II CIKLUS

1. JEDNADŽBE STAT. STRUJNOG POLJA I UVJETI NA GRANICI DVAJU VODIČA

→ u vodiču postoji stat uvjeti kada u njemu nema usmjerenog gibanja slob. elektrona (tj. uk. struja = 0)

→ ako unesemo neki naboj u vodič on stvara el. polje, a naboj i njegovo gibanje uzrokuje struju koja mora zadovoljiti jed. kontinuiteta

$$\nabla \vec{J} = -\frac{\partial \rho}{\partial t} \overset{\text{ZA STATIČKO (ISTOSMJERNA STRUJA)}}{=} 0 \qquad \text{pri čemu je} \quad \vec{J} = K \cdot \vec{E}$$

OHMOV ZAKON U EL. OBLIKU

$$\nabla (K \cdot \vec{E}) = -\frac{\partial \rho}{\partial t}$$

$$\underbrace{\overset{0}{\nabla K} \cdot \vec{E}}_{\text{HOMOGEN}} + K \cdot \nabla \vec{E}$$

$$K \cdot \nabla \vec{E} = -\frac{\partial \rho}{\partial t} \Rightarrow \nabla \vec{E} = -\frac{1}{K} \frac{\partial \rho}{\partial t}$$

→ VRIJEDI I GAUSSOV ZAKON

$$\nabla \vec{D} = \rho_s \Rightarrow \nabla \vec{E} = \frac{\rho_s}{\varepsilon}$$

→ VRIJEDI ZA STAT. STRUJNO POLJE: $\nabla \vec{J} = 0 \quad \vec{J} = K \cdot \vec{E} \quad \vec{E} = -\nabla \varphi$

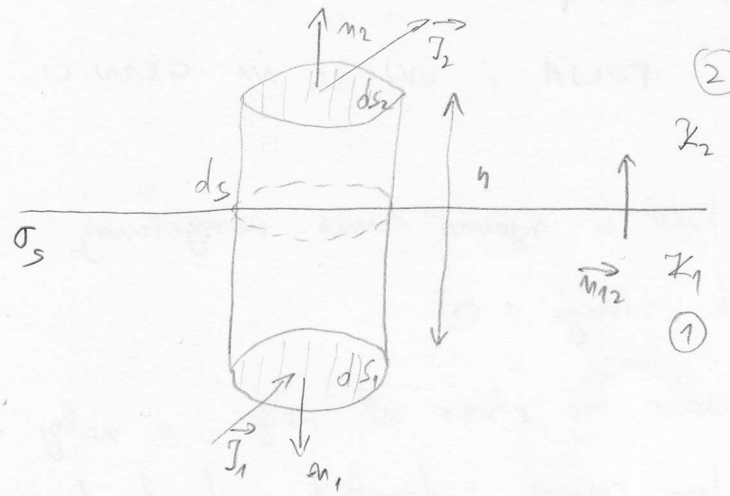
$$\Downarrow$$

$$\nabla \vec{J} = \nabla (K \vec{E}) = \nabla (-K \nabla \varphi) = 0$$

$$\Downarrow$$

$\Delta \varphi = 0$ → LAPLASOVA JEDNADŽBA

UVJETI NA GRANICI DVA VODIČA



→ dva vodiča

→ na granici slobodni naboj

$$\vec{n}_{12} = -\vec{n}_1 = \vec{n}_2$$

$$dS = dS_1 = dS_2$$

I ako primjenimo jedn. kont na mali cilindar

$$\oiint_S \vec{J}\vec{n}\,dS = -\frac{d}{dt}\iiint_V \rho_s \cdot dV$$

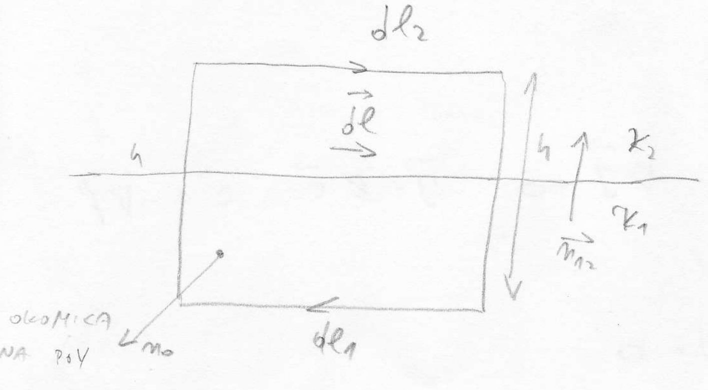
$$\vec{J}_1 \vec{n}_1\, dS_1 + \vec{J}_2 \vec{n}_2\, dS_2 + (\text{doprinos plašta}) = -\frac{d}{dt}(\rho_s \cdot h \cdot dS) \Big/ \lim_{h\to 0}$$

$$\vec{J}_1 n_1\, d\cancel{S}_1 + \vec{J}_2 n_2\, d\cancel{S}_2 + 0 = -\frac{d}{dt}\lim_{h\to 0}\{\underbrace{\rho_s\, h\, d\cancel{S}}_{\sigma}\} \Big/ \frac{1}{dS}$$

$$\boxed{\vec{n}_{12}(\vec{J}_2 - \vec{J}_1) = -\frac{d\sigma_s}{dt}}$$

→ NORMALNA KOMP. SE MIJENJA ZA IZNOS VREM. PROMJENE σ_s

II



→ POLJE HOMOGENO U PROMATRANOM MALO PRAVOKUTNIKU

$$\underline{\vec{J} = \kappa \cdot \vec{E}}$$

→ KONZERVATIVNOST $\nabla \times \vec{E} = 0$

$$\oint_C \vec{E}\,\vec{dl} = 0$$

$$\oint_l \vec{E}\,\vec{dl} = E_1\, \vec{dl}_1 + E_2\, \vec{dl}_2 + (\text{doprinos na stranicama } h) = 0 \Big/ \lim_{h\to 0}$$

$$(\vec{E}_1 - \vec{E}_2)\cdot \vec{l} = 0$$

$$(\vec{n} \times \vec{n}_0)(\vec{E}_1 - \vec{E}_2) = 0 \longrightarrow \boxed{\vec{n} \times (\vec{E}_2 - \vec{E}_1) = 0}$$

$$\vec{E}_2 = \frac{\vec{J}_2}{\kappa_2} \qquad \vec{E}_1 = \frac{\vec{J}_1}{\kappa_1}$$

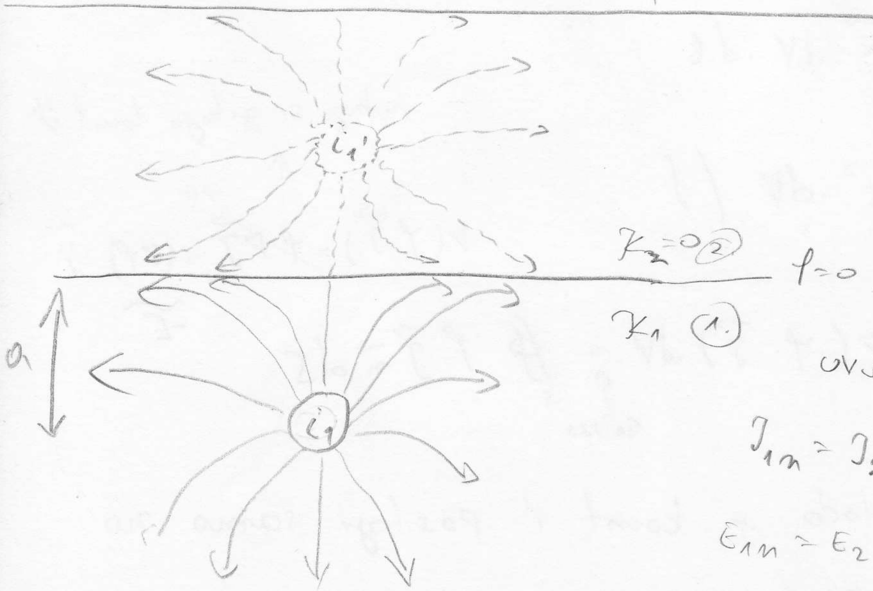
$$\frac{J_{1t}}{\kappa_1} = \frac{J_{2t}}{\kappa_2}$$

– TANG KOMP VEKTORA JAKOSTI EL. POLJA SE NE MIJENJA

– NORM KOMP GUSTOĆE STRUJE I TANG KOMP JAKOSTI EL POLJA NE MIJENJAJU SE

② ANALOGIJA STAT. STRUJNOG POLJA I STAT EL POLJA I ODSLIKAVANJE U STAT. STRUJNOM POLJU

	HOMOGENI DIELEKTRIK BEZ NABOJA $\rho_s = 0$	VODLJIVI MATERIJAL STAC. STRUJANJE $\partial \rho_s / \partial t = 0$
1. (GAUS)	$\nabla \vec{D} = 0$ $\vec{D} = \varepsilon \vec{E}$ $\vec{E} = -\nabla \varphi$	$\nabla \vec{J} = 0$ $\vec{J} = K \cdot \vec{E}$ $\vec{E} = -\nabla \varphi$
2. (LAPLAS)	$\Delta \varphi = 0$	$\Delta \varphi = 0$
3. (TOK)	$\Phi_{elektrin} = \oiint_S \vec{D} \vec{n}\, dS$	strujni tok vektor $\vec{J}$ $\Rightarrow$ $I = \iint_S \vec{J} \vec{n}\, dS$
	$C = \frac{Q}{U_{AB}} = \frac{\iint_S \vec{D} \vec{n}\, dS}{-\int_b^a \vec{E}\, d\vec{l}}$	VODLJIVOST $G = \frac{I}{U_{AB}} = \frac{\iint_S \vec{J} \vec{n}\, dS}{-\int_b^a \vec{E}\, d\vec{l}}$
	$\vec{D}$ $\vec{E}$ ε φ Φ_e C	$\vec{J}$ $\vec{E}$ K φ I $G = \frac{1}{R}$



→ i_1 se naloži u sredstvu provodnosti K_1

→ udaljenost od dielektrika je a

UVJETI NA GRANICI:

$J_{1n} = J_{2n}$ $\quad J_{2n} \overset{\text{DIELEKTRIK}}{=} 0 \Longrightarrow J_{1n} = 0$

$E_{1n} = E_{2n} = 0 \quad$ i $\quad \frac{\partial \varphi_1}{\partial n} = \frac{\partial \varphi_2}{\partial n} = 0$

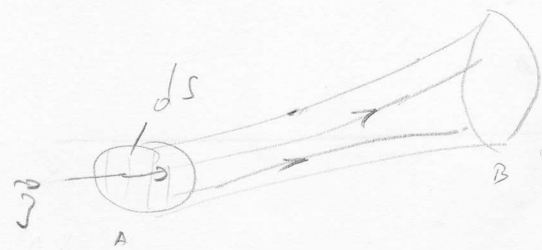
→ uvjeti su zadovoljeni da se odslika isti takav naboj u dielektrik

