MAXWELOVE JEDN (ZADACI ZA VJEŽBU)

PR1)

$$\vec{E} = E_m \sin ax \cos(\omega t - \beta z)\vec{a}_y$$

$$\vec{H} = ?$$

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} = -\frac{\partial}{\partial t}(\mu \vec{H}) = -\mu \frac{\partial}{\partial t}\vec{H}$$

$$\nabla \times \vec{E} = \begin{vmatrix} \vec{a}_x & \vec{a}_y & \vec{a}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 0 & E_y & 0 \end{vmatrix} = \vec{a}_x\left(-\frac{\partial}{\partial z}E_y\right) + \vec{a}_z\left(\frac{\partial E_y}{\partial x}\right)$$

$$= \vec{a}_x\left(-E_m \sin ax\,(-\sin(\omega t - \beta z)\cdot\beta)\right)$$

$$+ \vec{a}_z\left(E_m \cos(\omega t - \beta z)\, a \cos ax\right)$$

$$= \vec{a}_x\left(-\beta E_m \sin ax \sin(\omega t - \beta z)\right)$$

$$+ \vec{a}_z\left(E_m a \cos ax \cos(\omega t - \beta z)\right)$$

$$\vec{H} = -\frac{1}{\mu}\int\left(-\beta E_m \sin ax \sin(\omega t - \beta z)\right)\vec{a}_x + \left(E_m a \cos ax \cos(\omega t - \beta z)\right)$$

$$= -\frac{1}{\mu}\left[\vec{a}_x\left(-\beta E_m \sin ax\left(-\frac{1}{\omega}\cos(\omega t - \beta z)\right)\right) + \right.$$

$$\left. + \vec{a}_z\left(E_m a \cos ax \frac{1}{\omega}\sin(\omega t - \beta z)\right)\right]$$

$$\vec{H} = -\frac{\beta E_m}{\mu\omega}\sin ax \cos(\omega t - \beta z)\vec{a}_x$$

$$-\frac{a E_m}{\mu\omega}\cos ax \sin(\omega t - \beta z)\vec{a}_z + C$$

C=0 - za sinusna polja

PR2) $\mu = 3 \cdot 10^{-5}\ H/m$

$\varepsilon = 1{,}2 \cdot 10^{-10}\ F/m$

$\kappa = 0$

$\vec{H} = 2\cos(10^{10} t - \beta x)\vec{a_z}$, $10^{10} = w$

$\beta = ?$

$$\nabla \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t}$$

$$\nabla \times \vec{H} = \begin{vmatrix} \vec{a_x} & \vec{a_y} & \vec{a_z} \\ x & y & z \\ 0 & 0 & H_z \end{vmatrix} = \vec{a_x}\left(\frac{\partial H_z}{\partial y}\right) - \vec{a_y}\left(\frac{\partial H_z}{\partial x}\right) + \vec{a_z} \cdot 0$$

$$= \vec{a_x}(\cdot 0) - \vec{a_y}\left(2(-\sin(w t - \beta x) \cdot \beta\right)$$

$$= -2\beta \sin(wt - \beta x)\vec{a_y}$$

$$\vec{J} = \kappa \vec{E} = 0$$

$$\Rightarrow \quad -2\beta \sin(wt - \beta x) = \varepsilon \frac{\partial \vec{E}}{\partial t}$$

$$\vec{E} = -\frac{2\beta}{\varepsilon}\int \sin(wt - \beta x)\,dt$$

$$\vec{E} = -\frac{2\beta}{\varepsilon}\left(-\frac{1}{w}\cos(wt - \beta x)\right)\vec{a_y}$$

$$\vec{E} = \frac{2\beta}{\varepsilon w}\cos(wt - \beta x)\vec{a_y}$$

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} = -\mu \frac{\partial \vec{H}}{\partial t}$$

$$\nabla \times \vec{E} = \begin{vmatrix} \vec{a}_x & \vec{a}_y & \vec{a}_z \\ x & y & z \\ 0 & E_y & 0 \end{vmatrix} = \vec{a}_x \left(-\cancel{\frac{\partial E_y}{\partial z}} \right)^{0} - \vec{a}_y \cdot 0 + \vec{a}_z \left(\frac{\partial E_y}{\partial x} \right)$$

$$\left(\vec{E} = \frac{2B}{\varepsilon w} \cos(wt - Bx)\, \vec{a}_y \right)$$

$$\nabla \times \vec{E} = \vec{a}_z \frac{2B}{\varepsilon w} \left(-(-B) \sin(wt - Bx) \right)$$

$$= \vec{a}_z \left(+ \frac{2B^2}{\varepsilon w} \sin(wt - Bx) \right)$$

$$+ \frac{2B^2}{\varepsilon w} \sin(wt - Bx) = -\mu \frac{\partial}{\partial t} \left(2\cos(wt - Bx) \right)$$

$$= -\mu\, 2 \left(-w \sin(wt - Bx) \right)$$

$$+ \frac{\cancel{2}B^2}{\varepsilon w} \cancel{\sin(wt - Bx)} = \cancel{2} \mu w \cancel{\sin(wt - Bx)}$$

$$B^2 = \mu \varepsilon w^2$$

$$\underline{B = w\sqrt{\mu \varepsilon} = 600 \text{ rad/m}}$$

PR3 $\mu=\mu_0$

$\varkappa = 0$

$\vec{H} = H_0 \cos 2x \cos(\omega t - \beta y)\,\vec{a}_x$

$\frac{\partial \vec{D}}{\partial t} = ? \Rightarrow$ STRUJA POMAKA : $\vec{J}_{POM} = \frac{\partial \vec{D}}{\partial t}$

$\nabla \times \vec{H} = \cancel{\vec{J}} + \frac{\partial \vec{D}}{\partial t} \qquad (\vec{J} = \varkappa \vec{E})$

0 - jer je $\varkappa = 0$

$$\nabla \times \vec{H} = \begin{vmatrix} \vec{a}_x & \vec{a}_y & \vec{a}_z \\ x & y & z \\ H_x & 0 & 0 \end{vmatrix} = -\vec{a}_y\left(\cancel{-\frac{\partial H_x}{\partial z}}\right) + \vec{a}_z\left(-\frac{\partial H_x}{\partial y}\right)$$

$= H_0 \cos 2x\,(-\sin(\omega t - \beta y)(-\beta))\,\vec{a}_z$

$= H_0 \cos 2x \sin(\omega t - \beta y)\,\vec{a}_z = \frac{\partial \vec{D}}{\partial t}$

$\Rightarrow \vec{D} = H_0 \cos 2x \int \sin(\omega t - \beta y)\,dt\,\vec{a}_z$

$= H_0 \cos 2x \left(-\frac{1}{\omega}\cos(\omega t - \beta y)\right)\vec{a}_z$

$\vec{D} = -\frac{H_0}{\omega} \cos 2x \cos(\omega t - \beta y)\,\vec{a}_z$

$\vec{E} = -\frac{H_0}{\varepsilon\omega} \cos 2x \cos(\omega t - \beta y)\,\vec{a}_z$

$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$

$$= \begin{vmatrix} \vec{a}_x & \vec{a}_y & \vec{a}_z \\ x & y & z \\ 0 & 0 & E_z \end{vmatrix} = \vec{a}_x\left(\frac{\partial E_z}{\partial y}\right) - \vec{a}_y\left(\frac{\partial E_z}{\partial x}\right)$$

$= \vec{a}_x\left(-\frac{H_0}{\varepsilon\omega} \cos 2x\,[-\sin(\omega t - \beta y)\cdot(-\beta)]\right) - \vec{a}_y\left[-\frac{H_0}{\varepsilon\omega}\cos(\omega t - \beta y)(-\sin 2x\right.$

$$\nabla \times \vec{E} = -\vec{a}_x \frac{H_0 \beta}{\varepsilon\omega} \cos 2x \sin(\omega t - \beta y) - \frac{2H_0}{\varepsilon\omega} \sin 2x \cos(\omega t - \beta y) \vec{a}_z$$

$\Rightarrow$ polje ne zadovoljava maxwel. jed.

PR4

$$\vec{E} = \frac{E_0 \sin\alpha}{r} \cos(\omega t - \beta r) \vec{a}_\alpha$$

E_0, ω i $\beta = \omega\sqrt{\mu_0 \varepsilon_0}$ - konstante

$P = ?$

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} = -\mu_0 \frac{\partial \vec{H}}{\partial t}$$

$$\nabla \times \vec{E} = \vec{a}_r \frac{1}{r\sin\alpha} \left[\cancel{\frac{\partial}{\partial\alpha}(\sin\alpha E_\alpha)} - \cancel{\frac{\partial E_\alpha}{\partial\alpha}} \right]$$

$$+ \vec{a}_\alpha \frac{1}{r} \left[\cancel{\frac{1}{\sin\alpha} \frac{\partial E_r}{\partial\alpha}} - \cancel{\frac{\partial}{\partial r}(r E_\alpha)} \right]$$

$$+ \vec{a}_\alpha \frac{1}{r} \left[\frac{\partial}{\partial r}(r E_\alpha) - \cancel{\frac{\partial E_r}{\partial\alpha}} \right]$$

$$= -\vec{a}_r \frac{1}{r\sin\alpha} \frac{\partial}{\partial\alpha} \left(\cancel{\frac{E_0 \sin\alpha}{r} \cos(\omega t - \beta y)} \right)$$

$$+ \vec{a}_\alpha \frac{1}{r} \left[\frac{\partial}{\partial r} \left(\cancel{r} \frac{E_0 \sin\alpha}{\cancel{r}} \cos(\omega t - \beta r) \right) \right]$$

$$= \vec{a}_\alpha \frac{1}{r} E_0 \sin\alpha (-\sin(\omega t - \beta r) \cdot (-\beta)$$

$$= \vec{a}_\alpha \frac{\beta E_0}{r} \sin\alpha \sin(\omega t - \beta r) = -\mu_0 \frac{\partial \vec{H}}{\partial t}$$

$$\Rightarrow \vec{H} = -\frac{\beta E_0}{\mu r} \sin\alpha \int \sin(\omega t - \beta r)\, dt \; \vec{a}_\alpha$$

$$= -\frac{\beta E_0}{\mu r} \sin\alpha \left(-\frac{1}{\omega} \cos(\omega t - \beta r) \right) \vec{a}_\alpha$$

$$\vec{H} = \frac{\beta E_0}{\mu_0 r \omega} \sin\alpha \cos(\omega t - \beta r) \vec{a}_\alpha$$

$$\vec{N} = \vec{E} \times \vec{H} = \begin{vmatrix} \vec{a}_r & \vec{a}_\alpha & \vec{a}_\theta \\ 0 & 0 & E_\theta \\ 0 & H_\alpha & 0 \end{vmatrix} = \vec{a}_r\left(-H_\alpha E_\theta\right) - \cancel{\vec{a}_\alpha}(0) + \vec{a}_\theta \cdot 0$$

$$\boxed{\vec{N} = \vec{a}_r\left(-\frac{\beta E_0^2}{\mu_0 r^2 \omega} \sin^2\theta \cos^2(\omega t - \beta r)\right)}$$

$P = \oiint\limits_S \vec{N}\vec{n}\,ds$ $\quad$ S - uzimamo oplošje kugle (najjednostavniji slučaj)

$\vec{n} = \vec{a}_r$

$ds = r^2 \sin\theta \, d\theta \, d\alpha$

$$P = \frac{\beta E_0^2}{\mu_0 \omega} \int_{\alpha=0}^{2\pi} \int_{\theta=0}^{\pi} \frac{\sin^2\theta}{\cancel{r^2}} \cos^2(\omega t - \beta r) \cdot \cancel{r^2} \sin\theta \, d\theta \, d\alpha \cdot 1$$

$$= 2\pi \frac{\beta E_0^2}{\mu_0 \omega} \cos^2(\omega t - \beta r) \int_{\theta=0}^{\pi} \sin^3\theta \, d\theta$$

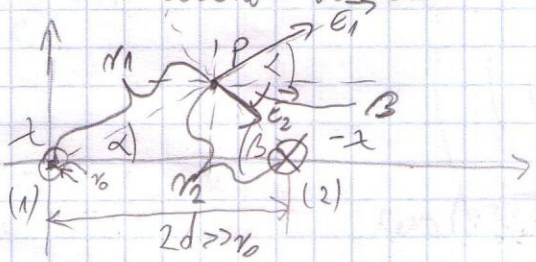
$$\int \sin^3 x \, dx = -\cos x + \frac{1}{3}\cos^3 x$$

$$P = 2\pi \frac{\beta E_0^2}{\mu_0 \omega} \cos^2(\omega t - \beta r) \underbrace{\left(-\cos\theta + \frac{1}{3}\cos^3\theta\right)\Big|_0^\pi}_{\frac{4}{3}}$$

$$\boxed{P = \frac{8}{3}\pi \frac{\beta E_0^2}{\mu_0 \omega} \cos^2(\omega t - \beta r)}$$

PR5

$\sigma \to \infty$; idealni vodiči



$\vec{E} = ?$

$$\varphi_1 = \frac{\lambda}{2\pi\varepsilon_0} \ln\frac{2d}{r_0}$$

$$\varphi_2 = \frac{-\lambda}{2\pi\varepsilon_0} \ln\frac{r_0}{2d}$$

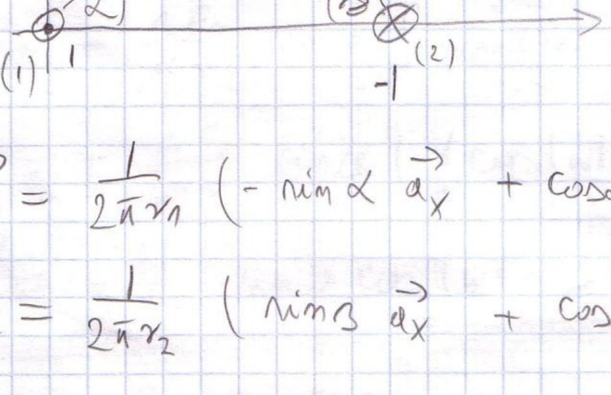
$$\Rightarrow U = \varphi_1 - \varphi_2 = \frac{\lambda}{\varepsilon_0 \pi} \ln\frac{2d}{r_0}$$

$$\Rightarrow \lambda = \frac{\pi\varepsilon_0 U}{\ln\frac{2d}{r_0}}$$

$$\vec{E}_1 = \frac{\lambda}{2\pi\varepsilon_0 r_1}\left(\cos\alpha\,\vec{a}_x + \sin\alpha\,\vec{a}_y\right)$$

$$\vec{E}_2 = \frac{\lambda}{2\pi\varepsilon_0 r_2}\left(\cos\beta\,\vec{a}_x - \sin\beta\,\vec{a}_y\right)$$

$$\vec{E} = \vec{E}_1 + \vec{E}_2 = \frac{\lambda}{2\pi\varepsilon_0}\left(\frac{\cos\alpha}{r_1} + \frac{\cos\beta}{r_2}\right)\vec{a}_x + \frac{\lambda}{2\pi\varepsilon_0}\left(\frac{\sin\alpha}{r_1} - \frac{\sin\beta}{r_2}\right)\vec{a}_y$$



