21 2007–2008

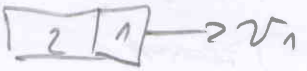
$k = 200\ N/m$

1.

$m_1 = 1\,kg$
$m_2 = 2\,kg$

$v = 4\ m/s$ $\quad \Delta x = ?$

$m_2 v = (m_1 + m_2) v'$



$v' = \frac{8}{3}\ m/s$

$E_c = E_{opruga}$

$\frac{(m_1+m_2)\,v'^2}{2} = \frac{k x^2}{2} \quad \Rightarrow \quad x = 0.3266\ m$

2.

ulje

$d = 12.3\ mm$

$h = 135\ mm$

$\rho_v = 998\ kg/m^3$

$\rho_{ulja} = ?$

$\vec{G}_1 = \vec{G}_2$

$m_{ulje} \cdot g = m_{voda} \cdot g$

$m_{ulje} = m_{voda}$

$\rho_{ulja} V_{ulje} = \rho_{H_2O} V_{H_2O}$

$\rho_{ulja} = \rho_{H_2O} \cdot \frac{V_{H_2O}}{V_{ulje}} = \rho_{H_2O} \cdot \frac{S \cdot h}{S \cdot (h+d)} = 914.66\ kg/m^3$

3. $n = 5\ mol$ $\quad i = 3$

$\Delta T = 20K$

$p = konst$

$\Delta Q = \Delta U + \Delta W$

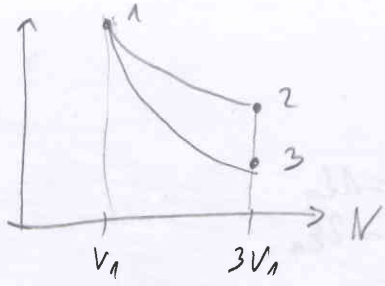
$\Delta U = \frac{i}{2} n R \Delta T = 1247.1\ J$

$\Delta W = p(V_2 - V_1)$

$\Delta Q = n c_p \Delta T = n \frac{i+2}{2} R \Delta T = 2078.5\ J$

$\rightarrow \Delta W = 831.4\ J$

4.



p_2, p_3, T_3, W chceme preko

p_1, V_1, T_1, R

$pV = nRT$

$i = 5$

$\varkappa = \frac{i+2}{i} = \frac{7}{5}$

$1 \to 2$ $\quad p_1 V_1 = p_2 V_2 \quad T = \text{konst}$

$$p_2 = \frac{p_1}{3}$$

$$W_{12} = \int p\,dV = \int_{(1)}^{(2)} \frac{nRT}{V}\,dV = nRT \ln\frac{V_2}{V_1} = nRT \ln 3 = p_1 V_1 \ln 3$$

$2 \to 3$ $\quad V = \text{konst}$

$$\frac{p_2}{T_2} = \frac{p_3}{T_3}$$

$$W_{2\to 3} = 0$$

$3 \to 1$ $\quad \Delta Q = \Delta U + \Delta W$

$\Delta Q = 0$

$$W_{31} = -\Delta U = -\frac{5}{2} nR\Delta T = -\frac{5}{2} nR(T_1 - T_3) = -\frac{5}{2}(p_1 V_1 - p_3 V_3)$$

$dU = -dW$

$\frac{5}{2} nR\,dT = -p\,dV$

$\frac{5}{2} nR\,dT = -\frac{nRT}{V}\,dV$

$\frac{5}{2} nR \frac{dT}{T} = -nR\frac{dV}{V}$

$\frac{5}{2} \ln\left(\frac{T_1}{T_3}\right) = -\ln\left(\frac{V_1}{V_3}\right) = \ln 3$