↳ utjecaj granice zamjenimo (i dalje vrijedi da postoji samo tang komponenta

3. GUBICI SNAGE U VODIČU U STAT. STRUJNOM POLJU

→ nastaje zbog otpora $R = \frac{U_{AB}}{I} = \frac{-\int_a^b \vec{E}\,d\vec{l}}{\iint_S \vec{J}\vec{n}\,dS} = \frac{-\int_a^b \vec{E}\,d\vec{l}}{k\iint \vec{E}\vec{n}\,dS}$

→ ako je jednolik presjek $R = \frac{1}{k}\frac{l}{S} = \rho \cdot \frac{l}{S}$



UK STRUJA

$I = \vec{J}\cdot\vec{n}\cdot S = J\cdot S = kE\cdot S$

$E = \frac{I}{k\cdot S}$

$U_{AB} = -\int_B^A E\,dl = E\cdot l = \frac{I}{k\cdot S}\cdot l = I\cdot \underbrace{\frac{1}{k}\cdot\frac{l}{S}}_{R} = I\cdot R$

→ konstantan je → ne mijenja se oblik vodiča

$\varphi(A) - \varphi(B) = \vec{E}\cdot d\vec{s}$

$dq = J\vec{n}\,dS\cdot dt$

$\underline{dW = dq\cdot[\varphi(A) - \varphi(B)]}$ $\quad W = q\cdot U$

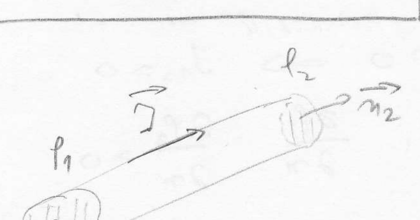
$dW = \vec{J}n\,dS\cdot dt\cdot\vec{E}\,d\vec{s} = \vec{J}\vec{E}\,dV\cdot dt$?

$dP = \frac{dW}{dt} = \frac{\vec{J}\cdot\vec{E}\cdot dV\cdot \cancel{dt}}{\cancel{dt}} = \vec{J}\cdot\vec{E}\cdot dV \quad /\int$

naboji se gibaju konst $\vec{J}$

$\nabla(\varphi\cdot\vec{J}) = \cancel{\varphi\nabla\vec{J}}^{=0} + \underbrace{(\nabla\varphi)}_{-\vec{E}}\cdot\vec{J}$

$\boxed{P = \iiint_V \vec{J}\cdot\vec{E}\cdot dV} = -\iiint_V \nabla(\varphi\cdot\vec{J})\,dV \underset{\text{GAUSS}}{=} \oint_S \varphi\cdot\vec{J}\vec{n}\,dS$



→ gustoća je konst i postoji samo na bazama

$P = -\oint \varphi\cdot\vec{J}\vec{n}\,dS = J\cdot S\,(\varphi_1 - \varphi_2) = I\cdot U = I^2\cdot R$

→ stat el. poljem ne možemo potjerat struju jer je sav naboj na pov a polje unutra 0

4. BIOT-SAVARTOV ZAKON i MAG. IND KRATKE RAVNE STRUJNICE

→ diferencijalni dio strujnice protjecan strujom I u vakuumu stvara u točki koja je od njega udaljena za R mag ind.:

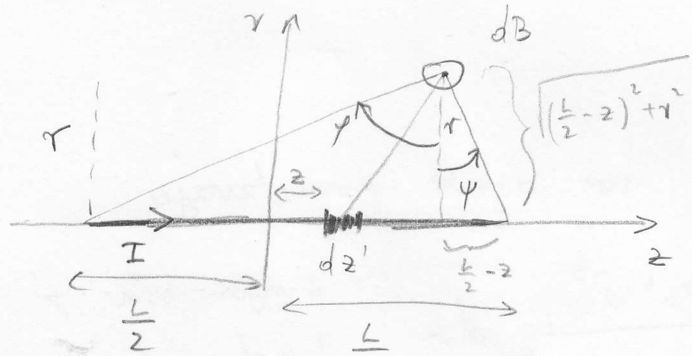
$$d\vec{B} = \frac{\mu_0 I}{4\pi} \cdot \frac{\vec{dl} \times \vec{R}}{R^3} \quad \Big/ \int$$

$\vec{r}\,'$ → položaj dijela strujnice

$\vec{r}$ → položaj točke → $\vec{R} = \vec{r} - \vec{r}\,'$

$$\boxed{\vec{B} = \int_l d\vec{B} = \frac{\mu_0}{4\pi} \int_l I \, \frac{\vec{dl} \times \vec{R}}{R^3}}$$

→ KRATKA STRUJNICA:



$\vec{dl} = dz' \, \vec{a_z}$

$\left.\begin{array}{l} \vec{r}\,' = z' \, \vec{a_z} \\ \vec{r} = r \, \vec{a_r} + z \vec{a_z} \end{array}\right\}$ $\vec{R} = \vec{r} - \vec{r}\,'$, $\vec{R} = r\vec{a_r} + (z - z')\vec{a_z}$

$\vec{dl} \times \vec{R} = (dz' \vec{a_z}) \times (r\vec{a_r} + (z-z')\vec{a_z})$

$= r \, dz' \vec{a_\alpha} + 0$

$\vec{a_z} \times \vec{a_r} = \vec{a_\alpha}$

$$\vec{B} = \frac{\mu_0 I}{4\pi} \int_l \frac{\vec{dl} \times \vec{R}}{R^3} = \frac{\mu_0 I}{4\pi} \int_l \frac{r \, dz' \vec{a_\alpha}}{[r^2 + (z-z')^2]^{\frac{3}{2}}}$$

$$\vec{B} = \frac{\mu_0 I r}{4\pi} \vec{a_\alpha} \int_{-\frac{L}{2}}^{\frac{L}{2}} \frac{dz'}{[r^2 + (z-z')^2]^{\frac{3}{2}}} \quad \ldots \text{sređiš}$$

$$\vec{B} = \vec{a_\alpha} \frac{\mu_0 I}{4\pi r} \Bigg[\underbrace{\frac{\frac{L}{2} + z}{\sqrt{(\frac{L}{2}+z)^2 + r^2}}}_{\sin\varphi} + \underbrace{\frac{\frac{L}{2} - z}{\sqrt{(\frac{L}{2}-z)^2 + r^2}}}_{\sin\psi} \Bigg]$$

$$\boxed{\vec{B} = \frac{\mu_0 I}{4\pi r} (\sin\varphi + \sin\psi) \, \vec{a_\alpha}}$$

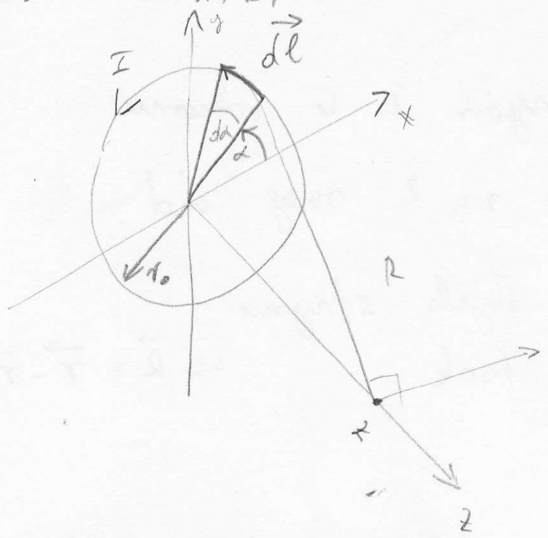
r → udaljenost do strujnice

$\vec{a_\alpha}$ → smjer modeliran vektorom smjera najkraće udaljenosti od točke do strujnice

→ kada $L \to \infty$ $(\varphi, \psi) \to \frac{\pi}{2}$

$$\vec{B} = \frac{\mu_0 I}{2\pi r} \cdot \vec{a_\alpha}$$

5. BIOT SAVARTOV ZAKON i MAG. IND NA OSI KRUŽNE STRUJNICE



$$\vec{dl} = r_0\, d\alpha\, \vec{a_\alpha}$$

$$\left.\begin{array}{l} \vec{r}' = r_0 \vec{a_r} \\ \vec{r} = z\vec{a_z} \end{array}\right\} \quad \begin{array}{l} R = \vec{r} - \vec{r}' \\ R = z\vec{a_z} - r_0 \vec{a_r} \end{array}$$

$$\vec{dl} \times \vec{R} = (r_0\, d\alpha\, \vec{a_\alpha}) \times (z\vec{a_z} - r_0\vec{a_r})$$

$$= r_0\, d\alpha\, z\, \vec{a_r} - r_0^2\, d\alpha\, (-\vec{a_z})$$

$$\vec{dl} \times \vec{R} = (r_0 z\, \vec{a_r} + r_0^2 \vec{a_z})\, d\alpha$$

$$\vec{B} = \frac{\mu_0 I}{4\pi} \int \frac{\vec{dl} \times \vec{R}}{R^3}$$

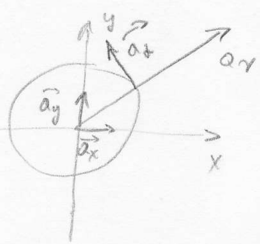
→ ostaje komp u smjeru $\vec{a_z}$ jer se sve ostale poništavaju

$$\vec{B} = \frac{\mu_0 I}{4\pi}\left[\int_0^{2\pi} \frac{r_0 z\, \vec{a_r}}{(r_0^2+z^2)^{\frac{3}{2}}}\, d\alpha + \int_0^{2\pi} \frac{r_0^2\, \vec{a_z}}{(r_0^2+z^2)^{\frac{3}{2}}}\, d\alpha\right]$$

konstanta

→ smjer osi je konst u odnosu na kut

$$\vec{B} = \frac{\mu_0 I}{4\pi}\left[\frac{r_0 z}{(r_0^2+z^2)^{\frac{3}{2}}} \underbrace{\int_0^{2\pi} \vec{a_r}\, d\alpha}_{\to 0} + \frac{r_0^2\, \vec{a_z}}{(r_0^2+z^2)^{\frac{3}{2}}} \int_0^{2\pi} d\alpha\right]$$



$$\vec{a_r} = \cos\alpha\, \vec{a_x} + \sin\alpha\, \vec{a_y}$$

$$\int_0^{2\pi} \cos\alpha\, a_x + \sin\alpha\, \vec{a_y} = 0 + (1-1) = 0$$

↳ RADIJALNA KOMP AKO IMAM CIJELU KRUŽNICU SE POKINE

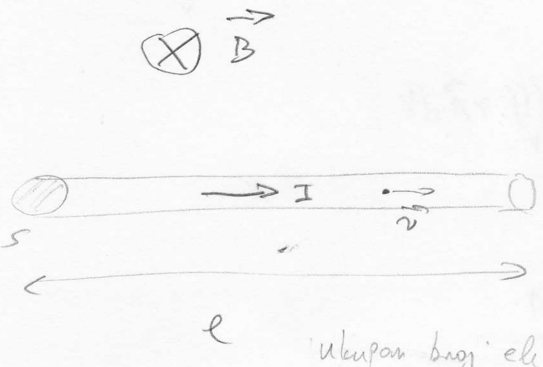
$$\vec{B} = \frac{\mu_0 I}{\cancel{4\pi}} \cdot \frac{r_0^2\, \vec{a_z}}{(r_0^2+z^2)^{\frac{3}{2}}} \cdot \cancel{2\pi}$$

$$\boxed{\vec{B} = \frac{\mu_0 I r_0^2}{2\cdot(r_0^2+z^2)^{\frac{3}{2}}} \cdot \vec{a_z}}$$

