Analysis of Massive Data Sets

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Community Detection in Social Network Graphs

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Outline

- **Social Networks**
  - Social graph
- **Detecting Communities**
  - Traditional approaches
- **Affiliation Graph Model**
  - Detecting communities using AGM
- **BigCLAM Approach**

Social Networks

- Modern social interactions

Social Networks

- **Modern social interactions**

- **Massive communities (Q4 2015)**
  - Google+ 2,200,000,000 (profiles, low activity)
  - Facebook 1,591,000,000
  - Instagram 400,000,000
  - Twitter 320,000,000

Social Networks

- **Vast amounts of data – immense opportunities**
  - Trend analysis, information cascades
  - Sentiment analysis
  - Social search
  - Recommendations
  - …
  - **Detecting communities**

Social Networks

- **What are social networks?**

1. Collection of entities (usually people, but not necessary)

2. At least one relationship exists between entities (friendship, follower, …)
    - Unidirectional/Bidirectional
    - Binary/Weighted

Social Networks

- **What are social networks?**
  3. Assumption of nonrandomness (locality)
     - If entity A is related to both B and C, there is higher then average probability that B and C are related

Facebook social graph visualization

Social Networks

- **Social Network Representation**
  - Social graph
    - Entities are nodes
    - Connections are edges
  - Connections can have a degree
    - Labeled edges
  - Connections can have a direction
    - Directed (G+, Twitter)
    - Undirected (Facebook)

Social Networks

- **Social Network Representation**
  - Social graph
    - Entities are nodes
    - Connections are edges
  - Entities can have different types
    - e.g. users, pages, tags
      - users can be related if they tag the same pages
    - k-partite social graph

Social Networks

- **Relationships other than "friendship"**
  - Telephone networks
  - Email networks
  - Collaboration networks

    ⋮

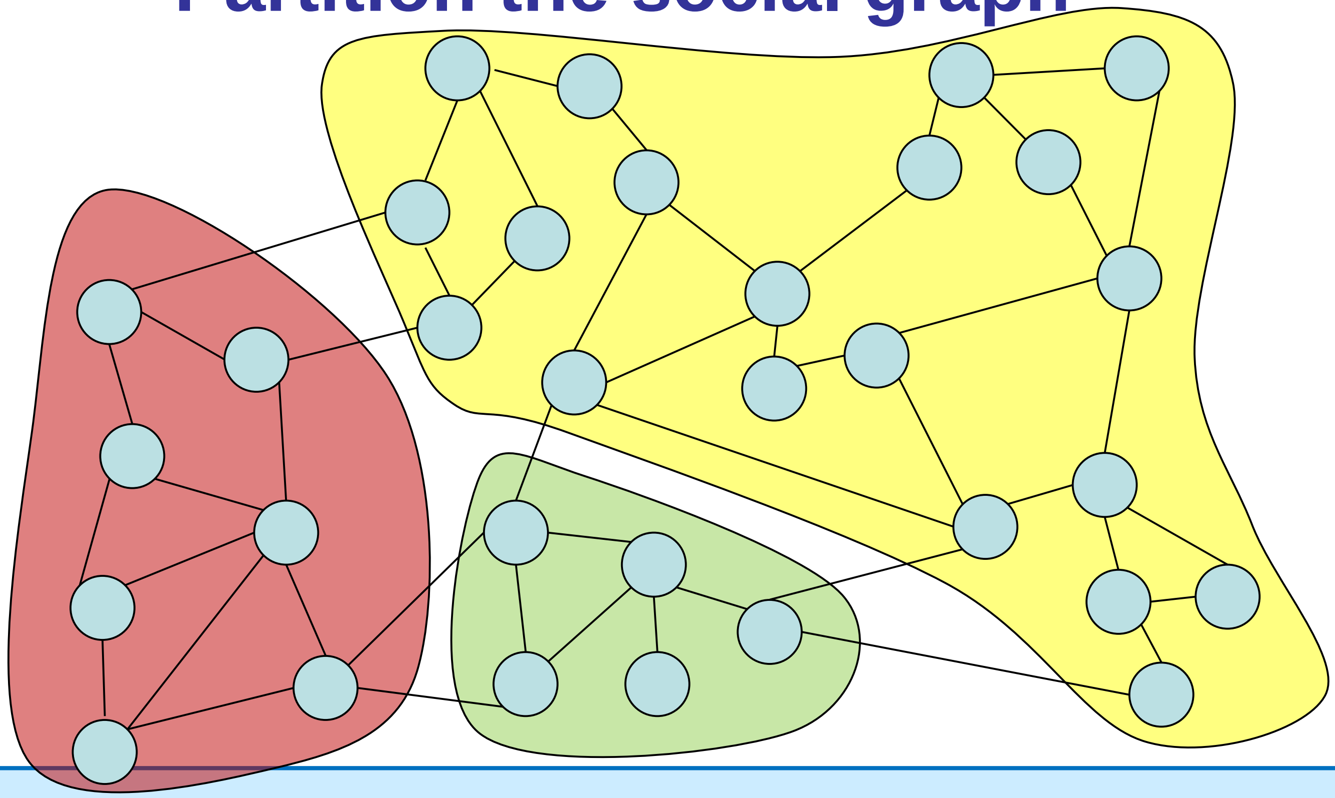
  - Information nets, infrastructure nets, …

