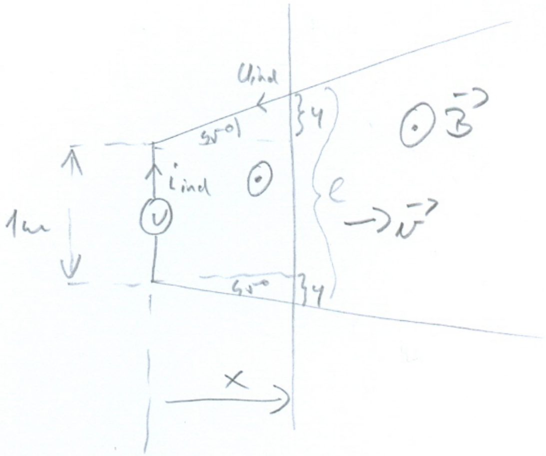
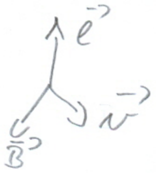
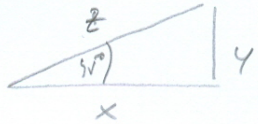
(I) Vodljivi štap A-B se giba po tračnicama jednolikom brzinom $v$ u homogenom mag. prostoru $B = 1\,T$. $t=0, x=0$




$l = 2y+1 = 1+2x$

$\boxed{l = 1+2x}$

$tg\,45 = \frac{y}{x} = 1$

$\Rightarrow \boxed{y = x}$

$\Rightarrow$ Uind po pravilu "desnog vijka" (palac $\vec{B}$, prsti smjer obilaženja)
$\Rightarrow$ struja se protivi promjeni toka pa je suprotnog predznaka

(1.) $t = 1s$, $e = u = 10V \Rightarrow v = ?$

$u = e = \frac{-d\Phi}{dt} \Rightarrow e = -\frac{d}{dt} B\,ds \Rightarrow dS = 10 = x \cdot l = x(1+2x) = x+2x^2$

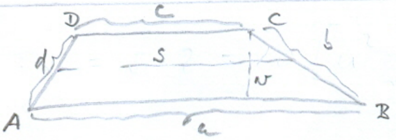
$\Rightarrow x + 2x^2 = 10 \Rightarrow 2x^2 + x - 10 = 0 \Rightarrow x_{1,2} = .. \Rightarrow \cancel{x_1 = \frac{-5}{2}}$

$\boxed{x_2 = 2\,[m]}$

$1 + 2x = l \Rightarrow \boxed{l = 5\,[m]} \Rightarrow x = v \cdot t \Rightarrow v = \frac{x}{t} = \frac{2m}{1s} = 2\left[\frac{m}{s}\right]$

$\boxed{v = 2\left[\frac{m}{s}\right]} \Rightarrow$ (B)

(2.) $\Phi(t=1s) = ?$ $\boxed{B = 1\,T}$



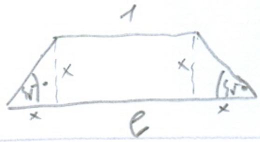

$P = s \cdot v = \frac{a+c}{2} \cdot v$

$\Phi = B \cdot S \Rightarrow \Phi = 1 \cdot S = 6\,[Wb] \Rightarrow$ (2.) (D)

$\begin{cases} l(t=1s) = 5\,[m] \\ x(t=1s) = 2\,[m] \end{cases}$

$\Rightarrow S$ je trapez:

ovdje $\Rightarrow \left(l = a,\ c = 1,\ v = x\right)$




$S_{trapeza} = \left(\frac{l+1}{2}\right) \cdot x = \left(\frac{6}{2}\right) \cdot 2 = 3 \cdot 2 = \underline{6\,[m^2]}$

(3.) $U_{AB}(t=3s) = ?$ $\Rightarrow$ $v = konst = 2\left[\frac{m}{s}\right] \Rightarrow x(t=3s) = x(t=1s) \cdot 3 = 2 \cdot 3 = 6\,[m]$

$\Rightarrow l = 2x+1 = 13\,[m]$

$\underline{U_{AB}} = B \cdot l \cdot v = 1 \cdot 13 \cdot 2 = \underline{26\,[V]} \Rightarrow \boxed{U_{AB} = 26\,[V]} \Rightarrow$ (D)

(4.) $R = 2\Omega$, $t = 2s \Rightarrow i = ?$ $l(t=2s) = 9\,[m]$ $x(t=2s) = 4\,[m]$

$u(t=2s) = B \cdot l(t=2s) \cdot v = 2 \cdot 1 \cdot 9 = 18\,[V]$ $\boxed{i = \frac{u}{R} = \frac{18}{2} = 9\,[A] \Rightarrow \text{(A)}}$

$\Rightarrow u = -d\Phi/dt \Rightarrow u$ smjer kazaljke na satu ....

