PRIMJER 3. – *MATLAB* filtdemo

Prijenosna funkcija (IIR)

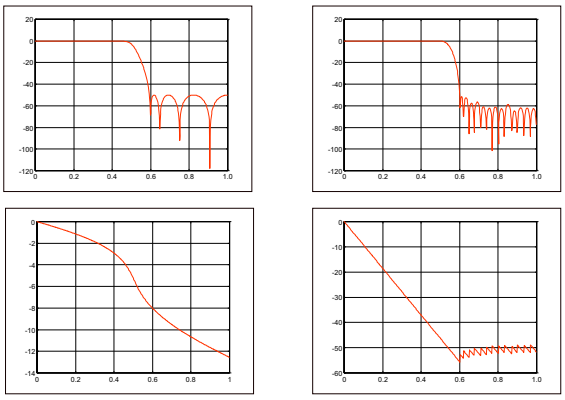
$$H(z)=\frac{0{,}06+0{,}30\cdot z^{-1}+0{,}78\cdot z^{-2}+1{,}30\cdot z^{-3}+1{,}53\cdot z^{-4}+1{,}30\cdot z^{-5}+0{,}78\cdot z^{-6}+0{,}30\cdot z^{-7}+0{,}06\cdot z^{-8}}{1-0{,}95\cdot z^{-1}+1{,}74\cdot z^{-2}+1{,}20\cdot z^{-3}+0{,}92\cdot z^{-4}+0{,}41\cdot z^{-5}+0{,}15\cdot z^{-6}+0{,}03\cdot z^{-7}+0{,}004\cdot z^{-8}}$$

Prijenosna funkcija (FIR)

$$H(z)=\sum_i b_i\cdot z^{-i}$$

1.	0.0004	21	-0.0176	41	-0.0152
2.	-0.0007	22	0.0270	42	0.0172
3.	-0.0004	23	0.0142	43	0.0067
4.	0.0015	24	-0.0432	44	-0.0148
5.	0.0002	25	-0.0042	45	-0.0010
6.	-0.0024	26	0.0673	46	0.0114
7.	0.0007	27	-0.0206	47	-0.0023
8.	0.0034	28	-0.1160	48	-0.0078
9.	-0.0023	29	0.1103	49	0.0038
10	-0.0040	30	0.4839	50	0.0047
11	0.0047	31	0.4839	51	-0.0040
12	0.0038	32	0.1103	52	-0.0023
13	-0.0078	33	-0.1160	53	0.0034
14	-0.0023	34	-0.0206	54	0.0007
15	0.0114	35	0.0673	55	-0.0024
16	-0.0010	36	-0.0042	56	0.0002
17	-0.0148	37	-0.0432	57	0.0015
18	0.0067	38	0.0142	58	-0.0004
19	0.0172	39	0.0270	59	-0.0007
20	-0.0152	40	-0.0176	60	0.0004

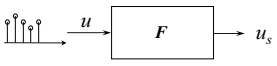
Usporedba IIR / FIR



Nerekurzivni digitalni filtri

Primjer - aritmetička sredina

$$u_s=\frac{u_1+u_2+\ldots+u_M}{M}$$



Interesantan je sustav koji služi za "glačanje" (usrednjavanje) slučajnih varijacija u signalu.

M-point moving average system

$$y[n]=\frac{u[n]+u[n-1]+u[n-2]+\ldots+u[n-M+1]}{M}$$ ili

$$y[n]=\sum_{i=0}^{M-1}\frac{1}{M}\cdot u[n-i]$$

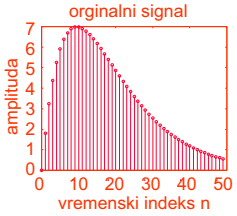
$$y[n]=\sum_{i=0}^{M-1}\frac{1}{M}u[n-i]=\sum_{i=0}^{M-1}h[i]\cdot u[n-i]$$

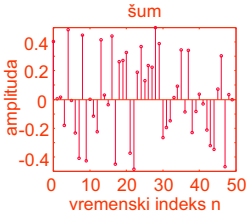
$$\updownarrow$$

$$h[i]$$

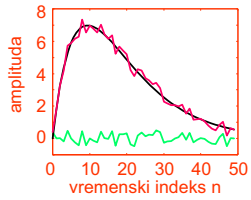
Sustav ima konačan impulsni odziv - FIR sustav

