1. OBRADA SLIKE - podvrgavanje numeričkih reprezentacija objekata seriji operacija s ciljem postizanja željenog rezultata

- PROBLEMI: predstavljanje i modeliranje slike, poboljšanje slike, obnavljanje slike, rekonstrukcija slike iz projekcija, kompresija

ANALIZA S.: - proces kojim iz slike dobivamo neke njene značajke

- PROBLEMI: ekstrakcija značajki s., segmentacija, analiza teksture/oblika/pokreta, registracija s.

RAZUMIJEVANJE SLIKE - proces zaključivanja o stanju okoline pribavljene slike

→ računalni vid

2. DIRAC

$$\delta(x,y) = \delta(x)\delta(y)$$

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(s,t)\,\delta(x-s,y-t)\,ds\,dt = f(x,y)$$

$$\lim_{e\to 0}\int_{-e}^{e}\int_{-e}^{e}\delta(x,y)\,dx\,dy = 1$$

KRONECK - kao Dirac f-ja ali u diskretnoj domeni

$$\delta(m,n) = \delta(m)\cdot\delta(n)$$

$$\sum_{i=-\infty}^{\infty}\sum_{j=-\infty}^{\infty} f(i,j)\,\delta(m-i,n-j) = f(m,n)$$

$$\sum_{i=-\infty}^{\infty}\sum_{j=-\infty}^{\infty}\delta(i,j) = 1$$

3. Ako je ulaz u 2-D sustav

$$x(m,n) = \delta(m-i, n-j)$$

~~ako~~ onda se pripadni odziv naziva impulsni odziv

$$h(m,n,i,j) = L[\delta(m-i, n-j)]$$

Linearni 2-D sustav je prostorno nepromjenjiv ako

$$h(m,n,i,j) = h(m-i, n-j, 0, 0) = h(m-i, n-j)$$

Odziv je u tom slučaju

$$y(m,n) = \sum_i \sum_j x(i,j)\, h(m-i, n-j)$$

4. 2-D sustav: Neka je $x(m,n)$ ulaz, a $y(m,n)$ izlaz 2-D sustava. 2-D sustav $L[\cdot]$ definiran je

$$y(m,n) = L[x(m,n)]$$

2-D linearni sustav: sustav je linearan ako za svaki a, b, x, y vrijedi

$$L[ax(m,n) + by(m,n)] = aL[x(m,n)] + bL[y(m,n)]$$

impulsni odziv: ako je ulaz 2-D sustava $x(m,n) = \delta(m-i, n-j)$ onda je izlaz impulsni odziv

$$h(m,n,i,j) = L[\delta(m-i, n-j)]$$

prostorno nep. sustav: linearni 2-D sustav $L[\cdot]$ je prostorno nepromjenjiv ako

$$h(m,n,i,j) = h(m-i, n-j, 0, 0) = h(m-i, n-j)$$

a odziv takvog sustava je

$$y(m,n) = \sum_i \sum_j x(i,j)\, h(m-i, n-j)$$

5. Operator $L[\cdot]$ je linearan ako vrijedi

$$L[ax(m,n)+by(m,n)] = aL[x(m,n)] + bL[y(m,n)]$$

$\forall a, b, m, n, x, y$

$$y(m,n) = L[x(m,n)] = 3x(m,n)+4$$

Pretpostavimo da je linearan.

neka je $x(m,n) = a\,z(m,n)$

$$L[x(m,n)] = L[a\,z(m,n)] = 3a\,z(m,n)+4$$

s druge strane zbog linearnosti

$$= aL[z(m,n)] = a(3z(m,n)+4)$$

$$= 3a\,z(m,n)+4a$$

$\Rightarrow$ kontradikcija, nije linearan.

6. $\nabla^2 = \dfrac{\partial^2}{\partial x^2} + \dfrac{\partial^2}{\partial y^2}$

$$\nabla^2[af(x,y)+bf(x,y)] =$$

$$= \frac{\partial^2}{\partial x^2}[af(x,y)+bf(x,y)] + \frac{\partial^2}{\partial y^2}[af(x,y)+bf(x,y)]$$

$$= a\frac{\partial^2}{\partial x^2}f(x,y) + b\frac{\partial^2}{\partial x^2}f(x,y) + a\frac{\partial^2}{\partial y^2}f(x,y) + b\frac{\partial^2}{\partial y^2}f(x,y)$$

$$= a\underbrace{\left[\frac{\partial^2}{\partial x^2}f(x,y) + \frac{\partial^2}{\partial y^2}f(x,y)\right]} + \ldots$$

$$= a\nabla^2 f(x,y) + b\nabla^2 f(x,y)$$

7. kontinuirana 2-D FT

$$F(\omega_1, \omega_2) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x,y)\, e^{-j(x\omega_1 + y\omega_2)}\, dx\, dy$$

SVOJSTVA:

a) rotacija $f(\pm x, \pm y) \multimap F(\pm\omega_1, \pm\omega_2)$

b) linearnost $a f(x,y) + b g(x,y) \multimap a F(\omega_1, \omega_2) + b G(\omega_1, \omega_2)$

c) separabilnost $f(x) g(y) \multimap F(\omega_1) G(\omega_2)$

d) skaliranje $f(ax, by) \multimap \dfrac{F(\omega_1/a, \omega_2/a)}{|ab|}$

e) pomak $f(x \pm a, y \pm b) \multimap e^{\pm j(a\omega_1 + b\omega_2)} F(\omega_1, \omega_2)$

f) modulacija $e^{-j(\eta_1 x + \eta_2 y)} f(x,y) \multimap F(\omega_1 \mp \eta_1, \omega_2 \mp \eta_2)$

g) linearna konvolucija $h(x,y) * f(x,y) \multimap H(\omega_1, \omega_2) F(\omega_1, \omega_2)$

h) multiplikacija $h(x,y) f(x,y) \multimap H(\omega_1, \omega_2) F(\omega_1, \omega_2)$

8. matrica rotacije

$$R(\varphi) = \begin{bmatrix} \cos\varphi & -\sin\varphi \\ \sin\varphi & \cos\varphi \end{bmatrix}$$