II Slobodni postor $K = 0$, $\varepsilon_r = 1$, $\mu_r = 1$ vlada polje :

$$\vec{E} = E_{ox}\, e^{\alpha y - bt}\, \vec{a_x} + E_{oy}\, e^{\beta x - bt}\, \vec{a_y} \quad . \quad E_{ox}, E_{oy}, b \text{ zadani :}$$

[2.]

$$\vec{E} = E_{ox} \cdot e^{\alpha y} \cdot e^{-bt}\, \vec{a_x} + E_{oy}\, e^{\beta x} \cdot e^{-bt}\, \vec{a_y}$$

$\Rightarrow K = 0 \Rightarrow$ val u sredstvu bez gubitaka $\Rightarrow \alpha = \beta = \omega \sqrt{\mu_0 \varepsilon_0}$

$\Rightarrow$ iz $\vec{E}$ čitamo kružnu frekv. $\omega \Rightarrow e^{-bt} = e^{-\omega t} \Rightarrow b = \omega$

$\Rightarrow \boxed{\alpha = \beta = b\sqrt{\mu_0 \varepsilon_0}}$ $(\mu_r = \varepsilon_r = 1)$ 5. i 6. $\Rightarrow$ E

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7. $\rho_s = ?$ $\Rightarrow$ iz starih formula $\Rightarrow \boxed{\rho_s = \varepsilon_0\, \nabla \cdot \vec{E}}$

$\Rightarrow \rho_s = \varepsilon_0 \left( \frac{\partial \vec{E_x}}{\partial x} + \frac{\partial \vec{E_y}}{\partial y} + \frac{\partial \vec{E_z}}{\partial z} \right) \Rightarrow$ x i y komponenta uz $\vec{a_x}, \vec{a_y}$ $\Rightarrow$
z komponentu u ovom zad. nemamo

$$= \rho_s = \varepsilon_0 \left( \frac{\partial (E_{ox}\, e^{\alpha y} e^{-bt})}{\partial x} + \frac{\partial (E_{oy}\, e^{\beta x} e^{-bt})}{\partial y} \right) =$$

$$= \varepsilon_0 \left( E_{ox}\, e^{-bt} \underbrace{\left( \frac{\partial e^{\alpha y}}{\partial x} \right)}_{0} + E_{oy}\, e^{-bt} \underbrace{\left( \frac{\partial e^{\beta x}}{\partial y} \right)}_{0} \right) = \varepsilon_0 \cdot 0 = 0$$

$\Rightarrow \boxed{\rho_s = 0}$ C

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8. $$\nabla \times \vec{E} = \begin{vmatrix} \vec{a_x} & \vec{a_y} & \vec{a_z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ E_x & E_y & 0 \end{vmatrix} = \cancel{\vec{a_x}\left( -\frac{E_y}{\partial z} \right)} - \cancel{\vec{a_y}\left( -\frac{E_x}{\partial z} \right)} + \vec{a_z}\left( \frac{E_y}{\partial x} - \frac{E_x}{\partial y} \right) =$$

po z-u je der. $= 0$

$$\Rightarrow \vec{a_z} \left( E_{oy}\, e^{\beta x} \cdot e^{-bt} \cdot \beta - E_{ox}\, e^{\alpha y} \cdot e^{-bt} \cdot \alpha \right) = \nabla \times \vec{E} = \frac{-\partial \vec{B}}{\partial t} = -\mu_0 \frac{\partial \vec{H}}{\partial t}$$

$$/ : \mu_0 \Rightarrow \frac{\partial \vec{H}}{\partial t} = \vec{a_z} \left( -\frac{E_{oy}\beta}{\mu_0} e^{\beta x} e^{-bt} + \frac{E_{ox}\alpha}{\mu_0} e^{\alpha y} e^{-bt} \right) \Rightarrow / \int dt \Rightarrow$$

$$\Rightarrow \vec{H} = \vec{a_z} \left( \frac{\alpha E_{ox}}{\mu_0} e^{\alpha y} \frac{e^{-bt}}{-b} - \frac{E_{oy}\beta}{\mu_0} e^{\beta x} \frac{e^{-bt}}{-b} \right) \Rightarrow$$