$T_1 = 1.5518\,T_3$

$$\frac{T_1}{T_3} = \left(\frac{V_3}{V_1}\right)^{\varkappa - 1}$$

$$T_1 = 1.5518\,T_3$$

$$\frac{T_1}{T_3} = \left(\frac{p_1}{p_3}\right)^{\frac{\varkappa-1}{\varkappa}} = \left(\frac{p_1}{p_3}\right)^{\frac{2}{7}}$$

$$\frac{p_1}{p_3} = \left(\frac{T_1}{T_3}\right)^{7/2}$$

$$p_3 = 0.2148\,p_1$$

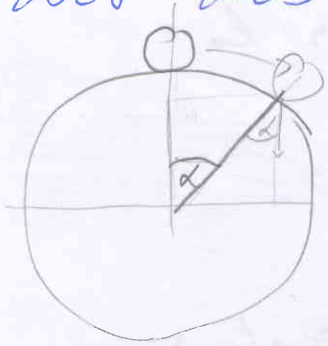
$$W_{31} = \frac{nR}{\varkappa - 1}(T_3 - T_1) = \frac{nR}{\frac{2}{5}} \cdot (-0.355\,T_1) = -0.89\,nRT_1 = -0.89\,p_1 V_1$$

PROVJERA $\quad p_2 = p_3 \frac{T_2}{T_3} = p_3 \frac{T_1}{T_3} = 1.5518\,p_3 = 1.5518 \cdot 0.2148\,p_1 = 0.33\,p_1$ ✓

$$W_{uk} = W_{12} + W_{23} + W_{31} = p_1 V_1 (\ln 3 - 0.89) = 0.21\,p_1 V_1$$

Z1 2008-2009

1.



$$mg\left(R+\frac{R}{10}\right) = mgh + \frac{mv^2}{2} + \frac{I\omega^2}{2}$$

$$h = \cos\alpha \cdot \left(R+\frac{R}{10}\right)$$

$$I = \frac{2}{5}mr^2 \qquad I\omega^2 = \frac{2}{5}mv^2$$

$$mg\left(R+\frac{R}{10}\right)(1-\cos\alpha) = \frac{mv^2}{2} + \frac{1}{5}mv^2 = \frac{7}{10}mv^2$$

$$v^2 = \frac{10}{7}g\left(R+\frac{R}{10}\right)(1-\cos\alpha)$$

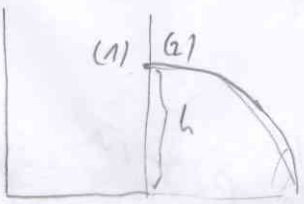
$F_{cf} = F_{G\perp}$

$$\frac{mv^2}{R+\frac{R}{10}} = mg\cos\alpha \quad \Rightarrow \quad \cos\alpha = \frac{10}{7}(1-\cos\alpha)$$

$$\frac{17}{7}\cos\alpha = \frac{10}{7} \qquad \alpha = 53.96^\circ = 0.9417\ \text{rad}$$

$$s = R\cdot\alpha = 0.9417R$$

2.



max donet

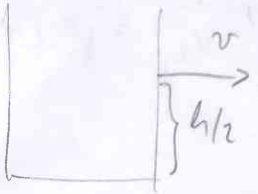
$$D = \frac{v_0^2 \cdot \sin 2\alpha}{g} \qquad \frac{dD}{d\alpha} = \frac{2v_0^2\cos 2\alpha}{g} = 0$$

$2\alpha = 90^\circ$

$\alpha = 45^\circ$

$$H = \frac{v_0^2 \cdot \sin^2\alpha/2}{2g} = \frac{v_0^2}{4g} = \frac{(\sqrt{2gh})^2}{4g} = \frac{h}{2}$$

usmena isteca mja



$$v = \sqrt{2g\,h/2}$$

$$\frac{h}{2} = \frac{gt^2}{2}$$

$$x = v_0 \cdot t = \sqrt{2g\frac{h}{2}}\sqrt{\frac{h}{g}} = h$$

$$p_1 + \rho g h_1 + \rho\frac{v_1^2}{2} = p_2 + \rho g h_2 + \rho\frac{v_2^2}{2}$$

$$v_2^2 = 2gh_1$$

$$v = \sqrt{2gh}$$

3. $l_1^* = l_0/\sqrt{1-\beta^2} = l_0/\sqrt{1-0.9^2} = 2.294\, l_0$

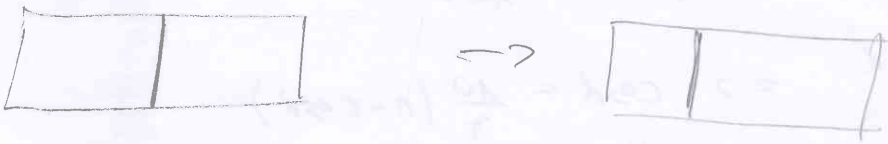
$V = S \cdot l$ $\quad V_1 = 2.294\, V_0$

$m = \rho \cdot V$

$\rho_0 V_0 = \rho_1 V_1 = \rho_1 \cdot 2.294 V_0$

$\rho_1 = 0.43588\, \rho_0 = 1.1765\ g/cm^3$

4.