They all exhibit locality of "friendship"

Social Networks

- **Goal**
  - **Extract** knowledge of social communities

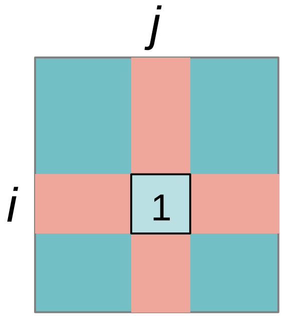
Partition the social graph



Detecting Communities

Traditional methods

- Minimal cut
  - One of the oldest methods
  - Minimize number of edges between communities
  - Problem: Will find communities even if they do not manifest
- Hierarchical clustering
  - Apply similarity measure on the adjacency matrix
    - Cosine similarity, Jaccard similarity, Hamming distance



  - Single-linkage clustering
    - All node pairs in different communities have similarity below a certain threshold

Detecting Communities

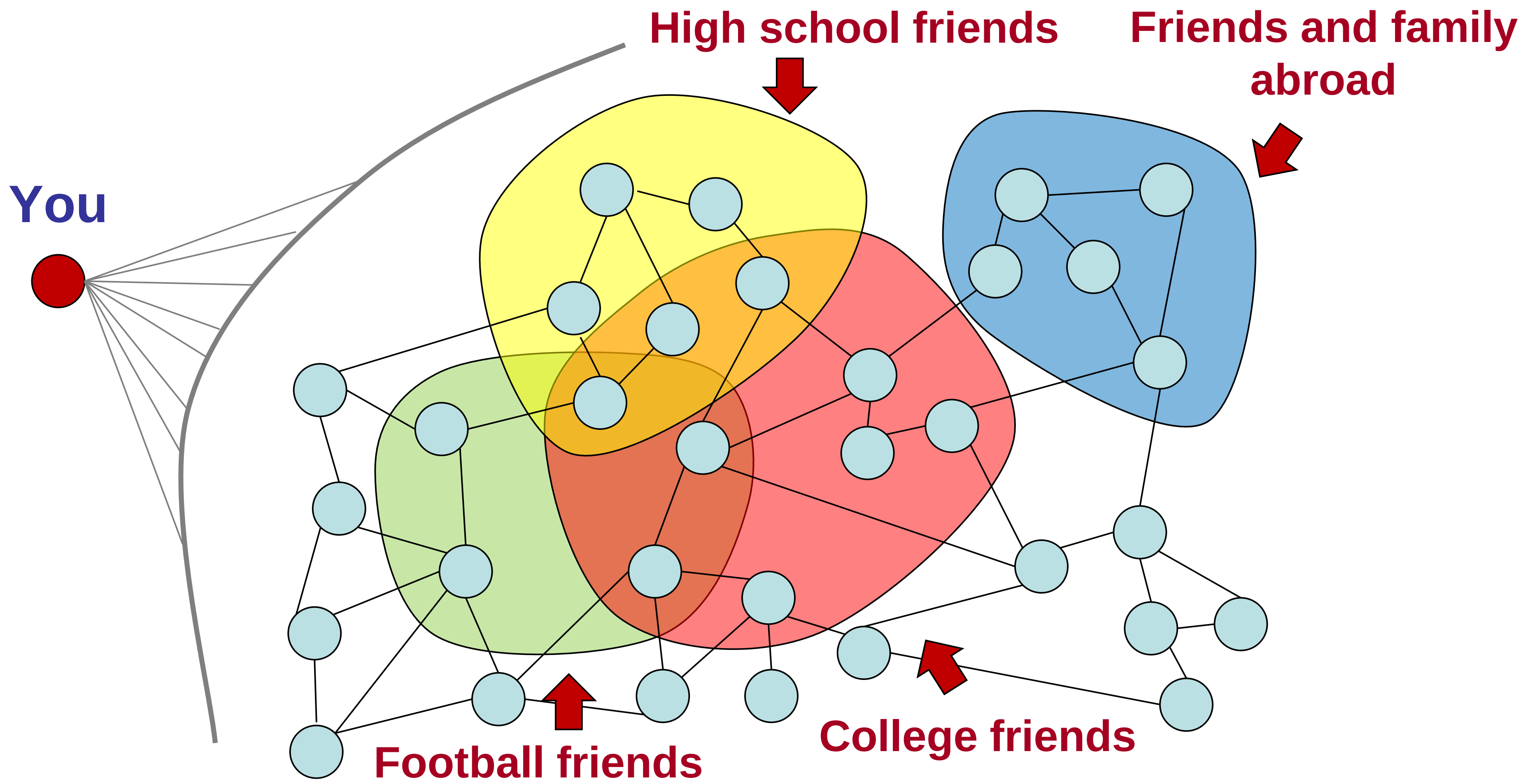
- **Traditional methods**
  - Girvan-Newman Algorithm
    - Identifies edges that lie between the communities and removes them
    - Measure: **betweenness**
    - Betweenness – number of times a node acts as a connection along the shortest path between other nodes in the graph
    - Edge betweenness – number of times an edge is within a shortest path between any two nodes in a graph

