Kulišić(bez 6.4. , 6.8. , 6.11.)

6.Rotacija krutog tijela

Bilješke

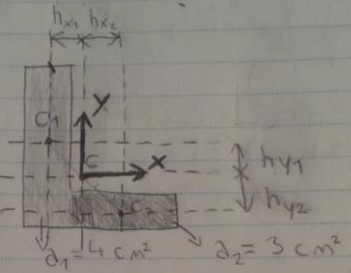
Rotacija krutog tijela

6.1.

$a = b = 4\,cm$

$c = 1\,cm$

$m = 7\,g$



$C_1(x_1, y_1)$, $C_2(x_2, y_2)$

$C_1(0.5, 2)$, $C_2(2.5, 0.5)$

$C(\bar{x}, \bar{y})$

$$\bar{x} = \frac{a_1 x_1 + a_2 x_2}{a_1 + a_2} = 1.357\,cm \qquad \bar{y} = \frac{a_1 y_1 + a_2 y_2}{a_1 + a_2} = 1.357\,cm$$

$$h_{y_1} = y_1 - \bar{y} = 2 - 1.357 = 0.643\,cm$$

$$h_{y_2} = \bar{y} - y_2 = 1.357 - 0.5 = 0.857\,cm$$

$$I_x = I_{x_1} + I_{x_2} = \frac{1 \cdot 4^3}{12} + (4 \cdot 1) \cdot 0.643^2 + \frac{3 \cdot 1^3}{12} + 3 \cdot 1 \cdot 0.857^2$$

