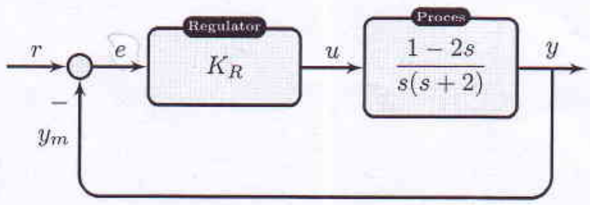
4. domaća zadaća

Stabilnost i točnost linearnih kontinuiranih sustava upravljanja

Priprema za vježbu

Zadatak 1

Na Slici 1 prikazan je zatvoreni regulacijski krug. Potrebno je:



Slika 1: Zatvoreni regulacijski krug.

a) Odrediti Hurwitzovim kriterijem stabilnosti interval vrijednosti parametra K_R za koje je zatvoreni sustav stabilan;

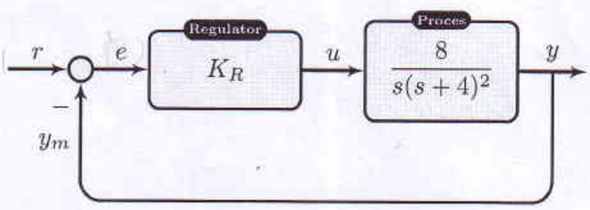
b) Odrediti polove zatvorenog kruga uz $K_R = 0.5$. Pronaći iznos maksimalnog propada y_p, vrijeme maksimalnog propada t_p, iznos maksimalnog nadvišenja σ_m, vrijeme prvog maksimuma t_m i vrijeme ustaljivanja $t_{1\%}$;

c) Odrediti regulacijsko odstupanje u ustaljenom stanju e_∞ za pobudu oblika $R(s) = \frac{5}{s}$ i uz $K_R = 0.5$;

d) Odrediti regulacijsko odstupanje u ustaljenom stanju e_∞ za pobudu oblika $R(s) = \frac{2}{s^2}$ i uz $K_R = 0.5$.

Zadatak 2

Na Slici 2 prikazan je zatvoreni regulacijski krug. Potrebno je



Slika 2: Zatvoreni regulacijski krug.

a) Odrediti pojačanje K_R tako da vrijeme prvog maksimuma prijelazne funkcije zatvorenog kruga bude $t_m \approx 3.75$ s. Koristite se pritom približnim relacijama između pokazatelja kvalitete sustava upravljanja u vremenskom i frekvencijskom području.

b) Nacrtati Bodeov i Nyquistov dijagram otvorenog regulacijskog kruga uz K_R određen pod a) te na temelju tih dijagrama odrediti je li zatvoreni regulacijski krug stabilan.

c) Analitički odrediti iznos amplitudnog i faznog osiguranja sustava, A_r i γ, te temeljem njih procijeniti iznos nadvišenja σ_m prijelazne funkcije zatvorenog regulacijskog kruga.

d) Procijeniti mjesta dominantnog para polova zatvorenog kruga $s_{p1,2}$, tj. odredite pripadne veličine ζ i ω_n na temelju σ_m i t_m.

e) Odrediti kritični iznos pojačanja K_R regulatora pri kojem je zatvoreni regulacijski krug na rubu stabilnosti te frekvenciju trajnih oscilacija sustava na rubu stabilnosti.

Napomena: Bodeov i Nyquistov dijagram nije dovoljno precrtati iz Matlaba. Kod crtanja Bodeovog dijagrama koristite aproksimacije pravcima.

4. DOMAĆA ZADAĆA

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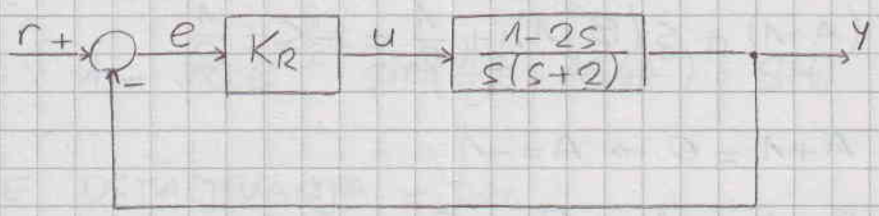
• ZADATAK 1.



