquad 5 - \frac{5}{4} = 1.2499(t - 1.386) \qquad t = 4.3865$$

$$t_a = t - t_z = 4s$$

$$K_R = 0.9\,\frac{t_a}{t_z \cdot k_s} = 1.865 \qquad T_I = 3.33\,t_z = 1.286s$$

c)



$$\overset{0}{\cancel{u_A\left(\frac{1}{R_1} \quad \frac{1}{R_2 + \frac{1}{sC}}\right)}} = e\frac{1}{R_1} + u\frac{1}{R_2 + \frac{1}{sC}}$$

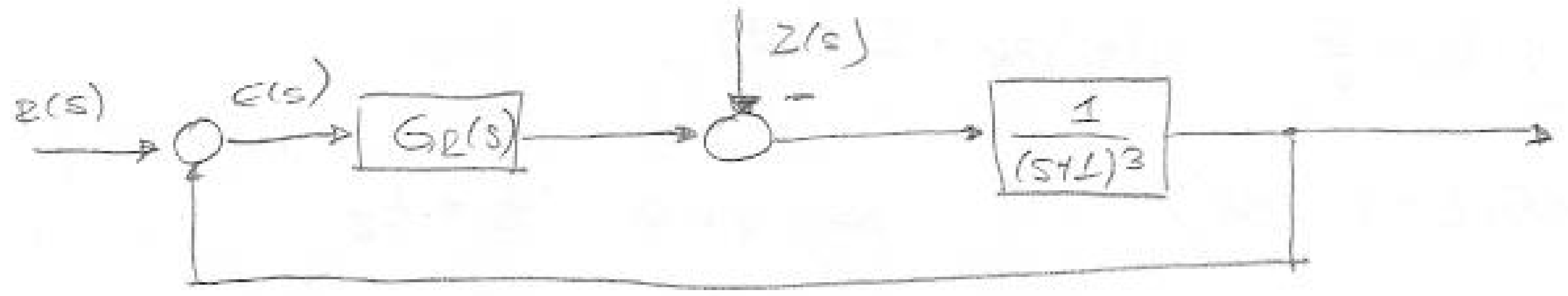
$$-e\frac{1}{R_1} = u\,\frac{1}{\frac{sCR_2 + 1}{sC}}$$

$$\frac{u}{e} = -\frac{\frac{sCR_2+1}{sC}}{\frac{R_1}{1}} = -\frac{sCR_2+1}{sCR_1} = -\frac{R_2}{R_1} + \frac{1}{sCR_1} = -\frac{R_2}{R_1}\left(1 + \frac{\cancel{R_1}}{R_2}\,\frac{1}{sC\cancel{R_1}}\right)$$

$$= -\frac{R_2}{R_1}\left(1 + \frac{1}{sCR_2}\right) = K_R\left(1 + \frac{1}{T_i s}\right)$$

$$K_R = \frac{R_2}{R_1} \Rightarrow R_2 = 500k\Omega \qquad T_i = CR_2 \qquad C = \frac{T_i}{R_2} = \frac{2}{500\,000} = 4\mu F$$

9. $G_R(s) = PI$ regulator



$$\frac{\partial I_{3,4}}{\partial k_R} = \frac{(8-k_R)^2(2k_R+1)T_i + 9k_R(k_R-16)}{\left(16T_ik_R - 2k_R^3T_i + 2k_R^2(7T_i-9)\right)^2 = A} = \emptyset \qquad T_i = \frac{9k_R(16-k_R)}{(8-k_R)^2(2k_R+1)}$$

$$\frac{\partial I_{3,4}}{\partial T_i} = \frac{(8-k_R)(k_R+1)T_i - 18k_R}{\left(16T_ik_R - 2k_R^3T_i + 2k_R^2(7T_i-9)\right)^2} = \emptyset \qquad T_i = \frac{18k_R}{(8-k_R)(k_R+1)}$$

$$\frac{9\cancel{k_R}(16-k_R)}{(8-k_R)^{\cancel{2}}(2k_R+1)} = \frac{18\cancel{k_R}}{\cancel{(8-k_R)}(k_R+1)} \qquad 9(16-k_R)(1+k_R) = 18(8-k_R)(2k_R+1)$$

$$16+16k_R-k_R-k_R^2 = 2(16k_R+8-2k_R^2-k_R)$$

$$16+15k_R-k_R^2 = 2(-2k_R^2+15k_R+8)$$

$$\cancel{16}+15k_R-k_R^2 = -4k_R^2+30k_R+\cancel{16}$$

$$-k_R^2+4k_R^2+15k_R-30k_R = 0 \qquad k_R(3k_R-15) = \emptyset$$

$$\cancel{k_R \neq \emptyset} \qquad k_R = 5 \qquad T_i = \frac{18k_R}{(8-k_R)(k_R+1)} = 5$$