$\vec{H} = ?$

$$\vec{H}_1 = \frac{I}{2\pi r_1}\left(-\sin\alpha\,\vec{a}_x + \cos\alpha\,\vec{a}_y\right)$$

$$\vec{H}_2 = \frac{I}{2\pi r_2}\left(\sin\beta\,\vec{a}_x + \cos\beta\,\vec{a}_y\right)$$

$$\vec{H} = \vec{H}_1 + \vec{H}_2 = \frac{1}{2\pi}\left(-\frac{\sin\alpha}{r_1} + \frac{\sin\beta}{r_2}\right)\vec{a}_x + \frac{1}{2\pi}\left(\frac{\cos\alpha}{r_1} + \frac{\cos\beta}{r_2}\right)\vec{a}_y$$

$$\vec{N} = \vec{E} \times \vec{H} = \left(E_x \vec{a}_x + E_y \vec{a}_y\right) \times \left(H_x \vec{a}_x + H_y \vec{a}_y\right)$$
$$= \vec{a}_z \left(E_x H_y - H_x E_y\right)$$
$$= \vec{a}_z \frac{VI}{4\pi \ln\frac{2d}{r_0}} \left[\left(\frac{\cos\alpha}{r_1} + \frac{\cos\beta}{r_2}\right)^2 + \left(\frac{\sin\alpha}{r_1} - \frac{\sin\beta}{r_2}\right)^2\right]$$

provjera:
$$\vec{N} = \vec{E} \times \vec{H} = \begin{vmatrix} \vec{a}_x & \vec{a}_y & \vec{a}_z \\ E_x & E_y & 0 \\ H_x & H_y & 0 \end{vmatrix} = \vec{a}_x(0) - \vec{a}_y(0) + \underline{\underline{\vec{a}_z(E_x H_y - H_x E_y)}}$$

4.1

$$\vec{E} = 2x\, \vec{a}_y \quad kV/m$$

$w_e(4m, 5m, 7m) = ?$

$$\underline{\underline{w_e}} = \frac{1}{2}\varepsilon_0 |\vec{E}|^2 = \frac{1}{2}\cdot 8{,}854\cdot 10^{-12}\cdot (2000\cdot 4)^2 = \underline{\underline{283{,}33\ \mu J/m^3}}$$

4.2

$\gamma = 5\ S/m$

$\varepsilon_r = 1{,}5$

$E = 250 \sin(10^{10} t)\ V/m$

$f(GHz) = ?$

$$\nabla \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t}$$

$I_{POM} = I_{PROV} \Rightarrow J_{POM} = J_{PROV}$

$$\Rightarrow \quad \frac{\partial D}{\partial t} = \gamma E$$

$$\varepsilon \frac{\partial}{\partial t} E = \gamma E$$

$$\varepsilon_0 \varepsilon_r \cancel{250} \cos(10^{10} t)\cdot 10^{10} = \gamma \cancel{250} \sin(10^{10} t)$$

$10^{10} = w$

$$\varepsilon_0 \varepsilon_r\, w \cos(wt) = \gamma \sin(wt)$$

$w = 2\pi f$

- za iste amplitude:

$$\Rightarrow \quad \varepsilon_0 \varepsilon_r w = \gamma \Rightarrow 2\pi f = \frac{\gamma}{\varepsilon_0 \varepsilon_r}$$

$$f = \frac{\gamma}{\varepsilon_0 \varepsilon_r 2\pi} = 59{,}92\ GHz$$

4.3

$\vec{E} = E_m \sin(\omega t - \beta z)\, \vec{a}_y$

$B_m = ?\ [mT]$

$E_m = 10\ V/m$ $\quad \nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$

$\beta = 0{,}6\ m^{-1}$

$\omega = 10^4$

$$\nabla \times \vec{E} = \begin{vmatrix} \vec{a}_x & \vec{a}_y & \vec{a}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 0 & E_y & 0 \end{vmatrix} = \vec{a}_x\left(-\frac{\partial E_y}{\partial z}\right) - \vec{a}_y(\cdot 0) + \vec{a}_z\left(\frac{\partial E_y}{\partial x}\right.$$

$$= -E_m \cos(\omega t - \beta z) \cdot (-\beta)\, \vec{a}_x + 0\vec{a}_z$$

$$= \beta E_m \cos(\omega t - \beta z)\, \vec{a}_x = -\frac{\partial}{\partial t}\vec{B}$$

$$\vec{B} = -\beta E_m \int \cos(\omega t - \beta z)\, dt\, \vec{a}_x$$

$$= -\beta E_m \frac{1}{\omega} \sin(\omega t - \beta z)\, \vec{a}_x$$

$$\Rightarrow \underline{\underline{|B_m|}} = \frac{\beta E_m}{\omega} = 6 \cdot 10^{-4} = \underline{\underline{0{,}6\ mT}} \checkmark$$

EM VALOVI

4.4

$$\vec{E}(z,t) = 50\cos(\omega t - \beta z)\,\vec{a}_x \quad V/m$$

$P = ?$

S - krug ; $r = 2{,}5$ m

$z = 2$ m

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} = -\mu \frac{\partial \vec{H}}{\partial t}$$

$$= \begin{vmatrix} \vec{a}_x & \vec{a}_y & \vec{a}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ E_x & 0 & 0 \end{vmatrix} = -\vec{a}_y\left(\frac{\partial E_x}{\partial z}\right) + \vec{a}_z \cancel{\left(-\frac{\partial E_x}{\partial y}\right)}^{0}$$

$$= -50\left(-\sin(\omega t - \beta z)(-\beta)\right)\vec{a}_y$$

$$= -50\beta\sin(\omega t - \beta z)\,\vec{a}_x = -\mu\frac{\partial \vec{H}}{\partial t}$$

$$\vec{H} = \frac{50\beta}{\mu}\int \sin(\omega t - \beta z)\,dt\,\vec{a}_y$$

$$= \frac{50\beta}{\mu}\left(-\frac{1}{\omega}\cos(\omega t - \beta z)\right)\vec{a}_y$$

$$\boxed{\vec{H} = -\frac{50\beta}{\omega\mu}\cos(\omega t - \beta z)\,\vec{a}_y}$$

$$\vec{N} = \vec{E} \times \vec{H} = \begin{vmatrix} \vec{a}_x & \vec{a}_y & \vec{a}_z \\ E_x & 0 & 0 \\ 0 & H_y & 0 \end{vmatrix} = \vec{a}_x \cdot 0 - \vec{a}_y(\cdot 0) + \vec{a}_z E_x H_y$$

$$\boxed{\vec{N} = -\frac{50^2\beta}{\omega\mu}\cos^2(\omega t - \beta z)\,\vec{a}_z}$$

$$\vec{N}_{sr} = \frac{1}{T}\int_0^T \vec{N}\,dt = \frac{1}{2}\vec{N} = \frac{50^2 \beta}{2\mu w}\,\vec{a}_z$$

$$P_{sr} = \oiint_S \vec{N}_{sr}\,\vec{n}\,ds = N_{sr}\,S = \frac{50^2 \beta\, r^2 \pi}{2\mu_0 w} = \frac{50^2\, w\sqrt{\varepsilon_0\mu_0}\, r^2\pi}{2\,\mu_0\, w} = 65{,}15\ \mathrm{W}$$

$\vec{n} = \vec{a}_z$ $\qquad \beta = w\sqrt{\mu_0\varepsilon_0}$

$S = r^2\pi$

4.8

$\kappa = 10^{-3}\ S/m$

$\varepsilon_r = 2{,}5$

$E = 6\cdot 10^{-6} \sin(9\cdot 10^9 t)\ V/m$

$|J_{prov}| = ?$

$J = \kappa E$

$J = 10^{-3}\cdot 6\cdot 10^{-6} \sin(9\cdot 10^9 t)$

$J = \underbrace{6\cdot 10^{-9}}_{|J_{prov}|} \sin(wt)$

4.9

$\vec{E}(z,t) = 10 \sin(wt - \beta z)\,\vec{a}_x - 15 \sin(wt-\beta z)\,\vec{a}_y\ V/m$

$t = 0$

$z = 0{,}75\lambda = \frac{3}{4}\lambda$

$E = ?$

$E = \sqrt{E_x^2 + E_y^2} = 18{,}03\ V/m$

$E_{x_{max}} = 10 \qquad E_{y_{max}} = -15$

4.12

$\kappa = 10^{-3}$ S/m

$\varepsilon_r = 2{,}5$

$E = 6 \cdot 10^{-6} \sin(9 \cdot 10^{9} t)$ V/m

J_{pom} [$\mu A/m^2$] = ?

$$J_{pom} = \frac{\partial D}{\partial t} = \varepsilon \frac{\partial E}{\partial t}$$

$$J_{pom} = 8{,}854 \cdot \cancel{10^{-12}}_{10^{-9}} \cdot 2{,}5 \cdot 6 \cdot \cancel{10^{-6}} \cdot \cos(9 \cdot 10^{9} t) \cdot \cancel{10^{9}} \cdot 9$$

$$J_{pom} = 1{,}195 \cos(9 \cdot 10^{9} t) \ \mu A/m^2$$

AMPLITUDA

4.14.

$$\vec{E}_A = E_m \sin x \sin t \ \vec{a}_y$$

$$\vec{H}_A = \frac{E_m}{\mu_0} \cos x \cos t \ \vec{a}_z$$

da li $\vec{E}$ i $\vec{H}$ zadovoljavaju MAXWELLOVE jed.?

mora vrijediti:

$$\nabla \times \vec{E}_A = -\frac{\partial \vec{B}}{\partial t} = -\mu_0 \frac{\partial \vec{H}}{\partial t}$$

$$= \begin{vmatrix} \vec{a}_x & \vec{a}_y & \vec{a}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 0 & E_y & 0 \end{vmatrix} = \cancel{\vec{a}_x\left(-\frac{\partial E_y}{\partial z}\right)}^{0} - \vec{a}_y \cdot 0 + \vec{a}_z\left(\frac{\partial E_y}{\partial x}\right)$$

$$= E_m \sin t \cos x \ \vec{a}_z = -\mu_0 \frac{\partial \vec{H}}{\partial t}$$

$$\Rightarrow \vec{H} = -\frac{E_m}{\mu_0} \cos x \int \sin t \, dt \ \vec{a}_z$$

$$= -\frac{E_m}{\mu_0} \cos x \, (-\cos t) \ \vec{a}_z$$

$$\vec{H} = \frac{E_m}{\mu_0} \cos x \cos t \ \vec{a}_z$$

mora vrijedit: $\vec{H} = \vec{H}_A \Rightarrow \frac{E_m}{\mu_0} \cos x \cos t = \frac{E_m}{\mu_0} \cos x \cos t$

2) $\nabla \times \vec{H}_A = \frac{\partial \vec{D}}{\partial t} = \varepsilon_0 \frac{\partial \vec{E}}{\partial t}$

$$= \begin{vmatrix} \vec{a}_x & \vec{a}_y & \vec{a}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 0 & 0 & H_z \end{vmatrix} = \vec{a}_x\left(\cancel{\frac{\partial H_z}{\partial y}}\right) - \vec{a}_y\left(\frac{\partial H_z}{\partial x}\right)$$

$$= -\frac{E_m}{\mu_0}(-\sin x)\cos t\ \vec{a}_y$$

$$= \frac{E_m}{\mu_0}\sin x \sin t\ \vec{a}_x = \varepsilon_0 \frac{\partial \vec{E}}{\partial t}$$

$$\Rightarrow \vec{E} = +\frac{E_m}{\mu_0\varepsilon_0}\sin x \int \cos t\, dt\ \vec{a}_y$$

$$= \frac{E_m}{\mu_0\varepsilon_0}\sin x \sin t\ \vec{a}_y$$

$\vec{E} \neq \vec{E}_A \Rightarrow$ nie wijeole NAYW. jed.

4.17 i 4.18

$$\vec{E} = E_m \cos(\omega t - k_2 z)\,\vec{a}_x$$

$$\vec{H} = \frac{E_m}{k_1}\cos(\omega t - k_2 z)\,\vec{a}_y$$

$k_1, k_2 =$ konst. $k_1 = ?$

$\nabla \times \vec{H} = \frac{\partial \vec{D}}{\partial t}$ - vakuum

$$= \begin{vmatrix} \vec{a}_x & \vec{a}_y & \vec{a}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 0 & H_y & 0 \end{vmatrix} = \vec{a}_x\left(-\frac{\partial H_y}{\partial z}\right) + \vec{a}_z\left(\cancel{\frac{\partial H_y}{\partial x}}\right)$$

$$= -\frac{E_m}{k_1}(-\sin(\omega t - k_2 z)\cdot(-k_2)\ \vec{a}_x$$

$$= -\frac{E_m}{k_1}k_2 \sin(\omega t - k_2 z)\ \vec{a}_x = \varepsilon_0 \frac{\partial \vec{E}}{\partial t}$$

$$\frac{\partial \vec{E}}{\partial t} = E_m\left(-\sin(\omega t - k_2 z)\,\omega\right)\vec{a_x} = -E_m\,\omega\,\sin(\omega t - k_2 z)$$ -14-

$$\cancel{+}\cancel{E_m}\,\frac{k_2}{k_1}\,\cancel{\sin(\omega t - k_2 z)} = \cancel{+}\varepsilon_0\,\cancel{E_m}\,\omega\,\cancel{\sin(\omega t - k_2 z)}$$

$$\boxed{k_2 = k_1\,\varepsilon_0\,\omega}$$

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} = -\mu_0\frac{\partial \vec{H}}{\partial t}$$

$$\nabla \times \vec{E} = \begin{vmatrix} \vec{a_x} & \vec{a_y} & \vec{a_z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ E_x & 0 & 0 \end{vmatrix} = -\vec{a_y}\left(-\frac{\partial E_x}{\partial z}\right) + \cancel{\vec{a_z}\left(-\frac{\partial E_x}{\partial y}\right)}^{0}$$

$$= E_m\left(-\sin(\omega t - k_2 z)(-k_2)\right)\vec{a_y}$$

$$= E_m\,k_2\,\sin(\omega t - k_2 z)\,\vec{a_y}$$

$$-\mu_0\frac{\partial \vec{H}}{\partial t} = \frac{E_m}{k_1}\left(-\sin(\omega t - k_2 z)\cdot\omega\right)\vec{a_y}$$

$$= +\mu_0\frac{E_m}{k_1}\,\omega\,\sin(\omega t - k_2 z)\,\vec{a_y}$$

$$\cancel{E_m}\,k_2\,\cancel{\sin(\omega t - k_2 z)} = +\mu_0\frac{E_m}{k_1}\,\omega\,\cancel{\sin(\omega t - k_2 z)}$$

$$\boxed{k_2 = +\mu_0\frac{\omega}{k_1}}$$

$$k_1\,\varepsilon_0\,\cancel{\omega} = \frac{\cancel{\omega}}{k_1}\,\mu_0 \qquad \boxed{k_2 = \sqrt{\frac{\mu_0}{\varepsilon_0}}\,\varepsilon_0\,\omega}\ \checkmark$$

$$k_1^2 = \frac{\mu_0}{\varepsilon_0}$$

$$\boxed{k_1 = \sqrt{\frac{\mu_0}{\varepsilon_0}}}\ \checkmark$$

↓

zapravo

valna impedancija

4.21

$$\vec{E} = 100 \cos\left(\omega t + \frac{2\pi}{5} x\right) \vec{a}_z \quad V/m$$

$$\vec{H} = \frac{100}{120\pi} \cos\left(\omega t + \frac{2\pi}{5} x\right) \vec{a}_y \quad A/m$$

$$\underline{\vec{N}} = ?$$

$$\underline{\vec{E}} = 100\, e^{j\frac{2\pi}{5}x}\, \vec{a}_z$$

$$\underline{\vec{H}} = \frac{100}{120\pi}\, e^{j\frac{2\pi}{5}x}\, \vec{a}_y \Rightarrow \underline{\vec{H}}^* = \frac{100}{120\pi}\, e^{-j\frac{2\pi}{5}x}\, \vec{a}_y$$

$$\underline{\vec{N}} = \underline{\vec{E}} \times \underline{\vec{H}}^* = \begin{vmatrix} \vec{a}_x & \vec{a}_y & \vec{a}_z \\ 0 & 0 & \underline{E}_z \\ 0 & \underline{H}_y^* & 0 \end{vmatrix}$$

$$= \vec{a}_x \left(-\underline{E}_z \underline{H}_y^*\right) + \vec{a}_z \cdot 0$$

$$\underline{\vec{N}} = -100 \cdot \frac{100}{120\pi}\, \cancel{e^{j\frac{2\pi}{5}x}} \cdot \cancel{e^{-j\frac{2\pi}{5}x}}\, \vec{a}_x$$

$$\boxed{\underline{\vec{N}} = -26{,}53\, \vec{a}_x}$$

4.22

$$\vec{E} = 100 \cos\left(\omega t + \frac{4\pi}{3} x\right) \vec{a}_z \quad V/m$$

$$\vec{H} = \frac{100}{120\pi} \cos\left(\omega t + \frac{4\pi}{3} x\right) \vec{a}_y \quad A/m$$

a) $\underline{\vec{E}} = ?$ $\quad \omega = 200\,\pi$

b) $\underline{\vec{H}} = ?$

c) $\underline{\vec{N}}(x=1, t=2) = ?$ u smjeru $\vec{a}_x$

d) $P_{sr} = ?$

$A(0,0,0)$ $\quad D(0,0,2)$

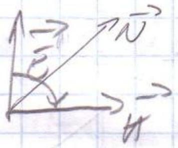
$B(0,2,0)$ $\quad$ i $\vec{n} = \vec{a}_x$

