STRUJNICE

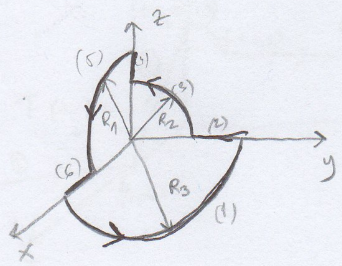
$$V = \frac{a\sqrt{3}}{2} \qquad P = \frac{a v_a}{2}$$

(jednakostrani)

pr. 1.2.1
1.2.5

STRUJNICE

LJ 14/15 H u ishodištu?



pr. 1.2.5.

(1) $\overline{H}_1 = \frac{I}{8R_3}\,\overline{a_z}$

(3) $\overline{H}_3 = \frac{I}{8R_2}\,\overline{a_x}$

(5) $\overline{H}_5 = \frac{I}{8R_1}\,\overline{a_y}$

$\underbrace{\qquad\qquad\qquad}$

$H_{uk} = \overline{H_1} + \overline{H_3} + \overline{H_5}$

$H_{uk} = \frac{I}{8}\left(\frac{1}{R_2}\overline{a_x} + \frac{1}{R_1}\overline{a_y} + \frac{1}{R_3}\overline{a_z}\right)$

(2) $\overline{r} = 0 \longrightarrow$ ishodište

$\overline{r'} = y\overline{a_y}$

$\overline{R} = \overline{r} - \overline{r'} = -y\overline{a_y}$

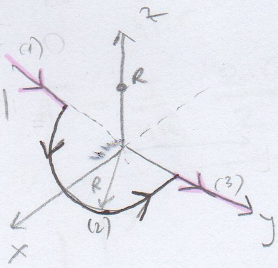
$d\overline{l} = \overline{a_y}\,dy$

$d\overline{l} \times \overline{R} = 0$

$dH = \frac{1}{4\pi}\int \frac{d\overline{l} \times \overline{R}}{R^3}$

$\boxed{H_2 = 0 = H_4 = H_6 = 0}$

3.13.

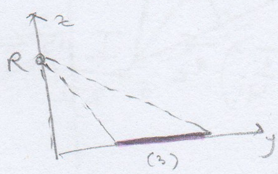


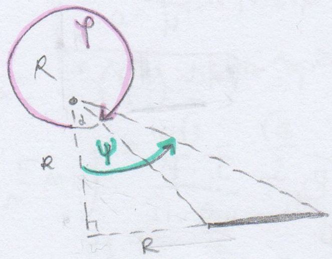
pr. 1.2.1.

pr. 1.2.5.

$H(0,0,R) = ?$ $\quad \overline{r} = R\overline{a_z}$

(3)





$\varphi = 360^\circ - \alpha$, $\alpha = 45^\circ$

$\varphi = 315^\circ$

$\psi = 90^\circ$ (jer ide u ∞)

$H_3 = \frac{I}{4\pi R}(\sin\varphi + \sin\psi)$

$H_3 = \frac{I}{4\pi R}(\sin 315^\circ + \sin 90^\circ)$

$\overline{H}_3 = \frac{I}{4\pi R}\,\frac{2-\sqrt{2}}{2}\,\overline{a_x} = \overline{H}_1 = \frac{1}{4\pi}\,\frac{2-\sqrt{2}}{2}\,\overline{a_x} = 0{,}023\,\overline{a_x}$

---

(2) $\overline{r} = R\overline{a_z}$, $\overline{r'} = R\overline{a_r}$ $\quad \overline{R} = \overline{r} - \overline{r'} = R(\overline{a_z} - \overline{a_r})$

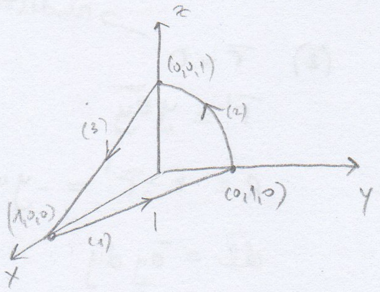
pr. 1.2.5 $\quad \overline{H}_2 = \frac{I}{4\pi}\int \frac{\overline{a_r}z + \overline{a_z}r_0}{(r_0^2 + z^2)^{\frac{3}{2}}}\,r_0\,d\alpha = \left| \begin{array}{l} \overline{a_r} = \frac{\overline{a_x}y + \overline{a_y}x}{\sqrt{x^2+y^2}} \\ x = r\cos\alpha,\ y = r\sin\alpha \end{array}\right.$ $\quad \overline{a_r} = \overline{a_x}\sin\alpha + \overline{a_y}\cos\alpha$

