1

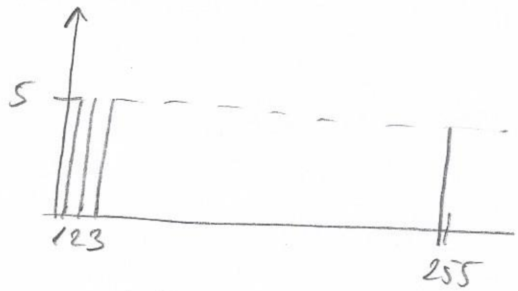
1) Rastezanje

Ogranicavanje intervala

Thresholding

2) Histogram je relativna frekvencija pojave različitih vrijednosti točaka u slici

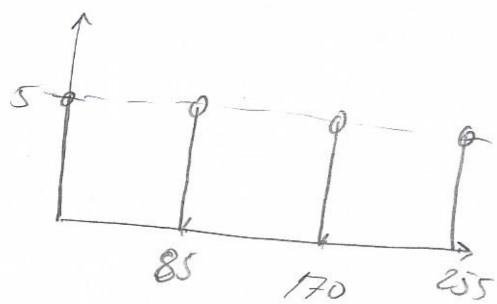
3) $S = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 2 & 2 & 2 & 2 & 2 \\ 3 & 3 & 3 & 3 & 3 \\ 255 & 255 & 255 & 255 & 255 \end{bmatrix}$



4)

5) Imamo 4 uzorka koji moraju biti ravnomjerno razmaknuti između 0 i 255

$255/3 = 85$ - korak



(2)

1)

$$v(k,l) = \sum_{m=0}^{N-1}\sum_{n=0}^{N-1} a(m,n)\, u(m,n) \qquad 0 \leq k,l \leq N-1$$

→ jezgra transformacije

$$u(m,n) = \sum_{k=0}^{N-1}\sum_{l=0}^{N-1} b(k,l)\, v(k,l) \qquad 0 \leq m,n \leq N-1$$

→ jezgra inverzne transformacije

2)

Jezgra je separabilna ako vrijedi

$$a_{k,l}(m,n) = c_k(m)\, d_l(n)$$

Jezgra je simetrična ako vrijedi

$$c_k(m) = d_l(n)$$

3) DA Jezgra 2D DFT

$$a_{k,l}(m,n) = \frac{1}{N} e^{-j\frac{2\pi(mk+nl)}{N}} = \underbrace{\frac{1}{\sqrt{N}} e^{-j\frac{2\pi km}{N}}}_{c_k(m)} \cdot \underbrace{\frac{1}{\sqrt{N}} e^{-j\frac{2\pi ln}{N}}}_{d_l(n)}$$

4)

$$V = A U A^T$$

$$U = A^{T*} V A^* \quad , A \text{ je unitarna matrica}$$

5)

1) Očuvanje energije
2) Raspodjela energije
3) Nekoreliranost koeficijenata transformacije

6)

$$v = Au \qquad \|v\| = \|u\|$$

$$\|v\|^2 = v^H v = A^H u^H A u = u^H u = \|u\|$$

7) DA

8) NE

9) DA

3.

1) $C = \begin{bmatrix} \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \\ \frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} \end{bmatrix}$ $S = \begin{bmatrix} 1 & 1 & 2 & 2 & 3 & 3 & 4 & 4 & 5 & 5 \\ 1 & 1 & 2 & 2 & 3 & 3 & 4 & 4 & 6 & 6 \\ 6 & 6 & 7 & 7 & 8 & 8 & 9 & 9 & 0 & 0 \\ 6 & 6 & 7 & 7 & 8 & 8 & 9 & 9 & 0 & 0 \end{bmatrix}$ $C^T = C = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \frac{1}{\sqrt{2}}$