Detecting Communities

- **Traditional methods**
  - Girvan-Newman Algorithm
    1. Calculate edge betweenness
    2. Remove edge with highest betweenness
    3. Recalculate betweenness
    4. Repeat steps 1-3 until graph brakes down into communities

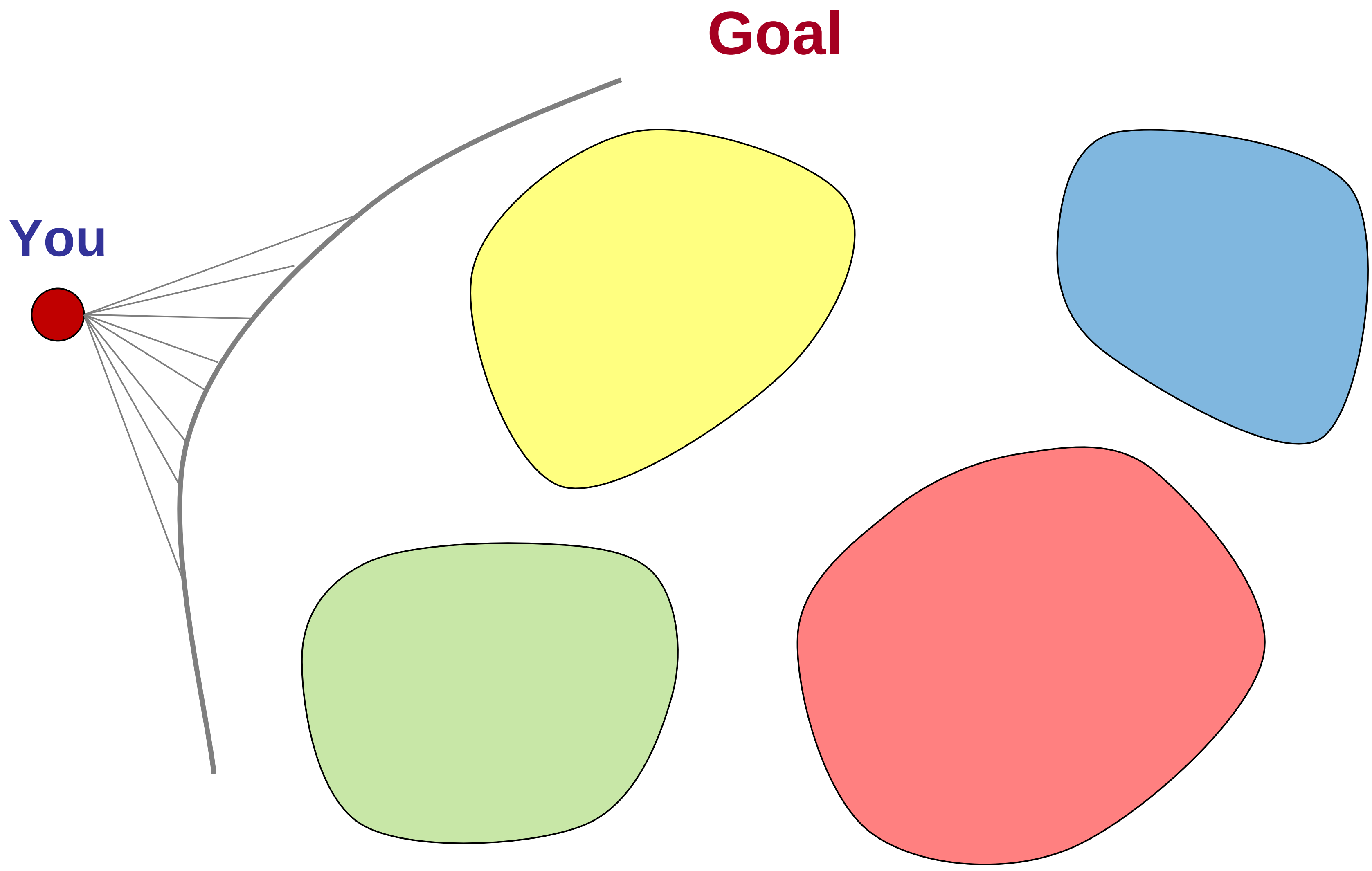
    - Effective but computationally heavy: $O(m^2n)$, m - edges, n - vertices

Detecting Communities



Communities can overlap!

Detecting Communities



Detecting Communities

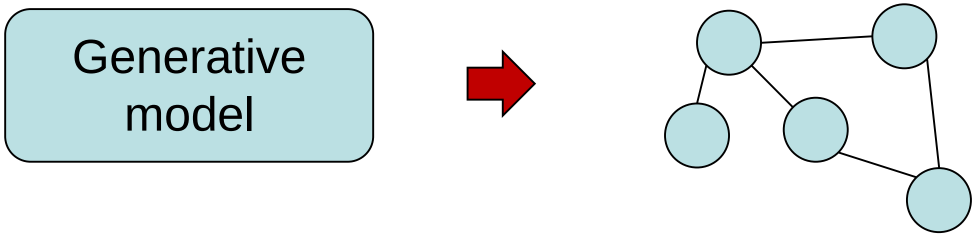
- **Problem**
  - Communities can overlap!

