Digitalna obradba signala – Zadaci za domaću zadaću

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Zavod za elektroničke sustave i obradbu informacija

1. Signal $y(t)$ ima CTFT transformaciju zadanu izrazom:

$$H(\Omega) = \begin{cases} -8j, & -4\pi \leq \omega < -3\pi \\ -2j, & -3\pi \leq \omega \leq -2\pi \\ 2j, & 2\pi \leq \omega \leq 3\pi \\ 8j, & 3\pi < \omega \leq 4\pi \\ 0, & inače. \end{cases}$$

a) Kojom frekvencijom treba očitati zadani signal da ne dođe do preklapanja?

b) Izračunajte izraz $y(t)$ koji opisuje signal u vremenskoj domeni.

c) Zadani signal očitajte frekvencijom 3 Hz.

d) Skicirajte amplitudni i fazni spektar signala koji se dobiva nakon očitavanja.

e) Skicirajte amplitudni i fazni spektar signala koji se dobiva idealnom interpolacijom očitanog signala.

2. Izračunajte vremenski diskretnu Fourierovu transformaciju (DTFT) signala $x[n] = cos(n\frac{\pi}{8})sin(n\frac{\pi}{8})\pi^{-n}\mu[n]$.

3. Zadan je signal $x[n] = \{\underline{1}, -2, 3, -4, 5\}$ i $h[n] = x[n]$. Uzorci signala koji nisu zadani su jednaki nuli.

a) Izračunajte $x[n] * h[n]$ u postupkom u vremenskoj domeni.

b) Izračunajte $x[n] \textcircled{5} h[n]$ korištenjem teorema o cirkularnoj konvoluciji.

c) Za koji N vrijedi $\big(x * h\big)[n] = \big(x \textcircled{N} h\big)[n]$, $n = 0, 1, \ldots, N-1$? Objasnite!

d) Ukoliko linearnu konvoluciju $x[n] * h[n]$ računamo metodom PREKLOPI-I-ZBROJI gdje ulazni signal $x[n]$ dijelimo u blokove duljine 3 uzorka, nađite rezultat obradbe prvog bloka i naznačite koji dobiveni uzorci su konačni, a koji se preklapaju sa sljedećim blokom.

4. Vaš matični broj sastoji se od 10 znamenki. Neka se signal $x_1[n]$ sastoji od zadnjih 8 znameki vašeg matičnog broja, neka se signal $x_2[n]$ sastoji od prva četiri uzorka signala $x_1[n]$ te neka se signal $x_3[n]$ sastoji od poslijednja četiri uzorka signala $x_1[n]$. Primjerice, ako je vaš matični broj 0036431234 tada je $x_1[n] = \{\underline{3}, 6, 4, 3, 1, 2, 3, 4\}$, $x_2[n] = \{\underline{3}, 6, 4, 3\}$, $x_3[n] = \{\underline{1}, 2, 3, 4\}$.

a) Ukratko objasnite što su REDFFT i RE2FFT postupci.

b) Korištenjem REDFFT postupka izračunajte DFT_8 transformaciju signala $x_1[n]$ korištenjem jedne DFT_4 transformacije.

c) Korištenjem RE2FFT postupka izračunajte DFT_4 transformacije signala $x_2[n]$ i $x_3[n]$.

5. Impulsni odziv vremenski diskretnog LTI sustava zadan je s $h[n] = \{-1, \underline{-2}, 2, 1\}$.

a) Odredite jednadžbu koja opisuje sustav u vremenskoj domeni. Radi li se o FIR ili IIR filtru?

b) Je li sustav kauzalan i na temelju čega to zaključujete?

c) Izračunajte prijenosnu funkciju sustava $H(z)$ te odredite nule, polove i red sustava.

d) Izračunajte i nacrtajte amplitudno-frekvencijsku i fazno-frekvencijsku karakteristiku sustava.

e) Ima li sustav linearnu fazu? Kako ste to mogli zaključiti iz uzoraka impulsnog odziva?

f) Izračunajte grupno vrijeme kašnjenja sustava.

