Analysis of Massive Data Sets

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Advertising on the Web

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Outline

- **Motivation**
  - Advertising on the Web, Issues
- **Online algorithms**
  - Online Bipartite Matching
  - Greedy algorithm
- **Advertising on the Web**
  - Adwords problem
  - Solutions: Greedy algorithm, Balance algorithm

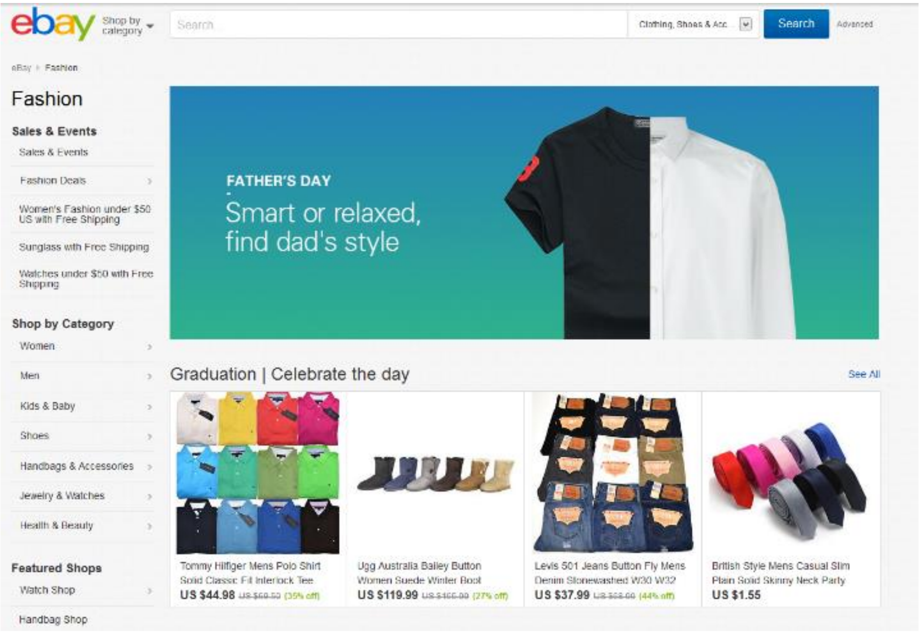
Motivation

- **Web applications support themselves through advertising**
  - One of the big surprises of the 21st century
  - Multi-billion dollar market

- **Other media revenue sources**
  - Radio and television
    - Advertising (primary)
  - Most media (newspapers and magazines)
    - Subscription (primary)
    - Hybrid approach – advertising and subscriptions

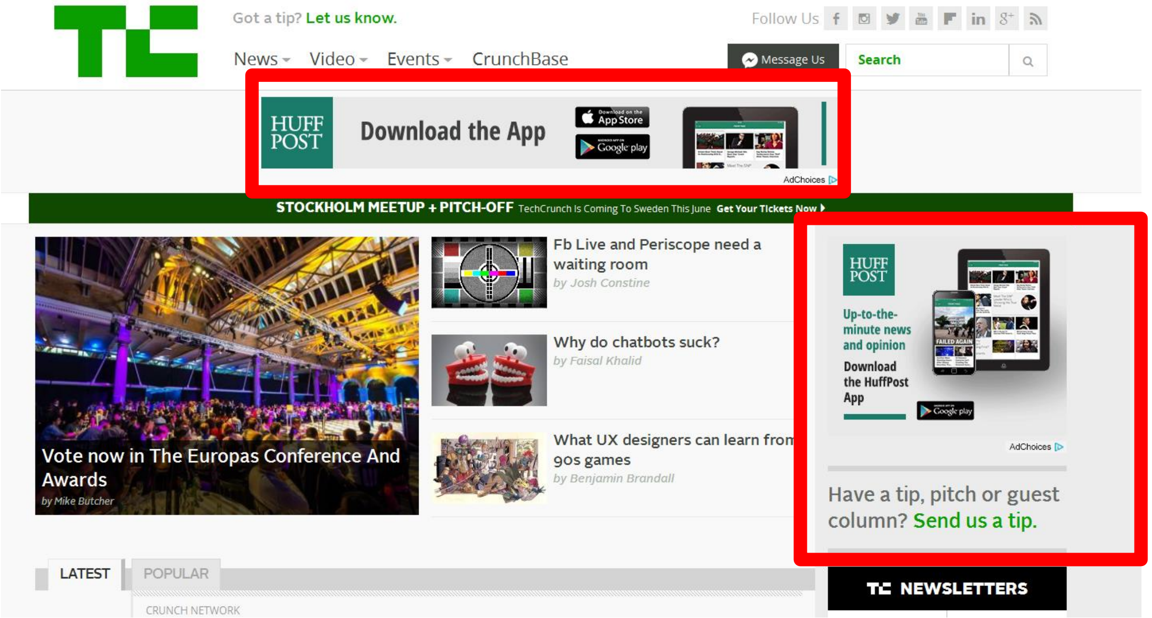
Motivation: Ad Types

- **Direct Placement of Ads**
  - Directly placed by advertisers
    - Free, for a fee or commission
  - eBay, Craig's List



