1.

Fundamentalne konstante:

Planckova konstanta: $\hbar = 1.055 \cdot 10^{-34}$ Js

Brzina svjetlosti: $c = 2.998 \cdot 10^{8}$ m/s

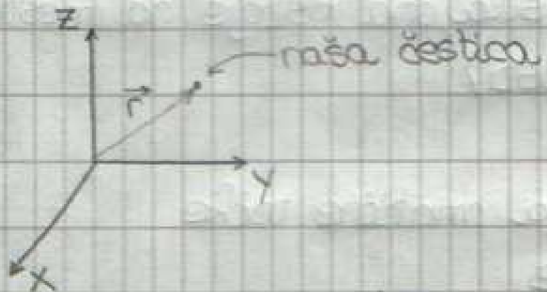
Konstanta gravitacije: $G = 6.672 \cdot 10^{-11} \, Nm^2 kg^{-2}$

$N = kg\,ms^{-2}$

Izvedena Planckova duljina (najfundamentalnija duljina za rad, manje od toga fizika ne zna)

$$l_p = \hbar^{\alpha} c^{\beta} G^{\gamma} = \dots = 1.6 \cdot 10^{-35} m$$

Kinematika točke



Položaj: $x(t)\vec{i} + y(t)\vec{j} + z(t)\vec{k}$

Brzina: $\vec{v}(t) = \dfrac{d\vec{r}(t)}{dt} = \dot{\vec{r}}(t) = \dot{x}(t)\vec{i} + \dot{y}(t)\vec{j} + \dot{z}(t)\vec{k} = v_x(t)\vec{i} + v_y(t)\vec{j} + v_z(t)\vec{k}$

Integralni zapis: $\vec{v}(t) = \dfrac{d\vec{r}(t)}{dt} \Big/ \cdot dt$

$$d\vec{r}(t) = \vec{v}(t)dt \Big/ \int_p^k$$

$$\int_p^k d\vec{r}(t) = \int_p^k \vec{v}(t)dt$$

$p = t_0 \quad k = t$

$$\vec{r}\Big|_{t_0}^{t} = \int_{t_0}^{t} \vec{v}(t)\,dt$$

$$\vec{r}(t)-\vec{r}(t_0) = \int_{t_0}^{t} \vec{v}(t)\,dt$$

$$\boxed{\vec{r}(t) = \vec{r}(t_0) + \int_{t_0}^{t} \vec{v}(t)\,dt}$$

Akceleracija: $\vec{a}(t) = \dfrac{d\vec{v}(t)}{dt} = \dot{\vec{v}}(t) = \dot{v}_x(t)\vec{i} + \dot{v}_y(t)\vec{j} + \dot{v}_z(t)\vec{k}$

$= a_x(t)\vec{i} + a_y(t)\vec{j} + a_z(t)\vec{k}$

$$\vec{v}(t) = \vec{v}(t_0) + \int_{t_0}^{t} \vec{a}(t)\,dt$$

Newtonovi aksiomi

Prvi: Kada na matičnu točku ne djeluje sila, ona ostaje na mjestu ili se giba po pravcu (princip tromosti)

Drugi: Vremenska promjena količine gibanja matične točke razmjerna je sili koja na nju djeluje

$\vec{p} = m\vec{v}$, $\vec{F} = \dfrac{d\vec{p}}{dt} = \dfrac{d(m\vec{v})}{dt} = m\dfrac{d\vec{v}}{dt} = m\vec{a}$

Treći: Ako jedno tijelo djeluje na drugo nekom silom, onda drugo djeluje na prvo silom istog iznosa, ali drugog smjera.

$\vec{F}_{1/2} = -\vec{F}_{2/1}$, $|\vec{F}_{1/2}| = |\vec{F}_{2/1}|$

2.

Rješavanje jednadžbi gibanja

Uz stalnu silu, tj. u bilo kojem položaju i u bilo kojem vremenu sila je kon $\vec{F_0}$

$$\frac{d\vec{p}}{dt} = \vec{F_0} = m\frac{d\vec{v}}{dt} = m\frac{d}{dt}\frac{d\vec{r}}{dt} = m\frac{d^2\vec{r}}{dt^2}$$

