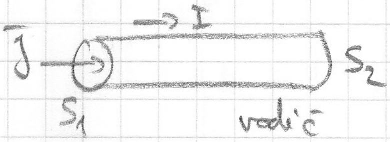
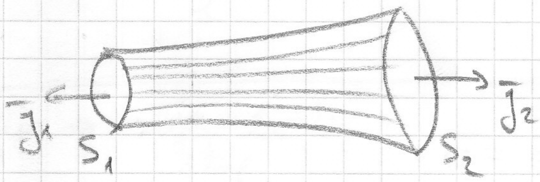
2. CIKLUS

STATIČKO STRUJNO POLJE

statičko el. polje

$\nabla \cdot \bar{J} = -\frac{\partial \varphi}{\partial t}$ jednadžba kontinuiteta





$\nabla \cdot \bar{J} = 0$

I. KIRCHOFF

$$\oiint_S \bar{J} n \, dS = 0 \rightarrow \iint_{S_1} \bar{J}_1 \bar{n} \, dS + \iint_{S_2} \bar{J}_2 \bar{n} \, dS = 0$$

$I_1 - I_2 = 0$

$\sum_k I_k = 0$ mora postojati polje

polje i gustoća struje

$\bar{J} = k \bar{E}$ $\quad \bar{E} = -\nabla \cdot \varphi$

$\nabla \times \bar{E} = 0 \quad \nabla \cdot \bar{J} = 0$

$\Delta \varphi = 0$

statičko el. polje

$\bar{E} = -\nabla \varphi$

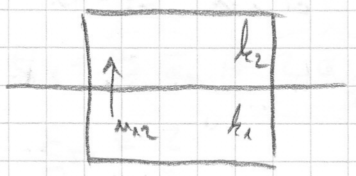
$\bar{D} = \varepsilon \bar{E}$

$\nabla \cdot \bar{D} = 0$ (za prostor bez naboja)

$\nabla \times \bar{E} = 0$

$\bar{D} \leftrightarrow \bar{J}$

UVJETI NA GRANICI



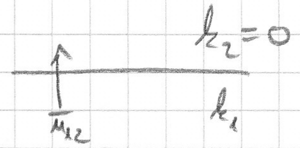
$$\bar{n}_{12} \times (\bar{E}_2 - \bar{E}_1) = 0$$

$$\bar{n}_{12} \times \left(\frac{\bar{J}_2}{k_2} - \frac{\bar{J}_1}{k_1} \right) = 0$$

$$\bar{n}_{12} \, (\bar{J}_2 - \bar{J}_1) = 0$$

---

specijalni slučaj



$$\Rightarrow \quad \bar{J}_2 = k_2 \bar{E} = 0$$

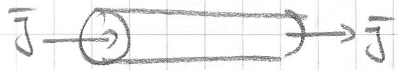
$$\bar{n}_{12} \cdot (\bar{J}_2 - \bar{J}_1) = 0$$

$$n_{12} \cdot \bar{J}_1 = 0$$

$$U = E \cdot l$$

$$U = I \cdot \frac{l}{\varepsilon S} \qquad R = \frac{1}{k} \cdot \frac{l}{S}$$

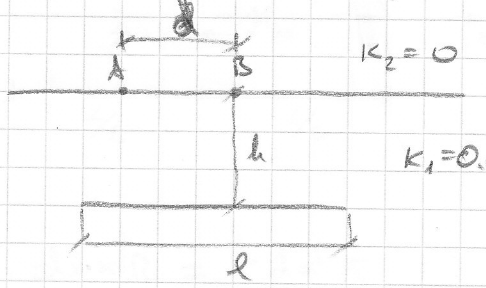
$$\rho \, [\Omega m]$$



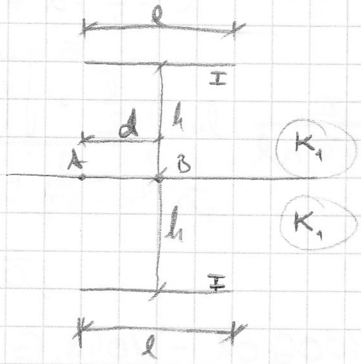
$$W_p = Q \cdot \varphi$$

$$\Delta W_p = Q(\varphi_A - \varphi_B) = Q \cdot U_{AB}$$

Zad. uzemivač polumjera $r_0 = 1\,cm$ duljine $l = 5\,m$ ukopan je u zemlju provodnosti $K_1 = 0.01\ S/m$ na dubini $h = 1\,m$ paralelno s pov. zemlje.
Odredi otpor rasprostiranja vodiča i razliku potencijala između A i B na pov. zemlje razmaknutih za $d = 1\,m$ ako iz vodiča teče struja $I = 100\,A$



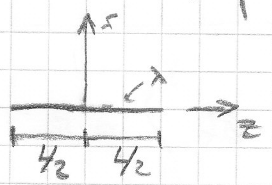
$\equiv$



predznaci se čuvaju
↓
nije kao u elektrostatici

!!!

analogija → nabijeni tanki štap



$$\lambda = \frac{Q}{L}$$

$$\varphi(r,z) = \frac{\lambda}{4\pi\varepsilon_0}\ln\left\{\frac{z+\frac{l}{2}+\sqrt{\left(z+\frac{l}{2}\right)^2+r^2}}{z-\frac{l}{2}+\sqrt{\left(z-\frac{l}{2}\right)^2+r^2}}\right\}$$

$D \Leftrightarrow J$ $\qquad$ $\varepsilon \Leftrightarrow K$

$Q \Leftrightarrow I$

$\lambda = \frac{Q}{L} \Leftrightarrow i = \frac{I}{l}$

$$\varphi(r,z) = \frac{i}{4\pi K}\ln\left\{\frac{z+\frac{l}{2}+\sqrt{\left(z+\frac{l}{2}\right)^2+r^2}}{z-\frac{l}{2}+\sqrt{\left(z-\frac{l}{2}\right)^2+r^2}}\right\}$$

(I) OTPOR RASPROSTIRANJA

$$R = \frac{\varphi_v}{I}$$ → potencijal vodiča

potencijal na površini ukopanog vodiča

$$\begin{pmatrix} z = 0 \\ r = r_0 \end{pmatrix} \qquad \varphi_v = \frac{I/l}{4\pi K}\ln\left\{\frac{\frac{l}{2}+\sqrt{\left(\frac{l}{2}\right)^2+r_0^2}}{-\frac{l}{2}+\sqrt{\left(\frac{l}{2}\right)^2+r_0^2}}\right\}$$

doprinos njega samog

$$+\ \frac{I/l}{4\pi K}\ln\left\{\frac{\frac{l}{2}+\sqrt{\left(\frac{l}{2}\right)^2+(2h)^2}}{-\frac{l}{2}+\sqrt{\left(\frac{l}{2}\right)^2+(2h)^2}}\right\}$$

zbog preslikavanja

$$R = 23.116\ \Omega$$

$$\varphi_v = \frac{I}{4\pi\cdot 0.01\cdot 5}\cdot 14.524$$

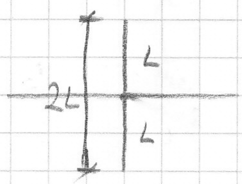
(II) RAZLIKA POTENCIJALA TOČAKA A i B

z koord.