$\Rightarrow$

$$\vec{H} = \vec{a}_z \left( \frac{E_{0x}\alpha}{\mu_0} \frac{e^{-bt}}{-b} e^{\alpha y} - \frac{E_{0y}}{\mu_0} \frac{e^{-bt}}{-b} e^{\beta x} \right) = \{ \alpha = \beta \}$$

$$= \vec{a}_z \left( E_{0y} e^{-bt} e^{\beta x} - E_{0x} e^{-bt} e^{\alpha y} \right) \cdot \frac{\alpha}{\mu_0 \cdot b} = \{ \alpha = \beta = b\sqrt{\mu_0 \varepsilon_0} \} \Rightarrow$$

$$= \vec{a}_z \left( E_{0y} e^{+b(x\sqrt{\mu_0\varepsilon_0} - t)} - E_{0x} e^{b(y\sqrt{\mu_0\varepsilon_0} - t)} \right) \cdot \sqrt{\frac{\varepsilon_0}{\mu_0}}$$

$$\Rightarrow \frac{b\sqrt{\mu_0\varepsilon_0}}{\mu_0 b} = \sqrt{\frac{\varepsilon_0}{\mu_0}}$$

$$\Rightarrow \vec{H} = \left( E_{0y} e^{b(x\sqrt{\mu_0\varepsilon_0} - t)} - E_{0x} e^{b(y\sqrt{\mu_0\varepsilon_0} - t)} \right) \cdot \sqrt{\frac{\varepsilon_0}{\mu_0}} \cdot \vec{a}_z$$

A

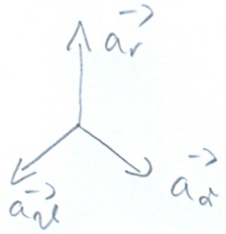
III Vektor jakosti mag. polja antene smještene u središtu sfernog koord. sustava je $\vec{H} = \frac{\sin^2 \vartheta}{r} \cos(\omega t - \beta r) \vec{a_\alpha}$.

$\varepsilon = \varepsilon_0$, $\mu = \mu_0$, $f = 100$ MHz $\Rightarrow \omega = 2\pi f = 200\pi \cdot 10^6 \left[\frac{rad}{s}\right]$

$\Rightarrow$ općenito za sferni koord. sustav u ishodištu ....

$\Rightarrow$ za izvor : $\vec{E} = \frac{V \sin \vartheta}{r} \cos(\omega t - \beta r) \vec{a_\vartheta}$

$\vec{H} = \frac{V \sin \vartheta}{|Z| \cdot r} \cos(\omega t - \beta r) \vec{a_\alpha}$

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$\Rightarrow$ zadano : $\vec{H} = \frac{\sin^2 \vartheta}{\cancel{r}} \cancel{\cos(\omega t - \beta r)} = \frac{V \cancel{\sin \vartheta}}{|Z| \cdot \cancel{r}} \cancel{\cos(\omega t - \beta r)} \Rightarrow$

$\Rightarrow \sin \vartheta = \frac{V}{|Z|} \Rightarrow \boxed{V = |Z| \cdot \sin \vartheta = 120\pi \cdot \sin \vartheta}$

$\Rightarrow$ val bez gubitaka $\varepsilon = \varepsilon_0, \mu = \mu_r \Rightarrow |Z| = Z_0 = \sqrt{\frac{\mu_0}{\varepsilon_0}} = 120\pi$

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9. $E(r = 100\,m, \vartheta = \frac{\pi}{4}\,rad) = \frac{V \sin \vartheta}{r} = \frac{120\pi \cdot \sin^2 \vartheta}{r} = \frac{120\pi \cdot (\sin(\frac{\pi}{4}))^2}{100} \Rightarrow$

$\boxed{E = 1,885 \left[\frac{V}{m}\right]} \Rightarrow$ B

---

10. $|\vec{N}| (r = 150\,m, \vartheta = \frac{\pi}{3} = 60°, t = 1\,\mu s) = ?$

PAZI! **ne** uračunavaš cosinuse

$\vec{N} = \vec{E} \times \vec{H} = \left(\frac{120\pi \sin^2 \vartheta}{r} \cdot \frac{\cancel{120\pi} \cdot \sin^2 \vartheta}{\cancel{120\pi} \cdot r}\right) \vec{a_r} \Rightarrow \boxed{|\vec{N}| = 9,42 \left[\frac{mW}{m^2}\right]}$

D

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11., 12.