Brzina: $\frac{d\vec{p}}{dt} = \vec{F_0} \Rightarrow m\frac{d\vec{v}}{dt} = \vec{F_0}$

$$\frac{d\vec{v}}{dt} = \frac{\vec{F_0}}{m} \quad / \cdot dt$$

$$d\vec{v} = \frac{\vec{F_0}}{m} dt \quad / \int_p^k$$

$$\vec{v}\Big|_p^k = \frac{1}{m}\int_p^k \vec{F_0}\, dt$$

$$\vec{v}_k - \vec{v}_p = \frac{1}{m}\vec{F_0}\Big|_p^k$$

$$\vec{v}_k - \vec{v}_p = \frac{1}{m}\vec{F_0}(k-p)$$

$p = t_0 \quad k = t$

$$\boxed{\vec{v}(t) = \vec{v}(t_0) + \frac{\vec{F_0}}{m}(t-t_0)}$$

Položaj: $\frac{d\vec{r}}{dt} = \vec{v}(t) \quad / \cdot dt$

$$d\vec{r} = \vec{v}(t)dt \quad / \int_p^k$$

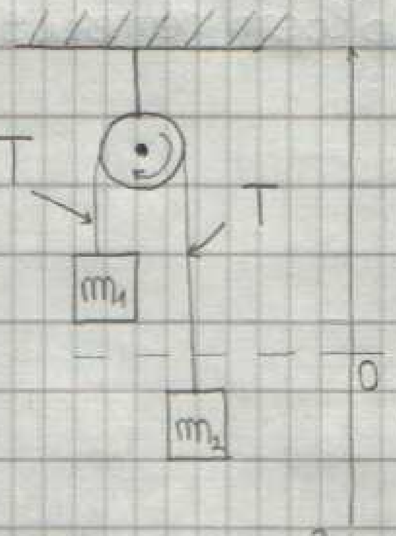
$p = t_0 \quad k = t$

$$\vec{r}(t) - \vec{r}(t_0) = \int_{t_0}^{t} \vec{v}(t)dt$$

$$\vec{r}(t) = \vec{r}(t_0) + \int_{t_0}^{t}\left[\vec{v}(t_0) + \frac{\vec{F_0}}{m}(t-t_0)\right]dt$$

$$\boxed{\vec{r}(t) = \vec{r}(t_0) + \vec{v}(t_0)(t-t_0) + \frac{\vec{F_0}}{2m}(t-t_0)^2}$$

Atwoodov padostroj



$$\vec{r}_1(t) = x_1(t)\vec{i}$$
$$\vec{r}_2(t) = x_2(t)\vec{i}$$

$$x_1(t) = -x_2(t)$$
$$\dot{x}_1(t) = -\dot{x}_2(t)$$
$$\ddot{x}_1(t) = -\ddot{x}_2(t)$$

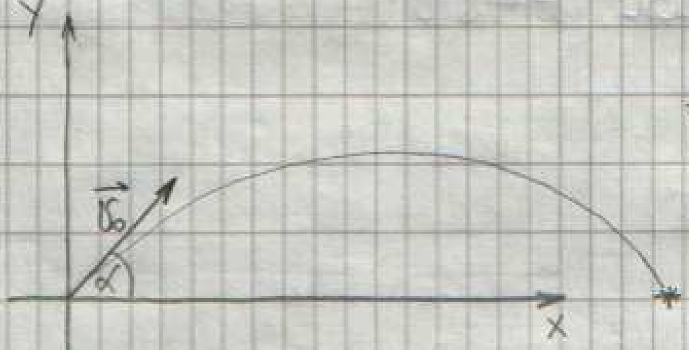
$$\left.\begin{aligned} m_1\ddot{x}_1 &= T - m_1 g \\ m_2\ddot{x}_2 &= T - m_2 g \end{aligned}\right\} -$$

$$m_1\ddot{x}_1 - m_2\ddot{x}_2 = m_2 g - m_1 g$$
$$m_1\ddot{x}_1 + m_2\ddot{x}_1 = g(m_2 - m_1)$$

$$\ddot{x}_1 = g\left(\frac{m_2 - m_1}{m_1 + m_2}\right)$$ ...akceleracija sustava

3.

Kosi hitac



$\vec{F_0} = -mg\vec{j}$

Početni uvjeti: $t_0 = 0$, $\vec{v}(t_0) = v_0(\cos\alpha\vec{i} + \sin\alpha\vec{j})$