$V = 2l$

$V_1 = 1\,l$, $p_1 = 2\,bar$ $\quad T_1 = T_2 = 300K$ $\quad p_1', T_1', p_2', T_2' = ?$

$V_2 = 1\,l$, $p_2 = 1\,bar$ $\quad p_1' = p_2'$

$i = 5 \quad \kappa = \frac{i+2}{i} = \frac{7}{5}$

$\frac{p_1'}{p_1} = \left(\frac{V_1'}{V_1}\right)^\kappa \quad p_1' V_1^\kappa = p_1 V_1'^\kappa$

$\frac{p_2'}{p_2} = \left(\frac{V_2'}{V_2}\right)^\kappa \quad p_2' V_2^\kappa = p_2 V_2'^\kappa$

$V_1 = V_2$

$p_1' = p_2'$

$\left.\begin{array}{l} p_1 V_1'^\kappa = p_2 V_2'^\kappa \\ 2 V_1'^\kappa = V_2'^\kappa \end{array}\right\} \quad | \ ^{5/7}$

$V_2' = 1.64\, V_1'$

$V_2' + V_1' = 2.64\, V_1' = 2l$

$V_1' = 0.757\, l$

$V_2' = 1.242\, l$

$\Rightarrow p_1' = p_1 \cdot \left(\frac{V_1'}{V_1}\right)^\kappa = 1.354\ bar = p_2'$

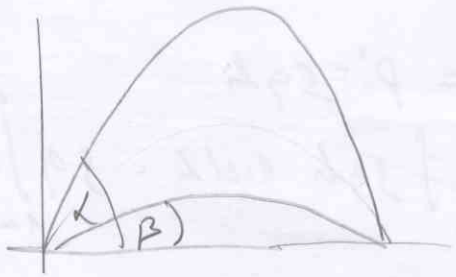
$\frac{T_1'}{T_1} = \left(\frac{V_1'}{V_1}\right)^{\kappa-1} = \left(\frac{V_1'}{V_1}\right)^{\frac{2}{5}}$

$T_1' = T_1 \cdot 0.757^{2/5} = 268.38\,K$

$T_2' = T_2 \cdot 1.242^{2/5} = 327.167\,K$

Z1 2009 - 2010

1.



I. $D_1 = 3H_1$

$$\frac{v_0^2 \sin 2\alpha}{g} = \frac{3 \cdot v_0^2 \sin^2 \alpha}{2g}$$

$$2 \sin\alpha \cos\alpha = \frac{3}{2} \sin^2\alpha$$

$$\operatorname{tg}\alpha = \frac{4}{3} \rightarrow \alpha = 53.13°$$

$$D_1 = \frac{v_0^2 \sin 2\alpha}{g} = D_2$$

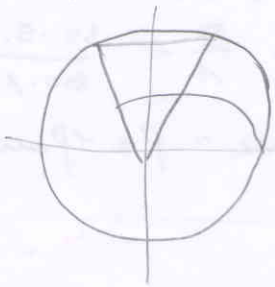
$$\frac{v_0^2 \sin 2\alpha}{g} = \frac{v_0^2 \sin 2\beta}{g}$$

$$\sin 2\alpha = \sin 2\beta = \sin 106.26° = 0.96$$

$$\arcsin 0.96 = 73.74°$$

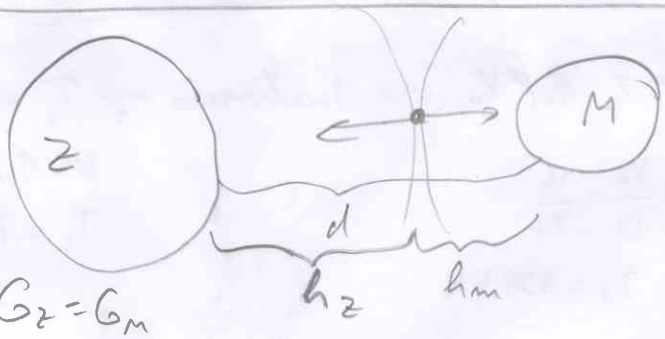
$$2\beta = 73.74°$$

$$\beta = 36.87° \Rightarrow$$



$$\Rightarrow \frac{H_2}{D_2} = \frac{v_0^2 \sin^2\alpha / 2g}{v_0^2 \sin 2\alpha / g} = \frac{\sin^2\alpha}{4 \sin\alpha \cos\alpha} = \frac{1}{4}\operatorname{tg}\alpha = \frac{3}{16}$$

2.