$$\varphi_A = 2 \cdot \frac{I}{4\pi k l} \ln \left\{ \frac{\frac{l}{2} - d + \sqrt{\left(\frac{l}{2} - d\right)^2 + h^2}}{-\frac{l}{2} - d + \sqrt{\left(-\frac{l}{2} - d\right)^2 + h^2}} \right\}$$

$$\varphi_B = 2 \cdot \frac{I}{4\pi k l} \ln \left\{ \frac{\frac{l}{2} + \sqrt{\left(\frac{l}{2}\right)^2 + h^2}}{-\frac{l}{2} + \sqrt{\left(\frac{l}{2}\right)^2 + h^2}} \right\}$$

$$U_{AB} = \varphi_A - \varphi_B = 1006{,}01 - 1048{,}6 = -42{,}65\ V$$



$$i = \frac{2I}{2L}$$

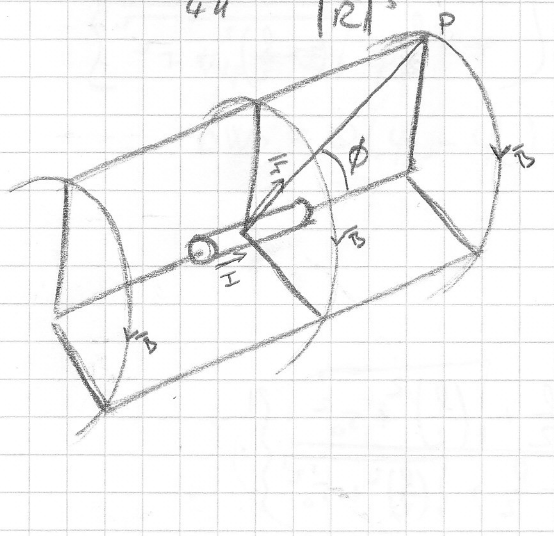
$$R = \frac{\varphi}{I}$$

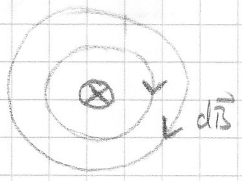
otpor se računa
samo sa realnim dijelom
vodiča → nesmije se uzimati i odslikani dio

MAGNETSKO POLJE

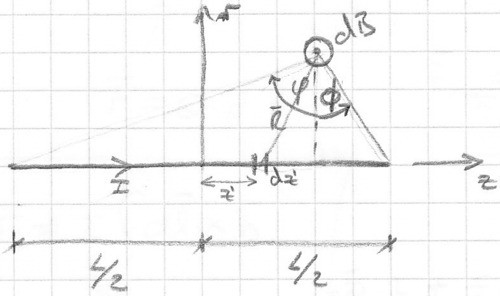
- Biot Savartov zakon

$$d\bar{B} = \frac{\mu_0}{4\pi} \frac{I d\bar{l} \times \bar{R}}{|\bar{R}|^3} \qquad \bar{R} = \bar{r} - \bar{r}' \qquad \mu_0 = 4\pi \cdot 10^{-7}$$





Zad] Odredi mag. ind. točke ravne strujnice duljine L kojom protječe istosmjerna struja I.



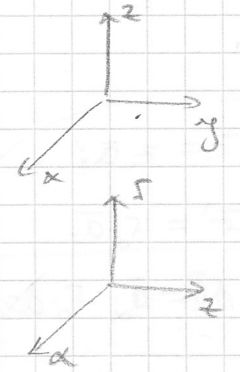
$$\bar{R} = \bar{r} - \bar{r}'$$

$$\bar{r} = r\,\bar{a}_r + z\,\bar{a}_z$$

$$\bar{r}' = z'\,\bar{a}_z$$

$$\bar{R} = r\,\bar{a}_r + (z - z')\,\bar{a}_z$$

$$d\bar{l} = dz' \cdot \bar{a}_z$$



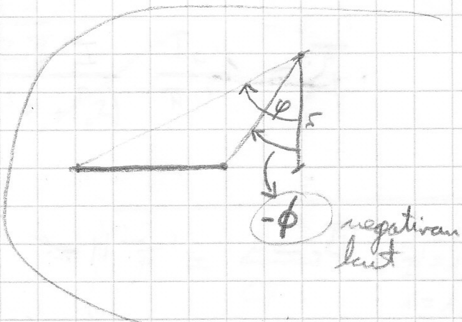
$$d\bar{l} \times \bar{R} = r\,dz'\,\bar{a}_\alpha$$

$$\boxed{d\bar{B} = \frac{\mu_0 \cdot I}{4\pi} \frac{d\bar{l} \times \bar{R}}{|\bar{R}|^3}} = \frac{\mu_0 I}{4\pi} \frac{r\,dz'\,\bar{a}_\alpha}{\left[r^2 + (z - z')^2\right]^{3/2}}$$

$$\bar{B} = \int_{z'=-\frac{L}{2}}^{\frac{L}{2}} d\bar{B} = \frac{\mu_0 I\,\bar{a}_\alpha\, r}{4\pi} \int_{-\frac{L}{2}}^{\frac{L}{2}} \frac{dz'}{\left[r^2 + (z - z')^2\right]^{3/2}}$$

$$\bar{B} = \bar{a}_\alpha \frac{\mu_0 I}{4\pi r} \left[\frac{\frac{L}{2} + z}{\sqrt{\left(\frac{L}{2} + z\right)^2 + r^2}} + \frac{\frac{L}{2} - z}{\sqrt{\left(\frac{L}{2} - z\right)^2 + r^2}} \right]$$

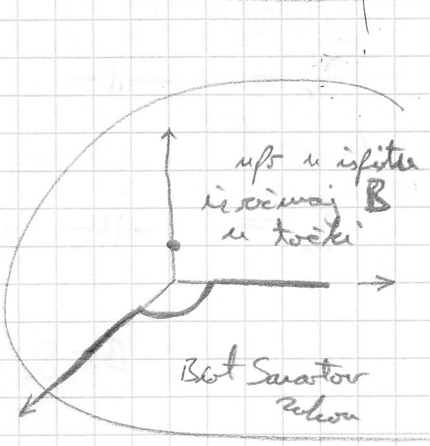
$$\bar{B} = \bar{a}_\alpha \frac{\mu_0 I}{4\pi r} \left[\sin\varphi + \sin\phi \right]$$