$\vec{r}(t_0) = 0$

$$\vec{v}(t) = \vec{v}(t_0) + \frac{\vec{F_0}}{m}(t - t_0)$$

uvrstimo u ↗

$$\vec{v}(t) = v_0(\cos\alpha\vec{i} + \sin\alpha\vec{j}) - gt\vec{j} = \underbrace{v_0\cos\alpha}_{v_x}\vec{i} + \underbrace{(v_0\sin\alpha - gt)}_{v_y}\vec{j}$$

$$\vec{r}(t) = \vec{r}(t_0) + \vec{v}(t_0)(t - t_0) + \frac{\vec{F_0}}{2m}(t - t_0)^2$$

uvrstimo u ↗

$$\vec{r}(t) = v_0 t(\cos\alpha\vec{i} + \sin\alpha\vec{j}) - \frac{g}{2}t^2\vec{j} = \underbrace{v_0 t\cos\alpha}_{x(t)}\vec{i} + \underbrace{\left(v_0 t\sin\alpha - \frac{g}{2}t^2\right)}_{y(t)}\vec{j}$$

$$t = \frac{x(t)}{v_0\cos\alpha}$$

$$y(x) = v_0\frac{x(t)}{v_0\cos\alpha}\sin\alpha - \frac{g}{2}\left(\frac{x(t)}{v_0\cos\alpha}\right)^2 = x\,tg\alpha - \frac{g}{2}\frac{x^2}{v_0^2\cos\alpha^2} \quad (u = tg\alpha)$$

$$= xu - \frac{gx^2}{2v_0^2}(1 + u^2)$$

Rješavanje jednadžbi gibanja

Kada je sila otpora razmjerna brzini, $\vec{F} = -\underset{\text{koeficijent}}{\gamma} \vec{v}$ ($\vec{F}$ je domutog smjera od $\vec{v}$)

$$\frac{d\vec{p}}{dt} = \vec{F} = \boxed{m\dot{\vec{v}} = -\gamma\vec{v}}$$

$$m(\dot{v}_x\vec{i} + \dot{v}_y\vec{j} + \dot{v}_z\vec{k}) = -\gamma(v_x\vec{i} + v_y\vec{j} + v_z\vec{k}) \quad \left.\right\} \quad \begin{aligned} m\dot{v}_x &= -\gamma v_x \\ m\dot{v}_y &= -\gamma v_y \\ m\dot{v}_z &= -\gamma v_z \end{aligned}$$

Rješavamo samo za X komponentu (analogno y i z).

Brzina:

$$m\frac{dv_x}{dt} = -\gamma v_x \quad / \cdot dt$$

$$m\,dv_x = -\gamma v_x\,dt \quad / : m v_x$$

$$\frac{dv_x}{v_x} = \frac{-\gamma}{m}dt \quad / \int_p^k$$

$$\int_p^k \frac{1}{v_x}dv_x = -\frac{\gamma}{m}\int_p^k dt$$

$$\ln v_x \Big|_p^k = -\frac{\gamma}{m}(k-p)$$

$$\ln v_x(t) = \ln v_x(t_0) - \frac{\gamma}{m}(t-t_0) \quad / e^{\wedge} \quad / \cdot \vec{i}$$

$$\boxed{\vec{v}(t) = \vec{v}(t_0) \cdot e^{-\frac{\gamma}{m}(t-t_0)}}$$

Položaj:

$$\vec{r}(t) = \vec{r}(t_0) + \int_{t_0}^{t} \vec{v}(t)\,dt$$

$$= \vec{r}(t_0) + \int_{t_0}^{t}\left[\vec{v}(t_0)e^{-\frac{\gamma}{m}(t-t_0)}\right]dt$$

$$= \vec{r}(t_0) + \vec{v}(t_0)\int_{t_0}^{t}\frac{m}{\gamma}e^{-\frac{\gamma}{m}(t-t_0)}\frac{\gamma}{m}\,dt$$

$$= \vec{r}(t_0) - \frac{m}{\gamma}\vec{v}(t_0)e^{-\frac{\gamma}{m}(t-t_0)}\Big|_{t_0}^{t}$$

$$\boxed{= \vec{r}(t_0) + \frac{m}{\gamma}\vec{v}(t_0)\left(1 - e^{-\frac{\gamma}{m}(t-t_0)}\right)}$$

Rješavanje jednadžbi gibanja

Uz konstantnu silu i otpor (koji je proporcionalan brzini)

$$\frac{d\vec{p}}{dt} = m\dot{\vec{v}} = \vec{F_0} - \gamma\vec{v}$$

Opet rješavamo za x komponentu.

