2014 - DIR

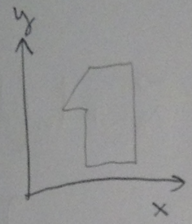
3. Odredite silu između strujnice i petlje prema slici.

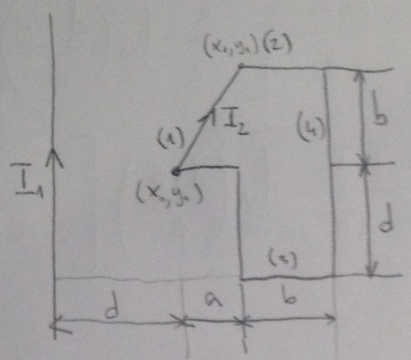
$d = 1\,m$

$a = 0{,}5\,m$

$b = 0{,}75\,m$

I_1





$$F = \int I \cdot (dl \times B)$$

$$B = \mu_0 \cdot \frac{I}{2r\pi}$$

$$\boxed{B = \mu_0 \cdot \frac{I}{2x\pi}}$$

smjer $B \Rightarrow -\vec{a_z}$

(1) $d\vec{l} = dx\,\vec{a_x} + dy\,\vec{a_y}$

(1) pravac

$(x_0, y_0) = (d, d) \Rightarrow T_0(1, 1)$

$(x_1, y_1) = (d+a, d+b) \Rightarrow T_1(1{,}5\ ,\ 1{,}75)$

$$y - y_0 = k(x - x_0)$$

$$k = \frac{y_1 - y_0}{x_1 - x_0} = \frac{1{,}75 - 1}{1{,}5 - 1} = \frac{0{,}75}{0{,}5}$$

$$\boxed{k = \frac{3}{2}}$$

$$y - 1 = \frac{3}{2}(x - 1)$$

$$\boxed{y = \frac{3}{2}x - \frac{1}{2}} \Rightarrow \boxed{dy = \frac{3}{2}dx}$$

$$F_{m1} = \int I_2 \cdot \begin{vmatrix} a_x & a_y & a_z \\ dx & dy & 0 \\ 0 & 0 & -\mu_0 \cdot \frac{I_1}{2x\pi} \end{vmatrix} = \int I_2 \cdot \left[a_x \left(-\mu_0 \cdot \frac{I_1}{2x\pi}\right) dy - a_y \cdot \left(-\mu_0 \cdot \frac{I_1}{2x\pi}\right) dx \right]$$

$$= I_2 \left[\int -\mu_0 \cdot \frac{I_1}{2x\pi}\, dy\ \vec{a_x} - \int -\mu_0 \frac{I_1}{2x\pi}\, dx\ \vec{a_y} \right]$$

$$= \frac{I_1 \cdot I_2}{2\pi} \cdot \mu_0 \left[-\vec{a_x} \cdot \int \frac{1}{x}\, dy + \vec{a_y} \int \frac{1}{x}\, dx \right]$$

$$= \frac{I_1 \cdot I_2}{2\pi} \cdot \mu_0 \left[-\vec{a_x} \cdot \frac{3}{2} \int_d^{d+a} \frac{dx}{x} + \vec{a_y} \int_d^{d+a} \frac{1}{x}\, dx \right]$$

$$F_{m1} = \mu_0 \frac{I_1 I_2}{2\pi} \cdot \left[-\frac{3}{2} \vec{a_x} \cdot \ln x \Big|_d^{d+a} + \vec{a_y} \ln x \Big|_d^{d+a} \right]$$

$$= \mu_0 \frac{I_1 I_2}{2\pi} \left[-\frac{3}{2} \cdot \ln\left(\frac{d+a}{d}\right) \cdot \vec{a_x} + \ln\left(\frac{d+a}{a}\right) \vec{a_y} \right]$$

$$F_{m1} = \mu_0 \frac{I_1 I_2}{2\pi} \cdot \ln\left(\frac{d+a}{d}\right) \left[-\frac{3}{2} \vec{a_x} + \vec{a_y} \right]$$