$C(0,2,2)$

a) $\boxed{\underline{\vec{E}} = 100\, e^{j\frac{4\pi}{3}x}\,\vec{a}_z \quad V/m}$ ✓

b) $\vec{H} = \frac{100}{120\pi}\, e^{j\frac{4\pi}{3}x}\,\vec{a}_y$ ✓ $\quad \underline{\vec{H}}^* = \frac{100}{120\pi}\, e^{-j\frac{4\pi}{3}x}\,\vec{a}_y$

c) $\vec{N} = \underline{\vec{E}} \times \underline{\vec{H}} = -\vec{a}_x E_z H_y = -\vec{a}_x \frac{100^2}{120\pi} \cos^2\left(\omega t + \frac{4\pi}{3}x\right)$



$\vec{N}(x=1, t=2) = -\frac{100^2}{120\pi} \cos^2\left(400\pi\cdot 10^3 \cdot 2 + \frac{4\pi}{3}\right)\vec{a}_x$

$\omega = 2\pi f = 400\pi \cdot 10^3$

$\boxed{\vec{N} = -10{,}14\ \vec{a}_x}$ 2 ✓

d) $P_{sr} = \oint\!\!\oint_S \underline{\vec{N}_{sr}}\,\vec{n}\,dS \qquad \underline{\vec{N}_{sr}} = \frac{1}{2}\left(\underline{\vec{E}} \times \underline{\vec{H}}^*\right) = \underline{-13{,}265\ \vec{a}_x}$

$\vec{n} = \vec{a}_x$

$S = 2\cdot 2 = 4$

$\underline{P_{sr}} = 4\cdot(-13{,}265)\cdot\underbrace{\vec{a}_x\cdot\vec{a}_x}_{1} = \underline{-53{,}06\ W/m^2}$

4.23

$\mu_r = 1$

$\vec{E} = 3 \sin(2 \cdot 10^8 t - 2x)\, \vec{a}_y$ V/m $\Rightarrow$ $\omega = 2 \cdot 10^8$ $\beta = 2$

a) smjer u kojem se giba val = ?

b) $\lambda = ?$

c) $\varepsilon_r = ?$

d) $\vec{H} = ?$

a) $\vec{a}_x$

b) $\lambda = \frac{2\pi}{\beta} = \frac{2\pi}{2} = \pi$ ✓

c) $\beta = \omega\sqrt{\varepsilon\mu}$

$\beta^2 = \omega^2 \varepsilon_0 \varepsilon_r \mu_0 \mu_r \Rightarrow$ $\varepsilon_r = \left(\frac{\beta}{\omega}\right)^2 \cdot \frac{1}{\varepsilon_0 \mu_0 \mu_r} = 8{,}99 \approx 9$ ✓

d) $\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} = -\mu \frac{\partial \vec{H}}{\partial t}$

$$= \begin{vmatrix} \vec{a}_x & \vec{a}_y & \vec{a}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 0 & E_y & 0 \end{vmatrix} = \cancel{\vec{a}_x\left(-\frac{\partial E_y}{\partial z}\right)}^{0} + \vec{a}_z\left(\frac{\partial E_y}{\partial x}\right)$$

$= 3\cos(\omega t - \beta x)(-\beta)\, \vec{a}_z$

$= -3\beta\cos(\omega t - \beta x)\, \vec{a}_z = -\mu \frac{\partial \vec{H}}{\partial t}$

$\Rightarrow \vec{H} = \frac{3\beta}{\mu} \int \cos(\omega t - \beta x)\, dt\, \vec{a}_z$

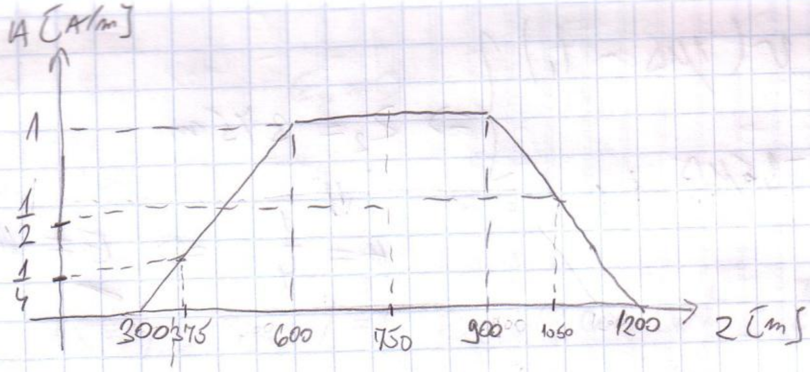
$\vec{H} = \frac{3\beta}{\mu\omega} \sin(\omega t - \beta x)\, \vec{a}_z = 0{,}024 \sin(2 \cdot 10^8 t - 2x)\, \vec{a}_z$ A/m ✓

4.25

$\mu_r = 1$

$\varepsilon_r = 4$

$t = 1\mu s$



- val se giba u +z smjeru
- el. polje ima samo x komponentu
- mag. -//- -//- y -//-

a) $E(t=-1\mu s, z=0) = ?$

b) $E(t=-1{,}5\mu s, z=0) = ?$

c) $E(t=-4\mu s, z=0) = ?$

d) $E(t=-6\mu s, z=0) = ?$

a) $\vec{E} = E_x \vec{a_x}$ $\qquad Z = \sqrt{\frac{\mu}{\varepsilon}} = \sqrt{\frac{\mu_0 \mu_r}{\varepsilon_0 \varepsilon_r}} = \underline{188{,}4\ \Omega}$

$\underline{E_m = H_m Z = 188{,}4\ V/m}$

$H_m = 1$ - vidi se na slike

$v = \frac{z_2 - z_1}{t_2 - t_1}$ $\qquad v = \frac{c}{\sqrt{\mu_r \varepsilon_r}} = \frac{1}{\sqrt{\mu \varepsilon}} = \underline{1{,}5 \cdot 10^8\ m/s}$

$\boxed{t = t_2}$

$\Rightarrow \boxed{z_2 - z_1 = v(t - t_1)}$

$z_1 = 0$ - zadano u pitanju! (pozicija početnog položaja)

$t_1 = -1\mu s$ - vrijeme početnog položaja

$\Rightarrow \quad z_2 - 0 = v(t_2 - t_1)$

$z_2 = 1{,}5 \cdot 10^8 (1\mu s - (-1\mu s)) = 300\ m$

$\Rightarrow$ sa slike $(z = 300) \Rightarrow H = 0 \Rightarrow \underline{E = 0}$ ✓

b) $z_2 = v(1\mu s - t_1)$, $t_1 = -1{,}5\mu s$ $\Rightarrow z_2 = 375\,m$

$\Downarrow$

$H = \frac{1}{4} \Rightarrow E = Hz = \frac{188{,}4}{4} = \underline{47{,}1\ V/m}$

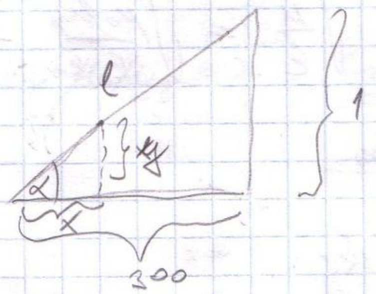
c) $t_1 = -4\mu s$, $z_2 = v(1\mu s - t_1)$ $\Rightarrow z_2 = 750\,m$

$\Rightarrow H = 1 \Rightarrow \underline{E = Hz = 188{,}4\ V/m}$

d) $t_1 = -6\mu s$, $z_2 = v(1\mu s - t_1)$ $\Rightarrow z_2 = 1050\,m$

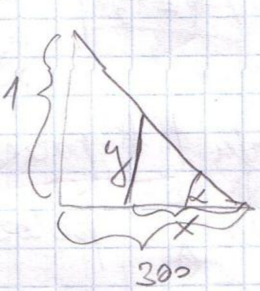
$\Rightarrow H = \frac{1}{2} \Rightarrow \underline{E = Hz = 94{,}2\ V/m}$

b) $H = \frac{1}{4}$ zbog:



$\sin\alpha = \frac{1}{300} = \frac{y}{75} \Rightarrow \underline{y = \frac{75}{300} = \frac{1}{4}} \Rightarrow H = \frac{1}{4}$

d) $H = \frac{1}{2}$ zbog



$x = 150$

$\sin\alpha = \frac{1}{300} = \frac{y}{150} \Rightarrow y = \frac{150}{300} = \frac{1}{2}$

4.29

$f = 10\ MHz$ $\qquad E_z = 0$

$\varphi = 30^\circ$ na $+z$ osi

$\psi = 45^\circ$ -||- $+x$ osi

$t = 10^{-9}\ s \qquad (x=0, y=0, z=0) \Rightarrow E = 10 \cos(\omega t - \frac{\pi}{6})\ V/m$

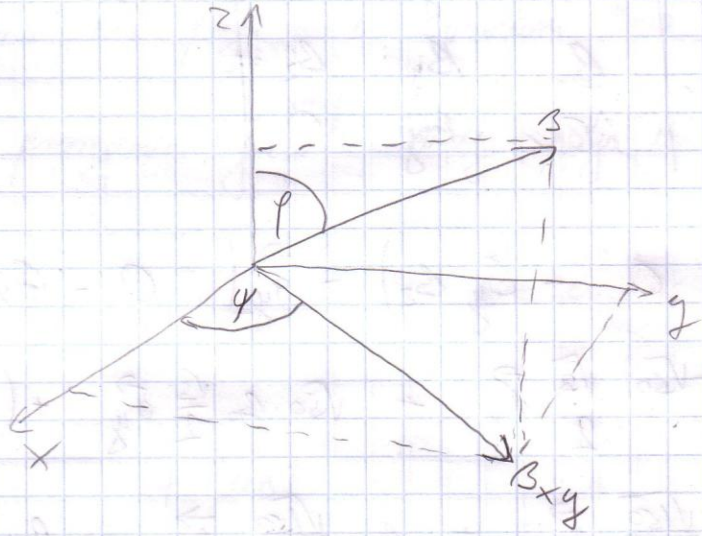
pretpostavka: $E_{ox} > 0$!

a) $E_{ox} = ?$

b) $E_{oy} = ?$

c) $H_{ox} = ?$

d) $H_{oy} = ?$



$$\vec{\beta} = [\sin\varphi \cos\psi\ \vec{a}_x + \sin\varphi \sin\psi\ \vec{a}_y + \cos\varphi\ \vec{a}_z]\ \beta_o$$

$$= \beta_o[\sin 30^\circ \cos 45^\circ\ \vec{a}_x + \sin 30^\circ \sin 45^\circ\ \vec{a}_y + \cos 30^\circ\ \vec{a}_z]$$

$$\beta_o = \omega\sqrt{\mu_o \varepsilon_o} = \frac{\omega}{c} = \frac{2\pi \cdot 10 \cdot 10^6}{3 \cdot 10^8} = \frac{2\pi}{30} = \underline{0{,}209}$$

za $\beta \perp E$ (ORTOGONALNI) $\Rightarrow \vec{\beta} \cdot \vec{E} = 0$

$E_o = 10\ V/m$

$E_{oz} = 0 \Rightarrow \qquad E_{ox}^2 + E_{oy}^2 = E_o^2 = 10^2$

$$\frac{\sqrt{2}}{4} E_{ox} + \frac{\sqrt{2}}{4} E_{oy} = 0 \Rightarrow \underline{E_{ox} = -E_{oy}}$$

$\Rightarrow \quad 2E_{ox}^2 = 100$

$$\boxed{E_{ox} = \sqrt{50} = 7{,}07\ V/m} \qquad \boxed{E_{oy} = -E_{ox} = -7{,}07\ V/m}$$

$$\vec{E} = \left(\sqrt{50}\,\vec{a_x} - \sqrt{50}\,\vec{a_y}\right)\cos\left(\omega t - \frac{\sqrt{2}}{4}\beta_{0x} - \frac{\sqrt{2}}{4}\beta_{0y} - \frac{\sqrt{3}}{2}\beta_{0z} - \frac{\pi}{6}\right) \quad -21-$$

$$\vec{\beta} = \beta_0 \frac{\sqrt{2}}{4}\vec{a_x} + \beta_0 \frac{\sqrt{2}}{4}\vec{a_y} + \beta_0 \frac{\sqrt{3}}{2}\vec{a_z}$$

$$\vec{H} = \frac{1}{\mu\omega}\,\vec{\beta}\times\vec{E} \qquad \frac{1}{\mu\omega} = 12{,}67\cdot 10^{-3}$$

$$\vec{\beta}\times\vec{E} = \begin{vmatrix} \vec{a_x} & \vec{a_y} & \vec{a_z} \\ \beta_x & \beta_y & \beta_z \\ E_x & E_y & 0 \end{vmatrix}$$

$$= \vec{a_x}\left(0 - E_y\beta_z\right) - \vec{a_y}\left(0 - E_x\beta_z\right) + \vec{a_z}\left(\beta_x E_y - E_x\beta_y\right)$$

$$= -\beta_0(-\sqrt{50})\frac{\sqrt{3}}{2}\vec{a_x} + \sqrt{50}\,\beta_0\frac{\sqrt{3}}{2}\vec{a_y} + \left(\beta_0\frac{\sqrt{2}}{4}(-\sqrt{50}) - \sqrt{50}\,\beta_0\frac{\sqrt{2}}{4}\right)\vec{a_z}$$

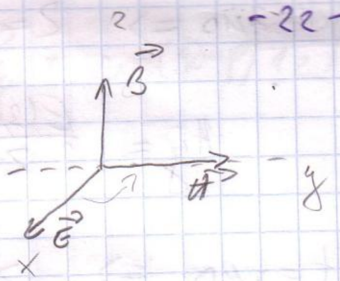
$$= \beta_0\frac{\sqrt{150}}{2}\vec{a_x} + \beta_0\frac{\sqrt{150}}{2}\vec{a_y} - \beta_0\frac{\sqrt{100}}{2}\vec{a_z}$$

$$\vec{H} = \frac{1}{\mu\omega}\left(\beta_0\frac{\sqrt{150}}{2}\vec{a_x} + \beta_0\frac{\sqrt{150}}{2}\vec{a_y} - \beta_0\frac{\sqrt{100}}{2}\vec{a_z}\right)$$

$$\boxed{H_{0x} = \frac{1}{\mu\omega}\beta_0\frac{\sqrt{150}}{2} = 16{,}21\ \text{mA/m}}$$