→ SAMO AKSIJALNA KOMPONENTA

6. SILA NA STRUJNI ELEMENT U MAGNETSKOM POLJU

→ Ako se vodič protjecan strujom naloži u mag polju, na slobodne el u vodiču čije gibanje predstavlja struju djeluje mag F



→ na svaki pojedini naboj djeluje:

$$\vec{f} = q\cdot(\vec{v}\times\vec{B})$$

ukupan broj elektrona

ukupna sila $\vec{F} = N\cdot f = n\cdot l\cdot S\cdot q\,(\vec{v}\times\vec{B})$

broj e po jedinici volumena

→ količina naboja koja u dt prođe presjekom:

$$dQ = N q = n\cdot \underbrace{l}_{v\cdot dt}\cdot S\cdot q = n\cdot v\cdot dt\cdot S\cdot q$$

$$\Downarrow$$

$$\boxed{I = \frac{dQ}{dt} = n\cdot v\cdot S\cdot q}$$

→ sada je sila $\vec{F} = \overbrace{n\cdot S\cdot q\cdot \vec{v}}^{I}\,(\vec{l}\times\vec{B})$ → CIKLIČKO PRAVILO

$$\boxed{\vec{F} = I\,(\vec{l}\times\vec{B})}$$ – UKUPNA SILA

→ ako nemamo homogeno mag polje ili ako vodič nije ravan

$$dF = I\,(d\vec{l}\times\vec{B}) \quad // \int$$

$$\boxed{\vec{F} = I\int_l d\vec{l}\times\vec{B}} \qquad \vec{F} = \iiint_V (\vec{J}\times\vec{B})\, dV$$

17. JEDNADŽBE STAT. MAG. POLJA U DIF. I INT. OBLIKU

I GAUSSOV ZAKON

→ ne postoji mag monopoli

za el. polje vrijedi

$$\oint \vec D \vec n \, dS = \iiint_V \rho_s \, dV = \Sigma Q$$

$$\boxed{\oint \vec B \vec n \, dS = 0} = \iiint \nabla \vec B \cdot dV = 0$$

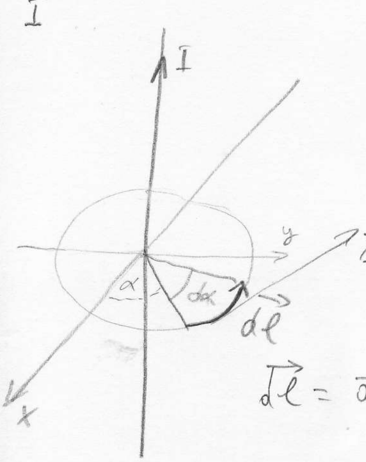
↑ GAUSSOV ZAKON $\oint \vec A \vec n \, dS = \iiint_V \nabla \vec A \, dV$

$$\boxed{\nabla \vec B = 0}$$

II AMPEROV ZAKON KRUŽNOG PROTJECANJA

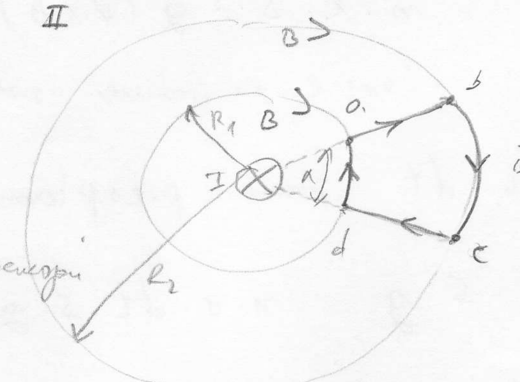
→ integral po krivulji oko strujnice sa strujom I

I



$$\vec{dl} = \vec a_\alpha \, r \, d\alpha$$

II



$$\vec{dl}_{a-b} = \vec a_r \, dr$$

$$\vec{dl}_{b-c} = \vec a_\alpha R_2 \, d\alpha$$

$$\vec{dl}_{c-d} = -\vec a_r \, dr$$

$$\vec{dl}_{d-a} = -\vec a_\alpha R_1 \, d\alpha$$

I

$$B_{STRUJNICE} = \frac{\mu_0 I}{2\pi r} a_\alpha \Rightarrow \oint_c \vec B \vec{dl} = \oint \vec a_\alpha \frac{\mu_0 I}{2\pi \cancel{r}} \vec a_\alpha \cancel{r} \, d\alpha =$$

$$\oint \vec B \vec{dl} = \frac{\mu_0 I}{2\pi} \int_0^{2\pi} d\alpha \Rightarrow \boxed{\oint_c B \, dl = \mu_0 I}$$

→ INT. mag ind po zatvorenoj krivulji jednak obuhvaćenoj struji

II

$$\oint_c \vec B \vec{dl} = \int_{r=R_1}^{R_2} \vec a_\alpha \frac{\mu_0 I}{2\pi r} \cdot \vec a_r \, dr + \int_b^c \vec a_\alpha \frac{\mu_0 I}{2\pi \cancel{R_2}} \vec a_\alpha \cancel{R_2} \, d\alpha - \int_{r=R_1}^{R_2} \vec a_\alpha \frac{\mu_0 I}{2\pi r} \vec a_r \, dr - \int_d^a \vec a_\alpha \frac{\mu_0 I}{2\pi \cancel{R_1}} \vec a_\alpha \cancel{R_1} \, d\alpha$$

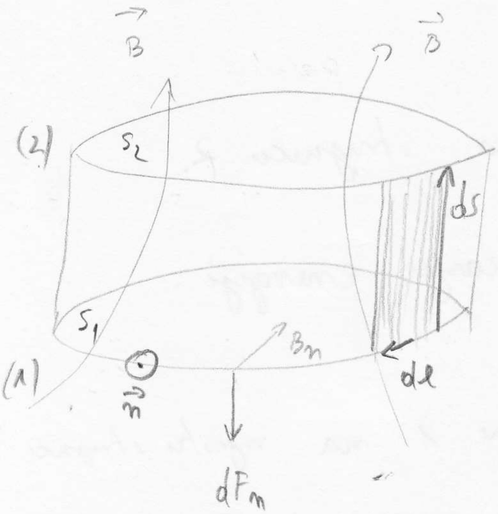
okomiti → $\cos 90 = 0$ $\quad$ $\cos 0 = 1$

$$= 0 + \int_b^c \frac{\mu_0 I}{2\pi} \alpha - 0 + \int_d^a \frac{\mu_0 I}{2\pi} d\alpha = \frac{\mu_0 I}{2\pi} \left[\underbrace{\int_b^c a_\alpha}_{\alpha} - \underbrace{\int_d^a a_\alpha}_{\alpha} \right] \to 0$$

→ AKO KRIVULJA NE OBUHVAĆA STRUJNICU $\oint \vec B \vec{dl} = 0$

OPĆENITO: $\boxed{\oint \vec B \vec{dl} = \mu_0 \iint_S \vec J_s \vec n \, dS}$

8. ENERGIJA POHRANJENA U MAG POLJU IZRAŽENA POMOĆU MAG. TOKA



na diferencijalni dio strujnice djeluje

$$d\vec{F}_m = I(d\vec{l}\times\vec{B})$$

→ moramo primijeniti vanjsku silu da bi savladali dF_m i pomak δr pretvori se u povećanje energiji strujnice

$$\delta W = d\vec{F}_v\cdot\delta\vec{r} = -dF_m\,\delta r = -I(d\vec{l}\times\vec{B})\cdot\delta r$$

CIKLIČKO PRAVILO $\vec{A}\cdot(\vec{B}\times\vec{C}) = \vec{C}\cdot(\vec{A}\times\vec{B})$

$$\delta W = -I\cdot\vec{B}\cdot(\delta\vec{r}\times d\vec{l}) = -I\cdot\vec{B}\cdot\vec{n}\,d\vec{S}$$ → šrafirano pov

→ kroz plašt valjka uloži dio toka (opisan pomakom iz 1. u 2.)

$$\delta\phi_{PLAŠT} = \vec{B}(-\vec{n})\,dS \Longrightarrow \delta W = I\cdot\delta\phi_{PLAŠT}$$

$$\boxed{dW = I\cdot d\phi_{PLAŠT}}$$

$d\phi_{PLAŠT}$ = UKUPNA PROMJENA TOKA $= \phi_{(2)} - \phi_{(1)}$

→ ako strujnicu unesemo u polje iz prostora di nema polja

$$W = I(\phi_2 - \phi_1) = I(\phi_2 - 0)$$

$$\boxed{W = I\cdot\phi}$$

mag energija strujnice u mag. polju

(9.) MAG. ENERGIJA SUSTAVA STRUJNIH PETLJI IZRAŽENA POMOĆU VEKTORSKOG MAG. POTENCIJALA

→ u magnetsko polje strujnice 1. dovodimo strujnicu 2.

