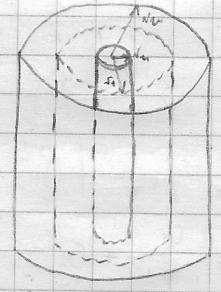
2.51. $r_a = 0.005$ m , $\varphi_a = 0$

$r_b = 0.03$ m , $\varphi_b = \varphi_0$

$\vec{E} = -4\cdot 10^3 \, \vec{a_r} \left[\frac{V}{m}\right]$, $r_1 = 0.015$ m

$\varphi_0 = ?$



rub. uv. : $\Delta \varphi = 0$

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r \frac{\partial \varphi}{\partial r}\right) = 0 \Rightarrow \varphi(r) = C_1 \ln r + C_2$$

$$\varphi(r_a) = \ln 0.005\, C_1 + C_2 = 0 \Rightarrow C_2 = -\ln 0.005\, C_1 = \ln 200\, C_1$$

$$\varphi(r_b) = \ln 0.03\, C_1 + C_2 = \varphi_0$$

$$\ln 0.03\, C_1 + \ln 200\, C_1 = \varphi_0$$

$$C_1 = \frac{\varphi_0}{\ln(0.03\cdot 200)} = 0.558\,\varphi_0 \Rightarrow C_2 = 2.957\,\varphi_0$$

$$\varphi(r) = 0.551\,\varphi_0 \cdot \ln r + 2.957\,\varphi_0 \ [V]$$

$$\vec{E} = -\nabla\varphi = -\frac{\partial \varphi}{\partial r}\vec{a_r} = -\frac{0.558\,\varphi_0}{r}\vec{a_r}$$

$$-\frac{0.558\,\varphi_0}{0.015} = -4\cdot 10^3 \Rightarrow \varphi_0 = \frac{4000\cdot 0.015}{0.558}$$

$$\varphi_0 = 107.5 \ V$$

2.52. $\sigma_b = ?$, podaci iz 2.51.

$$\oint\!\!\oint_{S_G} \vec{E}\,\vec{n}\, dS_G = \frac{1}{\varepsilon_0}\iint_S \sigma\, dS$$

$$\underbrace{E\,\vec{a_r}\cdot\vec{a_r}}_{=1} \cdot \cancel{2 r\pi\, l} = \frac{1}{\varepsilon_0}\,\sigma \cdot \cancel{2 r\pi\, l} \quad / \cdot \varepsilon_0$$

duljina cilindra

$$\sigma = \varepsilon_0 E ,$$

$$\vec{E} = -\nabla\varphi = -\frac{0.558\cdot 107.5}{r}\vec{a_r} = -\frac{60}{r}\vec{a_r}$$

$$E(r = r_b) = -\frac{60}{0.03} = 2000 \ \frac{V}{m}$$

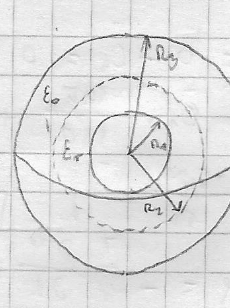
$$\sigma_b = 17.7 \ \frac{nC}{m^2}$$

2.53. $R_1 = 8$ cm $= 0.08$ m , $+Q$

$R_3 = 24$ cm $= 0.24$ m , $-Q$ $\quad Q = 15$ nC

$\varepsilon_r = 4$ na $8 \text{ cm} \le r \le 16$ cm $\quad R_2 = 16$ cm

zrak na $16 \text{ cm} \le r \le 24$ cm



a) $E = ?$ $\quad r = 0.14$ m

$r_a = 0.14$ m $\Rightarrow R_1 < r_a < R_2$, $\varepsilon_r = 4$

Gaussov zakon: $\oint\!\!\oint_{S_G} \vec{D}\,\vec{n}\, dS_G = Q$

$$\varepsilon_0 \varepsilon_r E \overbrace{\vec{a_r}\,\vec{a_r}}^{=1} \cdot 4 r_a^2 \pi = Q$$

$$E = \frac{Q}{\varepsilon_0 \varepsilon_r \cdot 4 r_a^2 \pi} = \frac{15 \cdot 10^{-9}}{8.854 \cdot 10^{-12} \cdot 4 \cdot 4 \cdot 0.14^2 \pi} = 1719.59$$

$$E = 1720 \ \frac{V}{m}$$

b) $E = ?$ $\quad r = 0.19 \ m$

$r_b = 0.19 \ m \Rightarrow R_2 < r_b < R_3$

Gaussov zakon: $\oiint_{S_G} \vec{D} \, \vec{n} \, dS_G = Q$

$$\varepsilon_0 E \overbrace{\vec{a_r} \, \vec{a_r}}^{=1} \cdot 4 r_b^2 \pi = Q$$

$$E = \frac{Q}{\varepsilon_0 \cdot 4 r_b^2 \pi} = \frac{15 \cdot 10^{-9}}{8.854 \cdot 10^{-12} \cdot 4 \cdot 0.19^2 \cdot \pi}$$

$$E = 4161 \ \frac{V}{m}$$

c) $\vec{P} = ?$ na vanjskoj površini dielektrika

$$\vec{P} = \vec{D} - \varepsilon_0 \vec{E} = \varepsilon_0 \varepsilon_r \vec{E} - \varepsilon_0 \vec{E} = \varepsilon_0 (\varepsilon_r - 1) \vec{E}$$

vanjska površina dielektrika: $E = \frac{Q}{\varepsilon_0 \varepsilon_r \cdot 4 R_2^2 \pi}$

$$P = \cancel{\varepsilon_0} (\varepsilon_r - 1) \cdot \frac{Q}{\cancel{\varepsilon_0} \varepsilon_r \cdot 4 R_2^2 \pi} = \frac{\varepsilon_r - 1}{\varepsilon_r} \cdot \frac{Q}{4 R_2^2 \pi} = \left(1 - \frac{1}{\varepsilon_r}\right) \frac{Q}{4 R_2^2 \pi} = 34.97 \cdot 10^{-9}$$

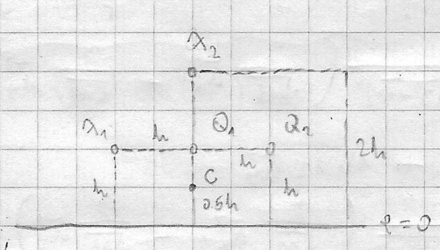
$$P = 35 \ \frac{nC}{m^2}$$

d) $Q_{vez} = ?$ na vanjskoj površini dielektrika

$$Q_{vez} = \oiint_S \vec{P} \, \vec{n} \, dS = P \cdot 4 R_2^2 \pi = 35 \cdot 10^{-9} \cdot 4 \cdot 0.16^2 \cdot \pi$$

