Poopćenje elemenata električkih krugova

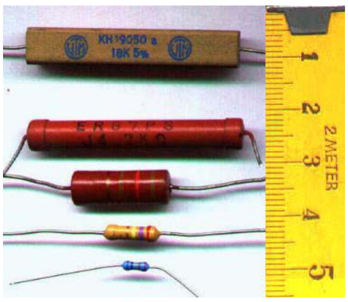
Poopćeni induktivitet

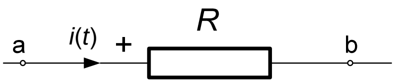
1

Koncept krugova

- Matematički opis kruga:

1. Matematički opis fizikalnih svojstava svakog električnog elementa – opis svojstava na priključnicama dvopolnog elementa preko električkih veličina





$$u_{ab}(t) = i(t) \cdot R$$

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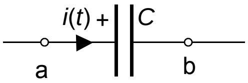
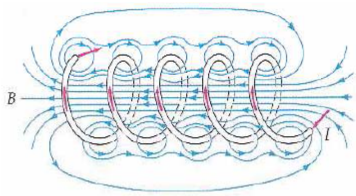
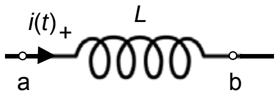


$$u_{ab}(t)=\frac{1}{C}\int_{t'=0}^{t} i(t')\cdot \mathrm{d}t'$$





$$u_{ab}(t)=L\frac{\mathrm{d}i(t)}{\mathrm{d}t}$$

2. Veze između veličina koje opisuju način povezivanja elemenata – Kirchhoffovi zakoni:

I KZ (za struje):

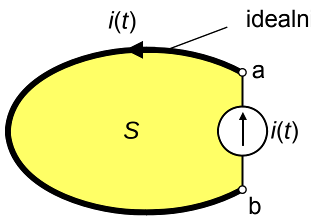
$$\sum_{j=1}^{n} i_j(t)=0 \;\; ; \;\; \sum_{j=1}^{n_{ul}} i_j(t)=\sum_{k=1}^{n_{iz}} i_k(t)$$ - za svaki čvor

II KZ (za napone):

$$\sum_{j=1}^{n_{iz}} u_{iz}(t)=\sum_{k=1}^{n_{pas}} u_{pas}(t)$$ - za svaku konturu

Poopćeni induktivitet

- Idealni magnetski sustav bez gubitaka je zatvorena petlja (strujni krug) sastavljena od idealnog vodiča i strujnog izvora spojenih između dvije priključnice:



Napon na priključnicama:

$$u_{ab}(t) = u(t) = \frac{\mathrm{d}\Psi}{\mathrm{d}t} = \frac{\mathrm{d}\Phi}{\mathrm{d}t}$$

$$\Phi = \int_A \vec{B} \cdot \vec{n} \cdot \mathrm{d}S$$

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- Pretpostavimo da je geometrija sustava nepromjenjiva osim jednog pokretnog dijela čija se trenutna pozicija može opisati pomakom *x* u odnosu na referentni položaj. Magnetski tok je onda:

$$\Psi = \Psi(i, x)$$

- Napon na priključnicama je:

$$u(t) = \frac{\mathrm{d}\Psi}{\mathrm{d}t} = \frac{\partial \Psi}{\partial i}\frac{\mathrm{d}i}{\mathrm{d}t} + \frac{\partial \Psi}{\partial x}\frac{\mathrm{d}x}{\mathrm{d}t} = u_T(t) + u_G(t)$$

  - $u_T(t)$ – napon transformacije (sustav miruje)
  - $u_G(t)$ – napon gibanja (struja je konstantna)

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- Ako je sustav s magnetski linearnim materijalom, to je električki linearan sustav:

$$\Psi(i,x) = L(x)\cdot i$$

$$u(t) = L(x)\frac{\mathrm{d}i}{\mathrm{d}t} + i\frac{\mathrm{d}L(x)}{\mathrm{d}x}\frac{\mathrm{d}x}{\mathrm{d}t}$$

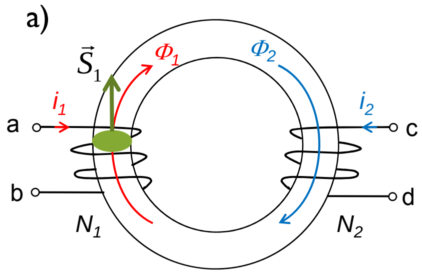
- Ako je geometrija nepromjenjiva:

$$u(t) = L\frac{\mathrm{d}i}{\mathrm{d}t}$$

- Elektromehanički sustavi često imaju:
  - više od jednog para električkih priključnica
  - više od jednog mehaničkog pomaka

- Dva strujna kruga mogu imati magnetske tokove istoga smjera a) ili suprotnog b):

a)



$$\Psi_{1uk} = \int_{A_1} \vec{B}\cdot\vec{n}\cdot\mathrm{d}S = N_1(\Phi_1 + \Phi_{12})$$

$$\Psi_{1uk} = \Psi_1 + \Psi_{12}$$

  - Φ_{l2} – dio magnetskog toka proizvedenog strujom i_2 a obuhvaćenog konturom struje i_l

$$L_1 = \frac{\Psi_1}{i_1} \;;\; M_{12} = \frac{\Psi_{12}}{i_2} \qquad \Psi_{1uk} = L_1 i_1 + M_{12} i_2$$

- Ako je geometrija nepromjenjiva:

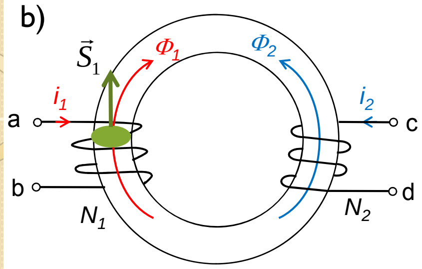
$$u_1(t) = u_{ab}(t) = \frac{\mathrm{d}\,\Psi_{1uk}}{\mathrm{d}t} = L_1 \frac{\mathrm{d}i_1}{\mathrm{d}t} + M_{12}\frac{\mathrm{d}i_2}{\mathrm{d}t}$$

- Vrijedi i obratno:

$$\Psi_{2uk} = \Psi_2 + \Psi_{21} = L_2 i_2 + M_{21} i_1$$

$$u_2(t) = u_{cd}(t) = \frac{\mathrm{d}\,\Psi_{2uk}}{\mathrm{d}t} = L_2 \frac{\mathrm{d}i_2}{\mathrm{d}t} + M_{21}\frac{\mathrm{d}i_1}{\mathrm{d}t} \;\; ; \; M_{21} = M_{12}$$

b)



$$\Psi_{1uk} = \int_{A_1} \vec{B}\cdot\vec{n}\cdot\mathrm{d}S = N_1(\Phi_1 - \Phi_{12})$$

$$\Psi_{1uk} = \Psi_1 - \Psi_{12} = L_1 i_1 - M_{12} i_2$$

$$u_1(t) = u_{ab}(t) = L_1 \frac{\mathrm{d}i_1}{\mathrm{d}t} - M_{12}\frac{\mathrm{d}i_2}{\mathrm{d}t}$$

