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3DZ iz AUTOMATSKOG UPRAVLJANJA

Zadatak-1

$$G(s) = 27\,\frac{s}{(s+3)(s+9)} = \frac{27}{3\cdot 9}\,\frac{s}{\left(1+\frac{s}{3}\right)\left(1+\frac{s}{9}\right)}$$

$$= \underbrace{\frac{j\omega}{1}}_{G_1} \cdot \underbrace{\frac{1}{1+j\frac{\omega}{3}}}_{G_2} \cdot \underbrace{\frac{1}{1+j\frac{\omega}{9}}}_{G_3}$$

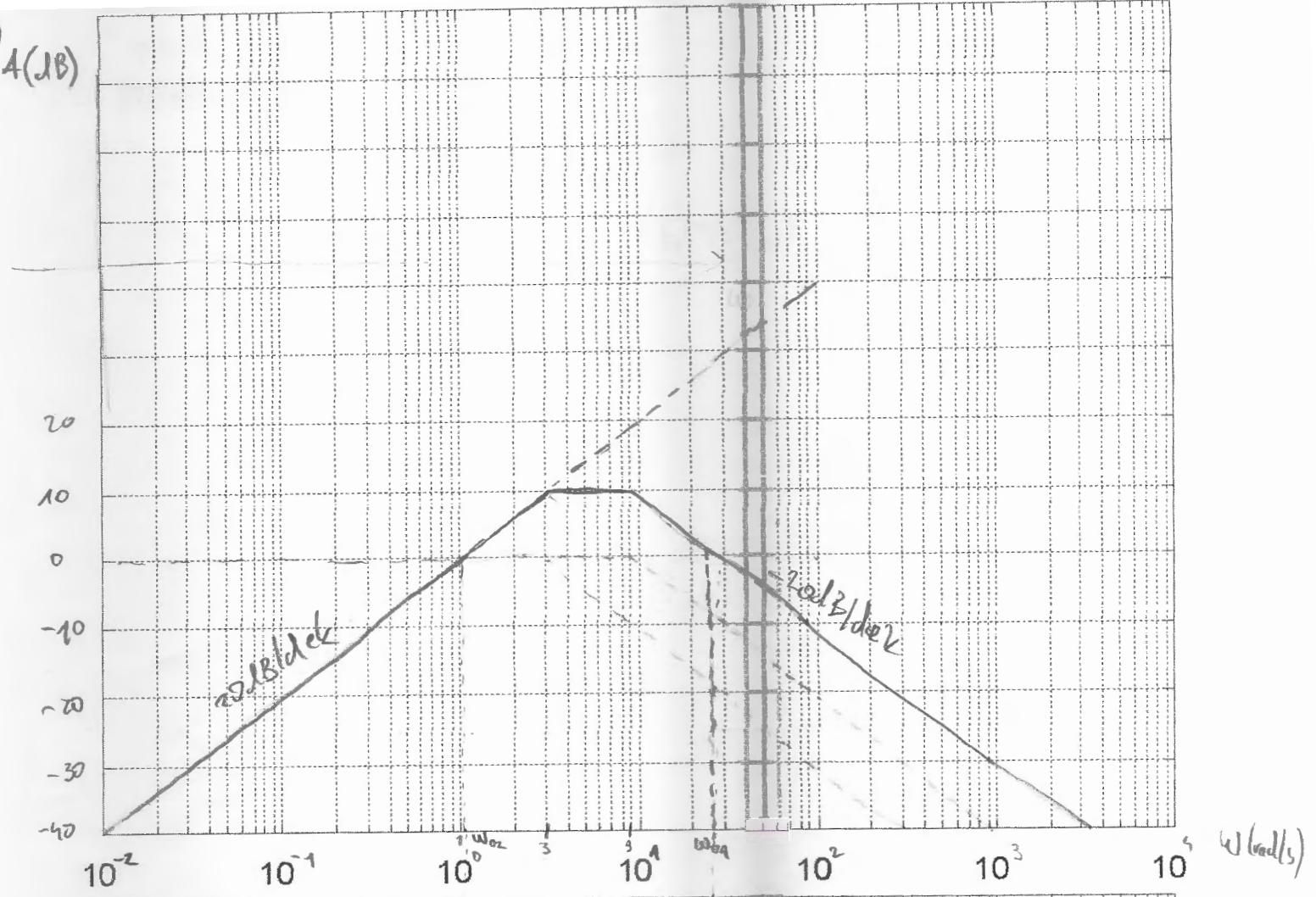
$\omega_{z1} = 1\ rad/s$

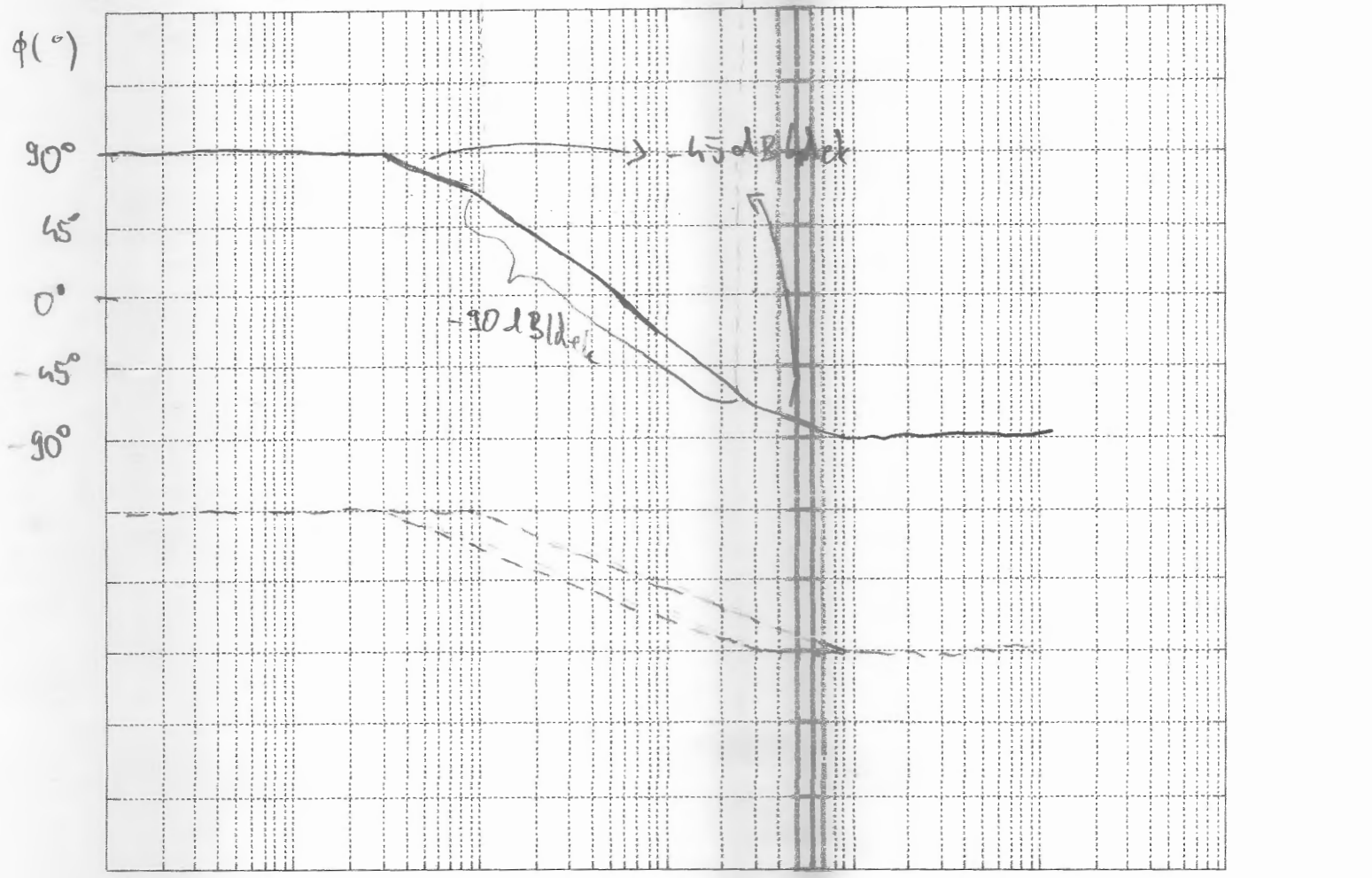
$A_0 = 1 \Rightarrow A_{0dB} = 20\log(1) = 0\ dB$

$\omega_{p1} = 3\ rad/s$

$\omega_{p2} = 9\ rad/s$

a)


A(dB)
20
10
0
-10
-20
-30
-40
+20dB/dec
-20dB/dec
10^{-2}
10^{-1}
10^{0}
10^{1}
10^{2}
10^{3}
10^{4}
ω_{0z}
3
9
ω_{0g}
ω (rad/s)