Primjer - *MATLAB*





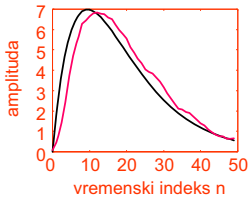
Primjer - nastavak



– orginalni signal

– šum

– signal + šum



– orginalni signal

– y[n] - izlaz iz *moving average* filtra

FIR sustav za M=2

Pogledajmo o kojem se sustavu radi:

Uzmimo $M=2$

$$y[n]=\sum_{i=0}^{1}h[i]\cdot u[n-i]=\left|h[i]=\frac{1}{M}=\frac{1}{2}\right|=$$

$$=\sum_{i=0}^{1}\frac{1}{2}\cdot u[n-i]=\frac{1}{2}\cdot\sum_{i=0}^{1}u[n-i]=$$

FIR sustav za M=2 ...

$$y[n]=\frac{1}{2}\cdot\sum_{i=0}^{1}u[n-i]=\left|\begin{matrix}u[n]=e^{j\omega n}\\-\infty\le n\le\infty\end{matrix}\right|$$

$$=\frac{1}{2}\left(e^{j\omega n}+e^{j\omega(n-1)}\right)=\frac{1}{2}e^{j\omega n}\left(1+e^{-j\omega}\right)=$$

$$=\underbrace{\frac{1}{2}\left(1+e^{-j\omega}\right)}_{H\left(e^{j\omega}\right)}\cdot e^{j\omega n}$$

FIR sustav za M=2 ...

Isto pomoću Z-transformacije

$$Y(z)=\underbrace{\frac{1}{2}\left(1+z^{-1}\right)}_{H(z)}\cdot U(z)$$

$$H(z)\ \Rightarrow\ z=e^{j\omega}\ \Rightarrow\ H\left(e^{j\omega}\right)$$

FIR sustav za M=2 ...

$$H\left(e^{j\omega}\right)=\frac{1}{2}\left(1+e^{-j\omega}\right)=$$

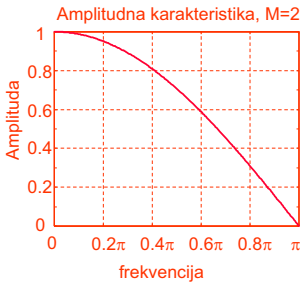
$$=\frac{1}{2}\left(e^{j\frac{\omega}{2}}\cdot e^{-j\frac{\omega}{2}}+e^{-j\frac{\omega}{2}}\cdot e^{-j\frac{\omega}{2}}\right)=$$

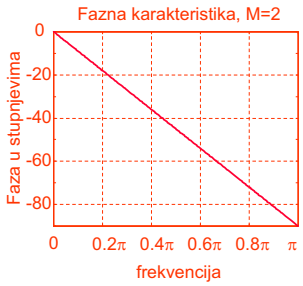
$$=e^{-j\frac{\omega}{2}}\cdot\underbrace{\frac{1}{2}\left(e^{j\frac{\omega}{2}}+e^{-j\frac{\omega}{2}}\right)}_{\cos\left(\frac{\omega}{2}\right)}=\underbrace{\cos\left(\frac{\omega}{2}\right)\cdot e^{-j\frac{\omega}{2}}}$$

Niskopropusni filtar

FIR sustav za M=2 ...

$$H\left(e^{j\omega}\right)=\cos\left(\frac{\omega}{2}\right)\cdot e^{-j\frac{\omega}{2}}$$





FIR sustav za proizvoljni M

Za proizvoljni M vrijedi:

$$H\left(e^{j\omega}\right)=\sum_{n=0}^{\infty}h[n]\cdot e^{-j\omega n}=\left|h[n]=\begin{cases}\frac{1}{M}, & 0\le n\le M-1\\ 0, & \text{inače}\end{cases}\right|=$$

$$=\frac{1}{M}\sum_{n=0}^{M-1}e^{-j\omega n}=\ \ldots\ldots$$

$$=\frac{1}{M}\cdot\frac{\sin\frac{M\cdot\omega}{2}}{\sin\frac{\omega}{2}}e^{-j\cdot(M-1)\frac{\omega}{2}}$$

FIR sustav za proizvoljni M ...