(2)

$$d\vec{l} = dx\, \vec{a_x}$$

$$F_{m2} = I_2 \int \begin{vmatrix} a_x & a_y & a_z \\ dx & 0 & 0 \\ 0 & 0 & -\mu_0 \frac{I_1}{2x\pi} \end{vmatrix} = I_2 \cdot \int_{d+a}^{d+a+b} -a_y \left(-\mu_0 \frac{I_1}{2\pi x}\right) dx = \mu_0 \cdot \frac{I_1 \cdot I_2}{2\pi} \int_{1,5}^{2,25} \frac{dx}{x} \vec{a_y}$$

$$= \mu_0 \frac{I_1 \cdot I_2}{2\pi} \cdot \ln \frac{2,25}{1,5} \vec{a_y}$$

(3)

$$F_{m2} = F_{m3} = \mu_0 \cdot \frac{I_1 I_2}{2\pi} \cdot \ln \frac{2,25}{1,5} \vec{a_y}$$

(4) $d\vec{l} = -dy\, \vec{a_y}$

$$F_{m4} = I_2 \int \begin{vmatrix} a_x & a_y & a_z \\ 0 & -dy & 0 \\ 0 & 0 & -\mu_0 \frac{I_1}{2\pi x} \end{vmatrix} = I_2 \cdot \int a_x \left(\mu_0 \cdot \frac{I_1}{2\pi x}\right) dy = \mu_0 \frac{I_1 I_2}{2\pi \cdot x} \int_0^{d+b} dy\, \vec{a_x}$$

$$= \mu_0 \cdot \frac{I_1 I_2}{2\pi \cdot (d+a+b)} \cdot (d+b)\, \vec{a_x}$$

$$\boxed{F_{uk} = F_{m1} + F_{m2} + F_{m3} + F_{m4} = \mu_0 \cdot \frac{I_1 I_2}{2\pi} \left[\vec{a_x} \left(\frac{d+b}{d+a+b} - \frac{3}{2} \ln\left(\frac{d+a}{d}\right)\right) + \vec{a_y} \left(2 \ln \frac{2,25}{1,5} + \ln \frac{d+a}{d} \right) \right]}$$

2014. – MR

5. Električno i magnetsko polje unutar beskonačno dugog cilindra $a \le r \le b$ su u cilindričnom koordinatnom sustavu zadani kao:

$\boxed{\text{cilindrični koordinatni sustav}}$

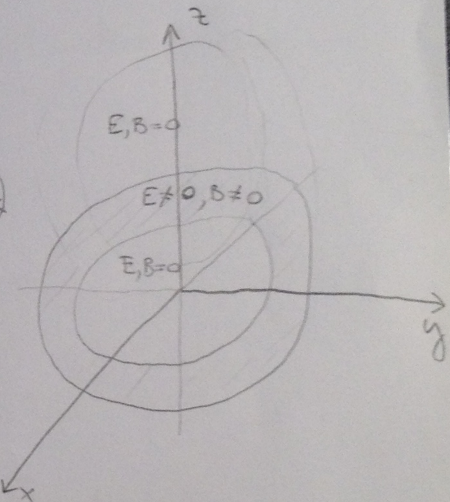
$$\vec{E} = \begin{cases} \overbrace{\dfrac{U_0}{r\ln\left(\frac{b}{a}\right)}\cos(\beta z)\cos(\omega t)}^{\text{BROJ 1}}\,\vec{a}_r & ;\ a \le r \le b \\ \underbrace{0}_{\text{BROJ 2}} & ;\ r<a\ ;\ r>b \end{cases}$$

$$B = \mu_0 \mu_r \cdot H$$

$$\mu_r = 1$$

$$B = \mu_0 \cdot H$$

$$\vec{B} = \begin{cases} \dfrac{\mu_0 \cdot I_0}{r\ln\left(\frac{b}{a}\right)}\sin(\beta z)\cos(\omega t)\,\vec{a}_\alpha & ;\ \dots\ a \le r \le b \\ 0 & ;\ r<a\ ,\ r>b \end{cases}$$



$$\vec{N} = \vec{E}\times\vec{H} = (\text{BROJ 1}\ \vec{a}_r)\times(\text{BROJ 2}\cdot\vec{a}_\alpha) =$$

$$\text{BROJ 1}\cdot\text{BROJ 2}\,(\vec{a}_r\times\vec{a}_\alpha)$$

$$\text{BROJ 1}\cdot\text{BROJ 2}\ \vec{a}_z$$

$$\vec{N} = \begin{vmatrix} \vec{a}_r & \vec{a}_\alpha & \vec{a}_z \\ E_r & 0 & 0 \\ 0 & H_\alpha & 0 \end{vmatrix}$$