- Vrijedi i obratno:

$$\Psi_{2uk} = \Psi_2 - \Psi_{21} = L_2 i_2 - M_{21} i_1$$

$$u_2(t) = u_{cd}(t) = \frac{\mathrm{d}\,\psi_{2uk}}{\mathrm{d}t} = L_2 \frac{\mathrm{d}i_2}{\mathrm{d}t} - M_{21}\frac{\mathrm{d}i_1}{\mathrm{d}t}$$

- Za magnetski sustav s N parova električkih priključnica (strujnih krugova) s N struja $(i_1,i_2,...,i_N)$ i M mehaničkih pomaka $(x_1,x_2,...,x_M)$ obuhvaćeni magnetski tok za neki k-ti strujni krug je:

$$\Psi_k = \int_{S_k} \vec{B}\cdot\vec{n}_k \cdot \mathrm{d}S_k$$

  - S_k je površina okružena k-tom konturom
  - $\vec{B}$ je ukupna magnetska indukcija proizvedena svim strujama $(i_1,i_2,...,i_N)$

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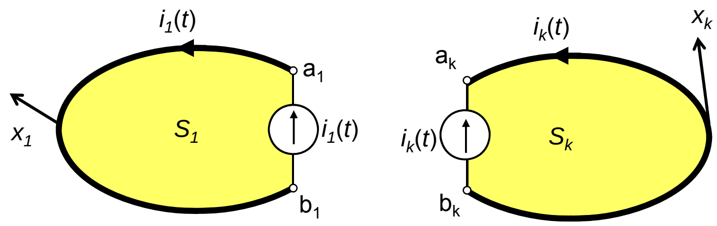

- Općenito je magnetski tok k-tog strujnog kruga:

$$\Psi_k = \Psi_k(i_1,i_2,...,i_N;x_1,x_2,...,x_M)\ \ ;\ \ k=1,2,...,N$$

- Inducirani napon k-tog strujnog kruga:

$$u_k(t) = \sum_{j=1}^{N}\frac{\partial \Psi_k}{\partial i_j}\frac{\mathrm{d}i_j}{\mathrm{d}t} + \sum_{j=1}^{M}\frac{\partial \Psi_k}{\partial x_j}\frac{\mathrm{d}x_j}{\mathrm{d}t} = u_{Tk}(t)+u_{Gk}(t)\ ;\ k=1,2,...,N$$

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- Magnetski tok *k*-tog strujnog kruga možemo izraziti i kao:

$$\Psi_k = L_k i_k \pm \sum_{\substack{i=1 \\ i \neq k}}^{N} M_{ki} i_i$$

- Inducirani napon *k*-tog strujnog kruga:

$$u_k(t) = L_k \frac{\mathrm{d}i_k}{\mathrm{d}t} \pm \sum_{\substack{i=1 \\ i \neq k}}^{N} M_{ki} \frac{\mathrm{d}i_i}{\mathrm{d}t} + i_k \frac{\partial L_k}{\partial x_k} \frac{\mathrm{d}x_k}{\mathrm{d}t} \pm$$

$$\pm \sum_{\substack{j=1 \\ j \neq k}}^{M} \sum_{i=1}^{N} i_i \frac{\partial M_{ki}}{\partial x_j} \frac{\mathrm{d}x_j}{\mathrm{d}t} \; ; \; k = 1,2,\ldots,N$$

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- Varijable i_i i x_j su neovisne pa mogu zamijeniti mjesta:

$$u_k(t) = L_k \frac{\mathrm{d}i_k}{\mathrm{d}t} \pm \sum_{\substack{i=1 \\ i \neq k}}^{N} M_{ki} \frac{\mathrm{d}i_i}{\mathrm{d}t} + i_k \frac{\partial L_k}{\partial x_k} \frac{\mathrm{d}x_k}{\mathrm{d}t} \pm$$

$$\pm \sum_{i=1}^{N} \sum_{\substack{j=1 \\ j \neq k}}^{M} i_i \frac{\partial M_{ki}}{\partial x_j} \frac{\mathrm{d}x_j}{\mathrm{d}t} \; ; \; k = 1,2,\ldots,N$$

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- U stvarnim sustavima imamo i gubitke snage u vodičima (Jouleovi gubici) i magnetskom materijalu (histereza i vrtložne struje)
- Premda su gubici važni u konstrukciji i radu (stupanj iskorištenja, termička naprezanja,...) imaju mali utjecaj na elektromehaničke interakcije
- Stoga ih modeliramo preko vanjskih otpora izvan elektromehaničkog sustava bez gubitaka

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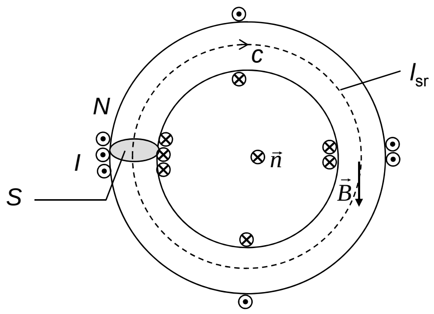
- Primjer 1: Odrediti induktivitet zavojnice namotane s N zavoja na torusnu jezgru načinjenu od feromagnetskog materijala s μ = konst., srednjeg opsega l_{sr}, kružnog poprečnog presjeka S. Koliki je inducirani napon, ako je zavojnica protjecana strujom $i = I_m \sin(\omega t)$.



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- Pretpostavke:
  - Cijeli magnetski tok prolazi kroz presjek torusa
  - Magnetsko polje je po cijelom presjeku torusa homogeno: H = konst.
  - Računamo sa srednjom vrijednosti magnetskog polja na srednjem polumjeru
- Ampereov zakon:

$$\oint_c \vec{H}\cdot \mathrm{d}\vec{l} = \sum I \Rightarrow H\cdot l_{sr} = NI \Rightarrow H = \frac{NI}{l_{sr}}$$

$$\Phi = \int_S \vec{B}\cdot\vec{n}\cdot \mathrm{d}S = \mu HS = \mu\frac{NI}{l_{sr}}S \;\;;\;\; \Psi = N\Phi = \mu\frac{N^2 I}{l_{sr}}S$$

$$L = \frac{\Psi}{I} = N^2\mu\frac{S}{l_{sr}} = N^2\mu_0\mu_r\frac{S}{l_{sr}} \;\;;\;\; u = L\frac{\mathrm{d}i}{\mathrm{d}t} = \omega L I_m \cos(\omega t)$$