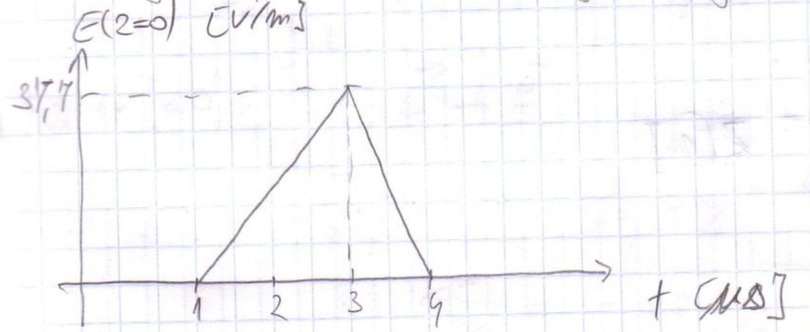
$$\boxed{H_{0y} = \frac{1}{\mu\omega}\beta_0\frac{\sqrt{100}}{2} = 16{,}21\ \text{mA/m}}$$

PR1



- slobodan prostor ; ε_0 i μ_0
- val se giba u + z smjeru
- el. polje ima samo x komponentu
- vremenska promjena el. polja u ravnini z=0 zadana je slikom

Skiciraj prostornu promjenu jakosti mag. polja u t = 2μs



$\vec{B} = \vec{E} \times \vec{H}$

$Z = \sqrt{\frac{\mu_0}{\varepsilon_0}} = 377\,\Omega$ $\quad c = \frac{1}{\sqrt{\varepsilon_0 \mu_0}} = 3 \cdot 10^8\, m/s$

H - u y smjeru ; $H_{y,max} = \frac{E_{x,max}}{Z} = \frac{37,7}{377} = 0,1\ A/m$

$z=0: \quad H(t, z=0) = f\left(t - \frac{0}{c}\right)$

$t = 2\mu s: \quad H_y(t = 2\cdot 10^{-6}, z) = f\left(2\cdot 10^{-6} - \frac{z}{c}\right)$

uvjeti: $H_y(t, 0) = H_y(2\cdot 10^{-6}, z) \Rightarrow z = (2\cdot 10^{-6} - t)\, c$

$\Rightarrow H_y(t, 0) = H_y\left[2\cdot 10^{-6}, (2\cdot 10^{-6} - t)\, c\right]$

$\Rightarrow t = 1\mu s \Rightarrow z = (2-1)\cdot \cancel{10^{-6}} \cdot 3 \cdot 10^{\cancel{8}\,2} = 300\, m$

$H_y(2\mu s, 300m) = \frac{E(1\mu s)}{Z} = 0\ A/m$

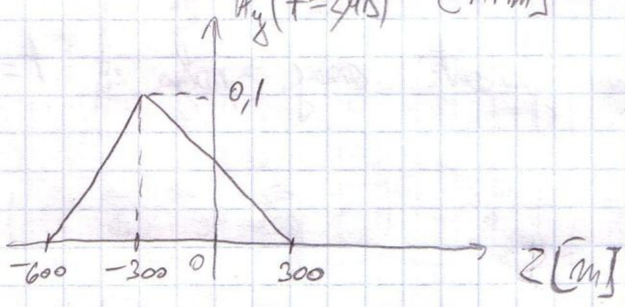
$\Rightarrow t = 2\mu s \Rightarrow z = (2-2)\, c = 0 \qquad H_y = \frac{E(2\mu s)}{Z} = \frac{37,7}{2Z}$

$t = 3\mu s \Rightarrow z = (2-3)c = -300\ m$

$$H_y = \frac{E(3\mu s)}{Z} = 0{,}1\ A/m$$

$t = 4\mu s \Rightarrow z = (2-4)c = -600\ m$

$$H_y = \frac{E(4\mu s)}{Z} = 0$$



PR2

$f = 12\ MHz$

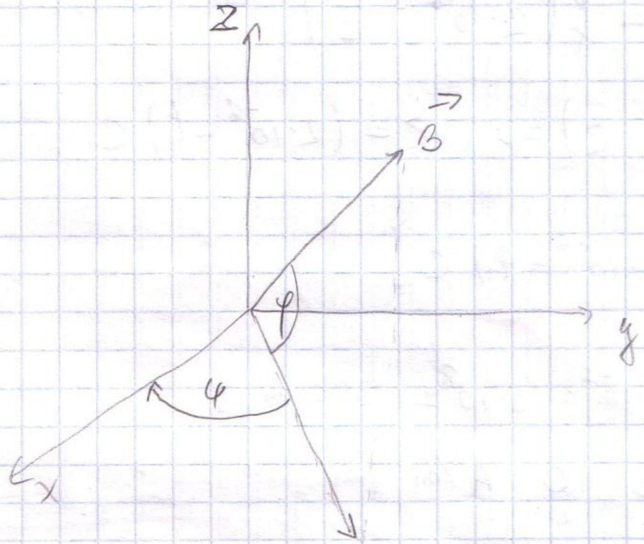
$\psi = 30°$

$\varphi = 60°$

- el. polje nema z komponente

$E(t=0, x=0, y=0, z=0) = 10\cos(\omega t - 30°)\ V/m$

$E, H = ?$



$$\vec{B} = B_0 \left[\sin(90^\circ - \varphi) \cos\varphi \, \vec{a}_x + \sin(90^\circ - \varphi) \sin(\varphi) \, \vec{a}_y + \cos(90 - \varphi) \, \vec{a}_z \right.$$

$$B_0 = \omega \sqrt{\mu_0 \varepsilon_0} = \frac{\omega}{c} = \frac{2\pi \cdot 12 \cdot 10^6}{3 \cdot 10^8} = \frac{8\pi}{100}$$

$$\vec{B} = \frac{8\pi}{100} \cdot \frac{1}{2} \frac{1}{2} \vec{a}_x + \frac{8\pi}{100} \frac{1}{2} \cdot \frac{\sqrt{3}}{2} \vec{a}_y + \frac{8\pi}{100} \frac{\sqrt{3}}{2} \vec{a}_z$$

$$\boxed{\vec{B} = \frac{\pi}{50} \vec{a}_x + \frac{\sqrt{3}\pi}{50} \vec{a}_y + \frac{2\sqrt{3}\pi}{50} \vec{a}_z}$$

$$E_{0z} = 0 \Rightarrow E_0^2 = E_{0x}^2 + E_{0y}^2 = 10^2$$

$$\vec{B} \cdot \vec{E} = 0 \quad \text{za} \quad \vec{B} \perp \vec{E}$$

0,0106

$$\Rightarrow \frac{\pi}{50} E_{0x} + \frac{\sqrt{3}\pi}{50} E_{0y} + 0 = 0$$

$$E_{0x} = -\sqrt{3} E_{0y}$$

$$\Rightarrow 4 E_{0y}^2 = 100 \Rightarrow \boxed{E_{0y} = 5 \ \mathrm{V/m}}$$

$$\Rightarrow \boxed{E_{0x} = -5\sqrt{3} \ \mathrm{V/m}}$$

$$\boxed{\vec{E} = (-5\sqrt{3} \, \vec{a}_x + 5 \vec{a}_y) \cos\left(\omega t - \frac{\pi}{50} - \frac{\sqrt{3}\pi}{50} + \frac{2\sqrt{3}\pi}{50} - 30^\circ\right)}$$

$$\vec{H}_0 = \frac{1}{\omega\mu} \vec{B} \times \vec{E}_0 = \frac{1}{\omega\mu} \begin{vmatrix} \vec{a}_x & \vec{a}_y & \vec{a}_z \\ B_x & B_y & B_z \\ E_x & E_y & 0 \end{vmatrix} = \vec{a}_x (-E_y B_z) - \vec{a}_y (-E_x B_z) + \vec{a}_z (B_x E_y - B_y E_x)$$

$$= \vec{a}_x \left(-5 \cdot \frac{2\sqrt{3}\pi}{50}\right) + \vec{a}_y \left(-5\sqrt{3} \, \frac{2\sqrt{3}\pi}{50}\right) + \vec{a}_z \left(5 \frac{\pi}{50} - \frac{\sqrt{3}\pi}{50}(-5\sqrt{3})\right)$$

$$\vec{H} = \left(-\frac{\sqrt{3}\pi}{5} \vec{a}_x - \frac{3\pi}{5} \vec{a}_y + \vec{a}_z \left(\frac{\pi}{10} + \frac{3\pi}{10}\right)\right) \frac{1}{\mu\omega}$$

$$H_{0x} = -\frac{1}{\mu\omega} \frac{\sqrt{3}\pi}{5} = 11{,}5 \ \mathrm{mA/m} \qquad H_{0z} = \frac{2\pi}{5} \frac{1}{\mu\omega}$$

$$H_{0y} = -\frac{3\pi}{5} \frac{1}{\mu\omega}$$

PR3

$\mu_r = 1$

$\vec{E} = 2\cos(10^8 t - x)\,\vec{a_y} + 4\cos(10^8 t - x)\,\vec{a_z}$

$\lambda, \varepsilon_r, H = ?$

$\beta = 1 \qquad \underline{\underline{\lambda}} = \frac{2\pi}{\beta} = \underline{\underline{2\pi}}$

$\omega = 10^8 = 2\pi f \Rightarrow \quad f = \frac{10^8}{2\pi}$

$$v = f\lambda = \frac{10^8}{\cancel{2\pi}} \cdot \cancel{2\pi} = 10^8 = \frac{1}{\sqrt{\mu\varepsilon}}$$

$$\mu\varepsilon v^2 = 1$$

$$\underline{\underline{\varepsilon_r}} = \frac{1}{v^2 \mu_0 \mu_r \varepsilon_0} = 8{,}99 \approx \underline{\underline{9}}$$

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} = -\mu \frac{\partial \vec{H}}{\partial t}$$

$$\nabla \times \vec{E} = \begin{vmatrix} \vec{a_x} & \vec{a_y} & \vec{a_z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 0 & E_y & E_z \end{vmatrix} = \vec{a_x}\left(\overset{0}{\cancel{\frac{\partial E_z}{\partial y}}} - \overset{0}{\cancel{\frac{\partial E_y}{\partial z}}}\right) - \vec{a_y}\left(\frac{\partial E_z}{\partial x}\right) + \vec{a_z}\left(\frac{\partial E_y}{\partial x}\right)$$

$$= -\vec{a_y} \cdot 4(-\sin(10^8 t - x)(-1)) + \vec{a_z}\, 2(-\sin(10^8 t - x)(-1))$$

$$= -4\sin(10^8 t - x)\,\vec{a_y} + 2\sin(10^8 t - x)\,\vec{a_z} = -\mu \frac{\partial \vec{H}}{\partial t}$$

$$\vec{H} = \frac{1}{\mu} 4 \int \sin(10^8 t - x)\,dt\,\vec{a_y} - \frac{2}{\mu}\int \sin(10^8 t - x)\,dt\,\vec{a_z}$$

$$= \frac{4}{\mu}\left(-\frac{1}{10^8}\cos(10^8 t - x)\right)\vec{a_y} - \frac{2}{\mu}\left(-\frac{1}{10^8}\cos(10^8 t - x)\right)\vec{a_z}$$

$$\vec{H} = -\frac{4}{\mu\, 10^8}\cos(10^8 t - x)\,\vec{a_y} + \frac{2}{\mu \cdot 10^8}\cos(10^8 t - x)\,\vec{a_z}$$

PR 4:

$E_0 = 100\ V/m$

$f = 300\ MHz$

- val putuje u +z os

$\varepsilon_r = 9 \quad \mu_r = 1$

$\vec{E}, \vec{H} = ? \quad \vec{E} = E_x \vec{a_x}$

srednja gustoća snage = ? => $N_{sr} = ?$

$\omega = 2\pi f = 1{,}885 \cdot 10^9$

$\beta = \omega \sqrt{\varepsilon\mu} = 18{,}86$

$$\vec{E} = E_0 \cos(\omega t - \beta z)\, \vec{a_x}$$

$$\boxed{\vec{E} = 100 \cos(1{,}885 \cdot 10^9 t - 18{,}86 z)\, \vec{a_x}}$$

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$= \begin{vmatrix} \vec{a_x} & \vec{a_y} & \vec{a_z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ E_x & 0 & 0 \end{vmatrix} = -\vec{a_y}\left(-\frac{\partial E_x}{\partial z}\right) + \vec{a_z}\cancel{\left(-\frac{\partial E_x}{\partial y}\right)}^{0}$$

$$= E_0 \left[-\sin(\omega t - \beta z)(-\beta)\right] \vec{a_y}$$

$$= E_0 \beta \sin(\omega t - \beta z)\, \vec{a_y} = -\mu \frac{\partial \vec{H}}{\partial t}$$

$$\Rightarrow \vec{H} = -\frac{E_0 \beta}{\mu} \int \sin(\omega t - \beta z)\, dt\, \vec{a_y}$$

$$= -\frac{E_0 \beta}{\mu}\left(-\frac{1}{\omega}\cos(\omega t - \beta z)\right) \vec{a_y}$$

$$\vec{H} = \frac{E_0 \beta}{\mu \omega} \cos(\omega t - \beta z)\, \vec{a_y}$$

$$\boxed{\vec{H} = 0{,}8 \cos(1{,}885 \cdot 10^9 t - 18{,}86 z)\, \vec{a_y}}$$

ili na

JEDNOSTAV

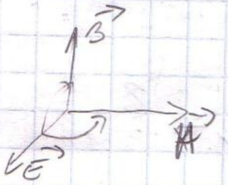
NAČIN =

JEDNOSTAVNIJI NAČIN:

$H = \frac{E}{Z}$ $\qquad Z = \sqrt{\frac{\mu}{\varepsilon}} = 125{,}58$

$H = 0{,}8$

$\Rightarrow$



$\Rightarrow \vec{H} = H_y \vec{a_y}$

$$\Rightarrow \boxed{\vec{H} = 0{,}8 \cos(1{,}885 \cdot 10^9 t - 18{,}86 z)\,\vec{a_y}}$$

$$\vec{N_{sr}} = \frac{1}{T}\int_0^T (\vec{E} \times \vec{H})\,dt = \frac{1}{2}(\vec{E} \times \vec{H})$$

$$= \frac{1}{2}\begin{vmatrix} \vec{a_x} & \vec{a_y} & \vec{a_z} \\ E_x & 0 & 0 \\ 0 & H_y & 0 \end{vmatrix} = \frac{1}{2} E_x H_y \vec{a_z} = 40\,\vec{a_z}$$

$$\boxed{N_{sr} = 40}$$

PR5

- val u +x smjeru

$\vec{E} = E_0 \cos(\omega t - \beta x)\vec{a_z}$

$E_0 = 100$ V/m

$\vec{E} = E_0 \vec{a_z}$

$\lambda = \frac{2\pi}{\beta} \Rightarrow \underline{\beta} = \frac{2\pi}{\lambda} = \underline{8\pi}$

$\lambda = 25$ cm

$v = \lambda f \Rightarrow \boxed{\underline{f} = \frac{v}{\lambda} = \underline{8 \cdot 10^8 \; Hz}}$

$v = 2 \cdot 10^8$ m/s

$\mu_r = 1$

$f, \; \varepsilon_r = ?$

$\omega = 2\pi f = 16\pi \cdot 10^8$

$\vec{E}, \vec{H} = ?$

$v = \frac{1}{\sqrt{\mu\varepsilon}} \Rightarrow \boxed{\underline{\varepsilon_r} = \frac{1}{v^2 \mu_0 \mu_r \varepsilon_0} = \underline{2{,}25}}$