$$H\left(e^{j\omega}\right)=\frac{1}{M}\cdot\frac{\sin\frac{M\cdot\omega}{2}}{\sin\frac{\omega}{2}}e^{-j\cdot(M-1)\frac{\omega}{2}}$$

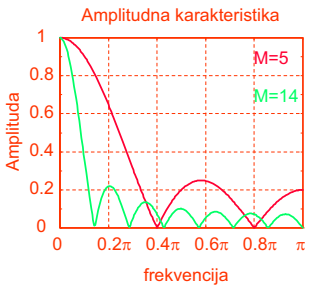
Amplitudno-fazna karakteristika

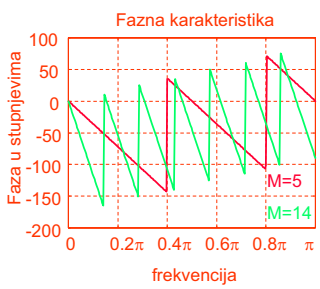
Amplitudna karakteristika $\left|H\left(e^{j\omega}\right)\right|=\left|\frac{1}{M}\cdot\frac{\sin\left(\frac{M\cdot\omega}{2}\right)}{\sin\left(\frac{\omega}{2}\right)}\right|$

Fazna karakteristika $\theta(\omega)=-\frac{(M-1)\cdot\omega}{2}+\pi\cdot\sum_{i=0}^{\left\lfloor\frac{M}{2}\right\rfloor}\mu\left(\omega-\frac{2\pi i}{M}\right)$

gdje je $\mu(\omega)$ step u ω

FIR sustav za proizvoljni M ...





Grupno kašnjenje

Daljnji parametar za karakterizaciju filtara je grupno kašnjenje.

mjera linearnosti fazne funkcije

$\tau(\omega)=-\frac{d\theta_c(\omega)}{d\omega}$, gdje je $\theta_c(\omega)$ faza

za prije navedeni primjer:

$$\tau(\omega)=\frac{M-1}{2}$$

Projektiranje FIR filtra

Pretpostavimo signal koji je suma kosinusnih signala

$$u[n] = u_1[n] + u_2[n] = \{\cos(0{,}1 \cdot n) + \cos(0{,}4 \cdot n)\}\,\mu(n)$$

Pretpostavimo da želimo sustav (filtar) koji će:

- gušiti signal $u_1[n]$ (kosinus kutne frekv. 0,1 rad/sec)
- propuštati signal $u_2[n]$ (kosinus kutne frekv. 0,4 rad/sec)

Radi jednostavnosti uzmimo:

- red filtra N=2 (tri uzorka impulsnog odziva)
- impulsni odziv filtra $h[0] = h[2] = \alpha \quad \wedge \quad h[1] = \beta$

Projektiranje FIR filtra ...

Jednadžba diferencija ovog sustava je:

$$y[n] = h[0] \cdot u[n] + h[1] \cdot u[n-1] + h[2] \cdot u[n-2] =$$
$$= \alpha \cdot u[n] + \beta \cdot u[n-1] + \alpha \cdot u[n-2]$$

a pripadna frekvencijska karakteristika

$$H(e^{j\omega}) = h[0] + h[1] \cdot e^{-j\omega} + h[2] \cdot e^{-j2\omega} =$$
$$= \alpha \cdot (1 + e^{-j2\omega}) + \beta \cdot e^{-j\omega} =$$
$$= 2\alpha \cdot \left(\frac{e^{j\omega} + e^{-j\omega}}{2}\right) \cdot e^{-j\omega} + \beta \cdot e^{-j\omega} =$$
$$= (2\alpha \cdot \cos(\omega) + \beta) \cdot e^{-j\omega}$$

Projektiranje FIR filtra ...