$$\vec{N} = \vec{a}_z\cdot\left(E_r - H_\alpha\right) = \vec{a}_z\left(\vec{E}_r - \frac{\vec{B}_\alpha}{\mu_0}\right)$$

$$= \vec{a}_z\cdot\left(\frac{U_0}{r\ln\left(\frac{b}{a}\right)}\cos(\beta z)\cos(\omega t) - \frac{I_0}{r\ln\left(\frac{b}{a}\right)}\underbrace{\sin(\beta z)}_{0}\cos(\omega t)\right)$$

$$N_{se} = \frac{1}{2}\vec{N}$$

za $\cos(\beta z) = 1$ i $\cos(\omega t) = 1$, $\sin(\beta z) = 0$

$$\boxed{\vec{N}_{se} = \frac{1}{2}\frac{U_0}{r\ln\left(\frac{b}{a}\right)}\vec{a}_z}$$

$$P_{se} = \frac{1}{2}\oiint_S \vec{N}\cdot n\,dS = N_{se}\cdot\left[2\pi\left(b^2-a^2\right)\right]$$

$$P_{se} = \frac{1}{\cancel{2}}\frac{U_0}{r\ln\left(\frac{b}{a}\right)}\cdot\cancel{2}\pi\left(b^2-a^2\right) = \frac{U_0\cdot\left(b^2-a^2\right)}{r\ln\left(\frac{b}{a}\right)}$$

$$\boxed{P_{se} = \begin{cases} \dfrac{u_o\left(b^2 - a^2\right)}{r \ln\left(\frac{b}{a}\right)} & ; \; r \geq a, \; r \leq b \\[2ex] 0 & ; \; r < 0, \; r > b \end{cases}}$$

2014 – JIR

2. Jakost električnog polja antene koja se nalazi u ishodištu sfernog koordinatnog sustava u slobodnom prostoru je

$$\vec{E} = E_0 \frac{\sin\vartheta \cos\vartheta}{r} \cos(\omega t - \beta r) \vec{a}_\vartheta$$

$\vec{H}$ je u smjeru $\vec{a}_\alpha$

$$H_0 = \frac{E_0}{Z} = \frac{2}{2\pi} = \frac{1}{\pi} \text{ A/m} \qquad Z = \frac{\omega\mu}{\beta} = \frac{10^{7} \cdot 4\pi\, 10^{-7}}{2}$$

$$\boxed{Z = 2\pi\ \Omega}$$

$$\vec{H} = \frac{1}{\pi} \cdot \frac{\sin\vartheta \cos\vartheta}{r} \cdot \cos(\omega t - \beta r) \vec{a}_\alpha$$

$$\boxed{\vec{H} = 0{,}069\ \vec{a}_\alpha \text{ A/m}}$$

2015 - DIR

3. Jakost električnog polja sinusno promjenjivog ravnog vala koji se prostire u idealnom dielektriku zadana je s:

$$\vec{E} = \left(\vec{a}_x + \vec{a}_y\right)\cos\left[2\sqrt{2}\cdot 10^8 t - 2z\right] \ V/m$$

$\boxed{\mu_r = 1}$

a) vektor magnetskog polja

$\boxed{\beta_z = 2}$

$$\vec{H}_0 = \frac{1}{\omega\mu}\cdot\vec{\beta}\times\vec{E}_0$$

$$= \frac{1}{2\sqrt{2}\cdot 10^8\cdot 4\pi\cdot 10^{-7}}\cdot\left[2\vec{a}_z \times \left(\vec{a}_x + \vec{a}_y\right)\right]$$

$$= 2{,}81\cdot 10^{-3}\cdot\begin{vmatrix} a_x & a_y & a_z \\ 0 & 0 & 2 \\ 1 & 1 & 0 \end{vmatrix} = 2{,}81\cdot 10^{-3}\cdot\left(a_x(-2) - a_y(-2)\right)$$