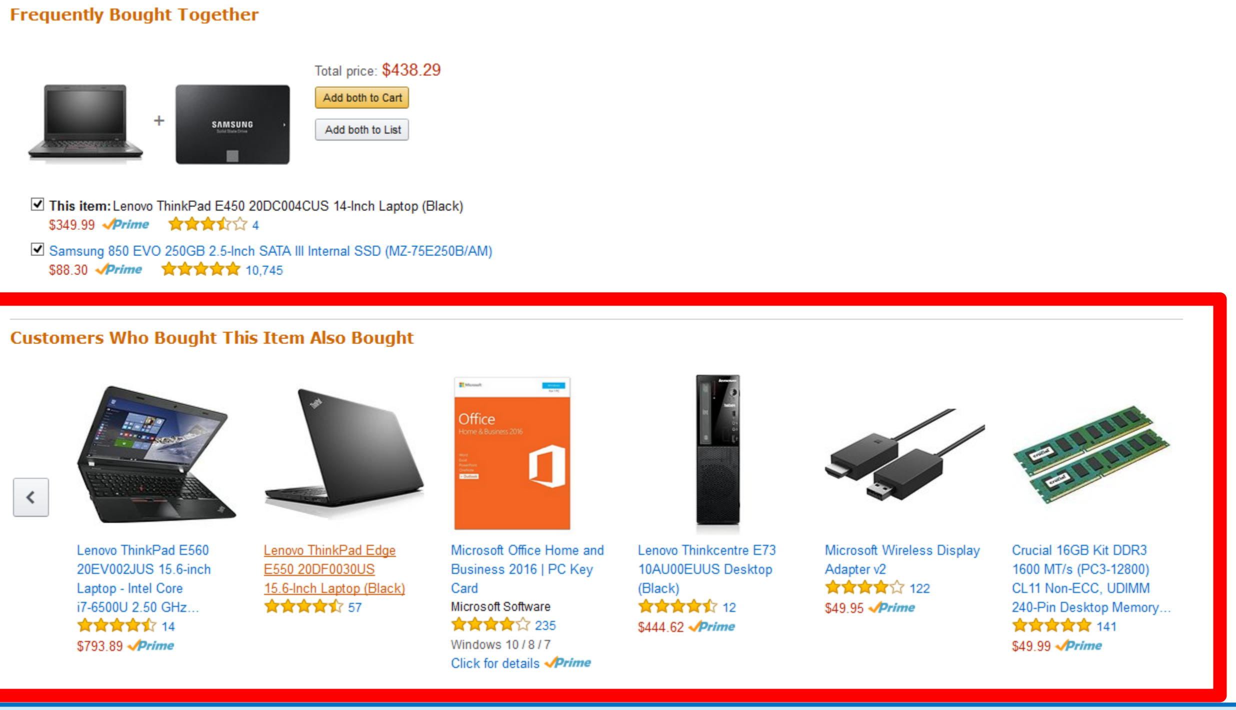
Motivation: Ad Types

- **Display ads (banners)**
  - Earliest form of Web advertising
  - E.g. fixed rate for *impression* (total cost defined by the number of times an ad has been rendered in a browser)



Motivation: Ad Types

- **Recommendation systems**
  - Amazon
  - Ad is selected by the store to maximize the probability that a customer will be interested in a product



Motivation: Ad Types

- **Search ads**
  - Placed along with the results of a search query
  - Bids for the right to have ad shown in response to certain queries
    - Payed only if the ad is clicked on

Motivation: Issues

- **Direct Placement of Ads**
  - **Identifying ads:** displayed in response to query terms
    - Inverted index of words (search engine)
    - Filters (user specifies a set of predefined parameters)
  - **Assigning importance to ads:** display order
    - Most recent first (njuskalo.hr)
      - Display the latest ad
      - Possible abuse – minimal ad changes at frequent intervals
    - Measure the “**attractiveness**” of an add
      - Record how many times the ad has been clicked on
      - Ads that are clicked on more frequently are presumed to be more attractive

Motivation: Issues

- **Direct Placement of Ads**
  - Measuring ad “**attractiveness**” is not that straightforward
    - The position of an ad in a list: first ad in a list has by far the greatest probability to be clicked on
    - Attractiveness can depend on the query terms
    - All ads should have opportunity to be shown initially (until their click probability can be estimated their attractiveness is unknown)

Motivation: Issues

- **Display Ads**
  - Resemble advertising in traditional media
  - Problem: lack of focus
    - User might not be interested (e.g. just bought a new mobile phone or is generally not into tech)
    - Low click through rates (banner advertising 1995-2001 offered low return of investment)
  - Printed media / TV cannot solve this issue **but** web ads can!

