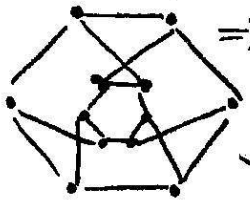
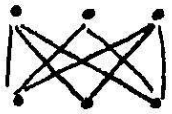
7. PLANARNOST

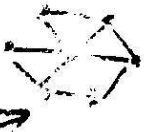
7.10 KURATOWSKI → ako se graf može svesti na $K_{3,3}$ ili K_5, je planaran



⇒ Je li planaran?



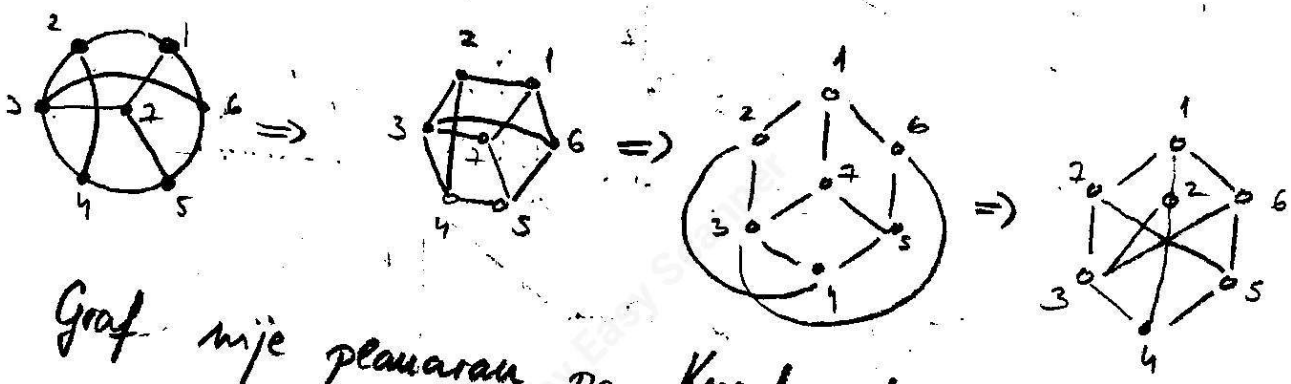
to je isto



dobiven iz ovoga

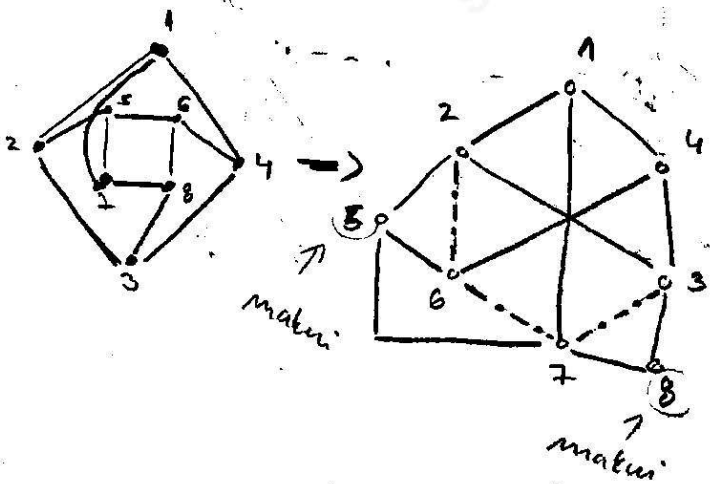
Graf nije planaran po Kuratowski teoremu

7.11 Planaran?



Graf nije planaran po Kuratowskom

7.12 Planaran?



Nije planaran po Kuratowski teoremu

7.13 ?

7.15 Križni broj K_6?

$Cr(K_6) = ?$

$$Cr(K_p) \leq \frac{1}{4} \cdot \left\lfloor \frac{p}{2} \right\rfloor \cdot \left\lfloor \frac{p-1}{2} \right\rfloor \cdot \left\lfloor \frac{p-2}{2} \right\rfloor \cdot \left\lfloor \frac{p-3}{3} \right\rfloor$$

↳ HILLOVA FORMULA

$$Cr(K_6) \leq \frac{1}{4} \cdot \left\lfloor \frac{6}{2} \right\rfloor \cdot \left\lfloor \frac{6-1}{2} \right\rfloor \cdot \left\lfloor \frac{6-2}{2} \right\rfloor \cdot \left\lfloor \frac{6-3}{2} \right\rfloor$$

$$Cr(K_6) \leq \frac{1}{4} \cdot 3 \cdot 2 \cdot 2 \cdot 1$$

$Cr(K_6) \leq 3$ ✓ $\longrightarrow$ $Cr(K_6) = 3$ ✓

⇒ također koristimo:

$$Cr(K_{r,s}) = \left\lfloor \frac{r}{2} \right\rfloor \cdot \left\lfloor \frac{r-1}{2} \right\rfloor \cdot \left\lfloor \frac{s}{2} \right\rfloor \cdot \left\lfloor \frac{s-1}{2} \right\rfloor$$