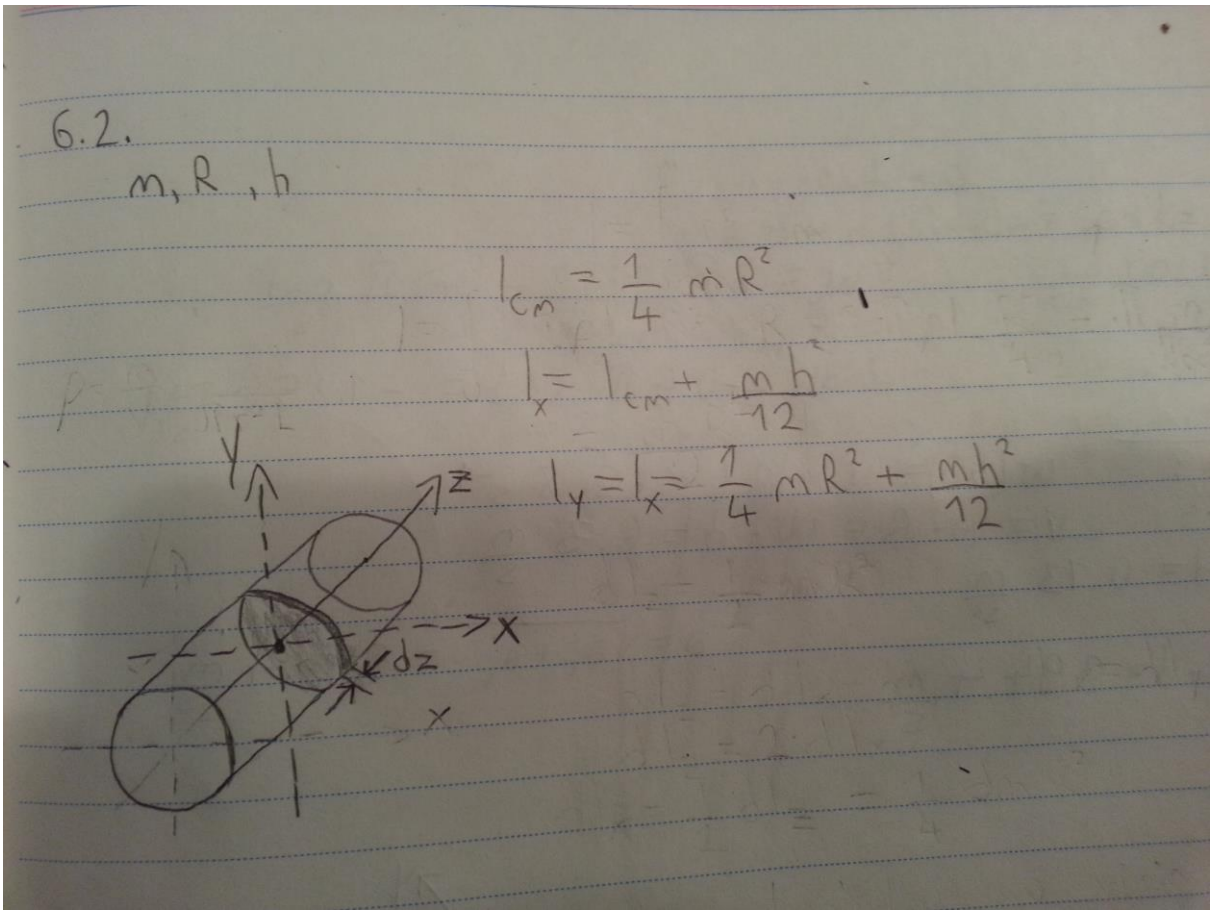
$$I_x = 9.44\ g\,cm^2$$

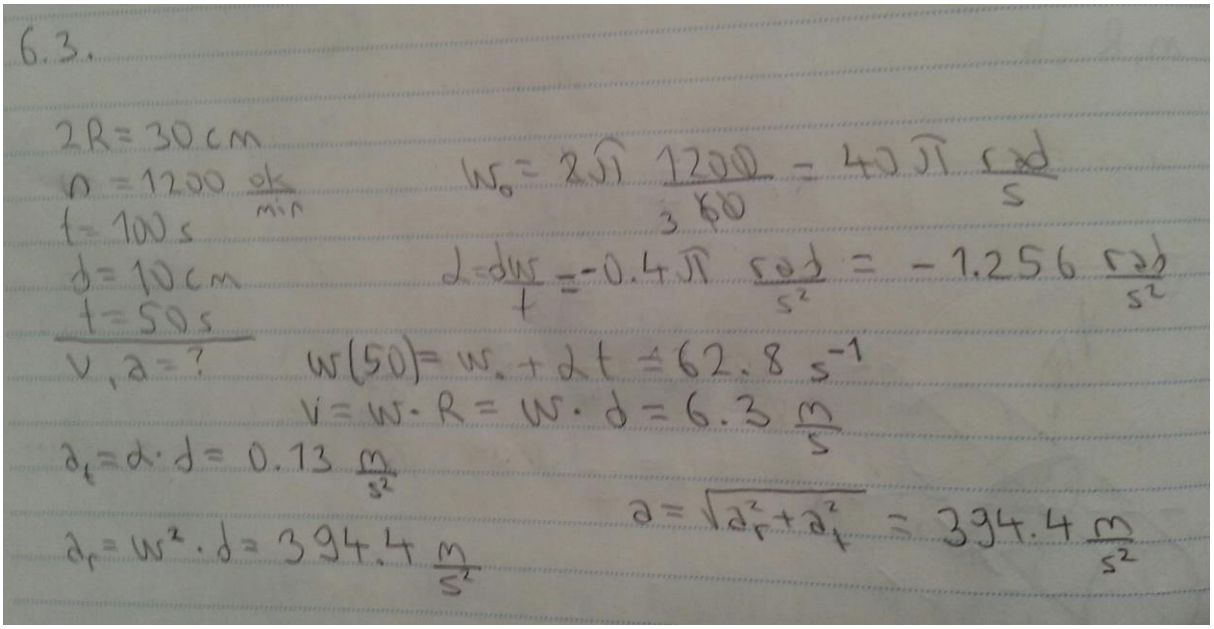
$$I_y = I_{y_1} + I_{y_2} = \frac{4 \cdot 1^3}{12} + 4 \cdot 1\,(1.357 - 0.5)^2 + \frac{1 \cdot 3^3}{12} + 3 \cdot 1 \cdot (2.5 - 1.357)^2$$

$$I_y = 9.44\ g\,cm^2$$

6.2. U ovom zadatku bi trebalo napisati cijeli postupak(malo integriranja).Ja sam to gledao da kad se rotira svejedno bilo oko x ili y osi da je moment tromosti Icm=1/4mR^2 što je zapravo moment tromosti diska,a ovaj drugi je zapravo moment tromost štapa i zbroj ta dva jednak je ukupnom momentu tromosti valjka prema Steinerovom poučku.Ako nekog zanima više ,tu je sličan zadatak riješen samo se valjak rotira oko jedne od njegovi baza.

http://hyperphysics.phy-astr.gsu.edu/hbase/icyl.html

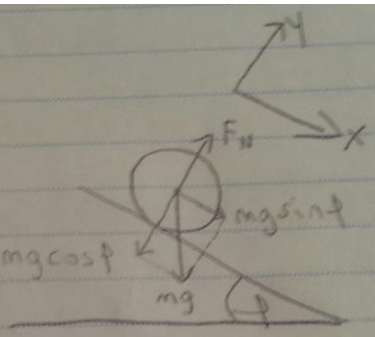




6.5.