IV Neki izvor proizvodi u vakuumu ravni val valne duljine $\lambda = 2\pi\,[m]$. Kada se val prostire u idealnom dielektriku, nepoznatih značajki, valna duljina se smanji 20 puta, a omjer $\frac{E_m}{H_m}$ se poveća 10 puta.

$$\frac{E_{m1}}{H_{m1}} \cdot 10 = \frac{E_{m2}}{H_{m2}}$$

$\lambda_1 = 2\pi\,[m] \Rightarrow$ za $\varepsilon_0, \mu_0$ , $\lambda_2 = \frac{\lambda_1}{20} = \frac{2\pi}{20} = \frac{\pi}{10}\,[m] \Rightarrow$ za $\mu_r, \varepsilon_r$

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$$\lambda = \frac{2\pi}{\omega\sqrt{\mu\varepsilon}} \Rightarrow \lambda_1 = \frac{2\pi}{\omega\sqrt{\mu_0\varepsilon_0}} \Rightarrow \underline{\underline{\omega}} = \frac{2\pi}{\lambda_1\sqrt{\mu_0\varepsilon_0}} = \frac{\cancel{2\pi}}{\cancel{2\pi}\sqrt{\mu_0\varepsilon_0}} = \underline{\underline{3\cdot10^8\left[\frac{rad}{s}\right]}}$$

$$\boxed{\omega = 3\cdot10^8\left[\frac{rad}{s}\right]} \Rightarrow \text{15. B}$$

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$$\frac{E_{m1}}{H_{m1}}\cdot 10 = \frac{E_{m2}}{H_{m2}}, \quad \frac{E_{m1}}{E_{m2}} = \sqrt{\frac{\mu}{\varepsilon}} = \sqrt{\frac{\mu_0}{\varepsilon_0}} = Z_0 = \underline{\underline{120\pi}}$$

$$\Rightarrow 1200\pi = \frac{E_{m2}}{H_{m2}}, \quad \frac{E_{m2}}{H_{m2}} = \sqrt{\frac{\mu}{\varepsilon}} = \sqrt{\frac{\mu_0\mu_r}{\varepsilon_0\varepsilon_r}} = \sqrt{\frac{\mu_0}{\varepsilon_0}}\sqrt{\frac{\mu_r}{\varepsilon_r}} = Z_0\cdot\sqrt{\frac{\mu_r}{\varepsilon_r}}$$

$$\Rightarrow \frac{\cancel{1200}\cancel{\pi}}{10} = \cancel{120}\cancel{\pi}\sqrt{\frac{\mu_0}{\varepsilon_r}} \Rightarrow 10 = \sqrt{\frac{\mu_r}{\varepsilon_r}} \Rightarrow \varepsilon_r\cdot100 = \mu_r$$

$$\Rightarrow \lambda_2 = \frac{2\pi}{\omega\sqrt{\mu\varepsilon}} \Rightarrow \sqrt{\mu\varepsilon} = \frac{2\pi}{\lambda_2\omega} = \frac{2\pi}{\frac{\pi\omega}{10}} = \frac{20}{\omega} = 6{,}667\cdot10^{-8}$$

$$\Rightarrow \sqrt{\mu_0\mu_r\varepsilon_0\varepsilon_r} = 6{,}667\cdot10^{-8} \Rightarrow \sqrt{\mu_r}\sqrt{\varepsilon_r} = \frac{6{,}667\cdot10^{-8}}{\sqrt{\mu_0\varepsilon_0}} \Rightarrow \sqrt{\mu_r}\sqrt{\varepsilon_r} = 20$$

$$\varepsilon_r\cdot100 = \mu_r, \quad \varepsilon_r = \frac{400}{\mu_r} \Rightarrow \mu_r = \frac{400}{\varepsilon_r}$$

$$\varepsilon_r\cdot\cancel{100} = \frac{\overset{4}{\cancel{400}}}{\varepsilon_r} \Rightarrow \varepsilon_r^2 = 4 \Rightarrow \boxed{\varepsilon_r = 2} \Rightarrow \text{13. D}$$

$$\Rightarrow \mu_r = 100\,\varepsilon_r = 100\cdot2 = 200 \Rightarrow \boxed{\mu_r = 200} \Rightarrow \text{14. A}$$

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$$\beta = \omega\sqrt{\mu\varepsilon} = \omega\sqrt{\mu_0\varepsilon_0\mu_r\varepsilon_r} = \text{uvrstimo brojeve} \Rightarrow \boxed{\beta = 20{,}014}$$

$$\boxed{\beta = 20\left[\frac{1}{m}\right]} \Rightarrow \text{16. C}$$

(V) Ravni val frekv. 10 MHz prostire se u real. sredstvu $\varepsilon_r = 4$, $\mu_r = 1$. Ako je konst. (fazna) $= \beta = 0{,}6\ \frac{1}{m}$, a max. vr. polja (el.) $E_{max}(x=0) = 100\ \frac{V}{m}$. Odredi:

$\omega = 2\pi f$ $\quad E_{max} = 100\frac{V}{m}$
$f = 10\ MHz$ $\quad \varepsilon_r = 4$
$\beta = 0{,}6$ $\quad \mu_r = 1$

(17.) $K = ?$

$$\beta = \frac{\omega}{c_1\sqrt{2}}\sqrt{\sqrt{1+\left(\frac{K}{\omega\varepsilon}\right)^2}+1} = 0{,}6 \Rightarrow \frac{c_1\sqrt{2}\cdot 0{,}6}{\omega} = \sqrt{\sqrt{1+\left(\frac{K}{\omega\varepsilon}\right)^2}+1} \quad /()^2 \Rightarrow$$

$$\Rightarrow \left(\frac{c_1\sqrt{2}\cdot 0{,}6}{\omega}\right)^2 - 1 = \sqrt{1+\left(\frac{K}{\omega\varepsilon}\right)^2} \Rightarrow 3{,}1035 = \sqrt{1+\left(\frac{K}{\omega\varepsilon}\right)^2} \Rightarrow$$

$$\left\{ \begin{array}{l} c_1 = \frac{c_0}{\sqrt{\mu_r\varepsilon_r}} = \frac{3\cdot 10^8}{2} = 1{,}5\cdot 10^8\ \frac{m}{s} \\ \varepsilon = \varepsilon_0\varepsilon_r = 8{,}854\cdot 10^{-12}\cdot 4 \\ \omega = 2\pi f = 2\pi\cdot 10\cdot 10^6\ rad/s \end{array} \right\}$$

$\Rightarrow$ $c_0$ se uvrsti kad je $\mu_r = \varepsilon_r = 1$, inače računamo $c_1$ ....

$$\Rightarrow \left(\frac{K}{\omega\varepsilon}\right)^2 = 8{,}6318 \Rightarrow K = \omega\varepsilon\cdot\sqrt{8{,}6318} = 6{,}5378\ \left[\frac{mS}{m}\right]$$ (D)

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(18.) Prigušna konst. vala $\alpha = ?$

$$\alpha = \frac{\omega}{c_1\sqrt{2}}\sqrt{\sqrt{1+\left(\frac{K}{\omega\varepsilon}\right)^2}-1} \underset{\text{brojeve}}{\overset{\text{Uvrstimo}}{=}} \ldots = 0{,}4297 = 0{,}43$$ (C)

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(19.) Aps. vr. valne impedancije $|Z| = ?$ (B)

$$|Z| = \frac{\omega\mu}{\sqrt{\alpha^2+\beta^2}} = \frac{4\pi\cdot 10^{-7}\cdot 1\cdot 2\cdot\pi\cdot 10\cdot 10^6}{\sqrt{0{,}545}} = \frac{78{,}957}{0{,}738} = 106{,}988 = 107\ \Omega$$

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(20.) Sr. vr. Re dijela Poyntingovog vektora za $x = d$ ... $d$ = dubina prodiranja

$d = \frac{1}{\alpha} = \frac{1}{0{,}43} = 2{,}326$ [m] $\Rightarrow$ dubina prodiranja

$\Rightarrow$ općenito za $\vec{E}$: $\boxed{\vec{E} = E_m e^{-\alpha x}\cos(\omega t - \beta x)\,\vec{a}_x}$ $\quad \overset{\text{ovako}}{\Rightarrow}$

$\Rightarrow \boxed{x = d} \Rightarrow \vec{E}(x=d) = 100\, e^{-0{,}43\cdot d}\cos(20\pi\cdot 10^6 - 0{,}6d)\,\vec{a}_y \Rightarrow \boxed{Re(\vec{E}) = 36{,}788\left(\frac{V}{m}\right)}$

$$\vec{H} = \frac{1}{\mu\omega}\left(\vec{\beta} \times \vec{E}\right) = \vec{a_z}\left(\frac{1}{\omega\mu}\right) \cdot (0,6) \cdot (36,788) \Rightarrow \mu = \mu_0 \mu_r \Rightarrow \mu = \mu_0 \cdot 1 = \mu_0$$

$$\boxed{\vec{H} = 0,279\ \vec{a_z}}$$

$$\left|\vec{N}_{sr}\right| = \left|\frac{1}{2}\left(\vec{E} \times \vec{H}\right)\right| = \frac{1}{2} \cdot 0,279 \cdot 36,788 = 5,13193\ \frac{W}{m^2}$$

$$\boxed{\left|\vec{N}_{sr}\right| = 5,14\ \frac{W}{m^2}}$$

E