→ moramo uložiti rad koji je jednak povećanju energije

$$W = \phi_{12} \cdot I_2$$

→ tok stvoren od struje 1 na mjestu strujnice 2

$$W = \phi_{21} \cdot I_1$$

→ tok struje 2 na mjestu 1

→ UKUPNA ENERGIJA $W = \frac{1}{2}\left(\phi_{12} \cdot I_2 + \phi_{21} \cdot I_1\right)$

→ AKO IMAM n VODIČA DI TEKU STRUJE GUSTOĆE $\vec{J}_i$ TOK KROZ i-ti VODIČ

$$\phi_i = \sum_{j=1}^{n} \oint \vec{A}_j \, d\vec{l}_i$$

A_i → vekt. mag potencijal stvoren tim vodičem

$$I_i = \iint_{S_i} \vec{J}_i \vec{n}_i \, dS_i$$

I_i → struja kroz i-ti vodič

$$W_{uk} = \frac{1}{2} \sum_{i=1}^{n} I_i \phi_i = \frac{1}{2} \sum_{i=1}^{n} \iint_{S_i} \vec{J}_i \vec{n} \, dS_i \left(\sum_{j=1}^{n} \oint \vec{A}_j \, d\vec{l}_i \right)$$

$n_i \, dl_i = dl_i$

$dS_i \cdot dl_i = dV_i$

$$\Downarrow$$

$$\boxed{W = \frac{1}{2} \iiint_V \vec{J} \cdot \vec{A} \cdot dV}$$

→ UKUPNI MAG POT. KOJEG STVARAJU STRUJE NA MJESTU dV

OVAJ IZRAZ SADRŽI KONTINUIRANE RASPODJELE STRUJA PA SADRŽI I VLASTITE ENERGIJE STRUJNICA

10. MAGNETSKA ENERGIJA SUSTAVA STRUJNIH PETLJI IZRAŽENA POMOĆU VEKTORA MAGNETSKOG POLJA

$$W = \frac{1}{2} \iiint_V \vec{J} \cdot \vec{A} \cdot dV$$

$$\nabla (\vec{H} \times \vec{A}) = \vec{A} (\underbrace{\nabla \times \vec{H}}_{\vec{J}}) - H (\underbrace{\nabla \times \vec{A}}_{\vec{B}}) = \vec{J}\vec{A} - \vec{B} \cdot \vec{H}$$

$$\vec{J}\vec{A} = \vec{B} \cdot \vec{H} + \nabla (\vec{H} \times \vec{A})$$

$$W = \frac{1}{2} \iiint_V \vec{J} \cdot \vec{A} \, dV = \frac{1}{2} \iiint_V \vec{B} \cdot \vec{H} \, dV + \frac{1}{2} \iiint_V \nabla (\vec{H} \times \vec{A}) \, dV$$

$$\overset{GAUS}{=} \frac{1}{2} \oiint (\vec{H} \times \vec{A}) \vec{n} \, dS$$

$$W = \frac{1}{2} \iiint_V \vec{B} \cdot \vec{H} \cdot dV + \frac{1}{2} \oiint (\vec{H} \times \vec{A}) \vec{n} \, dS$$

energija sadržana u volumenu

energija u prostoru izvan

→ ako obuhvatimo cijeli prostor S se zatvara u ∞

A pada sa $\frac{1}{r}$ i H pada sa $\frac{1}{r^2}$ i S raste sa r^2 → sve pada

$$\boxed{W = \frac{1}{2} \iiint_V \vec{B} \cdot \vec{H} \, dV}$$

→ struje koje stvaraju mag. polje ne moraju biti u volumenu V

→ ako i nema struja postoji energija

→ ako je materijal LHI vrijedi $\vec{B} = \mu \vec{H}$

$$W = \frac{1}{2} \mu \iiint_V |H|^2 dV = \frac{1}{2\mu} \iiint_V |B|^2 dV$$

11. MAGNETSKA ENERGIJA U NELINEARNIM MATERIJALIMA I GUBICI ZBOG HISTEREZE

→ imamo linearan materijal i povećavam struju od 0 do I → time raste i B od nule do B. Kod neke struje povećanji obuhvaćenog mag toka za $d\phi$ dat će prirast mag energije

$$dW = i \cdot d\phi \qquad d\phi = \iint_S d\vec{B}\,\vec{n}\,dS \qquad i = \oint_l \vec{H}\,dl \qquad dS\,dl = dV$$

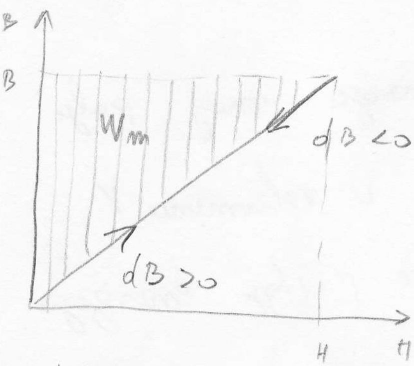
$$dW = \iiint_V \vec{H} \cdot d\vec{B} \cdot dV$$

$$\boxed{W = \iiint_V \left(\int_{B=0}^{B} \vec{H} \cdot d\vec{B} \right) dV}$$

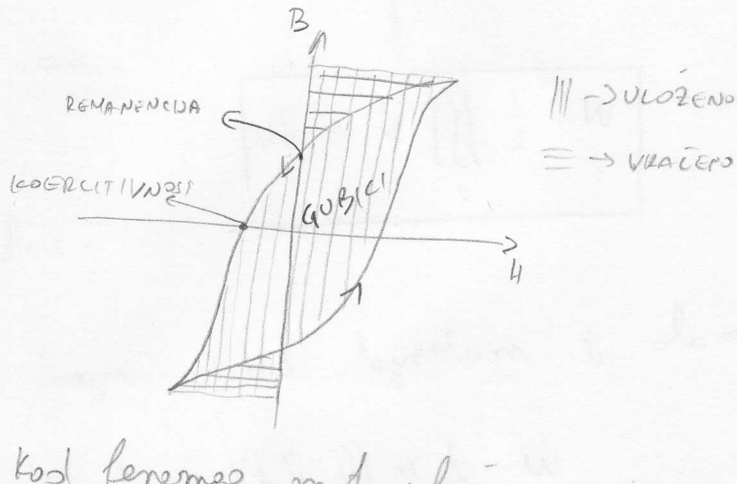
GUSTOĆA ENERGIJE MAG. POLJA

→ ZA LHI vrijedi (μ = konst) $\quad w = \int_{B=0}^{B} \vec{H} \cdot d\vec{B} = \frac{1}{\mu} \int_{B=0}^{B} B \cdot dB = \frac{1}{2\mu} B^2 \ (J/m^3)$

→ ako nije linearan moramo po dijelovima prostora računati

→ 

LINEARAN



→ pri magnetiziranju izvor daje E materijalu

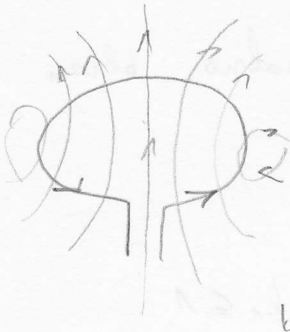
→ sva energija se u cijelosti vraća

$$w = \int_{B=0}^{B=B} \vec{H}\,d\vec{B} = -\int_{B=B}^{B=0} H\,dB$$

→ kod feromag mat uložena je E veća od one koju dobijemo razmag.

→ gubitke stvara trenje domena u feromag. materijalu

(12.) INDUKTIVITET STRUJNE PETLJE



→ struja koja teče stvara mag. polje (Biot-Savartov zakon)

→ struja teče zatvorenim krugovima po

kroz taj krug prolazi mag tok stvoren el. strujom

→ OMJER TOKA i STRUJE (KOEF. SAMOIND, SAMOIND ODNOSNO)

INDUKTIVITET $L = \frac{\phi}{I}$ [H] $\left[\frac{Vs}{A}\right]$

→ ako je petlja načinjena od N zavoja svaki zavoj obuhvaća

tok ϕ → ULANČENI TOK: $\psi = N\phi$

↗ UKUPNI TOK

$$L = \frac{\psi}{I} = \frac{N \cdot \phi}{I} = \frac{N \cdot N \cdot \phi_1}{I} = N^2 \frac{\phi_1}{I} = N^2 L_1$$

↗ ind jednog zavoja

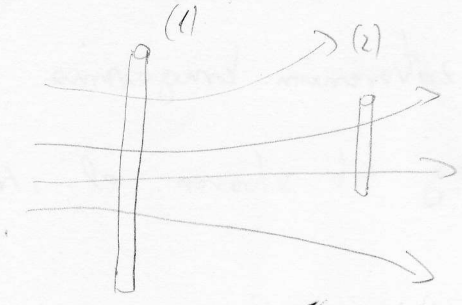
ϕ_1 - TOK STVOREN JEDNIM ZAVOJEM

-vanjski ind iz dijela toka koji prolazi izvan vodiča

→ za nelinearne (feromagnetski materijali zbog $\mu = \mu(H)$) ind. ovisi i o jakosti mag polja i struji i nelinearno je veličina

13. MEĐUINDUKTIVITET

→ ako imamo dva strujna kruga koji su međusobno blizu onda tok koji proizvede I_1 prođe i kroz petlju 2



$$\Phi_{21} = k_1 \cdot \Phi_1 \qquad k_1 \leq 1$$

od toka Φ_1 samo dio prođe kroz (2)

→ OMJER OBUHVAĆENOG TOKA I STRUJE KOJA GA JE PROIZVELA JE MEĐUIND

$$M_{21} = \frac{\Phi_{21}}{I_1}$$

→ ako (1) ima N_1 zavoja

$$\Phi_{1uk} = N_1 \cdot \Phi_1 \quad \Rightarrow \quad \Phi_{21u} = N_1 \Phi_{21} = k_1 N_1 \cdot \Phi_1 = k \cdot \Phi_{1u}$$

→ ako (2) ima N_2 zavoja ulančeni tok

$$\Psi_{21u} = N_2 \Phi_{21u} = N_2 \cdot N_1 \cdot \Phi_{21}$$

$$\Downarrow$$

$$\boxed{M_{21} = \frac{\Psi_{21u}}{I_1} = \frac{N_2 N_1 \Phi_{21}}{I_1}}$$

→ analogno vrijedi i za $M_{12} = \frac{N_1 N_2 \Phi_{12}}{I_2} = M_{21}$

14. ODNOS MEĐUINDUKTIVITETA I SAMOINDUKTIVITETA DVIJU STRUJNIH PETLJI

neki koeficijent

$$M_{21} = \frac{N_1 N_2 \phi_{12}}{I_2} = \frac{N_1 N_2 \cdot k_2 \phi_2}{I_2} = N_1 k_2 \frac{N_2 \phi_2}{I_2} = k_2 N_1 \frac{L_2}{N_2}$$

$$M_{21} = \frac{N_2 N_1 \phi_{21}}{I_1} = \frac{N_2 N_1 k_1 \phi_1}{I_1} = N_2 k_1 \frac{N_1 \phi_1}{I_1} = N_2 k_1 \frac{L_1}{N_1}$$

$$M_{12} \cdot M_{21} = M^2 = k_1 k_2 L_1 L_2 = k_2 L_1 L_2$$

$$\boxed{L_1 = \frac{N_1 \cdot N_1 \cdot \phi_1}{I_1} \quad L_2 = \frac{N_2 \cdot N_2 \phi_2}{I_2}}$$

$$\frac{N_1 \phi_1}{I_1} = \frac{L_1}{N_1}$$

$$\Downarrow$$

$$\boxed{M = k \cdot \sqrt{L_1 L_2}}$$

- koef. mag. sprege
- smanjuje se udaljavanjem petlji

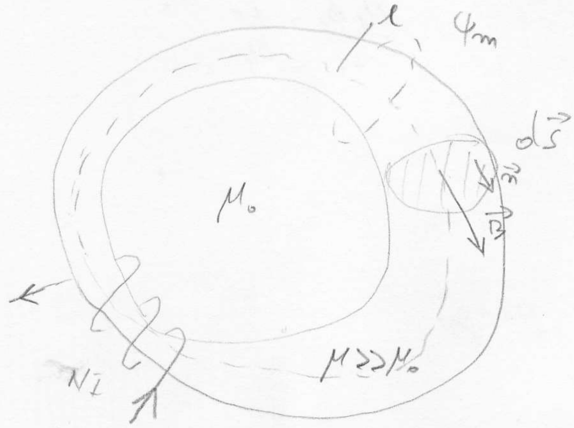
15. MAGNETSKI KRUG

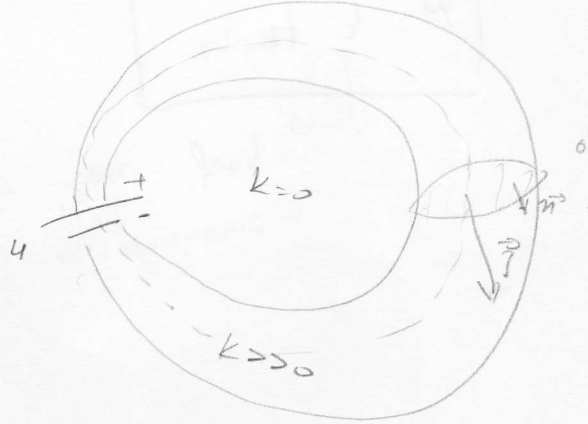
→ feromag. materijali zgušnjavaju silnice