$\frac{Irz}{4\pi(r^2+z^2)^{\frac{3}{2}}} = \frac{1}{4\pi 2\sqrt{2}}$ $\quad = \frac{Ir}{4\pi(r^2+z^2)^{\frac{3}{2}}}\int(\overline{a_x}\sin\alpha\,z + \overline{a_y}\cos\alpha\,z + \overline{a_z}r)\,d\alpha$

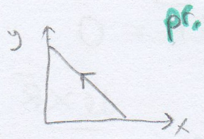
$H_{2x} = \frac{1}{8\sqrt{2}\pi}\int_0^{\pi} z\sin\alpha\,d\alpha = \frac{1}{8\sqrt{2}\pi}(-\cos\alpha)\Big|_0^{\pi} = \frac{1}{8\sqrt{2}\pi}\cdot 2 = \frac{1}{4\sqrt{2}\pi}$

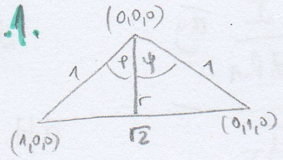
$H_{2y} = \frac{1}{8\sqrt{2}\pi}\int_0^{\pi} z\cos\alpha\,d\alpha = 0$ $\qquad H_{2z} = \frac{1}{8\sqrt{2}\pi}\cdot r\int_0^{\pi} d\alpha = \frac{r\pi}{8\sqrt{2}\pi} = \frac{1}{8\sqrt{2}}$

LJ 13/14 $I = 1A$, odrediti vektor jakosti mg. polja u ishodištu koord. sustava



(1) **pr. 1.2.1.**





$\bar{n} = \bar{a}_z$

pr. 1.2.1. $\rightarrow \bar{B}_{ravni} = \dfrac{I \cdot \mu}{4\pi r} (\sin\varphi + \sin\psi)\bar{n}$

$\varphi = \psi = \dfrac{\pi}{4}$

$r = \sqrt{1 - \left(\frac{\sqrt{2}}{2}\right)^2} = \dfrac{\sqrt{2}}{2}$

$\bar{B}_1 = \dfrac{I \cdot \mu}{4\pi \cdot \frac{\sqrt{2}}{2}} \left(\sin\frac{\pi}{4} + \sin\frac{\pi}{4}\right)\bar{a}_z = \dfrac{I \mu_0}{2\pi} \bar{a}_z$

(2) **pr. 1.2.5.** (pr. 1.2.6.)

$\bar{n} = \bar{a}_x$

$\bar{B}_{prsten} = \dfrac{I\mu}{2} \dfrac{r_0^2}{(r_0^2 + z^2)^{\frac{3}{2}}} \bar{n} = \dfrac{\mu I}{2} \dfrac{r_0^2}{(r_0^2 + x^2)^{\frac{3}{2}}} = \dfrac{\mu I}{2 r_0}$

nula jer se traži u ishodištu

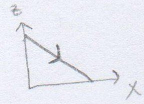
$\bar{B}_3 = \dfrac{1}{4} \dfrac{\mu I}{2 r_0}$, $r_0 = 1$

$\bar{B}_3 = \dfrac{I\mu}{8} \bar{a}_x$

$\bar{B}_{uk} = \bar{B}_1 + \bar{B}_2 + \bar{B}_3 = I\mu \left(\dfrac{\bar{a}_x}{8} + \dfrac{\bar{a}_y}{2\pi} + \dfrac{\bar{a}_z}{2\pi} \right)$

$\bar{H}_{uk} = I \left(\dfrac{\bar{a}_x}{8} + \dfrac{\bar{a}_y}{2\pi} + \dfrac{\bar{a}_z}{2\pi} \right)$