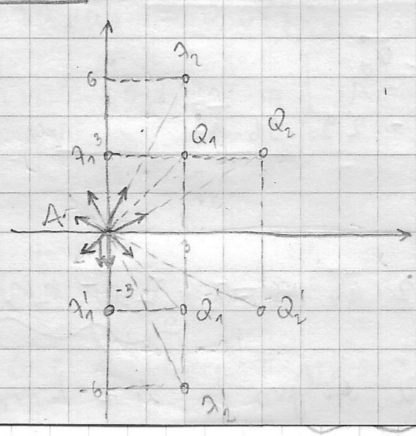
$$Q_{vez} = 11.3 \ nC$$

2.54. $\rho = 0$
$h = 3 \ m$
$Q_1 = 20 \ nC = 20 \cdot 10^{-9} \ C$
$Q_2 = -40 \ nC = -40 \cdot 10^{-9} \ C$
$\lambda_1 = 45 \ nC/m = 45 \cdot 10^{-9} \ \frac{C}{m}$
$\lambda_2 = -10 \ nC/m = -10 \cdot 10^{-9} \ \frac{C}{m}$



a) $E_A = ?$

zrcaljenje $\Rightarrow$



$Q_1' = -Q_1$
$Q_2' = -Q_2$
$\lambda_1' = -\lambda_1$
$\lambda_2' = \lambda_2$

$$E_{Q_1} = \frac{Q_1}{4\pi\varepsilon_0 r^2} \left(-\cos\alpha \, \vec{a_x} - \sin\alpha \, \vec{a_y}\right)$$

$$E_{Q_2} = \frac{Q_2}{4\pi\varepsilon_0 r^2} \left(\cos\alpha \, \vec{a_x} + \sin\alpha \, \vec{a_y}\right)$$

$$E_{\lambda_1} = \frac{\lambda_1}{4\pi\varepsilon_0 r} \left(-\vec{a_y}\right)$$

$$E_{\lambda_2} = \frac{\lambda_2}{4\pi\varepsilon_0 r} \left(\cos\alpha \, \vec{a_x} + \sin\alpha \, \vec{a_y}\right)$$

$$\vec{E}_{\lambda_1} = \frac{\lambda_1}{2\pi\varepsilon_0 \cdot 3}\left(-\vec{a_y}\right) \qquad \vec{E}_{\lambda_1'} = \frac{\lambda_1}{2\pi\varepsilon_0 \cdot 3}\left(-\vec{a_y}\right)$$

$$\vec{E}_{Q_1} = \frac{Q_1}{4\pi\varepsilon_0 (\sqrt{3^2+3^2})^2}\left(-\frac{\sqrt{2}}{2}\vec{a_x} - \frac{\sqrt{2}}{2}\vec{a_y}\right) \qquad \vec{E}_{Q_1'} = \frac{-Q_1}{2\pi\varepsilon_0 (\sqrt{18})^2}\left(+\frac{\sqrt{2}}{2}\vec{a_x} - \frac{\sqrt{2}}{2}\vec{a_y}\right)$$

$$\vec{E}_{Q_2} = \frac{Q_2}{4\pi\varepsilon_0 (\sqrt{6^2+3^2})^2}\left(+\frac{6}{\sqrt{6^2+3^2}}\vec{a_x} + \frac{3}{\sqrt{6^2+3^2}}\vec{a_y}\right) \qquad \vec{E}_{Q_2'} = \frac{-Q_2}{2\pi\varepsilon_0 (\sqrt{45})^2}\left(-\frac{6}{\sqrt{45}}\vec{a_x} + \frac{3}{\sqrt{45}}\vec{a_y}\right)$$

$$\vec{E}_{\lambda_2} = \frac{\lambda_2}{2\pi\varepsilon_0 \cdot \sqrt{3^2+6^2}}\left(+\frac{3}{\sqrt{6^2+3^2}}\vec{a_x} + \frac{6}{\sqrt{6^2+3^2}}\vec{a_y}\right) \qquad \vec{E}_{\lambda_2'} = \frac{\lambda_2}{2\pi\varepsilon_0 \cdot \sqrt{45}}\left(-\frac{3}{\sqrt{45}}\vec{a_x} + \frac{6}{\sqrt{45}}\vec{a_y}\right)$$

$$\vec{E}_{\lambda_1} = -269.63\,\vec{a_y}\ \left[\frac{V}{m}\right] \qquad \vec{E}_{Q_2} = +7.14\,\vec{a_x} + 3.57\,\vec{a_y}\ \left[\frac{V}{m}\right]$$

$$\vec{E}_{\lambda_1'} = -269.63\,\vec{a_y}\ \left[\frac{V}{m}\right] \qquad \vec{E}_{Q_2'} = -7.14\,\vec{a_x} + 3.57\,\vec{a_y}\ \left[\frac{V}{m}\right]$$

$$\vec{E}_{Q_1} = -7.06\,\vec{a_x} - 7.06\,\vec{a_y}\ \left[\frac{V}{m}\right] \qquad \vec{E}_{\lambda_2} = +11.98\,\vec{a_x} + 23.97\,\vec{a_y}\ \left[\frac{V}{m}\right]$$

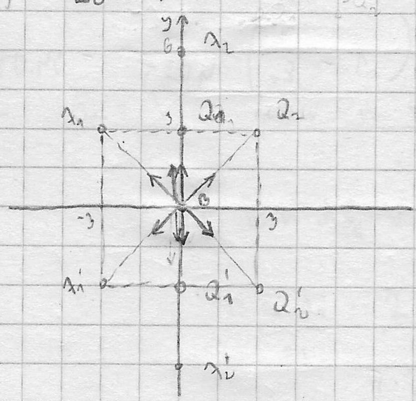
$$\vec{E}_{Q_1'} = +7.06\,\vec{a_x} - 7.06\,\vec{a_y}\ \left[\frac{V}{m}\right] \qquad \vec{E}_{\lambda_2'} = -11.98\,\vec{a_x} + 23.97\,\vec{a_y}\ \left[\frac{V}{m}\right]$$