(1b)

$$Y(\Omega) = \begin{cases} -8j & -4\pi \le \omega < -3\pi \\ -2j & -3\pi \le \omega \le -2\pi \\ 2j & 2\pi \le \omega \le 3\pi \\ 8j & 3\pi < \omega \le 4\pi \\ 0 & \text{inače} \end{cases}$$

a)

$$\omega_{MAX} = 4\pi \Rightarrow f_{MAX} = 2Hz \rightarrow f_s \geq 2f_{MAX} \rightarrow f_s \geq 4Hz$$

b)

$$y(t) = ICTFT(Y(\Omega)) = \frac{1}{2\pi}\left[\int_{-4\pi}^{-3\pi} -8j e^{j\omega t} d\omega + \int_{-3\pi}^{-2\pi} -2j e^{j\omega t} d\omega + \int_{2\pi}^{3\pi} 2j e^{j\omega t} d\omega + \int_{3\pi}^{4\pi} 8j e^{j\omega t} d\omega\right] =$$

$\omega = t$

ups

$$= \frac{1}{2\pi}\left[-8\frac{1}{\omega}e^{j\omega t}\Big|_{-4\pi}^{-3\pi} - 2\frac{1}{\omega}e^{j\omega t}\Big|_{-3\pi}^{-2\pi} + 2\frac{1}{\omega}e^{j\omega t}\Big|_{2\pi}^{3\pi} + 8\frac{1}{\omega}e^{j\omega t}\Big|_{3\pi}^{4\pi}\right] =$$

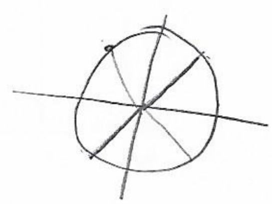
$$= \frac{\cancel{2}}{\cancel{2}\omega\pi}\left[-4\left(e^{-j3\pi\omega} - e^{-j4\pi\omega}\right) - \left(e^{-j2\pi\omega} - e^{-j3\pi\omega}\right) + \left(e^{j3\pi\omega} - e^{j2\pi\omega}\right) + 4\left(e^{j4\pi\omega} - e^{j3\pi\omega}\right)\right]$$

$$= \frac{1}{\omega\pi}\left[4\left(-e^{j3\pi\omega} - e^{-j3\pi\omega}\right) + 4\left(e^{j4\pi\omega} + e^{-j4\pi\omega}\right) + \left(e^{j3\pi\omega} + e^{-j3\pi\omega}\right) - \left(e^{j2\pi\omega} + e^{-j2\pi\omega}\right)\right]$$

$$y(t) = \frac{1}{\cancel{\omega}t\pi}\left[-8\cos(3\pi t) + 8\cos(4\pi t) + 2\cos(3\pi t) - 2\cos(2\pi t)\right]$$

c) $f = 3Hz \Rightarrow \omega_s = 6\pi \Rightarrow T = \frac{1}{3} = 0{,}3$

$$\boxed{y[n] = y(nT)}$$

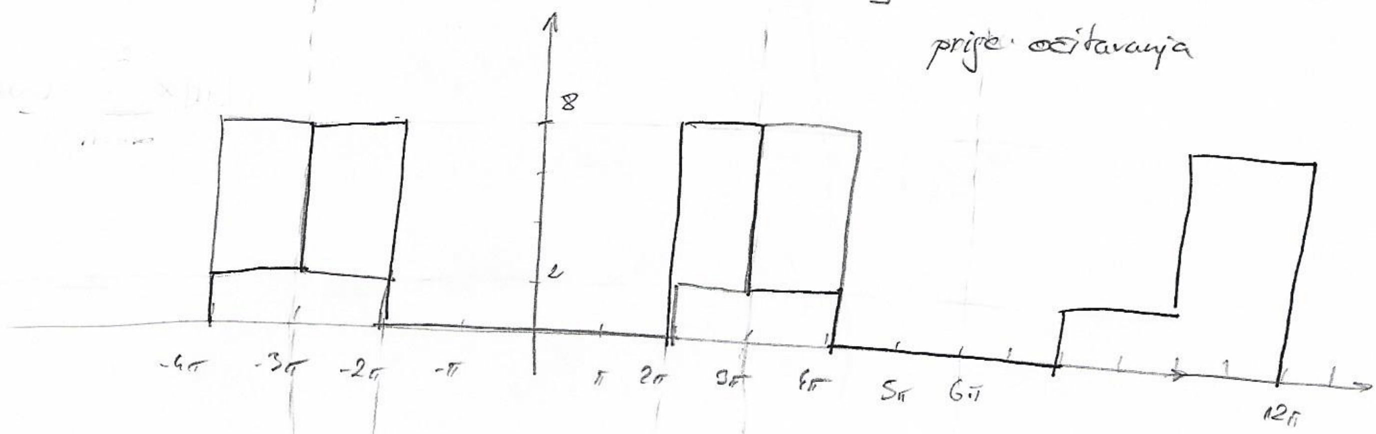


$$y[n] = \frac{3}{n\pi}\left[8\cos\left(\frac{4\pi}{3}n\right) - \underbrace{8\cos(\pi n)}_{-1} + 2\cos(\pi n) - 2\cos\left(\frac{2\pi}{3}n\right)\right]$$

$$= \frac{3}{n\pi}\left[8\cos\left(4n\frac{\pi}{3}\right) - 6\cos(n\pi) - 2\cos\left(2n\frac{\pi}{3}\right)\right]$$

$$y[n] = \frac{3}{n\pi}\left[8\cos\left(n\frac{4\pi}{3}\right) - 6\cos(n\pi) - 2\cos\left(n\frac{2\pi}{3}\right)\right]$$