DCT se radi po blokovima $C \cdot S \cdot C^{-1}$, a s obzirom da je C unitarna (jezgra dct je unitarna) vrijedi $C^{-1} = C^T$

$$\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \cdot \frac{1}{\sqrt{2}} \cdot \frac{1}{\sqrt{2}} = \begin{bmatrix} 2 & 2 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} = \begin{bmatrix} 4 & 0 \\ 0 & 0 \end{bmatrix} \frac{1}{2} = \begin{bmatrix} 2 & 0 \\ 0 & 0 \end{bmatrix}$$

$$V = \begin{bmatrix} 2 & 0 & 4 & 0 & 6 & 0 & 8 & 0 & 10 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 12 & 0 & 14 & 0 & 16 & 0 & 18 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

2) MSE = 0 jer radimo restrikciju baze 1x1 kvadratnu, što znači da samo gornji lijevi element ostavljamo, a ostali idu u 0, a kod nas je to već tako pa pri inverzu ništa ne gubimo

3) 0 2, 4, 6 8 10 12 14 16 18

3 bita → $2^3 = 8$ razina $\frac{18}{2^3-1} = 2{,}57$

0	000	0, 2
2,57	001	5
5,14	010	6
7,71	011	8, 10
10,28	100	12
12,85	101	14
15,42	110	16
18	111	18

4) Nakon što istosmjernu komponentu izbacimo, a potom provedemo inverznu transformaciju, rezultat (slika) će biti umanjena za srednju vrijednost svih piksela

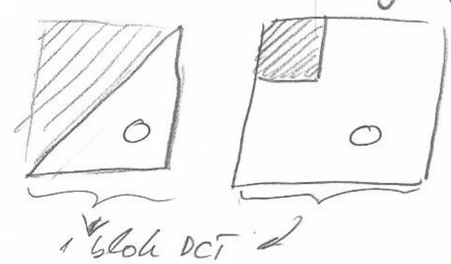
5) 1-D KL transformacija definirana je izrazom

$v = A^H u$, gdje je A matrica koja dijagonalizira R, autokorelacijsku matricu vektora u

Inverzna KL transformacija $u = Av = \sum_{k=0}^{N-1} v(k) a_k$

Restrikcija baze je redukcija broja uzoraka u frekvencijskoj domeni

-> može biti trokutasta i kvadratna



(4.) Linearnost

$$L[a\,x(m,n) + b\,y(m,n)] = a\,L[x(m,n)] + b\,L[y(m,n)]$$

1) $y(m,n) = 3x(m,n) + 9$

Pretp. da je linearan $x(m,n) = a\,z(m,n)$

$$L[x(m,n)] = L[a\,z(m,n)] = a\,z(m,n) + 9$$

$$\neq$$

$$a\,L[z(m,n)] = a\,(z(m,n) + 9) = a\,z(m,n) + 9a$$

nije linearan

2) ?

3) $$\nabla^2 = \frac{\delta^2}{\delta x^2} + \frac{\delta^2}{\delta y^2}$$

$$\nabla^2\left[af(x,y) + bf(x,y)\right] = \frac{\delta^2}{\delta x^2}\left[af(x,y) + bf(x,y)\right] + \frac{\delta^2}{\delta y^2}\left[af(x,y) + bf(x,y)\right]$$

$$= a\frac{\delta^2}{\delta x^2} f(x,y) + b\frac{\delta^2}{\delta x^2} f(x,y) + a\frac{\delta^2}{\delta y^2} f(x,y) + b\frac{\delta^2}{\delta y^2} f(x,y) =$$

$$= a\left[\frac{\delta^2}{\delta x^2} f(x,y) + \frac{\delta^2}{\delta y^2} f(x,y)\right] + b\left[\frac{\delta^2}{\delta x^2} f(x,y) + \frac{\delta^2}{\delta y^2} f(x,y)\right] =$$

$$= a\nabla^2(f(x,y)) + b\nabla^2(f(x,y))$$ Linearan!