$\vec{E}_A = ?$

$$\vec{E}_A = \vec{E}_{\lambda_1} + \vec{E}_{\lambda_1'} + \vec{E}_{Q_2} + \vec{E}_{\lambda_2} + \vec{E}_{Q_1} + \vec{E}_{Q_1'} + \vec{E}_{Q_2'} + \vec{E}_{\lambda_2'}$$

$$\vec{E}_A = -498.3\,\vec{a_y} \Rightarrow E_A = 498.3\ \frac{V}{m}$$

b) $E_B = ?$



$Q_1' = -Q_1$
$Q_2' = -Q_2$
$\lambda_1' = -\lambda_1$
$\lambda_2' = -\lambda_2$

$$\vec{E}_{Q_1} = \frac{|Q_1|}{4\pi\varepsilon_0 r^2}\left(-\vec{a_y}\right) = \vec{E}_{Q_1'}$$

$$\vec{E}_{Q_2} = \frac{|Q_2|}{4\pi\varepsilon_0 r^2}\left(\cos\alpha\,\vec{a_x} + \sin\alpha\,\vec{a_y}\right)$$

$$\vec{E}_{\lambda_1} = \frac{|\lambda_1|}{2\pi\varepsilon_0 r}\left(\cos\alpha\,\vec{a_x} - \sin\alpha\,\vec{a_y}\right)$$

$$\vec{E}_{\lambda_1'} = \frac{|\lambda_1|}{2\pi\varepsilon_0 r}\left(-\cos\alpha\,\vec{a_x} - \sin\alpha\,\vec{a_y}\right)$$

$$\vec{E}_{\lambda_2} = \vec{E}_{\lambda_2'} = \frac{|\lambda_2|}{2\pi\varepsilon_0 r}\,\vec{a_y}$$

$$\vec{E}_{Q_1} = -19.97\,\vec{a_y}$$

$$\vec{E}_{Q_1'} = -19.97\,\vec{a_y}$$

$$\vec{E}_{Q_2} = \frac{40\cdot 10^{-9}}{4\pi\cdot 8.854\cdot 10^{-12}\cdot(\sqrt{3^2+3^2})^2}\left(\frac{3}{\sqrt{18}}\vec{a_x} + \frac{3}{\sqrt{18}}\vec{a_y}\right) = 14.12\,\vec{a_x} + 14.12\,\vec{a_y}$$

$$\vec{E}_{Q_2'} = -14.12\,\vec{a_x} + 14.12\,\vec{a_y}$$

$$\vec{E}_{\lambda_1} = \frac{45\cdot 10^{-9}}{2\pi\cdot 8.854\cdot 10^{-12}\cdot\sqrt{3^2+3^2}}\left(\frac{3}{\sqrt{18}}\vec{a_x} - \frac{3}{\sqrt{18}}\vec{a_y}\right) = 134.82\,\vec{a_x} - 134.82\,\vec{a_y}$$

$$\vec{E}_{\lambda_1'} = -134.82\,\vec{a_x} - 134.82\,\vec{a_y}$$

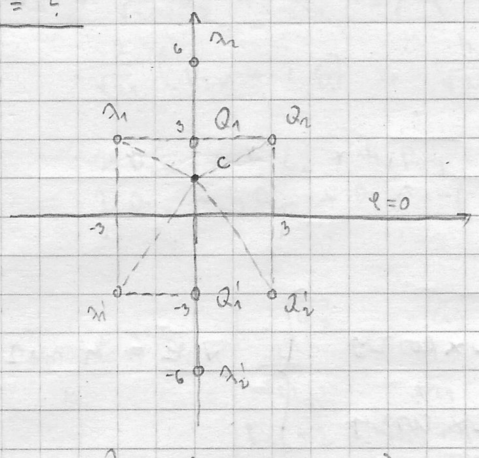
$$\vec{E}_{\lambda_2} = \frac{10\cdot 10^{-9}}{2\pi\cdot 8.854\cdot 10^{-12}\cdot 6}\,\vec{a_y} = 29.96\,\vec{a_y}$$

$$\vec{E}_{\lambda_2'} = 29.96\,\vec{a_y}$$

$$\vec{E}_B = \sum E_i = -221.42\,\vec{a_y} \Rightarrow E_B = 221.42\ \frac{V}{m}$$

* već je na slici vidljivo da su se x komponente $\vec{a_x}$ poništile!

c) $\varphi_c = ?$



$$\varphi_Q = \frac{Q}{4\pi\varepsilon_0}\left(\frac{1}{r} - \frac{1}{r_{REF}}\right)$$

$$\varphi_\lambda = \frac{\lambda}{2\pi\varepsilon_0} \ln \frac{r_{REF}}{r}$$

$$\varphi_{Q_1} = \frac{Q_1}{4\pi\varepsilon_0}\left(\frac{1}{1.5} - \frac{1}{3}\right) = 59.92 \text{ V}$$

$$\varphi_{Q_1'} = \frac{-Q_1}{4\pi\varepsilon_0}\left(\frac{1}{1.5+3} - \frac{1}{3}\right) = 19.97 \text{ V}$$

$$\varphi_{Q_2} = \frac{Q_2}{4\pi\varepsilon_0}\left(\frac{1}{\sqrt{3^2+1.5^2}} - \frac{1}{3}\right) = 12.65 \text{ V}$$

$$\varphi_{Q_2} = \frac{-Q_2}{4\pi\varepsilon_0}\left(\frac{1}{\sqrt{3^2+(1.5+3)^2}} - \frac{1}{3}\right) = -53.36 \text{ V}$$

$$\varphi_{\lambda_1} = \frac{\lambda_1}{2\pi\varepsilon_0} \ln \frac{3}{\sqrt{3^2+1.5^2}} = -90.25 \text{ V}$$

$$\varphi_{\lambda_1'} = \frac{-\lambda_1}{2\pi\varepsilon_0} \ln \frac{3}{\sqrt{3^2+(1.5+3)^2}} = 476.71 \text{ V}$$

$$\varphi_{\lambda_2} = \frac{\lambda_2}{2\pi\varepsilon_0} \ln \frac{6}{4.5} = -51.71 \text{ V}$$

$$\varphi_{\lambda_2'} = \frac{-\lambda_2}{2\pi\varepsilon_0} \ln \frac{6}{7.5} = -40.11 \text{ V}$$

$$\varphi_c = \sum \varphi = 333.82$$

$$\varphi_c = 334 \text{ V}$$

d) $Q = 15$ pC

$A_{AC} = ?$

$$A_{AC} = Q(\varphi_c - \overset{0}{\varphi_A}) = 15 \cdot 10^{-12} \cdot 334$$

$$A_{AC} = 5.01 \text{ nJ}$$

55. $\vec{E} = A\cos(kx)\cos(ky)\,\vec{a}_x + \sin(kx)\sin(ky)\,\vec{a}_y$

$k = 2$

a) $E = ?$ u $T_1(1, 2, 3)$

$$\vec{E} = A \cdot 0.272\,\vec{a}_x - 0.688\,\vec{a}_y$$

$$E = \sqrt{(A \cdot 0.272)^2 + (0.688)^2} = \left| A = 1 \right| = 0.740 \ \frac{\text{V}}{\text{m}}$$