→ prostor di je gustoća silnica veća nego nego u okolnoj izolaciji → MAGNETSKI KRUG

→ za vođenje mag toka proizvedenog zavojnicom kroz feromag. materijal potrebno je magnetizirati

16. ANALOGIJA MAG. KRUGA i ISTOSMJERNE STRUJE





→ NEHOMOGENO MAG POLJE ZBOG RAZL. PRESJEKA

→ NEHOMOGENO POLJE

→ TOK ISTI SVUGDJE

$$\phi = \iint_S \vec{B}\vec{n}\,dS$$

→ STRUJA ISTA

$$I = \iint_S \vec{J}\vec{n}\,ds$$

→ POMOĆU AMPEROVOG ZAKONA

$$\oint_c \vec{H}\cdot d\vec{l} = NI = \Theta$$

↓ MAGNETOMOTORNA SILA

$$\oint_c \vec{E}\,d\vec{l} = U$$

→ OMJER MAG POBUDE i TOKA

$$R_m = \frac{\Theta}{\phi} = \frac{\oint_c \vec{H}\,d\vec{l}}{\iint_S \vec{B}\vec{n}\,dS} \quad \boxed{\vec{B} = \mu\cdot\vec{H}}$$

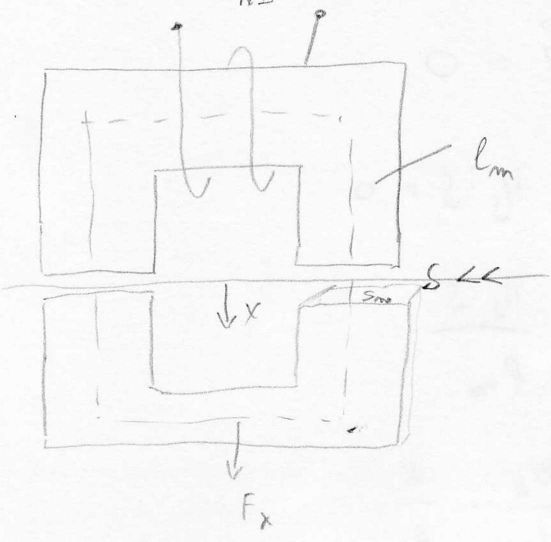
$$R = \frac{U}{I} = \frac{\oint_c \vec{E}\,d\vec{l}}{\iint \vec{J}\vec{n}\,dS} \quad \boxed{\vec{J} = \kappa\cdot\vec{E}}$$

→ MAG. VODLJIVOST $= \frac{1}{R_M}$

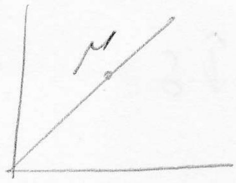
→ EL. VODLJIVOST $G = \frac{1}{R}$

$\frac{H_{SR}\cdot l_{SR}}{\ldots} \quad \frac{1}{\ldots}\frac{l_{SR}}{\ldots}$

17. MAGNETSKI KRUG ELEKTROMAGNETA



→ pretpostavimo da je μ linearan



→ TOK ISTI

$$H_1 l_1 + H_2 l_2 = NI$$

$$R_m = \frac{H_1 l_1}{B_1 S_1}$$

$$\phi = \frac{NI}{\frac{1}{\mu}\frac{l_m - 2\delta}{S_m} + 2\cdot\frac{1}{\mu_0}\frac{\delta}{S_m}}$$

OTPOR MAGNETA

ZRAČNI RASPORI

$$W = \frac{1}{2} N I \phi = \frac{1}{2} L I^2$$

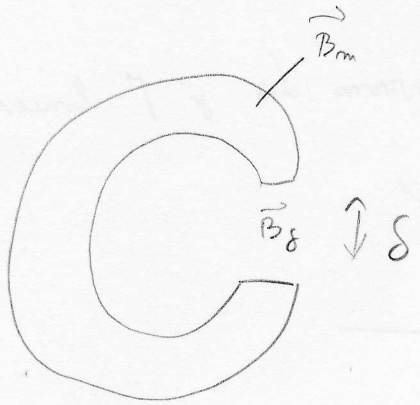
ODKUDA OVO ?

$$L = \frac{N^2}{\frac{1}{\mu}\frac{l_m}{S_m} + \frac{2}{\mu_0}\frac{\delta}{S_m}}$$

→ neizoliran sustav → I je konstantan ?

$$F_{smjer\ \delta} = \vec{a}_\delta \frac{\partial W}{\partial \delta} = \frac{1}{2} N^2 I^2 \vec{a}_\delta \frac{\partial}{\partial \delta}\left(\frac{1}{\frac{1}{\mu}\frac{l_m}{S_m} + 2\frac{1}{\mu_0}\frac{\delta}{S_m}}\right)$$

(18.) MAGNETSKI KRUG



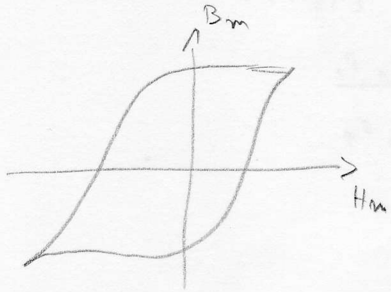
$$\oint \vec{H}\, d\vec{l} = 0$$

$$H_m l_m + H_\delta \cdot \delta = 0$$

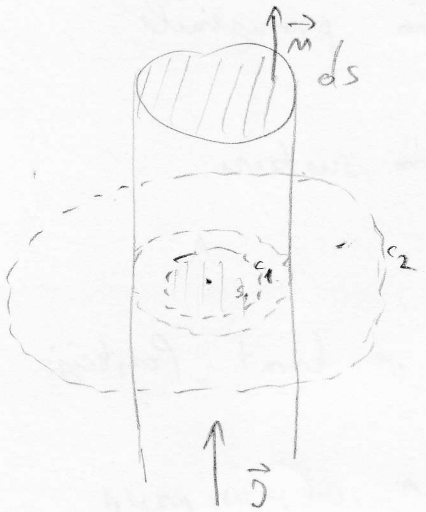
$$H_m = -\frac{H_\delta \cdot \delta}{l_m}$$

$$H_m = \frac{-B_\delta}{\mu_0} \cdot \frac{\delta}{l_m}$$

permanentni magneti imaju široku petlju histereze



(19.) AMPEROV KRUŽNI ZAKON I POLJE BESKONAČNO DUGOG RAVNOG VODIČA POLUMJERA R PROTJECANOG STRUJOM I JEDNOLIKO RASPOREĐENOM PO PRESJEKU VODIČA



I AMPEROV ZAKON ZA c_1

$$\oint_{c_1} \vec{B}\, d\vec{l} = \mu_0 \iint_{s_1} \vec{J}\, \vec{n}\, ds$$

→ unutar vrijedi za prostornu gustoću struje po B-S

$$\vec{B} = \frac{\mu_0}{4\pi} \iiint_V \frac{\vec{J} \times \vec{R}}{R^3}\, dV$$

II kada uzmemo c_2 kao da umotamo strujnicu jer smo obuhvatili punu struju I

$$\vec{B} = \frac{\mu_0 I}{2\pi r} \vec{a_\alpha} \qquad \oint \vec{B}\, d\vec{l} = \mu_0 I_{uk}$$

$$\oint \underbrace{\vec{B}}_{\frac{\mu_0 I}{2\pi r} a_\alpha} \overbrace{d\vec{l}}^{a_\alpha\, r\, d\alpha} = \frac{\mu_0 I}{2\pi r} \cdot r \int_0^{2\pi} d\alpha = \mu_0 I$$

20. VEKTORSKI MAGNETSKI POTENCIJAL, DIFERENCIJALNA JEDNADŽBA I PRORAČUN TOKOVA U MAGNETSKOM POLJU

GAUSSOV ZAKON $\nabla \vec{B} = 0$ i uz $\nabla(\nabla \times \vec{F}) = 0$ skužimo

$B = \nabla \times \vec{A}$ $\Rightarrow$ $\nabla(\nabla \times \vec{A}) = 0$ → ništa nismo promijenili

$$B = \text{rot}\,\vec{A} = \begin{vmatrix} a_x & a_y & a_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ A_x & A_y & A_z \end{vmatrix}$$ → u pravokutnom sustavu

→ kako je $\vec{B}$ derivacija $\vec{A}$, vekt mag. pot je kont. funkcija.