Brzina:

$$m\dot{v}_x = F_{0x} - \gamma v_x$$

$$m\frac{dv_x}{dt} = F_{0x} - \gamma v_x \quad / \cdot \frac{dt}{m}$$

$$dv_x = \left(\frac{F_{0x}}{m} - \frac{\gamma v_x}{m}\right) dt$$

$$\frac{dv_x}{\frac{F_{0x}}{m} - \frac{\gamma v_x}{m}} = dt \quad / \int_{t_0}^{t}$$

$$-\frac{m}{\gamma} \ln\left(F_{0x} - \gamma v_x\right)\Big|_{t_0}^{t} = t\Big|_{t_0}^{t}$$

$$\ln(F_{0x} - \gamma v_x(t)) = \ln(F_{0x} - \gamma v_x(t_0)) - \frac{\gamma}{m}(t - t_0) \quad / e^{\wedge}$$

$$F_{0x} - \gamma v_x(t) = (F_{0x} - \gamma v_x(t_0))\, e^{-\frac{\gamma}{m}(t-t_0)}$$

$$\vec{v}_x(t) = \frac{F_{0x}}{\gamma} - \left(\frac{F_{0x}}{\gamma} - v_x(t_0)\right) e^{-\frac{\gamma}{m}(t-t_0)}$$

$$\boxed{\vec{v}(t) = \vec{v}(t_0)\, e^{-\frac{\gamma}{m}(t-t_0)} + \frac{\vec{F_0}}{\gamma}\left(1 - e^{-\frac{\gamma}{m}(t-t_0)}\right)}$$

Položaj: $\boxed{\vec{r}(t) = \vec{r}(t_0) + \frac{\vec{F_0}}{\gamma}(t - t_0) + \left(\vec{v}(t_0) - \frac{\vec{F_0}}{m}\right)\frac{m}{\gamma}\left(1 - e^{-\frac{\gamma}{m}(t-t_0)}\right)}$

Rješavanje jednadžbi gibanja

Kada je sila otpora razmjerna kvadratu brzine.

$$\frac{d\vec{p}}{dt} = m\dot{v}_x = -k v_x^2$$

Brzina:

$$m\frac{dv_x}{dt} = -k v_x^2 \quad / \frac{dt}{m}$$

$$dv_x = -\frac{k}{m} v_x^2 \, dt \quad / : v_x^2$$

$$\frac{dv_x}{v_x^2} = -\frac{k}{m} dt \quad / \int_{t_0}^{t}$$

$$-\frac{1}{v_x}\Big|_{t_0}^{t} = -\frac{k}{m} t \Big|_{t_0}^{t}$$

$$-\frac{1}{v_x(t)} + \frac{1}{v_x(t_0)} = -\frac{k}{m}(t-t_0)$$

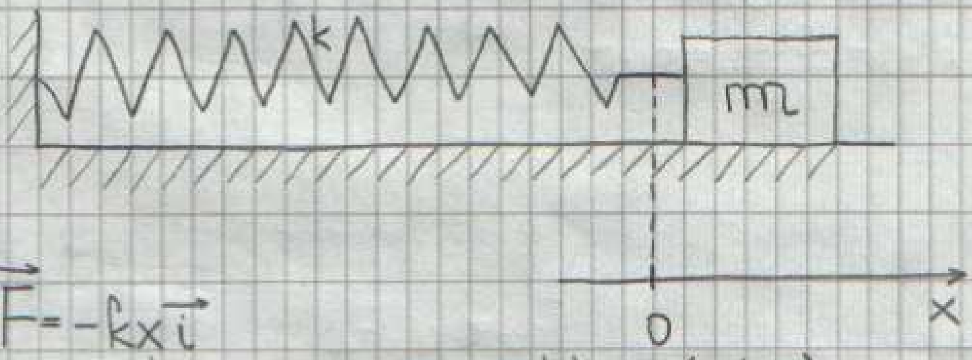
$$\boxed{v(t) = \frac{v(t_0)}{1 + \frac{k}{m} v(t_0)(t-t_0)}}$$

Položaj:

$$\boxed{\vec{r}(t) = \vec{r}(t_0) + \frac{m}{k} \ln\left(1 + \frac{k}{m} v(t_0)(t-t_0)\right)}$$

Rješavanje jednadžbi gibanja

Uz harmoničku silu (silu opruge)



$\vec{F} = -kx\vec{i}$

↳ otklon od ravn. položaja (od 0)

$\frac{d\vec{p}}{dt} = \vec{F} = -kx\vec{i}$

$m\ddot{x}(t) = -kx(t)$

$\ddot{x}(t) = -\frac{k}{m}x(t)$

$\omega^2 = \frac{k}{m}$

$\boxed{\ddot{x}(t) + \omega^2 x(t) = 0}$ jednadžba harmoničkog oscilatora

opće rješenje: $x(t) = A\cos\omega t + B\sin\omega t$ (1)