Teorem o rotaciji:

$$f(x,y) \multimap F(\omega_1, \omega_2) \Leftrightarrow f(x\cos\theta - y\sin\theta, x\cos\theta + y\sin\theta) \multimap F(\omega_1\cos\theta - \omega_2\sin\theta, \omega_1\cos\theta + \omega_2\sin\theta)$$

Dokaz:

$$\begin{bmatrix} x_R \\ y_R \end{bmatrix} = \begin{bmatrix} \cos\varphi & -\sin\varphi \\ \sin\varphi & \cos\varphi \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} x\cos\varphi - y\sin\varphi \\ x\sin\varphi - y\cos\varphi \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \cos\varphi & \sin\varphi \\ -\sin\varphi & \cos\varphi \end{bmatrix}\begin{bmatrix} x_R \\ y_R \end{bmatrix} = \begin{bmatrix} x_R\cos\varphi + y_R\sin\varphi \\ -x_R\sin\varphi + y_R\cos\varphi \end{bmatrix}$$

$$\mathcal{F}\{f(x_R, y_R)\} = \iint f(x_R, y_R)\, e^{-j(\omega_1 x + \omega_2 y)}\, dx\, dy$$

$$= \iint f(x_R, y_R)\, e^{-j(\omega_1 x_R\cos\varphi + \omega_1 y_R\sin\varphi - \omega_2 x_R\sin\varphi + \omega_2 y_R\cos\varphi)}\, dx_R\, dy_R$$

$$= \iint f(x_R, y_R)\, e^{-j[x_R(\omega_1\cos\varphi - \omega_2\sin\varphi) + y_R(\omega_1\sin\varphi + \omega_2\cos\varphi)]}\, dx_R\, dy_R$$

$$= \mathcal{F}\{\omega_1\cos\varphi - \omega_2\sin\varphi, \omega_1\sin\varphi + \omega_2\cos\varphi\}$$

9. 2-D Z transformacija

$$X(z_1, z_2) = \sum_{m=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} x(m,n) z_1^{-m} z_2^{-n}$$

ako se Z transformacija računa na jediničnoj kružnici

$|z_1| = |z_2| = 1$

dobiva se F. tran.

tj za $z_1 = e^{j\omega_1}$, $z_2 = e^{j\omega_2}$ $\Rightarrow$ 2-D F.T.

10. a) $f(x,y) = \text{sinc}\, x \;\text{sinc}\, y$

$$F(\omega_1, \omega_2) = \iint f(x,y) e^{-j(x\omega_1 + y\omega_2)} \, dx\, dy$$

$$= \int \text{sinc}\, x \, e^{-jx\omega_1} dx \int \text{sinc}\, y \, e^{-jy\omega_2} dy$$

$$= \text{rect}\frac{\omega_1}{2\pi} \, \text{rect}\frac{\omega_2}{2\pi}$$

b) $f(x,y) = \sin(x-y) + \sin(x+y)$

$$F(\omega_1, \omega_2) = \iint f(x,y) e^{-j(\omega_1 x + \omega_2 y)} dx\, dy$$

$$\cancel{= -2 \iint \text{sinc}\, y}$$

$$= 2 \iint \sin x \cos y \; e^{-j(\omega_1 x + \omega_2 y)} dx\, dy$$

$$= 2 \int \sin x \, e^{-j\omega_1 x} dx \int \cos y \, e^{-j\omega_2 y} dy$$

$$= -2j\pi^2 \left[\delta(\omega_1 - 1) - \delta(\omega_1 + 1) \right] \left[\delta(\omega_2 - 1) + \delta(\omega_2 + 1) \right]$$

$$\mathrm{sinc}\,x = \frac{\sin \pi x}{\pi x}$$

c) $$f(x,y) = \frac{\cos(x-y) - \cos(x+y)}{xy}$$

$$= \frac{2 \sin x \sin y}{xy}$$

$$F(\omega_1, \omega_2) = 2 \int \frac{\sin x}{x} e^{-j\omega_1 x} dx \int \frac{\sin y}{y} e^{-j\omega_2 y} dy$$

$a = \frac{1}{\pi}$

$$= 2 \int \frac{\sin \pi \frac{x}{\pi}}{\pi \cdot \frac{x}{\pi}} e^{-j\omega_1 x} dx \int \frac{\sin \pi \frac{y}{\pi}}{\pi \frac{y}{\pi}} e^{-j\omega_2 y} dy$$

$$= 2 \int \mathrm{sinc} \frac{x}{\pi} e^{-j\omega_1 x} dx \int \mathrm{sinc} \frac{y}{\pi} e^{-j\omega_2 y} dy$$

$$= 2 \pi^2 \,\mathrm{rect}\, \frac{\omega_1 \pi}{2\pi} \,\mathrm{rect}\, \frac{\omega_2 \pi}{2\pi}$$

$$= 2\pi^2 \,\mathrm{rect}\, \frac{\omega_1}{2} \,\mathrm{rect}\, \frac{\omega_2}{2}$$

11. DFT (2-D)

$$V(k,l) = \frac{1}{N} \sum_{m=0}^{N-1} \sum_{n=0}^{N-1} u(m,n)\, W_N^{km} W_N^{ln}$$

$$0 \leq k, l \leq N-1$$

$$W_N = e^{-j\frac{2\pi}{N}}$$

$$u(m,n) = \frac{1}{N} \sum_{k=0}^{N-1} \sum_{l=0}^{N-1} V(k,l)\, W_N^{-km} W_N^{-ln}$$

12. Linearna konv.

$$h(m,n) * u(m,n) = \sum_{i=-\infty}^{\infty} \sum_{j=-\infty}^{\infty} u(i,j) \cdot h(m-i, n-j)$$

cirkularna konv.

$$h(m,n) \circledast u(m,n) = \sum_{i=0}^{N-1} \sum_{j=0}^{N-1} h\big((m-i) \bmod N, (n-j) \bmod N\big)\, u(i,j)$$

$$h(m,n) = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \quad u(m,n) = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \quad u_{rot} = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$$

$$h(m,n) * u(m,n) = \begin{bmatrix} 1 & 3 & 2 \\ 3 & 8 & 6 \\ 0 & 3 & 4 \end{bmatrix}$$