$$E = 740 \ \frac{\text{mV}}{\text{m}}$$

b) $\rho_s = ?$ u $T_2(1, 2, 4)$

$$\Delta\varphi = -\frac{\rho_s}{\varepsilon} \Rightarrow \rho_s = \varepsilon \cdot -\Delta\varphi = \varepsilon - \nabla(\nabla\varphi) = \varepsilon\nabla\overbrace{(-\nabla\varphi)}^{\vec{E}}$$

$$\rho_s = \varepsilon \nabla \vec{E}$$

$$\nabla \vec{E} = \frac{\partial E_x}{\partial x} + \frac{\partial E_y}{\partial y}$$

$$E_x = -\cos(2x)\cos(2y)$$
$$E_y = \sin(2x)\sin(2y)$$

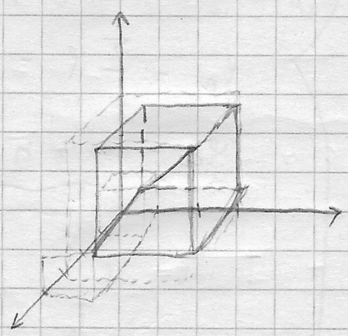
$$\left.\begin{aligned} \frac{\partial E_x}{\partial x} &= -\cos 2y \cdot [-\sin(2x)] \cdot 2 = 2 \sin 2x \cos 2y \\ \frac{\partial E_y}{\partial y} &= \sin 2x \cdot \cos 2y \cdot 2 = 2 \sin 2x \cos 2y \end{aligned}\right\} \nabla \vec{E} = 4 \sin 2x \cos 2y$$

$$\rho_s = 4 \varepsilon_0 \sin 2x \cos 2y = 4 \cdot 8.854 \cdot 10^{-12} \cdot \sin 2 \cdot \cos 4$$

$$\rho_s = -21.05 \ \frac{pC}{m^3}$$

c) $\Phi = ?$

przestrzeń ograniczona kubem (1,1,1) i (3,3,3)



$$\Phi = \iiint_V \rho \, dV = \iiint_V \varepsilon \nabla \vec{E} \, dV =$$
$$= 4 \varepsilon_0 \cdot \int_1^3 \sin 2x \, dx \int_1^3 \cos 2y \, dy \int_1^3 dz =$$
$$= 4 \varepsilon_0 \cdot \left. \frac{-\cos 2x}{2} \right|_1^3 \cdot \left. \frac{\sin 2y}{2} \right|_1^3 \cdot \left. z \right|_1^3 =$$
$$= 29 \ pC$$

d) $\nabla \times \vec{E} = ?$ w $T_3(2,1,2)$

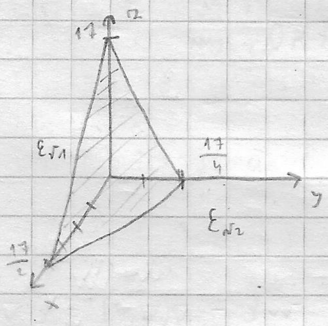
$$\nabla \times \vec{E} = \begin{vmatrix} \vec{a_x} & \vec{a_y} & \vec{a_z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ -\cos 2x \cos 2y & \sin 2x \sin 2y & 0 \end{vmatrix} = 0 \cdot \vec{a_x} - 0 \cdot \vec{a_y} + (2 \cos 2x \sin 2y - 2 \cos 2x \sin 2y) \vec{a_z}$$

$$= 0$$

$$\nabla \times \vec{E} = 0 \quad \text{w} \ \forall T(x,y,z)$$

2.56. $\varepsilon_{r1} = 3$
$\varepsilon_{r2} = 2$
$2x + 4y + z = 17$ - granica 2 dielektryków
$\vec{E_1} = -2\vec{a_x} - \vec{a_y} + 2\vec{a_z} \ \left[\frac{V}{m}\right]$

$\vec{D_2} = ?$ $U_{AB} = ?$ $A(0,0,0)$ i $B(1,0,0)$



$$\vec{n} = \frac{2\vec{a_x} + 4\vec{a_y} + \vec{a_z}}{\sqrt{4+16+1}} = \frac{2}{\sqrt{21}} \vec{a_x} + \frac{4}{\sqrt{21}} \vec{a_y} + \frac{1}{\sqrt{21}} \vec{a_z}$$

warunki graniczne: $\vec{n} \cdot (\vec{D_2} - \vec{D_1}) = \sigma_s = 0$
$\vec{n} \times (\vec{E_2} - \vec{E_1}) = 0$

$$\left(\frac{2}{\sqrt{21}}\vec{a_x} + \frac{4}{\sqrt{21}}\vec{a_y} + \frac{1}{\sqrt{21}}\vec{a_z}\right)\cdot\left(D_{2x}\vec{a_x} + D_{2y}\vec{a_y} + D_{2z}\vec{a_z} + 2\varepsilon_1\vec{a_x} + \varepsilon_1\vec{a_y} - 2\varepsilon_1\vec{a_z}\right) = 0$$

$$\frac{2}{\sqrt{21}}\left(D_{2x} + 2\varepsilon_1\right) + \frac{4}{\sqrt{21}}\left(D_{2y} + \varepsilon_1\right) + \frac{1}{\sqrt{21}}\left(D_{2z} - 2\varepsilon_1\right) = 0 \quad /\cdot\sqrt{21}$$

$$2D_{2x} + 4\varepsilon_1 + 4D_{2y} + 4\varepsilon_1 + D_{2z} - 2\varepsilon_1 = 0$$

$$2D_{2x} + 4D_{2y} + D_{2z} = -6\varepsilon_1 \qquad (1)$$

$$\vec{n}\times(\vec{E_2} - \vec{E_1}) = \begin{vmatrix} \vec{a_x} & \vec{a_y} & \vec{a_z} \\ \frac{2}{\sqrt{21}} & \frac{4}{\sqrt{21}} & \frac{1}{\sqrt{21}} \\ E_{2x}+2 & E_{2y}+1 & E_{2z}-2 \end{vmatrix} = \left[\frac{4}{\sqrt{21}}(E_{2z}-2) - \frac{1}{\sqrt{21}}(E_{2y}+1)\right]\vec{a_x} -$$