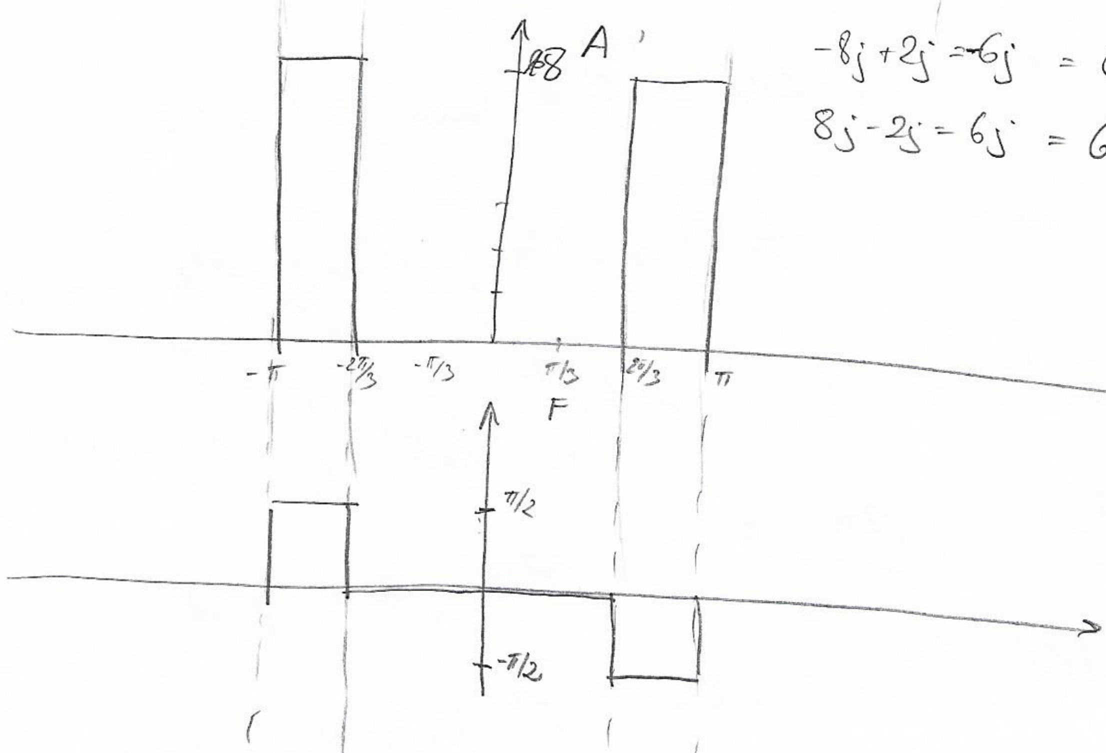
prije očitavanja



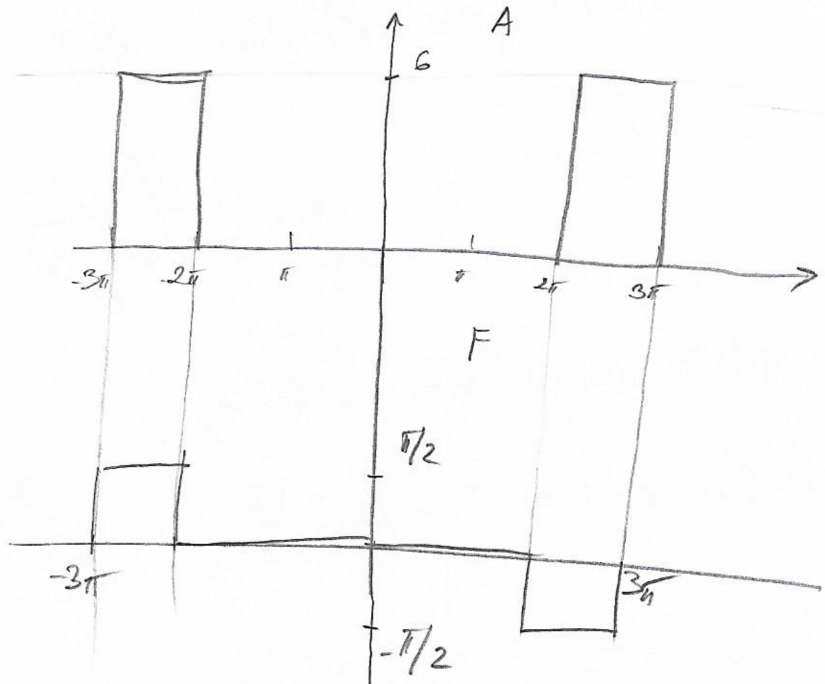
$-8j + 2j = -6j = 6e^{j\pi/2}$

$8j - 2j = 6j = 6e^{j\pi/2}$

POSLIJE



e)