$$E(s) = R(s) - Y(s) \quad , Y(s) = (E\,G_R(s) + Z(s))\cdot G_P = E\cdot G_R\cdot G_P + Z\cdot G_P$$
$$= E\cdot G_0 + Z\cdot G_P$$

$$E(s) = R(s) - E\cdot G_0 - Z\cdot G_P$$

$$E(1+G_0) = R(s) - Z\cdot G_P$$

$$E(s) = \frac{R(s) - Z\cdot G_P(s)}{1+G_0}$$

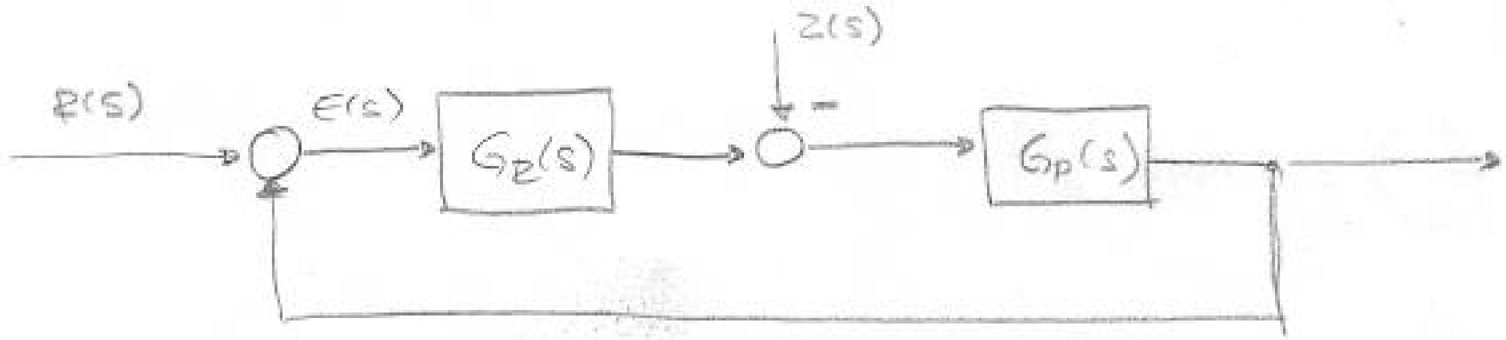
b) $Z(s) = 0$

$$E(s) = \frac{R(s)}{1+G_0} = \frac{\frac{1}{s}}{1+5\left(1+\frac{1}{5s}\right)\cdot\frac{1}{(s+1)^3}}$$

$$E(s) = \frac{\frac{1}{s}}{1+\frac{5s+1}{\cancel{5}s}\cdot\cancel{5}\cdot\frac{1}{(s+1)^3}} = \frac{\frac{1}{s}}{1+\frac{5s+1}{s}\cdot\frac{1}{(s+1)^3}} = \frac{\frac{1}{\cancel{s}}}{\frac{s(s+1)^3+5s+1}{\cancel{s}(s+1)^3}} = \frac{(s+1)^3}{s(s+1)^3+5s+1}$$

$$I_K = \int_0^\infty e(t)\,dt = \lim_{s\to 0} s\cdot\frac{1}{s}\cdot E(s) = 1$$

10.



G_R = PI regulator $\qquad G_P$ - proces II reda

$20 \log K = 25 \quad K = 17.78...$

a) $G_P = 17.78 \cdot \dfrac{1}{1+s} \cdot \dfrac{1}{1+\frac{1}{5}s} = \dfrac{17.78}{(1+s)(1+0.2s)}$

b) $\zeta = \dfrac{1}{\sqrt{2}}$ U TEXTU PIŠE: vremenskom konstantom regulatora kompenzira se veća vremenska konstanta procesa

$T_{I1} = 1 > T_{I2} = 0.2$

$G_R = K_R\left(1 + \dfrac{1}{T_I s}\right) = K_R \dfrac{s+1}{s} \qquad G_0 = K_R \cdot K_P \dfrac{\cancel{s+1}}{s} \cdot \dfrac{1}{\cancel{(1+s)}(1+0.2s)}$