(a) HURWITZOV KRITERIJ STABILNOSTI

$$G_0(s) = \frac{K_R(1-2s)}{s(s+2)}$$

$$\mathcal{L}_{CE} = K_R(1-2s) + s(s+2)$$

$$\mathcal{L}_{CE} = s^2 + s(2-2K_R) + K_R$$

(1) $K_R > 0, \quad 1 > 0, \quad 2-2K_R > 0$

$$K < 1$$

(2) $D_1 = 2 - 2K_R > 0 \rightarrow K_R < 1$

$$K_R \in \langle 0, 1 \rangle$$

(b) $K_R = 0.5$

$$G(s) = \frac{G_0(s)}{1+G_0(s)}$$

$$G(s) = \frac{-s+\frac{1}{2}}{s^2+s+\frac{1}{2}} \rightarrow H(s) = \frac{-s+\frac{1}{2}}{s(s^2+s+\frac{1}{2})}$$

$$H(s) = \frac{C_{11}}{s} + \frac{As+B}{s^2+s+\frac{1}{2}}$$

$$C_{11} = H(s)\cdot s \Big|_{s=0} = 1$$

$$H(s) = \frac{1}{s} + \frac{As+B}{s^2+s+\frac{1}{2}} = \frac{s^2(A+1) + s(B+1) + \frac{1}{2}}{s(s^2+s+\frac{1}{2})}$$

$$s^2(A+1) + s(B+1) + \frac{1}{2} = -s + \frac{1}{2}$$

$$A+1 = 0 \rightarrow A = -1$$

$$B+1 = -1 \rightarrow B = -2$$

$$H(s) = \frac{1}{s} - \frac{s+2}{(s+\frac{1}{2})^2 + \frac{1}{4}}$$

$$\frac{1}{s} \multimap S(t)$$

$$\frac{s+2}{(s+\frac{1}{2})^2+\frac{1}{4}} = \frac{s+\frac{1}{2}}{(s+\frac{1}{2})^2+\frac{1}{4}} + \frac{3}{2} \cdot 2 \frac{\frac{1}{2}}{(s+\frac{1}{2})^2+\frac{1}{4}}$$

$$\multimap e^{-\frac{1}{2}t}\left(\cos(\tfrac{1}{2}t) + 3\sin(\tfrac{1}{2}t)\right)$$

$$h(t) = \left\{1 - e^{-\frac{1}{2}t}\left(\cos(\tfrac{1}{2}t) + 3\sin(\tfrac{1}{2}t)\right)\right\} S(t)$$

$$\cos(\tfrac{1}{2}t) = \sin(\tfrac{1}{2}t + \tfrac{\pi}{2}) \rightarrow 1 \measuredangle \tfrac{\pi}{2}$$

$$a+bj := 1 \measuredangle \pi/2 \qquad \left.\begin{array}{l} a^2+b^2 = 1^2 \\ \frac{b}{a} = tg\frac{\pi}{2} \end{array}\right\} \begin{array}{l} a=0 \\ b=1 \end{array}$$

$$\cos(\tfrac{1}{2}t) \rightarrow 0+1j$$

$$3\sin(\tfrac{1}{2}t) \rightarrow 3 \measuredangle 0 \qquad \left.\begin{array}{l} a^2+b^2 = 3^2 \\ \frac{b}{a} = tg\, 0° \end{array}\right\} \begin{array}{l} a=3 \\ b=0 \end{array}$$

$$3\sin(\tfrac{1}{2}t) \rightarrow 3+0j$$

$$\cos\left(\tfrac{1}{2}t\right) + 3\sin\left(\tfrac{1}{2}t\right) \rightarrow 0 + 1j + 3 + 0j = 3 + j$$

$$3 + j = \sqrt{10} \measuredangle 18.44^\circ \rightarrow \sqrt{10}\sin\left(\tfrac{1}{2}t + 18.44^\circ\right)$$

$$\cdot\ h(t) = \left\{ 1 - \sqrt{10}\, e^{-\frac{1}{2}t} \sin\left(\tfrac{1}{2}t + 18.44^\circ\right) \right\} S(t)$$