$\dot{x}(t) = -A\omega\sin\omega t + B\omega\cos\omega t = \omega(-A\sin\omega t + B\cos\omega t)$ (2)

$\ddot{x}(t) = -\omega^2(A\cos\omega t + B\sin\omega t) = -\omega^2 x(t)$

1. slučaj: Mirovanje u ravnotežnom položaju ($t_0 = 0$, $x_0 = x(t_0) = 0$, $v(t_0) = 0$)

uvrštavanjem u (1): $x(t_0) = A \Rightarrow A = 0$

uvrštavanjem u (2): $v(t_0) = \omega B \Rightarrow B = 0$

uvrštavanjem u (1): $x(t) = 0$

2. slučaj: Mirovanje izvan ravnotežnog položaja ($t_0 = 0$, $x_0 = x(t_0) \neq 0$, $v(t_0) = 0$)

uvrštavanjem u (1): $x(t_0) = A \quad A \neq 0$

uvrštavanjem u (2): $v(t_0) = \omega B \Rightarrow B = 0$

uvrštavanjem u (1): $x(t) = A\cos\omega t = x(t_0)\cos\omega t$

3. slučaj: Početna brzina u ravnotežnom položaju ($t_0=0, x_0=0, v_0\neq 0$)

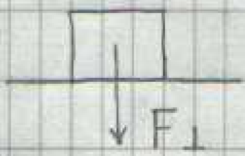
uvrštavanjem u (1): $x(t)=A \Rightarrow A=0$

uvrštavanjem u (2): $v(t_0)=\omega B \Rightarrow B=\frac{v(t_0)}{\omega}$

uvrštavanjem u (1): $x(t)=\frac{v(t_0)}{\omega}\sin\omega t$

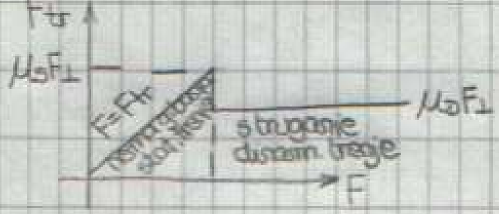
Trenje

1) Statičko (u mirovanju)

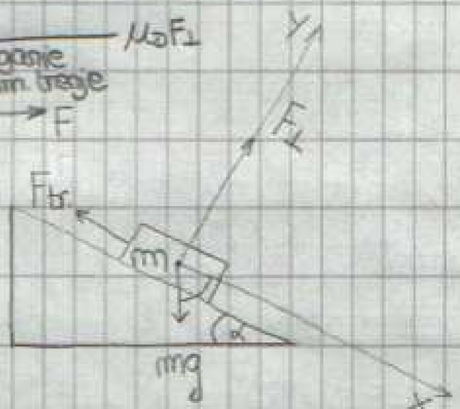


$F_{tr}\leq\mu_s F_\perp$

2) Dinamičko (pri struganju) $F_{tr}=\mu_D F_\perp$



KOSINA:



$\mu_{tr}=tg\alpha$

$$\frac{d\vec{p}}{dt}=m\ddot{\vec{r}}=\vec{F}=m\vec{g}+\vec{F}_\perp+\vec{F}_{tr} \quad \left(m\vec{g}=mg(\vec{i}\sin\alpha-\vec{j}\cos\alpha),\ \vec{F}_\perp=|F_\perp|\vec{j},\ \vec{F}_{tr}=\pm\mu|F_\perp|\vec{i}\right)$$

$$m(\ddot{x}\vec{i}+\ddot{y}\vec{j})=mg\sin\alpha\,\vec{i}-mg\cos\alpha\,\vec{j}+|F_\perp|\vec{j}\pm\mu|F_\perp|\vec{i})$$

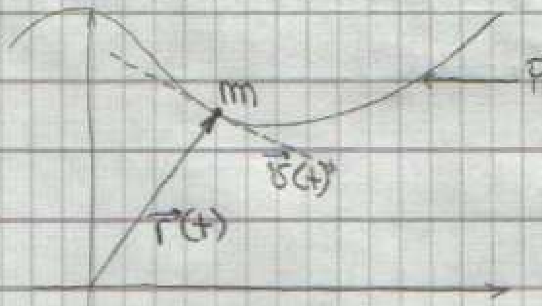
$$=\vec{i}(mg\sin\alpha\pm\mu|F_\perp|)+\vec{j}(|F_\perp|-mg\cos\alpha) \quad (\ddot{y}=0,\ |F_\perp|=mg\cos\alpha)$$

$$m\ddot{x}=mg\sin\alpha\pm\mu mg\cos\alpha$$

$$\boxed{a=g(\sin\alpha\pm\mu\cos\alpha)}$$

+ (uzbrdo)
− (nizbrdo)