(2.) $x[n] = \cos\left(n\frac{\pi}{8}\right)\sin\left(n\frac{\pi}{8}\right)\pi^{-n}\mu[n] =$

$$= \frac{1}{2}\left(\overbrace{\sin\left(n\frac{\pi}{8}-n\frac{\pi}{8}\right)}^{0} + \sin\left(n\frac{\pi}{4}\right)\right)\cdot\left(\frac{1}{\pi}\right)^n \mu[n] =$$

$$= \frac{1}{2}\left(\frac{1}{\pi}\right)^n \sin\left(n\frac{\pi}{4}\right)\mu[n] \;\circ\!\!-\!\!\bullet\; \frac{1}{2}\,\frac{\frac{1}{\pi}e^{j\omega}\sin\left(\frac{\pi}{4}\right)}{e^{2j\omega}-\frac{2}{\pi}e^{j\omega}\cos\left(\frac{\pi}{4}\right)+\frac{1}{\pi^2}}$$

(3.) $x[n] = \{\underline{1}, -2, 3, -4, 5\} = h[n]$

a)

	1	-2	3	-4	5	
5 -4 3 -2	1					1
5 -4 3	-2	1				-4
5 -4	3	-2	1			10
5	-4	3	-2	1		-20
	5	-4	3	-2	1	35
		5	-4	3	-2	-44
			5	-4	3	46
				5	-4	-40
					5	25

b)

$x[n] ⑤ x[n] \;\circ\!\!-\!\!\bullet\; X[k]\cdot X[k]$

$X[k] = W_5^0 - 2W_5^k + 3W_5^{2} - 4W_5^{3k} + 5W_5^{4k}$

$X[k]\cdot X[k] = (W_5^0 - 2W_5^k + 3W_5^{2k} - 4W_5^{3k} + 5W_5^{4k})(W_5^0 - 2W_5^k + 3W_5^{2k} - 4W_5^{3k} + 5W_5^{4k})$

$= W_5^0 - 2W_5^k + 3W_5^{2k} - 4W_5^{3k} + \boxed{5W_5^{4k}} - 2W_5^k + 4W_5^{2k} - 6W_5^{3k} + \boxed{8W_5^{4k}} - 10W_5^{5k} + 3W_5^{2k} - 6W_5^{3k} + \boxed{9W_5^{4k}}$
$- 12W_5^{5k} + 15W_5^{6k} - 4W_5^{3k} + \boxed{8W_5^{4k}} - 12W_5^{5k} + 16W_5^{6k} - 20W_5^{7k} + \boxed{5W_5^{4k}} - 10W_5^{5k} +$
$+ 15W_5^{6k} - 20W_5^{7k} + 25W_5^{8k} =$

$= W_5^0 - 4W_5^k + 10W_5^{2k} - 20W_5^{3k} + 35W_5^{4k} - 44W_5^{\overset{0}{5k}} + 46W_5^{\overset{1k}{6k}} - 40W_5^{\overset{2k}{7k}} + 25^{\overset{3k}{8k}}$

$x[n] ⑤ x[n] = \{\underline{-43}, 42, 30, 5, 35\}$

c) $(x*h)[n] = (x \textcircled{N} h)[n] \quad , n = 0, 1, \dots, N-1$

$N = L_x + L_h - 1$ zato što je cirkularna konvolucija x i h

L_x ↓ duljina od x, L_h ↓ duljina od h

jednaka $\sum (x*h)[n+kN]$, a za ovakav N za svaki $k>0$ je $(x*h)[n+kN]=0$

d) $x[n] = \{\underline{1}, -2, 3, -4, 5\}$

$N = 3 \Rightarrow$

$x_1[n] = \{1, -2, 3\}$

$x_2[n] = \{-4, 5, 0\}$

	1	-2	3	
5 -4 3 -2	1			1
5 -4 3	-2	1		-4
5 -4	3	-2	1	10
5	-4	3	-2	-16
	5	-4	3	22
		5	-4	-22
			5	15