(1) VRIJEME USTALJIVANJA – $t_{1\%}$

$$h(t) = 0{,}99\, h(\infty) \quad , \quad h(\infty) = 1$$

$$h(t) = 1 - \sqrt{10}\, e^{-\frac{1}{2}t} \sin\left(\tfrac{1}{2}t + 18.44^\circ\right) = 0{,}99$$

$$\sqrt{10}\, e^{-\frac{1}{2}t} = 0{,}01 \rightarrow t_{1\%} = 11.513\ s$$

(2) EKSTREMI

$\dot{h}(t) = g(t) = 0$ – NULTOČKE SU VREMENA EKSTREMA

$$g(t) = e^{-\frac{1}{2}t}\left(2\sin\left(\tfrac{1}{2}t\right) - \cos\left(\tfrac{1}{2}t\right)\right)$$

$$g(t) = 0 \rightarrow 2\sin\left(\tfrac{1}{2}t\right) - \cos\left(\tfrac{1}{2}t\right) = 0$$

RADIJANI

$$tg\left(\tfrac{1}{2}t + k\pi\right) = \tfrac{1}{2} \quad , \quad k \in \mathbb{Z}$$

$$\tfrac{1}{2}t + k\pi = \operatorname{arctg}\tfrac{1}{2} \rightarrow t_m = 0.9273 + 2k\pi$$

$t_{m1} = 0{,}9273\ s \quad , \quad k = 0$

$t_{m2} = 7{,}2105\ s \quad , \quad k = 1$

$t_{m3} = 13{,}4936\ s \quad , \quad k = 2$

$$\dot{g}(t) = \ddot{h}(t) = \tfrac{1}{2} e^{-\frac{1}{2}t}\left(3\cos\left(\tfrac{1}{2}t\right) - \sin\left(\tfrac{1}{2}t\right)\right)$$

$\ddot{h}(t) < 0 \quad , \quad t$ – MAKSIMUM

$\ddot{h}(t) = 0 \quad , \quad t$ – TOČKA PREGIBA

$\ddot{h}(t) > 0 \quad , \quad t$ – MINIMUM

$\ddot{h}(t_{m1}) > 0$, t_{m1} - TOČKA MINIMUMA

$\ddot{h}(t_{m2}) < 0$, t_{m2} - TOČKA MAKSIMUMA

$h(t_{m1}) = -0.406 \rightarrow y_p = -0.406$ - IZNOS PROPADA

$h(t_{m2}) = +1.061 \rightarrow y_m = +1.061$ - IZNOS MAKS. NADVIŠENJA

$$\sigma_m = \frac{y_m - y_{ss}}{y_{ss}} \cdot 100\ [\%] \rightarrow \sigma_m = 6.1\ \%$$

(c) $R(s) = \frac{5}{s}$, $K_R = 0.5$

$$Y(s) = E(s) \cdot G_o(s)$$

$$Y(s) = R(s) \cdot G(s) \quad , \quad G(s) = \frac{G_o(s)}{1 + G_o(s)}$$

$$E(s) = \frac{R(s)}{1 + G_o(s)} \rightarrow E(s) = \frac{10s + 20}{2s^2 + 2s + 1}$$

$$e_\infty = \lim_{s \to 0} sE(s) = 0 \rightarrow e_\infty = 0$$

(d) $R(s) = \frac{2}{s^2}$, $K_R = 0.5$

$$E(s) = \frac{2s + 4}{s\left(s^2 + s + \frac{1}{2}\right)}$$

$$e_\infty = \lim_{s \to 0} sE(s) = 8 \rightarrow e_\infty = 8$$

4. DOMAĆA ZADAĆA

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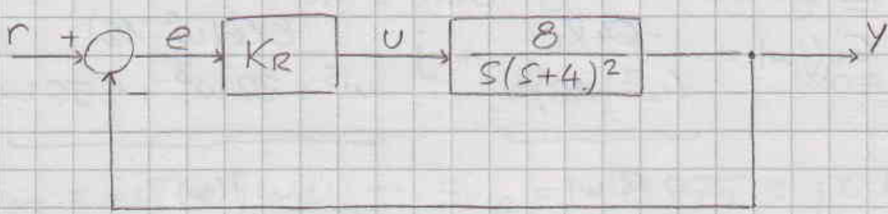
• ZADATAK 2.