Sila okomita na brzinu (Gibanje po kružnici)



$$\frac{d\vec{p}}{dt} = m\vec{a} = \vec{F}$$

ima smjer brzine

$$\vec{a} = \frac{d\vec{v}}{dt} = \frac{d}{dt}\left(|\vec{v}|\cdot\hat{\vec{v}}\right) = \frac{d}{dt}\left(v\cdot\hat{\vec{v}}\right) = \boxed{\frac{dv}{dt}\hat{\vec{v}} + v\frac{d\hat{\vec{v}}}{dt}}$$

okomit na

Rastavili smo akceleraciju na 2 vektora : $\vec{a} = \vec{a}_{tang.} + \vec{a}_{\perp}$

Na tangencijalnu i transverzalnu.

Tangencijalna je u smjeru tangente putanje i govori nam o promjeni brzine, tj. koliko čestica ubrzava gibajući se duž vlastite putanje.

Transverzalna (radijalna, centralna,...) je okomita na smjer gibanja i govori o promjeni smjera brzine.

TVRDNJA: Ako je sila okomita na brzinu ($\vec{F}\cdot\vec{v}=0$) onda se iznos brzine ne mijenja $\left(\frac{dv}{dt}=0\right)$

DOKAZ:

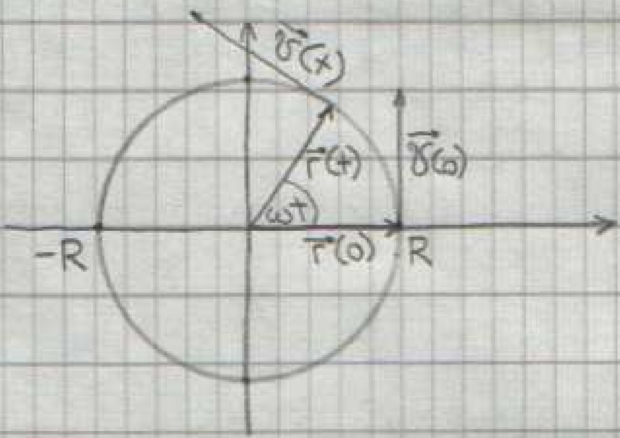
$$\vec{a} = \frac{dv}{dt}\hat{\vec{v}} + \frac{v\,d\hat{\vec{v}}}{dt} = \frac{\vec{F}}{m} \quad / \cdot \hat{\vec{v}}$$

$$\vec{a}\cdot\hat{\vec{v}} = \frac{dv}{dt}\hat{\vec{v}}\hat{\vec{v}} + v\frac{d\hat{\vec{v}}}{dt}\hat{\vec{v}} = \frac{F\hat{\vec{v}}}{m}$$

$$\vec{a}\cdot\vec{v} = \frac{dv}{dt} + 0 = \frac{0}{m}$$ iz tvrdnje

$$\frac{dv}{dt} = 0$$

Jednoliko kružno gibanje



$$\boxed{\vec r(t) = R(\cos\omega t\,\vec i + \sin\omega t\,\vec j)}$$

$$R = |\vec r(t)|$$

$$x(t) = R\cos\omega t$$

$$y(t) = R\sin\omega t$$

$$\vec v(t) = \dot{\vec r}(t) = R\omega(-\sin\omega t\,\vec i + \cos\omega t\,\vec j)$$

$|\vec v(t)| = R\omega = \bar v$ (Iznos brzine se ne mijenja)

$$\vec a(t) = \dot{\vec v}(t) = -\omega^2 R(\cos\omega t\,\vec i + \sin\omega t\,\vec j) = -\omega^2 \vec r(t)$$

$$\frac{d\vec p}{dt} = m\vec a = \vec F$$

$$\vec F = -m\omega^2 R \equiv \vec F_{cp} = \frac{m v^2}{R}$$

$\vec F_{cp} \perp \vec v$ ... dokaz: $\vec F_{cp}\cdot\vec v = (-m\omega^2 R(\cos\omega t\,\vec i + \sin\omega t\,\vec j))(R\omega(-\sin\omega t\,\vec i + \cos\omega t\,\vec j))$

$$= -m\omega^3 R^2 (\underbrace{-\sin\omega t\cos\omega t + \sin\omega t\cos\omega t}_{0})$$

$$= \underline{0}$$

Kutna brzina kao vektor ($\vec\omega$)

iznos: $\omega = \frac{v}{r}$

smjer: $\hat{\vec\omega} = \hat{\vec r} \times \hat{\vec v}$

$$\hat{\vec\omega} \times \hat{\vec r} = (\hat{\vec r} \times \hat{\vec v}) \times \vec r = \hat{\vec v}\underbrace{(\hat{\vec r}\cdot\hat{\vec r})}_{1} - \hat{\vec r}\underbrace{(\hat{\vec v}\cdot\hat{\vec r})}_{0} = \frac{1}{v}$$

$$\boxed{\vec v = \vec\omega \times \vec r}$$

7.