$M_Z = 81 M_M$ $\quad R_Z = 6400$ km

$d = 60 R_Z$ $\quad R_M = 1740$ km

$G_Z = G_M$

$$G\frac{m \cdot M_Z}{(R_Z + h_Z)^2} = \frac{G \cdot m \cdot M_M}{(R_M + h_M)^2}$$

$$81(R_M + h_M)^2 = (R_Z + h_Z)^2$$

$d = h_M + h_Z \rightarrow h_M = d - h_Z$

$$81(R_M + d - h_Z)^2 = (R_Z + h_Z)^2$$

$$9(R_M + d - h_Z) = R_Z + h_Z$$

$$3471660 - 9h_Z = 6400 + h_Z$$

$$h_Z = 346526 \text{ km}$$

$$E_Z = \frac{mv^2}{2} = E_P \quad / 346526 \text{ km}$$

$$E_P = \int F_g \cdot dh = \int_{h=6400\text{km}} \frac{GmM}{h^2} dh =$$

$$= GmM\left(-\frac{1}{h}\right)\Big|_{6400\text{km}}^{346526\text{km}} =$$

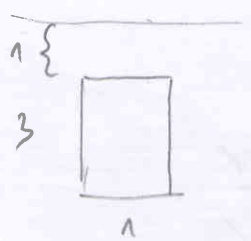
$$= m \cdot 60\,967183$$

$$E_Z = E_P$$

$$\frac{mv^2}{2} = m \cdot \searrow$$

$$v = 11042.4 \text{ m/s}$$

3.



$F = p \cdot A$

$p = p_0 + \rho g h$

→ tlak zbog tekućine $= p' = \rho g h$

$F = \int dF = \int \rho g h \cdot dA = \int \rho g h \cdot 1 \cdot dh = \rho g \int_{1m}^{4m} h\, dh$

$= \frac{15}{2} \rho g = 73575\ N$

4. $m = 35\ kg$, $A = 60\ cm^2$

$p_{masa} = \frac{F}{A} = \frac{35 \cdot 9.81}{60 \cdot 10^{-4}} = 57225\ Pa$

$p_{gume} = p_A + p_{max} = 158550\ Pa$

$V_0 = 40\ cm^3$, $p_0 = p_A$

→ UPUMPAVAMO

$V_1 = ?$ $p_1 = 158550\ Pa$

$p_0 V_0 = p_1 V_1$

$V_1 = 25.56\ cm^2$

$n = \frac{2000}{V_1} = 78.24$

$\Rightarrow n = 78$ puta

5. $n = 2\ mol$, $i = 3$

$V_0, T_0 = 273K$ → izobarno → $V_1 = 1.8 V_0$ → izohorno → $T_2 = T_0$

$V = 1.8 V_0$

$T_2 = 273K$

$Q = ?$

$\Delta Q = \Delta U + \Delta W$

$\Delta Q = n c \Delta T$

$\frac{V_0}{T_0} = \frac{V_1}{T_1}$

$T_1 = 491.4K$

$Q_{0\to1} = n \cdot c_p \cdot (T_1 - T_0)$

$Q_{1\to2} = n \cdot c_V (T_2 - T_1)$

$Q = Q_{01} + Q_{12} = n \cdot c_p (T_1 - T_0) + n c_V (T_0 - T_1) = n (c_p - c_V)(T_1 - T_0)$

$= n R (T_1 - T_0) =$

$= 3631.5\ J$

(17061)

12. $\Delta Q = \Delta U - \Delta W$, $\Delta W = 0$

$\Delta Q = \Delta U = \frac{5}{2} nR\Delta T = \frac{5}{2} \cdot nR \cdot 4T_1 = 10 p_1 V_1$

