1. Dokazi
$$G(a,b) \leq L(a,b) \leq A(a,b) \tag{1}$$
gdje su $G(a,b)$ geometrijska, $A(a,b)$ aritmeticka, a $L(a,b) = \frac{b-a}{\ln b - \ln a}$ logaritamska sredina.
Tezina =0/5

2. Izvedi
$$-\frac{\ln 1-x}{1-x} = \sum_{n=1}^{\infty} H_n x^n \tag{2}$$
gdje je
$$H_n = \textstyle\sum_{k=1}^{n} \frac{1}{k}$$
te zatim izvedi
$$\ln^2(1-x) = \sum_{n=1}^{\infty} \frac{2H_n}{n+1} x^n \tag{3}$$
Tezina = 2/5

3. Izvedi
$$\ln(1+x)\ln(1-x) = \sum_{n=1}^{\infty} \frac{x^{2n}}{n} \sum_{k=1}^{2n-1} \frac{(-1)^k}{k} = \sum_{n=1}^{\infty} \frac{1}{n}(H_n - H_{2n} - \frac{1}{2n}) x^{2n} \tag{4}$$
Tezina =2.5/5

4. Izvedi
$$\sum_{d|n} d \leq H_n + e^{H_n} ln(H_n) \tag{5}$$
Tezina $=10^{10}/5$

5. Dokazi
$$\sum_{n=1}^{\infty} \frac{H_n}{n^2} = 2\sum_{n=1}^{\infty} \frac{1}{n^3} \tag{6}$$
te
$$\sum_{n=1}^{\infty} \frac{H_n}{n^3} = \frac{\pi^4}{72} \tag{7}$$

6. Promotrimo integrale oblika $I(n,m) = \int x^n \ln^n x dx$.
Familija $\{x^i ln^j x\}; i,j \in \mathbf{N}_0$ je zatvorena s obzirom na integriranje.
Izvedi sljedece jednakosti:
$$I(n,n) = \frac{1}{n+1} x^{n+1} ln^n x - \frac{n}{n+1} I(n-1,n) \tag{8}$$

Tezina = 1/5

$$\int x^n lnxdx = \frac{x^{n+1}lnx}{n+1} - \frac{x^{n+1}}{(n+1)^2} \tag{9}$$

Tezina = 1/5

$$\int ln^n xdx = (-1)^n x \sum_{k=0}^{n} \frac{n!}{k!}(-lnx)^k \tag{10}$$

$$I(m,n) = \frac{x^n+1}{m+1} \sum_{k=0}^{m} \frac{(-1)^k (m+1-k)^{\underline{k}}}{(n+1)^k} \ln^{m-k} x \tag{11}$$

gdje je $(m+1-k)^{\underline{k}}$ k puta padajuci faktorijel.