$$-\left[\frac{2}{\sqrt{21}}(E_{2z}-2) - \frac{1}{\sqrt{21}}(E_{2x}+2)\right]\vec{a_y} + \left[\frac{2}{\sqrt{21}}(E_{2y}+1) - \frac{4}{\sqrt{21}}(E_{2x}+2)\right]\vec{a_z}$$

$$4(E_{2z}-2) = E_{2y}+1 \Rightarrow 4E_{2z} = E_{2y}+9 \quad /\cdot\varepsilon_2$$
$$2(E_{2z}-2) = E_{2x}+2 \Rightarrow 2E_{2z} = E_{2x}+6 \quad /\cdot\varepsilon_2$$
$$2(E_{2y}+1) = 4E_{2x}+8 \Rightarrow 2E_{2y} = 4E_{2x}+6 \quad /\cdot\varepsilon_2$$

$$2D_{2z} = D_{2x} + 6\varepsilon_2 \Rightarrow D_{2z} = 0.5D_{2x} + 3\varepsilon_2 \qquad (2)$$
$$2D_{2y} = 4D_{2x} + 6\varepsilon_2 \Rightarrow D_{2y} = 2D_{2x} + 3\varepsilon_2 \qquad (3)$$

(2) i (3) u (1) $\Rightarrow$ $2D_{2x} + 4(2D_{2x} + 3\varepsilon_2) + (0.5D_{2x} + 3\varepsilon_2) = 6\varepsilon_1$

$$10.5\,D_{2x} = 6\varepsilon_1 - 15\varepsilon_2 \quad /:10.5$$

$$D_{2x} = \varepsilon_0\cdot\frac{-6\varepsilon_{r1} - 15\varepsilon_{r2}}{10.5}$$

$$\underline{D_{2x} = -40.48\ \frac{pC}{m^2}} \Rightarrow \underline{D_{2y} = -27.83\ \frac{pC}{m^2}}$$

$$\Rightarrow \underline{D_{2z} = 32.89\ \frac{pC}{m^2}}$$

$$U_{AB} = -\int_B^A \vec{E_1}\,d\vec{l}$$

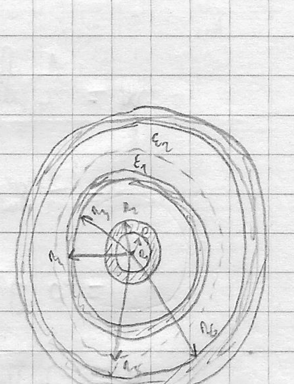
$$\left.\begin{matrix} x \Rightarrow 0 \to 1 \\ y = 0 \\ z = 0 \end{matrix}\right\}\ d\vec{l} = \vec{a_x}\,dx$$

$$U_{AB} = -\int_1^0 -2\,dx = 2\cdot x\Big|_1^0 = \underline{-2\ V}$$

2.57. cilindrični kondenzator, 2 sloja dielektrika

$$\rho = \begin{cases} 0, & r < r_1 \\ \frac{1}{r^4+81}, & r_1 \le r \le r_2 \quad \left[\frac{C}{m^3}\right] \\ 0, & r > r_2 \end{cases}$$

$\varepsilon_{r1} = 4$
$\varepsilon_{r2} = 2$
$r_1 = 1\ cm = 0.01\ m$
$r_2 = 3\ cm = 0.03\ m$
$r_3 = 4.5\ cm = 0.045\ m$
$r_4 = 5\ cm = 0.05\ m$
$r_5 = 10\ cm = 0.1\ m$
$r_6 = 20\ cm = 0.2\ m$



a) $E = ?$ na $r = 9\ cm = 0.09\ m$

* za $r < R_3$ i $R_1 \leq r \leq R_2$ vrijedi:

Gaussov zakon: $\varepsilon_0 E_1 \cdot 2 r \pi \cdot l = \int_{0.01}^{0.03} \frac{r}{r^4+81} dr \int_0^{2\pi} d\varphi \int_0^l dz$

$$\varepsilon_0 E_1 \cdot \cancel{2 r \pi} \cdot \cancel{l} = \frac{1}{2 \cdot 9} \operatorname{arctg} \frac{r^2}{9} \Big|_{0.01}^{0.03} \cdot \cancel{2\pi} \cdot \cancel{l}$$

$$\vec{E}_1 = \frac{557744.7014}{r} \vec{a}_r$$

$$n \cdot (\vec{D}_2 - \vec{D}_1) = \sigma_s = 0$$

$\vec{D}_1$: $r < R_3$, $\varepsilon_{r1} = 1$

$\vec{D}_2$: $r < R_3$, $\varepsilon_{r2} = 4$

$$\vec{a}_r (\varepsilon_2 E_2 \vec{a}_r - \varepsilon_1 E_1 \vec{a}_r) = 0$$

$$\varepsilon_0 \varepsilon_{r2} E_2 - \varepsilon_0 E_1 = 0 \Rightarrow E_2 = \frac{E_1}{\varepsilon_{r2}} = \frac{E_1}{4}$$

$$E_2 = \frac{557744.7014}{4 r} \Big/ r = 0.09\ m$$

$$\underline{E_2 = 1549.3\ \frac{kV}{m}}$$

b) $\vec{P} = ?$ na $r = 0.12\ m$

iz a) $\Rightarrow \vec{E}_2 = \frac{139436.1754}{r} \vec{a}_r$

$$\vec{E}_3 = \frac{\varepsilon_{r2}}{\varepsilon_{r3}} \vec{E}_2 = \frac{4}{2} \vec{E}_2 = 2 \vec{E}_2 = \frac{278872.3507}{r} \vec{a}_r$$

$$\vec{P} = \varepsilon_0 (\varepsilon_{r3} - 1) \vec{E}_3 = \varepsilon_0 (2 - 1) \vec{E}_3$$

$$\vec{P} = \varepsilon_0 \cdot \frac{278872.7500}{r} \vec{a}_r \Big/ r = 0.12\ m$$

$$\underline{P = 20.58\ \frac{\mu C}{m^2}}$$

c) $\underline{W' = ? \left[\frac{J}{m}\right]}$

$$W = \underbrace{\frac{1}{2} \oint_S \varphi \overset{0}{\vec{D}} \vec{n}\, dS}_{=0} + \frac{1}{2} \iiint_V \vec{D} \vec{E}\, dV = \frac{1}{2} \iiint_V \varepsilon |\vec{E}|^2 dV$$

$$W = \frac{1}{2} \iiint_V \varepsilon_0 (\varepsilon_{r2} E_2^2 + \varepsilon_{r3} E_3^2) dV = \frac{1}{2} \varepsilon_0 \left[\int_{0.05}^{0.1} \varepsilon_{r2} \frac{139436^2}{r^{\cancel{2}}} \cancel{r} dr + \int_{0.1}^{0.2} \varepsilon_{r3} \frac{278872^2}{r} dr \right]$$

$$= \frac{1}{2} \varepsilon_0 \varepsilon_{r2} \cdot 139436^2 \cdot \ln \frac{0.1}{0.05} + \frac{1}{2} \varepsilon_0 \varepsilon_{r3} \cdot 278872^2 \cdot \ln \frac{0.2}{0.1} =$$

$$= 0.72\ J$$

$$\underline{W' = 2\pi W = 4.5\ \frac{J}{m}}$$

d) $C' = ?$

$$U = -\int_B^A \vec{E}\, d\vec{l} = -\int_{R_5}^{R_4} E_2\, dr - \int_{R_6}^{R_5} E_3\, dr = 139436 \ln \frac{R_5}{R_4} + 278872 \ln \frac{R_6}{R_5}$$

