3. Međuispit iz Elektromagnetskih polja 27.06.2011.

Ime i prezime_____ Matični broj_____ INAČICA B

Ispit se sastoji od pet cjelina, u kojima se točan odgovor na svako pitanje nezavisno boduje, te se sastoji od ukupno 10 pitanja. Ukoliko želite odgovoriti na neko pitanje, zacrnite odgovor na obrascu za test. U ispitu postoje pitanja za 1, 2, 3 i 4 boda. Svaki točan odgovor na pitanje za 1 bod donosi 1 bod, a netočan -0,2 boda. Svaki točan odgovor na pitanje za 2 boda donosi 2 boda, a netočan -0,4 boda. Svaki točan odgovor na pitanje za 3 boda donosi 3 boda, a netočan -0,6 bodova. Svaki točan odgovor na pitanje za 4 boda donosi 4 boda, a netočan -0,8 bodova. Napišite ime na svim papirima s postupcima i predajte ih na kraju ispita zajedno s primjerkom testa u košuljici, dok se Obrazac za test posebno predaje.

I Neka je u sredstvu s $\mu_r = 1$ jakost magnetskog polja zadana jednadžbom:

$$\boldsymbol{H} = \frac{e^{-200y}}{10}\cos(2\pi \cdot 10^{10}t - 350y)\boldsymbol{a}_x[\mathrm{A/m}].$$

1. (1 bod) Odredite valnu impedanciju sredstva u [Ω].

A	196∠30°	B	219∠34°	C	304∠42°	D	247∠39°	E	101∠24°	F	152∠28°

2. (3 boda) Odredite jakost električnog polja u trenutku t = 2ns i y=0,1m u [nV/m].

A	-28,3	B	-18,9	C	12,8	D	35,1	E	-40,3	F	24,1

II Neka su jakost električnog polja i magnetska indukcija ravnog vala u slobodnom prostoru dane jednadžbama:

$$\boldsymbol{E}(x,y,z,t) = \boldsymbol{E_0}(x,y)\cdot e^{j(1{,}5\cdot 10^8\cdot t - k\cdot z)}$$
$$\boldsymbol{B}(x,y,z,t) = \boldsymbol{B_0}(x,y)\cdot e^{j(1{,}5\cdot 10^8\cdot t - k\cdot z)}$$

Pri tom je k konstanta, a $\boldsymbol{E_0}$ i $\boldsymbol{B_0}$ su vektori u xy ravnini.

3. (3 boda) Odredite konstantu k u [m^{-1}] tako da su zadovoljene Maxwellove jednadžbe.

A	1,00	B	0,66	C	0,17	D	0,75	E	0,33	F	0,50

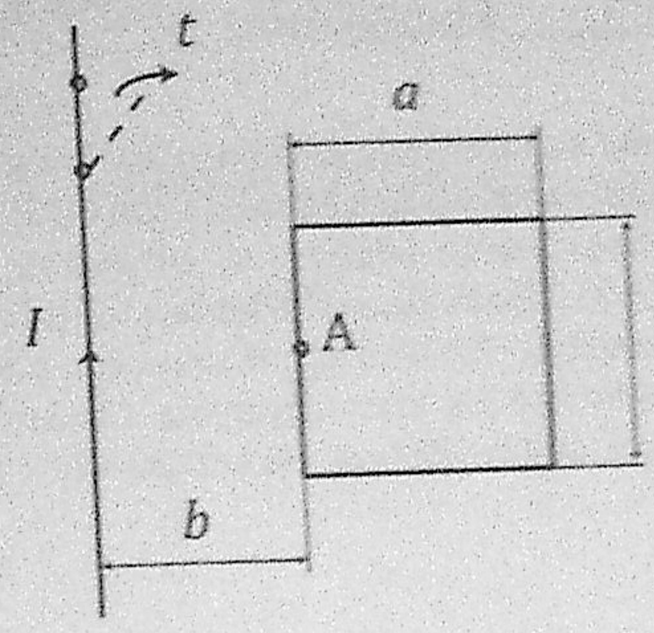
F

4. (1 bod) Odredite valnu duljinu vala u [m].

A	3π	B	9π	C	12π	D	4π	E	π	F	6π

D

III Žičana petlja kvadratnog oblika, stranice a i ukupnog otpora žice R, nalazi se na udaljenosti b od beskonačno dugog vodiča kojim teče struja I prema slici. U trenutku t, struja kroz žicu je prekinuta. (a=1m, b=1m, I=1A, R=1Ω)



5. (4 boda) Odredite ukupni naboj u [nC] koji prođe točkom A na petlji za vrijeme u kojem struja teče petljom.