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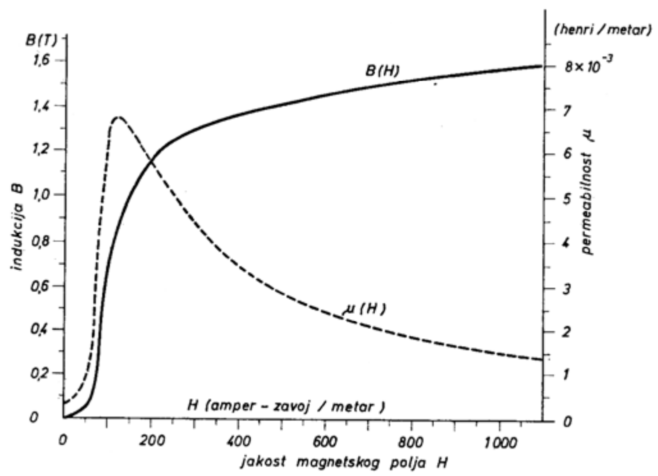
- Za feromagnetske materijale: $\mu = \mu(H)$
  - induktivitet je nelinearna veličina: $L = L(H) = L(I)$



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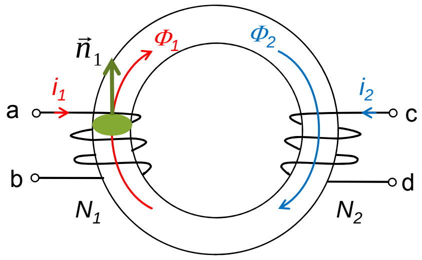
- Primjer 2: Odrediti međuinduktivitet dvije zavojnice s N_I *i* N_2 zavoja namotane na torusnu jezgru načinjenu od feromagnetskog materijala s μ = konst., kružnog poprečnog presjeka *S*, srednjeg opsega l_{sr} . Koliki su inducirani naponi na zavojnicama ako su protjecane strujama:

$$i_1 = I_{m1}\sin(\omega_1 t)\,, i_2 = I_{m2}\cos(\omega_2 t)$$



- Računamo međuinduktivitet ako je zavojnica "2" protjecana strujom i_2:

$$M_{12} = \frac{\Psi_{12}}{i_2} = \frac{N_1 \Phi_{12}}{i_2}$$

- Ampereov zakon:

$$\oint_c \vec{H}_2 \cdot \mathrm{d}\vec{l} = \sum I \Rightarrow H_2 \cdot l_{sr} = N_2 i_2 \Rightarrow H_2 = \frac{N_2 i_2}{l_{sr}}$$

$$\Phi_{12} = \int_{S_1} \vec{B}_2 \cdot \vec{n}_1 \cdot \mathrm{d}S_1 = \mu H_2 S_1 = \mu \frac{N_2 i_2}{l_{sr}} S\,; \Psi_{12} = N_1 \Phi_{12} = \mu \frac{N_1 N_2 i_2}{l_{sr}} S$$

$$M_{12} = M_{21} = M = \frac{\Psi_{12}}{i_2} = N_1 N_2 \mu \frac{S}{l_{sr}} = N_1 N_2 \mu_0 \mu_r \frac{S}{l_{sr}}$$

  - Međuinduktivitet je također nelinearan:

$$M = M(H) = M(I)$$

- Induktiviteti zavojnica su:

$$L_1 = N_1^2 \mu \frac{S}{l_{sr}} \quad ; \quad L_2 = N_2^2 \mu \frac{S}{l_{sr}}$$

- Inducirani naponi su:

$$u_1(t) = u_{ab}(t) = L_1 \frac{\mathrm{d}i_1}{\mathrm{d}t} + M_{12} \frac{\mathrm{d}i_2}{\mathrm{d}t}$$

$$u_1(t) = u_{ab}(t) = \omega_1 L_1 I_{m1} \cos(\omega_1 t) - \omega_2 M I_{m2} \sin(\omega_2 t)$$

$$u_2(t) = u_{cd}(t) = L_2 \frac{\mathrm{d}i_2}{\mathrm{d}t} + M_{21} \frac{\mathrm{d}i_1}{\mathrm{d}t}$$

$$u_2(t) = u_{cd}(t) = -\omega_2 L_2 I_{m2} \sin(\omega_2 t) + \omega_1 M I_{m1} \cos(\omega_1 t)$$

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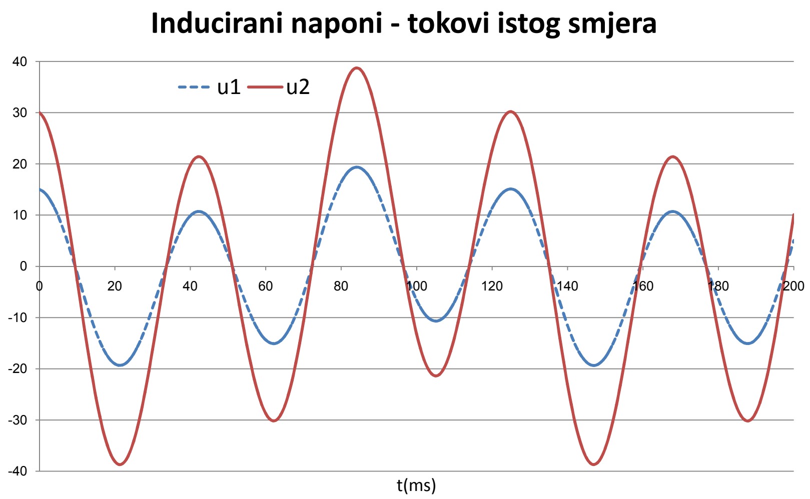

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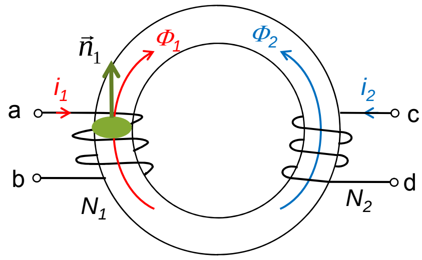
- Ako magnetski tokovi nisu istog smjera:



$$u_1(t) = u_{ab}(t) = L_1 \frac{di_1}{dt} - M_{12}\frac{di_2}{dt}$$

$$u_1(t) = u_{ab}(t) = \omega_1 L_1 I_{m1}\cos(\omega_1 t) + \omega_2 M I_{m2}\sin(\omega_2 t)$$

$$u_2(t) = u_{cd}(t) = L_2 \frac{di_2}{dt} - M_{21}\frac{di_1}{dt}$$

$$u_2(t) = u_{cd}(t) = -\omega_2 L_2 I_{m2}\sin(\omega_2 t) - \omega_1 M I_{m1}\cos(\omega_1 t)$$