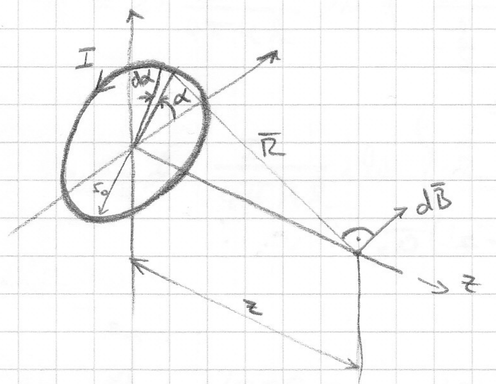
BESKONAČAN VODIČ

$$L/2 \to \infty$$

$$\left.\begin{array}{l} \varphi = \frac{\pi}{2} \\ \phi = \frac{\pi}{2} \end{array}\right\} \bar{B} = \bar{a}_\alpha \frac{\mu_0 I}{2\pi r}$$



Zad) Odredi mag. ind. na osi kružne strujnice polumjera r_0 kojom teče istosmjerna struja I



$$\bar{B} = \frac{\mu_0 I}{4\pi} \oint \frac{d\bar{l} \times \bar{R}}{|\bar{R}|^3}$$

$$\bar{B} = \frac{\mu_0 I}{4\pi} \int_{\alpha=0}^{2\pi} \frac{\bar{a}_r z + r_0 \bar{a}_z}{(r_0^2 + z^2)^{3/2}} r_0 \, d\alpha$$

$$\bar{B} = B_r \bar{a}_r + B_z \bar{a}_z$$

$\bar{r} = z \bar{a}_z$

$\bar{r}' = r_0 \bar{a}_r$

$d\bar{l} = \bar{a}_\alpha \cdot r_0 \, d\alpha$

$\bar{R} = z \bar{a}_z - r_0 \bar{a}_r$

$d\bar{l} \times \bar{R} = \bar{a}_\alpha \times (z \bar{a}_z - r_0 \bar{a}_r) \cdot r_0 \, d\alpha$
$= (\bar{a}_r z + r_0 \bar{a}_z) r_0 \, d\alpha$

$$B_z = \frac{\mu_0 I}{4\pi} \frac{r_0^2}{(r_0^2 + z^2)^{3/2}} \int_0^{2\pi} d\alpha$$

$$B_z = \frac{\mu_0 I \cdot r_0^2}{2 (r_0^2 + z^2)^{3/2}}$$

za $z = 0$

$$\boxed{B_z = \frac{\mu_0 I}{2 r_0}}$$

$$\bar{B}_r = \frac{\mu_0 I}{4\pi} \frac{I}{(r_0^2 + z^2)^{3/2}} r_0 \int_{\alpha=0}^{2\pi} \bar{a}_r \, d\alpha$$

$$\bar{a}_r = \bar{a}_x \cos\alpha + \bar{a}_y \sin\alpha$$

$$\bar{B}_r = \text{-\,||\,-} \quad \int_0^{2\pi} \bar{a}_x \cos\alpha \, d\alpha + \int_0^{2\pi} \bar{a}_y \sin\alpha \, d\alpha$$

$$\bar{B}_r = \text{-\,||\,-} \quad \left(\bar{a}_x \underbrace{\sin\alpha \Big|_0^{2\pi}}_{0} - \bar{a}_y \underbrace{\cos\alpha \Big|_0^{2\pi}}_{0} \right)$$

$$\bar{B}_r = \text{-\,||\,-} \cdot 0$$

$$B = B_z \bar{a}_z = \frac{\mu_0 I \cdot r_0^2}{2 (r_0^2 + z^2)^{3/2}} \bar{a}_z \quad //$$

$$\bar{F} = Q(\bar{E} + \bar{v} \times \bar{B})$$

$$\bar{F} = Q \cdot \bar{E}$$

$$\bar{F}_m = Q(\bar{v} \times \bar{B})$$

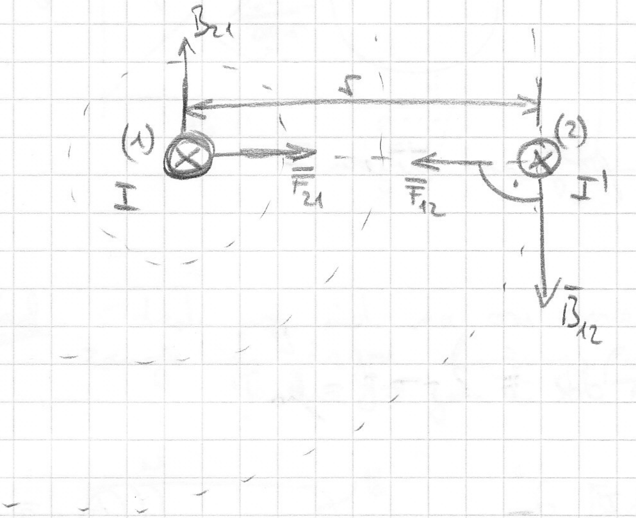
$$\boxed{\bar{F} = I \int_{žici} d\bar{l} \times \bar{B}}$$

struja koja prolazi kroz dS

$$I = \frac{dq}{dt} = \rho \cdot v\, dS$$

gustoća struje

$$J = \frac{I}{dS} = \rho \cdot v$$



$$B_{12} = \frac{\mu_0 I}{2\pi r}$$

$B_{21} = B_{12}$

$$\bar{F}_{12} = I' \bar{l} \times \bar{B}$$

$$\bar{F}_{12} = I' \cdot l \cdot \frac{\mu_0 I}{2\pi r}$$

$$\bar{F}_{12}' = \frac{\bar{F}_{12}}{l} = \frac{\mu_0 I I'}{2\pi r}$$

$$B_{21} = \frac{\mu_0 I'}{2\pi r}$$

$$\bar{F}_{21} = I \cdot l \times B$$

$$\bar{F}_{21} = I \cdot l \frac{\mu_0 I'}{2\pi r}$$

$$\boxed{\bar{F}_{21}' = \frac{\mu_0 I' \cdot I}{2\pi r}}$$

$$F' = \frac{\mu_0}{2\pi}$$

$I = 1A$
$I' = 1A$
$r = 1m$

$$F' = 2 \cdot 10^{-7}\ N/m$$

$\boxed{\nabla \times [\,] \to \text{rot}}$

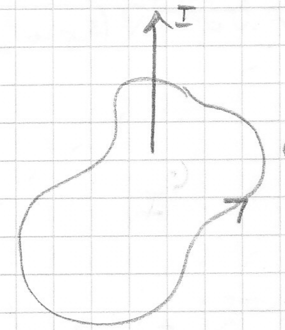
$$\oiint_S B \cdot n \, dS = 0$$

$$\nabla \cdot B = 0$$

$\boxed{\nabla \times \bar{B} = \mu_0 \bar{J}}$ Amperov zakon

$$\boxed{\oint_C \bar{B} \, d\bar{l} = \mu_0 \iint_S \bar{J} \, \bar{n} \, dS = \mu_0 I}$$

Amperov zakon

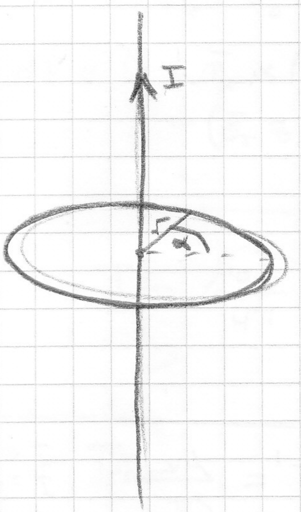


pravilo desne ruke

za pozitivni smjer struje

Zad.