- **Solution**
  - Clique detection methods
    - All nodes within in a clique are directly connected (dense graph)
    - Clique percolation method (CPM)
  - Affiliation graph model (AGM)
    - Yang, Leskovec 2012 [3]

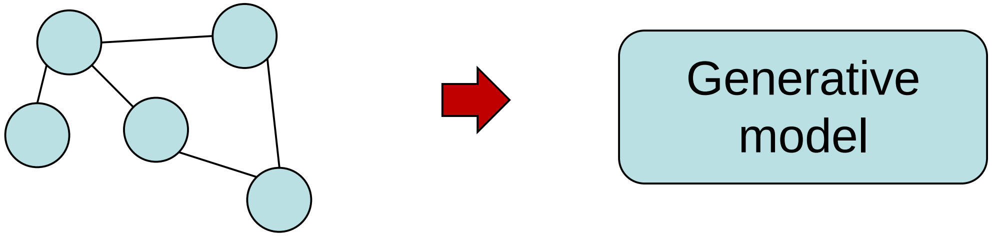
Affiliation Graph Model

- **Approach: Utilize generative models**

1. Using a model generate the social network

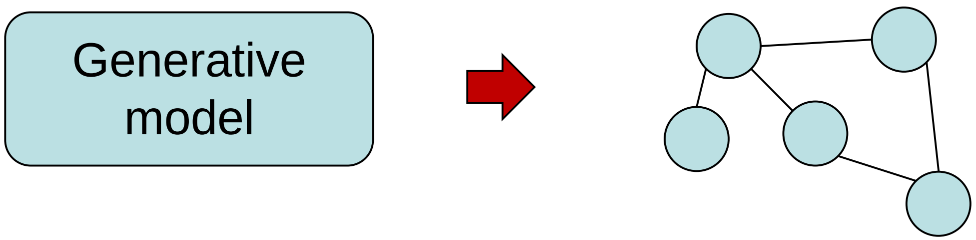


2. Given a social network, derive the most appropriate model that describes it



Affiliation Graph Model

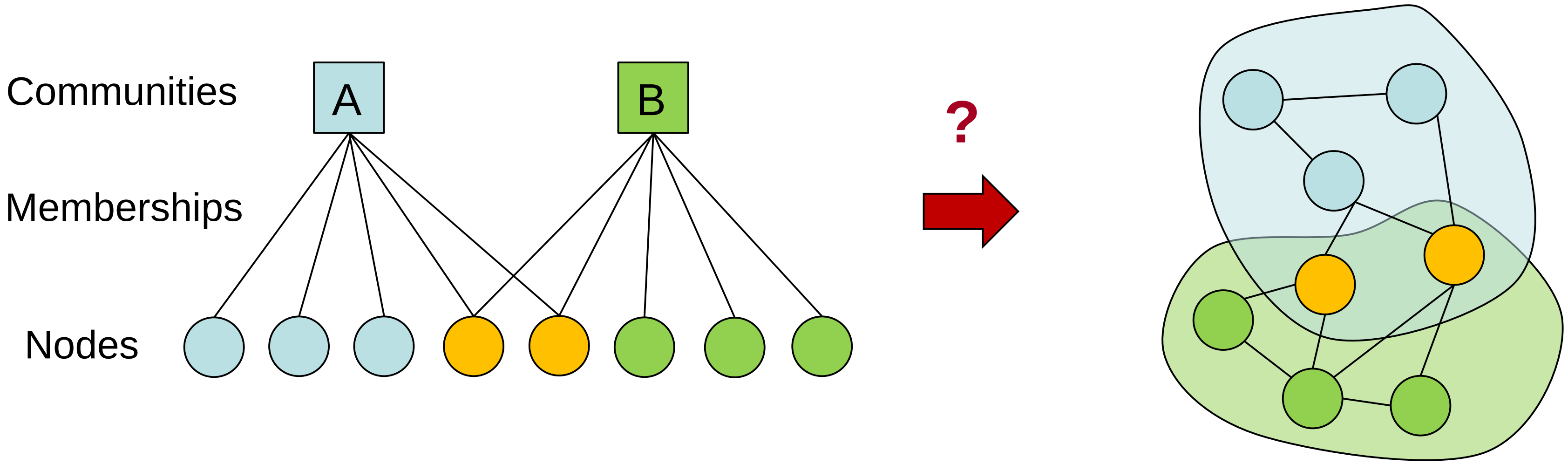
- **Goal: Derive a model that generates social networks**
  - The model has a set of parameters that need to be estimated
    - In doing so we in effect detect communities



  - Given network nodes (entities), how do communities (defined by AGM) generate the edges of the network?