$$= -2{,}81\cdot 10^{-3}\left(2\vec{a}_x - 2\vec{a}_y\right)$$

$$\boxed{\vec{H} = -5{,}63\cdot 10^{-3}\left(\vec{a}_x - \vec{a}_y\right)\cdot\cos\left[2\sqrt{2}\cdot 10^8 t - 2z\right] \ A/m}$$

b)

$$\vec{N} = \vec{E}\times\vec{H} = \begin{vmatrix} \vec{a}_x & \vec{a}_y & \vec{a}_z \\ 1 & 1 & 0 \\ -5{,}63\cdot 10^{-3} & 5{,}63\cdot 10^{-3} & 0 \end{vmatrix} = \vec{a}_z\left(5{,}63\cdot 10^{-3} - \left(-5{,}63\cdot 10^{-3}\right)\right)\cdot\cos\ldots$$

$\cos\left(2\sqrt{2}\cdot 10^8 t - 2z\right)$

$$= a_z\cdot 1{,}126\cdot 10^{-2}\cdot\cos\left[2\sqrt{2}\cdot 10^8 t - 2z\right] \ W/m^2$$

c) valna impedancija sredstva

$$\boxed{|Z| = \frac{|E_0|}{|H_0|} = \frac{|1|}{|-5{,}63 \cdot 10^{-3}|} = 177{,}62\ \Omega}$$

d) relativna dielektrična konstanta sredstva

$$\beta = \omega\sqrt{\mu\varepsilon} \Rightarrow \frac{\beta^2}{\omega^2 \mu_0 \mu_r \varepsilon_0} = \varepsilon_r$$

$$\boxed{\varepsilon_r = 4{,}49}$$

2014_DIR

4. Jakost električnog polja ravnog elektromagnetskog vala koji se širi dielektrikom relativne magnetske permeabilnosti $\mu_r = 1$ zadana je izrazom:

$$\vec{E} = 2{,}8 \sin(2{,}11 \cdot 10^8 t - 1{,}9x)\ \vec{a_y}\ \frac{V}{m}$$

Odredite smjer širenja vala, relativnu dielektričnost sredstva, valnu duljinu i jakost magnetskog polja.

$$\vec{E} = E_m \cdot \sin(\omega t - \beta x)$$

(1) Smjer širenja vala jest $\boxed{+\vec{a_x}}$

(2)

$$\beta = \omega\sqrt{\mu \cdot \varepsilon} \rightarrow \left(\frac{\beta}{\omega}\right)^2 = \mu_0 \mu_r \cdot \varepsilon_0 \varepsilon_r \qquad \boxed{\beta = 1{,}9}$$

$$\boxed{\varepsilon_r = \frac{\beta^2}{\omega^2 \cdot \mu_0 \cdot \mu_r \cdot \varepsilon_0} = 7{,}29} \qquad \boxed{\omega = 2{,}11 \cdot 10^8}$$

(3)

$$\boxed{\lambda = \frac{2\pi}{\beta} = 3{,}31\ m}$$

(4)

$$Z = \sqrt{\frac{\mu_0 \mu_r}{\varepsilon_0 \varepsilon_r}} = 139{,}53\ \Omega \Rightarrow \frac{E_0}{H_0} = 139{,}53 \Rightarrow H_0 = \frac{E_0}{139{,}53} = \boxed{0{,}02}$$

$$\boxed{\vec{H} = 0{,}02 \cdot \sin(2{,}11 \cdot 10^8 t - 1{,}9x)\ \vec{a_z}\ A/m}$$