Motivation: Issues

- **Display Ads**
  - User-tailored ads: adds fitted to a specific user
    - use information about users to determine which ad they should be shown
  - **Problem:** need to get to know users
    - Activity on Facebook
    - E-mail
    - Time spent on a particular site, bookmarks etc.
    - **Search queries**
  - Significant privacy issues

Motivation: PBA

- Performance-based Advertising

- Concept introduced by Overture (2000)
  - Advertisers place bids on search keywords
  - When a keyword is searched, the **highest bidder's ad is shown**
  - Advertiser is charged **only if** the ad is clicked on

Motivation: PBA

- **Google adopted the Overture PBA model around 2002 and modified it**
  - **Adwords**
  - The value of PBA was proven as web advertising started to get a lot of traction

Motivation: PBA

- **What ads are to be displayed for a given user query?**

- **What search terms advertisers should bid on?**
- **How much to bid for a particular search term?**
  - Out of scope of this lecture

Motivation: PBA

- **Problem**
  - Advertisers usually have a limited budget
  - How to display ads in an optimal way if all search queries are **not known** in advanced?
  - **Online algorithms**

Online algorithms

- **Classic (“offline”) algorithms**
  - The entire input is available
  - Can access data set in any order and compute some function over all input values

- **Online Algorithms**
  - Do not have access the entire data set
  - Input is read piece-by-piece
  - Output is produced for each input data set piece (cannot wait for the entire data set to arrive!)

Online algorithms

- **Examples:**
  - Stream mining algorithms
  - Insertion sort
  - BFR clustering algorithm (if centroids are known)
  - …

- **Many online algorithms are greedy**
  - Output is produced by maximizing some function of the current input and the past (comprising of previous input elements)

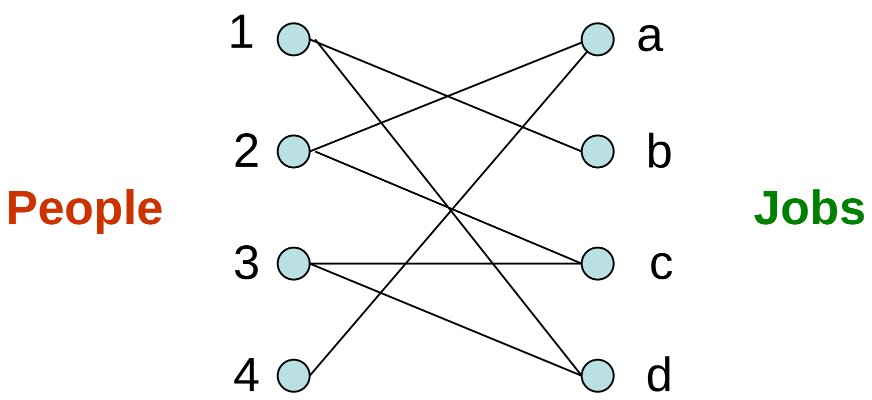
Online algorithms

- **Online algorithms can (and usually do) return result that is not as good as the result of the best offline algorithm**

- **Competitive ratio**
  - Given a solution quality for an offline algorithm $\boldsymbol{o}$ there exists such constant $\boldsymbol{c}$, $0 \leq \boldsymbol{c} \leq 1$, so that $\boldsymbol{c} \cdot \boldsymbol{o}$ is the solution quality of the online algorithm
  - $\boldsymbol{c}$ is the ***competitive ratio*** for the online algorithm

Online algorithms: Example

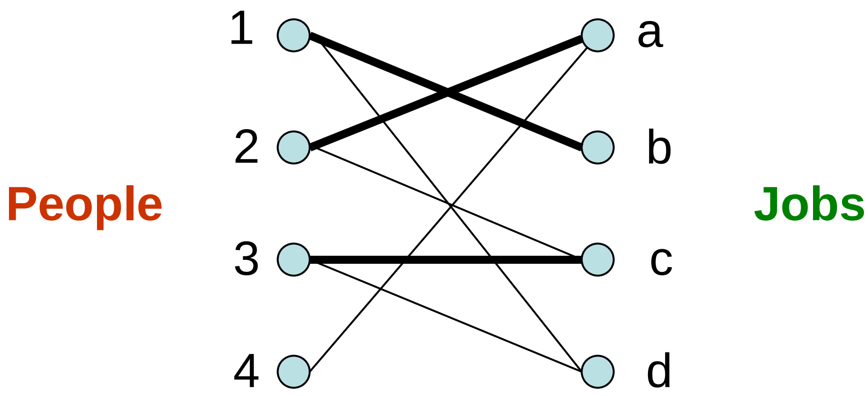
- **The Bipartite Matching Problem**
  - Assignment of entities from two ("different") sets
  - E.g. assign people to jobs, tasks to servers
  - OR **ads** to web site renderings!



