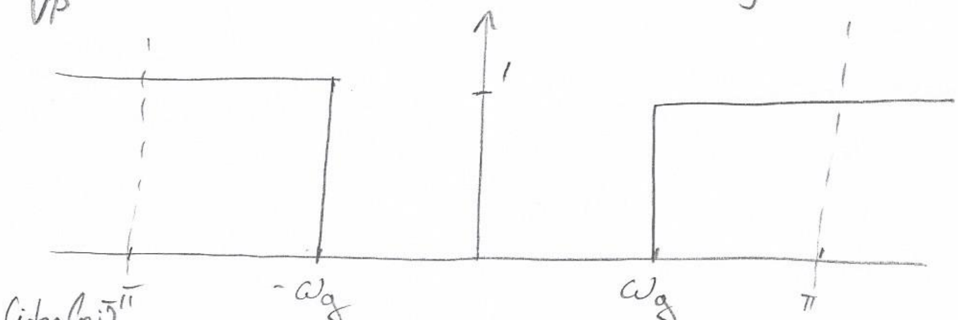
(1) $N=6$, $\omega g = \omega_g$

$\nearrow$ ostvariv jer VP filtar zahtijeva paran broj reda

a) VP



Takav filtar nije ostvariv jer nemamo načina pomoću sinusnih signala napraviti tako pravilan pravokutnik. Zbog toga dolazi do prijelaznog područja i zaobljenih rubova. Uz to, nemoguće je postići tako savršeno gušenje. Pri projektiranju filtra uvijek važemo između širine prijelaznog područja i gušenja.

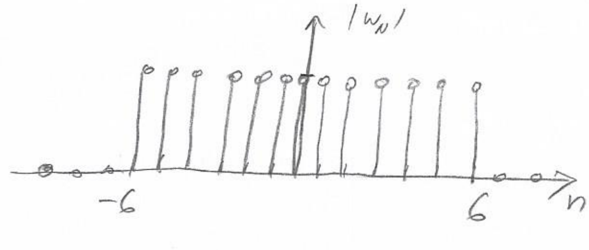
b)

$$h_{ID}[n] = \frac{1}{2\pi}\left[\int_{-\pi}^{-\omega_g} 1 \cdot e^{j\omega n} d\omega + \int_{\omega_g}^{\pi} 1 \cdot e^{j\omega n} d\omega\right] = \frac{1}{2\pi} \cdot \frac{1}{jn}\left[e^{-j\omega_g n} - e^{-j\pi n} + e^{j\pi n} - e^{j\omega_g n}\right] =$$

$$= \frac{1}{\pi n}\left[-\sin(\omega_g n) + \underbrace{\sin(n\pi)}_{0}\right] = -\frac{\sin(\omega_g n)}{\pi n} \qquad h_{ID}[0] = \frac{1}{2\pi}\left[-\omega_g + \pi + \pi - \omega_g\right] = \frac{-\omega_g + \pi}{\pi}$$