A	102,7	B	60,5	C	138,6	D	25,3	E	229,7	F	180,9

IV Sinusno promjenjivi ravni val se širi u realnom sredstvu za koje je zadano ε_r=6, μ_r=1, κ=0,5 S/m. Frekvencija vala je f=150 MHz, a početna amplituda je E_0=200 V/m.

6. (1 bod) Odredite omjer iznosa električnog polja $E(x=0)/E(x=2\lambda)$ gdje je λ valna duljina.

A	34746	B	59879	C	25922	D	41895	E	86757	F	70218

7. (1 bod) Odredite valnu impedanciju u [Ω].

A	48,5∠42°	B	31,5∠43°	C	73,5∠38°	D	62,4∠40°	E	40,1∠39°	F	56,0∠41°

8. (2 boda) Odredite srednju vrijednost realnog dijela Poyntingova vektora na udaljenosti x=0,8d (d je dubina prodiranja) u [W/m²].

A	61,7	B	49,4	C	54,3	D	41,6	E	72,8	F	32,9

U ishodištu sfernog koordinatnog sustava nalazi se izvor polja:

$$E = 50\sin(\vartheta)\cdot r^{-1}\cdot\cos(10^9 t - 2r)a_\vartheta \text{ V/m}$$

$$H = \frac{50}{120\pi}\sin(\vartheta)\cdot r^{-1}\cdot\cos(10^9 t - 2r)a_\alpha \text{ A/m}$$

9. (2 boda) Odredite iznos Poyntingova vektora N na udaljenosti r=3m u trenutku t=3ns za $\vartheta = \pi/5$ u [mW/m²].

A	44,1	B	249,5	C	108,8	D	80,2	E	162,7	F	214,6

10. (2 boda) Odredite ukupnu srednju snagu izvora u [W].

A	14,6	B	83,4	C	45,9	D	96,1	E	27,8	F	70,2

[11] $\mu_r = 1$

$$H = \frac{e^{-200y}}{10}\cos(2\pi\cdot 10^{10}t - 350y)\,a_x \quad [A/m]$$

$$\Rightarrow \alpha = 200 \qquad \beta = 350$$

(1) $$Z = \frac{\omega\mu}{\sqrt{\alpha^2+\beta^2}}\, e^{j\arctg\left(\frac{\alpha}{\beta}\right)} = \frac{2\pi\cdot 10^{10}\cdot 1.4\pi\cdot 10^{-7}}{\sqrt{200^2+350^2}}\, e^{j\arctg\left(\frac{200}{350}\right)}$$

$$= 196\angle 30^\circ$$

(2.) $\epsilon = ?$ $\quad t = 2ns \quad y = 0.1m \quad [\pi V/m]$

$$Z = \frac{E}{H} \Rightarrow \epsilon = 2.14$$

$$= Re\{z\} = 196\cdot\cos(30^\circ)\cdot\frac{e^{-20}}{10}\cos(2\pi\cdot 10^{10}\cdot 2\cdot 10^{-9} - 35)$$

$$= -40.3$$

II $E(x,y,z,t) = E_0(x,y)\, e^{j(1.5\cdot 10^8 t - \beta z)}$

$B(x,y,z,t) = B_0(x,y) e^{j(1.5\cdot 10^8 t - \beta z)}$

(B.)

$$\nabla \times E = \frac{\partial B}{\partial t}$$

$$\left(\cancel{\frac{\partial}{\partial y} E_z} + \frac{\partial}{\partial z} E_y\right) a_x + \left(\cancel{\frac{\partial}{\partial x} E_z} - \frac{\partial}{\partial z} E_x\right) \vec{a_y} = + \frac{\partial B}{\partial t}$$

$= 0$ $\quad = 0$

samo po jednoj koordinati uzmemo

$$E_0 - j\beta e^{j(\omega t - \beta z)} = \frac{\partial B}{\partial t} \Big/ \int$$

$$B = \frac{-E_0 \beta e^{j(\omega t - \beta z)} j}{j\omega}$$

$B = \mu_0 H$

$H = \frac{B}{\mu_0}$

$$\nabla \times H = \frac{\partial D}{\partial t}$$

$$\frac{\partial H}{\partial t} = \varepsilon \frac{\partial E}{\partial t}$$

$$\frac{-1}{\mu_0} \frac{E_0^2 \beta^2 e^{j(\omega t - \beta z)}}{\omega} = \frac{\varepsilon E_0 \omega e^{j(\omega t - \beta z)}}{j\omega}$$

$$\beta^2 = \omega^2 \varepsilon_0 \mu_0 \beta^2$$

$$\beta = \sqrt{\varepsilon_0 \mu_0}\, \omega = 0.5$$

alternativno:

iz formule vidimo

$k = \beta$

$\beta = \omega \sqrt{\varepsilon \mu}$

$k = \omega \sqrt{\mu_0 \varepsilon_0} = 0.5$

(4.) $\lambda = \frac{2\pi}{\beta} = \frac{2\pi}{0.5} = 4\pi$

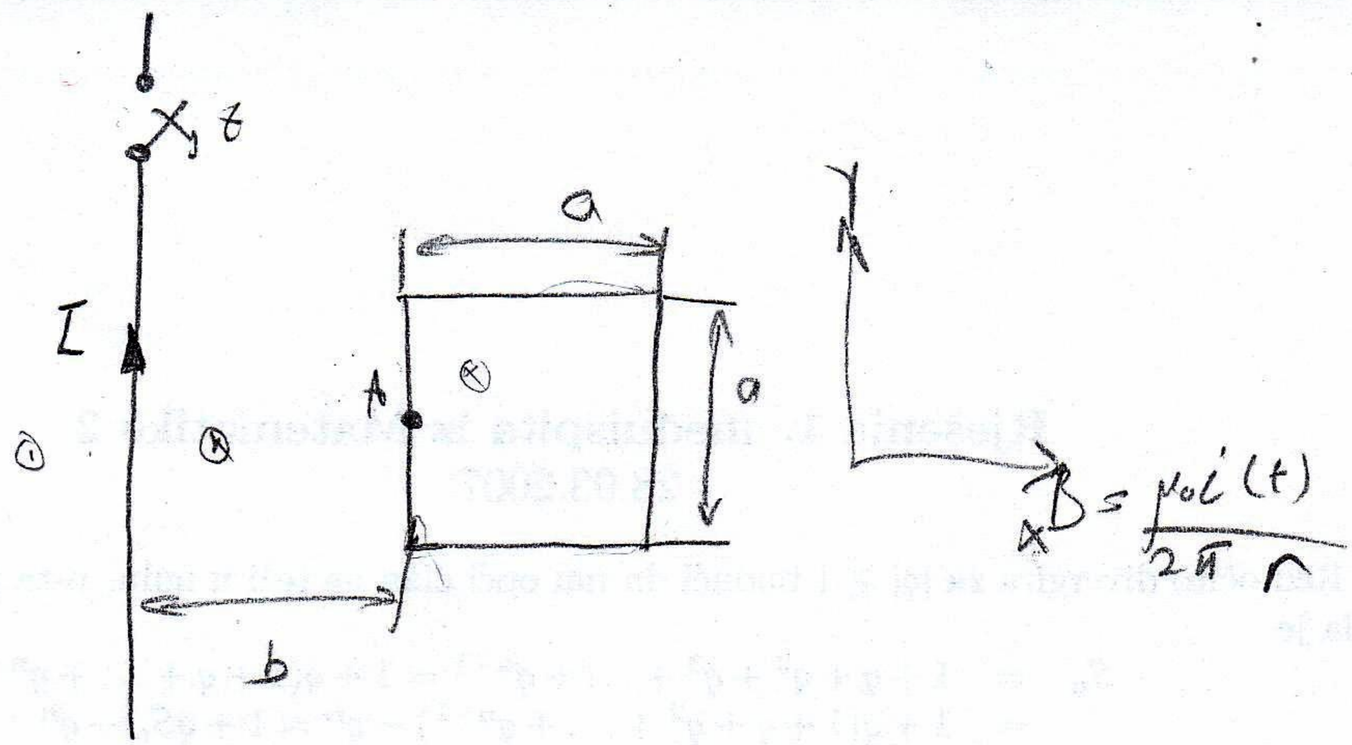
III

$a = 1\,m$

$b = 1\,m$

$I = 1\,A$

$R = 1\,\Omega$



$$\text{evtl} \quad \frac{\mu_0 \cdot i}{2\pi b} \cdot a^2 = \frac{2\pi \cdot 10^{-7} \cdot 1}{2\pi \cdot 1} = 200\,n\,\text{V}$$

$$Q = \int i(t)\,dt$$

$$= \int \frac{\Theta_{ind}(t)}{R}\,dt$$

$$= \int \frac{-\partial \Phi(t)}{\partial R}\,dt \qquad = \frac{\Phi}{R} = \Phi = \int B \cdot n\,ds$$

$$Q = \frac{\mu_0 I}{2\pi} \iint \frac{1}{x}\,dx\,dy \qquad = \frac{\mu_0 I}{2\pi} \int_1^2 \frac{1}{x} \int_0^1 dy\,dx = \frac{\mu_0 I}{2\pi} \cdot \ln\left(\frac{2}{1}\right) \cdot 1$$

$$= 200 \cdot 10^{-9} \cdot \ln(2) = 138.62\,nC$$

IV $\quad \varepsilon_r = 6 \quad \mu_r = 1 \quad \kappa = 0.5 \quad f = 150\,MHz \quad E_0 = 200\ V/m$

$$\frac{\kappa}{\omega\varepsilon} = 10$$

$$\alpha = \frac{\omega}{\sqrt{2}c}\sqrt{\sqrt{1+\left(\frac{\kappa}{\omega\varepsilon}\right)^2}-1} \qquad \beta = \frac{\omega}{\sqrt{2}c}\sqrt{\sqrt{1+\left(\frac{\kappa}{\omega\varepsilon}\right)^2}+1}$$

$$= \frac{\omega}{\sqrt{2}c}\,3 \qquad \beta = \frac{\omega}{\sqrt{2}c}\cdot 3.32$$