→ $\nabla \times \vec{A} = B$ → DA BI VEKT FUNK BILA DEFINIRANA MORAMO ZNATI I DIVERGENCIJU I ROTACIJU

$\nabla \cdot \vec{A} = 0$ → COULOMBOVO BAŽDARENJE

→ IZRAČUN MAG TOKA

$$\phi = \iint_S \vec{B}\vec{n}\,ds = \iint_S (\nabla \times \vec{A})\vec{n}\,ds = \oint_C \vec{A}\,dl$$

STOKES $\oiint (\nabla \times A)\, n\, ds = \oint_C F\, dl$

GAUS $\oiint \vec{F} \cdot \vec{n}\, ds = \iiint \nabla \vec{F}\, dV$

$$\boxed{\phi = \iint_S \vec{B}\vec{n}\,ds = \oint_C \vec{A}\,dl}$$

→ AMPEROV ZAKON U DIFERENC. OBLIKU $\nabla \times \vec{H} = \vec{J}$ $\quad \vec{H} = \frac{\vec{B}}{\mu}$ $\quad \vec{B} = \nabla \times \vec{A}$

$$\nabla \times \left(\frac{1}{\mu} \nabla \times \vec{A}\right) = \vec{J}_s$$

$$\nabla \times \nabla \times \vec{A} = \mu \vec{J}_s \Rightarrow \cancel{\nabla(\nabla \cdot \vec{A})}^{0} - \Delta \vec{A} = \mu \cdot \vec{J}_s$$

$$\boxed{\Delta \vec{A} = \mu \cdot \vec{J}}$$ POISSONOVA

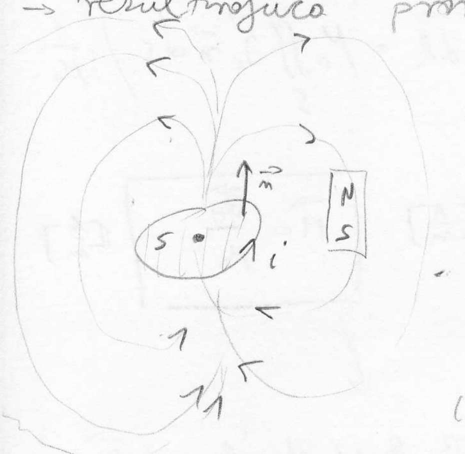
→ ako nema prostornih struja $\boxed{\Delta \vec{A} = 0}$ LAPLASOVA

21: MAGNETIZACIJA i AMPERSKE STRUJE

→ neki materijali unijeti u polje djeluju na polje

→ odziv određen MAGNETSKIM DIPOLIMA koje stvara gibanje vezanih e^-

→ rezultirajuća promjena je MAGNETIZIRANJE



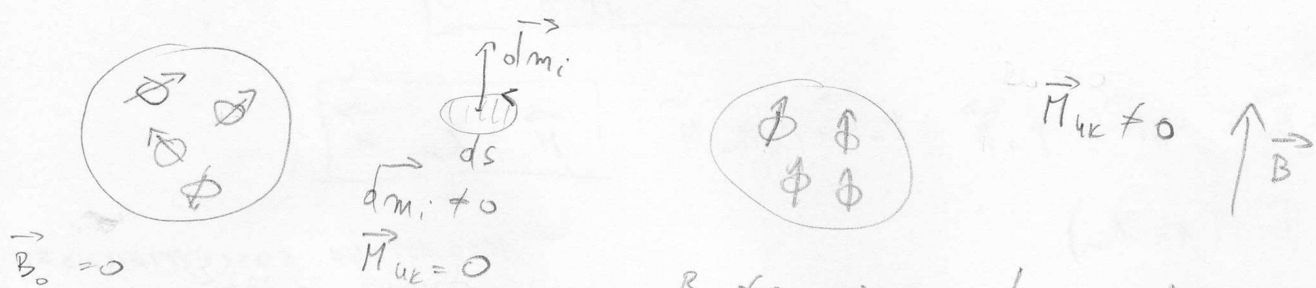
MAGNETSKI MOMENT

$\vec{m} = \vec{n} \cdot i \cdot S$

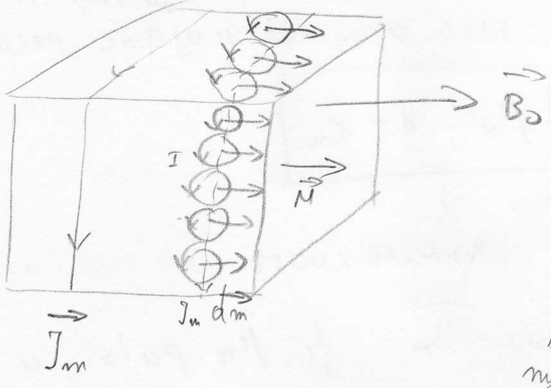
$\vec{n}$ → normala određena pravilom desne ruke

i → struja nastala gibanjem naboja

– KADA NEMAMO POLJE → KADA NAMETNEMO POLJE



$B_0 \neq 0$ → nametnuto vanjsko polje



→ MATERIJAL U VANJSKOM $\vec{B}_0$ koje je uzrokovano plošnom strujom gustoće $\vec{J}_s$ namotanom oko materijala

nije $\vec{J}_m$!

→ domene se orijentiraju u istom smjeru (stvaraju mag. momente)

→ u unutrašnjosti se petlje susjednih petlji poništavaju

→ jedino na površini ostaje struja magnetizacije gustoće J_M (AMPERSKE STRUJE)

→ ukupno polje je određeno vanjskim slobodnim strujama $\vec{J}_s$ koje stvaraju $\vec{B}_0$ i amperskim strujama J_s koje stvaraju $\vec{B}_M$

(22.) JAKOST MAG. POLJA i PONAŠANJE MATERIJALA U MAG. POLJU

UKUPNA MAG. IND: $\vec{B} = \vec{B_0} + \vec{B}_{Magnetizacije} \Rightarrow \vec{B_0} = \vec{B} - \vec{B_M}$

→ AMPEROV KRUŽNI NA VANJSKU ZAVOJNICU: $\oint_C \vec{B_0}\, d\vec{l} = \mu_0 \iint \vec{J_s}\,\vec{n}\, ds \Rightarrow \oint (\vec{B} - \vec{B_M})\, d\vec{l} = \mu_0 \iint_S \vec{J_s}\,\vec{n}\, ds \Big/ : \frac{1}{\mu_0}$

$$\oint \left(\frac{\vec{B}}{\mu_0} - \frac{\vec{B_M}}{\mu_0}\right) d\vec{l} = \iint_S \vec{J}\,\vec{n}\, ds$$

$$\boxed{\vec{H} = \frac{\vec{B_0}}{\mu_0}} \left[\frac{A}{m}\right] \qquad \boxed{\vec{M} = \frac{\vec{B_M}}{\mu_0}} \left[\frac{A}{m}\right]$$

$$= \frac{\vec{B}}{\mu_0} - \vec{M}$$

OMJER IND B_0 (stvorene od $\vec{J}_{SLOBODNO}$) i μ_0

OMJER B_M (stvorile se $\vec{J}_{MAGNET}$)

AMPEROV ZAKON U MATERIJALU $\boxed{\oint \vec{H}\, d\vec{l} = \iint \vec{J_s}\,\vec{n}\, ds}$

$$\vec{B_{UK}} = \vec{B_0} + \vec{B_M} = \mu_0 \vec{H} + \mu_0 \vec{M} = \mu_0 \vec{H} + \mu_0 \chi_m \vec{H}$$

$$\vec{B_{UK}} = \mu_0 \vec{H} \cdot (1 + \chi_m)$$

$$\boxed{\vec{M} = \chi_m \cdot \vec{H}}$$

↓

MAGNETSKA SUSCEPTIBILNOST

KVANTIFICIRA UNUTARNJU MAG MATERIJALA ZBOG UTJECAJA VANJSKOG POLJA

$$\boxed{\vec{B} = \mu_0 \mu_r \cdot \vec{H}}$$

$$\boxed{\mu_r = 1 + \chi_m}$$

↓

PERMEABILNOST

→ vanjsko $\vec{B_0}$ se u materijalu mijenja uslijed magnetizacije za iznos $\vec{B_M}$, tj μ_r puta u odnosu na mag ind u vakuumu

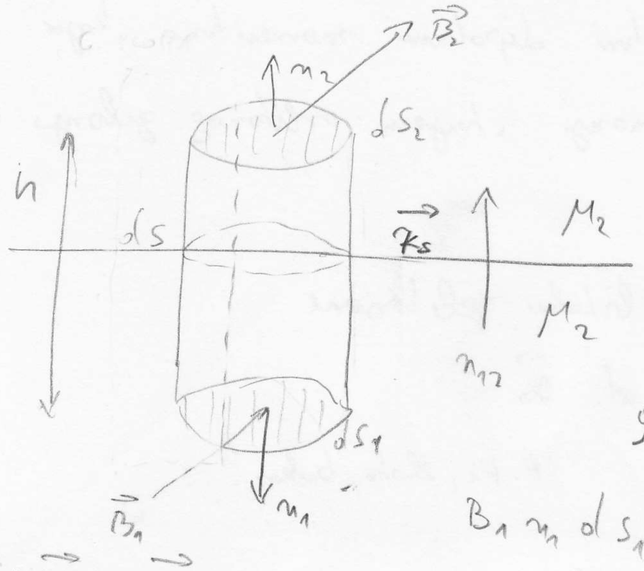
→ u materijalu vrijedi:

B–S: $d\vec{H} = \frac{I}{4\pi} \frac{d\vec{l} \times R}{R^3}$

AMP: $\oint \vec{H}\, d\vec{l} = \iint \vec{J_s}\,\vec{n}\, ds$

GAUS: $\nabla \times \vec{H} = \vec{J_s}$

23. UVJETI ZA VEKTORE MAGNETSKOG POLJA NA GRANICI DVA MATERIJALA



→ na granici se nalazi strujni oblog (struja plošne gustoće $\vec{K}_s$)

I GAUS NA ZATVORENI CILINDAR

$$\oint \vec{B}\vec{n}\, dS = 0$$ ← nema mag monopola

$$B_1 n_1\, dS_1 + B_2 n_2\, dS_2 + (\text{doprinos plašta}) = 0 \Big/ \lim_{h \to 0}$$

$$-\vec{n}_1 = \vec{n}_2 = \vec{n}_{12}$$

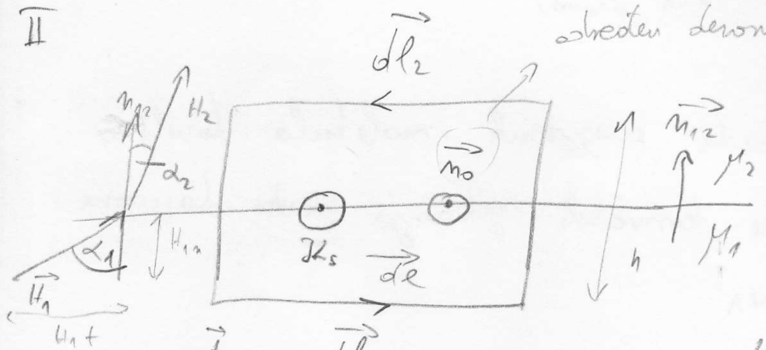
$$-B_1 \vec{n}_{12}\, dS + \vec{B}_2 \vec{n}_{12}\, dS = 0 \;/ : dS$$

$$\boxed{\vec{n}_{12}\,(\vec{B}_2 - \vec{B}_1) = 0} \qquad \boxed{B_{1n} = B_{2n}} \qquad \mu_1 H_{1n} = \mu_2 H_{2n}$$

II

obuhvaćen strujom rubom (obloženi petlje)