(3) pr. 1.2.1

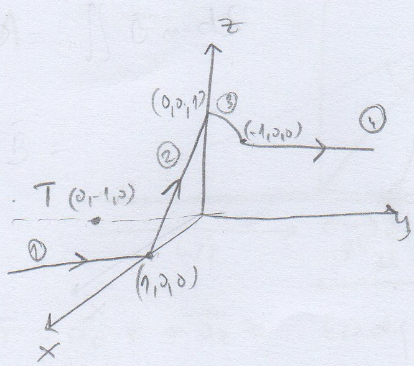


$\bar{n} = \bar{a}_y$

(isto kao i (1))

$\bar{B}_3 = \dfrac{I \mu_0}{2\pi} \cdot \bar{a}_y$

21 11/12



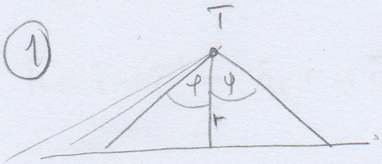
$\bar{B}(0,-1,0) = ?$

$I = 12{,}7\ A$

ravni dio: 1.2.1 $\bar{H} = \frac{I}{4\pi r}(\sin\varphi + \sin\psi)$

krivulje: 1.2.5 $\bar{H} = \frac{I}{2}\frac{r_0^2}{(r_0^2+z^2)^{\frac{3}{2}}}$

①



$\varphi = 90^\circ \quad \psi = 45^\circ \quad r = 1$

$\bar{n} = \bar{a}_z$ (u xy ravnini)

$\bar{H}_1 = \frac{I}{4\pi r}(\sin\varphi + \sin\psi)\,\bar{a}_z$

$\bar{H}_1 = \frac{I}{4\pi}\,\frac{2+\sqrt{2}}{2}\,\bar{a}_z$

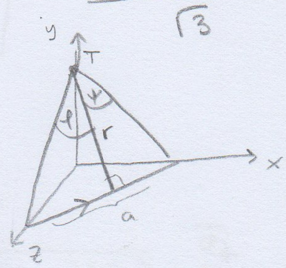
②

A (1,0,0)
B (0,0,1)
T (0,-1,0)

$$\begin{vmatrix} x-1 & y & z \\ -1 & 0 & 1 \\ -1 & -1 & 0 \end{vmatrix} = 0$$

$x-1-y+z=0$

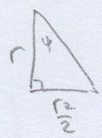
$\frac{\bar{a}_x - \bar{a}_y + \bar{a}_z}{\sqrt{3}} = \bar{n}$



$a = \sqrt{1+1} = \sqrt{2}$

r → visina trokuta

$r = \frac{a\sqrt{3}}{2} = \frac{\sqrt{6}}{2}$



$\operatorname{tg}\psi = \frac{\frac{\sqrt{2}}{2}}{\frac{\sqrt{6}}{2}}$ $\quad \psi = 30^\circ$ $\quad \varphi = 30^\circ$

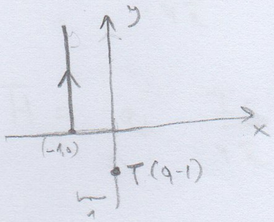
$\bar{H}_2 = \frac{I}{4\pi\frac{\sqrt{6}}{2}}(\sin 30^\circ + \sin 30^\circ)\frac{(\bar{a}_x - \bar{a}_y + \bar{a}_z)}{\sqrt{3}}$

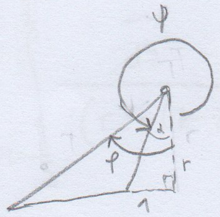
$= \frac{I}{6\sqrt{2}\pi}(\bar{a}_x - \bar{a}_y + \bar{a}_z)$

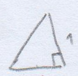
④

(-1,0,0)
(-1,1,0)
(0,-1,0)

$$\begin{vmatrix} x+1 & y & z \\ 0 & 1 & 0 \\ 1 & -1 & 0 \end{vmatrix} = z, (-\bar{a}_z)$$







$r = 1$

$\bar{n} = -\bar{a}_z$

$\operatorname{tg}\alpha = 1$

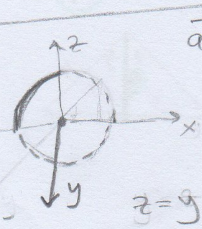
$\alpha = 45^\circ$

$\psi = 360^\circ - \alpha = 315^\circ$

$\varphi = 90^\circ$

$\bar{H}_4 = \frac{I}{4\pi r}(\sin 90^\circ + \sin 315^\circ) = \frac{I}{4\pi}\,\frac{2-\sqrt{2}}{2}(-\bar{a}_z)$