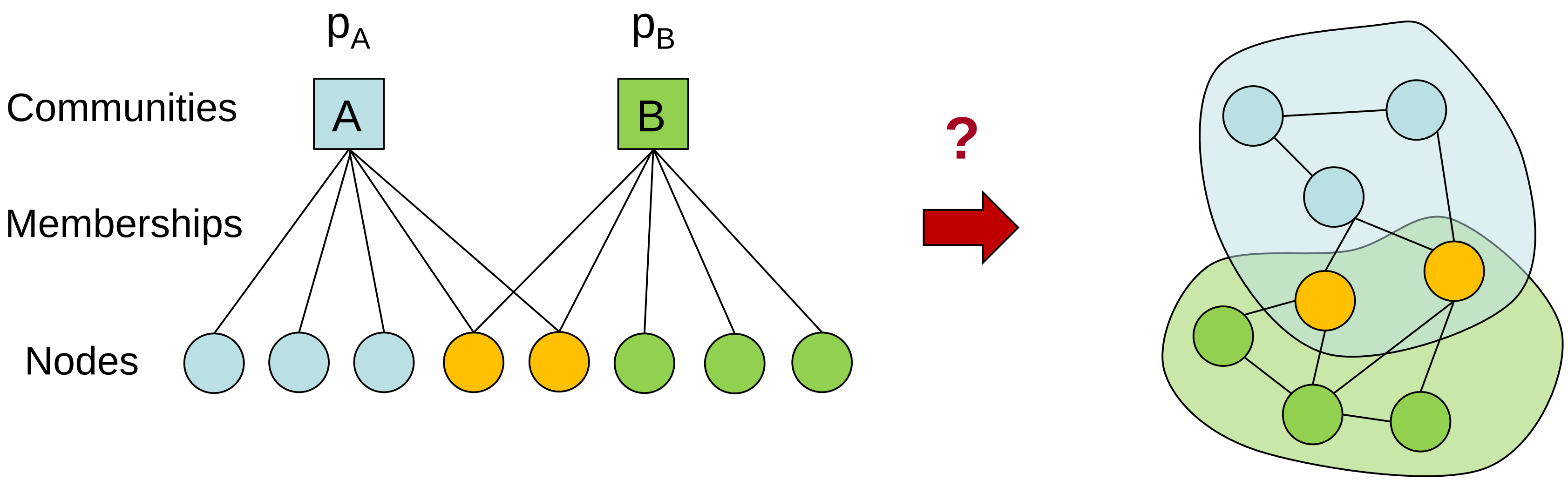
Affiliation Graph Model

- **Model**
  - Two types of nodes
    - Social network entities (nodes)
    - Communities
  - Edges
    - Community membership



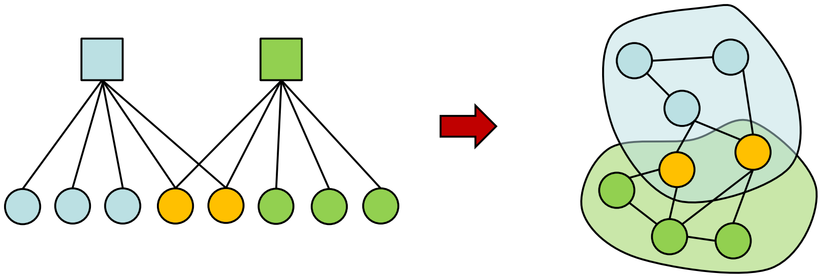
Affiliation Graph Model

- **Model**
  - Probability that a node links to other nodes in community C
    - p_c
  - AGM(N, C, M, $\{p_c\}$)



Affiliation Graph Model

- **Generative process**



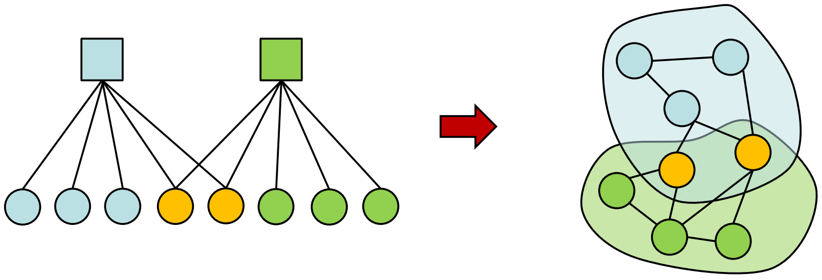
  - Each pair of nodes in community C is connected with probability p_c

- **Total probability**
  - Nodes ***u*** and ***v*** are connected

$$P(u,v) = 1 - \prod_{c \in M_u \cap M_v} (1 - p_c)$$

Affiliation Graph Model

- Generative process

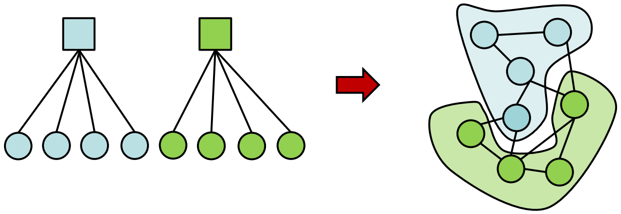


  - Go through all node pairs ***(u, v)*** and generate a connection (edge) with probability:

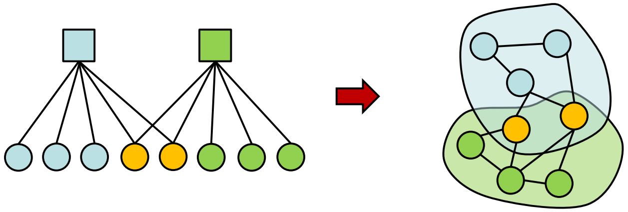
$$P(u,v) = 1 - \prod_{c \in M_u \cap M_v} (1 - p_c)$$