$\alpha = 45°$

$s = 10\,m$



a, $v = ?$

$x\ldots \quad ma = mg\sin\alpha - F_{tr} \qquad y\ldots \quad F_N - mg\cos\alpha = 0$

$M = I \cdot \varepsilon = F_{tr} \cdot R$

$I = \frac{mR^2}{2} \qquad \varepsilon = \frac{a}{R}$

$a = g\sin\alpha - \frac{F_{tr}}{m} = g\sin\alpha - \frac{\frac{mR^2}{2}\cdot\frac{a}{R}}{m} = g\sin\alpha - \frac{a}{2}$

$a = g\sin\alpha - \frac{a}{2}$

$\frac{3}{2}a = g\sin\alpha \longrightarrow a = \frac{2}{3}g\sin\alpha = 4.62\,\frac{m}{s^2}$

$v = \sqrt{2as} = 9.61\,\frac{m}{s}$

6.6.

$v_0 = 1\,\frac{m}{s}$

$\alpha = 30°$

$m = 1\,kg$

$s, \mu_{min} = ?$

1... $M = I \cdot \varepsilon = F_{tr} \cdot R$ $\qquad \varepsilon = \frac{a}{R}$

$F_{tr} \cdot R = \frac{mR^2}{2} \cdot \frac{a}{R}$

$F_{tr} = \frac{m \cdot a}{2}$

2... $mg\sin(\alpha) + F_{tr} = ma$

$mg\sin\alpha = ma - \frac{1}{2}ma$

$mg\sin\alpha = \frac{1}{2}ma \Rightarrow a = 2g\sin\alpha = 9.81\,\frac{m}{s^2}$

$a = \frac{dv}{dt} \Rightarrow t = \frac{1}{9.81} = 0.1\,s$

$F_{tr} = \frac{m \cdot a}{2} = 4.905\,N$ $\qquad F_{tr\,MAX} = \mu m g$

poglejdat primjer 6.13. za objašnjenje

$F_{tr} \leq F_{tr\,MAX}$

$4.905 \leq \mu m g$

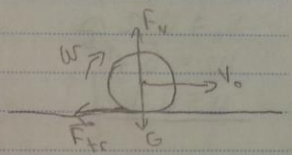
$\mu \geq \frac{4.905}{m \cdot g} \Rightarrow \mu \geq 0.5$

6.7.

$v_0 = 6.5 \frac{m}{s}$

$\mu_{kl} = 0.3$

uvjet kotrljanja: $v_{cm} = \omega \cdot R$



translacija:

$m \frac{dv}{dt} = -F_{tr}$

$m \frac{dv}{dt} = -\mu m g$

$\frac{dv}{dt} = -\mu g$

$dv = -\mu g \, dt \,/ \int$

$\int dv = \int -\mu g \, dt$

$v(t) = -\mu g t + C$

$C = v(0) = 6.5 \frac{m}{s}$ $\Big\} v(t) = 6.5 - \mu g t$

rotacija: $I_{kugle} = \frac{2}{5} m R^2$

$M = I \cdot \alpha = F_{tr} \cdot R$

$I \frac{d\omega}{dt} = R \cdot \mu m g$

$d\omega = \frac{R \mu m g}{I} dt$

$d\omega = \frac{R \mu m g}{\frac{2}{5} m R^2} dt = \frac{5}{2} \frac{\mu g}{R} dt \,/ \int$

$\omega(t) = \frac{5}{2} \frac{\mu g}{R} t + C$

$C = \omega(0) = 0$ $\Big\} \omega(t) = \frac{5}{2} \frac{\mu g}{R} t$

$v_{cm} = R\omega$

$6.5 - \mu g t = R \cdot \frac{5}{2} \frac{\mu g}{R} t$

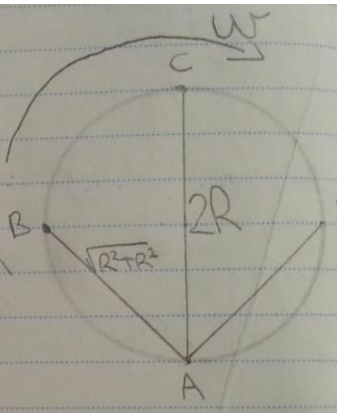
$t\left(\frac{5}{2}\mu g + \mu g\right) = 6.5 \Rightarrow t = \frac{6.5}{\frac{7}{2}\mu g} = 0.63 s$

6.9.