$y_1[n] = \{1, -4, 10\}$

preklapanje

(4) 0036449369

↳ 4. predavanje

$x_1[n] = \{3, 6, 4, 4, 9, 3, 6, 9\}$

$x_2[n] = \{3, 6, 4, 4\}$

$x_3[n] = \{9, 3, 6, 9\}$

a) REDFFT - transformacija realnog niza $v(n)$ duljine $2N$ pomoću DFT duljine N

RE2FFT - FFT dvaju realnih nizova uz pomoć jedne kompleksne FFT

b) $x_1[n] = \{3, 6, 4, 4, 9, 3, 6, 9\}$ → $y_1[n] = x_1[2n] = \{3, 4, 9, 6\}$

→ $y_2[n] = x_1[2n+1] = \{6, 4, 3, 9\}$

$$X(k) = \sum_{n=0}^{2N-1} x_1[n]\, W_{2N}^{nk} = \sum_{n=0}^{N-1} x_1[2n]\, W_{2N}^{2nk} + \sum_{n=0}^{N-1} x_1[2n+1]\, W_{2N}^{(2n+1)k} =$$

$$= \sum_{n=0}^{N-1} x_1[2n]\, W_{2N}^{2nk} + W_{2N}^{k} \sum_{n=0}^{N-1} x_1[2n+1]\, W_{2N}^{2nk} = \left| W_{2N}^{2nk} = e^{-j\frac{2 \cdot 2n\pi k}{2N}} = W_N^{nk} \right| =$$

$$= \underbrace{\sum_{n=0}^{N-1} y_1[n]\, W_N^{nk}} + W_{2N}^{k} \underbrace{\sum_{n=0}^{N-1} y_2[n]\, W_N^{nk}} \Rightarrow \boxed{X(k) = Y_1(\langle k \rangle_N) + W_{2N}^{k}\, Y_2(\langle k \rangle_N)}$$

$$Y_1(k) = 3W_4^0 + 4W_4^k + 9W_4^{2k} + 6W_4^{3k}$$
$$= 3 + 4e^{-j\frac{2\pi k}{4}} + 9e^{-j\frac{4\pi k}{4}} + 6e^{-j\frac{6\pi k}{4}}$$

$$Y_1[0] = 3+4+9+6 = 22$$
$$Y_1[1] = 3 + 4e^{-j\frac{\pi}{2}} + 9e^{-j\pi} + 6e^{-j\frac{3\pi}{2}} = -6+2j$$
$$Y_1[2] = 3 + 4e^{-j\pi} + 9e^{-j2\pi} + 6e^{-j3\pi} = 2$$
$$Y_1[3] = 3 + 4e^{-j\frac{3\pi}{2}} + 9e^{-j3\pi} + 6e^{-j\frac{9\pi}{2}} = -6-2j$$

$$Y_2(k) = 6 + 4e^{-j\frac{2\pi k}{4}} + 3e^{-j\frac{4\pi k}{4}} + 9e^{-j\frac{6\pi k}{4}}$$

$$W_8^k = e^{-j\frac{2\pi k}{8}}$$

$$Y_2[0] = 6+4+3+9 = 22$$
$$Y_2[1] = 6 + 4e^{-j\frac{\pi}{2}} + 3e^{-j\pi} + 9e^{-j\frac{3\pi}{2}} = 3+5j$$
$$Y_2[2] = 6 + 4e^{-j\pi} + 3e^{-j2\pi} + 9e^{-j3\pi} = -4$$
$$Y_2[3] = 6 + 4e^{-j\frac{3\pi}{2}} + 3e^{-j3\pi} + 9e^{-j\frac{9\pi}{2}} = 3-5j$$

$$X[0] = 22+22 = 44$$
$$X[1] = -6+2j + e^{-j\frac{\pi}{4}}(3+5j) = -6+4\sqrt{2}+j(2+2\sqrt{2})$$
$$X[2] = 2 + e^{-j\frac{\pi}{2}}(-4) = 2+j4$$
$$X[3] = -6-2j + e^{-j\frac{3\pi}{4}}(3-5j) = -6-4\sqrt{2}+j(-2+\sqrt{2})$$
$$X[4] = 22 + e^{-j\pi}(22) = 0$$
$$X[5] = -6+2j + e^{-j\frac{5\pi}{4}}(3+5j) = -6-4\sqrt{2}+j(2-\sqrt{2})$$
$$X[6] = 2 + e^{-j\frac{6\pi}{4}}(-4) = 2-j4$$
$$X[7] = -6-2j + e^{-j\frac{7\pi}{4}}(3-5j) = -6+4\sqrt{2}-j(2+\sqrt{2})$$