c) $\omega_g = \frac{\pi}{3}$

$$w_N = \begin{cases} 1, & -6 \leq n \leq 6 \\ 0, & \text{inače} \end{cases}$$

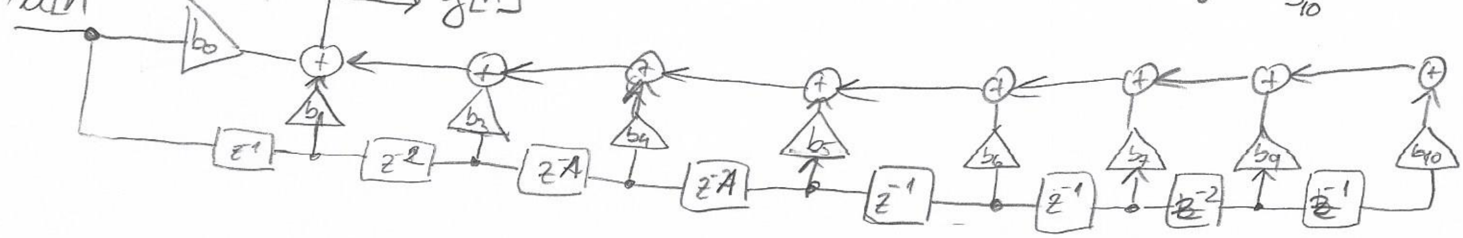


$$h_k[0] = \frac{\pi - \frac{\pi}{3}}{\pi} = \frac{2}{3}$$

$$h_k[n] = h_{ID}[n] \cdot w_N[n] = \{0, \underbrace{0.0551}_{b_0}, \underbrace{0.0689}_{b_1}, \underbrace{0}_{b_2}, \underbrace{-0.137}_{b_3}, \underbrace{-0.275}_{b_4}, \underbrace{\tfrac{2}{3}}_{b_5}, \underbrace{-0.275}_{b_6}, \underbrace{-0.137}_{b_7}, \underbrace{0}_{b_8}, \underbrace{0.0689}_{b_9}, \underbrace{0.0551}_{b_{10}}, 0\}$$

d) Što je red filtra veći, to je manje prijelazno područje.

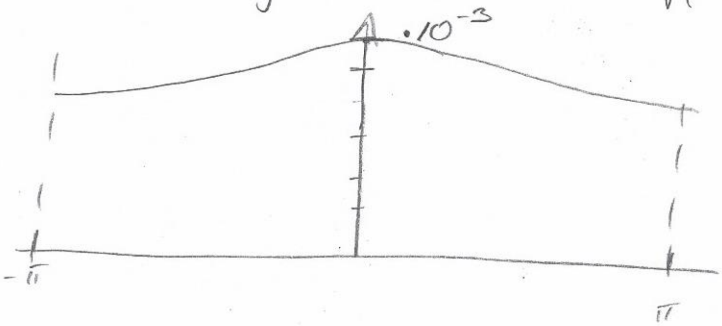
e)



(2.) $h(t) = t^2 e^{-7t}\mu(t)$

a) $H(s) = \dfrac{2}{(s+7)^3}$, $Re(s) > -7$

$$H(\Omega) = \frac{2}{(j\Omega+7)^3} = \frac{2}{(343-21\Omega^2)+j(147\Omega-\Omega^3)} \Rightarrow |H(\Omega)| = \frac{2}{\sqrt{(343-21\Omega^2)^2+(147\Omega-\Omega^3)^2}}$$



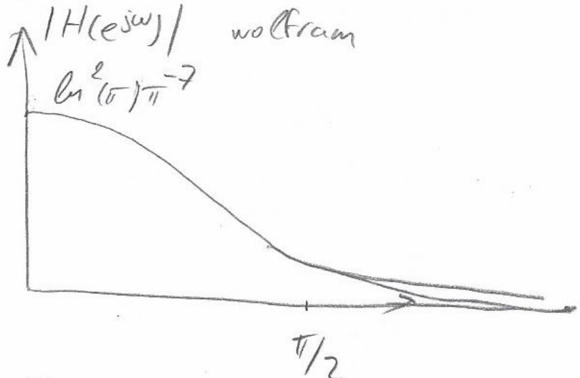
b) $T_s = \ln(\pi)$ $t = nT_s$

$$h[n] = h(t=nT_s) = n^2\ln^2\pi\, e^{-7\ln(\pi)n} = \underbrace{\ln^2(\pi)}_{konst}\cdot n^2\cdot\underbrace{\pi^{-7n}}_{\underbrace{a^n}_{f(n)}}\mu[n]$$

$$H(z) = \frac{\pi^7 z(\pi^7 z+1)}{(\pi^7 z-1)^3}\cdot \ln^2(\pi)$$

$$n f[n] \circ\!\!-\!\!\bullet\; -z\frac{dF(z)}{dz} = F_1(z) \bullet\!\!-\!\!\circ f_1[n]$$

$$\underbrace{n\cdot n f[n]}_{f_1[n]} \circ\!\!-\!\!\bullet\; -z\frac{dF_1(z)}{dz} = F_2(z)$$



c) Povećanje perioda očitavanja uzrokuje povećanje prijelaznog područja

d) Bilinearna transformacija čuva i stabilnost i tip filtra dok MJIO čuva stabilnost ali tip filtra ne nužno

