1. $\binom{4321}{1234} = \dfrac{4321!}{1234!\,3087!}$

$\left\lfloor \frac{4321}{5} \right\rfloor + \left\lfloor \frac{4321}{25} \right\rfloor + \left\lfloor \frac{4321}{125} \right\rfloor + \left\lfloor \frac{4321}{625} \right\rfloor + \left\lfloor \frac{4321}{3125} \right\rfloor = 864+172+34+6+1=1077$

$\left\lfloor \frac{1234}{5} \right\rfloor + \left\lfloor \frac{1234}{25} \right\rfloor + \left\lfloor \frac{1234}{125} \right\rfloor + \left\lfloor \frac{1234}{625} \right\rfloor = 246+49+9+1=305$

$\left\lfloor \frac{3087}{5} \right\rfloor + \left\lfloor \frac{3087}{25} \right\rfloor + \left\lfloor \frac{3087}{125} \right\rfloor + \left\lfloor \frac{3087}{625} \right\rfloor = 617+123+24+4=768$

BROJ NULA $= 1077-(305+768)=4$

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2. $175x \equiv 252 \pmod{294}$ $(175, 294)=7$ $7 \mid 252$

skraćivanje: $25x \equiv 36 \pmod{42}$

$42 = 1\cdot 25 + 17$
$25 = 1 \cdot 17 + 8$
$17 = 2 \cdot 8 + 1$

$1 = 17 - 2\cdot 8 = 17 - 2\cdot(25 - 1\cdot 17) = 3\cdot 17 - 2\cdot 25$
$= 3(42-25) - 2\cdot 25 = -\underline{5}\cdot 25 + 3\cdot 42$

$x \equiv -5\cdot 36 \equiv (-5)\cdot(-6) \equiv 30 \pmod{42}$

SVA RJEŠENJA: $x \equiv 30, 72, 114, 156, 198, 240, 282 \pmod{294}$.

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3. $x \equiv 2 \pmod 4$, $x \equiv 1 \pmod 9$, $x \equiv 4 \pmod{11}$

$x_0 = 99x_1 + 44x_2 + 36x_3$

$99x_1 \equiv 2 \pmod 4 \Rightarrow 3x_1 \equiv 2 \pmod 4 \Rightarrow x_1 = 2$
$44x_2 \equiv 1 \pmod 9 \Rightarrow 8x_2 \equiv 1 \pmod 9 \Rightarrow x_2 = 8$
$36x_3 \equiv 4 \pmod{11} \Rightarrow 3x_3 \equiv 4 \pmod{11} \Rightarrow x_3 = 5$

$x_0 \equiv 99\cdot 2 + 44\cdot 8 + 36\cdot 5 \equiv 730$
$\equiv 334 \pmod{396}$

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4. $\varphi(n) = p_1^{\alpha_1-1} p_2^{\alpha_2-1} \cdots p_k^{\alpha_k-1}(p_1-1)(p_2-1)\cdots(p_k-1) = 42 \Rightarrow p_i - 1 \mid 42 \Rightarrow p_i \in \{2, 3, 4, 7, 8, 15, 22, 4$
p_i prost $\Rightarrow p_i \in \{2, 3, 7, 43\}$. Isprobavanjem se vidi da su moguće kombinac
prostih faktora:

43 $n = 43^\alpha \Rightarrow \varphi(n) = 43^{\alpha-1}\cdot 42 = 42 \Rightarrow \alpha = 1$ $\boxed{n=43}$
$43, 2$ $n = 43^\alpha 2^\beta \Rightarrow \varphi(n) = 43^{\alpha-1}\cdot 2^{\beta-1}\cdot 42\cdot 1 = 42 \Rightarrow \alpha=1, \beta=1$ $\boxed{n=86}$
$7,$ $n = 7^\alpha$ $\varphi(n) = 7^{\alpha-1}\cdot 6 = 42 \Rightarrow \alpha = 2 \Rightarrow$ $\boxed{n=49}$
$7, 2$ $n = 7^\alpha 2^\beta$ $\varphi(n) = 7^{\alpha-1}\cdot 2^{\beta-1}\cdot 6 \Rightarrow \alpha=2, \beta=1$ $\boxed{n=98}$

ČETIRI RJEŠENJA

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5. a) ~~prostih~~ prim. korijena mod 31 ima $\varphi(31-1) = \varphi(30) = \varphi(2\cdot3\cdot5) = (2-1)(3-1)(5-1) = 8$

b) UOČIMO: $2^5 \equiv 32 \equiv 1 \pmod{31}$, pa 2 NIJE PRIMITIVNI KORIJEN
DOKAŽIMO: 3 JE PRIMITIVNI KORIJEN: NUŽNO I DOVOLJNO $3^{\frac{30}{2}}, 3^{\frac{30}{3}}, 3^{\frac{30}{5}} \not\equiv 1$ (mod
$3^4 \equiv 81 \equiv 19 \pmod{31}$, $3^5 \equiv 19\cdot 3 \equiv 57 \equiv 26 \equiv -5 \pmod{31}$, $3^{10} \equiv (-5)^2 \equiv 25 \not\equiv 1 \pmod{31}$
$3^6 \equiv -15 \equiv 16 \not\equiv 1 \pmod{31}$, $3^{15} \equiv 3^{10}\cdot 3^5 \equiv (-6)\cdot(-5) \equiv 30 \not\equiv 1 \pmod{31}$

c) $26^x \equiv 25 \pmod{31} \Rightarrow x \cdot \mathrm{ind}_3 26 \equiv \mathrm{ind}_3 25 \pmod{30}$. Zbog b) slijedi
$5x \equiv 10 \pmod{30}$ $(5, 30) = 5 \mid 10$, skraćivanje $x \equiv 2 \pmod 6$
SVA RJEŠENJA $x \equiv 2, 8, 14, 20, 26 \pmod{30}$

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6. a) p prost, $a \in \mathbb{Z}$

$\left(\frac{a}{p}\right) = \begin{cases} 1, & a \text{ kvadr. ostatak mod } p \\ -1, & a \text{ kvadr. neostatak mod } p \\ 0, & p \mid a \end{cases}$

b) Broj kv. ostataka mod 233 je
$\frac{233-1}{2} = \frac{232}{2} = 116$

c) PRAVILA . . .

$\left(\frac{-35}{233}\right) = \underbrace{\left(\frac{-1}{233}\right)}_{1}\cdot\left(\frac{35}{233}\right) = \left(\frac{233}{35}\right) = \left(\frac{23}{35}\right)$
$= -\left(\frac{35}{23}\right) = -\left(\frac{12}{23}\right) = -\left(\frac{2}{23}\right)^2\cdot\left(\frac{3}{23}\right)$
$= -\left(\frac{3}{23}\right) = \left(\frac{23}{3}\right) = \left(\frac{2}{3}\right) = -1$