c) $x_2[n] = \{3,6,4,4\}$, $x_3[n] = \{9,3,6,9\}$ } stvaramo novi kompleksni niz

$$z[n] = x_2[n] + jx_3[n]$$

$$z(k) = \sum_{n=0}^{N-1}\left[x_2[n]+jx_3[n]\right]e^{-j\frac{2\pi nk}{N}} = \sum_{n=0}^{N-1}\left[x_2[n]+jx_3[n]\right]\left[\cos\left(\frac{2\pi nk}{N}\right) - j\sin\left(\frac{2\pi nk}{N}\right)\right]$$

$$= \underbrace{\sum_{n=0}^{N-1} x_2[n]\cos\left(\frac{2\pi nk}{N}\right)}_{Re\{X(k)\}} + \underbrace{\sum_{n=0}^{N-1} x_3[n]\sin\left(\frac{2\pi nk}{N}\right)}_{-Im\{Y(k)\}} + j\left[\underbrace{\sum_{n=0}^{N-1} x_3[n]\cos\left(\frac{2\pi nk}{N}\right)}_{Re\{Y(k)\}} \underbrace{-\sum_{n=0}^{N-1} x_2[n]\sin\left(\frac{2\pi nk}{N}\right)}_{Im\{X(k)\}}\right]$$

(1) $Re\{z(k)\} = Re\{X(k)\} - Im\{Y(k)\}$
(2) $Im\{z(k)\} = Re\{Y(k)\} + Im\{X(k)\}$
(3) $Re\{z(N-k)\} = Re\{X(N-k)\} - Im\{Y(N-k)\}$
(4) $Im\{z(N-k)\} = Re\{Y(N-k)\} + Im\{X(N-k)\}$

$$Re\{X(k)\} = \left(Re\{z(k)\} + Re\{z(N-k)\}\right)\frac{1}{2}$$
$$Im\{X(k)\} = \left(Im\{z(k)\} - Im\{z(N-k)\}\right)\frac{1}{2}$$
$$Re\{Y(k)\} = \left(Im\{z(k)\} + Im\{z(N-k)\}\right)\frac{1}{2}$$
$$Im\{Y(k)\} = \left(Im\{z(N-k)\} - Re\{z(k)\}\right)\frac{1}{2}$$

$$z(k) = \underbrace{\sum_{n=0}^{3} x_2[n] \cos\left(\frac{2nk\pi}{4}\right)}_{Re\{X(k)\}} + \underbrace{\sum_{n=0}^{3} x_3[n] \sin\left(\frac{2\pi nk}{4}\right)}_{-Im\{Y(k)\}} + j\Bigg[\underbrace{\sum_{n=0}^{3} x_3[n] \cos\left(\frac{2\pi nk}{4}\right)}_{Re\{Y(k)\}} - \underbrace{\sum_{n=0}^{3} x_2[n] \sin\left(\frac{2\pi nk}{4}\right)}_{Im\{X(k)\}}\Bigg]$$

$$z(0) = 3+6+4+4+j(9+3+6+9) = 17+j27 \qquad x_2 = \{3,6,4,4\} \quad x_3 = \{9,3,6,9\}$$

$0 \ (\sin(0)) \qquad 0$

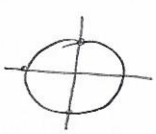
$$z(1) = 3+6\cancel{\cos\left(\tfrac{\pi}{2}\right)}+4\cos(\pi)+4\cancel{\cos\left(\tfrac{3\pi}{2}\right)}+9+3\sin\left(\tfrac{\pi}{2}\right)+6\cancel{\sin(\pi)}+9\sin\left(\tfrac{3\pi}{2}\right)+$$
$$+j\left(9+3\cancel{\cos\left(\tfrac{\pi}{2}\right)}+6\cos(\pi)+9\cancel{\cos\left(\tfrac{3\pi}{2}\right)}-3\cancel{\sin(0)}-6\sin\left(\tfrac{\pi}{2}\right)-4\cancel{\sin(\pi)}-4\sin\left(\tfrac{3\pi}{2}\right)\right) = -3+j$$

$$z(2) = 3\cos(0)+6\cos(\pi)+4\cos(2\pi)+4\cos(3\pi)+\underbrace{\cdots}_{\text{svi sin su } 0}+j\,9\cos(0)+3\cos(\pi)+6\cos(2\pi)+9\cos(3\pi) = -3+j3$$