$\bar{a}_r = \frac{-\bar{a}_x z + \bar{a}_z x}{\sqrt{x^2+z^2}}$



$z = y$ (u form), $y = 1$

$r_0 = 1 \quad \to \bar{a}_z = \bar{a}_y$

③ $\bar{H}_3 = \frac{I}{4\pi}\int \frac{\bar{a}_r z + \bar{a}_z r_0}{(r_0^2+z^2)^{\frac{3}{2}}} r_0\, d\alpha = \Big| \bar{a}_r = \frac{-\bar{a}_x z + \bar{a}_z x}{\sqrt{x^2+z^2}} = -\bar{a}_x \sin\alpha + \bar{a}_z\cos\alpha$

$x = r\cos\alpha,\ z = r\sin\alpha$

$= \frac{I r_0}{4\pi(r_0^2+y^2)^{\frac{3}{2}}}\int_0^{\frac{\pi}{2}}(-\bar{a}_x \sin\alpha\cdot y + \bar{a}_z\cos\alpha\cdot y + \bar{a}_y r_0)\,d\alpha$

$= \frac{I}{4\pi\, 2^{\frac{3}{2}}}\left(-\bar{a}_x\cos\alpha + \bar{a}_z\sin\alpha + \bar{a}_y\alpha\right)\Big|_0^{\frac{\pi}{2}} = \frac{I}{8\sqrt{2}\pi}\left(-\bar{a}_x + \frac{\pi}{2}\bar{a}_y + \bar{a}_z\right)$

$\bar{H}_{3x} = \frac{I}{8\sqrt{2}\pi}(-\bar{a}_x) \qquad \bar{H}_{3y} = \frac{I}{16\sqrt{2}}\bar{a}_y \qquad \bar{H}_{3z} = \frac{I}{8\sqrt{2}\pi}\bar{a}_z$

gdje kružnica

(3) $H_{y0} = \frac{I}{2} \frac{r_0^2}{(r_0^2+y^2)^{\frac{3}{2}}} = \frac{I}{2} \frac{1}{(1+1)^{\frac{3}{2}}}$

četvrtina kruga

$H_y = \frac{I}{8} \frac{1}{2^{\frac{3}{2}}}$

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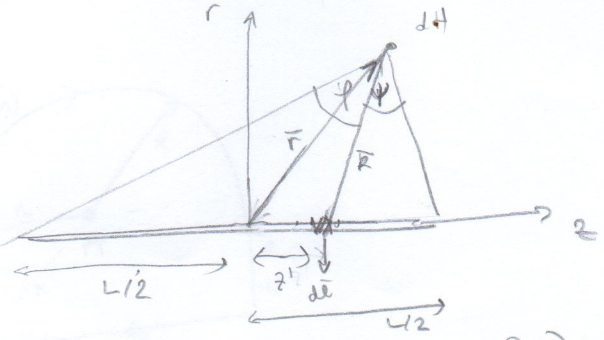
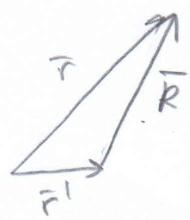
x i y komponente:

$\bar{H} = \frac{I}{4\pi}$

pr. 1.2.1





$\bar{r} = \bar{a}_r\, r + \bar{a}_z\, z$ (radij vektor točke)

$\bar{r}' = \bar{a}_z\, z'$ (radij vektor vodiča)

$\bar{R} = \bar{r} - \bar{r}' = \bar{a}_r\, r + \bar{a}_z (z - z')$ $\quad d\bar{l} = dz'\, a_z$ $\quad R = \sqrt{r^2 + (z-z')^2}$

$$d\bar{l} \times \bar{R} = \begin{vmatrix} \bar{a}_r & \bar{a}_\phi & \bar{a}_z \\ 0 & 0 & dz' \\ r & 0 & z-z' \end{vmatrix} = -a_\phi(-dz' \cdot r) = a_\phi\, r\, dz'$$