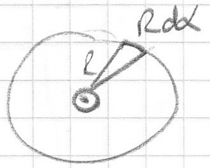
Odredi magnetsku indukciju beskonačne strujnice protjecane strujom I



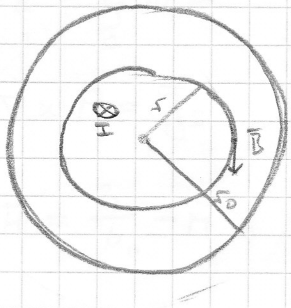
$$\bar{B} = \bar{B} \, \bar{a}_\alpha$$

$$\oint_C \bar{B} \, d\bar{l} = \oint_C B \, \bar{a}_\alpha \cdot \bar{a}_\alpha \, r \, d\alpha = 2 r \pi B = \mu_0 I$$

$$\bar{B} = \frac{\mu_0 I}{2 \pi r} \bar{a}_\alpha$$



Zad. Odredi mag. ind. unutar beskonačno dugog ravnog vodiča kružnog presjeka polumjera r_0 protjecan strujom I.



$r < r_0$

$\bar B = B \bar a_\alpha$
$d l = r d\alpha \bar a_\alpha$

$$\oint_C \bar B d\bar l = \mu_0 \iint_S \bar J \bar n dS$$

$$B \cdot 2r\pi = \mu_0 \bar J \iint_S dS$$

$S = r^2\pi$

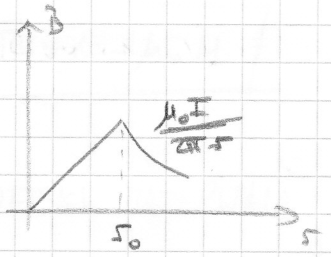
$$B \cdot 2\pi r = \mu_0 \bar J r^2 \pi$$

$$\bar J = \frac{I}{r_0^2 \pi}$$

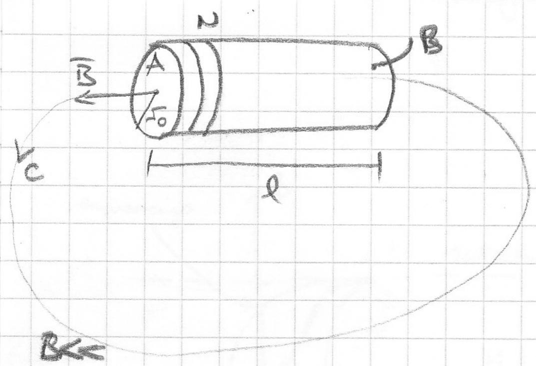
$$B \cdot 2\pi \cancel{r} = \mu_0 \frac{I}{r_0^2 \cancel{\pi}} r^{\cancel{2}} \cancel{\pi}$$

$$\boxed{\bar B = \bar a_\alpha \frac{\mu_0 I r}{2\pi r_0^2}}$$

MAG. IND UNUTAR BESKONAČNOG VODIČA



Zad. Odredi mag. ind. na osi vrlo duge zavojnice duljine l namotane s N zavoja na jezgru kružnog poprečnog presjeka polumjera r_0 protjecane strujom I.



$$\oint_C \bar B d\bar l = \sum \mu_0 I$$

$$\int_B^A \bar B d\bar l + \cancel{\int_B^A \bar B d\bar l}^{\,0} = \mu_0 \cdot N \cdot I$$

UNUTAR ZAVOJNICE

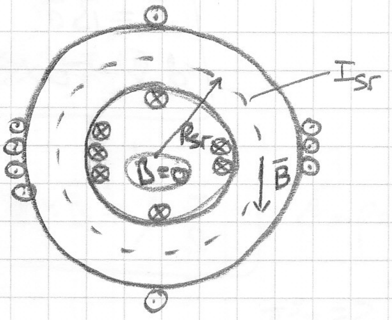
IZVAN ZAVOJNICE

indukcija opada u području izvan zavojnice - približno 0.

$$B \cdot l = \mu_0 \cdot N \cdot I$$

$$\boxed{B = \frac{\mu_0 \cdot N \cdot I}{l}}$$

Zad Odredi mag. ind. torusne zavojnice srednjeg opsega l_{sr} namotane s N zavoja na jezgru kružnog poprečnog presjeka polumjera r_0 protjecane strujom I.



$$\oint \bar{B} d\bar{l} = \sum \mu_0 I$$

$$B \cdot l_{sr} = \mu_0 \cdot N \cdot I$$

$\hookrightarrow 2\pi R_{SR}$

$$\boxed{B = \frac{\mu_0 N \cdot I}{2\pi R_{SR}}}$$ TORUSNA ZAVOJNICA

$B=0$ → FORA SA TRAFO-OM
unutra nije nula
uvijek ima rasipnog toka

$$\boxed{\begin{aligned} E &= -\nabla \varphi \\ \varphi &= -\int_{r_0}^{r} \bar{E} d\bar{l} \end{aligned}}$$

Vektorski magnetski potencijal

$\nabla \cdot B = 0$ gaussov zakon

$\nabla \times B = \mu_0 \bar{J}$ Amperov zakon

$$\boxed{\bar{B} = \nabla \times \bar{A}}$$ A - vektorski magnetski potencijal

$$\nabla \cdot \bar{A} = 0$$

$$\bar{A}(\bar{r}) = \frac{\mu_0 I}{4\pi} \int_c \frac{d\bar{l}}{|\bar{r} - \bar{r}'|}$$ - linijske struje

$$\phi = \iint_S \bar{B} \cdot \bar{n}\, dS = \iint_S (\nabla \times \bar{A})\, \bar{n}\, dS = \oint_c \bar{A}\, d\bar{l}$$