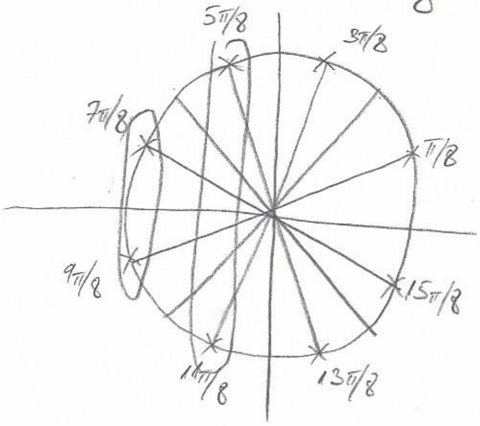
(3.) $T=2 \quad N=4$

a) $\Omega_g = \frac{2}{T} tg\left(\frac{\omega_g}{2}\right)$

$\omega_g = \frac{\pi}{6} \Rightarrow \Omega_g = \frac{2}{2} tg\left(\frac{\pi}{12}\right) = 2-\sqrt{3}$

b) $|H_B(s)|^2 = \frac{s^8}{s^8 + \Omega_g^8}$

$s_k = \Omega_g \exp\left\{j \frac{\pi + 2k\pi}{8}\right)$



$s_0 = \Omega_g e^{j\frac{\pi}{8}} \qquad s_4 = \Omega_g e^{j\frac{9\pi}{8}}$

$s_1 = \Omega_g e^{j\frac{3\pi}{8}} \qquad s_5 = \Omega_g e^{j\frac{11\pi}{8}}$

$s_2 = \Omega_g e^{j\frac{5\pi}{8}} \qquad s_6 = \Omega_g e^{j\frac{13\pi}{8}}$

$s_3 = \Omega_g e^{j\frac{7\pi}{8}} \qquad s_7 = \Omega_g e^{j\frac{15\pi}{8}}$

$S_2 = \Omega_g(-0{,}38 + 0{,}92j) \qquad S_5 = \Omega_g(-0{,}38 - 0{,}92j)$

$S_3 = \Omega_g(-0{,}92 + 0{,}38j) \qquad S_4 = \Omega_g(-0{,}92 - 0{,}38j)$

$$H(s) = \frac{s^4}{\underbrace{(s+0{,}38\Omega_g - 0{,}92\Omega_g j)(s+0{,}38\Omega_g + 0{,}92\Omega_g j)}_{(s+0{,}38\Omega_g)^2+(0{,}92\Omega_g)^2}\underbrace{(s+0{,}92\Omega_g - 0{,}38\Omega_g j)(s+0{,}92\Omega_g + 0{,}38\Omega_g j)}_{(s+0{,}92\Omega_g)^2+(0{,}38\Omega_g)^2}}$$

$$= \frac{s^4}{s^4 + 2{,}6 s^3 \Omega_g + 3{,}38 s^2 \Omega_g^2 + 2{,}576 s \Omega_g^3 + 0{,}98 \Omega_g^4} =$$

$$= \frac{s^4}{s^4 + 0{,}692 s^3 + 0{,}243 s^2 + 0{,}496 s + 0{,}005}$$

$$s = \frac{2}{T}\,\frac{1-z^{-1}}{1+z^{-1}} \Rightarrow H(z) = \frac{\left(\frac{1-z^{-1}}{1+z^{-1}}\right)^4}{\left(\frac{1-z^{-1}}{1+z^{-1}}\right)^4 + 0{,}697\left(\frac{1-z^{-1}}{1+z^{-1}}\right)^3 + 0{,}243\left(\frac{1-z^{-1}}{1+z^{-1}}\right)^2 + 0{,}496\left(\frac{1-z^{-1}}{1+z^{-1}}\right) + 0{,}005} =$$

$$= \frac{(1-z^{-1})^4}{(1-z^{-1})^4 + 0{,}697(1-z^{-1})^3(1+z^{-1}) + 0{,}243(1-z^{-1})^2(1+z^{-2})^2 + 0{,}496(1-z^{-1})(1+z^{-2})^3 + 0{,}005(1+z^{-1})^4}$$

$$H(z) = \frac{z^{-4} - 4z^{-3} + 6z^{-2} - 4z^{-1} + 1}{0,5\, z^{-4} - 2,69\, z^{-3} + 5,56\, z^{-2} - 5,27\, z^{-1} + 1,99} \qquad z^{-1} = e^{-j\omega}$$

d)