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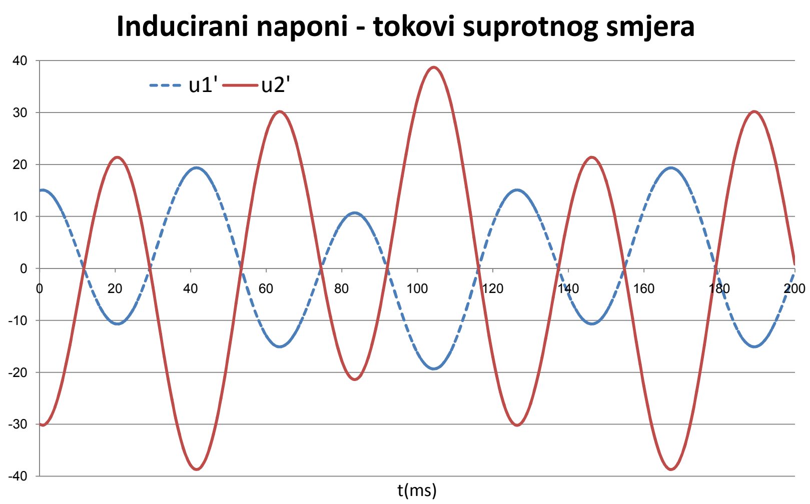

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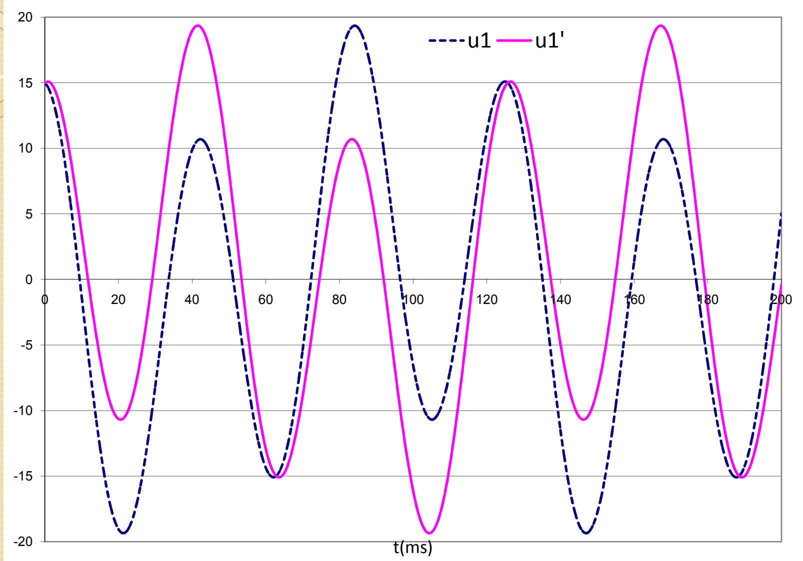

u1
u1'
20
15
10
5
0
-5
-10
-15
-20
0
20
40
60
80
100
120
140
160
180
200
t(ms)

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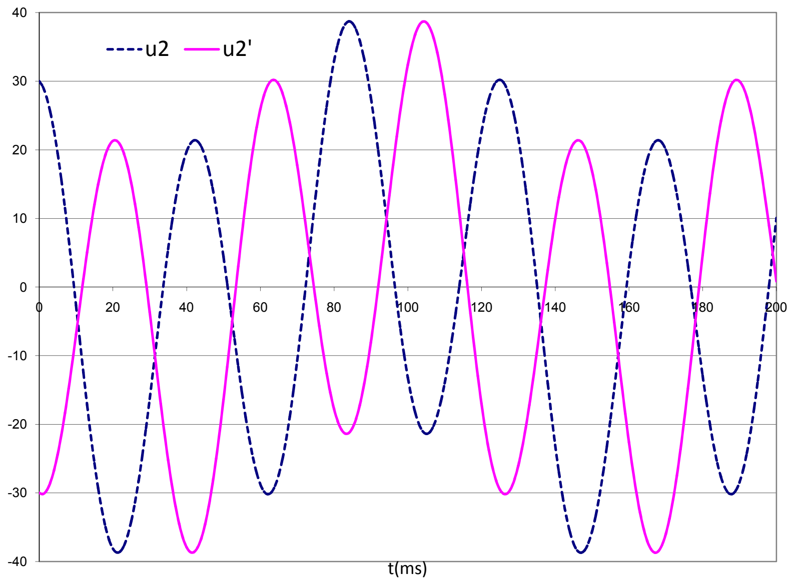

u2
u2'
40
30
20
10
0
-10
-20
-30
-40
0
20
40
60
80
100
120
140
160
180
200
t(ms)

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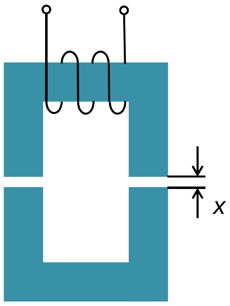
- Primjer 3: Odrediti induktivitet zavojnice s *N* zavoja namotane na jezgru načinjenu od feromagnetskog materijala s μ = konst., poprečnog presjeka *S*, srednje duljine linija polja u željezu l_{Fe} i razmaka *x*. Koliki je inducirani napon ako je zavojnica protjecana strujom $i = I_m \sin(\omega t)$ a razmak *x* se povećava brzinom *v*.



-Magnetsko polje u željezu: H_{Fe}
-Magnetsko polje u zraku: H_x

- Ampereov zakon:

$$\oint_c \vec{H} \cdot \mathrm{d}\vec{l} = \sum I \Rightarrow H_{Fe} \cdot l_{Fe} + H_x \cdot 2x = NI$$

$$B_{Fe} = B_x = B \Rightarrow H_{Fe} = \frac{B}{\mu_0 \mu_r} \ ; \ H_x = \frac{B}{\mu_0}$$

$$\frac{B}{\mu_0 \mu_r} l_{Fe} + \frac{B}{\mu_0} 2x = NI \Rightarrow B = \mu_0 \frac{NI}{\frac{l_{Fe}}{\mu_r} + 2x}$$

$$\Psi = N\Phi = NBS = \mu_0 \frac{N^2 SI}{\frac{l_{Fe}}{\mu_r} + 2x} \qquad L = \frac{\Psi}{I} = \mu_0 \frac{N^2 S}{\frac{l_{Fe}}{\mu_r} + 2x}$$

$$u(t) = L(x)\frac{\mathrm{d}i}{\mathrm{d}t} + i\frac{\mathrm{d}L(x)}{\mathrm{d}x}\frac{\mathrm{d}x}{\mathrm{d}t}$$