ϕ(°)
90°
45°
0°
-45°
-90°
-45dB/dec
-90dB/dec

$$G(j\omega) = \frac{27j\omega}{(j\omega+3)(j\omega+9)} = \frac{27j\omega}{-\omega^2+12j\omega+27} = \frac{27j\omega}{(27-\omega^2)-j12\omega} \cdot \frac{(27-\omega^2)-j12\omega}{(27-\omega^2)-j12\omega} =$$

$$= \frac{27\cdot 12\omega^2}{(27-\omega^2)^2+144\omega^2} + j\,\frac{27\omega(27-\omega^2)}{(27-\omega^2)^2+144\omega^2} = \underbrace{\frac{324\omega^2}{\omega^4+90\omega^2+729}}_{Re(\omega)} + j\,\underbrace{\frac{27\omega(27-\omega^2)}{\omega^4+90\omega^2+729}}_{Im(\omega)}$$

$Re(\omega=0^+)=0$

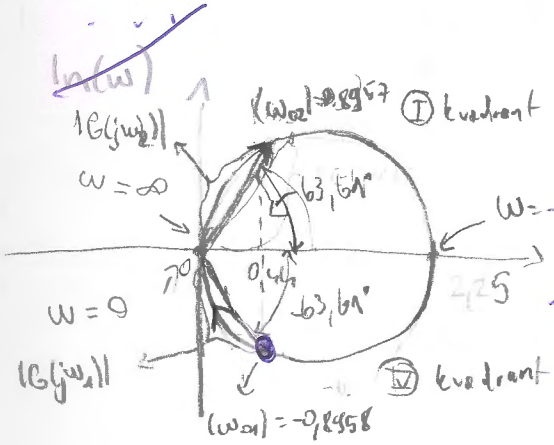
$Im(\omega=0^+)=0$

$Re(\omega=\infty)=0$

$Im(\omega=\infty)=0$

$Im=0 \Rightarrow 27-\omega^2=0 \Rightarrow \omega^2=27 \Rightarrow Re(\omega^2=27)=2{,}25$

5)



Nyquistov dijagram

$Re(G(j\omega))$ $G(j\omega_{01})$

dijagram prolazi kroz I i IV kvadrant

c) $u(t) = 3\sin(\omega_0 t)$

$y(t) = 3\sin(\omega_0 t + \phi)$

$\omega_0, \phi = ?$

$U_m \cdot G(j\omega) = Y_m e^{j\phi}$

$G(j\omega) = \frac{Y_m e^{j\phi}}{U_m} = \frac{3e^{j\phi}}{3} = e^{j\phi}$

$|G(j\omega)| = |e^{j\phi}| = 1$

$27 \frac{|j\omega_0|}{|j\omega_0+3|\,|j\omega_0+9|} = 1$

$27 \frac{\omega_0}{\sqrt{9+\omega_0^2} \cdot \sqrt{81+\omega_0^2}} = 1$

$27\omega_0 = \sqrt{729 + 90\omega_0^2 + \omega_0^4} \quad |\,.^2$

$729\omega_0^2 = 729 + 90\omega_0^2 + \omega_0^4$

$\omega_0^4 - 639\omega_0^2 + 729 = 0$

$\omega_0^2 = t$

$t^2 - 639t + 729 = 0$

$t_{1,2} = \frac{639 \pm \sqrt{639^2 - 4 \cdot 729}}{2}$

$t_1 = 637{,}857\ s^{-2}$

$t_2 = 1{,}142889\ s^{-2}$

$\omega_{01} = 25{,}256\ s^{-1}$

Zadatak-2

$$G(s) = 27\,\frac{as+1}{(s+3)(s+9)}$$

a) $h(t) = ?$

$g(t) = ?$

$$H(s) = \frac{1}{s}G(s) = \frac{27(as+1)}{s(s+3)(s+9)}$$

$$= \frac{A}{s} + \frac{B}{s+3} + \frac{C}{s+9}$$

$$27(as+1) = A(s+3)(s+9) + Bs(s+9) + Cs(s+3)$$

Za $s=0$:

$27 = 27A$

$A = 1$

Za $s=-3$

$27(-3a+1) = -18B$

$B = \frac{9a-3}{2}$

Za $s=-9$:

$27(-9a+1) = 54C$

$C = \frac{1-9a}{2}$

$$H(s) = \frac{1}{s} + \frac{9a-3}{2}\cdot\frac{1}{s+3} + \frac{1-9a}{2}\cdot\frac{1}{s+9}$$

$$h(t) = \left(1 + \frac{9a-3}{2}e^{-3t} + \frac{1-9a}{2}\cdot e^{-9t}\right)\mu(t)$$

$$g(t) = \frac{dh(t)}{dt}$$

$$= \left(\frac{9-27a}{2}e^{-3t} + \frac{81a-9}{2}e^{-9t}\right)\mu(t)$$

b) $a_1 = \frac{1}{3}$:

$$h(t) = (1 - e^{-9t})\mu(t)$$

ne vidi se e^{-3t}

$a_2 = \frac{1}{9}$:

$$h(t) = (1 - e^{-3t})\mu(t)$$

ne vidi se e^{-9t}

c)

$$\lim_{s\to\infty} sH(s) = \frac{27(as+1)}{(s+3)(s+9)} = \lim_{s\to 0} \frac{27as+27}{(s+3)(s+9)} = \frac{27}{27} = 1$$

- nula ne utječe jer će za svaki iznos a rezultat biti 1

d) $$\lim_{t\to 0^+} g(t) = \lim_{s\to\infty} sG(s) = \lim_{s\to\infty} \frac{27s(as+1)}{(s+3)(s+9)}$$

$$= \lim_{s\to\infty} \frac{27as^2 + 27as}{s^2 + 12s + 27} = 27a$$

- nula ovisi o a

$-\infty < a < -1 \Rightarrow g(0^+)$ je velikog negativnog iznosa, nula je na (+) realnoj osi blizu ishodišta

$-1 < a < 0 \Rightarrow g(0^+)$ je malog negativnog iznosa, nula je na (+) realnoj osi daleko od ishodišta

$0 < a < 1 \Rightarrow g(0^+)$ je malog pozitivnog iznosa, nula je na (-) realnoj osi daleko od ishodišta

$1 < a < +\infty \Rightarrow g(0^+)$ je velikog pozitivnog iznosa, nula je na (-) realnoj osi blizu ishodišta

<u>uvjet za podbačaj</u>

$h'(0^+) < 0$ $\quad 27a < 0 \Rightarrow$ za $a < 0$ ima podbačaja

$h'(t) = g(t)$

d) $A_1(\omega_{o1} = 25{,}256) = 2\,dB$ $\quad \varphi_1(\omega_0) = -60°$

$A_2(\omega_{o2} = 1{,}069) = 0\,dB$ $\quad \varphi(\omega_0) = 60°$

e) za $\omega_{o1} = 25{,}256\,s^{-1}$:

$G(j\omega_{o1}) = 0{,}444 - j\,0{,}8958$

za $\omega_{o2} = 1{,}069$:

$G(j\omega_{o2}) = 0{,}444 + j\,0{,}8957$

$|G(j\omega_{o1})| = |G(j\omega_{o2}| = \sqrt{0{,}444^2 + 0{,}895^2} = 0{,}9991$

$\arg[G(j\omega_{o1})] = \operatorname{arctg}\left(\frac{-0{,}895}{0{,}444}\right) = -63{,}61°$

$\arg[G(j\omega_{o2})] = \operatorname{arctg}\left(\frac{0{,}895}{0{,}444}\right) = 63{,}61°$

e) $h(t) = \frac{1}{s} + \frac{9a-3}{2} e^{-3t} + \frac{1-9a}{2} e^{-9t}$

$\lim_{t \to \infty} h(t) = 1$

$h(t) - h(\infty) = 0$

ako postoje nultočke, onda postoji i nadvišenje

$\frac{9a-3}{2} e^{-3t} + \frac{1-9a}{2} e^{-9t} = 0$

$e^{-3t} \cdot \frac{9a-3}{2} = e^{-9t} \cdot \frac{9a-1}{2}$

$e^{6t} = \frac{9a-1}{9a-3}$

$t = \frac{1}{6} \ln \frac{9a-1}{9a-3}$

$\ln \frac{9a-1}{9a-3} > 0$ → da i nadvišenje postoji

$\frac{9a-1}{9a-3} > 1 \Rightarrow \frac{9a-1}{9a-3} - 1 > 0 \Rightarrow \frac{9a-1-9a+3}{9a-3} > 0$

$\frac{2}{9a-3} > 0 \Rightarrow 9a-3 > 0$

$9a > 3$

$a > \frac{1}{3}$, $a \in \langle \frac{1}{3}, +\infty \rangle$

1.) $27\left(\frac{1}{3}s+1\right) = 0 \Rightarrow s = -3$

2.) $27(\infty \cdot s + 1) = \infty (s \neq 0) \Rightarrow s = 0$

nule u rasponu $\langle -3, 0 \rangle$