$2R = 30 cm = 0.3 m$

$v_{cm} = 0.43 \frac{m}{s}$

$\omega = \frac{v_{cm}}{R} = 2.87 \frac{rad}{s}$



A $v_A = 0 \frac{m}{s}$ zbog uvjeta da tijelo ne klizi

C $v_c = 2R\omega = 0.86 \frac{m}{s}$

B, D $v_B = v_D = \sqrt{R^2+R^2} \cdot \omega = 0.61 \frac{m}{s}$

6.10.

$2R = 30\,cm = 0.3\,m$

$v_0 = 0.2\,\frac{m}{s}$

$w = 6\,s^{-1}$

$\mu = 0.1$

Uvjet kotrljanja:

$v_{cm} = R w$

translacija:

$m\frac{dv}{dt} = -F_{tr}$

$m\frac{dv}{dt} = -\mu m g$

$dv = -\mu g\,dt \,/\int$

$v(t) = -\mu g t + C$

$v(0) = 0.2\,\frac{m}{s}$

$v(t) = 0.2 - \mu g t$

rotacija:

$M = I \cdot \alpha = F_{tr} \cdot R$

$I\frac{dw}{dt} = \mu m g \cdot R$

$dw = \frac{\mu m g R}{I}\,dt$

$dw = \frac{\mu m g R}{\frac{m R^2}{2}}\,dt \,/\int$

$w(t) = \frac{2\mu g}{R} t^2 + C$

$w(0) = 6\,s^{-1}$

$w(t) = \frac{2\mu g t}{R} + 6$

$v_{cm} = R w$

$0.2 - \mu g t = R \cdot \left(\frac{2\mu g t}{R} + 6\right)$

$2\mu g t + \mu g t = 0.2 - R \cdot 6$?

$3\mu g t = 0.2 - R \cdot 6$

$t = \frac{0.2 - R \cdot 6}{3\mu g} = -0.24\,s$

$v(-0.24) = 0.43\,\frac{m}{s}$

$w(-0.24) = 2.86\,\frac{rad}{s}$

6.12.

$n = 10 \frac{ok}{s}$

$\mu = 0.1$

$R = 30\,cm$

$\omega = 2\pi \cdot 10 = 20\pi = 62.83\,s^{-1}$

$I = mr^2$ $\qquad N_2 = F_{tr1}$

$M = I \cdot \alpha = F_{tr_1} \cdot R + F_{tr_2} \cdot R$

$I \frac{d\omega}{dt} = \mu m g R + \mu N_2 R$

$I \frac{d\omega}{dt} = \mu m g R + \mu^2 m g R$

$d\omega = \frac{\mu m g R + \mu^2 m g R}{m R^2} dt \quad / \int$

$\omega(t) = \frac{(\mu g + \mu^2 g)}{R} t + C$

$C = \omega(0) = 62.83\,s^{-1}$

$\omega(t) = \frac{(\mu + \mu^2) g}{R} t + 62.83$

Uvjet: $\omega = 0\,s^{-1}$

$t = -\frac{62.83 \cdot R}{(\mu + \mu^2) g} = -17.5\,s$?

$\alpha = \frac{d\omega}{dt} = \frac{-62.83}{17.5} = -3.59\,\frac{rad}{s^2}$

$\varphi = \omega_0 t + \frac{1}{2} \alpha t^2 = 62.83 \cdot 17.5 + \frac{1}{2}(-3.59) \cdot 17.5^2$

$\varphi = 549.81\,rad$

$N = \frac{\varphi}{2\pi} = 87.55$

6.13.

$m_1 = 30\,kg$ $\quad I_{c_1} = 2.34\,kg m^2$ $\quad I_{c_2} = 0.98\,kg m^2$

$R = 1\,m$

$f = 20\,\frac{okr}{min}$ $\quad \omega_1 = 2\pi \frac{20}{60} = 2.1\,\frac{rad}{s}$

$$\sum_i I_i \omega_i = konst$$

$$I_1 = m_1 \frac{R^2}{2} + I_{c_1}$$

$$I_2 = m_1 \frac{R^2}{2} + I_{c_2}$$

$$I_1 \omega_1 = I_2 \omega_2$$

$$\omega_2 = \frac{\left(m_1 \frac{R^2}{2} + 2.94\right)}{m_1 \frac{R^2}{2} + 0.98} \omega_1 = 2.2\,\frac{rad}{s}$$

$$\omega_2 = 2\pi f_2 \Rightarrow f_2 = 0.35\,Hz$$