Kutna količina gibanja

$\hat{\vec{\omega}} = \hat{\vec{r}} \cdot \hat{\vec{v}}$ / $\cdot\, rmv$

$rmv\,\hat{\vec{\omega}} = \vec{r} \times m\vec{v} = \vec{r} \times \vec{p}$

$$\vec{L} = \vec{r} \times \vec{p} = rmv\,\hat{\vec{\omega}} = rmv\frac{\vec{\omega}}{\omega} = r^2 m\vec{\omega} = I\vec{\omega}$$

$I = r^2 m$ (moment tromosti)

TEOREM: Ako se čestica giba u polju centralne sile (usmjerena u 1 točku koordinatnog sustava) kutna količina gibanja (L) je očuvana veličina.

DOKAZ:
$$\frac{d\vec{L}}{dt} = \frac{d}{dt}(\vec{r} \times \vec{p}) = \frac{d\vec{r}}{dt} \times \vec{p} + \vec{r} \times \frac{d\vec{p}}{dt} = \vec{v} \times (m\vec{v}) + \vec{r} \times \vec{F}$$
$$= m\vec{v} \times \vec{v} + \vec{r} \times (\vec{F}_{(r)} \cdot \vec{r})$$
$$= m\underbrace{\vec{v} \times \vec{v}}_{0} + \vec{F}_{(r)}\underbrace{\vec{r} \times \vec{r}}_{0} = 0$$

$$\frac{d\vec{L}}{dt} = \vec{r} \times \vec{F} \equiv \vec{M}$$

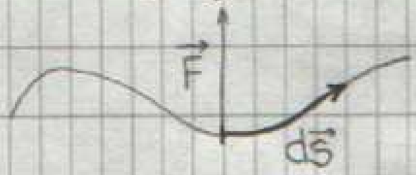
$$\frac{d\vec{L}}{dt} = I \cdot \dot{\vec{\omega}} = I \cdot \vec{\alpha} \equiv \vec{M}$$

$\dot{\vec{\omega}}$ ↓ kutna brzina; $\vec{\alpha}$ ↓ kutna akceleracija

Newtonova jednadžba za moment sile: $\frac{d\vec{L}}{dt} = I \cdot \vec{\alpha} \equiv \vec{M}$

Rad

Sila $\vec{F}$ djeluje duž diferencijala $d\vec{s}$.



Diferencijal obavljenog rada: $dW = \vec{F} \cdot d\vec{s}$

$W = \int \vec{F} \cdot d\vec{s}$

Rad za savijanje opruge: $W = \frac{1}{2} k x^2$

Kinetička energija

Neka sila F djeluje na tijelo mase m duž x osi dok se tijelo giba

$$F = \frac{d\vec{p}}{dt} = m\frac{d\vec{v}}{dt} \quad \Big/ \cdot \frac{dx}{dx}$$

promjena brzina brzine po prevaljenom putu

$$= \frac{m\,d\vec{v}}{dt} \cdot \frac{dx}{dx} = m\frac{d\vec{v}}{dx} \cdot \frac{dx}{dt} = m\vec{v} \cdot \frac{d\vec{v}}{dx}$$

brzina

Rad: $W = \int_A^B F\,ds = \int_A^B \left(mv\frac{dv}{dx}\right)dx = m\int_A^B v\frac{dv}{dx}\cdot dx = \frac{mv^2}{2}\Big|_A^B$

$$= \frac{mv_B^2}{2} - \frac{mv_A^2}{2} = E_{kin\,B} - E_{kin\,A} = \Delta E_{kin}$$

Teorem o radu i kinetičkoj energiji: $W = \Delta E_{kin}$

Obavljeni rad = promjeni kin. energije

Snaga

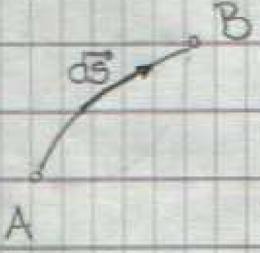
Mjera količine rada obavljene u jedinici vremena.