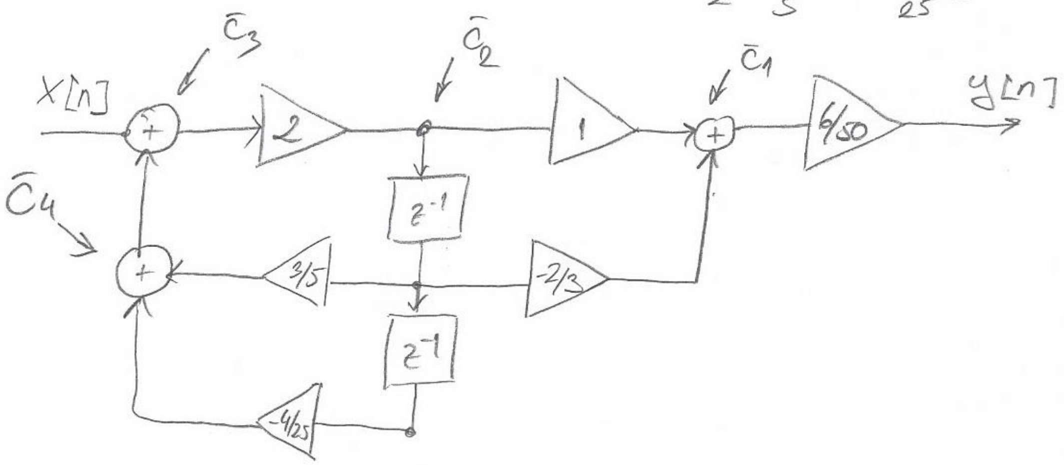
$\omega = 0 \Rightarrow 0$

$\omega = \pi/3$

(4) $25y[n] - 30y[n-1] + 8y[n-2] = 6u[n] - 4u[n-1]$

a) $$H(z) = \frac{6 - 4z^{-1}}{25 - 30z^{-1} + 8z^{-1}} = \frac{6}{50} \cdot \frac{1 - \frac{2}{3}z^{-1}}{\frac{1}{2} - \frac{3}{5}z^{-1} + \frac{4}{25}z^{-2}}$$

b)



c) $$H_1(z) = \frac{1 - \frac{2}{3}z^{-1}}{\frac{1}{2} - \frac{3}{5}z^{-1} + \frac{4}{25}z^{-2}} \qquad H_3(z) = \frac{H_2(z)}{2} = \frac{1}{1 - \frac{6}{5}z^{-1} + \frac{8}{25}z^{-2}}$$

$$H_2(z) = \frac{1}{\frac{1}{2} - \frac{3}{5}z^{-1} + \frac{4}{25}z^{-2}} \qquad H_4(z) = \frac{\frac{3}{5}z^{-1} - \frac{4}{25}z^{-2}}{\frac{1}{2} - \frac{3}{5}z^{-1} + \frac{4}{25}z^{-2}}$$

d)

$$y[n] = x[n] * h[n] = \sum_{i=0}^{\infty} x[i]\, h[n-i] \Rightarrow \sum_{i=0}^{\infty} x[i]\, h[n-i] \leq \sum_{i=0}^{\infty} h[i]\ \text{signum}\big[h[i]\big]$$

$$= \sum_{i=0}^{\infty} h[i]$$

e)