- Uz $v=\frac{\mathrm{d}x}{\mathrm{d}t}=\text{konst.}\Rightarrow x(t)=x_0+v\cdot t$ inducirani napon je:

$$u(t)=\mu_0\frac{N^2S}{\frac{l_{Fe}}{\mu_r}+2x}\omega I_m\cos(\omega t)-\mu_0\frac{N^2S}{\left(\frac{l_{Fe}}{\mu_r}+2x\right)^2}2I_m\sin(\omega t)\cdot v$$

$$u(t)=\mu_0\frac{N^2S}{\frac{l_{Fe}}{\mu_r}+2x}I_m\left(\omega\cdot\cos(\omega t)-\frac{2\sin(\omega t)\cdot v}{\frac{l_{Fe}}{\mu_r}+2x}\right)$$

$$u(t)=\mu_0\frac{N^2S}{\frac{l_{Fe}}{\mu_r}+2(x_0+vt)}I_m\left(\omega\cdot\cos(\omega t)-\frac{2\sin(\omega t)\cdot v}{\frac{l_{Fe}}{\mu_r}+2(x_0+vt)}\right)$$

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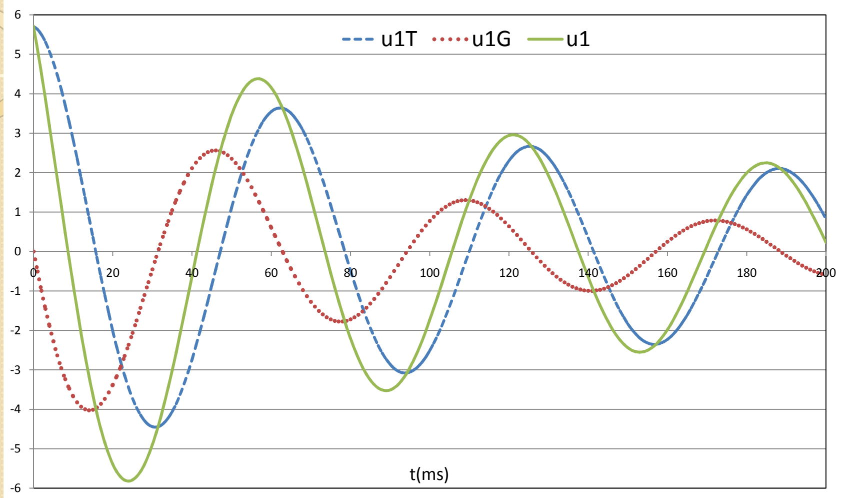

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- Ako je npr. $l_{Fe} = 100\ \mathrm{mm},\ x = 0{,}1\ \mathrm{mm},\ \mu_r = 5000$

tada je: $\dfrac{l_{Fe}}{\mu_r} = 0{,}02\ \mathrm{mm},\ 2x = 0{,}2\ \mathrm{mm}$

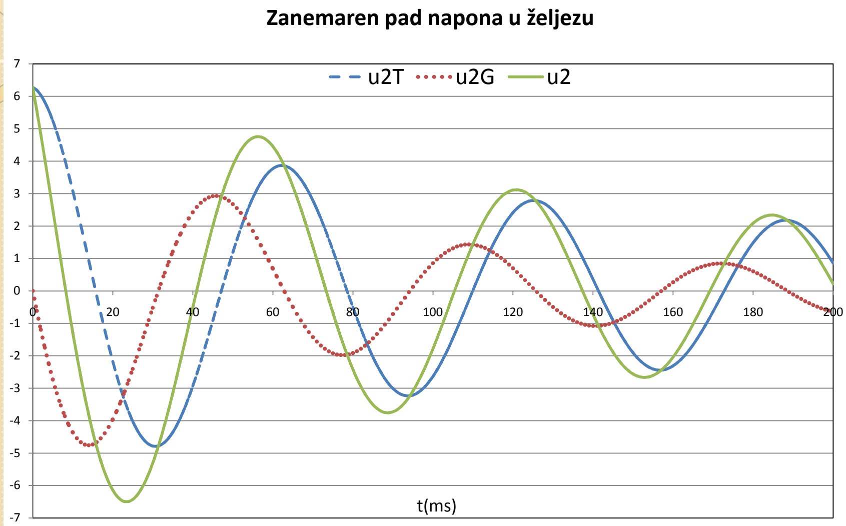
i magnetski pad napona u željezu može se zanemariti u odnosu na zrak pa je:

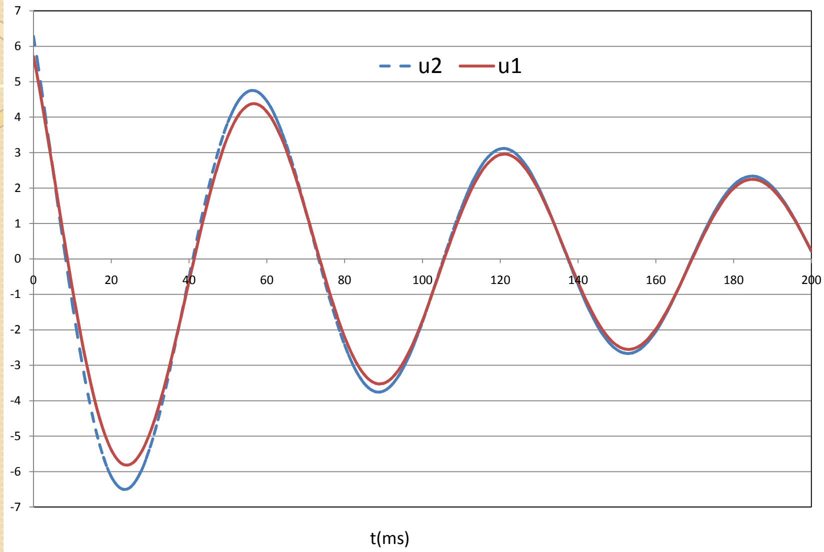
$$H_{Fe} \approx 0 \Rightarrow B \approx \mu_0 \frac{NI}{2x} \Rightarrow H_x \approx \frac{NI}{2x} \qquad L \approx \mu_0 \frac{N^2 S}{2x}$$

$$u(t) \approx \mu_0 \frac{N^2 S}{2x} \omega I_m \cos(\omega t) - \mu_0 \frac{N^2 S}{(2x)^2} 2 I_m \sin(\omega t) \cdot v$$

$$u(t) \approx \mu_0 \frac{N^2 S}{2x} I_m \left(\omega \cdot \cos(\omega t) - \frac{\sin(\omega t) \cdot v}{x} \right)$$

$$u(t) \approx \mu_0 \frac{N^2 S}{2(x_0 + vt)} I_m \left(\omega \cdot \cos(\omega t) - \frac{\sin(\omega t) \cdot v}{(x_0 + vt)} \right)$$