Veza magnetizacije i amperskih struja

$$I_a = \oint_c \bar{M}\, d\bar{l} = \iint_{S_c} \bar{J}_a \bar{n}_{S_c}\, dS$$

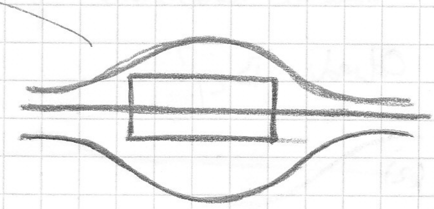
$$\nabla \times \bar{M} = \bar{J}_s$$

$$\bar{H} = \frac{B}{\mu_0} - \bar{M}$$ jakost mag. polja $\left[\frac{A}{m}\right]$

$$\nabla \times \bar{H} = \bar{J}_s$$

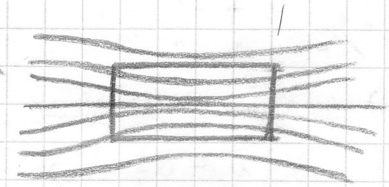
$$\boxed{\bar{B} = \mu_r \mu_0 \bar{H}}$$

MAGNETSKI MATERIJALI

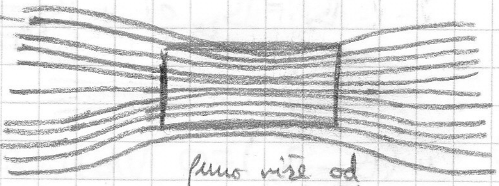


Dijamagnet - susceptibilnost mali negativni broj -10^{-5}

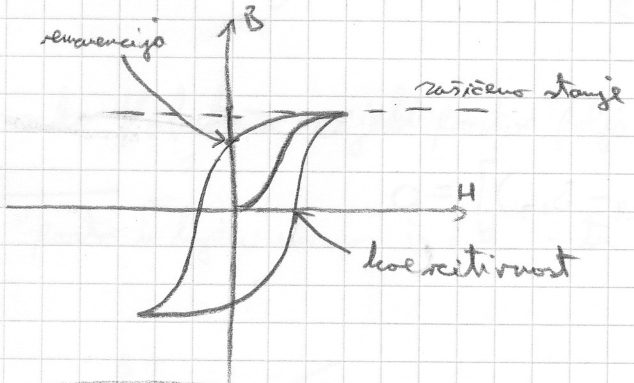
Paramagnet - mali pozitivni broj $10^{-2} \div 10^{-6}$



Feromagnet - veliki pozitivni broj $10^{2} \div 10^{6}$



puno više od paramagneta



Jednadžbe statičkog mag. polja u materijalu

$$\nabla \times \bar{H} = \bar{J}_s$$

$$\Downarrow$$

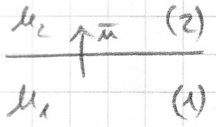
$$\oint_c \bar{H} \cdot d\bar{l} = \iint_S \bar{J}_s \cdot \bar{n}\, dS = I$$ ← amperov zakon u materijalima

$$\nabla \cdot \bar{B} = 0$$

$$\Downarrow$$

$$\oiint \bar{B}\, \bar{n}\, dS = 0$$

Uvjeti na granici



plošna struja

$$\boxed{\begin{aligned}\bar{n} \times (\bar{H}_2 - \bar{H}_1) &= \bar{K}_s \\ \bar{n}\,(\bar{B}_2 - \bar{B}_1) &= 0\end{aligned}}$$

tang komponenta $\bar{H}$ - skok za iznos gustoće slobodnih plošnih struja na granici

okomita komponenta $\bar{B}$ - kontinuirano prelazi granicu

$\bar{B}_1 = \bar{B}_2$

FORMULE

H_{tang} se ne mijenja

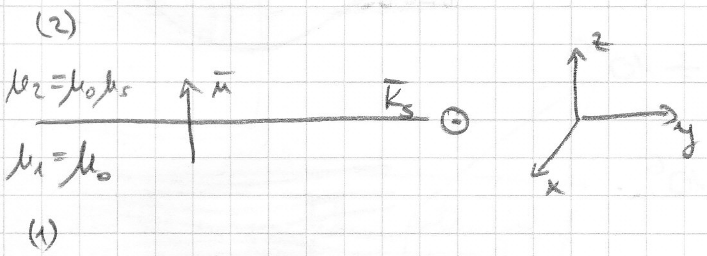
$B_{normalno}$ se ne mijenja

$$\boxed{\bar{B} = \mu \bar{H}}$$

Zad. Mag. polje ulazi iz zraka $\mu_1 = \mu_0$ gdje vlada $B_1 = \bar{a}_x\,0.01 + \bar{a}_z\,0.02$ [T] u feromag. materijal $\mu_2 = 100\mu_0$

Granica sredstava je ravnina $z=0$ na kojoj je $K_s = a_x \cdot 5000$ [A/m]

Odredi $\bar{B}_2$?



$$\bar{n} = \bar{a}_z$$

$$\bar{B}_2 = B_{2x}\bar{a}_x + B_{2y}\bar{a}_y + B_{2z}\bar{a}_z$$

I $\quad \bar{n}\cdot(\bar{B}_2 - \bar{B}_1) = 0$

$$\bar{a}_z \cdot \left[\bar{a}_x(B_{2x} - B_{1x}) + \bar{a}_y(B_{2y} - B_{1y}) + \bar{a}_z(B_{2z} - B_{1z})\right] = 0$$

$$B_{2z} - B_{1z} = 0$$

$$\boxed{B_{2z} = B_{1z} = 0.02\,T}$$

II $\quad \bar{n} \times (\bar{H}_2 - \bar{H}_1) = \bar{K}_s$

$$\bar{a}_z \times \left[\bar{a}_x\left(\frac{B_{2x}}{\mu_0 \cdot \mu_{r2}} - \frac{B_{1x}}{\mu_0}\right) + \bar{a}_y\left(\frac{B_{2y}}{\mu_0\mu_{r2}} - \frac{B_{1y}}{\mu_0}\right) + a_z\left(\frac{B_{2z}}{\mu_0\mu_{r2}} - \frac{B_{1z}}{\mu_0}\right)\right] = 5000$$

$$\underbrace{\bar{a}_y\left(\frac{B_{2x}}{\mu_0\mu_{r2}} - \frac{B_{1x}}{\mu_0}\right)}_{0} - \bar{a}_x\left(\frac{B_{2y}}{\mu_0\mu_{r2}} - \frac{B_{1y}}{\mu_0}\right) = 5000\bar{a}_x$$

$$\frac{B_{2x}}{\cancel{\mu_0}\mu_{r2}} = \frac{B_{1x}}{\cancel{\mu_0}} \Rightarrow B_{2x} = 0.01 \cdot 100$$