  - **Constraints:** number of ways entities can be matched is limited → **defined** by graph edges
    - Limited number of job listings, job limitations, …

Online algorithms: Example

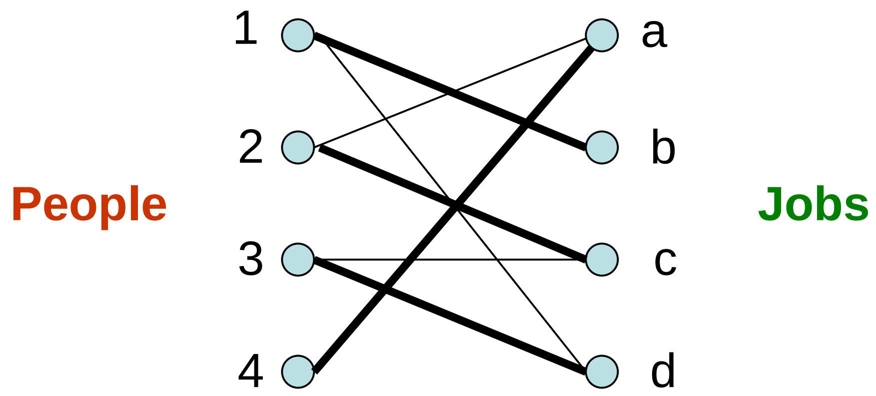
- **The Bipartite Matching Problem**
  - **Task:** prune the graph so each vertex in set s_1 is connected to **at most one** vertex in set s_2



  - Matched pairs (1,b), (2,a), (3,c)
  - Cardinality of matching $|\mathbf{M}| = \mathbf{3}$

Online algorithms: Example

- **The Bipartite Matching Problem**
  - **Maximum matching:** largest possible number of matches → **Goal**
  - **Perfect matching:** all graph vertices are matched



  - Perfect matching = maximum matching
  - More than one maximum/perfect matching can exist

Online algorithms: Example

- **The Bipartite Matching Problem**
  - **Offline algorithms** solve the problem of finding maximal matching in polynomial time
  - Hopcroft-Karp algorithm ($O\left(E\sqrt{V}\right)$)
  - Can be treated as a max-flow problem
  - What if entire data set is not known in advance?

Online algorithms: Example

- **Online approach to bipartite matching problem**
  - Initially, set s_1 is known (e.g. people that look for jobs, possible ads)
  - Data from set s_2 becomes known gradually (job offers, ad rendering possibilities)
  - Choice:
    - Pair available items using current knowledge
    - Do not pair items (a better match could become available)

Online algorithms: Greedy Approach

- **Matching pairs in a greedy manner**
  - Pair the newly discovered entity with any available (but eligible) entity in the other set
    - Pair the job offering with the first available person
    - If eligible person does not exist, do not match
  - Is this approach any good?

Online algorithms: Greedy Approach

- Let D be the input data set
- Let be M_G and M_0 the cardinality of matching for greedy and offline algorithms respectively

- Competitive ratio c is defined as:

$$c = \min_{D}\left(\frac{M_G}{M_0}\right)$$

  - c represents the **worst case** scenario for the greedy approach

Online algorithms: Greedy Approach

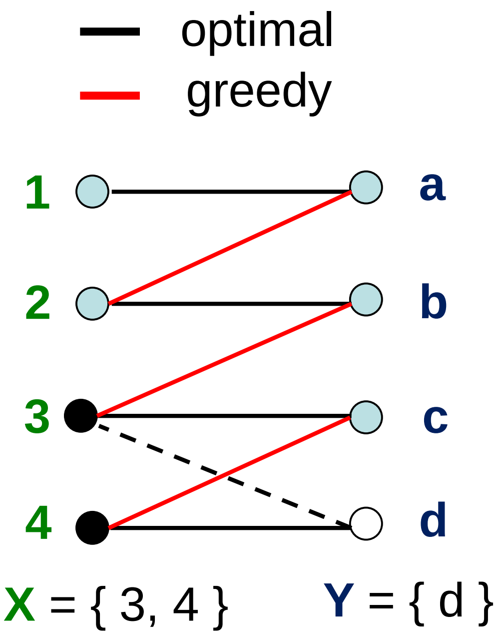
- **Let $M_G \neq M_O$**

- Y – a set of unmatched entries in M_G (e.g. jobs)
- X – a set of entities adjacent to Y

$$M_o = M_G + |Y| \qquad (1)$$

$$M_G \geq |X| \qquad (2)$$

As all elements in X are apparently matched by the greedy algorithm. Otherwise, elements in Y would have been matched!