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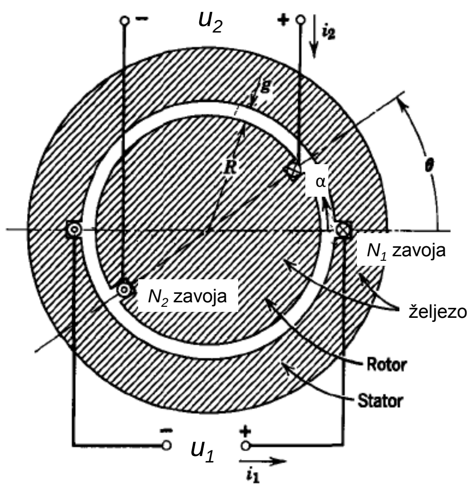
- Primjer 4: Magnetski sustav s dvije uzbude
  - Stator: N_1 zavoja protjecanih strujom $i_1(t)$
  - Rotor: N_2 zavoja protjecanih strujom $i_2(t)$



- aksijalna dužina l

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- Za $0 \le \Theta \le \pi$, uz zanemarenje magnetskog pada napona u željezu, magnetsko polje je:

$$\vec{H}_1 = \vec{a}_r \frac{N_1 i_1 - N_2 i_2}{2g} = \vec{a}_r H_1 , \qquad 0 < \alpha < \Theta$$

$$\vec{H}_2 = \vec{a}_r \frac{N_1 i_1 + N_2 i_2}{2g} = \vec{a}_r H_2 , \qquad \Theta < \alpha < \pi$$

$$\vec{H}_3 = -\vec{a}_r \frac{N_1 i_1 - N_2 i_2}{2g} = -\vec{a}_r H_1 , \qquad \pi < \alpha < \pi + \Theta$$

$$\vec{H}_4 = -\vec{a}_r \frac{N_1 i_1 + N_2 i_2}{2g} = -\vec{a}_r H_2 , \qquad \pi + \Theta < \alpha < 2\pi$$

$$H_1 = \frac{N_1 i_1 - N_2 i_2}{2g} \quad ; \quad H_2 = \frac{N_1 i_1 + N_2 i_2}{2g}$$

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- Za $-\pi \le \Theta \le 0$, uz zanemarenje magnetskog pada napona u željezu, magnetsko polje je:

$$\vec{H}_1 = \vec{a}_r \frac{N_1 i_1 + N_2 i_2}{2g} = \vec{a}_r H_1 , \qquad 0 < \alpha < \pi + \Theta$$

$$\vec{H}_2 = \vec{a}_r \frac{N_1 i_1 - N_2 i_2}{2g} = \vec{a}_r H_2 , \qquad \pi + \Theta < \alpha < \pi$$

$$\vec{H}_3 = -\vec{a}_r \frac{N_1 i_1 + N_2 i_2}{2g} = -\vec{a}_r H_1 , \qquad \pi < \alpha < 2\pi + \Theta$$

$$\vec{H}_4 = -\vec{a}_r \frac{N_1 i_1 - N_2 i_2}{2g} = -\vec{a}_r H_2 , \qquad 2\pi + \Theta < \alpha < 0$$

$$H_1 = \frac{N_1 i_1 + N_2 i_2}{2g} \quad ; \quad H_2 = \frac{N_1 i_1 - N_2 i_2}{2g}$$

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- Za $0 \le \Theta \le \pi$ obuhvaćeni magnetski tokovi su:

$$\Psi_1 = N_1 \Phi_1 = N_1 \int_S \vec{B} \cdot \vec{n} \cdot \mathrm{d}S \;\; ; \; \vec{B} = \mu_0 \vec{H} \;\; ; \; \vec{n} = \vec{a}_r \;\; ; \; \mathrm{d}S = R \cdot \mathrm{d}\alpha \cdot l$$

$$\Psi_1 = \mu_0 N_1 \left(\int_{\alpha=0}^{\Theta} \vec{H}_1 \cdot \vec{n} \cdot \mathrm{d}S + \int_{\alpha=\Theta}^{\pi} \vec{H}_2 \cdot \vec{n} \cdot \mathrm{d}S \right)$$

$$\Psi_1 = \mu_0 R l N_1 \left(\int_{\alpha=0}^{\Theta} H_1 \mathrm{d}\alpha + \int_{\alpha=\Theta}^{\pi} H_2 \mathrm{d}\alpha \right)$$

$$\Psi_1 = \frac{\mu_0 R l N_1^2 \pi}{2g} i_1 + \frac{\mu_0 R l N_1 N_2 (\pi - 2\Theta)}{2g} i_2$$

$$\Psi_1 = L_1 i_1 + L_m i_2$$

$$L_1 = N_1^2 L_0 \;\; ; \; L_0 = \frac{\mu_0 R l \pi}{2g} \;\; ; \; L_m = N_1 N_2 L_0 \left(1 - \frac{2\Theta}{\pi} \right)$$

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$$\Psi_2 = N_2 \Phi_2 = N_2 \int_S \vec{B} \cdot \vec{n} \cdot \mathrm{d}S \;\; ; \; \vec{B} = \mu_0 \vec{H} \;\; ; \; \vec{n} = \vec{a}_r \;\; ; \; \mathrm{d}S = R \cdot \mathrm{d}\alpha \cdot l$$

$$\Psi_2 = \mu_0 N_2 \left(\int_{\alpha=\Theta}^{\pi} \vec{H}_2 \cdot \vec{n} \cdot \mathrm{d}S + \int_{\alpha=\pi}^{\pi+\Theta} \vec{H}_3 \cdot \vec{n} \cdot \mathrm{d}S \right)$$

$$\Psi_2 = \mu_0 R l N_2 \left(\int_{\alpha=\Theta}^{\pi} H_2 \mathrm{d}\alpha - \int_{\alpha=\pi}^{\pi+\Theta} H_1 \mathrm{d}\alpha \right)$$

$$\Psi_2 = \frac{\mu_0 R l N_2^2 \pi}{2g} i_2 + \frac{\mu_0 R l N_1 N_2 (\pi - 2\Theta)}{2g} i_1$$

$$\Psi_2 = L_2 i_2 + L_m i_1 \;\; ; \; L_2 = N_2^2 L_0$$

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- Za $-\pi \le \Theta \le 0$ obuhvaćeni magnetski tokovi su:

$$\Psi_1 = N_1\Phi_1 = N_1\int_S \vec{B}\cdot\vec{n}\cdot \mathrm{d}S \ ; \ \vec{B} = \mu_0\vec{H} \ ; \ \vec{n} = \vec{a}_r \ ; \ \mathrm{d}S = R\cdot \mathrm{d}\alpha \cdot l$$

$$\Psi_1 = \mu_0 N_1\left(\int_{\alpha=0}^{\pi+\Theta}\vec{H}_1\cdot\vec{n}\cdot\mathrm{d}S + \int_{\alpha=\pi+\Theta}^{\pi}\vec{H}_2\cdot\vec{n}\cdot\mathrm{d}S\right)$$

$$\Psi_1 = \mu_0 RlN_1\left(\int_{\alpha=0}^{\pi+\Theta}H_1\mathrm{d}\alpha + \int_{\alpha=\pi+\Theta}^{\pi}H_2\mathrm{d}\alpha\right)$$

$$\Psi_1 = \frac{\mu_0 RlN_1^2\pi}{2g}i_1 + \frac{\mu_0 RlN_1N_2(\pi+2\Theta)}{2g}i_2$$

$$\Psi_1 = L_1 i_1 + L_m i_2$$

$$L_1 = N_1^2 L_0 \ ; \ L_0 = \frac{\mu_0 Rl\pi}{2g} \ ; \ L_m = N_1N_2L_0\left(1+\frac{2\Theta}{\pi}\right)$$