$$\boxed{B_{2x} = 1\ T}$$

$$\frac{B_{1y}}{\mu_0} - \frac{B_{2y}}{\mu_0 \mu_{r2}} = 5000$$

$$B_{2y} = (\mu_0 \mu_{r2})\left(\frac{B_{1y}}{\mu_0} - 5000\right)$$

$$B_{2y} = B_{1y}\mu_{r2} - \mu_0 \mu_{r2} 5000$$

$$B_{2y} = \mu_{r2}(B_{1y} - \mu_0 \cdot 5000)$$

$$B_{2y} = 100(0 - \mu_0 5000)$$

$$B_{2y} = -500\,000\,\mu_0 \qquad \mu_0 = 4\pi \cdot 10^{-7}$$

$$\boxed{B_{2y} = -0.628\ T}$$

$$\bar{B}_2 = 1\bar{a}_x - 0.628\bar{a}_y + 0.02\bar{a}_z$$

ENERGIJA MAG. POLJA I INDUKTIVITETI

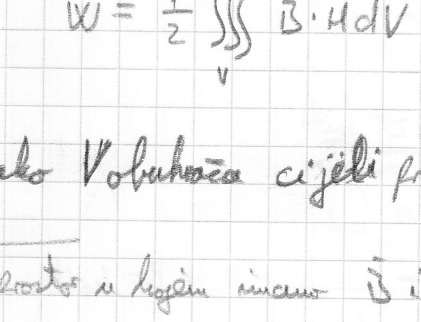
$$W = \frac{1}{2}\iiint_V \bar{J} \cdot A \cdot dV \qquad (dV = dS \cdot dl)$$

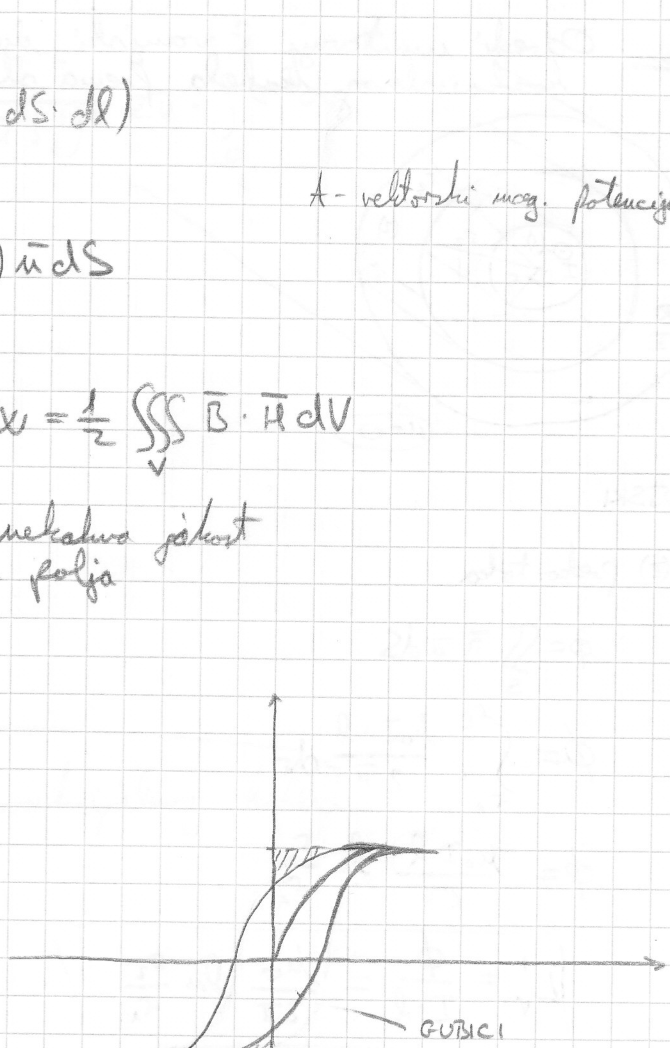
A - vektorski mag. potencijal

$$W = \frac{1}{2}\iiint_V \bar{B}\cdot\bar{H}\,dV + \frac{1}{2}\oiint_S (\bar{H}\times\bar{A})\,\bar{n}\,dS$$

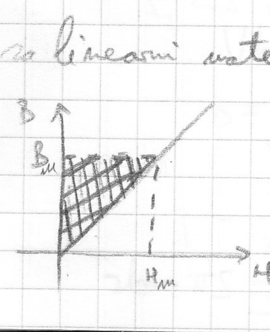
ako V obuhvaća cijeli prostor polja $\Rightarrow$ $W = \frac{1}{2}\iiint_V \bar{B}\cdot\bar{H}\,dV$

prostor u kojem imamo $\bar{B}$ i $\bar{H}$ - tu postoji nekakva jakost mag. polja





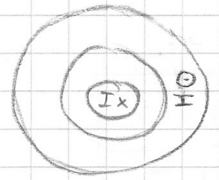
za linearni materijal



$$W = \frac{B_m \cdot H_m}{2}$$

Induktiviteti

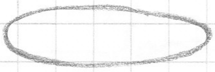
$$W=\frac{1}{2}LI^2$$



Induktivitet tankih strujnih petlji

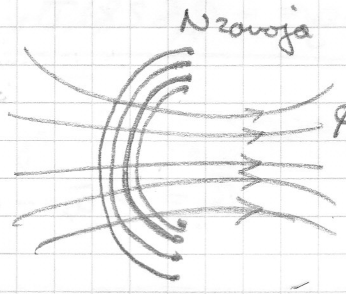
$$L=\frac{\oint_c \bar{A}\,dl}{I}=\frac{\phi}{I}$$

samo za petlju koja je dovoljno tanka



Obuhvaćeni tok

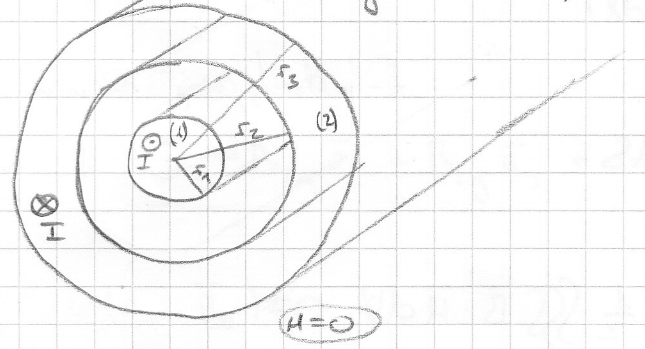
- kada imamo N zavoja



$$\psi=N\cdot\phi \Rightarrow \quad L=\frac{N\cdot\phi}{I}=\frac{\psi}{I}$$

induktivitet je omjer obuhvaćenog toka i struje

Zad. Odredi unutarnji i vanjski induktivitet po jedinici duljine koaksijalnog kabela prema slici