5)

1)
1) Osnovne transformacije vrijednosti točaka
2) Transformacije linearne po segmentima – za kontrast
3) Modeliranje histograma
4) Aritmetičke i logičke operacije

2) Median – salt & pepper

Avg – gauss

3) impulsni šum (0 i 255) – trebamo median
↳ s&p

$$S = \begin{bmatrix} 1 & 2 & 3 & 4 & 5 \\ 1 & 2 & 3 & 4 & 5 \\ 1 & 2 & 255 & 4 & 5 \\ 1 & 2 & 3 & 4 & 5 \\ 1 & 2 & 3 & 4 & 5 \end{bmatrix} \Rightarrow S_{filt} = \begin{bmatrix} 0 & 1 & 2 & 3 & 0 \\ 1 & 2 & 3 & 4 & 4 \\ 1 & 2 & 3 & 4 & 4 \\ 1 & 2 & 3 & 4 & 4 \\ 0 & 1 & 2 & 3 & 0 \end{bmatrix}$$

↳ npr. 2 2 2 3 3 4 4 4 255

4)

$$S_{neg} = \begin{bmatrix} 254 & 253 & 252 & 251 & 250 \\ 254 & 253 & 252 & 251 & 250 \\ 254 & 253 & 0 & 251 & 250 \\ 254 & 253 & 252 & 251 & 250 \\ 254 & 253 & 252 & 251 & 250 \end{bmatrix}$$

$L = 255$

negativ $v = f(u) = L - u$

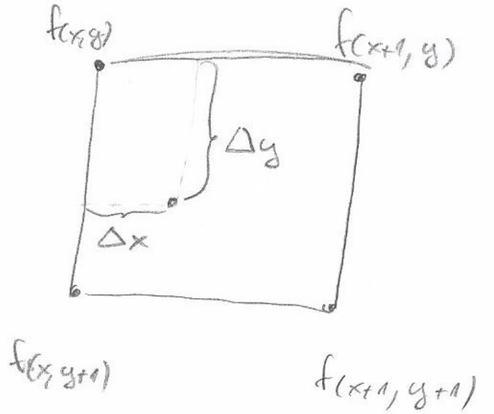
5) Roberts gradijent

$$g(x,y) = \left\{ \left(f(x,y) - f(x+1,y+1)\right)^2 - \left(f(x+1,y) - f(x,y+1)\right)^2 \right\}^{1/2}$$

aproks.

$$g(x,y) = \left| f(x,y) - f(x+1,y+1) \right| - \left| f(x+1,y) - f(x,y+1) \right|$$

6)



$$f(x+\Delta x, y+\Delta y) = f(x,y)(1-\Delta x)(1-\Delta y) + f(x+1,y)\Delta x(1-\Delta y) + f(x,y+1)\Delta y(1-\Delta x) + f(x+1,y+1)\Delta x \Delta y$$

6.

1) 1) Inverzni filtar

2) Pseudoinverzni filtar

3) Wienerov filtar

2) Osjetljivost na šum i problem dizajna zbog moguće nestabilnosti sustava (nule i polovi se zamijene)

3) Wienerov filtar je metoda obnavljanja slike koja u obzir uzima prisustvo šuma.

Potrebno je odrediti estimaciju $\hat{u}(m,n)$ takvu da se minimizira srednja kvadratna pogreška

$$\sigma_e^2 = E\left\{\left[u(m,n)-\hat{u}(m,n)\right]^2\right\}$$

$$H(\omega_1,\omega_2) = \frac{H^*(\omega_1,\omega_2)\, S_{uu}(\omega_1,\omega_2)}{|H(\omega_1,\omega_2)|^2 S_{uu} + S_{\eta\eta}(\omega_1,\omega_2)}$$

za $S_{\eta\eta} \ll S_{uu}$

$$H(\omega_1,\omega_2) = \frac{H^*(\omega_1,\omega_2)\,\cancel{S_{uu}(\omega_1,\omega_2)}}{\underbrace{|H(\omega_1,\omega_2)|^2}_{\hookrightarrow H^*(\omega_1,\omega_2)\cdot H(\omega_1,\omega_2)}\cancel{S_{uu}(\omega_1,\omega_2)}} = \underbrace{\frac{1}{H(\omega_1,\omega_2)}}$$

inverzni filtar

4)

1) translatiranje

2) skaliranje

3) rotacija

4) nelinearno iskrivljenje

5)