$\vec{E} = 100 \cos(16\pi \cdot 10^8 t - 8\pi x)\vec{a_z}$

$H_0 = \frac{E_0}{Z} \qquad Z = \sqrt{\frac{\mu}{\varepsilon}} = 251{,}16$

$\underline{H_0 = 0{,}4 \; A/m}$

$\boxed{\vec{H} = 0{,}4 \cos(16\pi \cdot 10^8 t - 8\pi x)\vec{a_z}}$

4,30

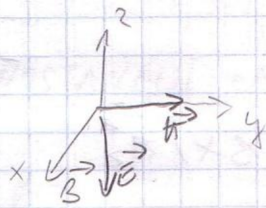
$\mu_r = 1 \quad \varepsilon_r = 2$

$\vec{H} = 10 \cos(\omega t - 3x)\vec{a_y}$ A/m

a) $\beta = ?$ $\qquad \boxed{\beta = 3}$

b) $v = ?$ $\qquad \underline{v} = \frac{1}{\sqrt{\mu\varepsilon}} = \frac{1}{\sqrt{\mu_0\mu_r\varepsilon_0\varepsilon_r}} = \frac{c}{\sqrt{\mu_r\varepsilon_r}} = \underline{2{,}12 \cdot 10^8 \; m/s}$

c) $\vec{E}(t = 10\,ns, \; x = 0{,}4\,m) = ?$



$$\Rightarrow \vec{E} = -E_0 \vec{a}_y$$

$$Z = \frac{E_0}{H_0} \qquad Z = \sqrt{\frac{\mu}{\varepsilon}} = 266{,}4\ \Omega$$

$$\Rightarrow E_0 = Z H_0 = 2664\ V/m$$

$$\vec{E} = -2664 \cos(\omega t - 3x)\,\vec{a}_y$$

$$\beta = \omega\sqrt{\mu\varepsilon} \Rightarrow \omega = \frac{\beta}{\sqrt{\mu\varepsilon}} = \frac{\beta c}{\sqrt{\mu_r\varepsilon_r}} = 6{,}36\cdot 10^9$$

$$E(t = 10\,ms,\ x = 0{,}4) = -2664\cos(6{,}36\cdot \cancel{10^9}\cdot \cancel{10^{-8}} - 3\cdot 0{,}4) = -1153\ V/m$$

d) val se širi u x smjeru

4.31

$$\vec{E} = 100 \sin\theta\ \frac{1}{r}\cos(10^{10}t - r)\,\vec{a}_\theta\ V/m$$

$$\vec{H} = \frac{100}{120\pi}\sin\theta\ \frac{1}{r}\cos(10^{10}t - r)\,\vec{a}_\alpha\ A/m$$

a) smjer prostiranja vala: $+\vec{a}_r$

b) $\vec{N}(r=2,\ t=1\,ms,\ \theta=\frac{\pi}{6})\ [mW/m^2] = ?$

$$\vec{N} = \vec{E}\times\vec{H} = \begin{vmatrix} \vec{a}_r & \vec{a}_\alpha & \vec{a}_\theta \\ 0 & 0 & E_\theta \\ 0 & H_\alpha & 0 \end{vmatrix} = \vec{a}_r(-E_\theta H_\alpha)$$

$$\vec{N} = -\frac{100^2}{120\pi}\sin^2\theta\ \frac{1}{r^2}\cos^2(10^{10}t - r)\,\vec{a}_r$$

$$\left.\begin{matrix} r=2 \\ t=1\,ms \\ \theta=\frac{\pi}{6} \end{matrix}\right\} \Rightarrow |N| = \left|-\frac{100^2}{120\pi}\sin^2\left(\frac{\pi}{6}\right)\frac{1}{4}\cos^2(\cancel{10^{10}}\cdot\cancel{10^{-3}} - 2)\right| = 35{,}1\ mW/m^2$$

c) $N_{sr}\left(r=4\,m,\ \theta=\frac{\pi}{4}\right)\ [mW/m^2] = ?$

$$N_{sr} = \frac{1}{T}\int_0^T N\,dt = \frac{1}{2}\,\frac{100^2}{120\pi}\,\sin^2\theta\,\frac{1}{r^2}$$

uz $\left.\begin{matrix} r=4 \\ \theta=\frac{\pi}{4} \end{matrix}\right\} \Rightarrow \underline{\underline{N_{sr}}} = \frac{1}{2}\,\frac{100^2}{120\pi}\,\sin^2\left(\frac{\pi}{4}\right)\,\frac{1}{16} = \underline{\underline{414{,}5\ mW/m^2}}$

d) $P_{IZVORA} = ?$ - ukupna srednja snaga izvora

$$P_{izvora} = P = \iint_S \vec{N}_{sr}\,\vec{n}\,dS \qquad \vec{n} = \vec{a}_r$$

$$dS = r^2 \sin\theta\, d\theta\, d\alpha = 2\pi R^2 \sin\theta\, d\theta$$

$$P = \int_{\theta=0}^{\pi} \frac{1}{\cancel{2}}\,\frac{100^2}{120\cancel{\pi}}\,\sin^2\theta\,\frac{1}{\cancel{R^2}}\cdot \cancel{R^2}\,\cancel{2\pi}\,\sin\theta\, d\theta$$

$$= \frac{100^2}{120}\int_0^{\pi} \sin^3\theta\, d\theta$$

$$\int_0^{\pi} \sin^3 x\, dx = -\frac{3}{4}\cos x\Big|_0^{\pi} + \frac{1}{12}\cos 3x\Big|_0^{\pi}$$

$$= -\frac{3}{4}(-1-1) + \frac{1}{12}(-1-1)$$

$$= \frac{3}{2} - \frac{1}{6} = \frac{8}{6} = \frac{4}{3}$$

$$\boxed{P = \frac{4}{3}\,\frac{100^2}{120} = 111\ W}$$

FARADEYEV ZAKON

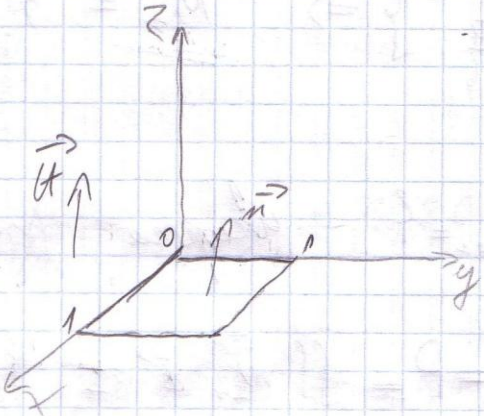
PR1

$$\vec{H} = 300 \cos(3\cdot10^8 t - y)\,\vec{a_z}$$

A(0,0,0) C(1,1,0)

B(1,0,0) D(0,1,0)

$U_{ind} = ?$



$$U_{ind} = -\frac{d\phi}{dt}$$

$$\phi = \iint_S \vec{B}\,\vec{n}\,dS \qquad \vec{n} = \vec{a_z}$$

$$\vec{B} = B\,\vec{a_z} = \mu_0 H\,\vec{a_z}$$

$$\phi = \int_0^1 dx \int_0^1 300\mu_0 \cos(\omega t - y)\cdot dy$$

$$= 300\mu_0 \cdot 1 \cdot (-1) \sin(\omega t - y)\Big|_0^1$$

$$\phi = -300\mu_0 \left[\sin(\omega t - 1) - \sin(\omega t)\right] \quad \Big/ \frac{d}{dt} \quad /(-1)$$

$$-\frac{d\phi}{dt} = U_{ind} = +300\mu_0 \left(\omega\cos(\omega t - 1) - \omega\cos\omega t\right)$$

$$\boxed{U_{ind} = 300\mu_0\omega\left[\cos(\omega t - 1) - \cos\omega t\right]}$$

PR 2

$a = 25$ cm

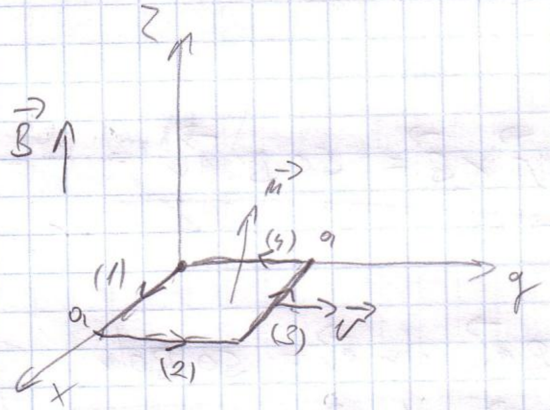
$t=0$; $z=0$: $A(0,0,0)$ $B(a, a, 0)$

$C(a, a, 0)$ $D(0, a, 0)$

$\vec{v} = 50\, \vec{a_y}$

$\vec{B} = 8 \cos(1{,}5 \cdot 10^8 t - 0{,}5x)\, \vec{a_z}$ μT

$U_{ind} = ?$



$$U_{ind} = \underbrace{-\iint_S \frac{\partial \vec{B}}{\partial t} \vec{n}\, ds}_{I_1} + \underbrace{\oint_C (\vec{v} \times \vec{B})\, d\vec{l}}_{I_2}$$

$$\frac{\partial \vec{B}}{\partial t} = 8\left(-\sin\left(\omega t - \frac{1}{2}x\right)\omega\right)\vec{a_z}$$

$$= -8\omega \sin\left(\omega t - \frac{1}{2}x\right)\vec{a_z}$$

$$I_1 = +8\omega \int_0^a dy \int_0^a \sin\left(\omega t - \frac{1}{2}x\right) dx$$

$$= 8\omega a \left(-\left(-\frac{1}{\frac{1}{2}}\right)\cos\left(\omega t - \frac{1}{2}x\right)\right)\Big|_0^a$$

$$= 8\omega a \cdot 2\left[\cos\left(\omega t - \frac{a}{2}\right) - \cos(\omega t)\right]$$

$$= 16\omega a \left[\cos\left(\omega t - \frac{a}{2}\right) - \cos(\omega t)\right]$$

$a = \frac{1}{4}$

$$I_1 = 4\omega \left[\cos\left(\omega t - \frac{1}{8}\right) - \cos(\omega t)\right]$$

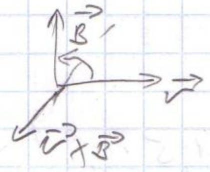
I_2: $\quad I_2 = \oint_c (\vec{v} \times \vec{B})\, d\vec{l}$

$d\vec{l}_1 = dx\, \vec{a}_x$

$d\vec{l}_2 = dy\, \vec{a}_y$

$d\vec{l}_3 = -dx\, \vec{a}_x$

$d\vec{l}_4 = -dy\, \vec{a}_y$



$$\left.\begin{array}{l} \vec{v} = 50\, \vec{a}_y \\ \vec{B} = B\, \vec{a}_z \end{array}\right\} \Rightarrow \quad \underline{\vec{v} \times \vec{B} = 50B\, \vec{a}_x}$$

$$(1) \quad \left.\begin{array}{l} (\vec{v} \times \vec{B})\, d\vec{l}_1 = 50B\vec{a}_x\, dx\, \vec{a}_x = 50B\,dx \\ (\vec{v} \times \vec{B})\, d\vec{l}_2 = 50B\,\vec{a}_x\, dy\, \vec{a}_y = 0 \\ (\vec{v} \times \vec{B})\, d\vec{l}_3 = 50B\,\vec{a}_x\, (-dx)\,\vec{a}_x = -50B\,dx \\ (\vec{v} \times \vec{B})\, d\vec{l}_4 = 0 \end{array}\right\} \Rightarrow$$

$$\Rightarrow \quad \underline{I_2} = \oint_c (\vec{v} \times \vec{B})\, d\vec{l} = 50B\,dx + 0 - 50B\,dx + 0 = \underline{0}$$

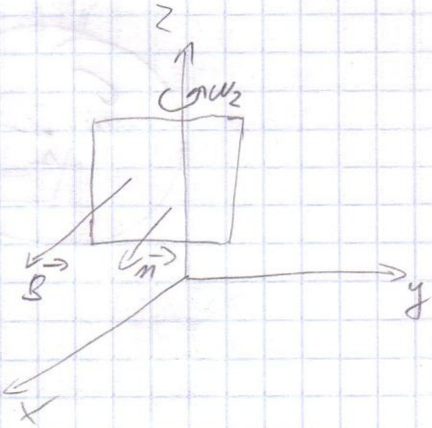
$$\Rightarrow \boxed{U_{ind} = I_1 + I_2 = 4\omega \left[\cos\left(\omega t - \frac{1}{8}\right) - \cos(\omega t) \right]}$$

✓

PR3

$\vec{B} = B_0 \cos(w_1 t)\, \vec{a_x}$

$U_{ind} = ?$



$\vec{n} = \cos\varphi\, \vec{a_x} + \sin\varphi\, \vec{a_y}$

$U_{ind} = -\dfrac{d\phi}{dt}$

$$\left.\begin{array}{l} \vec{B}\vec{n} = B_0 \cos\varphi \cos(w_1 t) \\ \varphi = \varphi_0 + w_2 t \end{array}\right\} \Rightarrow \vec{B}\vec{n} = B_0 \cos(\varphi_0 + w_2 t)\cos(w_1 t)$$

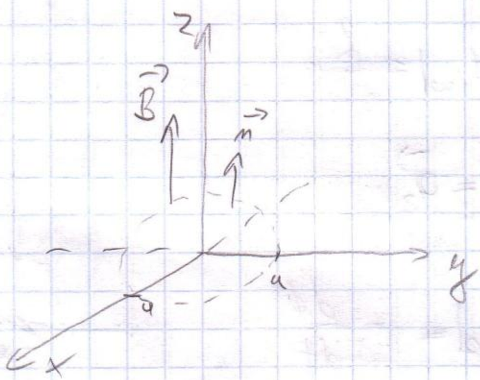
$\phi = \iint_S \vec{B}\vec{n}\, ds = \vec{B}\vec{n}\, S$

$U_{ind} = -\dfrac{d\phi}{dt} = -S\, B_0 \left[-w_2 \sin(\varphi_0 + w_2 t)\cos(w_1 t) - w_1 \sin(w_1 t)\cos(\varphi_0 + w\right.$

PR4

$\vec{B} = \begin{cases} B_0 \sin wt\, \vec{a_z} & , r < a \\ 0 & , r > a \end{cases}$

$\vec{E_i} = ?$



$r < a$:

$\nabla \times \vec{E} = -\dfrac{\partial \vec{B}}{\partial t}$

$-\dfrac{\partial \vec{B}}{\partial t} = -B_0 w \cos wt\, \vec{a_z}$

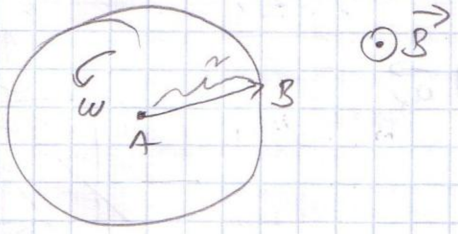
$\nabla \times \vec{E} = \vec{a_r}\left(\dfrac{1}{r}\dfrac{\partial E_z}{\partial \alpha} - \dfrac{\partial E_\alpha}{\partial z}\right) + \vec{a_\alpha}\left(\dfrac{\partial E_r}{\partial z} - \dfrac{\partial E_z}{\partial r}\right) + \vec{a_z}\dfrac{1}{r}\left(\dfrac{\partial}{\partial r}(rE_\alpha) - \dfrac{\partial}{\partial}\right.$

$\Rightarrow -B_0 w \cos wt = \dfrac{1}{r}\left[\dfrac{\partial}{\partial r}(rE_\alpha) - \dfrac{\partial E_r}{\partial \alpha}\right]$

?

$\Rightarrow E = 0$?