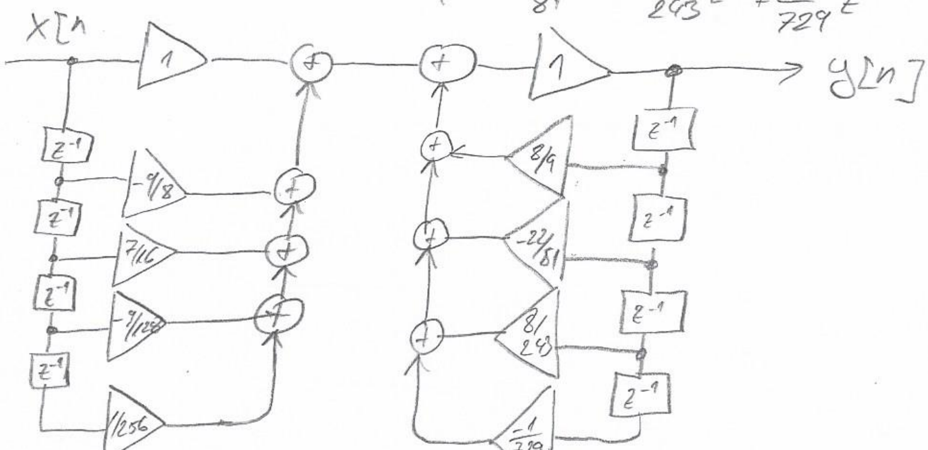
$$H(z) = \frac{1 - \frac{2}{3}z^{-1}}{\frac{1}{2} - \frac{3}{5}z^{-1} + \frac{4}{25}z^{-2}} = \frac{1 - \frac{2}{3}z^{-1}}{\left(1 - \frac{4}{5}z^{-1}\right)\left(1 - \frac{2}{5}z^{-1}\right)} = \frac{\frac{1}{3}}{1 - \frac{4}{5}z^{-1}} + \frac{\frac{2}{3}}{1 - \frac{2}{5}z^{-1}}$$

$$h[n] = \left[\frac{1}{3}\left(\frac{4}{5}\right)^n + \frac{2}{3}\left(\frac{2}{5}\right)^n\right]\mu[n] \quad k = \sum_{n=0}^{\infty} |h[n]| = \frac{1}{3}\frac{1}{1-\frac{4}{5}} + \frac{2}{3}\frac{1}{1-\frac{2}{5}} = \frac{5}{3} + \frac{10}{9} = \frac{25}{9}$$

(5)

$$H(z) = \frac{\left(1 - \frac{3}{4}z^{-1} + \frac{1}{8}z^{-2}\right)\left(1 - \frac{3}{8}z^{-1} + \frac{1}{32}z^{-2}\right)}{\left(1 - \frac{1}{3}z^{-1}\right)^2\left(1 - \frac{1}{9}z^{-1}\right)^2}$$

a)
$$H(z) = \frac{1 - \frac{9}{8}z^{-1} + \frac{7}{16}z^{-2} - \frac{9}{128}z^{-3} + \frac{1}{256}z^{-4}}{1 - \frac{8}{9}z^{-1} + \frac{22}{81}z^{-2} - \frac{8}{243}z^{-3} + \frac{1}{729}z^{-4}}$$



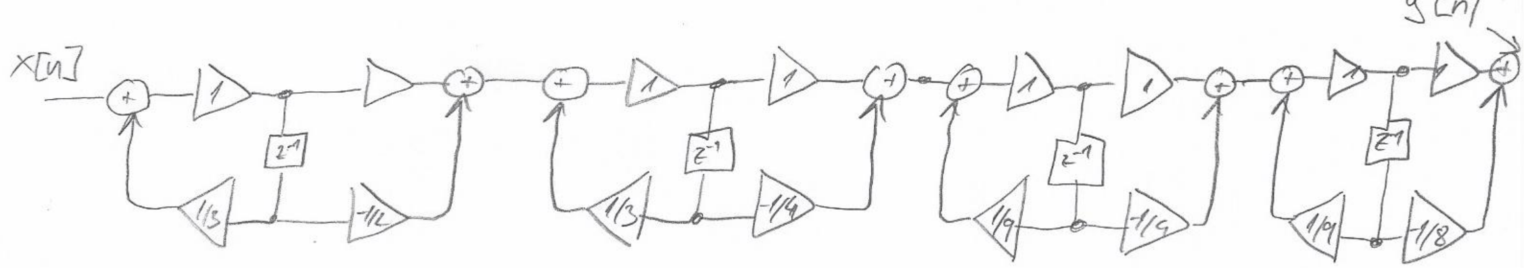
b)