$G_0(s) = K_R \cdot K_P \dfrac{1}{s(1+0.2s)} \qquad G_Z = \dfrac{G_0}{1+G_0} = \dfrac{\dfrac{K_R K_P}{\cancel{s(1+0.2s)}}}{\dfrac{s(1+0.2s)+K_R K_P}{\cancel{s(1+0.2s)}}}$

$G_Z(s) = \dfrac{17.78 K_P}{0.2s^2 + s + 17.78 K_P} = \dfrac{1}{\dfrac{0.2}{K_R K_P}s^2 + \dfrac{1}{K_R K_P} \cdot s + 1}$

$\dfrac{0.2}{K_R K_P} = \dfrac{1}{\omega_n^2} \quad \omega_n = \sqrt{5 K_R K_P} \qquad \dfrac{1}{K_R K_P} = \dfrac{2\zeta}{\omega_n} \qquad \omega_n = 2\zeta K_R K_P$

$2\zeta K_R \cdot K_P = \sqrt{5 K_R \cdot K_P}\ \Big/^2 \qquad 4\zeta^2 K_R^{\cancel{2}} K_P^{\cancel{2}} = 5 \cdot \cancel{K_R K_P} \qquad 4\zeta K_R K_P = 5$

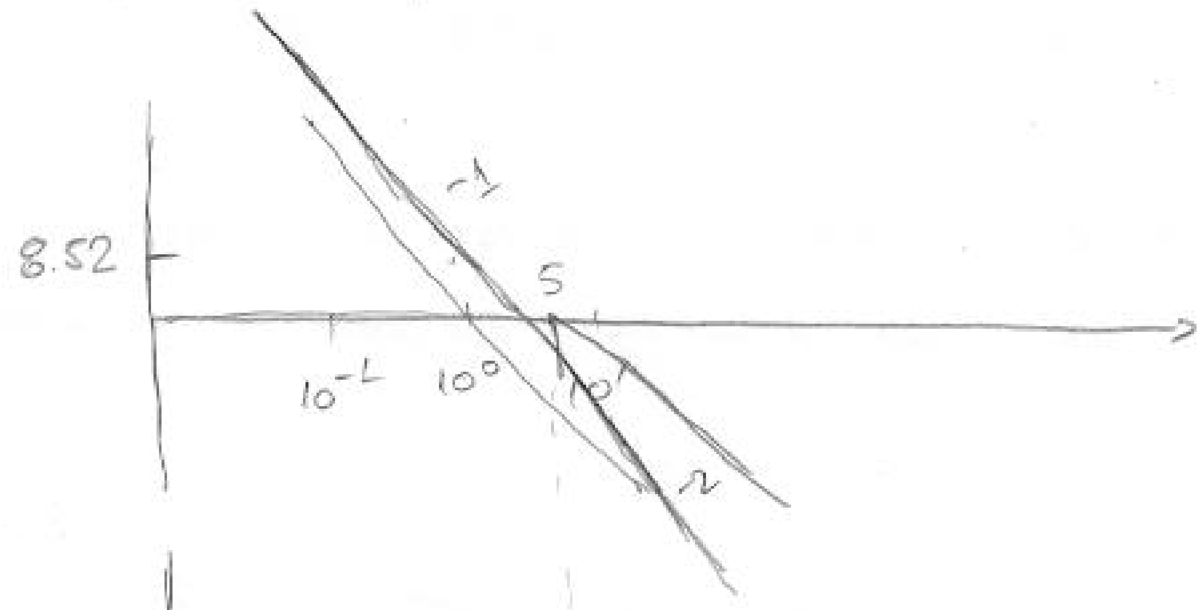
$K_R = \dfrac{5}{4\zeta^2 K_P} = 0.140585$