→ veličine slike kod lin. konv. je $M = M_h \cdot M_u - 1$, $N = N_h \cdot N_u - 1$, a kod cirkularne veličine $M = \max(M_h, M_u)$ i $N = \max(N_h, N_u)$

$$h(m,n) \circledast u(m,n) = \begin{bmatrix} 7 & 6 \\ 8 & 8 \end{bmatrix}$$

13. Linearna

$$h(m,n) * u(m,n) = \sum_{i=0}^{\infty} \sum_{j=0}^{\infty} u(i,j)\, h(m-i, n-j)$$

cirkularna

$$h(m,n) \circledast u(m,n) = \sum_{i=0}^{N-1} \sum_{j=0}^{N-1} u(i,j)\, h((m-i) \bmod N, (n-j) \bmod N)$$

$$(h \circledast u)(m,n) \xrightarrow{DFT} H(k,l)\, U(k,l)$$

računanje cirk. konvolucije pomoću 2D DFT

ima složenost $\mathcal{O}(N^2 \log_2 N)$ što je manje od $\mathcal{O}(N^4)$

14. $h = \begin{bmatrix} 1 & 1 \\ -1 & -1 \end{bmatrix}$

$u = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 2 \\ 3 & 2 & 1 \end{bmatrix}$

$\begin{bmatrix} -1 & -1 \\ 1 & 1 \end{bmatrix}$ 2 3

$h' = \begin{bmatrix} -1 & -1 \\ 1 & 1 \end{bmatrix}$

$$y = h * u = \begin{bmatrix} 1 & 3 & 5 & 3 \\ 1 & 2 & 0 & -1 \\ 1 & 0 & -2 & -1 \\ -3 & -5 & -3 & -1 \end{bmatrix}$$

15. DCT

$C = \{c(k,n)\}$

$$c(k,n) = \begin{cases} \frac{1}{\sqrt{N}}, & k=0, \quad 0 \le n \le N-1 \\ \sqrt{\frac{2}{N}} \cos \frac{\pi(2n+1)k}{2N}, & 1 \le k \le N-1 \\ & 0 \le n \le N-1 \end{cases}$$

- realna transformacija (koef. su realni brojevi)
- ortogonalna transformacija $C^* = C \Rightarrow C^{-1} = C^T$
- nije realni dio DFT-a
- DCT niza duljine N moguće je izračunati pomoću DFT-a simetrično proširenog niza duljine 2N

- odlično pakiranje energije za jako korelirane slike (veći dio energije se nalazi u nižim koef.)
- često se koristi u kompresiji slike

RESTRIKCIJA BAZE

$$u \Rightarrow \boxed{\underset{N\times N}{C}} \xrightarrow{v} \boxed{I_m} \xrightarrow{w} \boxed{\underset{N\times N}{D}} \Rightarrow z$$

- vektor w ima prvih m elemenata iz v, a ostalo su nule
- w je vektor iz $m \leq N$-dimenzionalnog prostora, a u i v su iz N-dimenzionalnog prostora

greška između u i z $\quad J_m = \frac{1}{N} E\left[\sum_{n=0}^{N-1} |u(n) - z(n)|^2\right]$

Za dani broj koef. m KL predstavlja niz u s najmanjom pogreškom

NP filtri za redukciju broja uzoraka u frek. domeni

16. Neka je R autokorelacijska matrica vektora u. Neka je A $N\times N$ matrica čiji su stupci ortonormalizirani vlastiti vektori matrice R. Matrica A je unitarna matrica koja reducira R na dijagonalnu formu $Q = A^H R A$

$$v = A^H u$$

inverzna KLT: $u = Av = \sum_{k=0}^{N-1} v(k) a_k$

SVOJSTVA: dekoreliranost koef. - koef. KL transf. imaju $v(k)$ imaju očekivanje 0 i dekorelirani su

restrikcija baze

17. Matrica R je autokorelacijska matrica vektora u. Matrica A dimenzije N×N čiji su stupci ortonormirani svojstveni vektori matrice R. A dijagonalizira R.

$$Q = A^H R A$$

$$v = A^H u$$

inverzna $u = Av = \sum_{k=0}^{N-1} a_k v(k)$

$$R = \begin{bmatrix} 4 & 0 & -2 \\ 0 & 2 & 0 \\ -2 & 0 & 4 \end{bmatrix}$$

$$\det(R - \lambda I) = 0$$

$$\lambda_1 = \lambda_2 = 2 \qquad \lambda_3 = 6$$

$\lambda_1 = 2$

$$(R - \lambda I \mid 0) = \left[\begin{array}{ccc|c} 2 & 0 & -2 & 0 \\ 0 & 0 & 0 & 0 \\ -2 & 0 & 2 & 0 \end{array}\right]$$

$$x_1 = x_3 \quad , \quad x_2$$

$\cancel{x_2 = 0}$

$$x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = x_1 \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} + x_2 \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$

$$v_2 = v_1 = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} \qquad \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$

$\lambda_3 = 6$

$$(R - \lambda I \mid 0) = \left[\begin{array}{ccc|c} -2 & 0 & -2 & 0 \\ 0 & -4 & 0 & 0 \\ -2 & 0 & -2 & 0 \end{array}\right]$$

$$x_1 = -x_3$$

$$x_2 = 0 \qquad \begin{bmatrix} \frac{1}{\sqrt{2}} \\ 0 \\ -\frac{1}{\sqrt{2}} \end{bmatrix}$$

$$T^H A T = \begin{bmatrix} 1/\sqrt{2} & 0 & -1/\sqrt{2} \\ 0 & 1 & 0 \\ -1/\sqrt{2} & 0 & 1/\sqrt{2} \end{bmatrix} \begin{bmatrix} 4 & 0 & -2 \\ 0 & 2 & 0 \\ -2 & 0 & 4 \end{bmatrix} \begin{bmatrix} \frac{1}{\sqrt{2}} & 0 & -1/\sqrt{2} \\ 0 & 1 & 0 \\ +1/\sqrt{2} & 0 & 1/\sqrt{2} \end{bmatrix}$$

$$= \begin{bmatrix} 6/\sqrt{2} & 0 & -6/\sqrt{2} \\ 0 & 2 & 0 \\ -6/\sqrt{2} & 0 & 6/\sqrt{2} \end{bmatrix} \begin{bmatrix} & & \\ & & \\ & & \end{bmatrix} = \begin{bmatrix} 6 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$$