Online algorithms: Greedy Approach

- **Let $M_G \neq M_O$**

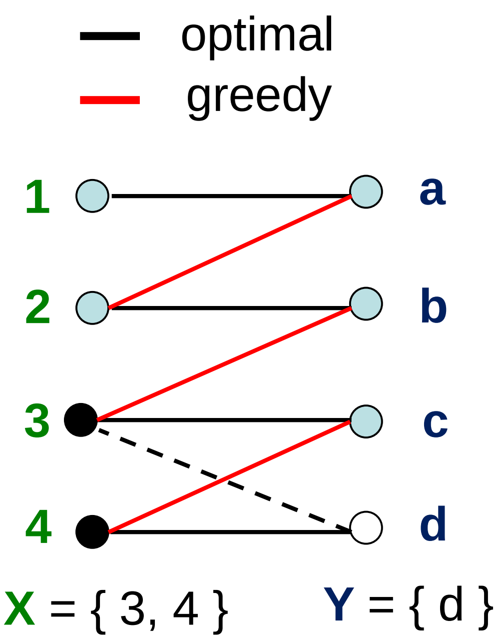
$$|Y| \leq |X| \qquad (3)$$

As optimal approach matched all elements in Y to some elements in X

$$(2) + (3) \rightarrow |Y| \leq |X| \leq M_G \qquad (4)$$

Worst case

$$|Y| = |X| = M_G \qquad (5)$$

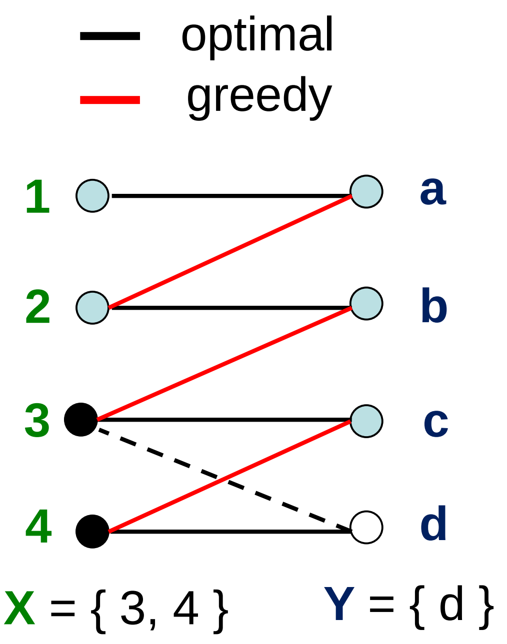


Online algorithms: Greedy Approach

- **Let $M_G \neq M_O$**

(1) + (5) → $M_O \leq 2M_G$

$$\boxed{\frac{M_G}{M_O} \geq 1/2}$$



Advertising: The Adwords Problem

- **How to match ads to search queries?**
  - Very similar to the general problem of bipartite graph matching

- **Search engine gets a set of queries as input values**
- **Advertisers bid on search keywords**
- **Upon answering the query, the search engine picks a subset of ads to display**
  - Usually more than one ad is shown

- **Goal is to maximize the profits from advertising**

Advertising: Adwords problem

Advertiser	Bid
A	$1.00
B	$0.75
C	$0.50
D	$0.25

Model used by Overture
Advertisers are sorted by bid values

Advertising: The Adwords Problem

- **Adwords introduced the click-through rate (CTR)**

- **CTR**
  - Number of times the ad has been clicked on as the result of being displayed divided by the total number of ad clicks

- **Expected revenue**
  - B * CTR, where B is the bidding value

Advertising: Adwords problem

Advertiser	Bid	CTR	Bid * CTR
A	$1.00	1%	1 cent
B	$0.75	2%	1.5 cents
C	$0.50	2.5%	1.125 cents
D	$0.25	8%	2 cents

Advertising: Adwords problem

Advertisers sorted by the expected revenue

Advertiser	Bid	CTR	Bid * CTR
D	$0.25	8%	2 cents
B	$0.75	2%	1.5 cents
C	$0.50	2.5%	1.125 cents
A	$1.00	1%	1 cent

Advertising: Adwords problem

- **Statement of the Adwords problem**

- **Having the following values:**

  1. A set of advertisers' bids on search queries
  2. A click-through rate for each ad-query(keyword) pair
  3. A budget for each advertiser (e.g. for 1 month)
  4. A limit on the number of ads that can be displayed per search query

Advertising: Adwords problem

- **Derive a subset of ads such that:**

1. The size of the set is not larger than the ad display limit
2. The advertiser has placed a bid on the search keywords
3. The advertiser has enough budget left to pay if the ad gets clicked on

Advertising: Adwords problem

- **It turns out that sorting the ads by the expected revenue is the optimal strategy, but only if**
  - The click-trough rate for each ad-query pair is known
  - Advertisers have an unlimited budget
- **In general, this is not the case!**
- **How to estimate the CTR and deal with limited advertiser budgets?**

Advertising: Estimating CTR

- **CTR can be measured historically**
  - Show the ad a large number of times
  - Compute the CTR estimate by observing the number of clicks

- **Problems**
  - CTR is very position dependent
    - First ad in the list has the greatest probability to be clicked on
  - Explore or Exploit
    - Exploit: continue displaying ads with a high CTR estimate
    - Explore: examine CTR of a new ad (might be worthwhile)

Advertising: Limited Budget

- **Simplified environment**
  - **1** ad is shown for each query
  - All advertisers have the same budget ***B***
  - All ads are equally likely to be clicked
  - All ads have the same price (e.g. 1 cent)

- **Greedy algorithm:**
  - Pick an advertiser that has bid on the query and has enough leftover budget
  - Competitive ratio is 0.5

Advertising: Limited Budget

- **Two advertisers A and B**
  - ***A*** bids only on queries $\boldsymbol{x}$, ***B*** bids both on $\boldsymbol{x}$ and $\boldsymbol{y}$
  - Both advertisers have budgets of **4 cents**
  - Cost of displaying an ad is 1 cent

- **Query stream: *x x x x y y y y***
  - Worst case for greedy: ***B B B B*** _ _ _ _
    - Earned 4 cents
  - Optimal solution: **A A A A B B B B**
    - Earned 8 cents

Advertising: Limited Budget

- **Is it possible to get a better competitive ratio?**
  - Problem with greedy approach is that it always breaks ties in the same way
  - This leads to exhausting budgets of certain advertisers and thus limiting the ways of efficiently utilizing other advertisers