$(s_x, s_y) = (3, \frac{1}{3})$

$(t_x, t_y) = (2, -2)$

$\varphi = \frac{\pi}{3}$

$$\begin{bmatrix} s_x & 0 & 0 \\ 0 & s_y & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ 1 \end{bmatrix} \qquad \begin{bmatrix} 1 & 0 & t_x \\ 0 & 1 & t_y \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} \cos(\varphi) & -\sin(\varphi) & 0 \\ \sin(\varphi) & \cos(\varphi) & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} 1/2 & -\sqrt{3}/2 & 0 \\ \sqrt{3}/2 & 1/2 & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & -2 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 3 & 0 & 0 \\ 0 & 1/3 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1/2 & -\sqrt{3}/2 & 0 \\ \sqrt{3}/2 & 1/2 & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 3 & 0 & 2 \\ 0 & 1/3 & -2 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 3/2 & -\frac{1}{2\sqrt{3}} & 1+\sqrt{3} \\ \frac{3\sqrt{3}}{2} & \frac{1}{6} & \sqrt{3}-1 \\ 0 & 0 & 1 \end{bmatrix}$$

a) $$\begin{bmatrix} 3/2 & -\frac{1}{2\sqrt{3}} & 1+\sqrt{3} \\ \frac{3\sqrt{3}}{2} & \frac{1}{6} & \sqrt{3}-1 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 3 \\ 5 \\ 1 \end{bmatrix} = \Big[\qquad \Big]$$

sumnjivo mi je ovo :)

b) Ne?

c) ~~i~~nverz ovog što smo dobili

d) jedinična matrica

6) Perspektivna transformacija preslikava svaku točku (X, Y, Z) iz trodim. prostora u dvodimenzionalnu projekciju (x, y) te točke u ravninu slike

$$x = \frac{fX}{f-Z} \qquad y = \frac{fY}{f-Z}$$

f — fokus

⑦

1) $f(x,y) = \mathrm{sinc}(x-3)\,\mathrm{sinc}(y-6)$

$\mathrm{sinc}(at) \circ\!-\!\bullet \frac{1}{a}\,\mathrm{rect}\left(\frac{\omega}{2\pi a}\right)$

$F(\omega_1,\omega_2) = e^{-j\omega 3}\,\mathrm{rect}\left(\frac{\omega_1}{2\pi}\right)\cdot \mathrm{rect}\left(\frac{\omega_2}{2\pi}\right) e^{-j6\omega}$ $\quad x(t-t_0) \circ\!-\!\bullet X(\omega)\, e^{-j\omega t_0}$

$F(\omega_1,\omega_2) = e^{-j9\omega}\,\mathrm{rect}\left(\frac{\omega_1}{2\pi}\right)\mathrm{rect}\left(\frac{\omega_2}{2\pi}\right)$

2) $f(x,y) = \mathrm{rect}(x)\,\mathrm{rect}(y)$ $\quad \mathrm{rect}\left(\frac{t}{T}\right) \circ\!-\!\bullet T\,\mathrm{sinc}\left(\frac{\omega T}{2\pi}\right)$

$F(\omega_1,\omega_2) = \mathrm{sinc}\left(\frac{\omega_1}{2\pi}\right)\mathrm{sinc}\left(\frac{\omega_2}{2\pi}\right)$

3) $f(x,y) = \sin(2x-2y) + \sin(2x+2y) = 2\sin(2x)\cos(2y)$

$F(\omega_1,\omega_2) = 2\cdot\left[-j\pi\left(\delta(\omega_1-\omega_0) - \delta(\omega_1+\omega_0)\right)\right]\left[\pi\left(\delta(\omega_2-\omega_0) + \delta(\omega_2+\omega_0)\right)\right]$

4) $f(x,y) = \frac{\cos(x-y) - \cos(x+y)}{xy} = \frac{-2\sin(x)\sin(-y)}{xy} = \frac{\sin x}{x}\,\frac{\sin y}{y} =$

sinc

$F(\omega_1,\omega_2) = \mathrm{rect}\left(\frac{\omega_1}{2\pi}\right)\mathrm{rect}\left(\frac{\omega_2}{2\pi}\right)$