18. 2-D ortogonalna transf. slike može se prikazati

$$v(k,l) = \sum_{m=0}^{N-1} \sum_{n=0}^{N-1} \underbrace{a_{k,l}(m,n)}_{\text{jezgra tr.}} u(m,n)$$

$$u(m,n) = \sum_{k=0}^{N-1} \sum_{l=0}^{N-1} a^*_{k,l}(m,n) v(k,l)$$

SVOJSTVA: separabilnost i simetričnost

trans. je separabilna ako vrijedi

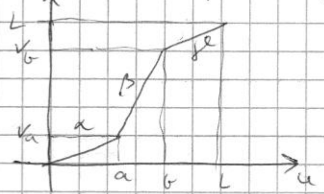
$$a_{k,l}(m,n) = a_k(m) a_l(n) = a(k,m) b(l,n)$$

$A = \{a(k,m)\}$ i $B = \{b(l,n)\}$ su unitarne matrice

ako je $A = B$ onda je matrica simetrična

19. rasterzanje kontrasta – uski pojas sivih tonova se rasteže na širi u svrhu dobivanja preglednije slike

preslikavanje $v = \begin{cases} \alpha u, & 0 \le u \le a \\ \beta(u-a) + v_a, & a \le u \le b \\ \gamma(u-b) + v_b, & b \le u \le L \end{cases}$



IZJEDNAČAVANJE HISTOGRAMA

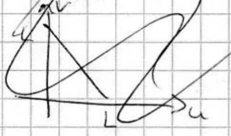
$$v = f(u) = \int_0^u p_u(w)\,dw = F_u(u) \Rightarrow \frac{dv}{du} = p_u(u)$$

$$p_v(v) = \left[p_u(u) \frac{du}{dv} \right]_{u=f^{-1}(v)} = \left[p_u(u) \cdot \frac{1}{p_u(u)} \right]_{u=f^{-1}(v)} = 1$$

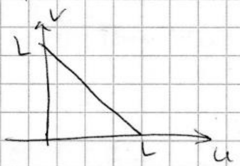
→ dobivena slika ima uniformnu distribuciju (histogram) sivih tonova

→ uniformna distribucija popravlja izgled slike.

20. ~~$u = V - L$ → negativ slike~~



$v = f(u) = L - u$ → negativ slike



$$p_u(x) = p(u = x) = \frac{n_x}{N}$$ → histogram prvog reda

n_x → broj točaka slike vrijednosti x

N → ukupan broj točaka slike

~~ZNAČAJKE: moment~~ $\cancel{m_i = \sum_{x=0}^{L-1} (x - m_i)^i p_u(x)}$

~~centralni moment~~ $\cancel{\mu_i = \sum_{x=0}^{L-1} (x}$

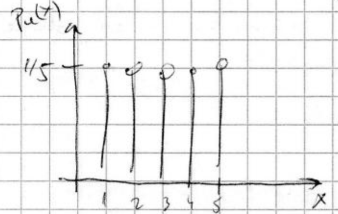
ZNAČAJKE: moment: $m_i = \sum_{x=0}^{L-1} x^i p_u(x)$

centralni moment $\mu_i = \sum_{x=0}^{L-1} (x - m_i)^i p_u(x)$

entropija $H = - \sum_{x=0}^{L-1} p_u(x) \log_2 p_u(x)$

- histogram se može računati lokalno i globalno

$$S = \begin{bmatrix} 1 & 2 & 3 & 4 & 5 \\ 2 & 3 & 4 & 5 & 1 \\ 3 & 4 & 5 & 1 & 2 \\ 4 & 5 & 1 & 2 & 3 \\ 5 & 1 & 2 & 3 & 4 \end{bmatrix} \Rightarrow S' = \begin{bmatrix} 5 & 4 & 3 & 2 & 1 \\ 4 & 3 & 2 & 1 & 5 \\ 3 & 2 & 1 & 5 & 4 \\ 2 & 1 & 5 & 4 & 3 \\ 1 & 5 & 4 & 3 & 2 \end{bmatrix}$$



→ isti histogrami

21. $p_u = 2 - 2u$

$p_v = 2v$

$v = f(u) = ?$

$$p_v(v)\,dv = p_u(u)\,du$$

$$2v\,dv = (2-2u)\,du$$

$$v^2 = 2u - u^2$$

$$v = \sqrt{2u-u^2}$$

$$f(u) = \sqrt{2u-u^2}$$

26. ako je slika dana funkcijom $f(x,y)$ u točki (x,y) gradijent G definira se kao

$$G\{f(x,y)\} = \begin{vmatrix} \frac{\partial f}{\partial x} \\ \frac{\partial f}{\partial y} \end{vmatrix}_{(x,y)} = \begin{bmatrix} f_x \\ f_y \end{bmatrix}$$

Robertsov, Sobelov...

- gradijent diskretne funkcije $f(x,y)$ može se procijeniti

$$g(x,y) = \{(f(x,y) - f(x+1,y))^2 + (f(x,y) - f(x,y+1))^2\}^{1/2}$$

a aproksimira

$$g(x,y) = |f(x,y) - f(x+1,y)| + |f(x,y) - f(x,y+1)|$$

Robertsov

$$g(x,y) = \{[f(x,y) - f(x+1,y+1)]^2 + [f(x+1,y) - f(x,y+1)]^2\}^{1/2}$$

aprok. pomoću apsolutnih

$$g(x,y) = |f(x,y) - f(x+1,y+1)| + |f(x+1,y) - f(x,y+1)|$$

IZNOS $\quad g(m,n) = \sqrt{f_1^2(m,n) + f_2^2(m,n)}$

SMJER $\quad \Theta(m,n) = \operatorname{arctg} \frac{f_2(m,n)}{f_1(m,n)}$

27. Robertsov operator

$$g(x,y) = \left[(f(x,y) - f(x+1, y+1))^2 + (f(x+1,y) - f(x,y+1))^2 \right]^{1/2}$$

aprok. pomoću aps.