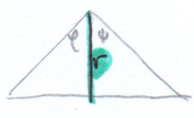
$d\bar{l} \times \bar{R} = a_\phi\, r\, d\bar{z}$

BIOT-SAVARTOV ZAKON

$$d\bar{H} = \frac{I}{4\pi} \frac{d\bar{l} \times \bar{R}}{R^3} = \frac{I}{4\pi} \frac{r\, dz'}{\left[r^2 + (z-z')^2\right]^{\frac{3}{2}}}\, \bar{a}_\phi \quad \Big/ \int \text{po strujnici } (z')$$

$$\bar{H} = a_\phi \frac{Ir}{4\pi} \int_{z'=-\frac{L}{2}}^{\frac{L}{2}} \frac{dz'}{\left(r^2 + (z-z')^2\right)^{\frac{3}{2}}} \quad \left| \begin{array}{l} r = \sqrt{x^2 + a^2} \\ a = r \\ x = z' - z \\ \int \frac{dx}{r^3} = \frac{x}{a^2 r} \end{array} \right| = a_\phi \frac{I\cancel{r}}{4\pi} \left[\frac{z' - z}{r^2 \sqrt{r^2 + (z'-z)^2}} \right]_{-\frac{L}{2}}^{\frac{L}{2}}$$

$$= a_\phi \frac{I}{4\pi r} \left[\frac{\frac{L}{2} - z}{\sqrt{r^2 + (\frac{L}{2} - z)^2}} - \frac{-\frac{L}{2} - z}{\sqrt{r^2 + (\frac{L}{2} + z)^2}} \right] = a_\phi \frac{I}{4\pi r} (\sin\varphi + \sin\psi)$$



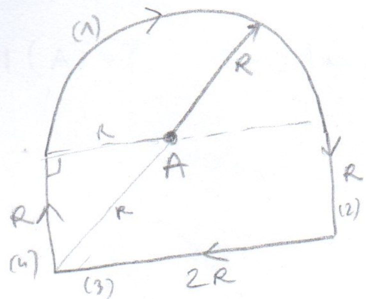
za ∞ dugu strujnicu $\quad \varphi = \psi = \frac{\pi}{2}$

$$\bar{H} = \bar{a}_\phi \frac{I}{2\pi r}$$

$\bar{r} \rightarrow$ od ishodišta do T

$\bar{r}' \rightarrow$ od ishodišta do $d\bar{l}$ (strujnice)

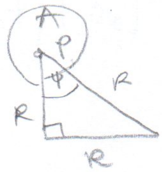
3.29.



svi H su istog smjera!

$H(A) = ?$ $\quad I = 1\,A,\ R = 1\,m$

$\bar{H}_1 = \frac{1}{2} \cdot \frac{I}{2} \cdot \frac{1}{R} = \frac{I}{4R}\,(-\bar{a}_z)$



$\frac{180^\circ - 90^\circ}{2} = 45^\circ$

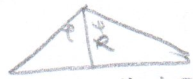
$\bar{H}_2 = \bar{H}_4 = \frac{I}{4\pi R} \sin 45^\circ\,(-\bar{a}_z)$

$\bar{H}_3 = \frac{I}{4\pi R}\, 2\sin 45^\circ\,(-\bar{a}_z)$

$\psi = 45^\circ$

$\varphi = 0$

(3)



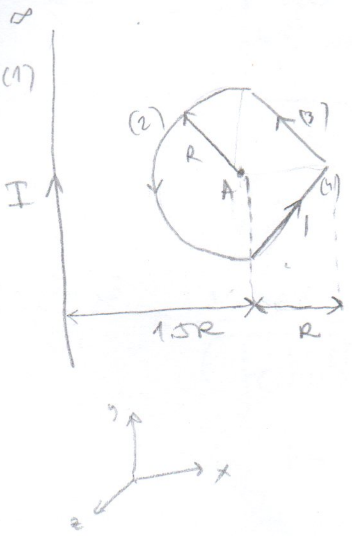
$\varphi = \psi = 45^\circ$

$\bar{H}_{uk} = \frac{I}{4R} + \cancel{2} \cdot \frac{I}{4\pi R} \cdot \frac{\sqrt{2}}{\cancel{2}} + \frac{I}{4\pi R}\, \cancel{2} \cdot \frac{\sqrt{2}}{\cancel{2}}$