Affiliation Graph Model

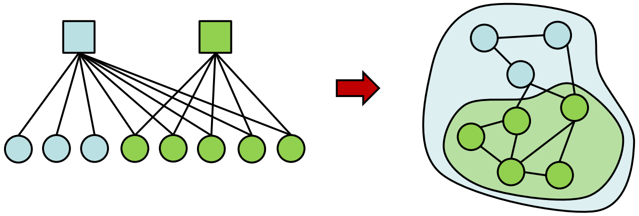
- **Advantage of AGM**
  - Flexible community representation
    - Non-overlapping



    - Overlapping

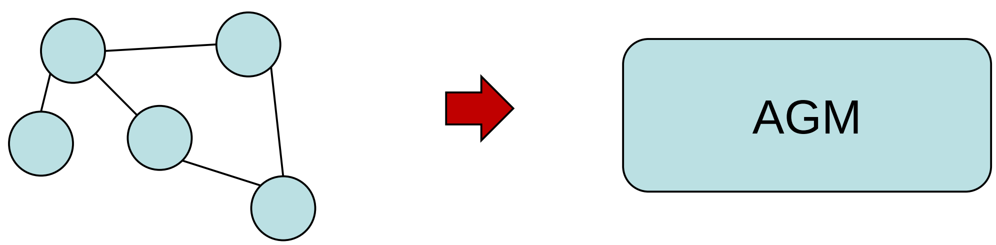


    - Nested



Detecting Communities using AGM

- **Finding the model = finding the communities**



Input: social network

Output: AGM

Community memberships

Detecting Communities using AGM

- **Find AGM parameters**

  - **N** → social net (entities) → **directly from graph**

  - **C** → number of comminutes → **estimate form graph [3]**

  - $\mathbf{p_c}$ → prob. that two nodes in community are connected

  - **M** → community memberships

Detecting Communities using AGM

- **Solution: Maximum Likelihood Estimation**
  - Given a social graph ***G***
  - Given a model ***f(param)***
  - We want to estimate $\boldsymbol{P_f(G \mid param)}$
    - Conditional probability
    - The probability that **AGM** generated ***G*** given parameters ***param***

- **Find the most likely model that generated graph G**

$$\arg\max_{param} \; P_f(G \mid param)$$

Detecting Communities using AGM

- **MLE: Example**
  - Suppose we have a sequence of events (e.g. coin toss, rainy days etc.)
  - X = [0, 1, 0, 0, 0, 1, 1, 0, 1, 0, 1, 0]
  - Model ***f*(*Y*) –** return 1 with probability ***Y***
  - What is $\boldsymbol{P_f(X \mid Y)}$ **?**

  - Assuming the events are independent
    - $\boldsymbol{P_f(X \mid Y) =} P_f(0 \mid Y) * P_f(1 \mid Y) * \ldots * P_f(0 \mid Y) = \boldsymbol{Y^5 (1-Y)^7}$
    - $\boldsymbol{P_f(X \mid Y = 5 / 12) = 0.0002886432}$

    - **X was most probably generated if Y = 5 / 12**

Detecting Communities using AGM

- **MLE for AGM**
  - Event = two network nodes are connected
    - $P(u, v)$
  - Likelihood of **AGM** generating some graph **G with** a set of edges **E**:

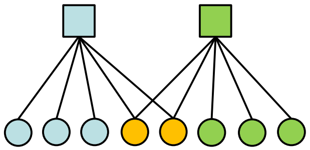
$$P(G \mid param) = \prod_{(u,v) \in E} P(u, v) \prod_{(u,v) \notin E} \left(1 - P(u, v)\right)$$

Detecting Communities using AGM

- **MLE for AGM**
  - **Goal:** find parameters ***param*** = (N, C, M, $\{p_c\}$) such that:

$$\arg\max_{param} \prod_{(u,v) \in E} P(u,v) \prod_{(u,v) \notin E} \left(1 - P(u,v)\right)$$

  - Problem
    - Finding ***param*** is equal to finding bipartite affiliation network
    - Too hard for large data sets!

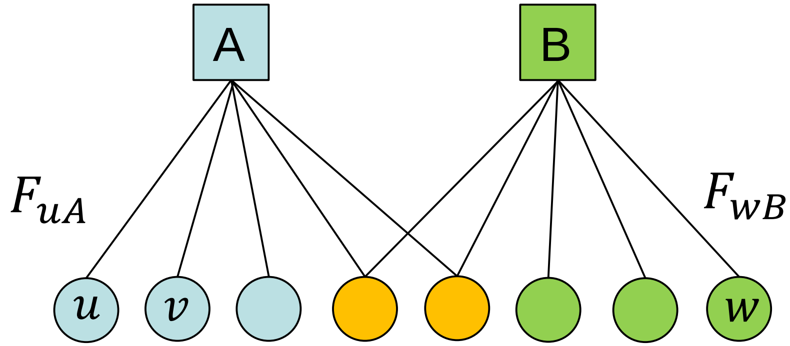


BigCLAM Approach

- **Solution: Relax the model**
- **BigCLAM (Yang, Leskovec 2013) [2]**
  - **C**luster **A**ffiliation **M**odel for **B**ig **N**etworks
  - Idea: avoid discrete memberships
    - Introduce strengths to memberships
    - Strengths are non negative values
    - If strength is **0**, the entity is not a member of the given community
    - If strength is **high**, the entity is a very active community member
  - Implications
    - Moving to continuous domain enables usage of very effective approaches, like gradient descent