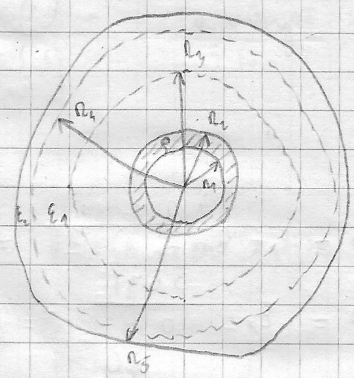
$$U = 289.95 \text{ kV}$$

$$W' = \frac{1}{2} C' U^2 \Rightarrow C' = \frac{2W'}{U^2}$$

$$C' = 10.7 \ \mu F$$

2.68.

$$\rho = \begin{cases} 0 & r < R_1 \\ \dfrac{10^{-6}}{r+1}, & R_1 \le r \le R_2 \\ 0 & r > R_2 \end{cases} \left[\frac{C}{m^3}\right]$$



*slane ljuske

$\varepsilon_{r1} = 4$ $R_1 = 0.01$ m $R_4 = 0.1$ m

$\varepsilon_{r2} = 2$ $R_2 = 0.03$ m $R_5 = 0.14$ m

$R_3 = 0.045$ m

a) $E_a = ?$ u $r = 0.02$ m

$R_1 \le r \le R_2 \Rightarrow$ Gaussov zakon

$$\varepsilon_0 E_a \cdot 4 r^2 \pi = \int_{0.01}^{r} \frac{10^{-6}}{r+1} r^2 dr \int_0^{\pi} \sin\theta d\theta \int_0^{2\pi} d\lambda$$

$$\varepsilon_0 E_a 4 r^2 \pi = 10^{-6} \int_{0.01}^{r} \frac{r^2}{r+1} dr \cdot (-\cos\theta)\Big|_0^{\pi} \cdot 2\pi$$

$$\varepsilon_0 E_a \cdot \cancel{4} r^2 \cancel{\pi} = 10^{-6} \cdot \int_{0.01}^{r} \frac{r^2}{r+1} dr \cdot \cancel{2} \cancel{2\pi}$$

$$r = 0.02 \Rightarrow \varepsilon_0 E_a \cdot 0.02^2 = 10^{-6} \cdot \int_{0.01}^{0.02} \frac{r^2}{r+1} dr$$

$$10^{-6} \int_{0.01}^{0.02} \frac{r^2}{r+1} dr = 10^{-6} \left[\frac{x^2}{2} - x + \ln(x+1) - \frac{3}{2}\right]\Big|_{0.01}^{0.02} =$$

$$= 2.296 \cdot 10^{-12}$$

$$E_a = \frac{2.296 \cdot 10^{-12}}{0.0004 \varepsilon_0} = 648.41 \ \frac{V}{m}$$

b) $D_b = ?$ u $r = 0.04$ m

Gaussov zakon: $\varepsilon_0 E_2 \cdot \cancel{4} r^2 \cancel{\pi} = \int_{0.01}^{0.03} \frac{10^{-6} r^2}{r+1} \cdot \cancel{2}\cancel{2\pi}$

$$D_2 = \frac{\int_{0.01}^{0.03} \frac{10^{-6} r^2}{r+1}}{r^2} = \frac{8.471 \cdot 10^{-12}}{r^2}$$

$$n \cdot (\vec{D_b} - \vec{D_2}) = \sigma_s = 0$$

$$\vec{D_b} = \vec{D_2} \Rightarrow D_b = D_2(r = 0.04) = \frac{8.471 \cdot 10^{-12}}{0.04^2}$$

$$D_b = 5.3 \ \frac{nC}{m^2}$$

c) $P = ?$ u $r = 13$ cm $= 0.13$ m

$R_4 \le r \le R_5 \Rightarrow$ sredstvo $\varepsilon_{r2} = 2$

do $r \le R_3 = 0.045$ m je $D_3 = \frac{8.471 \cdot 10^{-12}}{r^2} \Rightarrow E_3 = \frac{8.471 \cdot 10^{-12}}{\varepsilon_0 r^2} = \frac{0.957}{r^2}$

prva granica: $n(\vec{D}_4 - \vec{D}_3) = \sigma_s = 0$

$\varepsilon_0 \varepsilon_{r1} E_4 - \varepsilon_0 E_3 = 0 \Rightarrow E_4 = \frac{E_3}{\varepsilon_{r1}} = \frac{E_3}{4} = \frac{0.239}{r^2}$

druga granica: $n(\vec{D}_5 - \vec{D}_4) = \sigma_s = 0$

$\varepsilon_0 \varepsilon_{r2} E_5 - \varepsilon_0 \varepsilon_{r1} E_4 = 0 \Rightarrow E_5 = \frac{\varepsilon_{r1}}{\varepsilon_{r2}} E_4 = 2 E_4 = \frac{0.478}{r^2}$

$P = \varepsilon_0 (\varepsilon_{r2} - 1) E_5 = \varepsilon_0 \cdot (2-1) \cdot E_5$

$P = \varepsilon_0 \cdot \frac{0.478}{0.13^2} = 250.63 \cdot 10^{-12}$

$P = 251 \frac{pC}{m^2}$

d) $W = ?$ između R_2 i R_3

$W = \frac{1}{2} \iiint_V \varepsilon_0 |\vec{E}|^2 dV$

za $R_2 \le r \le R_3$: $\vec{E} = \frac{10^{-6} \int_{0.01}^{0.03} \frac{r^2}{r+1} dr}{\varepsilon_0 r^2} \vec{a}_r = \frac{0.957}{r^2} \vec{a}_r \Big|^2$