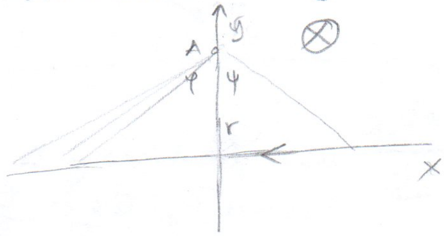
$|H_{uk}| = \frac{I}{4R}\left(1 + \frac{\sqrt{2}}{\pi} \cdot 2\right) = \frac{1}{4}\left(1 + \frac{2\sqrt{2}}{\pi}\right)\ A/m$

J. 14/15

1. $H(A) = ?$ $R = 0.1\,m,\ I = 2A$



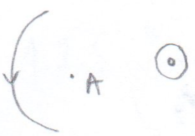
(1) beskonačna strujnica pr. 1.2.1.



$\varphi = \psi = \frac{\pi}{2}$

$$\bar{H}_1 = \frac{I}{4\pi \cdot r} \overbrace{(\sin\varphi + \sin\psi)}^{2} (-\bar{a}_z) = \frac{2I}{4\pi \cdot 1{,}5R} (-\bar{a}_z)$$

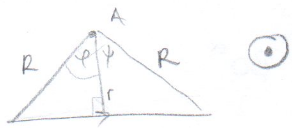
(2) polukružnica pr. 1.2.5.



$$\bar{H}_2 = \frac{I}{2} \cdot \frac{1}{2} \cdot \frac{R^2}{(R^2+z^2)^{\frac{3}{2}}} = \frac{I}{4R} \bar{a}_z$$

(3) pr. 1.2.1

$R^2 - \frac{R^2}{2} = \frac{R^2}{2} = \frac{R}{\sqrt{2}}$



$\cos\varphi = \frac{r}{R}$

$r = \cos\varphi R = \frac{\sqrt{2}}{2} \cdot R$

$\varphi = \psi = \frac{\pi}{4}$

$$\bar{H}_3 = \frac{I}{4\pi r} (\sin\varphi + \sin\psi) = \frac{I}{4\pi \frac{\sqrt{2}}{2} R} \cancel{2} \cdot \frac{\cancel{\sqrt{2}}}{\cancel{2}}$$

(4) $\bar{H}_4 = \bar{H}_3 = \frac{2I}{4\pi R} \cdot \bar{a}_z$

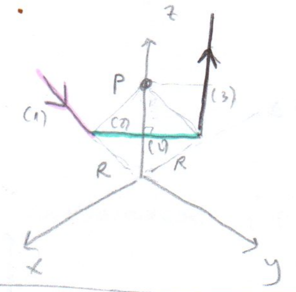
$$\bar{H}_{uk} = \bar{H}_1 + \bar{H}_2 + \bar{H}_3 + \bar{H}_4$$

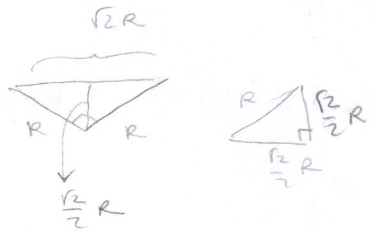
$$= \frac{I}{2\pi \cdot 1{,}5R} (-\bar{a}_z) + \frac{I}{4R} \bar{a}_z + 2 \cdot \frac{I}{4\pi R} \bar{a}_z =$$

$$|H_{uk}| = \frac{I}{R} \left(\frac{1}{4} + 2 \cdot \frac{2}{4\pi} - \frac{1}{2 \cdot 1{,}5\pi} \right) = \frac{2}{0{,}1} \cdot 0{,}4622 = 9{,}244\ A/m$$

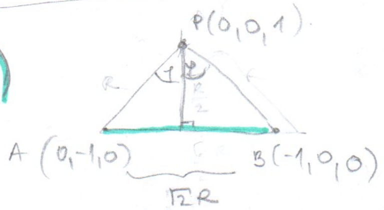
3.16. $I = 10\,A$, $P(0,0,R)$

$R = 1\,m$



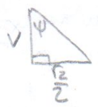


(2)



visina jednakostraničnog Δ

$v = \frac{a\sqrt{3}}{2} = \frac{\sqrt{2}R \cdot \sqrt{3}}{2} = \frac{\sqrt{6}}{2} R$



