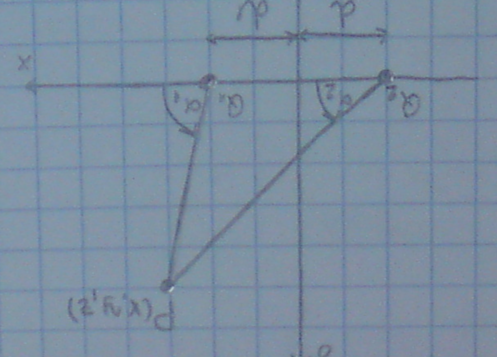
① Odredite električno polje i potencijal dva točkasta naboja



• odrediti radijvektor promatrane točke ($\vec{r}$) $\quad \vec{r} = \vec{a}_x x + \vec{a}_y y + \vec{a}_z z$

• imamo dva radijvektora izvora ($\vec{r}\,'$)

$\vec{r}\,'_1 = d\vec{a}_x \qquad \vec{R}_1 = \vec{r} - \vec{r}\,'_1 = x\vec{a}_x + y\vec{a}_y + z\vec{a}_z - d\vec{a}_x = (x-d)\vec{a}_x + y\vec{a}_y + z\vec{a}_z$

$\vec{r}\,'_2 = -d\vec{a}_x \qquad \vec{R}_2 = \vec{r} - \vec{r}\,'_2 = x\vec{a}_x + y\vec{a}_y + z\vec{a}_z - (-d\vec{a}_x) = (x+d)\vec{a}_x + y\vec{a}_y + z\vec{a}_z$

• odrediti električno polje $\vec{E} = \vec{E}_1 + \vec{E}_2$

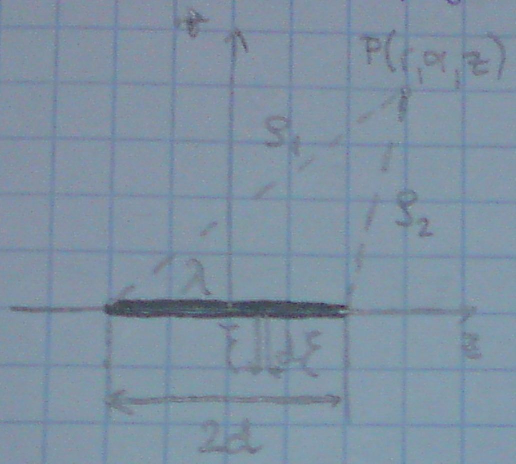
$$\vec{E} = \frac{1}{4\pi\varepsilon_0}\left[Q_1\frac{\vec{R}_1}{|\vec{R}_1|^3} + Q_2\frac{\vec{R}_2}{|\vec{R}_2|^3}\right] = \frac{1}{4\pi\varepsilon_0}\left[Q_1\frac{(x-d)\vec{a}_x + y\vec{a}_y + z\vec{a}_z}{\left((x-d)\vec{a}_x + y\vec{a}_y + z\vec{a}_z\right)^{3/2}} + Q_2\frac{(x+d)\vec{a}_x + y\vec{a}_y + z\vec{a}_z}{\left((x+d)\vec{a}_x - y\vec{a}_y + z\vec{a}_z\right)^{3/2}}\right]$$

• potencijal :

$$\varphi = \frac{1}{4\pi\varepsilon_0}\left[\frac{Q_1}{|\vec{R}_1|} + \frac{Q_2}{|\vec{R}_2|}\right] = \frac{1}{4\pi\varepsilon_0}\left[\frac{Q_1}{\sqrt{(x-d)^2 + y^2 + z^2}} + \frac{Q_2}{\sqrt{(x+d)^2 + y^2 + z^2}}\right]$$

② Odredite ele

(2.) Odredite električno polje i potencijal jednoliko nabijene dužine



$\vec{r} = r\vec{a}_r + z\vec{a}_z$ → osno simetrično → nema α komponente (uvijek je ista)