$|\vec{E}|^2 = \frac{0.915}{r^4}$

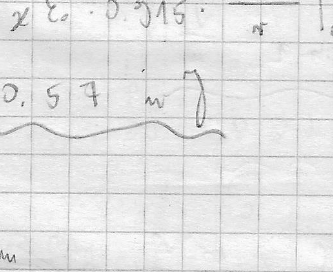
$W = \frac{1}{2} \varepsilon_0 \cdot 0.915 \cdot \int_{0.03}^{0.045} \frac{1}{r^4} \cdot r^2 dr \cdot \int_0^{\pi} \sin\theta \, d\theta \int_0^{2\pi} d\phi =$

$= \frac{1}{2} \varepsilon_0 \cdot 0.915 \cdot \frac{-1}{r} \Big|_{0.03}^{0.045} \cdot 2 \cdot 2\pi = 2\pi \varepsilon_0 \cdot 0.915 \left(\frac{1}{0.03} - \frac{1}{0.045} \right) = 5.68 \cdot 10^{-10}$

$= 0.57$ nJ

2.59. $d = 1$ m
$r = 0.5$ m
$\lambda = 17 \frac{nC}{m}$

a) $E_A = ?$



udaljenost točke od prstena

$E_{prsten} = \frac{\lambda \cdot r}{2 \varepsilon_0} \frac{z}{(r^2 + z^2)^{3/2}}$

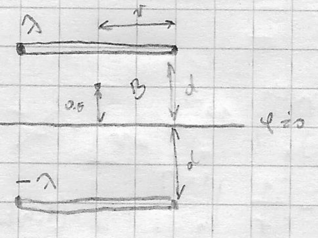
original: $E_{pr,0} = 0$ jer je $z = 0$

slika: $E_{prsten,2} = \frac{\lambda r}{2 \varepsilon_0} \cdot \frac{2d}{(r^2 + 4d^2)^{3/2}}$

$$E_A = E_{proton,o} + E_{proton,s} = 0 + \frac{17\cdot10^{-9}\cdot 0.5}{2\cdot 8.854\cdot10^{-12}}\cdot\frac{2d}{(0.5^2+2d^2)^{3/2}}$$

$$E_A = 109.6\ \frac{V}{m}$$

b)



$E_B = ?$

original: $$E_{y,original} = \frac{\lambda r}{2\varepsilon_0}\cdot\frac{d-0.5}{[r^2+(d-0.5)^2]^{3/2}} = \frac{17\cdot10^{-9}\cdot0.5}{2\varepsilon_0}\cdot\frac{0.5}{(0.5^2+0.5^2)^{3/2}} =$$

$$= 678.82\ \frac{V}{m}$$

slika: $$E_{y,slika} = \frac{\lambda r}{2\varepsilon_0}\cdot\frac{0.5+d}{[r^2+(0.5+d)^2]^{3/2}} = \frac{\lambda r}{2\varepsilon_0}\cdot\frac{1.5}{[0.5^2+1.5^2]^{3/2}} =$$

$$= 182.15\ \frac{V}{m}$$

$$E_B = E_{y,original} + E_{y,slika} = 890.97\ \frac{V}{m}$$

c) $W_{\infty\to A} = ?$

proton $\Rightarrow q_1 = e = 1.602\cdot10^{-19}$ C

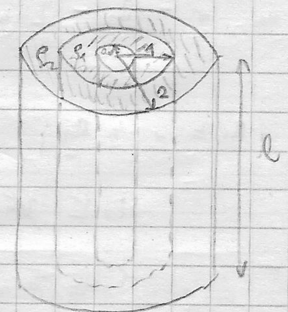
$$W_{\infty\to A} = -\int_{\infty}^{A} q\vec{E}\,d\vec{l} = q\cdot\varphi(A)$$

$$\varphi(A) = \frac{\lambda r}{2\varepsilon_0}\cdot\frac{1}{\sqrt{r^2+0^2}} + \frac{-\lambda r}{2\varepsilon_0}\cdot\frac{1}{\sqrt{r^2+4d^2}} = 727.16\ V$$

$$W_{\infty\to A} = 1.16\cdot10^{-16}\ J$$

60. ul. k. z. i

$$\rho = \begin{cases} 0, & r < 0.5\,m \\ \frac{10^{-9}}{r^4}, & 0.5\,m \le r \le 1\,m \\ \frac{10^{-9}}{r^3}, & 1\,m \le r \le 2\,m \end{cases} \quad \left[\frac{C}{m^3}\right]$$



$\varphi(r = 10\,m) = 0$

a) $\varphi(3,0,0) = ?$

Gaussov zakon: $$\varepsilon_0 E_1\cdot 2r\pi\cdot l = \left[\int_{0.5}^{1}\frac{10^{-9}}{r^{4}}\cdot r\,dr + \int_{1}^{2}\frac{10^{-9}}{r^{3}}\,r\,dr\right]\cdot 2\pi\cdot l$$

$$\varepsilon_0 E_1\cdot r = \left(-\frac{1}{2}\cdot\frac{1}{r^2}\Big|_{0.5}^{1} + \frac{-1}{r}\Big|_{1}^{2}\right)\cdot10^{-9}$$

$$E_1 = \frac{2\cdot10^{-9}}{\varepsilon_0 r}$$

$$\varphi(3) - \underbrace{\varphi(10)}_{=0} = \int_3^{10} \frac{2\cdot 10^{-9}}{\varepsilon_0 r}\,dr = \frac{2\cdot 10^{-9}}{8.854\cdot 10^{-12}} \cdot \ln\frac{10}{3}$$

$$\varphi(3) = 271.96\ \mathrm{V}$$

b) $E(1.4, 0, 0) = ?$

Gaussov zakon: $\varepsilon_0 E \cdot 1.4 \cdot 2\pi \cdot l = \left[\int_{0.5}^{1} \frac{10^{-9}}{r^3}\,dr + \int_1^{1.4} \frac{10^{-9}}{r^2}\,dr\right] \cdot \cancel{2\pi \cdot l}$

$$1.4\,\varepsilon_0 E = 10^{-9}\left[\frac{-1}{2}\cdot\frac{1}{r^2}\Big|_{0.5}^{1} + -\frac{1}{r}\Big|_1^{1.4}\right] \quad \Big/ : (1.4\,\varepsilon_0)$$

$$E(1.4) = 144.06\ \frac{\mathrm{V}}{\mathrm{m}}$$

c) $E(0.7, 0, 0) = ?$

Gaussov zakon: $\varepsilon_0 E \cdot 0.7 \cdot \cancel{2\pi l} = \int_{0.5}^{0.7} \frac{10^{-9}}{r^3}\,dr \cdot \cancel{2\pi l}$