$$g(x,y) = |f(x,y) - f(x+1, y+1)| + |f(x+1,y) - f(x,y+1)|$$

matrično $h_1 = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$ $h_2 = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$

$$g(x,y) = \sqrt{(1-0)^2 + 0} = 1$$

$$\Theta(x,y) = \operatorname{arctg} \frac{1}{0} = \frac{\pi}{2}$$

28. PROSTORNO USREDNJAVANJE

- zajednička značajka operacija za prostorno usrednjavanje je da izlaz ovisi o ulazu; operacije su memorijske, a izlaz se računa pomoću konvolucije slike i filtera

$$v(m,n) = \sum_{k=0}^{\infty} \sum_{l=0}^{\infty} a(k,l)\, u(m-k, n-l)$$

- u najjednostavnijem slučaju imp. odziv filtra je konst. $a(k,l) = \frac{1}{N_w}$, gdje je N_w broj točaka u prozoru w

$$v(m,n) = \frac{1}{N_w} \sum_k \sum_l u(m-k, n-l)$$

ako je slika $y(m,n) = u(m,n) + \eta(m,n)$ gdje je $\eta(m,n)$ Gaussov šum očekivanja nula i varijance σ^2 onda se prostornim usrednjavanjem dobije

$$v(m,n) = \frac{1}{N_w} \sum_k \sum_l u(m-k, n-l) + \eta_a(m,n)$$

gdje $\eta_a(m,n)$ ima varijancu $\frac{1}{N_w} \sigma_a^2$

PREDNOST: Što je maska veća, veći je N_w, pa je manji usrednjeni šum

MANA: originalna slika je zamućena zbog prostornog usrednjavanja

$$h = \frac{1}{9}\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

$$u = \begin{bmatrix} 1 & 2 & 3 & 4 & 5 \\ 2 & 3 & 4 & 5 & 1 \\ 3 & 4 & 5 & 1 & 2 \\ 4 & 5 & 1 & 2 & 3 \\ 5 & 1 & 2 & 3 & 4 \end{bmatrix}$$

$$y = \begin{bmatrix} 27 & 31 & 30 \\ 31 & 30 & 24 \\ 30 & 24 & 23 \end{bmatrix}$$

29. MEDIAN FILTAR

- izlazna vrijednost točke je jednaka medianu piksela sadržanih unutar prozora W
- median skupa brojeva se izračuna tako da se brojevi poredaju po veličini te se izabere onaj koji je rangiran kao srednji (N_w je neparan)
- nelinearan filtar
- korišten za uklanjanje izoliranih linija bez mijenjanja preostalog dijela slike
- dobar za binarni šum, a loš za Gaussov
- loše djeluje kada je broj točaka šuma veći od polovice ukupnog broja točaka

$$u = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 2 & 3 & 4 & 3 \\ 3 & 4 & 3 & 2 \\ 4 & 3 & 2 & 1 \end{bmatrix}$$

$w = 3 \times 3$

1 2 2 3 3 3 3 4 4

2 2 3 3 3 3

$$y = \begin{bmatrix} 3 & 3 \\ 3 & 3 \end{bmatrix}$$

30. 1) usrednjavanje - dobar za Gaussov šum

$$v(m,n) = \sum_k \sum_l u(m-k, n-l)\, a(k,l)$$

$$a(k,l) = \frac{1}{N_w}$$

$$y(m,n) = u(m,n) + \eta(m,n)$$

$$v(m,n) = \sum_k \sum_l u(m-k, n-l) + \eta(m,n)$$

2) MEDIAN - dobar za binarni šum

$$v(m,n) = \text{median}\,(u(m-k, n-l),\ k, l \in W)$$

W - prozor filtra

$$u = \begin{bmatrix} 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 1 \end{bmatrix}$$

1) $y = \frac{1}{9}\begin{bmatrix} 3 & 3 & 3 \\ 3 & 5 & 3 \\ 3 & 3 & 3 \end{bmatrix}$

0 0 0 0 0 1 1 1 1

2) $y = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

31. $\frac{\partial}{\partial x} f(x,y) \approx f(x-\frac{1}{2}, y) - f(x+\frac{1}{2}, y)$

$\frac{\partial}{\partial y} f(x,y) \approx f(x, y-\frac{1}{2}) - f(x, y+\frac{1}{2})$

$$\nabla^2 f(x,y) = \frac{\partial^2}{\partial x^2} f(x,y) + \frac{\partial^2}{\partial y^2} f(x,y)$$

$$\frac{\partial}{\partial x} f(x,y) = f(x-\tfrac{1}{2}, y) - f(x+\tfrac{1}{2}, y)$$

$$\frac{\partial^2}{\partial x^2} f(x,y) = \frac{\partial}{\partial x}\left[f(x-\tfrac{1}{2}, y) - f(x+\tfrac{1}{2}, y)\right]$$

$$= f(x-1, y) - f(x,y) - f(x,y) + f(x+1, y)$$

ANALOGNO

$$\frac{\partial^2}{\partial y^2} f(x,y) = f(x,y-1) - 2f(x,y) + f(x,y+1)$$

$$\nabla^2 f(x,y) = f(x-1,y) - 4f(x,y) + f(x+1,y) + f(x,y-1) + f(x,y+1)$$

$$\nabla^2 f(x,y) = \begin{bmatrix} 0 & 1 & 0 \\ 1 & -4 & 1 \\ 0 & 1 & 0 \end{bmatrix}$$

33. Mana kod inverznog filtriranja je velika osjetljivost na šum. Wienerov filtar je metoda koja uzima u obzir prisustvo šuma. Određuje estimaciju $\hat{u}(m,n)$ slučajnog niza $u(m,n)$ tako da minimizira srednju kvadratnu pogrešku

$$\sigma^2 = E\left[(u(m,n) - \hat{u}(m,n))^2\right]$$

IZRAZ W. F.:

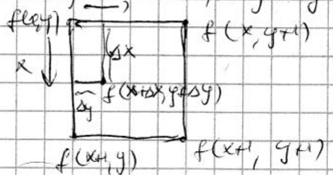
$$G(\omega_1,\omega_2) = \frac{H^*(\omega_1,\omega_2)\, S_{uu}(\omega_1,\omega_2)}{|H(\omega_1,\omega_2)|^2 S_{uu}(\omega_1,\omega_2) + S_{\eta\eta}(\omega_1,\omega_2)}$$

za $S_{\eta\eta} \ll S_{uu}(\omega_1,\omega_2)\,|H(\omega_1,\omega_2)|^2$

$$\Rightarrow G(\omega_1,\omega_2) = \frac{H^*(\omega_1,\omega_2)}{|H(\omega_1,\omega_2)|^2} = \frac{1}{H(\omega_1,\omega_2)} \rightarrow \text{inverzni filtar}$$

34. Interpolacija se provodi da bi se povećala rezolucija slike

$0 \le \Delta x, \Delta y \le 1$



$$f(x+\Delta x, y+\Delta y) = f(x,y)(1-\Delta x)(1-\Delta y) + f(x+1,y)(1-\Delta y)\Delta x$$
$$f(x+1,y+1)\Delta x \Delta y + f(x,y+1)\Delta y(1-\Delta x)$$

35. LINEARNA

$$\cancel{v(k,l) = \sum_{m=0}\sum_{n=0} u(m,n)\, a_{kl}(m,n)}$$

$$v(k,l) = \sum_{m=0}^{N-1}\sum_{n=0}^{N-1} a_{kl}(m,n)\, u(m,n)$$

$$u(m,n) = \sum_{k=0}^{N-1}\sum_{l=0}^{N-1} b_{mn}(k,l)\, v(k,l)$$

gdje je slika $u(m,n)$, a $a_{kl}(m,n)$ i $b_{mn}(k,l)$ su ~~matrice~~ ~~transformacije~~ jezgre transformacije odnosno inverzne transformacije

$$a_{k,l}(m,n) = \frac{1}{N} \exp\left(-j \frac{2\pi(mk+nl)}{N}\right)$$

$$c_k = \frac{1}{\sqrt{N}} \exp\left(-j \frac{2\pi mk}{N}\right)$$

$$d_l = \frac{1}{\sqrt{N}} \exp\left(-j \frac{2\pi nl}{N}\right)$$

NELINEARNE (izobličenje, rasterenje)

$$x = x(u,v) \qquad y = y(u,v)$$

$$u = u(x,y) \qquad v = v(x,y)$$

x, y, u, v su nelinearne funkcije druge varijable (npr. polinom n-tog stupnja)

- polinomsko izobličenje

$$x = \sum_{i=0}^{m} \sum_{j=0}^{m} a_{ij}\, u^i v^j$$

$$y = \sum_{i=0}^{m} \sum_{j=0}^{m} b_{ij}\, u^i v^j$$

38. $p_{u_1u_2}(x_1, x_2) = p(u_1 = x_1, u_2 = x_2) = \dfrac{N(x_1, x_2)}{N}$ → ocjena histograma 2. reda

ukupan broj točaka u slici

broj parova točaka (x_1, x_2) u slici

– koristi se u analizi teksture

ZNAČAJKE:

autokorelacija $S_A = \sum_{x_1=0}^{L-1} \sum_{x_2=0}^{L-1} x_1 x_2 \, p(x_1, x_2)$

kovarijanca $S_C = \sum_{x_1=0}^{L-1} \sum_{x_2=0}^{L-1} (x_1 - \bar{x}_1)(x_2 - \bar{x}_2) \, p(x_1, x_2)$

inercija $S_I = \sum_{x_1=0}^{L-1} \sum_{x_2=0}^{L-1} (x_1 - x_2)^2 \, p(x_1, x_2)$

apsolutna vrijednost razlike $S_V = \sum_{x_1=0}^{L-1} \sum_{x_2=0}^{L-1} |x_1 - x_2| \, p(x_1, x_2)$

energija $S_E = \sum_{x_1=0}^{L-1} \sum_{x_2=0}^{L-1} |p(x_1, x_2)|^2$

entropija $S_T = -\sum_{x_1=0}^{L-1} \sum_{x_2=0}^{L-1} p(x_1, x_2) \log_2 p(x_1, x_2)$

39. $u_1 = u(m, n) \quad u_2 = u(m+1, n+1)$

$$S = \begin{bmatrix} 1 & 2 & 3 & 4 & 5 \\ 2 & 3 & 4 & 5 & 1 \\ 3 & 4 & 5 & 1 & 2 \\ 4 & 5 & 1 & 2 & 3 \\ 5 & 1 & 2 & 3 & 4 \end{bmatrix} u_1$$

$$u_2 = \begin{bmatrix} 3 & 4 & 5 & 1 & 0 \\ 4 & 5 & 1 & 2 & 0 \\ 5 & 1 & 2 & 3 & 0 \\ 1 & 2 & 3 & 4 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$H = \frac{1}{25} \begin{array}{c} \\ 0 \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} \begin{array}{c} \begin{array}{cccccc} 0 & 1 & 2 & 3 & 4 & 5 \end{array} \\ \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 2 & 0 & 0 & 3 & 0 & 0 \\ 2 & 0 & 0 & 0 & 3 & 0 \\ 2 & 0 & 0 & 0 & 0 & 3 \\ 1 & 4 & 0 & 0 & 0 & 0 \\ 2 & 0 & 3 & 0 & 0 & 0 \end{bmatrix} \end{array}$$

$S_A = \sum_{x_1=0}^{L-1} \sum_{x_2=0}^{L-1} x_1 x_2 \, p(x_1, x_2) = 4.96$

$S_I = \sum_{x_1=0}^{L-1} \sum_{x_2=0}^{L-1} (x_1 - x_2)^2 \, p(x_1, x_2) = 6.72$