BigCLAM Approach

- **Membership strengths**



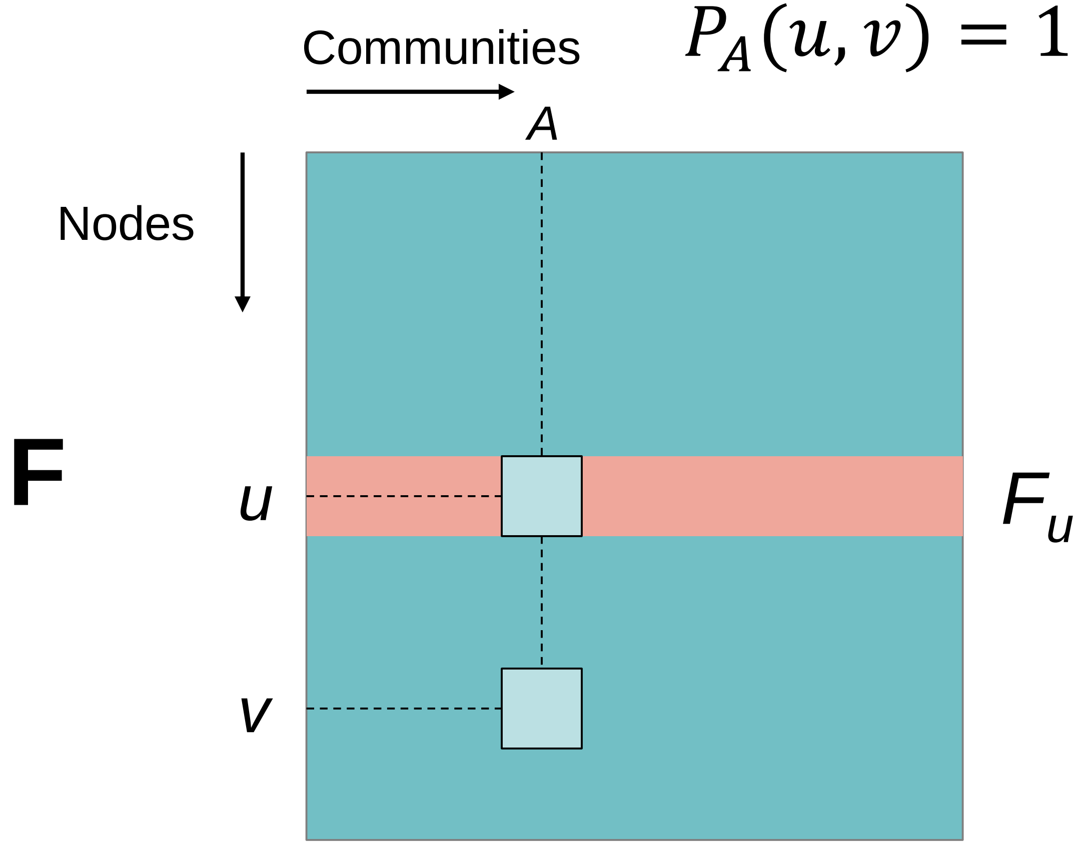
  - $F_{uA} > 0$: membership strength
  - Probability that nodes ***u*** and ***v*** are connected in **A**:

$$P_A(u, v) = 1 - e^{-F_{uA} \cdot F_{vA}}$$

BigCLAM Approach

- **Membership strength matrix F**

$$P_A(u,v) = 1 - e^{-F_{uA} \cdot F_{vA}}$$



BigCLAM Approach

- **Probability that at least one common node connects nodes *u* and *v***

$$P_A(u,v) = 1 - e^{-F_{uA} \cdot F_{vA}}$$

$$P(u,v) = 1 - \prod_{c} (1 - P_c(u,v))$$

$$P(u,v) = 1 - e^{-\sum_C F_{uC} \cdot F_{vC}}$$

$$= 1 - e^{-F_u \cdot F_v^T}$$

BigCLAM Approach

- **Goal: Find such F so that:** $P(u,v) = 1 - e^{-F_u \cdot F_v^T}$

$$\arg\max_{\mathbf{F}} \prod_{(u,v) \in E} P(u,v) \prod_{(u,v) \notin E} \left(1 - P(u,v)\right)$$

$$\arg\max_{\mathbf{F}} \prod_{(u,v) \in E} \left(1 - e^{-F_u \cdot F_v^T}\right) \prod_{(u,v) \notin E} e^{-F_u \cdot F_v^T}$$