$$0.7\,\varepsilon_0 E = 10^{-9}\cdot\frac{-1}{2}\,\frac{1}{r^2}\Big|_{0.5}^{0.7} \quad \Big/ : (0.7\,\varepsilon_0)$$

$$E(0.7) = 158.05\ \frac{\mathrm{V}}{\mathrm{m}}$$

d) $\varphi(0, 0, 0) = ?$

Gaussov zakon: $0 = 0$ ✓ $\Rightarrow$ ne možemo koristiti tu formulu

za $r \geq R_2$;

$$\varepsilon_0 E \cdot \cancel{2\pi}\, r \cdot \cancel{l} = \left[\int_{0.5}^{1} \frac{10^{-9}}{r^3}\,dr + \int_1^{2} \frac{10^{-9}}{r^2}\,dr\right] \cancel{2\pi l}$$

$$E_2 = \frac{2\cdot 10^{-9}}{\varepsilon_0 r}$$

za $r = R_2 = 2\ \mathrm{m} \Rightarrow \varphi(2) - \underbrace{\varphi(10)}_{=0} = \int_2^{10} \frac{2\cdot 10^{-9}}{\varepsilon_0 r} = \frac{2\cdot 10^{-9}}{\varepsilon_0} \ln\frac{10}{2} = 363.54\ \mathrm{V}$

za $R_1 \leq r \leq R_2$;

$$\varepsilon_0 E\, \cancel{2\pi}\, r\, \cancel{l} = \left[\int_{0.5}^{1} \frac{10^{-9}}{r^3}\,dr + \int_1^{r} \frac{10^{-9}}{r^2}\,dr\right] \cancel{2\pi l}$$

$$\varepsilon_0 E_1 \cdot r = 1.5\cdot 10^{-9} + 10^{-9}\cdot\left(1 - \frac{1}{r}\right) = 2.5\cdot 10^{-9} - \frac{10^{-9}}{r} \quad \Big/ : \varepsilon_0 r$$

$$E_1 = \frac{2.5\cdot 10^{-9}}{\varepsilon_0 r} - \frac{10^{-9}}{\varepsilon_0 r^2}$$

za $r = R_1 = 1\ \mathrm{m} \Rightarrow \varphi(1) - \varphi(2) = \int_1^2 \left(\frac{2.5\cdot 10^{-9}}{\varepsilon_0 r} - \frac{10^{-9}}{\varepsilon_0 r^2}\right) dr = 139.24\ \mathrm{V}$

$$\varphi(1) = 139.24 + 363.54 = 502.78$$

za $R_0 \leq r \leq R_1$; $\varepsilon_0 E \cdot \cancel{2\pi r l} = \int_{0.5}^{r} \frac{10^{-9}}{r^3}\,dr \cdot \cancel{2\pi l} \Rightarrow E_0 = \frac{2}{\varepsilon_0 r} - \frac{1}{2\varepsilon_0 r^3}$

za $r = R_0 = 0.5\ \mathrm{m} \Rightarrow \varphi(0.5) - \varphi(1) = \int_{0.5}^{1} 10^{-9}\left(\frac{2}{\varepsilon_0 r} - \frac{1}{2\varepsilon_0 r^3}\right) dr = 71.86\ \mathrm{V} \ \Big/ + \varphi(1)$

$$\varphi(0.5) = \varphi(0) = 574.64\ \mathrm{V}$$

2.61. $m = 10^{-12}$ kg

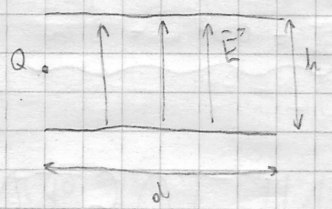
$\vec{v}_0 = \vec{a_x} - \vec{a_y}$ $\frac{m}{s}$

$E = 2$ $\frac{V}{m}$

$d = 0.01$ m

$h = 0.2$ m

$Q = 1 \cdot 10^{-9}$ C



a) $v_{iz} = ?$

$$\vec{F} = m\vec{a} \quad \Rightarrow \quad m\vec{a} = Q\vec{E}$$

$$m \cdot \frac{d\vec{v}}{dt} = Q\vec{E}$$

$$m\frac{dv_x}{dt}\vec{a_x} + m\frac{dv_y}{dt}\vec{a_y} = QE\vec{a_y}$$

(1.) $m\dfrac{dv_x}{dt} = 0 \quad \Big| : m \quad \Big| \int$

$$v_x = \text{konst.} \quad \Rightarrow \quad v_x(t=0) = v_{x0} = 1$$

(2.) $m\dfrac{dv_y}{dt} = QE$

$$\frac{dv_y}{dt} = \frac{QE}{m} \quad \Big| \int$$

$$v_y = \frac{QE}{m}t + \underbrace{v_{y0}}_{=-1} = \frac{QE}{m}t - 1 = 2000\,t - 1$$

$$\vec{v} = \vec{a_x} + (2000\,t - 1)\vec{a_y} \quad \left[\frac{m}{s}\right]$$

$$v_x = \frac{dx(t)}{dt} \quad \Rightarrow \quad x(t) = \int v_x(t)\,dt = v_{x0}t + \underbrace{x_0}_{=0} = v_{x0}t$$

$$t = \frac{x(t)}{v_{x0}}$$

Kraj je kretnja na ploči pa je $t_{iz} = \dfrac{x_{max}}{v_{x0}} = \dfrac{d}{v_{x0}} = 0.01$ s $\Rightarrow v_y = 19$

$$\vec{v} = \vec{a_x} + 19\vec{a_y} \quad \Rightarrow \quad v = \sqrt{1^2 + 19}$$

$$v = 19.03 \text{ m/s}$$

b) i c) $y_{min} = ?$ $\quad t_{min} = ?$

$$v_y(t) = \frac{dy(t)}{dt} \Rightarrow y(t) = \int v_y(t)\,dt = \int (2000\,t - 1)\,dt = 2000 \cdot \frac{t^2}{2} - t + \underbrace{y_0}_{=0} =$$

$$= 1000\,t^2 - t$$

– traži se minimum: $\dfrac{dy(t)}{dt} = 0$

$$1000 \cdot 2\,t_{min} - 1 = 0$$

$$t_{min} = \frac{1}{2000} = 0.5 \text{ ms} \quad \Rightarrow \quad y_{min} = -\frac{1}{4000} = -0.25 \text{ mm}$$

d) $y_{vr} = ?$

$$\left.\begin{array}{l} y(t) = 1000t^2 - t \\ t_{vr} = 0.010 \end{array}\right\} \quad y_{vr} = 0.09\ m = 9\ cm$$