→ primjenjujemo Amperov zakon za malu petlju



$$\oint \vec{H}\, \vec{dl} = \iint \vec{J}\vec{n}\, dS$$

$$\vec{dl} = -(\vec{n}_0 \times \vec{n}_{12})\, dl \qquad H_1\, \vec{dl}_1 + \vec{H}_2\, \vec{dl}_2 + (\text{doprinos na stranicama } h) = \vec{J}_s \cdot \vec{n}_0\, \underbrace{h \cdot dl}_{dS} \Big/ \lim_{h \to 0}$$

$$\lim_{h \to 0} \left\{ \vec{H}_1\, \vec{dl} - \vec{H}_2 \cdot \vec{dl} + (\text{doprinos na stranicama } h) \right\} = \lim_{h \to 0} \{ J_s\, n_0\, h \cdot dl \}$$

$$\vec{dl} = dl \cdot \vec{t}$$

$$(\vec{H}_1 - \vec{H}_2)\, \vec{t}_{tang} = n_0 \cdot \underbrace{\lim \{ J_s \cdot h \}}_{\vec{K}_s}$$

$$(\vec{H}_2 - \vec{H}_1)\, \overset{\to \text{ciklički}}{(\vec{n}_0 \times \vec{n}_{12})} = \vec{n}_0 \vec{K}_s$$

$$\cancel{n_0} \cdot \{ \vec{n}_{12} \times (\vec{H}_2 - \vec{H}_1) \} = \cancel{n_0}\, \vec{K}_s$$

$$\boxed{\vec{n}_{12} \times (\vec{H}_2 - \vec{H}_1) = \vec{K}_s}$$

$$\operatorname{tg} \alpha_1 = \frac{H_{1t}}{H_{1n}} \qquad \operatorname{tg} \alpha_2 = \frac{H_{2t}}{H_{2n}}$$

$$\frac{\operatorname{tg} \alpha_1}{\operatorname{tg} \alpha_2} = \frac{H_{1t} \cdot H_{2n}}{H_{2t} \cdot H_{1n}} = \frac{\mu_1}{\mu_2}$$

PODJELA MATERIJALA:

→ para i fero pretežno uzrokovani permanentni dipolni momentima koji su rezultat spinova elektrona (puno manji strujom orbitalnog gibanja e^-

$\boxed{\text{1. DIJAMAGNETIZAM}}$

→ posljedica djelovanja mag polja na orbitalu elektrona

→ kao da unamo $\vec{M}$ suprotnog smjera od $\vec{B_0}$

→ μ_r nešto manji od 1 $\mu_r = 0.9999xx$ (H, He, zlato, bakar...

$\boxed{\text{2. PARAMAGNETIZAM}}$

→ postoji permanentni mag dipolni spinovi i bez prisutstva $\vec{B_0}$ i nasumice su orijentirani

→ $\vec{B_0}$ ih usmjerava ($\vec{M}$ u smjeru vanjskog polja

→ μ_r nešto veći od 1 $\mu_r = 1.00xx$ (Al, Fe Cl, olovo)

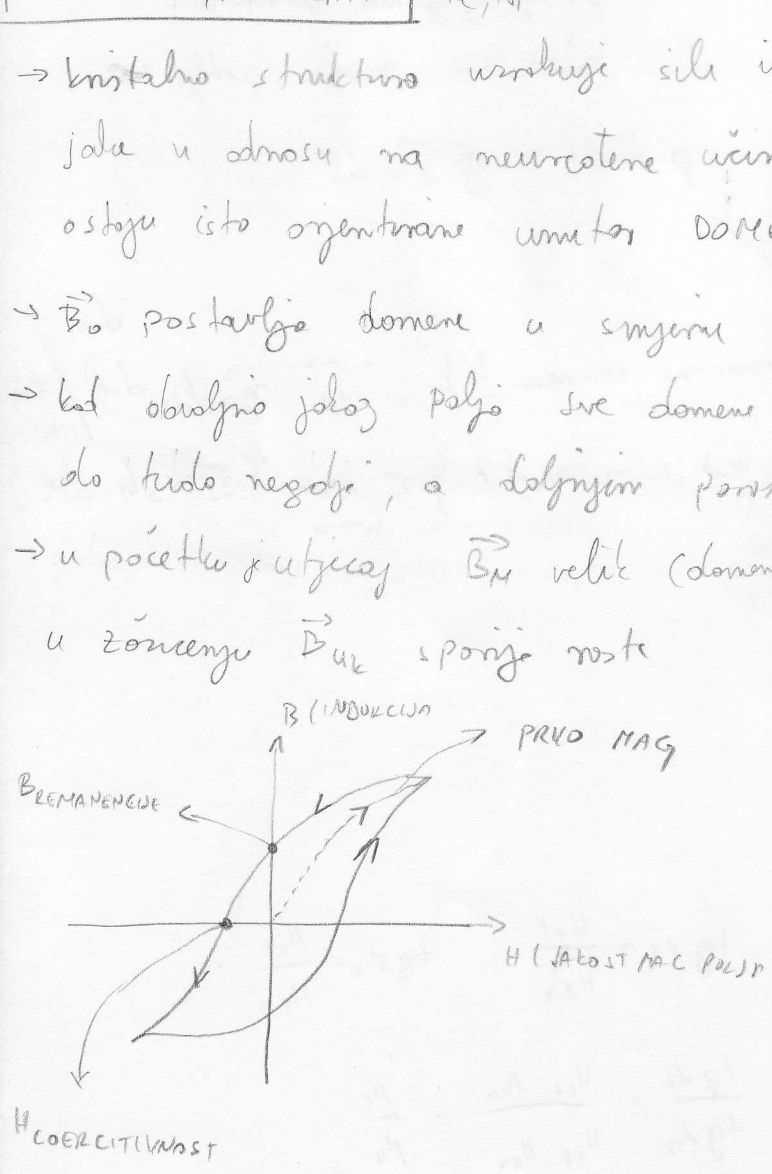
$\boxed{\text{3. FEROMAGNETIZAM}}$ Fe, Ni

→ kristalna struktura uzrokuje sile između susjednih molekula dovoljno jake u odnosu na neuređene učinke termičkih gibanja tako da one ostaju isto orijentirane unutar DOMENA

→ $\vec{B_0}$ postavlja domene u smjeru polja

→ kod dovoljno jakog polja sve domene su usmjerene → ZASIĆENJE (M/H) raste do točke nagiba, a daljnjim porastom $\vec{H}$ pada prema μ_0

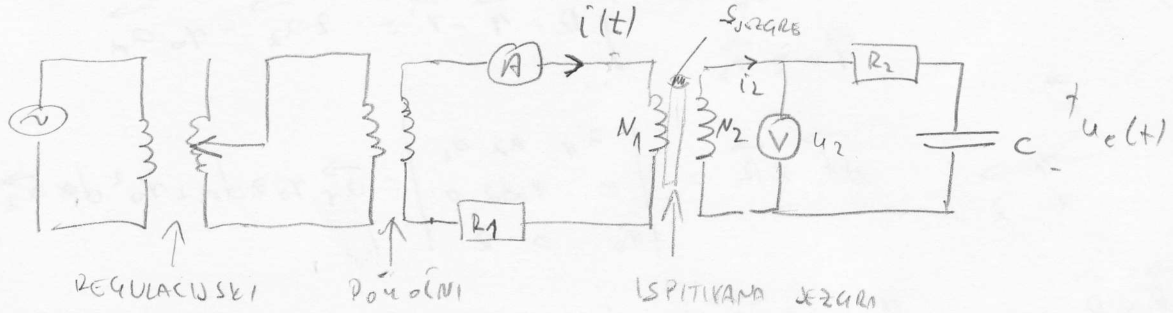
→ u početku utjecaj $\vec{B_M}$ velik (domene se usmjeruju), a kasnije pada i u zasićenju $\vec{B_{uk}}$ sporije raste



(24) INDIREKTNO MJERENJE MAG. POLJA U FEROMAGNETSKOJ TORUSNOJ JEZGRI U POKUSU SNIMANJA DINAMIČKE PETLJE HISTEREZE

(25.) MJERENJE IND

→ mjerenje din. petlje histereze pomoću izmjeničnih struja



↳ PODEŠAVAMO NAPON NA PRIMARU

- $i(t)$ se mjeri na otporu R_1
- RC član služi kao integrirajući član u svrhu dobivanja mag ind na temelju mjerenja ind napona na sekundaru (u_2)
- FARADAYEV ZAKON $\quad e_{IND}(t) = -N_2 \dfrac{d\phi}{dt} = -N_2 \dfrac{d\iint B_n \, dS}{dt} = -N_2 \cdot S \cdot \dfrac{dB(t)}{dt}$ (konst)

$u_2 = -e(t) \qquad \underline{u_2 = N_2 S \dfrac{dB}{dt}}$, te ako je $R_2 << \dfrac{1}{\omega C}$

$$i_2(t) = \frac{u_2(t)}{R_2} = \frac{N_2 S}{R_2}\frac{dB}{dt}$$

$B(t) = \dfrac{1}{N_2 S}\int u(t)\,dt$

$$\frac{dB}{dt} = \frac{R_2}{N_2 S} i_2(t) \Big/ \int \implies \boxed{B(t) = \frac{R_2}{N_2 S}\int i_2(t)\,dt}$$

→ napon na C $\quad u_c = \dfrac{1}{C}\int i_2(t)\,dt$

$$\boxed{B(t) = \frac{R_2 C}{N_2 S} u_c(t)}$$

→ vrijedi Amper

$\oint H\,dl = \Sigma I$

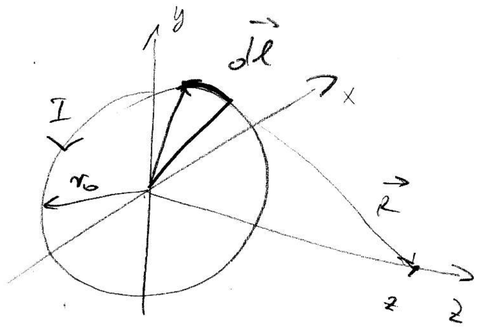
$H \cdot l_{SR} = N_1 \cdot I$

$H(t) = \dfrac{N_1 \cdot i_1(t)}{l_{SR}} \qquad l_{SR} = 2\pi \dfrac{R_1 + R_2}{2}$

$$\implies \boxed{H(t) = \frac{N_1 i_1(t)}{l_{SR}} = \frac{N_1}{l_{SR}} \cdot \frac{u_{R1}(t)}{R_1}}$$