(a) $\omega_c = \frac{3}{t_m} \rightarrow \omega_c = 0{,}8\ s^{-1}$

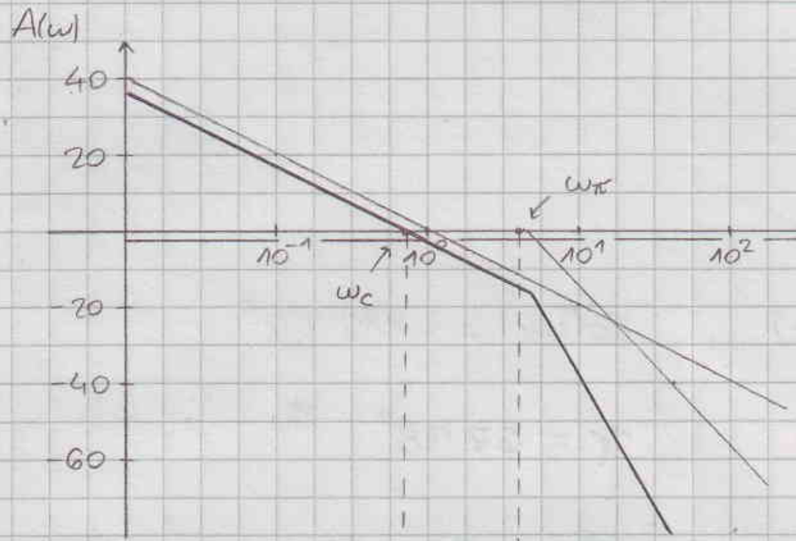
$$|G_o(j\omega_c)| = 1 \qquad G_o(s) = \frac{8K_R}{s(s+4)^2}$$

$$\frac{8K_R}{\omega_c\sqrt{(16-\omega_c^2)^2 + (8\omega_c)^2}} = 1$$

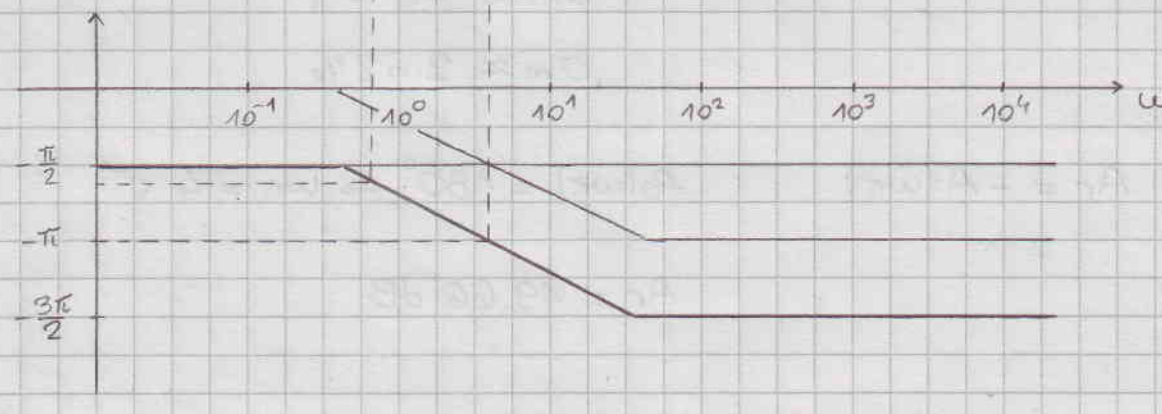
$$K_R = 1{,}664$$

(b) $A(\omega) = 20\log(0{,}832) - 20\log(\omega) - 40\log\sqrt{1+\left(\frac{\omega}{4}\right)^2}$

$\varphi(\omega) = -\frac{\pi}{2} - 2\,\mathrm{arctg}\left(\frac{\omega}{4}\right)$