Advertising: BALANCE Algorithm

- **BALANCE Algorithm (Mehta, Saberi, Vazirani, and Vazirani)**
  - Upon processing a query, choose the advertiser with the largest unspent budget
  - Break ties in an arbitrary way
    - But do it **deterministically**

Advertising: BALANCE Algorithm

- **Two advertisers A and B**
  - ***A*** bids only on queries x, ***B*** bids both on x and y
  - Both advertisers have budgets of **4 cents**
  - Cost of displaying an ad is 1 cent

- **Query stream: *x x x x y y y y***

	Break tie		Break tie					
Balance A	4	3	3	2	2	2	2	2
Balance B	4	4	3	3	2	1	0	0
Output	A	B	A	B	B	B	-	-

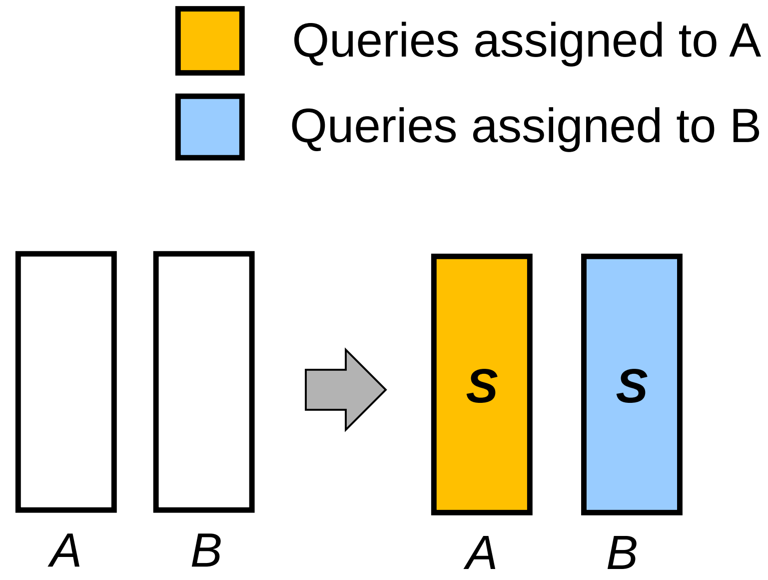
Advertising: BALANCE Algorithm

- **BALANCE output: A B A B B B _ _**
  - Optimal: A A A A B B B B
  - Cardinality of matching is ¾

- **In general: for BALANCE having 2 advertisers**
  - $\mathbf{Competitive\ ratio} = \frac{\mathbf{3}}{\mathbf{4}}$

Advertising: BALANCE Algorithm

- **Proof c = ¾**
  - Two advertisers A and B
  - They have the same budget S
  - Let all ads be priced the same (=1)
  - 2S queries
  - Optimal case
    - Both budgets get exhausted
    - The overall revenue is **2S**

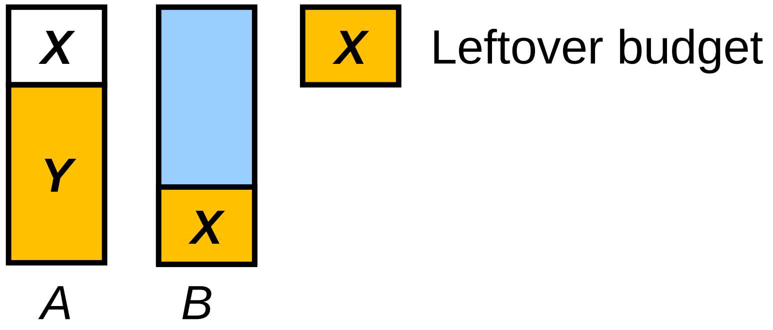


Advertising: BALANCE Algorithm

- **Proof c = ¾**
  - Budget in case of BALANCE algorithm
    - Budget of **one advertiser** will surely get **exhausted**!
      - Because optimal approach managed to perform a perfect match
      - Both advertisers placed a bid on at least half the queries
    - Assume without the loss of generality that B’s budget gets exhausted

Advertising: BALANCE Algorithm

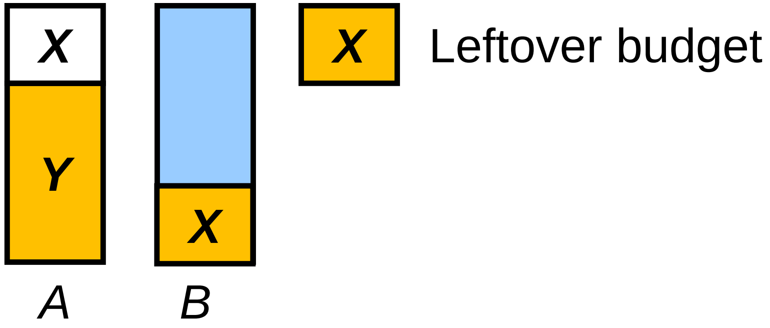
- **Proof c = ¾**
  - Budget in case of BALANCE algorithm
    - X – number of queries that were left unassigned
  - Total revenue in this case is
    - 2S – X
    - S + Y