$\vec{r}' = \xi \vec{a}_z$

$\vec{R} = \vec{r} - \vec{r}' = r\vec{a}_r + (z - \xi)\vec{a}_z$

- električno polje

$$\vec{E} = \frac{1}{4\pi\varepsilon_0} \int dQ \frac{\vec{R}}{R^3} \qquad dQ = \lambda d\xi \rightarrow Q = \lambda \int d\xi$$

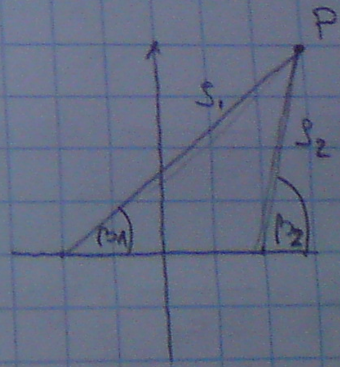
$$\vec{E} = \frac{\lambda}{4\pi\varepsilon_0} \int \frac{r\vec{a}_r + (z-\xi)\vec{a}_z}{\left(\sqrt{r^2 + (z-\xi)^2}\right)^3} d\xi = E_r \vec{a}_r + E_z \vec{a}_z$$

d $\quad [\quad d-z \quad]$

$$\vec{E} = \frac{\lambda}{4\pi\varepsilon_0} \int \frac{r\vec{a}_r + (z-\xi)\vec{a}_z}{\left(\sqrt{r^2 + (z-\xi)^2}\right)^3} d\xi = E_r\vec{a}_r + E_z\vec{a}_z$$

$$E_r = \frac{\lambda}{4\pi\varepsilon_0} \int_{-d}^{d} \frac{r}{\left[r^2 + (z-\xi)^2\right]^{3/2}} d\xi = \frac{\lambda}{4\pi\varepsilon_0 r} \left[\frac{d+z}{\sqrt{r^2 + (d+z)^2}} + \frac{d-z}{\sqrt{r^2 + (d-z)^2}} \right]$$

$$E_z = \frac{\lambda}{4\pi\varepsilon_0} \int_{-d}^{d} \frac{z-\xi}{\left[r^2 + (z-\xi)^2\right]^{3/2}} = \frac{\lambda}{4\pi\varepsilon_0 r} \left[\frac{r}{\sqrt{(d-z)^2 + r^2}} - \frac{r}{\sqrt{(d+z)^2 + r^2}} \right]$$



$$E_r = \frac{\lambda}{4\pi\varepsilon_0 r} \left[\frac{d+z}{s_1} - \frac{z-d}{s_2} \right] = \frac{\lambda}{4\pi\varepsilon_0 r} (\cos\beta_1 - \cos\beta_2)$$

$$E_z = \frac{\lambda}{4\pi\varepsilon_0 r} \left[\frac{r}{s_2} - \frac{r}{s_1} \right] = \frac{\lambda}{4\pi\varepsilon_0 r} (\sin\beta_2 - \sin\beta_1)$$

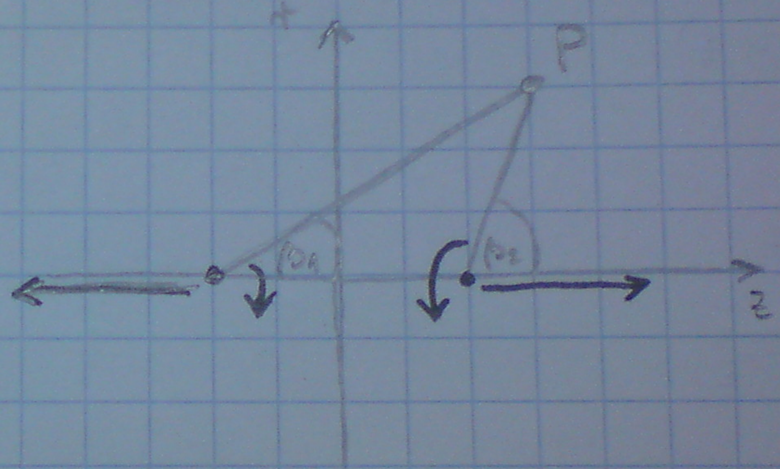
- potencijal

$$\varphi = \int \frac{dQ}{4\pi\varepsilon_0 R} = \frac{\lambda}{4\pi\varepsilon_0} \int_{-d}^{d} \frac{d\xi}{\sqrt{r^2 + (z-\xi)^2}} = \frac{\lambda}{4\pi\varepsilon_0} \ln \frac{d+z + \sqrt{(d+z)^2 + r^2}}{z-d + \sqrt{(z-d)^2 + r^2}}$$