$$P = \frac{dW}{dt} = \frac{d}{dt}(\vec{F} \cdot d\vec{s}) = \vec{F} \cdot \frac{d\vec{s}}{dt} = \vec{F} \cdot \vec{v}$$

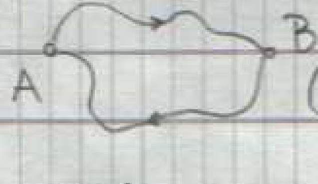
Konzervativne sile i potencijalna energija

$W = \int^{B} F d\vec{s}$ (rad obavljen od A do B)

Sila za koju vrijedi da je obavljeni rad W uvijek jednak neovisno o putanji zove se KONZERVATIVNA SILA.



$$dW_{AB} = \vec{F} \cdot d\vec{s}_{AB}$$
$$dW_{BA} = \vec{F} \cdot d\vec{s}_{BA} = \vec{F} \cdot (-d\vec{s}_{AB}) = -\vec{F} d\vec{s}_{AB} = -dW_{AB}$$



(krenemo od A i vratimo se u A, $W=0$)

$$\oint \vec{F} \cdot d\vec{s} = 0$$

↳ integral po zatvorenoj putanji

Kod konzervativne sile rad duž svake zatvorene krivulje iščezava.

Potencijalna energija (u polju konzervativne sile) definira se kao rad koji treba obaviti da se neko tijelo dovede u neku točku.

$$U(B) = U(A) - \int_{A}^{B} \vec{F}_{konz} d\vec{s} \qquad U(x) = U(x_0) - \int_{x_0}^{x} \vec{F}(x) dx$$

$$\vec{F}(\vec{r}) = -\frac{dU}{dx}\vec{i} - \frac{dU}{dy}\vec{j} - \frac{dU}{dz}\vec{k} = -\frac{dU}{d\vec{r}} = -\vec{\nabla} U \qquad F(x) = -\frac{d}{dx} U(x)$$

↓ konz. sila

Zakon očuvanja mehaničke energije

$W_{konz} = \Delta E_{kin.} = -\Delta U \quad \Rightarrow \quad \Delta(E_{kin} + U) = 0 \qquad \Delta E = 0$

Ukupna energija: $E = E_{kin} + U$

$$\frac{dE}{dt} = \frac{d}{dt}(E_{kin.} + U) = \frac{d}{dt}\left(\frac{m}{2}\vec{v}\vec{v} + U(\vec{r})\right)$$

$$= \frac{m}{2}\left(\dot{\vec{v}}\cdot\vec{v} + \vec{v}\dot{\vec{v}}\right) + \frac{dU}{d\vec{r}}\cdot\frac{d\vec{r}}{dt}$$

$$= m\dot{\vec{v}}\vec{v} + \vec{\nabla}U\vec{v}$$

$$= \vec{v}\left(m\dot{\vec{v}} + \vec{\nabla}U\right) = \vec{v}(m\vec{a} + \nabla U)$$

$$= \vec{v}(\vec{F} - \vec{F}) = 0$$

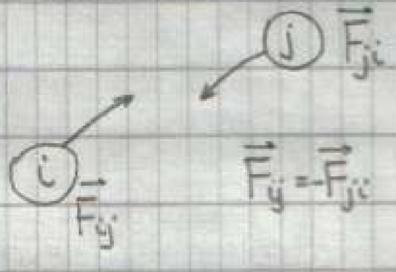
Zakon očuvanja količine gibanja

$\vec{p} = m\vec{v}$

Promatramo sustav čestica; ukupna količina gibanja: $\vec{P} = \sum_i \vec{p}_i$

$$\frac{d\vec{P}}{dt} = \frac{d}{dt}\sum_i \vec{p}_i = \sum_i \frac{d\vec{p}_i}{dt} = \sum_i \vec{F}_i$$

$$\vec{F}_i = \vec{F}_{i\,vanjska} + \sum_j^{j\neq i} \vec{F}_{ij}$$



$\vec{F}_{ij} = -\vec{F}_{ji}$

$$\frac{d\vec{P}}{dt} = \sum_i \vec{F}_i = \sum_i \left(\vec{F}_{i\,vanjska} + \sum_j^{j\neq i} \vec{F}_{ij}\right)$$

$$= \sum_i \vec{F}_{i\,vanjska} + \underbrace{\sum_i \sum_j^{j\neq i} \vec{F}_{ij}}_{0} = \sum_i F_{i\,vanjska}$$

Ako na sustav čestica ne djeluju vanjske sile ukupna količina gibanja je očuvana.