$$z(3) = 3\cos(0)+6\cancel{\cos\left(\tfrac{3\pi}{2}\right)}+4\cos(3\pi)+4\cancel{\cos\left(\tfrac{9\pi}{2}\right)}+9\cancel{\sin(0)}+3\sin\left(\tfrac{3\pi}{2}\right)+6\cancel{\sin(3\pi)}+9\sin\left(\tfrac{9\pi}{2}\right)+$$
$$+j\left(9\cos(0)+3\cancel{\cos\left(\tfrac{3\pi}{2}\right)}+6\cos(3\pi)+9\cancel{\cos\left(\tfrac{9\pi}{2}\right)}+3\cancel{\sin(0)}+6\sin\left(\tfrac{3\pi}{2}\right)+4\sin(3\pi)+4\sin\left(\tfrac{9\pi}{2}\right)\right) = 5+j$$

BOLJI NAČIN! Gore sam se pogubila

$$z(k) = \sum_{n=0}^{3} (x_2[n]+jx_3[n])\, e^{-j\frac{2\pi nk}{4}} = (3+j9)e^0+(6+j3)e^{-j\frac{\pi k}{2}}+(4+j6)e^{-j\pi k}+(4+j9)e^{-j\frac{3\pi k}{2}}$$



$$z(0) = 17+j27$$
$$z(1) = 3+j9+(6+j3)\underbrace{e^{-j\frac{\pi}{2}}}_{-j}+(4+j6)\underbrace{e^{-j\pi}}_{-1}+(4+j9)\underbrace{e^{-j\frac{3\pi}{2}}}_{j} = 3+j9-j6+3-4-j6+j4-9 = -7+j$$
$$z(2) = 3+j9+(6+j3)\underbrace{e^{-j\pi}}_{-1}+(4+j6)\underbrace{e^{-j2\pi}}_{1}+(4+j9)\underbrace{e^{-j3\pi}}_{-1} = 3+j9-6-j3+4+j6-4-j9 = -3+j3$$
$$z(3) = 3+j9+(6+j3)\underbrace{e^{-j\frac{3\pi}{2}}}_{j}+(4+j6)\underbrace{e^{-j3\pi}}_{-1}+(4+j9)\underbrace{e^{-j\frac{9\pi}{2}}}_{-j} = 3+j9+6j-3-4-j6-j4+9 = 5+j5$$
$$z(4) = 3+j9+(6+j3)\underbrace{e^{-j2\pi}}_{1}+(4+j6)\underbrace{e^{-j4\pi}}_{1}+(4+j9)\underbrace{e^{-j6\pi}}_{1} = 17+j27$$

$$Re(X_2(0)) = \frac{1}{2}\Big(Re(z(0))+Re(z(4))\Big) = 17 \qquad Im\{X_2(0)\} = \frac{1}{2}\Big(Im(z(0))-Im(z(4))\Big) = 0$$
$$Re(X_2(1)) = (-7+5)\cdot\frac{1}{2} = -1 \qquad Im\{X_2(1)\} = \frac{1}{2}(j-j5) = -2$$
$$Re(X_2(2)) = (-3-3)\cdot\frac{1}{2} = -3 \qquad Im\{X_2(2)\} = \frac{1}{2}(j3-j3) = 0$$
$$Re(X_2(3)) = (5-7)\cdot\frac{1}{2} = -1 \qquad Im\{X_2(3)\} = \frac{1}{2}(j5-j) = 2$$

$$Re(X_3(0)) = 27 \qquad Im\{X_3(0)\} = 5$$
$$Re(X_3(1)) = 3 \qquad Im\{X_3(1)\} = 6$$
$$Re(X_3(2)) = 3 \qquad Im\{X_3(2)\} = 3$$

(5.) $h[n] = \{-1, \underline{-2}, 2, 1\}$

a) $g[n] = -u[n+1] - u[n] + 2u[n-1] + u[n-2] \Rightarrow$ FIR jer ima konačni impulsni odziv

b) sustav nije kauzalan jer u obzir uzima ulaz koji se događa prije $n=0$

c) $H(z) = -z^{1} - 2z^{0} + 2z^{-1} + z^{-2} = -z^{1} - 2 + 2z^{-1} + z^{-2} = \dfrac{-z^3 - 2z^2 + 2z + 1}{z^2}$