Amplitudna i fazna karakteristika filtra su

$$|H(e^{j\omega})| = 2\alpha \cdot \cos(\omega) + \beta$$

$$\theta(\omega) = -\omega$$

Iz zahtjeva na filtar određujemo α i β

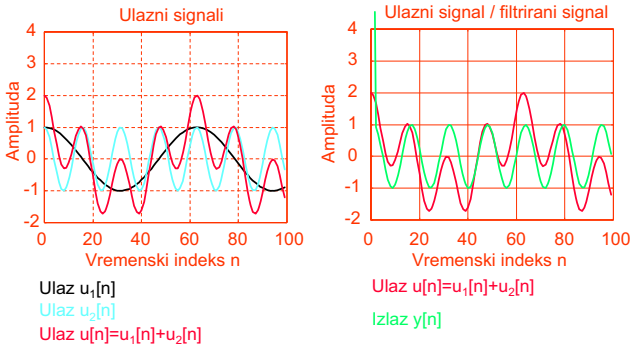
$$\left.\begin{aligned} |H(e^{j\cdot 0{,}1})| &= 2\alpha \cdot \cos(0{,}1) + \beta = 0 \\ |H(e^{j\cdot 0{,}4})| &= 2\alpha \cdot \cos(0{,}4) + \beta = 1 \end{aligned}\right\} \quad \begin{aligned} \alpha &= -6{,}76195 \\ \beta &= 13{,}456335 \end{aligned}$$

što daje

$$y[n] = -6{,}76195 \cdot (u[n] + u[n-2]) + 13{,}456335 \cdot u[n-1]$$

Projektiranje FIR filtra ...

Uz pobudu: $u[n] = \{\cos(0{,}1 \cdot n) + \cos(0{,}4 \cdot n)\}\,\mu(n)$



Ulaz $u_1[n]$
Ulaz $u_2[n]$
Ulaz $u[n]=u_1[n]+u_2[n]$

Ulaz $u[n]=u_1[n]+u_2[n]$
Izlaz $y[n]$

FIR-filtri

- filtri s linearnom fazom
- simetrija impulsnog odziva je nužan preduvjet za linearnu fazu
- ovisno o tipu simetrije impulsnog odziva definiramo četiri tipa FIR filtra sa realnim impulsnim odzivom duljine N+1

Tipovi FIR-filtara

–> Tip 1 - simetričan impulsni odziv
- neparan broj uzoraka impulsnog odziva

–> Tip 2 - simetričan impulsni odziv
- paran broj uzoraka impulsnog odziva

–> Tip 3 - antisimetričan impulsni odziv
- neparan broj uzoraka impulsnog odziva

–> Tip 4 - antisimetričan impulsni odziv
- paran broj uzoraka impulsnog odziva

Tip 1 FIR filtra

impulsni odziv zadovoljava slijedeći uvjet

$$h[n] = h[N-n], \qquad 0 \le n \le N$$

za daljnje razmatranje pretpostavimo N=8. U tom slučaju prijenosna funkcija filtra je

$$H(z) = h[0] + h[1]\cdot z^{-1} + h[2]\cdot z^{-2} + h[3]\cdot z^{-3} + h[4]\cdot z^{-4} + \\ + h[5]\cdot z^{-5} + h[6]\cdot z^{-6} + h[7]\cdot z^{-7} + h[8]\cdot z^{-8}$$

Tip 1 FIR filtra ...

Prema definiciji tipa 1 FIR filtra za N=8 vrijedi

$$h[0] = h[8],\ h[1] = h[7],\ h[2] = h[6],\ h[3] = h[5],$$

te se prethodni izraz može pojednostaviti

$$H(z) = h[0]\cdot\left(1 + z^{-8}\right) + h[1]\cdot\left(z^{-1} + z^{-7}\right) + \\ + h[2]\cdot\left(z^{-2} + z^{-6}\right) + h[3]\cdot\left(z^{-3} + z^{-5}\right) + h[4]\cdot z^{-4} = \\ = h[0]\cdot z^{-4}\cdot\left(z^{4} + z^{-4}\right) + h[1]\cdot z^{-4}\cdot\left(z^{3} + z^{-3}\right) + \\ + h[2]\cdot z^{-4}\cdot\left(z^{2} + z^{-2}\right) + h[3]\cdot z^{-4}\cdot\left(z + z^{-1}\right) + h[4]\cdot z^{-4} = \\ = z^{-4}\cdot\left\{h[0]\cdot\left(z^{4} + z^{-4}\right) + h[1]\cdot\left(z^{3} + z^{-3}\right) + \right. \\ \left. + h[2]\cdot\left(z^{2} + z^{-2}\right) + h[3]\cdot\left(z + z^{-1}\right) + h[4]\right\}$$

Tip 1 FIR filtra ...