$v = r$

$\bar{n} = \frac{\bar{a}_x + \bar{a}_y - \bar{a}_z}{\sqrt{1+1+1}} = \frac{\bar{a}_x + \bar{a}_y - \bar{a}_z}{\sqrt{3}}$

ravnina:

$$\begin{vmatrix} x-x_1 & y-y_1 & z-z_1 \\ x_2-x_1 & y_2-y_1 & z_2-z_1 \\ x_3-x_1 & y_3-y_1 & z_3-z_1 \end{vmatrix} = \begin{vmatrix} x & y+1 & z \\ -1 & 1 & 0 \\ 0 & 1 & 1 \end{vmatrix} = x+y+1-z \Rightarrow x+y-z=-1$$

$\mathrm{tg}\,\psi = \frac{\frac{\sqrt{2}}{2}}{\frac{\sqrt{6}}{2}} = 30°$

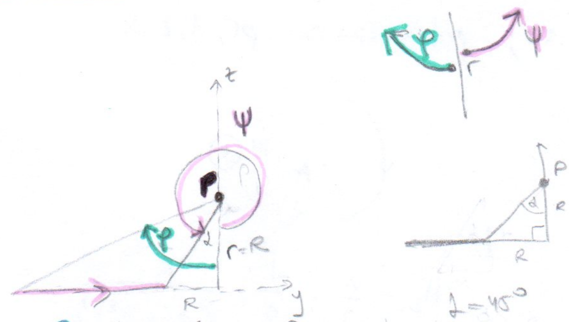
$\psi = 30°$, $\varphi = 30°$

$\bar{H}_2 = \frac{I}{4\pi r}(\sin\varphi + \sin\psi) \cdot \frac{\bar{a}_x + \bar{a}_y - \bar{a}_z}{\sqrt{3}}$

$\bar{H}_2 = \frac{I}{4\pi \frac{\sqrt{6}}{2} \cdot \sqrt{3}}(\sin 30° + \sin 30°)\ \bar{a}_x + \bar{a}_y - \bar{a}_z$

$\bar{H}_2 = \frac{I}{6\sqrt{2}\pi}(\bar{a}_x + \bar{a}_y - \bar{a}_z) = 0{,}375(a_x + \bar{a}_y - \bar{a}_z)$

(1)



$\varphi = 90°$ (ide u ∞)

$\psi = 360° - \alpha = 315°$

$r = R$

$\bar{H}_1 = \frac{I}{4\pi r}(\sin\varphi + \sin\psi)\,\bar{a}_x$

$= \frac{I}{4\pi R}(\sin 90° + \sin 315°)$

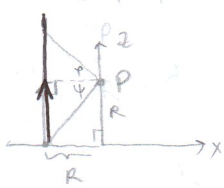
$\bar{H}_1 = \frac{I}{4\pi R}\,\frac{2-\sqrt{2}}{2}\,\bar{a}_x = 0{,}233\,\bar{a}_x$

(3) A(-1, 0, 0)
B(-1, 0, z)
P(0, 0, 1)

$$\begin{matrix} x+1 & y & z \\ 0 & 0 & z \\ 1 & 0 & 1 \end{matrix}$$

→ y komp.

$\bar{n} = \bar{a}_y$



$\psi = 45°$

$\varphi = 90°$

$\bar{H}_3 = \frac{I}{4\pi R}(\sin 90° + \sin 45°)\bar{a}_y = \frac{I}{4\pi R}\,\frac{2+\sqrt{2}}{2}$

$\bar{H}_3 = 1{,}358\,\bar{a}_y$

$\bar{H}_{uk} = \bar{H}_1 + \bar{H}_2 + \bar{H}_3 =$

$= 0{,}233\bar{a}_x + 0{,}375(\bar{a}_x + \bar{a}_y - \bar{a}_z) + 1{,}358\,\bar{a}_y$

$\bar{H}_{uk} = 0{,}608\,\bar{a}_x + 1{,}733\,\bar{a}_y - 0{,}375\,\bar{a}_z$

UVIJEK TRAŽI NORMALU PREKO RAVNINE DA BUDEŠ SIGURNA