$\omega_c < \omega_\pi$ – STABILAN SUSTAV



$$G_o(j\omega) = \frac{8K_R}{j\omega(16-\omega^2+8j\omega)}$$

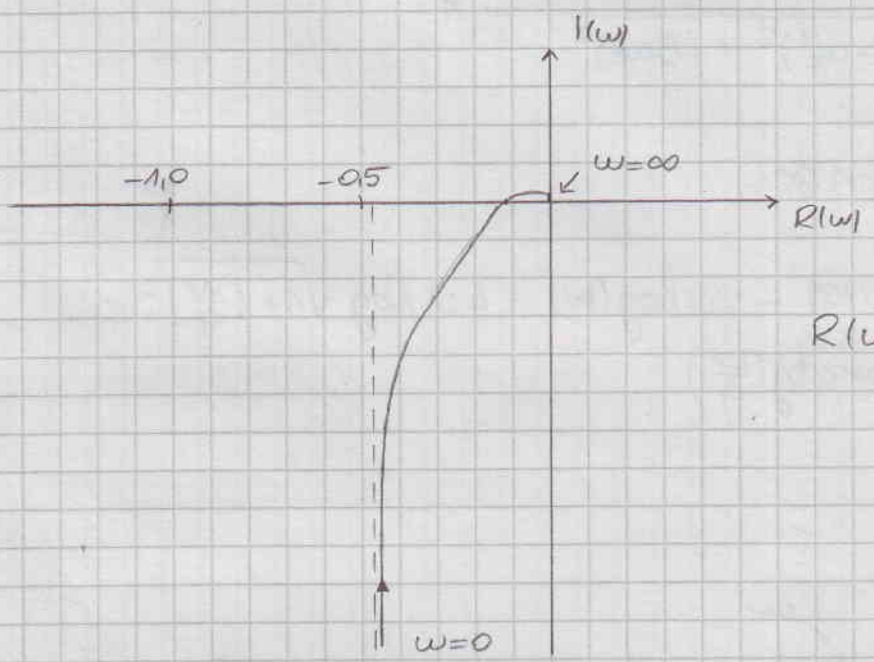
$$G_o(j\omega) = \underbrace{\frac{-64K_R}{(\omega^2+16)^2}}_{R(\omega)} + j\underbrace{\frac{8K_R(\omega^2-16)}{\omega^5+32\omega^3+256\omega}}_{I(\omega)}$$

(1) $R(\omega=0^+) = -0.416$ (2) $R(\omega=+\infty) = 0$

$I(\omega=0^+) = -\infty$ $I(\omega=+\infty) = 0$

(3) $R(\omega_1)=0 \rightarrow \omega_1=\infty$ (4) $I(\omega_2)=0 \rightarrow \omega_2=4$

$R(\omega_2) = -0.10418$



$R(\omega_2=\omega_\pi) > -1$

(c) $\gamma = 180 + \varphi_o(\omega_c)$, $\varphi_o(\omega_c) = -112.62^\circ$

$\gamma = 67.38^\circ$

$\sigma_m \approx 70^\circ - \gamma$

$\sigma_m \approx 2.62\%$

$A_r = -A(\omega_\pi)$ $\varphi_o(\omega_\pi) = 180^\circ \rightarrow \omega_\pi = 4\ s^{-1}$

$A_r = 19.66$ dB

(d) $\sigma_m = 2.62\,\%$ $\qquad t_m = \frac{\pi}{\omega_d} \rightarrow \omega_d = 0.834\ s^{-1}$

$t_m = 3.75s$

$$\sigma_m = 100 \cdot e^{\frac{-\pi \zeta}{\sqrt{1-\zeta^2}}} \rightarrow \zeta = 0.76$$

$$\omega_n = 1.283$$

$$s_{P1,2} = -\sigma \pm j\omega_d \rightarrow s_{P1,2} = -0.975 \pm j\,0.834$$

(e) $\omega_{KR} = \omega_\pi = 4\ s^{-1}$

$$G_P(j\omega) = \frac{8}{j\omega(4+j\omega)^2}$$

$$K_R \cdot |G_P(j\omega)| < 1 \ , \text{ ZA } \ \omega = \omega_\pi$$

$$K_R \cdot \frac{8}{\omega\sqrt{(16-\omega^2)^2+(8\omega)^2}} = 1 \ , \ \omega = \omega_\pi$$

$$K_R = 16$$