PR ZB 4.1



$\vec{B} = B\vec{a}_z$

$U_{BA} = ?$

$\vec{v} = \omega r \vec{a}_\alpha$

$\vec{v} \times \vec{B} = \omega r \vec{a}_\alpha \times B\vec{a}_z = \omega r B \vec{a}_r$

$$\underline{\underline{U_{BA}}} = \int_r (\vec{v}\times\vec{B})\, d\vec{l} = \int_{r=0}^{r_0} \omega r B\, dr\, \vec{a}_r = \underline{\underline{\omega B \frac{r_0^2}{2}}}$$

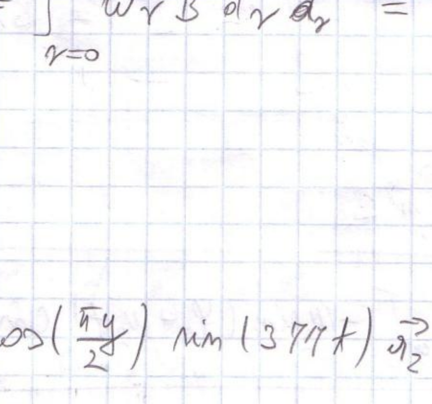
PR ZB 4.2

$\vec{B} = 0{,}1 \sin\left(\frac{\pi x}{2}\right)\cos\left(\frac{\pi y}{2}\right)\sin(377t)\,\vec{a}_z$ [T]

$A(0,0,0)$ $B(1,0,0)$ $C(1,1,0)$ $D(0,1,0)$

$U_{ind} = ?$

$\vec{n} = \vec{a}_z$



$U_{ind} = -\frac{d\phi}{dt}$

$$\phi = \iint_S \vec{B}\vec{n}\, ds = 0{,}1\sin(377t)\int_0^1 \sin\left(\frac{\pi x}{2}\right)dx \int_0^1 \cos\left(\frac{\pi y}{2}\right)dy$$

$$\phi = 0{,}1\sin(377t)\left[-\frac{2}{\pi}\left(\cancel{\cos}^{0}\left(\frac{\pi}{2}\right) - \cancel{\cos}^{1}\, 0\right)\right]\left[\frac{2}{\pi}\left(\cancel{\sin}^{1}\left(\frac{\pi}{2}\right) - \cancel{\sin}^{0}(0)\right)\right]$$

$$= 0{,}1\sin(377t)\cdot\frac{2}{\pi}\cdot\frac{2}{\pi}$$

$$\phi = \frac{0{,}4}{\pi^2}\sin(377t)$$

$$U_{ind} = -\frac{d\phi}{dt} = -\frac{0{,}4}{\pi^2} \cos(377t) \cdot 377$$

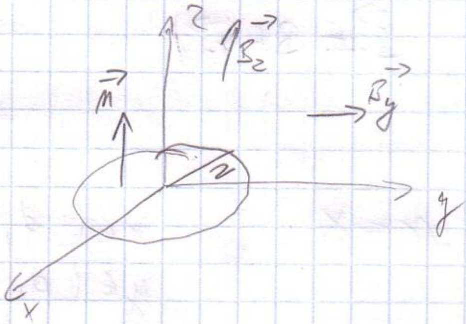
$$\boxed{U_{ind} = -\frac{150{,}8}{\pi^2} \cos(377t)}$$

4.13

$r = 5\ cm$

$\vec{B} = 0{,}5 \cos(377t)\,(4\,\vec{a_y} + 4\,\vec{a_z})\ [T]$

$U_{ind} = ?$



$$U_{ind} = -\frac{d\phi}{dt}$$

$\vec{n} = \vec{a_z}$

$$\phi = \iint_S \vec{B}\,\vec{n}\,dS = \iint_S 0{,}5\cos(377t)\,(4\vec{a_y} + 4\vec{a_z})\cdot\vec{a_z}\,dS$$

$$= \iint_S 2\cos(377t)\,dS$$

$$= 2\cos(377t)\,S = 2r^2\pi\cos(377t)$$

$$U_{ind} = -\frac{d\phi}{dt} = -2r^2\pi\,(-\sin(377t)\cdot 377) = 2\cdot 0{,}05^2\pi\cdot 377\,\sin(377t)$$

$$\boxed{U_{ind} = 5{,}92\,\sin(377t)}$$

4.15

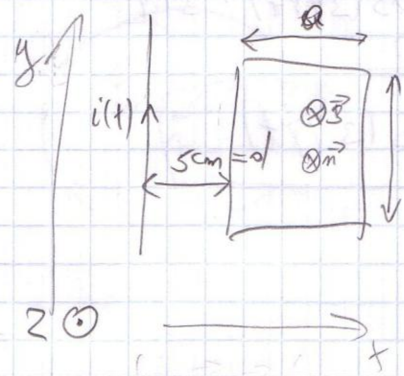
$a = 20\ cm$

$I = 1A$

$f = 5\ kHz$

$U_{ind}\ [mV] - ?$

$d = 5cm$



$$U_{ind} = -\frac{d\phi}{dt}$$

$$\vec{n} = -\vec{u_z}$$

$$\vec{B} = B(-\vec{a_z})$$

$$\phi = \iint_S \vec{B}\,\vec{n}\,ds$$

$$\vec{B} = \frac{\mu_0 I}{2\pi r}(-\vec{a_z}) \qquad r = x \qquad x \in (d,\ d+a)$$

$$y \in (0,\ a)$$

$$\phi = \iint_S \frac{\mu_0 I}{2\pi x} \underbrace{(-\vec{a_z})(-\vec{u_z})}_{1}\ ds$$

$$\omega = 2\pi f = 31{,}42 \cdot 10^3$$

$$= \frac{\mu_0 I}{2\pi} \iint \frac{1}{x}\,dx\,dy$$

$$= \frac{\mu_0 I}{2\pi} \int_d^{d+a} \frac{dx}{x} \int_0^a dy$$

$$I_{ef} = \frac{I_m}{\sqrt{2}} \Rightarrow I_m = I_{ef}\sqrt{2}$$

$$\left.\begin{aligned}\phi &= \frac{\mu_0 I}{2\pi} \ln\left(\frac{d+a}{d}\right) \cdot a \\ I &= \sqrt{2}\sin(\omega t)\end{aligned}\right\} \Rightarrow$$

$$I_{ef} = \frac{\sqrt{2}}{\sqrt{2}} = 1A$$

$$\Rightarrow \phi = \sqrt{2}\,\frac{\cancel{4\pi}^{\,2} \cdot 10^{-7}}{\cancel{2\pi}} \cdot 0{,}2 \cdot \ln\left(\frac{0{,}25}{0{,}05}\right) \sin(\omega t)$$

$$\phi = 6{,}4 \cdot 10^{-7} \sin(\omega t)$$

$$U_{ind} = -\frac{d\phi}{dt} = -6{,}4 \cdot 10^{-7}\,\omega \cos(\omega t) = \underline{\underline{-2{,}02\ mV}}$$

4.16

$R = 20\ m\Omega$

$w = 2$

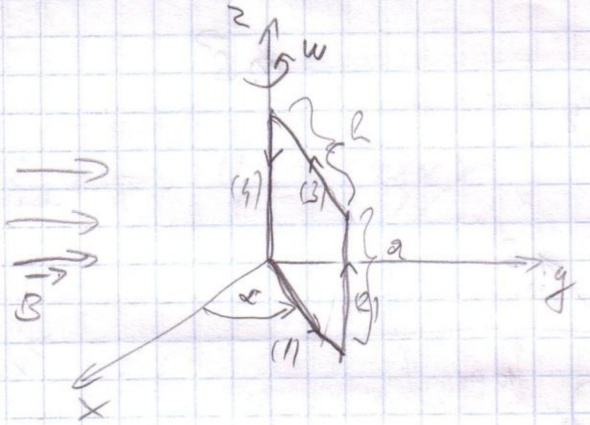
$\vec{B} = 10\ \vec{a_y}\ mT$

$I\ [mA] = ?$

$\alpha = \frac{\pi}{4}$

$a = 20\ mm$

$l = 10\ mm$



$\vec{a_y} = \sin\alpha\ \vec{a_r} + \cos\alpha\ \vec{a_\alpha}$

$\alpha = wt$

$U_{ind} = -\frac{d\phi}{dt} = -\frac{\partial}{\partial t}(\vec{B}\vec{S})$

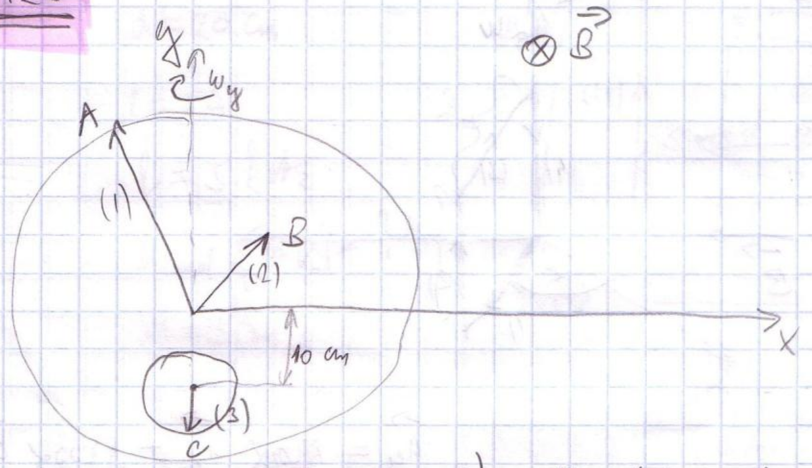
$= -\frac{\partial}{\partial t}(BS\cos\alpha) = -\frac{\partial}{\partial t}(BS\cos wt)$

$= BSw\sin(wt)$

$U_{ind} = BSw\sin\alpha$

$$\boxed{I_{ind} = \frac{U_{ind}}{R} = \frac{BSw\sin\alpha}{R} = 0{,}14\ mA}$$

4,26



a) $U_{AB}(t_1 = 2h\ 15min\ 30s) = ?$

$\vec{B} = -0{,}5\ \vec{a}_z$

$r_A = 20\ cm$

$r_B = 10\ cm$

$r_C = 5\ cm$

$$U_{ind} = \int_0^{r \text{ — do duljine kazaljke}} \underbrace{\omega r}_{v} B\, dr = \omega B \frac{r^2}{2}$$

$\omega_{minute} = \frac{2\pi}{60 \cdot 60} = \frac{2\pi}{3600}$ – brzina vrtnje kazaljke A

$\omega_{sati} = \frac{2\pi}{60 \cdot 60 \cdot 12} = \frac{2\pi}{12 \cdot 3600}$ – –//– –//– B

$$U_{AB} = \frac{B}{2}\left(\frac{2\pi}{3600} r_A^2 - \frac{2\pi}{12 \cdot 3600} r_B^2 \right) = 17{,}09\ \mu V$$

c) $U_{AC}(t_1 = 2h\ 15\ min\ 30s) = ?$

$\omega_{sekunde} = \frac{2\pi}{60}$ – brzina vrtnje kazaljke C

$$U_{AC} = \frac{B}{2}\left(\frac{2\pi}{3600} r_A^2 - \frac{2\pi}{60} r_C^2 \right) = -48\ \mu V$$

c) $t_4 = 3h \quad 30\ min \quad 30\ s$

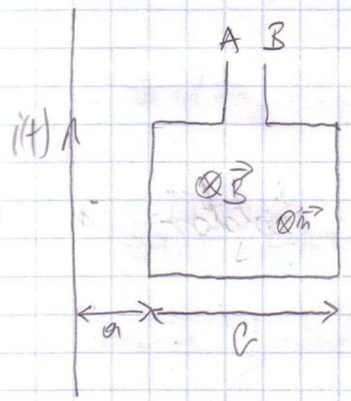
$\omega_4 = 0{,}5\pi \ rad/s$

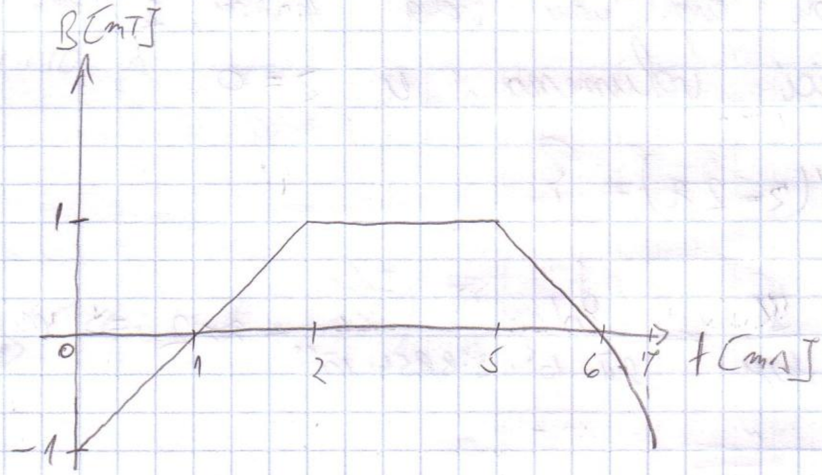
$U_{AC}(t = 3h \ 30\ min \ 32s) = ? \quad \Rightarrow U_{AC} = 48\mu V$

d) $U_{AC}(t = 3h \ 30 min \ 34s) = ?$

- u c i d $\quad U_{AC} = 48\mu V \qquad \Rightarrow U_{AC} = 48\mu V$

4.27





$a = 2$

$b = 1$

$c = 2$

$$B = \frac{\mu_0 i(t)}{2\pi r} \qquad r = 1$$

a) $t = 4ms \quad i(t) = ?$

$$\Rightarrow \underline{i(t)} = \frac{2\pi B}{\mu_0} = \underline{5\ kA}$$

$t = 4ms \Rightarrow B = 1 mT$

$$\vec{B} = -\frac{\mu_0 i(t)}{2\pi r}\vec{a}_2$$

e) $t = 1 ms \quad U_{AB} = ?$

$$\vec{n} = -\vec{a}_2$$

$$U_{AB} = U_{ind} = -\frac{d\phi}{dt} =$$

$$\phi = \iint_S \frac{\mu_0 i(t)}{2\pi x}(-1)(+1)\,dx\,dy = \frac{\mu_0 i(t)}{2\pi} \cdot c \int_a^{b-a} \frac{dx}{x} = \frac{\mu_0 i(t)}{2\pi} c \ln\left(\frac{b-a}{a}\right)$$

?

EM VALOVI U REALNIM DIELEKTRICIMA I VODIČIMA

PR1

$E_m(z=0) = 10 \ V/m$

$f = 100 \ MHz$

- prostire se u smjeru osi z

$\varepsilon_r = 9 \quad \mu_r = 1 \quad \kappa = 0,1 \ S/m$

Odredi amplitudu vala nakon 2 valne duljine i gubitke po jedinici volumena u $z=0$.