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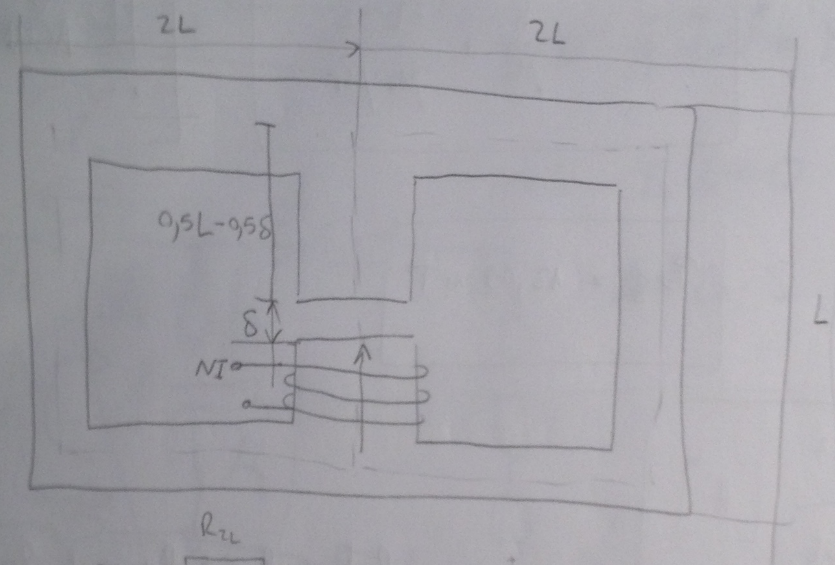
3. Magnetski krug zadan je slikom, pri čemu je duljina $L = 10$ cm. Permitivnost materijala iznosi $\mu_r = 1000$. Površina poprečnog presjeka magnetskog materijala je svugdje jednaka 20 cm². Odredite induktivitet zavojnice. Ako je struja zavojnice 0,1 A, a broj zavoja $N = 250$ odredite indukciju u zračnom rasporu.

$L = 10$ cm

$\mu_r = 1000$

$S = 20$ cm²

$\delta = 0{,}2$ cm



$$H = \frac{B}{\mu}$$

$$H_\delta = \frac{B_\delta}{\mu_0}$$

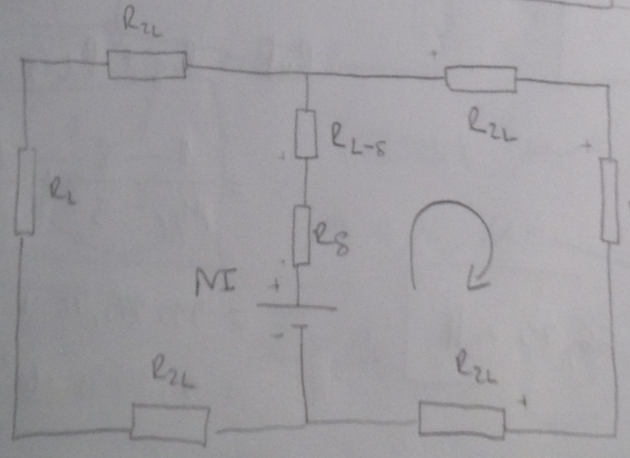
$B = $ konst.

$\phi = B \cdot S$

$B_\delta = B_j$

$$H_j = \frac{B_j}{\mu_0 \mu_r}$$

$$H_j = \frac{B_\delta}{\mu_0 \mu_r}$$



$$NI = \phi R_\delta + \phi R_{L-\delta} + 5 \cdot \phi R_L$$

$$N \cdot I = H_\delta \cdot \delta + H_j \cdot \left[L - \delta + 5L \right]$$

$$N \cdot I = \frac{B_\delta}{\mu_0} \cdot \delta + \frac{B_\delta}{\mu_0 \mu_r} \cdot (6L - \delta)$$

$$= B_\delta \cdot \left[\frac{\delta}{\mu_0} + \frac{6L - \delta}{\mu_0 \mu_r} \right]$$

$$\boxed{B_\delta = \frac{N \cdot I}{\frac{\delta}{\mu_0} + \frac{6L - \delta}{\mu_0 \cdot \mu_r}}}$$

[!! RAČUNAJ U METRIMA]

$$\boxed{B_\delta = 12{,}09 \text{ mT}}$$

$$\boxed{L = \frac{N^2}{R_{uk}} \; [H]}$$

$$R = \frac{1}{\mu_0 \mu_r} \cdot \frac{l}{S}$$

$$\boxed{L = \frac{250^2}{9{,}35831 \cdot 10^5} = 66{,}79 \text{ mH}}$$

$$R_{uk} = R_\delta + R_{L-\delta} + R_{5L} \| R_{5L}$$

$$= \frac{1}{\mu_0} \cdot \frac{\delta}{S} + \frac{1}{\mu_0 \mu_r} \left(\frac{L - \delta}{S} + \frac{1}{2} \cdot \frac{5L}{S} \right)$$