POLOVI
$z^2 = 0 \Rightarrow p_{1,2} = 0$

nakon dijeljenja $(-z^3 - 2z^2 + 2z + 1) : (z-1)$

NULE $n_1 = 1 \Rightarrow -z^2 - 3z - 1$

$n_2 = \dfrac{-3+\sqrt{5}}{2}$

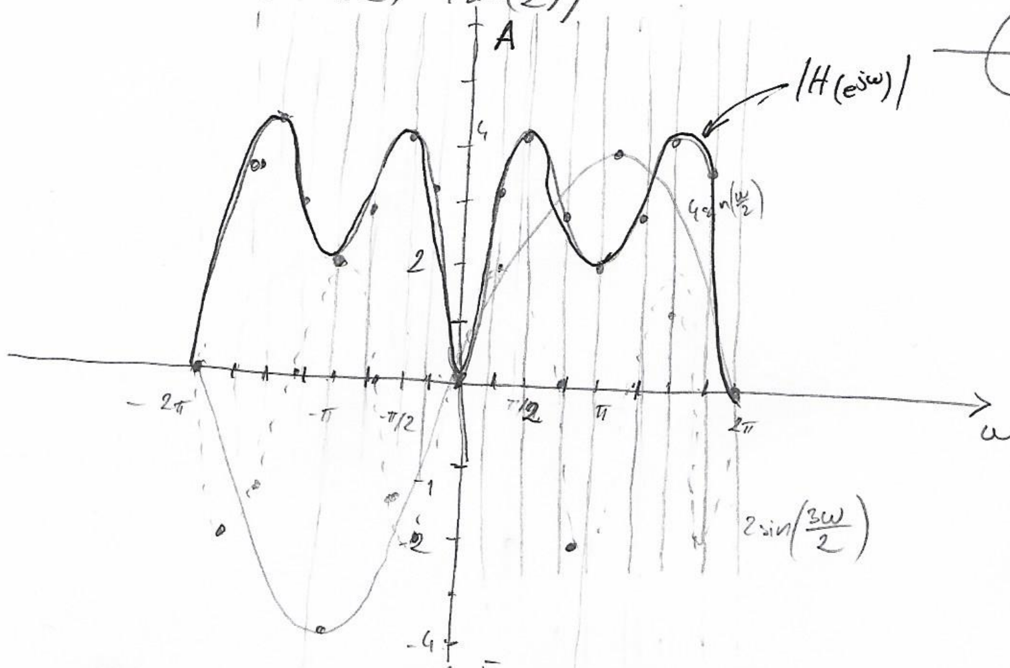
$n_3 = \dfrac{-3-\sqrt{5}}{2}$

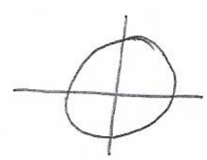
d) Amplituda $\Rightarrow |H(e^{j\omega})|$

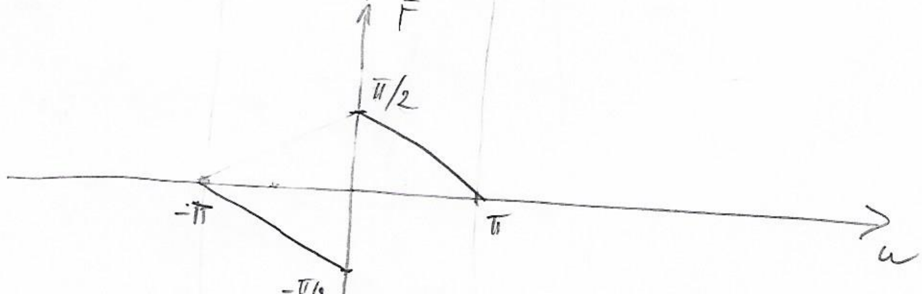
$$H(e^{j\omega}) = -e^{j\omega} - 2 + 2e^{-j\omega} + e^{-j2\omega} = e^{-j\frac{\omega}{2}}\left(-e^{j\frac{3\omega}{2}} - 2e^{j\frac{\omega}{2}} + 2e^{-j\frac{\omega}{2}} + e^{-j\frac{3\omega}{2}}\right)$$

$$= -e^{-j\frac{\omega}{2}}\left(2j\sin\left(\frac{3\omega}{2}\right) + 4j\sin\left(\frac{\omega}{2}\right)\right) = \underbrace{-j}_{e^{j\frac{\pi}{2}}} e^{-j\frac{\omega}{2}}\left(2\sin(3\omega/2) + 4\sin(\omega/2)\right) =$$

$$= e^{-j\left(\frac{\omega}{2}+\frac{\pi}{2}\right)}\left(2\sin\left(\frac{3\omega}{2}\right) + 4\sin\left(\frac{\omega}{2}\right)\right)$$







e) Ima jer su uzorci <u>antisimetrični</u>

f)

$$\tau = -\frac{\delta \varphi}{\delta \omega} = \frac{\delta \varphi}{\delta \omega}\left(\frac{\omega}{2} + \frac{\pi}{2}\right) = \frac{1}{2}$$