40. $u_1(m,n) \quad u_2 = (m+1, n)$

$\xrightarrow{\quad} m$, $n \downarrow$

$$u_1 = \begin{bmatrix} 1 & 5 & 3 & 2 \\ 5 & 3 & 2 & 4 \\ 3 & 2 & 4 & 1 \\ 2 & 4 & 1 & 5 \\ 2 & 1 & 5 & 3 \end{bmatrix}$$

$$u_2 = \begin{bmatrix} 5 & 3 & 2 & 4 \\ 3 & 2 & 4 & 1 \\ 2 & 4 & 1 & 5 \\ 2 & 1 & 5 & 3 \\ 1 & 5 & 3 & 4 \end{bmatrix}$$

$$H = \frac{1}{20} \begin{array}{c|ccccc|} & 1 & 2 & 3 & 4 & 5 \\ 1 & & & & & 4 \\ 2 & 4 & & & 4 & \\ 3 & & 3 & & 1 & \\ 4 & 3 & & & & \\ 5 & & & 4 & & \end{array}$$

$$H = \frac{1}{20} \begin{bmatrix} 0 & 0 & 0 & 0 & 4 \\ 1 & 0 & 0 & 4 & 0 \\ 0 & 3 & 0 & 1 & 0 \\ 3 & 0 & 0 & 0 & 0 \\ 0 & 0 & 4 & 0 & 0 \end{bmatrix}$$

$$S_H = \sum_{x_1=0}^{L-1} \sum_{x_2=0}^{L-1} p(x_1, x_2) \log_2 p(x_1, x_2) = 2.65 \rightarrow \text{entropija}$$

$$S_4 = \sum_{x_1=0}^{L-1} \sum_{x_2=0}^{L-1} |p(x_1, x_2)|^2 = 0.17 \rightarrow \text{energija}$$

41. Histogram prvog reda $p_u(x) = p(u=x) = \frac{n_x}{N}$ → broj točaka vrijednosti x

N → ukupan broj točaka

- može se računati lokalno i globalno
- značajke: moment: $m_i = \sum_{x=0}^{L-1} x^i p_u(x)$

centralni moment $\mu_i = \sum_{x=0}^{L-1} (x - m_i)^i p_u(x)$

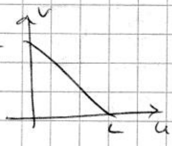
entropija $H = -\sum_{x=0}^{L-1} p_u(x) \log_2 p_u(x)$

$$S = \begin{bmatrix} 1 & 2 & 3 & 4 & 5 \\ 2 & 3 & 4 & 5 & 1 \\ 3 & 4 & 5 & 1 & 2 \\ 4 & 5 & 1 & 2 & 3 \\ 5 & 1 & 2 & 3 & 4 \end{bmatrix}$$

$\rightarrow$ histogram 1. reda

negativ slike

$$v = f(u) = L - u$$



$$S' = \begin{bmatrix} 5 & 4 & 3 & 2 & 1 \\ 4 & 3 & 2 & 1 & 5 \\ 3 & 2 & 1 & 5 & 4 \\ 2 & 1 & 5 & 4 & 3 \\ 1 & 5 & 4 & 3 & 2 \end{bmatrix}$$

$\rightarrow$ isti histogram 1. reda kao i za original

$u_1 = S \qquad u_2 = S'$

$$H = \begin{array}{c|c} & \begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} & \begin{bmatrix} 0 & 0 & 0 & 0 & 5 \\ 0 & 0 & 0 & 5 & 0 \\ 0 & 0 & 5 & 0 & 0 \\ 0 & 5 & 0 & 0 & 0 \\ 5 & 0 & 0 & 0 & 0 \end{bmatrix} \end{array} \cdot \frac{1}{25}$$

$\rightarrow$ histogram 2. reda

42. Zadatak vanije! (za histogram 1. reda)

HISTOGRAM 2. REDA $\quad p(x_1, x_2) = p(u_1 = x_1, u_2 = x_2) = \frac{N(x_1, x_2)}{N}$

ukupan broj točaka

ZNAČAJKE:

broj parova točaka

autokorelacija $\quad S_a = \sum_{x_1=0}^{L-1} \sum_{x_2=0}^{L-1} x_1 x_2 \, p(x_1, x_2)$

kovarijanca $\quad S_c = \sum_{x_1=0}^{L-1} \sum_{x_2=0}^{L-1} (x_1 - \bar{x}_1)(x_2 - \bar{x}_2) \, p(x_1, x_2)$

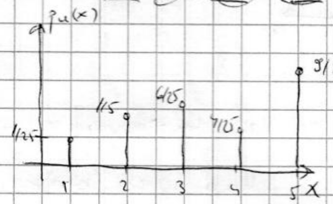
inercija $\quad S_I = \sum_{x_1=0}^{L-1} \sum_{x_2=0}^{L-1} (x_1 - x_2)^2 \, p(x_1, x_2)$

aps. vr. razlike $\quad S_v = \sum_{x_1=0}^{L-1} \sum_{x_2=0}^{L-1} |x_1 - x_2| \, p(x_1, x_2)$

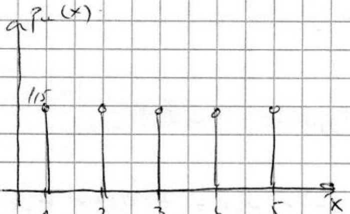
entropija $\quad S_T = -\sum_{x_1=0}^{L-1}\sum_{x_2=0}^{L-1} p(x_1, x_2)\log_2(x_1, x_2)$

energija $\quad S_a = \sum_{x_1=0}^{L-1}\sum_{x_2=0}^{L-1} |p(x_1, x_2)|^2$

$$u_1 = S_1 = \begin{bmatrix} 1 & 2 & 3 & 5 & 5 \\ 2 & 3 & 5 & 5 & 4 \\ 3 & 5 & 5 & 4 & 3 \\ 5 & 5 & 4 & 3 & 2 \\ 5 & 4 & 3 & 2 & 2 \end{bmatrix}$$



$$u_2 = S_2 = \begin{bmatrix} 1 & 5 & 3 & 2 & 4 \\ 5 & 3 & 2 & 4 & 1 \\ 3 & 2 & 4 & 1 & 5 \\ 2 & 4 & 1 & 5 & 3 \\ 2 & 1 & 5 & 3 & 4 \end{bmatrix}$$