$$= 795\,774{,}72 + 140\,056{,}35$$

$$= 9{,}35831 \cdot 10^5 \; \Omega$$

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3. U poluprostoru (1) $z>0$ vrijedi $K_1=0$, $\varepsilon_{r1}=1$, $\mu_{r1}=4$, a u poluprostoru (2) $z<0$ vrijedi $K_2=0$, $\varepsilon_{r2}=1$, $\mu_{r2}=2$. Magnetska indukcija u poluprostoru (1) je $B_1 = B_0\left(2\vec{a_x} + 4\vec{a_y} + 5\vec{a_z}\right) T$, a na granici dvaju poluprostora teče plošna struja gustoće $\vec{K} = \frac{B_0}{\mu_0}\left(\vec{a_x} - 2\vec{a_y}\right) A/m$

$$B = \mu \cdot H$$

$$H = \frac{B}{\mu} = \frac{B}{\mu_0 \mu_r}$$

$$\vec{n} \times \left(\vec{H_2} - \vec{H_1}\right) = \vec{K}$$

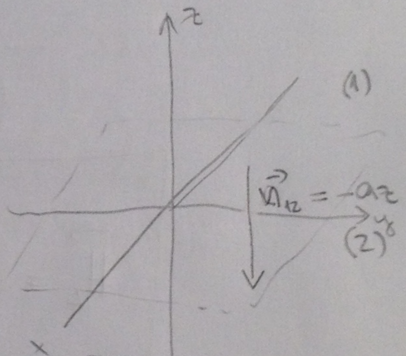
$$\vec{n} \cdot \left(\vec{B_2} - \vec{B_1}\right) = 0$$

$$B_1 = B_0 \cdot \left(2\vec{a_x} + 4\vec{a_y} + 5\vec{a_z}\right) \; T$$

$$B_2 = B_0 \left(x\vec{a_x} + y\vec{a_y} + z a_z\right) \; T$$

$$H_1 = \frac{B_0}{\mu_0 \cdot \mu_{r1}} \left(2\vec{a_x} + 4\vec{a_y} + 5 a_z\right) A/m$$

$$H_2 = \frac{B_0}{\mu_0 \cdot \mu_{r2}} \left(x\vec{a_x} + y\vec{a_y} + z\vec{a_z}\right) A/m$$



$$\boxed{\vec{n}_{12} = -a_z}$$

$$\vec{n}_{12} \times \left(\vec{H_2} - \vec{H_1}\right) = \begin{vmatrix} \vec{a_x} & \vec{a_y} & \vec{a_z} \\ 0 & 0 & -1 \\ \frac{x}{\mu_{r2}} - \frac{2}{\mu_{r1}} & \frac{y}{\mu_{r2}} - \frac{4}{\mu_{r1}} & \frac{z}{\mu_{r2}} - \frac{5}{\mu_{r1}} \end{vmatrix} \cdot \frac{B_0}{\mu_0}$$

$$= \left[\vec{a_x}\left(+\left(\frac{y}{\mu_{r2}} - \frac{4}{\mu_{r1}}\right)\right) - \vec{a_y}\left(+\left(\frac{x}{\mu_{r2}} - \frac{2}{\mu_{r1}}\right)\right) + 0\,\vec{a_z}\right] \frac{B_0}{\mu_0}$$

$$= \frac{B_0}{\mu_0}\left[\vec{a_x}\left(\frac{y}{2} - \frac{4}{4}\right) - \vec{a_y}\left(\frac{x}{2} - \frac{2}{4}\right) + 0\,\vec{a_z} = \vec{a_x}\left(\frac{y}{2} - 1\right) + \vec{a_y}\left(-\frac{x}{2} + \frac{1}{2}\right) + 0\,\vec{a_z}\right]$$

$$\frac{B_0}{\mu_0}\left[\vec{a_x}\left(\frac{y}{2}-1\right)+\vec{a_y}\left(-\frac{x}{2}+\frac{1}{2}\right)\right]=\vec{K}=\frac{B_0}{\mu_0}\left(\vec{a_x}-2\vec{a_y}\right)$$

$$\vec{a_x}\left(\frac{y}{2}-1\right)+\vec{a_y}\left(-\frac{x}{2}+\frac{1}{2}\right)=1\cdot\vec{a_x}+\vec{a_y}\cdot(-2)$$