Pripadna frekvencijska karakteristika

$$H\left(e^{j\omega}\right) = e^{-j4\omega}\cdot\{2h[0]\cdot\cos(4\omega) + 2h[1]\cdot\cos(3\omega) + \\ + 2h[2]\cdot\cos(2\omega) + 2h[3]\cdot\cos(\omega) + h[4]\}$$

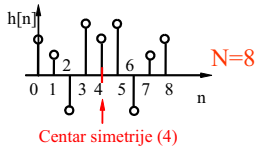
ili u općem slučaju

$$H\left(e^{j\omega}\right) = e^{-jN\omega/2}\cdot\left\{\sum_{m=0}^{N/2} a[m]\cdot\cos(\omega\cdot m)\right\}$$

$$a[0] = h\left[\frac{N}{2}\right], \quad a[m] = 2h\left[\frac{N}{2} - m\right], \quad 1 \le m \le \frac{N}{2}$$

Tipovi FIR-filtara ...

Tip 1
- simetričan impulsni odziv
- neparan broj uzoraka



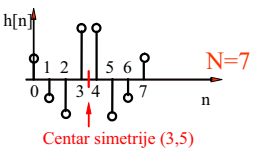
$$H\left(e^{j\omega}\right) = e^{-jN\omega/2}\cdot\left\{\sum_{m=0}^{N/2} a[m]\cdot\cos(\omega\cdot m)\right\}$$

$$a[0] = h\left[\frac{N}{2}\right], \quad a[m] = 2\cdot h\left[\frac{N}{2} - m\right], \quad 1 \le m \le \frac{N}{2}$$

$$\tau(\omega) = \frac{N}{2}$$

Tipovi FIR-filtara ...

Tip 2
- simetričan impulsni odziv
- paran broj uzoraka



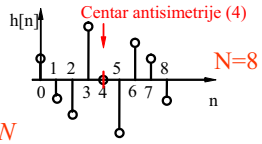
$$H\left(e^{j\omega}\right) = e^{-jN\omega/2}\cdot\left\{\sum_{m=1}^{(N+1)/2} b[m]\cdot\cos\left(\omega\cdot\left(m - \frac{1}{2}\right)\right)\right\}$$

$$b[m] = 2\cdot h\left[\frac{N+1}{2} - m\right], \quad 1 \le m \le \frac{N+1}{2}$$

$$\tau(\omega) = \frac{N}{2}$$

Tipovi FIR-filtara ...

Tip 3
- antisimetričan impulsni odziv
- neparan broj uzoraka



$$h[n] = -h[N-n], \quad 0 \le n \le N$$

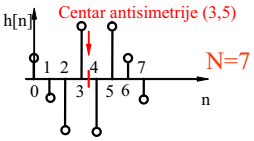
$$H\left(e^{j\omega}\right) = e^{-jN\omega/2}\cdot e^{j\pi/2}\cdot\left\{\sum_{m=1}^{N/2} c[m]\cdot\sin(\omega\cdot m)\right\}$$

$$c[m] = 2\cdot h\left[\frac{N}{2} - m\right], \quad 1 \le m \le \frac{N}{2}$$

$$\tau(\omega) = \frac{N}{2}$$

Tipovi FIR-filtara ...

Tip 4
- antisimetričan impulsni odziv
- paran broj uzoraka



N=7

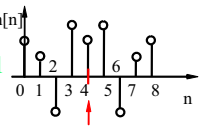
$$H\left(e^{j\omega}\right) = e^{-jN\omega/2} \cdot e^{j\pi/2} \cdot \left\{ \sum_{m=1}^{(N+1)/2} d[m] \cdot \sin\left(\omega \cdot \left(m - \frac{1}{2} \right) \right) \right\}$$

$$d[m] = 2 \cdot h\left[\frac{N+1}{2} - m \right], \quad 1 \le m \le \frac{N+1}{2}$$

$$\tau(\omega) = \frac{N}{2}$$

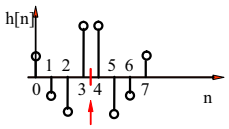
Tipovi FIR-filtara ...

Tip 1



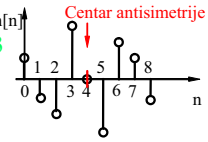
Centar simetrije (4)

Tip 2



Centar simetrije (3,5)

Tip 3



Tip 4