$$H = \frac{1}{25}\begin{array}{c|ccccc} & 1 & 2 & 3 & 4 & 5 \\ \hline 1 & 1 & & & & \\ 2 & & & 2 & 1 & 2 \\ 3 & & & 3 & & 3 \\ 4 & 4 & & & & \\ 5 & & 5 & & 4 & \end{array}$$

$$H = \frac{1}{25}\begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 2 & 1 & 2 \\ 0 & 0 & 3 & 0 & 3 \\ 4 & 0 & 0 & 0 & 0 \\ 0 & 5 & 0 & 4 & 0 \end{bmatrix}$$

$$S_a = \sum_{x_1}\sum_{x_2} |p(x_1, x_2)|^2 = 0{,}1504$$

43. Sobelov operator je gradijentni operator koji se koristi za detekciju rubova. Od gradijentnih operatora (računanje gradijenta u dva ortogonalna smjera) imamo još i Robertsov, Prewittov i Frei-Chenov. Postoje i kompas operatori (detekcija smjera rubova u više smjerova).

$$h_1 = \begin{bmatrix} -1 & 0 & 1 \\ -2 & 0 & 2 \\ -1 & 0 & 1 \end{bmatrix} \qquad h_2 = \begin{bmatrix} -1 & -2 & -1 \\ 0 & 0 & 0 \\ 1 & 2 & 1 \end{bmatrix}$$

$$u = \begin{bmatrix} 1 & 2 & 3 & 5 & 5 \\ 2 & 3 & 5 & 5 & 4 \\ 3 & 5 & 5 & 4 & 3 \\ 5 & 5 & 4 & 3 & 2 \\ 5 & 4 & 3 & 2 & 2 \end{bmatrix}$$

$$h_1' = \begin{bmatrix} 1 & 0 & -1 \\ 2 & 0 & -2 \\ 1 & 0 & -1 \end{bmatrix} \qquad h_2' = \begin{bmatrix} 1 & 2 & 1 \\ 0 & 0 & 0 \\ -1 & -2 & -1 \end{bmatrix}$$

$$g_1 = \begin{bmatrix} -10 & -6 & 2 \\ -6 & 2 & 7 \\ 2 & 7 & 7 \end{bmatrix}$$

$$g_2 = \begin{bmatrix} -10 & -6 & 2 \\ -6 & 2 & 7 \\ 2 & 7 & 7 \end{bmatrix}$$

44. Roberts, Sobel, Prewitt, Frei-Chen

detekcija rubova u 2 ortogonalna smjera

IZNOS: $g = \sqrt{g_1^2 + g_2^2}$

SMJER: $\theta = \mathrm{arctg} \frac{g_2}{g_1}$

- daju najbolje rezultate za oštre rubove
- problem: nemogućnost točne detekcije rubova u prisutnosti šuma
- rješenje: povećanje dimenzija maski da bi se postigao

efekt usrednjavanja vodi smanjenju utjecaja šuma

→ velika maska smanji utjecaj šuma, ali zamućuje sliku

Roberts : $h_1 = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$ $\qquad h_2 = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$

$$u = \begin{bmatrix} 1 & 2 & 3 & 5 & 5 \\ 2 & 3 & 5 & 5 & 4 \\ 3 & 5 & 5 & 4 & 3 \\ 5 & 5 & 4 & 3 & 2 \\ 5 & 4 & 3 & 2 & 2 \end{bmatrix}$$

$$g_1 = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \qquad h_1' = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$$

$$g_2 = \begin{bmatrix} +2 & +3 & +2 & -1 \\ +3 & +2 & -1 & -2 \\ +2 & -1 & -2 & -2 \\ -1 & -2 & -2 & -1 \end{bmatrix} \qquad h_2' = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$$

45. Gradijentne maske daju najbolje rezultate za oštre rubove, a kada ovi postupci blaži bolje rezultate daju druge derivacije.

→ Laplaceov operator

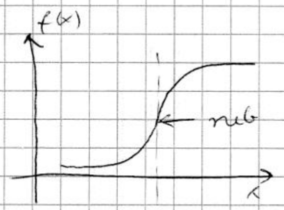
$$\nabla^2 f(x,y) = \frac{\partial^2}{\partial x^2} f(x,y) + \frac{\partial^2}{\partial y^2} f(x,y)$$

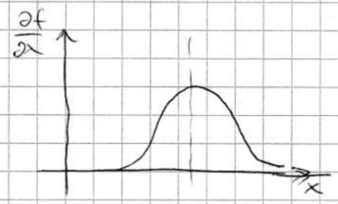
$$\begin{bmatrix} 0 & -1 & 0 \\ -1 & 4 & -1 \\ 0 & -1 & 0 \end{bmatrix} \quad \begin{bmatrix} -1 & -1 & -1 \\ -1 & 8 & -1 \\ -1 & -1 & -1 \end{bmatrix} \quad \begin{bmatrix} 1 & -2 & 1 \\ -2 & 4 & -2 \\ 1 & -2 & 1 \end{bmatrix}$$

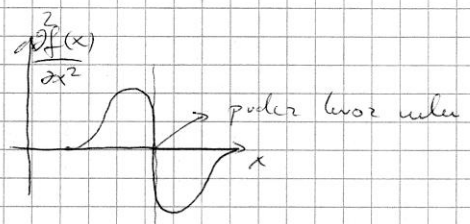
- detektira položaj rubova ne i jačinu

a) odziv blaže na prvog funkcija → proizvodi dvostruke rubove

b) nula funkcije → velika osjetljivost na šum







- Marr dodao N? filter da smanjio utjecaj šuma

$$L = \begin{bmatrix} 0 & -1 & 0 \\ -1 & 4 & -1 \\ 0 & -1 & 0 \end{bmatrix}$$

$$u = \begin{bmatrix} 1 & 2 & 3 & 5 & 5 \\ 2 & 3 & 5 & 5 & 4 \\ 3 & 5 & 5 & 4 & 3 \\ 5 & 5 & 4 & 3 & 2 \\ 5 & 4 & 3 & 2 & 2 \end{bmatrix}$$

$$g = \begin{bmatrix} -2 & 4 & 2 \\ 4 & 2 & 0 \\ 2 & 0 & 0 \end{bmatrix}$$

Thats all folks