$E_m(z = 2\lambda) = ?$

$$\frac{\kappa}{\omega\mu} = \frac{0,1}{2\pi \cdot 10^8 \cdot 9 \cdot 8,854 \cdot 10^{-12}} \approx 2 \Rightarrow \text{nije ni izolator ni vodič}$$

$$v = \frac{c}{\sqrt{\mu_r \varepsilon_r}} = \frac{1}{\sqrt{\mu\varepsilon}} = 10^8 \ m/s$$

$$\alpha = \frac{\omega}{\sqrt{2}c}\sqrt{\sqrt{1+\left(\frac{\kappa}{\omega\varepsilon}\right)^2} - 1} = 4,94 \ m^{-1}$$

$$\beta = \frac{\omega}{\sqrt{2}c}\sqrt{\sqrt{1+\left(\frac{\kappa}{\omega\varepsilon}\right)^2} + 1} = 8 \ m^{-1}$$

$$\lambda = \frac{2\pi}{\beta} = \frac{2\pi}{8} = \frac{\pi}{4}$$

$$E = E_m(z=0)\, e^{-\alpha z} \cos(\omega t - \beta z)$$

$$\frac{E_m(z=2\lambda)}{E_m(z=0)} = e^{\alpha(-2\lambda - (-0))} = e^{-2\lambda\alpha} = e^{-\frac{\pi}{2}\alpha}$$

$$\Rightarrow \boxed{E_m(z=2\lambda) = e^{-2\lambda} E_m(z=0) = e^{-\frac{\pi}{2}\alpha} \cdot 10 = 4,3 \ mV/m}$$

P22 Poluprostor $x>0$ ispunjen je bakrom ($\gamma = 5,73 \cdot 10^7$ S/m). -42-

Na površini poluprostora jakost el. polja iznosi:

$\vec{E} = 2 \cdot 10^3 \cos(10^4 t)\,\vec{a_y}$ V/m. Odredi E, J, i H u ravnini $x=3d$

DUBINA PRODIRANJA

$$\frac{\gamma}{\omega\varepsilon} = 6,5 \cdot 10^{14} \gg 1 \Rightarrow \text{dobar vodič!}$$

$$\alpha = \beta = \sqrt{\frac{1}{2}\omega\mu\gamma} = 600 \text{ m}^{-1}$$

$$d = \frac{1}{\alpha} = \frac{1}{600} = 1,67 \cdot 10^{-3} \text{ m}$$

$$x = 3d = 5 \cdot 10^{-3} \text{ m}$$

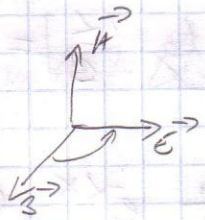
$$\vec{E}(x=3d) = E_m(x=0)\, e^{-\alpha x} \cos(\omega t - \beta x)\,\vec{a_y}$$

$$\alpha x = \frac{1}{\cancel{d}} \cdot 3\cancel{d} = 3$$

$$\vec{E}(x=3d) = 2000\, e^{-3} \cos(\omega t - \cancel{600} \cdot 3 \cdot \frac{1}{\cancel{600}})\,\vec{a_y}$$

$$\boxed{\vec{E} = 2000\, e^{-3} \cos(\omega t - 3)\,\vec{a_y}}$$

$$\boxed{\vec{J} = \gamma \vec{E} = 2000\,\gamma\, e^{-3} \cos(\omega t - 3)\,\vec{a_y}}$$



$$\vec{H} = \frac{E_m}{|Z|} e^{-\alpha x} \cos(\omega t - \beta x - \frac{\pi}{4})\,\vec{a_z}$$

jer su $\alpha = \beta$ pa su E i H pomaknuti u fazi za $\frac{\pi}{4}$

$$Z = (1+j)\sqrt{\frac{\omega\mu}{2\gamma}}$$

$$|Z| = \cancel{\sqrt{2}} \sqrt{\frac{\omega\mu}{\cancel{2}\gamma}} = \sqrt{\frac{\omega\mu}{\gamma}} = 14,81 \cdot 10^{-6}\ \Omega$$

$$\Rightarrow \boxed{\vec{H} = 6,72 \cdot 10^{6} \cos(\omega t - 3 - \frac{\pi}{4})\,\vec{a_z}}$$

✓

PR3. Ravni val se giba u sredstvu značajki $\varepsilon_r = 36$, $\mu_r = 4$, $\kappa = 1\ S/m$ u smjeru osi x. El. polje je:
$\vec{E} = 100 e^{-\alpha x} \cos(10^9 \pi t - \beta x)\, \vec{a}_z$ V/m. Odredi $\vec{H}$.

$$\frac{\kappa}{\omega \varepsilon} = 1 \Rightarrow \text{ni vodič ni izolator!}$$

$$\alpha = \frac{\omega}{\sqrt{2}\, c} \sqrt{\sqrt{1+\left(\frac{\kappa}{\omega\varepsilon}\right)^2} - 1} = 57{,}19\ m^{-1}$$

$$c = \frac{c_0}{\sqrt{\mu_r \varepsilon_r}} = 0{,}25 \cdot 10^8\ m/s$$

$$\beta = \frac{\omega}{\sqrt{2}\, c} \sqrt{\sqrt{1+\left(\frac{\kappa}{\omega\varepsilon}\right)^2} + 1} = 138{,}1\ m^{-1}$$

$$|Z| = \frac{\omega \mu}{\sqrt{\alpha^2 + \beta^2}} = 105{,}65\ \Omega$$

$$\varphi = \operatorname{arctg}\left(\frac{\alpha}{\beta}\right) = 22{,}5^\circ = 0{,}393\ rad$$

$$\vec{H} = \frac{E_m}{|Z|} e^{-\alpha x} \cos(10^9 \pi t - \beta x - 0{,}392)\,(-\vec{a}_y)$$

$$\boxed{\vec{H} = -0{,}95\, e^{-57{,}19 x} \cos(10^9 \pi t - 138{,}1 x - 0{,}392)\, \vec{a}_y}$$

a

$\varepsilon_r = 15$ Odredi udaljenost na kojoj amplituda

$\mu_r = 1$ polja padne 10 puta za f_1 i f_2.

$\varkappa = 10^{-2}$ S/m

$\gamma, \alpha, Z, = ?$ $a = ?$

$$\frac{E_m(x=a)}{E_m(x=0)} = \frac{1}{10}$$

a) $f_1 = 60$ Hz

b) $f_2 = 100$ MHz

a) $\underline{f_1 = 60 \text{Hz}}$ $\Rightarrow \frac{\varkappa}{\omega\varepsilon} = 200 \cdot 10^3 >> 1 \Rightarrow$ dobar vodič

$$\alpha = \beta = \sqrt{\frac{1}{2}\omega\mu\varkappa} = 1,54 \cdot 10^{-3} \text{ m}^{-1}$$

$$\gamma = \alpha + j\beta = \sqrt{\alpha^2 + \beta^2} = 2,18 \cdot 10^{-3} \text{ m}^{-1}$$

$$E = E_m e^{-\alpha x} \cos(\omega t - \beta x) \rightarrow \text{širi se u } x \text{ smjeru (ZADANO)}$$

$$E_m(x=a) = E_m(x=0)\, e^{-\alpha a}$$

$$\frac{E_m(x=a)}{E_m(x=0)} = \frac{1}{10} = e^{-\alpha a} \Rightarrow a = -\frac{1}{\alpha}\ln\frac{1}{10} = \frac{1}{\alpha}\ln(10) = 1495$$

b) $f_2 = 100$ MHz $\Rightarrow \frac{\varkappa}{\omega\varepsilon} = 0,12 < 0,2 \Rightarrow$ dobar izolator

$\hookrightarrow$ granica za dobar izolator

$$\Rightarrow \alpha = \frac{\varkappa}{2}\sqrt{\frac{\mu}{\varepsilon}} = 0,5 \text{ m}^{-1}$$

$$a = \frac{1}{\alpha}\ln(10) = 4,73 \text{ m}$$

PR 5 $\varepsilon_r = 80$

$\mu_r = 1$

$\varkappa = 4\ S/m$

a) $f_1 = 10\,000\ MHz$

b) $f_2 = 25\,000\ Hz$

$\alpha, \beta, Z, \lambda = ?$

a) $f_1 = 10\,000\ MHz$

$$\frac{\varkappa}{\omega\varepsilon} = 0{,}09 < 0{,}2 \Rightarrow \text{dobar izolator}$$

$$\alpha = \frac{\varkappa}{2}\sqrt{\frac{\mu}{\varepsilon}} = 84{,}3\ m^{-1}$$

$$\beta = \frac{\omega}{c} = 1873{,}3\ m^{-1}$$

$$c = \frac{c_0}{\sqrt{\mu_r\varepsilon_r}} = 0{,}34 \cdot 10^8$$

$$Z = \sqrt{\frac{\mu}{\varepsilon}} = 42{,}15\ \Omega$$

$$\lambda = \frac{2\pi}{\beta} = 3{,}35 \cdot 10^{-3}\ m$$

b) $f = 25\ kHz$ $\quad \frac{\varkappa}{\omega\varepsilon} = 35{,}95 \cdot 10^3 >> 1 \Rightarrow$ dobar vodič

$$\alpha = \beta = \sqrt{\frac{1}{2}\omega\mu\varkappa} = 560{,}5\ m^{-1}$$

$$|Z| = \sqrt{\frac{\omega\mu}{\varkappa}} = 0{,}07\ \Omega \qquad \lambda = \frac{2\pi}{\beta} = 11{,}2 \cdot 10^{-3}\ m$$

PR 6 Odredi maximalnu frekv. koju možemo koristiti za komunikaciju s podmornicama ako moramo ispuniti uvjet da se na dubini od 30m signal mora smanjiti 10 puta

$\varepsilon_r = 80$

$\mu_r = 1$

$\varkappa = 4\ S/m$

$$E = \underbrace{E_m e^{-\alpha z}}_{E_m(z)} \cos(\omega t - \beta z)$$

$d = 30\ m$

$$\frac{E_m(z=d)}{E_m(z=0)} = \frac{1}{10} = e^{-\alpha d} \geq \frac{1}{10} \Rightarrow -\alpha \geq \frac{1}{d}\ln\left(\frac{1}{10}\right) \quad / \cdot (-1)$$

$$\alpha \leq \frac{1}{d}\ln(10) = 76{,}75 \cdot 10^{-3}$$

- morska voda dobar vodič na niskim frekv.:

$$\alpha = \sqrt{\frac{1}{2}\omega\mu\varkappa}$$

$$2\alpha^2 = \omega\mu\varkappa \Rightarrow \quad \omega_g \leq \frac{2\alpha^2}{\mu\varkappa} = 2343{,}8$$

$$\Rightarrow \boxed{f_g \leq \frac{\omega_g}{2\pi} = 373\ Hz}$$

4.5 $\vec{E} = xz\,\vec{a}_y$ kV/m vakuum: $\varepsilon = \varepsilon_0$

$w_{el}(1,5,7) = ?$

$$w_{el} = \frac{1}{2}\vec{E}\vec{D} = \frac{1}{2}xz \cdot \vec{a}_y \cdot \varepsilon_0 \cdot xz\,\vec{a}_y$$

$$\underline{\underline{w_{el}}} = \frac{1}{2}x^2z^2\varepsilon_0 = \underline{0{,}217 \cdot 10^{-3}\ J/m} \quad \checkmark$$

4.6 $\vec{E}(z,t) = 10\cos(\omega t - \beta z)\,\vec{a}_x$ V/m

$r = 1{,}5$ m

$z = 2$ m

$P_{sr} = ?$ $[W] = ?$

$$\vec{N} = \vec{E} \times \vec{H}$$

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} = -\mu\frac{\partial \vec{H}}{\partial t} = \begin{vmatrix} \vec{a}_x & \vec{a}_y & \vec{a}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ E_x & 0 & 0 \end{vmatrix} = \vec{a}_x \cdot 0 - \vec{a}_y\left(-\frac{\partial E_x}{\partial z}\right) + \vec{a}_z\left(-\cancel{\frac{\partial E_x}{\partial y}}\right)$$

0

$$= \vec{a}_y\,10\,(-\sin(\omega t - \beta z)\cdot(-\beta))$$

$$= \beta 10\sin(\omega t - \beta z)\,\vec{a}_y = -\mu\frac{\partial \vec{H}}{\partial t} \Rightarrow$$

$$\vec{H} = -\frac{10\beta}{\mu_0}\int \sin(\omega t - \beta z)\,dt\,\vec{a}_y$$

$$= -\frac{10\beta}{\mu_0}\frac{1}{\omega}\left(-\cos(\omega t - \beta z)\right)\vec{a}_y$$

$$\boxed{\vec{H} = \frac{10\beta}{\mu_0 \omega}\cos(\omega t - \beta z)\,\vec{a}_y}$$

$$\vec{N} = \vec{E}\times\vec{H} = \begin{vmatrix} \vec{a}_x & \vec{a}_y & \vec{a}_z \\ E_x & 0 & 0 \\ 0 & H_y & 0 \end{vmatrix} = \vec{a}_x\cdot 0 + \vec{a}_y\cdot 0 + \vec{a}_z\,E_x H_y$$

$$\vec{N} = \frac{100\beta}{\mu_0 \omega}\cos^2(\omega t - \beta z)\,\vec{a}_z$$

$$N_{sr} = \frac{1}{T}\int_0^T N\,dt = \frac{1}{2}\frac{100\beta}{\mu_0 \omega}$$

$$P_{sr} = \oint_S \vec{N}_{sr}\vec{n}\,dS = N_{sr}\,r^2\pi = \frac{1}{2}\frac{100\beta r^2\pi}{\mu_0 \omega} = \frac{1}{2}\frac{100\,\cancel{\omega}\sqrt{\mu\varepsilon}\,r^2\pi}{\mu_0\cancel{\omega}}$$

$$\boxed{P_{sr} = \frac{50 r^2\pi\sqrt{\varepsilon_0\mu_0}}{\mu_0} = 0{,}938\ W}\ \checkmark$$

4.7

$\vec{H} = 200x\,\vec{a}_y$ A/m

$w_m(2,5,1)\ [mJ/m^3] = ?$

$$\boxed{w_m = \frac{1}{2}\vec{H}\vec{B} = \frac{1}{2}\mu_0 H^2 \underbrace{\vec{a}_y\cdot\vec{a}_y}_{1} = \frac{1}{2}\mu_0(200x)^2 = 100{,}53\ mJ/m^3}$$

10. $\vec{E}(z,t) = 150 \sin(\omega t - \beta z)\, \vec{a}_x \quad V/m$

$a = 3\,cm$

$b = 1{,}5\,cm$

$z = 2\,m$

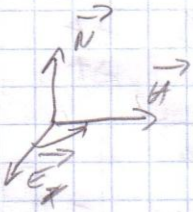
$P_{sr}\,[W] = ?$

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} = -\mu \frac{\partial \vec{H}}{\partial t}$$

$$\nabla \times \vec{E} = \begin{vmatrix} \vec{a}_x & \vec{a}_y & \vec{a}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ E_x & 0 & 0 \end{vmatrix} = \vec{a}_y \frac{\partial E_x}{\partial z} = 150 \cos(\omega t - \beta z)(-\beta)\, \vec{a}_y$$

$$\vec{H} = \frac{150}{\mu} \int \cos(\omega t - \beta z)\, dt\, \vec{a}_y = \frac{150\beta}{\mu} \frac{1}{\omega} \sin(\omega t - \beta z)\, \vec{a}_y$$

$$\vec{N} = \vec{E} \times \vec{H} = \begin{vmatrix} \vec{a}_x & \vec{a}_y & \vec{a}_z \\ E_x & 0 & 0 \\ 0 & H_y & 0 \end{vmatrix} =$$



$$= \vec{a}_z\, E_x H_y$$

$$\vec{N} = \frac{150^2 \beta}{\mu \omega} \sin^2(\omega t - \beta z)\, \vec{a}_z$$

$$N_{sr} = \frac{1}{2} \frac{150^2 \beta}{\mu \omega} = \frac{1}{2} \frac{150^2 \sqrt{\mu_0 \varepsilon_0}}{\mu_0}$$

$$P_{sr} = \iint_S \vec{N}_{sr}\, \vec{n}\, ds = N_{sr}\, S = N_{sr} \cdot a \cdot b = \frac{1}{2} \frac{150^2 \sqrt{\mu_0 \varepsilon_0}}{\mu_0} \cdot 0{,}03 \cdot 0{,}015$$

$$\boxed{P_{sr} = 0{,}0134\ W}$$

4.11

$$\vec{H} = 100xy\,\vec{a}_z \quad A/m$$

$$w_m(2,5,1) = ? \quad [mJ/m^3]$$

$$\underline{w_m} = \frac{1}{2}\vec{H}\vec{B} = \frac{1}{2}\mu_0 H^2 = \frac{1}{2}\mu_0\, 100^2 x^2 y^2 = \underline{628{,}32 \ mJ/m^3}$$