⇒ TURÁNOVA FORMULA

7.16 Heawoodov graf

$Cr(G) = ?$

$$Cr \geqslant \left\lfloor e - \frac{c(v-2)}{c-2} \right\rfloor$$

→ vrijedi za regularne grafove

c – duljina najkraćeg ciklusa

$$Cr \geqslant \left\lfloor 21 - \frac{6(14-2)}{6-2} \right\rfloor$$

$Cr(G) \geqslant 3$

$Cr(G) = 3$ ✓

7.17 Grötzschov graf

$Cr(G) = ?$ $\longrightarrow$ $Cr = 5$ ✓

⇒ samo izbroji, ovo se ne da izračunati, ali mi pojednostavite.

7.18 Protuprimjer: $K_{1,6}$

7.19 Jednostavni povezani bipartitni graf s $e \geq 2$

$\downarrow$

$2u - 4 \leq e$

Dokaži: $2f \leq e$

$u - e + f = 2$

$\swarrow$ $\qquad$ $\searrow$

$e = 2$ $\qquad$ **$e > 2$**

$u - 2 + f = 2$ $\qquad$ $u - e + f = 2$

$u + f = 4$ $\qquad$ $u + f = 2 + e$

$\Downarrow$

$u + f - 2 = e$

$2f = u + f - 2$

$2f - f = u - 2$

$f = u - 2$

$u = f + 2$ $\longrightarrow$ $2(f+2) - 4 \leq e$

$2f + 4 - 4 \leq e$

$2f \leq e$ ✓

$u = 3, f = 1$

$2 \cdot 1 \leq e$

$2 \leq 2$ ✓

7.20 $u \leq \frac{5}{3}(u-2)$ $\Rightarrow$ $u \leq \frac{c}{c-2}(u-2)$

$c = 5$

Dokaži!

$$\begin{cases} 2u = 5f \\ u - u + f = 2 \end{cases} \longrightarrow f \leq \frac{2}{5}u$$

$u - u + \frac{2}{5}u = 2$

$u - \frac{3}{5}u = 2$

$u - 2 = \frac{3}{5}u$

$\Downarrow$

$u \leq \frac{5}{3} \cdot \frac{5}{5}u$

$u \leq u$ ✓

7.21 $2u - 4 \geq f$

Dokaži!

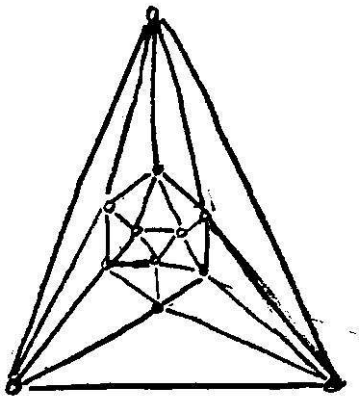
$$\begin{cases} u - e + f = 2 \\ e \leq 3u - 6 \end{cases} \longrightarrow u + f - 2 = e$$

$$u + f - 2 \leq 3u - 6$$

$$f \leq 2u - 4 \quad \checkmark$$

7.22 $(s-4)F + (t-4)N$

$t = 5, \; o = 3 \longrightarrow F = 20, \; N = 12 \Rightarrow$ ikosaedar



ikosaedar

7.23 ?

7.24 $t(K_7) = ?$

$$t(G) \geq \left\lceil \frac{e}{3u-6} \right\rceil \longrightarrow \text{vrijedi samo za } K_u \text{ i } K_{m,n}$$

$$t(G) \geq \left\lceil \frac{21}{21-6} \right\rceil$$

$$t(G) \geq 2 \longrightarrow t(K_7) = 2$$

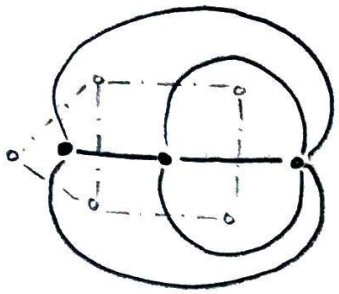
7.25 Petersenov graf?

$t(G) = 2$ (Graf nije planaran, min. $t(G) = 2$)

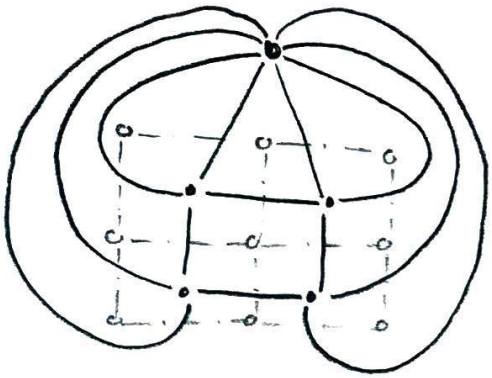
7.26 Heawoodov graf?

$t(G) = 2$ (Graf nije planaran, min $t(G) = 2$)

7.27 Dual!



7.28 Dual!



7.29 ?