$\frac{p_2}{T_2} = \frac{p_1}{T_1}$ $T_2 = 5T_1$

16. $\Delta U = \Delta Q - \Delta W = \Delta Q - p \cdot \Delta V$

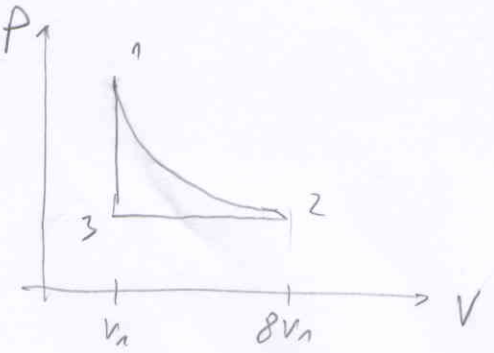
$\Delta Q = nc_p \Delta T$

$\Delta W = p \cdot \Delta V = n \cdot R \cdot \Delta T = \frac{\Delta Q}{c_p} \cdot R = \frac{\Delta Q}{4R} R = 5000\,J$

$\Delta U = 15000\,J$

17. 1 mol, $i = 3$

$p_1 = 10$ atm, $V_1 = 10^{-3}\,m^3$ $\eta = ?$



$\eta = \frac{W}{Q}$

$\varkappa = \frac{5}{3}$

1→2. $\Delta Q = 0$, $dW = -\Delta U$

$W_{12} = \frac{nR}{\varkappa - 1}(T_1 - T_2)$

$\frac{T_1}{T_2} = \left(\frac{V_2}{V_1}\right)^{\varkappa - 1}$

$p_1 V_1 = nRT_1 \rightarrow T_1 = 121.9\,K$

$T_2 = 30.47\,K$

$W_{12} = 1140\,J$

$\frac{p_1}{p_2} = \left(\frac{V_2}{V_1}\right)^{\varkappa} = 8^{\varkappa}$ $p_2 = 31664\,Pa$

2→3. $dp = 0$

$W_{23} = p_2(V_3 - V_2) = p_2(-7V_1) = -221.65\,J$

3→1. $\Delta V = 0 \rightarrow \Delta W = 0$

$\Delta Q = \Delta U = nC_v \Delta T = n \cdot \frac{3}{2} R (T_1 - T_3) = 1472.7\,J$

$p_3 V_3 = nRT_3 \rightarrow T_3 = 3.81\,K$

$p_3 = p_2$

$\eta = \frac{W_{12} + W_{23}}{Q_{31}} = \frac{1140 - 221}{1472} \approx 62\%$

10.11. Homogenim drvenim štapom duljine $l = 5m$, težine $G = 40N$ gustoće $\rho_1 = 796\ kg/m^3$ izmjeri se dubina jezera od $h = 4{,}75m$ Koliki je izvršeni rad ako je štap potapan vertikalno? $\rho_v = 10^3\ kg/m^3$

$l_1 \rightarrow$ duljina štapa u vodi u času kada je sila uzgona jednaka težini štapa.

$$U = G$$

$$\rho_v \cdot g \cdot S \cdot l_1 = \rho_š \cdot g \cdot S \cdot l$$

$$l_1 = l \cdot \frac{\rho_š}{\rho_v} = \dots$$

Sila kojom treba djelovati na štap F_y

$$F_y = \rho_v \cdot g \cdot S(l_1 + y) - G \qquad \rho_v \cdot g \cdot S \cdot l_1 = \rho_š \cdot g \cdot S \cdot l$$

$$F_y = \rho_v \cdot g \cdot S \cdot l_1 + \rho_š \cdot g \cdot S \cdot y - \rho_š \cdot g \cdot S \cdot l$$

$$F_y = \rho_v \cdot g \cdot S \cdot y$$

$$W = \int_0^{h-l_1} F_y\, dy = \int_0^{h-l_1} \rho_v \cdot g \cdot S \cdot y \cdot dy$$

$G = m \cdot g = \rho_1 \cdot S \cdot l \cdot g$

$\rightarrow g S = \frac{G}{\rho_š \cdot l}$

$$W = \frac{\rho_v \cdot G}{2 \cdot \rho_š \cdot l} \left(H - l \cdot \frac{\rho_š}{\rho_v}\right)^2 = 2{,}98\ J$$