$$\oint \bar{H}d\bar{l}=\Sigma I$$

$$H\,2\pi r=I$$

$$H=\frac{I}{2\pi r}$$

$$\bar{B}=\frac{\mu_0 I}{2\pi r}\bar{a}_\alpha$$

VANJSKI

(I) preko toka

$$dS=l\cdot dr \qquad B=\mu_0 H$$

$$\bar{n}=\bar{a}_\alpha \qquad dV=2\pi r\cdot l\,dr$$

$$\phi=\iint_S \bar{B}\,\bar{n}\,dS$$

$$\phi=\int_{r_1}^{r_2}\frac{\mu_0 I\cdot l}{2\pi r}dr$$

$$\phi=\frac{\mu_0 I l}{2\pi}\ln\frac{r_2}{r_1}$$

$$\boxed{L_v'=\frac{\phi}{I\cdot l}=\frac{\mu_0}{2\pi}\ln\frac{r_2}{r_1}}$$

(II) preko $\bar{B}$ i $\bar{H}$ (preko energije)

$$W_v'=\frac{L_v' I^2}{2}$$

$$W=\frac{1}{2}\iiint_V \bar{B}\cdot\bar{H}\,dV$$

$$W=\mu_0\iiint_V \frac{H^2}{2}dV$$

$$W'=\frac{W}{l}=\mu_0\int_{r_1}^{r_2}\frac{H^2}{2}2\pi r\,dr$$

$$w' = \frac{\mu_0}{2}\int_{r_1}^{r_2} \frac{I^2}{(2\pi)^2 r^2}\, 2\pi r\, dr = \frac{\mu_0 I^2}{2\cdot 2\pi} \ln\frac{r_2}{r_1}$$

$$\boxed{L_v' = \frac{2 w_v'}{I^2} = \frac{\mu_0}{2\pi}\ln\frac{r_2}{r_1}}$$

UNUTARNJI

L_{u1}' PREKO EN.

$$\bar{B} = \frac{\mu_0 I r}{2\pi r_1^2}\,\bar{a}_\alpha \qquad r < r_1$$

$$w_u = \frac{1}{2}\iiint_V \bar{B}\,\bar{H}\, dV \qquad B = \mu_0 H$$

$$w_u = \frac{1}{2\mu_0}\int_{r_1}^{r_2} \bar{B}^2\, 2\pi r \cdot l\, dr$$

$$w_u' = \frac{w_u}{l} = \frac{1}{2\mu_0}\,\frac{\mu_0^2 I^2}{(2\pi r_1^2)^2}\, 2\pi \cdot \int_0^{r_1} r^3 dr$$

$$w_u' = \frac{\mu_0 I^2}{2\cdot 2\pi r_1^4}\,\frac{r_1^4}{4}$$

$$w_u' = L_u' \frac{I^2}{2}$$

$$L_u' = \frac{2 w_u'}{I^2}$$

$$\boxed{L_{u1}' = \frac{\mu_0}{8\pi}}$$

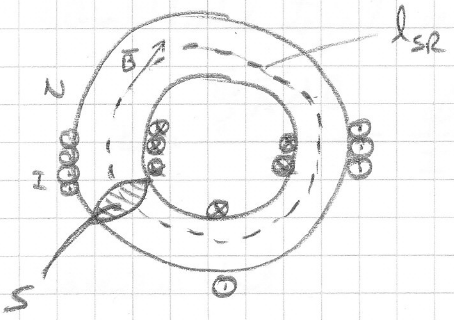
$L_{u2}' \ll$ - vrlo je malen
jer je vanjska linija koaksijalnog kabela
vrlo tanka

$$r_2 < r < r_3$$

$$B = \frac{\mu_0 I}{2\pi r}$$

$$J = \frac{I}{\pi(r_3^2 - r_2^2)}$$

Odredi induktivitet torusne zavojnice srednjeg opsega l_{sr}
namotane sa N zavoja
na jezgru od materijala s $\mu = konst$
kružnog poprečnog presjeka S
protjecane strujom I.



$$\oint \bar{H}\, d\bar{l} = \sum I$$

$$H \cdot l_{SR} = N \cdot I$$

$$B = \frac{\mu_0 N I}{l_{SR}}$$

$$\phi = B \cdot S = \frac{\mu_0 N I}{l_{SR}}$$ – aproksimacija
ako površinski presjek nije puno veći od l_{SR}

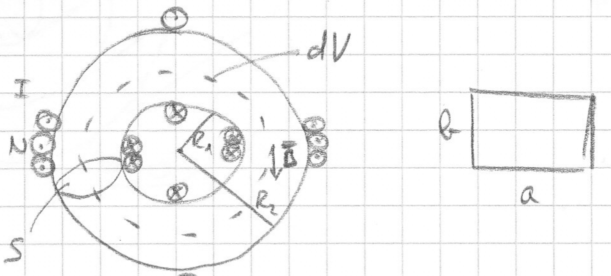
$$\boxed{L = \frac{\psi}{I} = \frac{N\phi}{I} = \frac{\mu \cdot N^2 S}{l_{SR}}}$$

Zad

Odredi mag. en. sadržanu u jezgri torusne zavojnice pravokutnog poprečnog presjeka ako je

a) $\mu_r = 5000$
b) krivulja magnetiziranja aproksimirana jednadžbom
$$B = k\sqrt{H}, \quad k = 0.1\, Vs\, A^{-\frac{1}{2}} m^{-\frac{3}{2}}$$

$N = 100$
$I = 1A$
$b = 1cm$
$a = 2cm$
$R_1 = 5cm$



a) $R_1 < r < R_2$

$$\oint \bar{H}\, d\bar{l} = \sum I$$

$$H \cdot 2\pi r = N \cdot I$$

$$H = \frac{NI}{2\pi r}$$

$$w = \frac{BH}{2} \quad \{B = \mu H\} = \frac{\mu H^2}{2}$$

$$W = \int_V w\, dV$$ – energija

$$dV = 2\pi r\, dr\, b$$

$$W = \int_{R_1}^{R_2} \frac{\mu H^2}{2} 2\pi r b\, dr = \mu \int_{R_1}^{R_2} \frac{N^2 I^2}{(2\pi r)^2} b \pi r\, dr$$

$$\boxed{W = \frac{\mu_0 \mu_r (NI)^2}{4\pi} b \ln R_2/R_1}$$ MAG. ENERGIJA

b) $B = k\sqrt{H}$

$B^2 = k^2 H$

$H = \frac{B^2}{k^2}$

$$w = \int_0^{B_0} H\,dB = \frac{1}{k^2}\int_0^{B_0} B^2\,dB = \frac{B_0^3}{3k^2} = \frac{k^3 H_0^{3}}{3k^2} = \frac{k}{3}\left(\frac{NI}{2\pi r}\right)^{3/2}$$