4.19

$$\vec{E} = E_m \cos(\beta x)\cos(\omega t)\,\vec{a}_z$$

$$\vec{H} = ?$$

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} = -\mu \frac{\partial \vec{H}}{\partial t}$$

$$\nabla \times \vec{E} = \begin{vmatrix} \vec{a}_x & \vec{a}_y & \vec{a}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 0 & 0 & E_z \end{vmatrix} = \vec{a}_x \underbrace{\cancel{\left(\frac{\partial E_z}{\partial y}\right)}}_{0} - \vec{a}_y \frac{\partial E_z}{\partial x} + 0$$

$$= -\vec{a}_y\, E_m \cos\omega t\,(-\sin\beta x \cdot \beta)$$

$$= \beta E_m \cos\omega t \sin\beta x\, \vec{a}_y = -\mu \frac{\partial \vec{H}}{\partial t}$$

$$\Rightarrow \vec{H} = -\frac{\beta E_m}{\mu} \sin\beta x \int \cos\omega t\, dt \ \vec{a}_y$$

$$= -\frac{\beta E_m}{\mu} \sin\beta x \frac{1}{\omega} \sin\omega t\, \vec{a}_y$$

$$= -\frac{\beta E_m}{\mu_0 \omega} \sin\beta x \sin\omega t\, \vec{a}_y$$

$$\beta = \omega\sqrt{\mu_0\varepsilon_0} \Rightarrow \mu_0 = \frac{\beta^2}{\omega^2\varepsilon_0}$$

$$\frac{\beta E_m}{\omega \cdot \frac{\beta^2}{\omega^2\varepsilon_0}} = \frac{E_m \varepsilon_0 \omega}{\beta}$$

$$\boxed{\vec{H} = -\frac{\omega}{\beta} E_m \varepsilon_0 \sin\beta x \sin\omega t\, \vec{a}_y}$$

5.20 - za prethodni zadatak odrediti omjer $\frac{\omega}{\beta}$!

$$\frac{\omega}{\beta} = \frac{\omega}{\omega\sqrt{\mu_0\varepsilon_0}} = \frac{1}{\sqrt{\mu_0\varepsilon_0}} = \underline{3\cdot10^8}$$

5.24

$R_u = 5\ mm = 5\cdot10^{-3}\ m$

$R_v = 6\cdot10^{-3}\ m$

$l = 500\cdot10^{-3}\ m$

$\varepsilon_r = 6,7$

$\varphi_u = 0\ V$

$\varphi_v = 250\sin(377t)\ \vec{a}_r\ V/m$

a) $\vec{E}(r=5,5\cdot10^{-3}) = ?$

$$\vec{E} = -\nabla\varphi$$

$$\vec{E} = -\frac{\partial\varphi}{\partial r} = \vec{a}_r\frac{\varphi_v}{r\ln\left(\frac{R_v}{R_u}\right)}$$

$$\boxed{\vec{E}(r=5,5\cdot10^{-3}) = \vec{a}_r\frac{250\sin(377t)}{5,5\cdot10^{-3}\cdot\ln\left(\frac{6}{5}\right)} = -249310\sin(377t)}$$

b) $\vec{J}_{POM}(r=5,2\cdot10^{-3}) = ?$

$$\vec{J}_{POM} = \frac{\partial\vec{D}}{\partial t} = \varepsilon\frac{\partial\vec{E}}{\partial t} = \varepsilon_r\varepsilon_0 E_0\cos(377t)\cdot377\ \vec{a}_r$$

$$\vec{E}(5,2\cdot10^{-3}) = \underbrace{-263693}_{E_0}\sin(377t)\ \vec{a}_r \quad \checkmark$$

$$\vec{J}_{POM}(r=5,2\cdot10^{-3}) = 6,7\cdot8,854\cdot10^{-12}\cdot(-263693)\,377\cdot\cos(377t)$$

$$\boxed{\vec{J}_{POM} = -5,9\cdot10^{-3}\cos(377t)\ \vec{a}_r}$$

c) $I_{UK} = ?$ $\qquad$ jer je $\vec{J} = \kappa \vec{E}$, a $\kappa = 0$ $\downarrow$ jer se radi o izolatoru

$$I_{UK} = \oiint_S (\cancel{\vec{J}} + \vec{J}_{POM}) \vec{n}\, ds$$

$$I_{UK} = \oiint_S \vec{J}_{POM} \vec{n}\, ds$$

- integrira se po plaštu cilindra radijusa r i duljine l
(između R_1, R_2)

$S = 2 r \pi l$

$$\left.\begin{aligned} R_1 &= 5 \cdot 10^{-3}\, m \\ l &= 0{,}5\, m \end{aligned}\right\} = \text{ZADANO!}$$

$$\vec{E}(R_1) = \frac{250 \sin(377t)}{5 \cdot 10^{-3} \ln\left(\frac{6}{5}\right)} (-\vec{a}_r) = -274240{,}75 \sin(377t)\, \vec{a}_r$$

$$\vec{J}_{POM} = \frac{\partial \vec{D}}{\partial t} = \varepsilon_0 \varepsilon_r \frac{\partial \vec{E}}{\partial t} = \varepsilon_0 \varepsilon_r E_0 \omega \cos(\omega t)\, \vec{a}_r$$

$$\vec{J}_{POM}(R_1) = -6{,}1333 \cdot 10^{-3} \cos(377t)\, \vec{a}_r$$

$$I_{UK} = I_{POM} = \oiint_S \vec{J}_{POM} \vec{n}\, ds = 6{,}1333 \cdot 10^{-3} \cos(377t) \underbrace{\iint_S ds}_{S}$$

$$= 6{,}1333 \cdot 10^{-3} \cos(377t) \cdot 2 R_1 \pi l$$

$$\boxed{I_{POM} = 9{,}634 \cdot 10^{-5} \cos(377t)}$$

d) $C = ?$

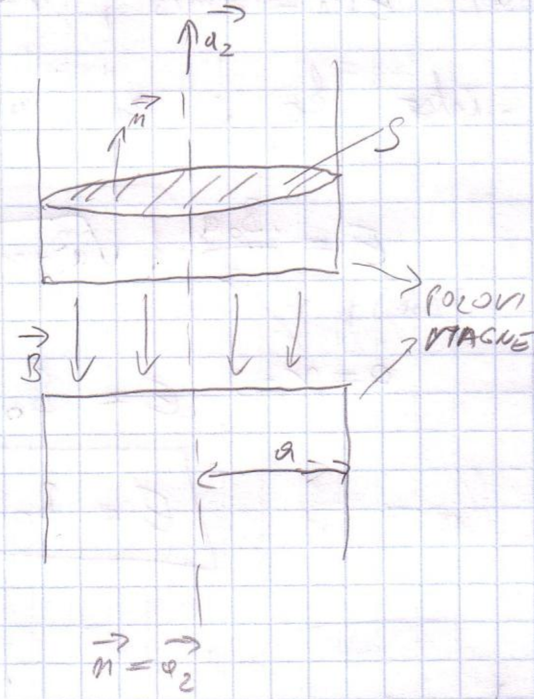
$$C = \frac{Q}{U} \qquad \boxed{C = \frac{2\pi \varepsilon_0 l}{\ln\left(\frac{R_2}{R_1}\right)}} \rightarrow \text{167 str. pri. 7.2.3}$$

$$C = 1{,}02 \cdot 10^{-9} F$$

$$\boxed{C \approx 1 nF}$$

28

$$\vec{B} = \begin{cases} -B_0 \dfrac{at}{\sqrt{a^2+r^2}} \vec{a_z}, & t>0 \\ 0, & t<0 \end{cases}$$



$a = 0{,}1\ m$ $\quad K = 0$

$B_0 = 1\ T$ $\quad \varepsilon_r = 1$

a) $\vec{E}(t = 3\ ms,\ r = 0{,}1\ m)\ [mV/m] = ?$

$$U_{ind} = -\frac{d\phi}{dt}$$

$$\oint_c \vec{E}\, d\vec{l} = -\frac{d}{dt} \iint_S \vec{B}\, \vec{n}\, ds$$

$$E\, \cancel{2\pi} r = \frac{d}{dt} \int_{x=0}^{r} B_0 \frac{at}{\sqrt{a^2+x^2}}\, \cancel{2\pi} x\, dx$$

$$E r = \frac{d}{dt}\left[B_0 a t \int_0^r \frac{x\, dx}{\sqrt{a^2+x^2}} \right]$$

vrijedi: $\boxed{\int \frac{x\, dx}{\sqrt{x^2+a^2}} = \sqrt{a^2+x^2}}$

$$\Rightarrow \quad E r = \frac{d}{dt}\left[B_0 a t \sqrt{a^2+x^2}\Big|_0^r \right] = \frac{d}{dt}\left[B_0 a t \left(\sqrt{a^2+r^2} - a\right)\right]$$

$$E r = B_0 a \sqrt{a^2+r^2} - B_0 a^2$$

$$\boxed{E = \frac{B_0 a}{r}\left(\sqrt{a^2+r^2} - a\right) = 23{,}6\ mV/m}$$ ✓

b) $E(t = 3\,ms,\ r = 0{,}1\,m)\ [mV/m] = ?$

- isto ko i za u zadatku,

$$E = \frac{B_0 a}{r}\left(\sqrt{a^2+r^2} - a\right)$$

$$a = r \Rightarrow E = B_0\left(a\sqrt{2} - a\right)$$

$$\boxed{E = B_0 a\left(\sqrt{2} - 1\right) = 41{,}4\ mV/m}$$

c) $\left.\begin{array}{l} t = -1\,ms \\ r = 0{,}05\,m \end{array}\right\} \Rightarrow B(t<0) = 0 \Rightarrow \boxed{E = 0}$

d) $t = 1\,ms$

$\phi\ [\mu Wb] = ?$

$$\phi = \iint_S \vec{B}\,\vec{n}\,dS = -\int_{x=0}^{a} B_0 \frac{at}{\sqrt{a^2+x^2}}\, 2\pi x\, dx$$

$$= -B_0\, a t\, 2\pi \int_0^a \frac{x\,dx}{\sqrt{a^2+x^2}}$$

$$= -B_0\, a t\, 2\pi\left(\sqrt{a^2+a^2} - \sqrt{a^2}\right)$$

$$= -B_0\, a t\, 2\pi\left(a\sqrt{2} - a\right)$$

$$\boxed{\phi = -B_0 a^2 t\, 2\pi\left(\sqrt{2} - 1\right) = -26\ \mu Wb}$$

c) $t = 3\,ms$

$U_{AB} = ?$ $\qquad U_{AB} = 0$ - nema promjene struje u 3 ms

d) $t = 5{,}5\,ms$

$U_{AB} = ?$ $\qquad B = \frac{\mu_0 i(t)}{2\pi r} \qquad r = x$

- analogno kao i za b slučaj

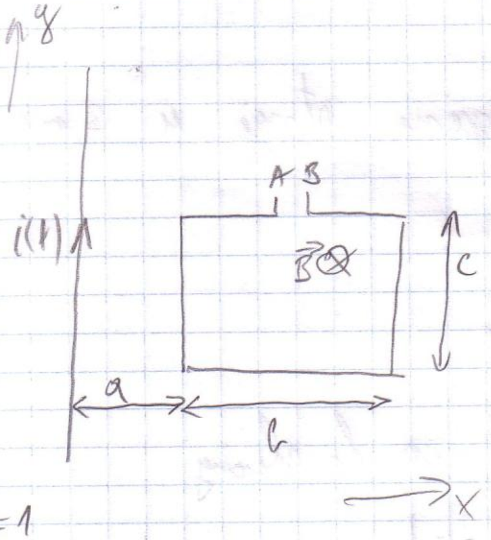
$$U_{AB} = -\frac{d}{dt}\iint_S \vec{B}\,\vec{n}\,dS = -\frac{d}{dt}\left(i(t)\cdot 1{,}62\cdot 10^{-7}\right)$$

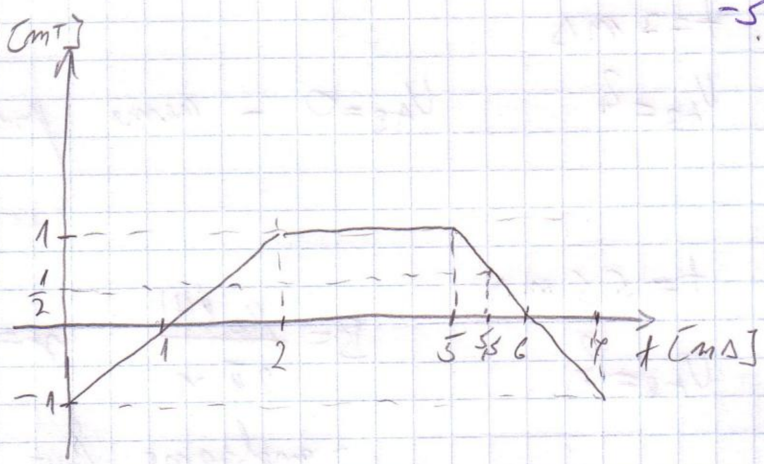
$$= 1{,}62\cdot 10^{-7}\,\frac{di}{dt}$$

$$\frac{di}{dt} = \frac{2{,}5\,kA - 5\,kA}{5{,}5\,ms - 5\,ms} = \frac{-2{,}5\cdot 10^{3}}{0{,}5\cdot 10^{-3}} = 5\cdot 10^{6}$$

$$\boxed{U_{AB} = 1{,}62\cdot \cancel{10^{-7}}_{10^{-1}}\cdot 5\cdot \cancel{10^{6}} = 0{,}81\,V}$$

-27





$r = 1$

$a = 2\ m$

$b = 1\ m$

$c = 2\ m$

a) $i(t = 4ms)\ [kA] = ?$

$t = 4ms \Rightarrow B = 1mT$

$$B = \frac{\mu_0\, i(t)}{2\pi r} \Rightarrow i(t) = \frac{B\, 2\pi r}{\mu_0}$$

$$\boxed{i(t=4ms) = 5\ kA}$$

b) $t = 1ms$ $\qquad B = \frac{\mu_0\, i(t)}{2\pi r}$ $\qquad r = x$

$U_{AB} = ?$

$$U_{ind} = U_{AB} = -\frac{\partial}{\partial t} \iint_S \vec{B}\,\vec{n}\,dS = -\frac{\partial}{\partial t} \int \frac{\mu_0\, i(t)}{2\pi x}\,dx\,dy$$

$$U_{ind} = U_{AB} = -\frac{\partial}{\partial t} \left(\frac{\mu_0}{2\pi}\, i(t) \int_a^{a+b} \frac{dx}{x} \int_0^c dy \right)$$

$$= -\frac{\partial}{\partial t} \left(\frac{\mu_0}{2\pi}\, i(t)\, \ln\left(\frac{a+b}{a}\right) \cdot c \right)$$

$$= -\frac{\partial}{\partial t} \left(i(t) \cdot 1{,}62 \cdot 10^{-7} \right) \qquad \frac{di}{dt} = \frac{5 \cdot 10^3}{1 \cdot 10^{-3}}$$

$$= -1{,}62 \cdot 10^{-7} \cdot \frac{di}{dt}$$

$$\boxed{U_{ind} = -1{,}62 \cdot \underset{10^{-1}}{\cancel{10^{-7}}} \cdot 5 \cdot \cancel{10^{6}} = -0{,}81\ V}$$