Advertising: BALANCE Algorithm

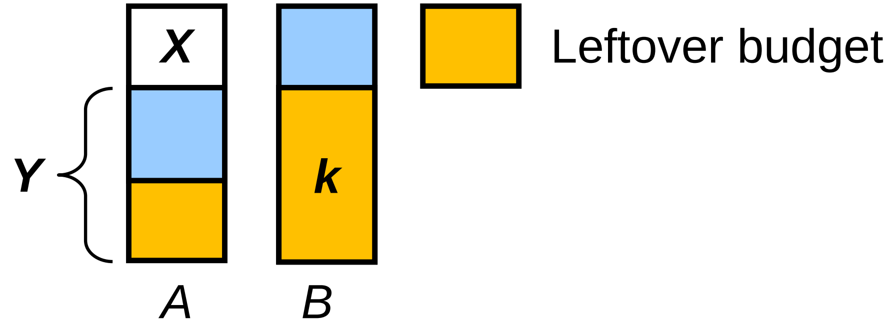
- **Proof $c = 3/4$**
  - Goal
    - Prove that $X \leq S/2$ or $Y \geq S/2$
    - Thus, $Y \geq X$
  - Case 1
    - Assume that less then S/2 queries of A were assigned to B
    - $X \leq S/2$
    - $X + Y = S$

$Y \geq X$



- **Proof c = ¾**
  - Case 2
    - Assume that k ≥ S/2 queries of A were assigned to B
    - At the time last query belonging to A was added to B, A's budget would have been greater or equal to B's budget
      - Otherwise the BALANCE algorithm **would not** make that assignment!
    - The only possibility is that some queries of B were assigned to A
    - Y is at least equal to k

Y ≥ X



Advertising: BALANCE Algorithm

- **Proof c = ¾**
  - Finally
    - X + Y = S
    - Y ≥ X
    - Worst case Y = X = S/2
    - Worst case revenue is S + S/2
    - Competitive ratio

$$c = \frac{\frac{3}{2}S}{2S} = \frac{3}{4}$$

Advertising: BALANCE Algorithm

- **For more then 2 advertisers the BALANCE algorithm has slightly lower competitive ratio – approx.. 0.63**

$$c = 1 - \frac{1}{e}$$

  - This appears to be the best solution for the Adwords problem – no online algorithm has a better competitive ratio!

Advertising: BALANCE Algorithm

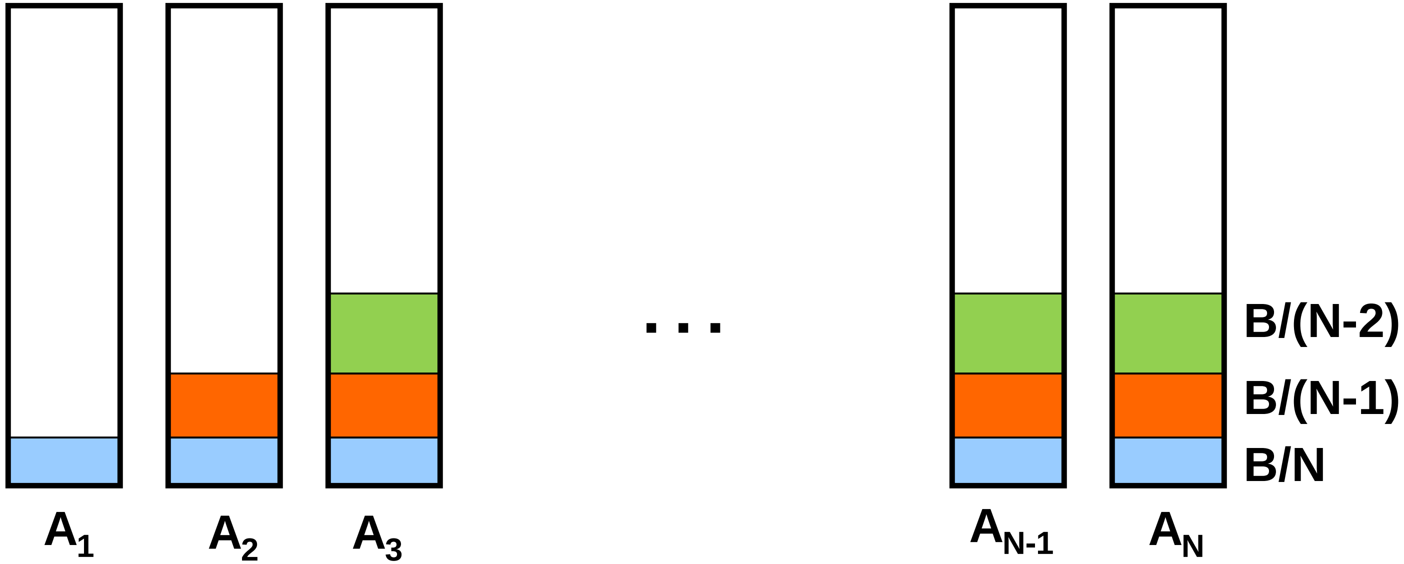
- **Worst case scenario $c = 1 - \frac{1}{e}$**
  - Let there be N advertisers $A_1, \ldots A_N$
  - Each advertiser has a budget B, B > N
  - N rounds, each containing B queries = N * B
  - Bidding
    - In i^{th} round, bids are placed by A_j where $j \geq i$
      - 1^{st} round $A_1, A_2, A_3, \ldots, A_N$ place bids
      - 2^{nd} round $A_2, A_3, \ldots, A_N$ place bids
      - …