⑥ $\quad \frac{E(x=0)}{E(x=2\lambda)} \qquad E = E_0 e^{-\alpha x}\cos(\omega t - \beta x)$

$$\lambda = \frac{2\pi}{\beta}$$

$$= \frac{E_0 e^{0}\cos(\omega t)}{E_0 e^{-2\alpha\lambda}\cos(\omega t - \beta\cdot 2\lambda)}$$

$$= \frac{\cos(\omega t)}{e^{-2\pi\frac{\alpha}{\beta}}\cdot\cos(\omega t - \beta\cdot\frac{2\cdot 2\pi}{\beta})} \qquad = \frac{\cos\omega t}{e^{4\pi\frac{3}{3.32}}\cos(\omega t - 4\pi)}$$

(isti' su)

$$= e^{4\pi\frac{3}{3.32}} = \frac{85000}{e^{11.35}} \simeq \frac{86757}{e^{11.37}}$$

→ odtud ta razlika

"1A" "0N1"

⑦ $\quad Z = \frac{\omega\mu\, e^{j\arctg(\frac{\alpha}{\beta})}}{\sqrt{\alpha^2+\beta^2}} \qquad = \frac{\omega\mu}{\sqrt{2}c}\, e^{j\arctg(\frac{3}{3.32})} \qquad \frac{\sqrt{2}c}{4.47}\, e^{j42}$

$$= \frac{\alpha}{\sqrt{2}c}\sqrt{3^2+3.32^2} \qquad = 48.5\angle 42°$$

⑧ $\quad d = \frac{1}{\alpha} \qquad x = 0.8d \qquad = \frac{0.8}{\alpha}$

$$N_{sr} = \frac{1}{2}\frac{E_0^2}{|Z|}e^{-2\alpha x}\cos\varphi = \frac{1}{2}\frac{200^2}{48.5}e^{-2\alpha\frac{0.8}{\alpha}}\cos(42°)$$

$$= \frac{1}{2}\frac{200^2}{48.5}e^{-1.6}\cdot\cos(42°) = \underline{\underline{27.8}}$$

IV/

$$\bar{E} = \frac{50 \sin\vartheta}{r} \cos(10^9 t - 2r)\, a_\vartheta \quad [V/m]$$

$$H = \frac{50 \sin\vartheta}{120\pi \cdot r} \cos(10^9 t - 2r)\, a_\varphi \quad [A/m]$$

(9.) $N = ?$ $r = 3\,m$ $t = 3\,ns$ $\vartheta = \pi/5$ $[mW/m^2]$

$$N = E \times H = \begin{vmatrix} a_r & a_\vartheta & a_\varphi \\ 0 & E & 0 \\ 0 & 0 & H \end{vmatrix} = \begin{vmatrix} a_r & a_\vartheta & a_\varphi \\ 0 & E & 0 \\ 0 & \vartheta & H \end{vmatrix} = HE\vec{a}_r$$

$$N = \frac{50^2 \sin^2\vartheta}{120\pi\, r^2} \cos^2(10^9 t - 2r)\, \vec{a}_r \Big|_{\substack{r=3 \\ \vartheta=\pi/5 \\ t=3ns}} = 0{,}249.5 = 249.5\ mW/m^2$$

(10.) $P_{uk} = ?$

$$dV = 2\pi R^2 \cdot \sin\vartheta\, d\vartheta$$

$$P = \int_V N_{sr}\, dV$$

$$N_{sr} = \frac{1}{2} \frac{50^2 \sin^2\vartheta}{120\pi \cdot R^2}$$

$$P = \frac{1}{2} \frac{50^2}{120\pi \cdot R^2} \cdot 4\pi R^2 \int_0^{\pi} \sin^3\vartheta\, d\vartheta = \frac{50^2 \cdot 4}{120 \cdot 3} = \frac{50}{90}$$

$$\int_0^{\pi} \sin^3\vartheta\, d\vartheta = 4/3$$

$$30$$

$$= 27.8$$

$$P = \frac{V^2}{90}$$ (za sferne kord.)

$$\bar{E} = \frac{V \sin\vartheta}{r} \cos(\omega t - \beta r)$$