$$\varphi = \frac{\lambda}{4\pi\varepsilon_0} \ln \frac{d+z+s_1}{z-d+s_2}$$

3. Odrediti električno polje beskonačno dugog naboja

~> specijalan slučaj !



$\boxed{d \to \infty}$

$\beta_1 \to 0$

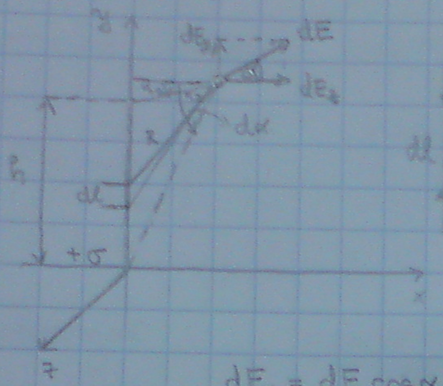
$\beta_2 \to \pi$

$$E_r = \frac{\lambda}{4\pi\varepsilon_0 r}\left[\cos 0 - \cos\pi\right] = \frac{\lambda}{4\pi\varepsilon_0 r}$$

$$E_z = \frac{\lambda}{4\pi\varepsilon_0 r}\left[\sin\pi - \sin 0\right] = 0$$

4. Odrediti električno polje beskonačno duge trake nabijene jednoliko

4. Odrediti električno polje beskonačno duge trake nabijene jednolikim nabojem gustoće σ.



$$dE = \frac{\lambda}{2\pi\varepsilon_0 R}$$

$$\lambda = \sigma\, dl$$

$$\cos\alpha = \frac{R\,d\alpha}{dl} \rightarrow R = \frac{dl\cos\alpha}{d\alpha}$$

$$dE_x = dE\cos\alpha = \frac{\lambda}{2\pi\varepsilon_0 R}\,\frac{R\,d\alpha}{dl} = \frac{\sigma\,dl}{2\pi\varepsilon_0 \cancel{R}}\cdot\frac{\cancel{R}\,d\alpha}{\cancel{dl}} = \frac{\sigma\,d\alpha}{2\pi\varepsilon_0}$$

$$dE_y = dE\sin\alpha = \frac{\lambda}{2\pi\varepsilon_0 R}\cdot\sin\alpha = \frac{\sigma\, dl}{2\pi\varepsilon_0}\,\frac{d\alpha}{dl\cos\alpha}\cdot\sin\alpha = \frac{\sigma\,d\alpha}{2\pi\varepsilon_0}\cdot\mathrm{tg}\,\alpha$$

$$E_x = \frac{\sigma}{2\pi\varepsilon_0}\int_{\alpha_1}^{\alpha_2} d\alpha = \frac{\sigma}{2\pi\varepsilon_0}(\alpha_2 - \alpha_1)$$

$$E_y = \frac{\sigma}{2\pi\varepsilon_0}\int_{\alpha_1}^{\alpha_2}\mathrm{tg}\,\alpha\, d\alpha = \frac{\sigma}{2\pi\varepsilon_0}\ln\frac{\cos\alpha_1}{\cos\alpha_2}$$

specijalni slučajevi:

1. točka na simetrali:

$$\alpha = \alpha_2 = -\alpha_1$$

$$E_x = \frac{\sigma}{2\pi\varepsilon_0}2\alpha \qquad E_y = 0$$

2. $h \to \infty \rightarrow \alpha_2 = \frac{\pi}{2} \quad \alpha_1 = -\frac{\pi}{2}$

$$E_x = \frac{\sigma}{2\varepsilon} \qquad E_y = 0$$