Advertising: BALANCE Algorithm

- **Worst case scenario $c = 1 - \frac{1}{e}$**
  - Optimal ad matching
    - In the 1^{st} round all ads are assigned to A_1
      - It's the only round where A_1 has placed bids!
    - In the 2^{nd} round all ads are assigned to A_2
    - etc.
    - Total revenue is **N** · **B**

Advertising: BALANCE Algorithm

- Worst case scenario $c = 1 - \frac{1}{e}$



  - k[th] advertiser will have the following allocation after **k** rounds

$$S_k = \sum_{i=1}^{k} \frac{B}{N - i + 1}$$

Advertising: BALANCE Algorithm

- **Worst case scenario $c = 1 - \frac{1}{e}$**

$$S_k = \sum_{i=1}^{k} \frac{B}{N - i + 1}$$

  - At some point S_k will become greater than B
    - No further allocations are possible at that point
    - Advertisers with i < k have no active bids, and advertisers i ≥ k have to small budgets
  - Goal
    - Find smallest k so that S_k ≥ B

Advertising: BALANCE Algorithm

- **Worst case scenario** $c = 1 - \frac{1}{e}$

$$S_k = \sum_{i=1}^{k} \frac{B}{N - i + 1}$$

$\geq B$

$\boxed{B/N + B/(N-1) + \ldots + B/(N-(k-1))} + \ldots + B/3 + B/2 + B/1$

≥ 1

$\boxed{1/N + 1/(N-1) + \ldots + 1/(N-(k-1))} + \ldots + 1/3 + 1/2 + 1/1$

Advertising: BALANCE Algorithm

- **Worst case scenario $c = 1 - \frac{1}{e}$**

$$S_k = \sum_{i=1}^{k} \frac{B}{N - i + 1}$$

  - Proof by Euler:

~ 0 for larger k

$$\sum_{i=1}^{k} \frac{1}{k} = \ln(k) + \boxed{\gamma + \varepsilon_k}$$

1	ln(N - k)
1/N + 1/(N-1) + … + 1/(N-(k-1))	+ … + 1/3 + 1/2 + 1/1

Advertising: BALANCE Algorithm

- **Worst case scenario $c = 1 - \frac{1}{e}$**

$$S_k = \sum_{i=1}^{k} \frac{B}{N - i + 1}$$

$$\ln(N - k) = \ln(N) - 1$$

$$\Downarrow$$

$$\ln(N / (N - k)) = 1$$

$$\Downarrow$$

$$k = N(1 - 1/e)$$

- After k rounds no more ads can be assigned
  - Total revenue is $k \cdot B = B \cdot N(1 - 1/e)$

Advertising: BALANCE Algorithm

- **Worst case scenario $\boldsymbol{c = 1 - \frac{1}{e}}$**
  - Thus, the competitive ratio equals to:

$$c = \frac{NB\left(1 - \frac{1}{e}\right)}{NB} = 1 - \frac{1}{e}$$

Advertising: BALANCE Algorithm

- **Generally, advertisers do not have the same budget and do not place the same bids**
  - This fact ruins the performance of BALANCE algorithm

- **Example**
  - 2 advertisers A and B, 5 queries
  - A: budget 100, bid: 1
  - B: budget 80, bid: 10
  - BALANCE will select A and earn 5
  - Optimal revenue 50

Advertising: BALANCE Algorithm

- **Generalized BALANCE**
  - Having query **q** and bidder **I**
  - Bid value $\boldsymbol{x_i}$
  - Budget size $\boldsymbol{b_i}$
  - Amount spent so far = $\boldsymbol{m_i}$
  - Fraction of budget left over $\boldsymbol{f_i = 1 - \frac{m_i}{b_i}}$
  - $\psi_{\boldsymbol{i}}(\boldsymbol{q}) = \boldsymbol{x_i(1 - e^{-f_i})}$

Advertising: BALANCE Algorithm

- **Generalized BALANCE**
  - Taking into account CTR
    - CTR = c

$$\psi_i(q) = c \cdot x_i(1 - e^{-f_i})$$

  - Also worth considering historical frequency of queries

Advertising: BALANCE Algorithm

- **Generalized BALANCE**
  - Query *q* is assigned to the bidder *i* that has the largest value of $\psi_{\boldsymbol{i}}(\boldsymbol{q})$
  - Exhibits the same competitive ratio $\mathbf{1}-\frac{\mathbf{1}}{\boldsymbol{e}}$

Literature

1. J. Leskovec, A. Rajaraman, and J. D. Ullman, "Mining of Massive Datasets", 2014, Chapter 8 : “Advertising on the Web" (link)