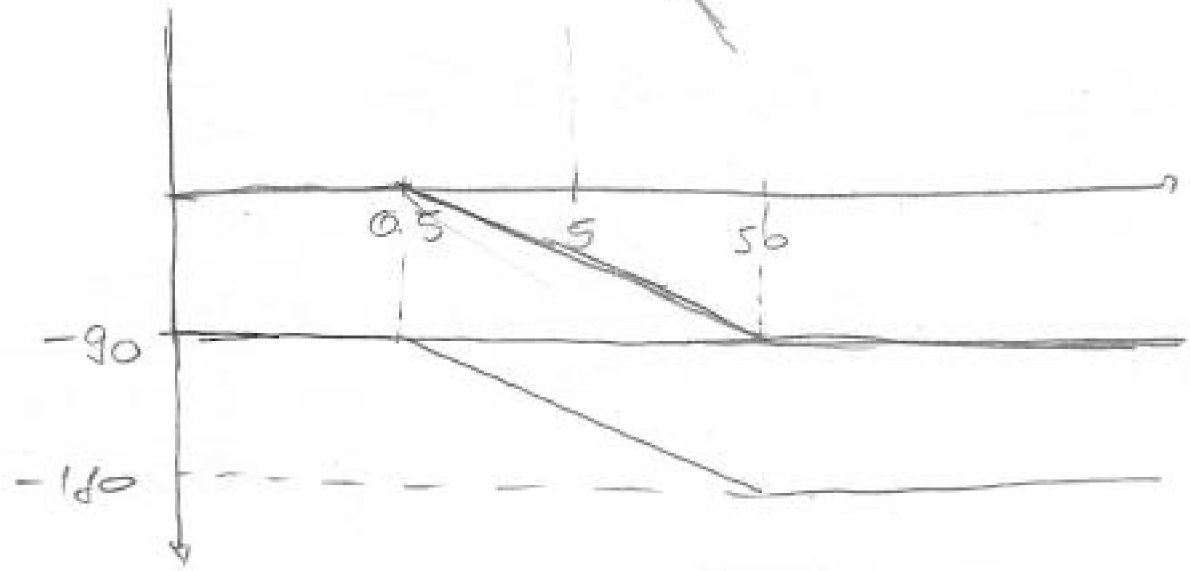
c) $k_R = 0.1405$

$$G_R = k_R\left(1+\frac{1}{T_i s}\right) = 0.1405\,\frac{s+1}{s}$$

$$G_0 = 0.15\cdot\frac{\cancel{s+1}}{s}\cdot 17.78\,\frac{1}{\cancel{(1+s)}(1+0.2s)}$$

$$= \frac{2.5}{s(1+0.2s)} = 2.5\,\frac{1}{s}\cdot\frac{1}{1+0.2s}$$





d) $k_R = 0.1405$ $\qquad G_R = 0.1405\,\frac{s+1}{s}$

$$s = \frac{2}{T}\,\frac{z-1}{z+1}$$

$$G_0 = \frac{2.5}{s(1+0.2s)} \qquad |G_0(j\omega_c)| = 1 = \frac{2.5}{\omega_c\sqrt{1+0.04\omega_c^2}}$$

$$\omega_c\sqrt{1+0.04\omega_c^2} = 2.5 \quad |^2 \qquad \omega_c^2 = t$$

$$\omega_c^2(1+0.04\omega_c^2) = 6.25 \qquad 0.04t^2+t-6.25=0$$

$$t = 5.1776 \qquad \omega_c = \sqrt{t} = 2.275449\ \text{rad/s}$$

$$T = (0.17 \div 0.34)\frac{1}{\omega_c} \qquad T = \frac{0.17+0.34}{2\cdot\omega_c} = 112\,\text{ms}$$

d) nastavak

$$G_R = 0.1405\left(1+\frac{1}{s}\right) = 0.1405\left(1+\frac{1}{\frac{2}{0.112}\cdot\frac{z-1}{z+1}}\right) = 0.1405\left(1+0.056\,\frac{z+1}{z-1}\right)$$

$$= 0.1405\left(\frac{z-1+0.056z+0.056}{z-1}\right) = 0.1405\,\frac{1.056z-0.944}{z-1}$$

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11. $$G_o(s) = \frac{0.4}{(1+125s)(1+50s)(1+25s)}$$

$$G_R = k_R\left(1+\frac{1}{T_i s}\right)$$, kompenzira dominantnu vre. kon

$$= k_R\left(\frac{T_i s+1}{T_i s}\right) \qquad T_i = 125\ \text{ms}$$

$$G_R(s) = k_R\,\frac{125s+1}{125s}$$

$$G_{oR} = k_R\,\frac{\cancel{125s+1}}{125s}\cdot\frac{0.4}{(\cancel{1+125s})(1+50s)(1+25s)} = \frac{0.0032\,k_R}{s(1+50s)(1+25s)}$$

$\sigma_m = 10\%$ $\qquad \gamma = 70 - \sigma_m = 60\%$

Bode $\quad \omega_c = 0.0061\ s^{-1}$, $\Rightarrow$ $A = 5.6$ dB $\quad$?

$20\log k = 5.6 \Rightarrow k = 1.905$

Pažljivo pogledat Bodea:

$T(\log 10^{-2}, -9.89)$, $k = -20$ dB/dek

$y + 9.89 = -20(x - \log 10^{-2})$ za $y=0$ $\quad x = -2.4945$ $\qquad \omega = 10^x = 0.00320\ \text{rad/s}$

ovo potvrdi i MATLAB