26. TOČNI PRORAČUN MAG. IND NA OSI JEDNOSLOJNE ZAVOJNICE

KRUŽNA STRUJNICA



$$\vec{dl} = \vec{a_\alpha}\, r_0\, d\alpha$$

$$\left.\begin{array}{l} \vec{r}' = \vec{a_r}\, r_0 \\ \vec{r} = z\,\vec{a_z} \end{array}\right\} \vec{R} = \vec{r} - \vec{r}' = z\vec{a_z} - r_0\vec{a_r}$$

$$\vec{dl} \times \vec{R} = \begin{vmatrix} a_r & a_\alpha & a_z \\ 0 & r_0 d\alpha & 0 \\ -r_0 & 0 & z \end{vmatrix} = \vec{a_r}\, r_0 z\, d\alpha + r_0^2\, d\alpha\, \vec{a_z}$$

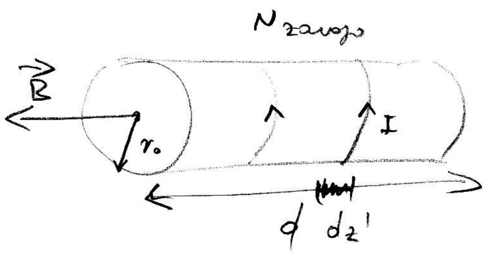
$$\vec{B} = \mu_0 \frac{I}{4\pi} \int \frac{\vec{dl} \times \vec{R}}{|\vec{R}|^3} = \mu_0 \frac{I}{4\pi} \int \left[\frac{\vec{a_r}\, r_0 z\, d\alpha}{(r_0^2+z^2)^{1.5}} + \frac{r_0^2\, d\alpha\, \vec{a_z}}{(r_0^2+z^2)^{1.5}} \right]$$

$$= \mu_0 \frac{I}{4\pi} \left[\int \underbrace{\overset{\text{KONST}}{\frac{\vec{a_r}\, z\, r_0}{(r_0^2+z^2)^{1.5}}}}_{\text{RADIJALNA}} d\alpha + \int \frac{r_0^2\, \vec{a_z}}{(r_0^2+z^2)^{1.5}}\, d\alpha \right]$$

$$\int_{\alpha=0}^{2\pi} \vec{a_r}\, d\alpha = \int_0^{2\pi} (\vec{a_x}\cos\alpha + \vec{a_y}\sin\alpha)\, d\alpha = 0$$

$$= \frac{\mu_0 I}{4\pi} \cdot \frac{r_0^2\, \vec{a_z}}{\left(\sqrt{r_0^2+z^2}\right)^3} = \boxed{\frac{\mu_0 I}{2} \frac{r_0^2}{(r_0^2+z^2)^{1.5}}\, \vec{a_z} = B_{AKSIJALNO}}$$

JEDNOSLOJNA ZAVOJNICA



→ elementarni dio ove zavojnice dz' možemo nadomjestiti kružnom petljom kojom teče struja $dI = \frac{NI}{d} dz'$

$$\vec{r}' = z'\,\vec{a_z}$$
$$\vec{r} = r_0\vec{a_r} + z\vec{a_z}$$

→ UKUPNA IND DOBIJE SE INTEGRACIJOM PO ZAVOJNICI

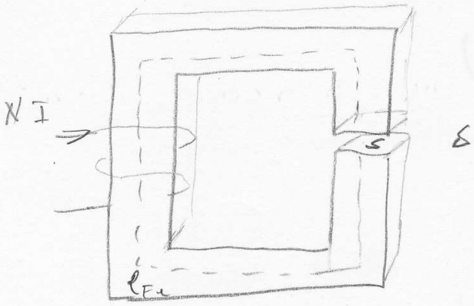
$$\vec{B_z} = \mu_0 \frac{NI\, r_0^2\, \vec{a_z}}{2d} \int_{z=0}^{d} \frac{dz'}{[r_0^2 + (z-z')^2]^{\frac{3}{2}}} = \mu_0 \frac{NI\,\vec{a_z}}{2d} \left(\frac{d-z}{\sqrt{r_0^2+(z-d)^2}} + \frac{z}{\sqrt{r_0^2+z^2}} \right)$$

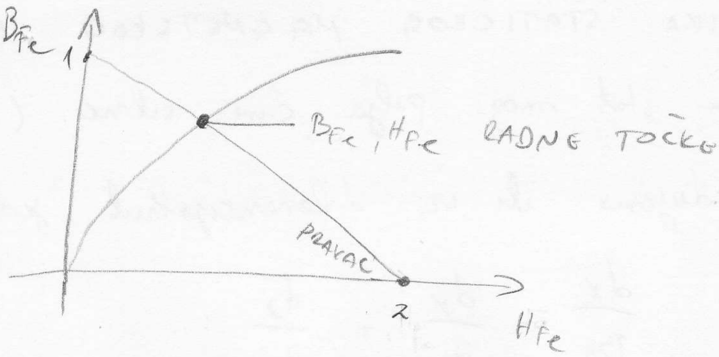
→ ako $L \to \infty$ $\boxed{\vec{B} = \vec{a_z} \frac{\mu_0 NI}{d}}$

→ isti izraz se dobije primjenom Amp. ako se za točke van zavojnice uzme da je ind 0.

$$\oint \vec{B} \cdot \vec{dl} = \mu_0 NI \implies B \cdot d = \mu_0 I N \implies B = \frac{\mu_0 NI}{d}$$

(27.)





$$NI = H_{Fe} \cdot l_{Fe} + H_\delta \delta \qquad \phi = B_{Fe} \cdot S = B_\delta \cdot S \Longrightarrow B_{Fe} = B_\delta$$

$$\mu_0 \mu_r H_{Fe} = \mu_0 H_\delta$$

$$H_{Fe} = \frac{B_{Fe}}{\mu_0 \mu_r}$$

$$NI = H_{Fe} l_{Fe} + \frac{B_{Fe}}{\mu_0} \cdot \delta \rightarrow \text{PRAVAC}$$

1. za $H_{Fe} = 0$ 2. za $B_{Fe} = 0$

$$NI = \frac{B_{Fe}}{\mu_0} \delta \qquad NI = H_{Fe} l_{Fe}$$

→ ako je materijal linearan (μ konstantan) vrijedi

$$NI = H_{Fe} l_{Fe} + \frac{B_{Fe}}{\mu_0} \delta = H_{Fe} l_{Fe} + \frac{\mu_0 \mu_r H_{Fe}}{\mu_0} \delta$$

$$NI = H_{Fe} l_{Fe} + \mu_r H_{Fe} \delta \Longrightarrow \boxed{H_{Fe} = \frac{NI}{l_{Fe} + \mu_r \delta}}$$

28. SLIKA STATIČKOG MAGNETSKOG POLJA – LINIJE POLJA

- slika stat mag. polja čine silnice (B- LINIJE)

→ određujemo ih iz diferencijalnih jednadžbi

$$\frac{dx}{B_x} = \frac{dy}{B_y} = \frac{dz}{B_z}$$

→ u 2D možemo ih odrediti izravno iz vekt mag. potencijala:

$$\vec{J} = a_z \vec{J}(x,y) \Longrightarrow \vec{A} = \vec{a_z} A(x,y)$$

$$A = \frac{\mu}{4\pi} \iiint_V \frac{J \, dv}{|\vec{R}|}$$

$$\vec{B} = \nabla \times \vec{A} = \vec{a_x} \underbrace{\frac{\partial A_z(x,y)}{\partial y}}_{B_x} - a_y \underbrace{\frac{\partial A_z(x,y)}{\partial x}}_{B_y} = a_x B_x + a_y B_y$$

iz $\frac{dx}{B_x} = \frac{dy}{B_y} \Longrightarrow dx\,B_y = B_x\,dy$

$$B_x \cdot dy - B_y \cdot dx = 0 \Longrightarrow \boxed{\frac{\partial A_z(x,y)}{\partial y} dy + \frac{\partial A_z(x,y)}{\partial x} dx = 0}$$

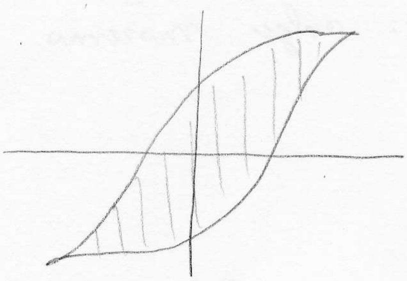
$$+$$

$$dA(x,y) = 0 \Longrightarrow A(x,y) = konst$$

$$\Downarrow$$

u 2D zaključamo linije konstantnog vektorskog potencijala jednake su silnicama

29. SNAGA GUBITAKA I SPECIFIČNI GUBICI U POKUSU SNIMANJA DINAMIČKE PETLJE HISTEREZE.



→ VOLUMNA GUSTOĆA GUBITAKA U JEZGRI U JEDNOM CIKLUSU MAGNETIZIRANJA JE POVRŠINA

$$\boxed{w = \oint_B H dB}$$

→ UKUPNA SNAGA GUBITAKA:

$$W = \iiint_V \left(\int_{B=0}^{B} \vec{H} \cdot d\vec{B} \right) dV$$

gustoća gubitaka

$$P = V \cdot w \cdot f = l_{sr} \cdot S_{r} \cdot w \cdot f$$

volumen jezgre

frekvencija izmjenične struje magnetiziranja

→ SPECIFIČNI GUBITCI

$$\boxed{p = \frac{P}{m} = \frac{w \cdot f}{\rho}}$$

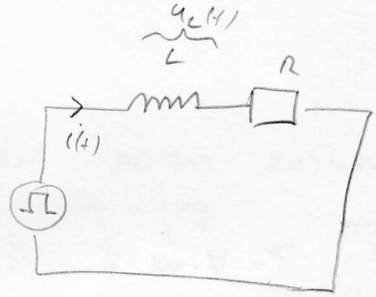
gustoća jezgre (kg/m³)

30. MJERENJE ENERGIJE LINEARNE ZAVOJNICE POMOĆU SPOJA ZAVOJNICE I OTPORNIKA

- u linearnom sredstvu energiju sadržanu u mag. polju možemo izraziti kao:

$$W = \frac{\mu}{2} \iiint_V |H|^2 dV = \frac{1}{2\mu} \iiint_V |B|^2 dV$$

- ukoliko to želimo izvoriti spojem R i L:



$$\tau = \frac{L}{R}$$

$$i(t) = \frac{U}{R}\left(1 - e^{-t/\tau}\right) \qquad u_L(t) = U \cdot e^{-\frac{t}{\tau}}$$

→ TRENUTNA SNAGA NA ZAVOJNICI JE

$$P_L(t) = u_L(t) \cdot i(t) = \frac{U^2}{R}\left(e^{-\frac{t}{\tau}} - e^{-\frac{2t}{\tau}}\right)$$

→ UKUPNU W DOBIJEMO INTEGRACIJOM SNAGE

$$W = \int_0^\infty P_L(t)\, dt = \frac{U^2}{R}\left[-\tau e^{-\frac{t}{\tau}} + \frac{\tau}{2} e^{-\frac{2t}{\tau}}\right]\Big|_0^\infty$$

⋮

$$W = \frac{L \cdot I^2}{2}$$