$$\frac{y}{2}-1=1 \qquad \boxed{y=4}$$

$$-\frac{x}{2}+\frac{1}{2}=-2 \quad /\cdot 2$$

$$-x+1=-4$$

$$-x=-5$$

$$\boxed{x=5}$$

$$\boxed{\vec{H_2}=\frac{B_0}{2\mu_0}\left(5\vec{a_x}+4\vec{a_y}+5\vec{a_z}\right)\ A/m}$$

$$(-a_z)\cdot\left((x-2)\vec{a_x}+(y-4)\vec{a_y}+(z-5)\vec{a_z}\right)=0$$

$$-z+5=0$$

$$\boxed{z=5}$$

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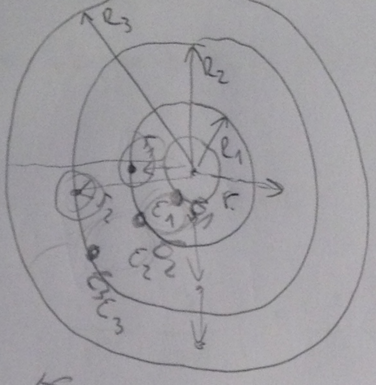
4. Zadan je troslojni kuglasti kondenzator prema slici. Odredite ε_{r1} i ε_{r2} pri kojima su najveće apsolutne vrijednosti jakosti električnog polja u sva tri sloja jednake

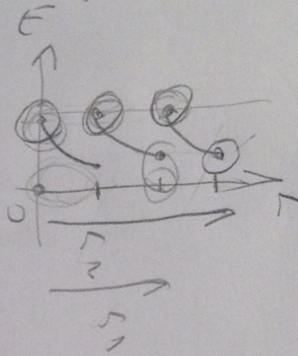
$R_1 = R$

$R_2 = 2R$

$R_3 = 3R$



$$\oint ds = 4r^2\pi$$



$$\oiint_S \vec{D} \cdot \vec{n}\, ds = \int_V \rho\, dv = Q$$

$$E_1 = E_2 = E_3$$

$$\oiint_S \varepsilon_0 \varepsilon_r \cdot E \cdot n\, ds = \varepsilon_0 \varepsilon_r \cdot E \cdot \oiint_S ds = \varepsilon_0 \cdot \varepsilon_r \cdot E \cdot 4r^2\pi = Q$$

$$\boxed{E = \frac{Q}{4r^2\pi \cdot \varepsilon_0 \varepsilon_r}}$$

$$\varepsilon_{r1} = \boxed{\varepsilon_{r1}}$$

$$\varepsilon_{r2} = \boxed{\varepsilon_{r1} \cdot \frac{1}{4}}$$

$$\varepsilon_{r3} = \varepsilon_{r2} \cdot \frac{16}{36} = \boxed{\frac{4}{36}\varepsilon_{r1}}$$

(1) $r = R_1 = R$

$\varepsilon_r = \varepsilon_{r1}$

$$E_1 = \frac{Q}{4R^2\pi \cdot \varepsilon_0 \varepsilon_{r1}}$$

(2) $r = R_2 = 2R$

$\varepsilon_r = \varepsilon_{r2}$

$$E_2 = \frac{Q}{4 \cdot 4R^2\pi \cdot \varepsilon_0 \varepsilon_{r2}} = \frac{Q}{16 R^2 \cdot \pi \cdot \varepsilon_0 \cdot \varepsilon_{r2}}$$

(3) $r = R_3 = 3R$

$\varepsilon_r = \varepsilon_{r3}$

$$E_3 = \frac{Q}{4 \cdot 9R^2\pi \cdot \varepsilon_0 \cdot \varepsilon_{r3}} = \frac{Q}{36 R^2 \cdot \pi \cdot \varepsilon_0 \cdot \varepsilon_{r3}}$$

$$\frac{1}{16\varepsilon_{r2}} = \frac{1}{36\varepsilon_{r3}} \Rightarrow \boxed{\varepsilon_{r3} = \frac{16}{36}\varepsilon_{r2}}$$

$$\frac{1}{4\varepsilon_{r1}} = \frac{1}{36\varepsilon_{r3}} = \frac{1}{36} \cdot \frac{\cancel{36}}{16\varepsilon_{r2}}$$

$$\boxed{\varepsilon_{r1} = 4\varepsilon_{r2}}$$