BigCLAM Approach

- **Modification: log likelihood**
  - Why?
    - Sums instead of products
    - Errors are less pronounced when summing small numbers

$$\log P(X)$$

BigCLAM Approach

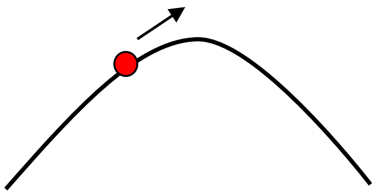
$$l(F) = \log P(G|F)$$

- **Goal**
  - Find ***F*** that maximizes:

$$\boxed{l(F) = \sum_{(u,v) \in E} \log\left(1 - e^{-F_u \cdot F_v^T}\right) - \sum_{(u,v) \notin E} F_u \cdot F_v^T}$$

BigCLAM Approach

- **Gradient descent**
  1. Compute a gradient for a single row
  2. Update row – move in the direction of gradient



  3. Repeat for all rows until F stops changing

BigCLAM Approach

- **Gradient descent**

$$l(F_u) = \sum_{v \in N(u)} \log\left(1 - e^{-F_u \cdot F_v^T}\right) - \sum_{v \notin N(u)} F_u \cdot F_v^T$$

$N(u)$ neighbors of node ***u*** (set of outgoing neighbors)

BigCLAM Approach

- **Gradient descent**

$$\nabla l(F_u) = \sum_{v \in N(u)} F_v \frac{e^{-F_u \cdot F_v^T}}{1 - e^{-F_u \cdot F_v^T}} - \sum_{v \notin N(u)} F_v$$

Update row

$$F_u \leftarrow F_u + \mu \cdot \nabla l(F_u)$$

If $F_{uC} < 0$: $F_{uC} = 0$

BigCLAM Approach

- **Gradient descent**

$$\nabla l(F_u) = \sum_{v \in N(u)} F_v \frac{e^{-F_u \cdot F_v^T}}{1 - e^{-F_u \cdot F_v^T}} - \sum_{v \notin N(u)} F_v$$

  - Computing $\nabla l(F_u)$ is slow!
    - Takes linear time on the size of network

However!

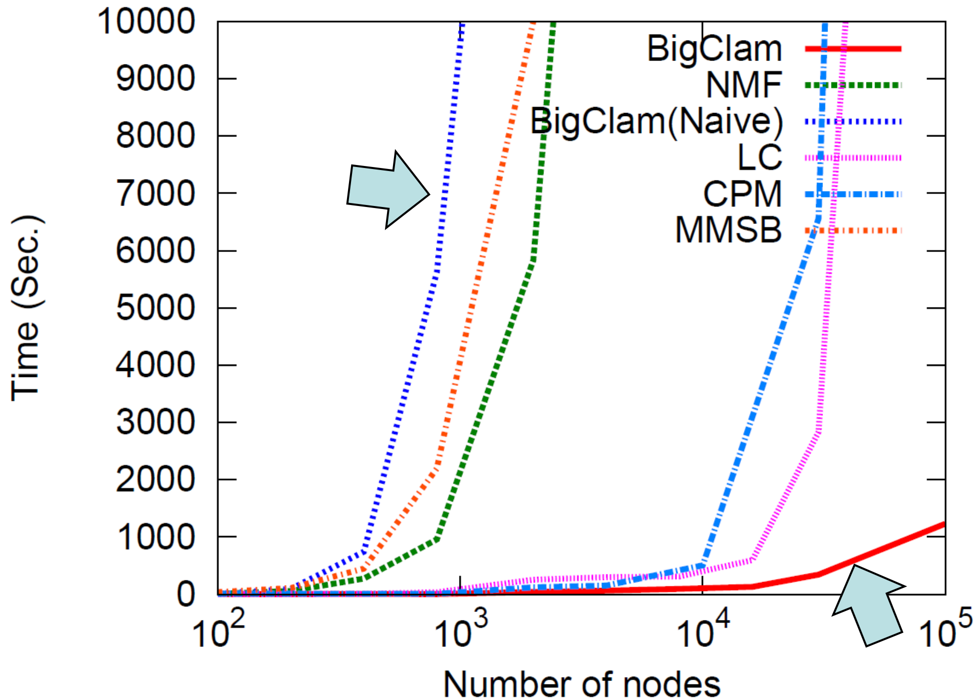
Compute once at the beginning of a pass

$$\sum_{v \notin N(u)} F_v = \sum_{v} F_v - F_u - \sum_{v \in N(u)} F_v$$

Computing $\sum_{v \notin N(u)} F_v$ now takes linear time in the degree of node u ($|N(u)|$)

Node degree is **much smaller** than the total number of nodes in the network!

BigCLAM Approach



[2] Yang, Leskovec 2013

~ 5 min for 300k nodes

~ 1 day for network with 100M edges

Literature

1. J. Leskovec, A. Rajaraman, and J. D. Ullman, "Mining of Massive Datasets", 2014, Chapter 6 : "Mining Social-Network Graphs" (link)

2. J. Yang, J. Leskovec: "Overlapping community detection at scale: a nonnegative matrix factorization approach ", WSDM '13 Proceedings of the sixth ACM international conference on Web search and data mining, Rome, Italy, February 2013, pp. 587 – 596. (link)

3. J. Yang, J. Leskovec: "Community-Affiliation Graph Model for Overlapping Network Community Detection", ICDM '12 Proceedings of the 2012 IEEE 12th International Conference on Data Mining, Brussels, Belgium, December 2012, pp. 1170–1175. (link)