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$$\Psi_2 = N_2\Phi_2 = N_2\int_S \vec{B}\cdot\vec{n}\cdot \mathrm{d}S \ ; \ \vec{B} = \mu_0\vec{H} \ ; \ \vec{n} = \vec{a}_r \ ; \ \mathrm{d}S = R\cdot \mathrm{d}\alpha \cdot l$$

$$\Psi_2 = \mu_0 N_2\left(\int_{\alpha=\Theta}^{0}\vec{H}_4\cdot\vec{n}\cdot\mathrm{d}S + \int_{\alpha=0}^{\pi+\Theta}\vec{H}_1\cdot\vec{n}\cdot\mathrm{d}S\right)$$

$$\Psi_2 = \mu_0 RlN_2\left(\int_{\alpha=\Theta}^{0}-H_2\mathrm{d}\alpha + \int_{\alpha=0}^{\pi+\Theta}H_1\mathrm{d}\alpha\right)$$

$$\Psi_2 = \frac{\mu_0 RlN_2^2\pi}{2g}i_2 + \frac{\mu_0 RlN_1N_2(\pi+2\Theta)}{2g}i_1$$

$$\Psi_2 = L_2 i_2 + L_m i_1 \ ; \ L_2 = N_2^2 L_0$$

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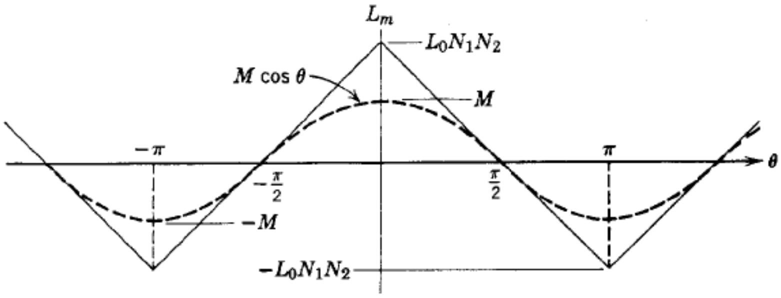
- Međuinduktivitet u ovisnosti o kutu Θ

$$L_m = L_0 N_1 N_2\left(1-\frac{2\Theta}{\pi}\right) \quad \text{za} \quad 0<\Theta<\pi$$

$$L_m = L_0 N_1 N_2\left(1+\frac{2\Theta}{\pi}\right) \quad \text{za} \quad -\pi<\Theta<0$$

$$L_0 = \frac{\mu_0 R l \pi}{2g}$$



41

- Kod projektiranja rotacijskih strojeva izmjenične struje nastoji se postići da je promjena međuinduktiviteta harmonička:

$$L_m = M\cos\Theta$$

- To se postiže s dodatnim namotima raspoređenim po obodu statora i rotora
- Induktiviteti su:

$$L_1 = N_1^2 L_0 \quad ; \quad L_2 = N_2^2 L_0$$

- Magnetski tokovi su:

$$\Psi_1 = L_1 i_1 + L_m i_2 \quad ; \quad \Psi_2 = L_m i_1 + L_2 i_2$$

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- Inducirani naponi su:

$$u_1 = \frac{\mathrm{d}\Psi_1}{\mathrm{d}t} = L_1\frac{\mathrm{d}i_1}{\mathrm{d}t} + \frac{\mathrm{d}}{\mathrm{d}t}(L_m i_2) = L_1\frac{\mathrm{d}i_1}{\mathrm{d}t} + L_m\frac{\mathrm{d}i_2}{\mathrm{d}t} + i_2\frac{\mathrm{d}L_m}{\mathrm{d}t}$$

$$u_1 = L_1\frac{\mathrm{d}i_1}{\mathrm{d}t} + M\cos\Theta\frac{\mathrm{d}i_2}{\mathrm{d}t} - i_2 M\sin\Theta\frac{\mathrm{d}\Theta}{\mathrm{d}t}$$

$$u_2 = \frac{\mathrm{d}\Psi_2}{\mathrm{d}t} = L_2\frac{\mathrm{d}i_2}{\mathrm{d}t} + \frac{\mathrm{d}}{\mathrm{d}t}(L_m i_1) = L_2\frac{\mathrm{d}i_2}{\mathrm{d}t} + L_m\frac{\mathrm{d}i_1}{\mathrm{d}t} + i_1\frac{\mathrm{d}L_m}{\mathrm{d}t}$$

$$u_2 = L_2\frac{\mathrm{d}i_2}{\mathrm{d}t} + M\cos\Theta\frac{\mathrm{d}i_1}{\mathrm{d}t} - i_1 M\sin\Theta\frac{\mathrm{d}\Theta}{\mathrm{d}t}$$

- Ako su struje sinusne u vremenu s kružnom frekvencijom ω:

$$i_1(t) = I_{m1}\sin(\omega t) \;\; ; \;\; i_2(t) = I_{m2}\sin(\omega t)$$

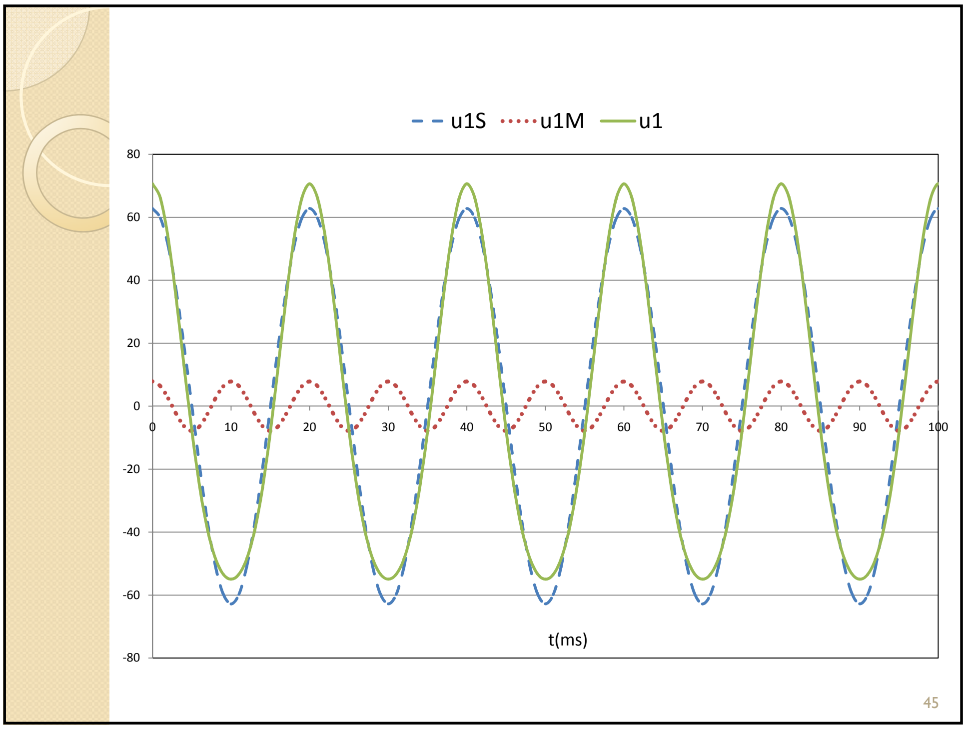
i rotor se vrti također kutnom brzinom ω, inducirani naponi su:

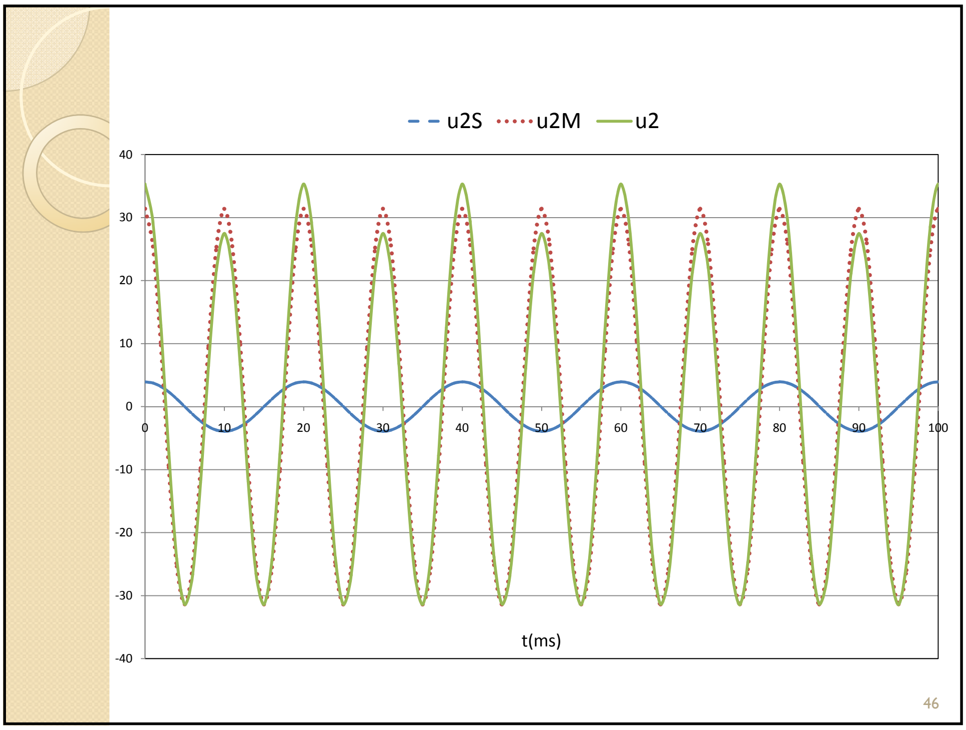
$$u_1(t) = u_{S1}(t) + u_{M1}(t)$$

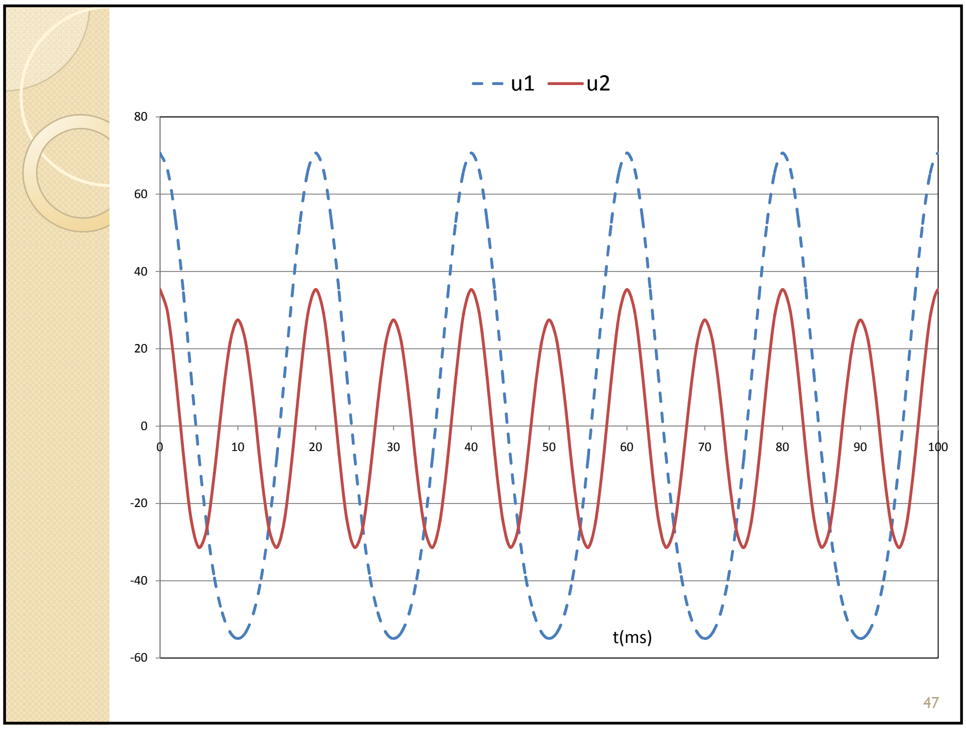
$$u_{S1}(t) = \omega L_1 I_{m1}\cos(\omega t) \;\; ; \;\; u_{M1}(t) = \omega M I_{m2}\cos(2\omega t)$$

$$u_2(t) = u_{S2}(t) + u_{M2}(t)$$

$$u_{S2}(t) = \omega L_2 I_{m2}\cos(\omega t) \;\; ; \;\; u_{M2}(t) = \omega M I_{m1}\cos(2\omega t)$$


u1S
u1M
u1
80
60
40
20
0
-20
-40
-60
-80
0
10
20
30
40
50
60
70
80
90
100
t(ms)
45


u2S
u2M
u2
40
30
20
10
0
-10
-20
-30
-40
0
10
20
30
40
50
60
70
80
90
100
t(ms)
46


u1
u2
80
60
40
20
0
-20
-40
-60
0
10
20
30
40
50
60
70
80
90
100
t(ms)
47