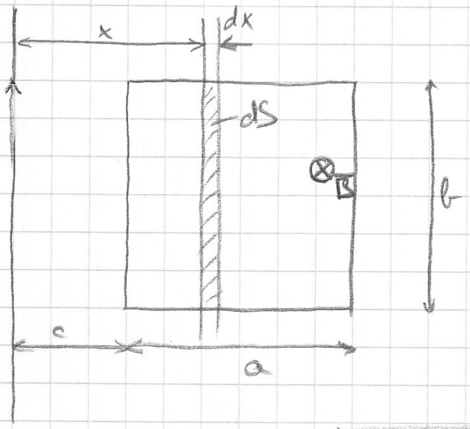
$$W = \int_V w\,dV = \int_{r=R_1}^{R_2} \frac{k}{3}\left(\frac{NI}{2\pi r}\right)^{3/2} b \cdot 2\pi r\,dr = \frac{k(NI)^{3/2} b}{3\sqrt{2\pi}}\,\frac{1}{\frac{1}{2}}\sqrt{r}\Big|_{R_1}^{R_2}$$

$$W = \frac{2kb(NI)^{3/2}}{3\sqrt{2\pi}}\left[\sqrt{R_1} - \sqrt{R_2}\right] = 10.89\ mJ$$

$H(R_1)$

$H(R_2)$

Zad. Odredi međuinduktivitet ravne, beskonačno duge strujnice i pravokutne vodljive petlje prema slici.



$$L_{12} = \frac{\phi}{I}$$

$$\phi = \iint \bar{B}\,\bar{n}\,dS$$

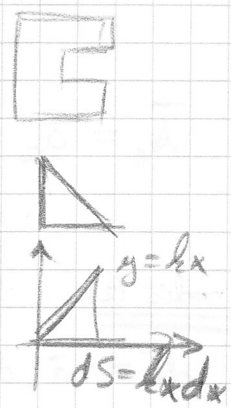
$$\phi = \iint \frac{\mu_0 I}{2\pi x}\,b\,dx$$

$$\phi = \frac{\mu_0 I b}{2\pi}\ln\frac{a+c}{c}$$

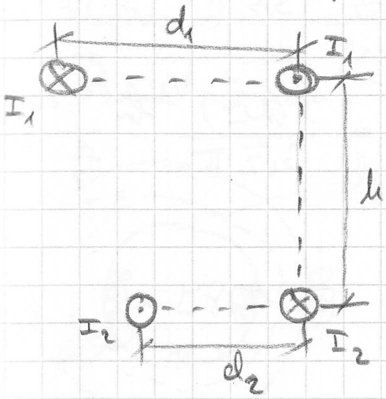
$$B = \frac{\mu_0 I}{2\pi x} = B(x)$$

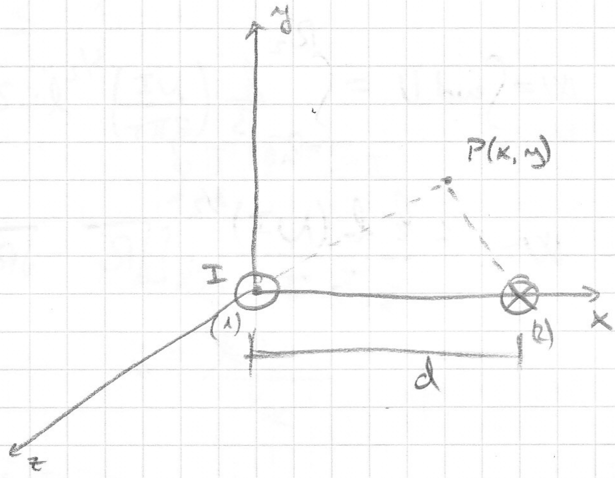
$$dS = b\,dx$$

$$\boxed{L_{12} = \frac{\mu_0 b}{2\pi}\ln\frac{a+c}{c}}$$



Zad. Odredi međuinduktivitet po jedinici duljine dvožičnog voda prema slici.





vodič (1)

$d\bar{l} = dz'\,\bar{a}_z$ - infinit. izvora

$\bar{r}' = \bar{a}_z z'$ - koord. izvora

$\bar{r} = \bar{a}_x x + \bar{a}_y y$ - koord. promatrane točke

$$\bar{R} = \bar{r} - \bar{r}' = \bar{a}_x x + \bar{a}_y y - \bar{a}_z z'$$

$$R = \sqrt{x^2 + y^2 + (z')^2} = \sqrt{r_1^2 + (z')^2}$$

$$\bar{A}_{(1)} = \frac{\mu_0 I}{4\pi}\,\bar{a}_z \int_{z'=-\infty}^{\infty} \frac{dz'}{\sqrt{r_1^2 + (z')^2}}$$

$$\bar{A}_{(1)} = \frac{2\mu_0 I}{4\pi}\,\bar{a}_z \int_0^{\infty} \frac{dz'}{\sqrt{r_1^2 + (z')^2}}$$

A je u smjeru struje izvora

$$\bar{A}_{(1)} = \bar{a}_z \frac{\mu_0 I}{2\pi} \ln\left(z' + \sqrt{r_1^2 + (z')^2}\right)\Bigg|_{z'=0}^{\infty}$$

vodič (2)

$d\bar{l} = -\bar{a}_z\,dz'$

$\bar{r} = \bar{a}_x x + \bar{a}_y y$

$\bar{r}' = \bar{a}_x d + \bar{a}_z z'$

$$\bar{R} = \bar{r} - \bar{r}' = \bar{a}_x (x-d) + \bar{a}_y y - \bar{a}_z z'$$

$$|\bar{R}| = \sqrt{(x-d)^2 + y^2 + (z')^2}$$

$$\bar{A}_{(2)} = \frac{-\mu_0 I}{4\pi}\,\bar{a}_z \cdot 2 \int_{z'=0}^{\infty} \frac{dz'}{\sqrt{r_2^2 + (z')^2}}$$

$$\bar{A}_{(2)} = -\frac{\mu_0 I}{2\pi}\,\bar{a}_z \ln\left(z' + \sqrt{r_2^2 + (z')^2}\right)\Bigg|_{z'=0}^{\infty}$$

$$\bar{A} = \bar{A}_{(1)} + \bar{A}_{(2)}$$

$$\bar{A} = \bar{a}_z \frac{\mu_0 I}{2\pi} \ln \frac{z' + \sqrt{\rho_1^2 + (z')^2}}{z' + \sqrt{\rho_2^2 + (z')^2}} \Bigg|_{z'=0}^{\infty}$$

$$\bar{A} = a_z \frac{\mu_0 I}{2\pi} \left(\ln \left\{ \lim_{z' \to \infty} \frac{1 + \sqrt{\left(\frac{\rho_1}{z'}\right)^2 + 1}}{1 + \sqrt{\left(\frac{\rho_2}{z'}\right)^2 + 1}} \right\} \right) - \ln \frac{\rho_1}{\rho_2}$$

$$\bar{A} = a_z \frac{\mu_0 I}{2\pi} \ln \frac{\rho_2}{\rho_1}$$