$$H(z) = \frac{\left(1-\frac{1}{2}z^{-1}\right)\left(1-\frac{1}{4}z^{-1}\right)\left(1-\frac{1}{8}z^{-1}\right)\left(1-\frac{1}{4}z^{-1}\right)}{\left(1-\frac{1}{3}z^{-1}\right)^2\left(1-\frac{1}{9}z^{-1}\right)^2}$$

$$\overbrace{\binom{4}{2}}^{\text{nazivnik}} \cdot 2 \cdot \overbrace{\binom{4}{2}}^{\text{brojnik}} = 72 \text{ načina}$$

c)

$$H(z) = \frac{1-\frac{1}{2}z^{-1}}{1-\frac{1}{3}z^{-1}} \cdot \frac{\left(1-\frac{1}{4}z^{-1}\right)}{1-\frac{1}{3}z^{-1}} \cdot \frac{1-\frac{1}{4}z^{-1}}{1-\frac{1}{9}z^{-1}} \quad \frac{1-\frac{1}{8}z^{-1}}{1-\frac{1}{9}z^{-1}}$$



d) D-II VS. D-I – manje memorijskih lokacija (blokova za kašnjenje)

KASKADA VS. DII – kvantizacija utječe samo na nule unutar pojedine sekcije

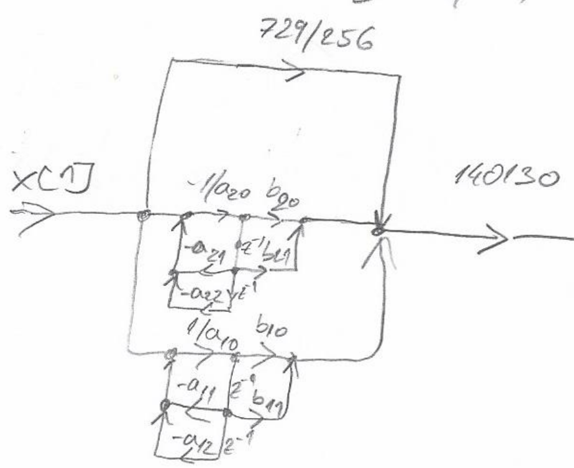
– lakše kontroliramo dinamiku pojedine sekcije

e)

$$H(z) = \frac{-\frac{473}{256} - \frac{1575}{2048}z^{-1} - \frac{43}{128}z^{-2} + \frac{3}{128}z^{-3}}{\left(1-\frac{2}{3}z^{-1}+\frac{1}{9}z^{-2}\right)\left(1-\frac{2}{9}z^{-1}+\frac{1}{81}z^{-2}\right)} + 1$$

ISTI STUPANJ POLINOMA!

$$H(z) = \frac{\frac{351}{1024}z^{-1} - \frac{1458}{1024}}{\left(1-\frac{2}{3}z^{-1}+\frac{1}{9}z^{-2}\right)} + \frac{\frac{17145}{1024}z^{-1} - \frac{140130}{1024}}{1-\frac{2}{9}z^{-1}+\frac{1}{81}z^{-2}} + \frac{729}{256}$$



$a_{10} = 1$ $\quad b_{10} = -\frac{1458}{143\,493\,120}$

$a_{11} = -\frac{2}{3}$ $\quad b_{11} = \frac{351}{143\,493\,120}$

$a_{12} = \frac{1}{9}$

$a_{20} = 1$ $\quad b_{20} = \frac{-1}{1024}$

$a_{21} = -\frac{2}{9}$ $\quad b_{21} = \frac{17145}{143\,493\,120}$

$a_{22} = \frac{1}{81}$