1.) $t_1 = \frac{t}{2}$, $v_1 = 16 km/h$

$\frac{S_0}{2}$, $v_2 = 24 km/h$

$\frac{S_2}{2}$, $v_3 = 9 km/h$

$t_1 + t_2 + t_3 = t$

$t_2 + t_3 = \frac{t_{ük}}{2}$

$S_1 + S_2 = S_{ük}$

$\overline{v} = \frac{S_{ük}}{t_{ük}}$

$S_1 = v_1 \frac{t}{2}$

$S_2 = 2 v_2 t_2$

$S_2 = 2 v_3 t_3$

$v_2 t_2 = v_3 t_3$

$t_3 = \frac{v_2}{2 v_3} t_2$

$t_2 + \frac{v_2}{2 v_3} t_2 = \frac{t_{ük}}{2}$

$t_2 \left(1 + \frac{v_2}{2 v_3}\right) = \frac{t_{ük}}{2}$

$t_{ük} = 2 t_2 \left(1 + \frac{v_2}{2 v_3}\right)$

$v_1 \frac{t_{ük}}{2} + 2 v_2 t_2 = S_{ük}$

$v_1 t_2 \left(1 + \frac{v_2}{2 v_3}\right) + 2 v_2 t_2 = S_{ük}$

$t_2 \left(v_1 + \frac{v_1 v_2}{2 v_3} + 2 v_2\right) = S_{ük}$

$$\bar{v} = \frac{s_c\left(v_1 + \frac{v_1 v_2}{v_3} + 2v_2\right)}{2t_c\left(1 + \frac{v_2}{v_3}\right)} = 1.7 \text{ km/h}$$

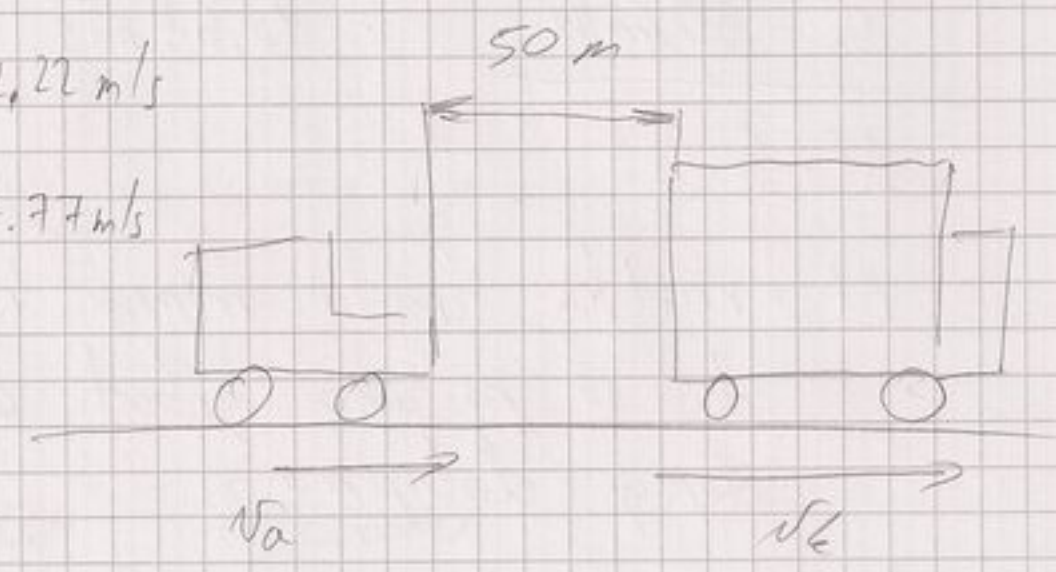
2.) $v_{0k} = 80 \text{ km/h} = 22,22 \text{ m/s}$

$v_{0a} = 100 \text{ km/h} = 27.77 \text{ m/s}$

$l = 50 \text{ m}$

$a_k = -1.6 \text{ m/s}^2$

$a_a = -7 \text{ m/s}^2$



OBA VOZILA SE ZAUSTAVLJAJU NA KRAJU

$$v^2 = 2as + v_0^2$$

GDJE JE v = BRZINA NA KRAJU GIBANJA, PA TO PREOSTALI v_0

$$0 = 2as + v_0^2$$

$$s = -\frac{v_k^2}{2a_k} = -\frac{22.21^2}{2 \cdot (-1.6)} = 154,29 \text{ m}$$

$$s_k = s + l = 204.29 \text{ m}$$

$$s_a = -\frac{v_{0a}^2}{2a_a} = 55.08 \text{ m}$$

$$\Delta s = s_k - s_a = 149.2 \text{ m}$$

$$v = \frac{s}{t} \qquad t = \frac{s}{v} = \frac{\Delta s}{v_{0a}} = 5.37 \text{ s}$$

3.) $v_1 = 600\ km/h = 166.67\ m/s$

$s_1 = 900\ m$

$t_2 = 4\ min = 240\ s$

$a_3 = 0.15\ m/s^2$

$v_3 = 60\ km/h = 16.67\ m/s$

$\Delta t = ?$

Δt = razlika između vremena koje bi mu trebalo da se nastavio gibati 60 km/h i ovog kojeg dobijemo

(1) $v^2 = 2as + v_0^2$

ZAUSTAVIO SE $v = 0$, $0 = 2as + v_0^2$

$$a = \frac{-v_0^2}{2s} = -0.154\ m/s^2$$

$v = v_0 + at$

$$t_1 = -\frac{v_0}{a} = 108.25\ s$$

(3) $v = v_0 + at$

$v_0 = 0$

$v = at$ $\quad t_3 = \frac{v_3}{a_3} = 111.13\ s$

$v^2 = 2as + v_0^2$

$$s = \frac{v^2}{2a} = 926.3\ m$$

$s_{uk} = s_1 + s_2 = 1826.3\ m$

$t_{BEZ SIGNALA} \dfrac{s_{uk}}{v_1} = \dfrac{1826.3}{16.67} = 109.56\ s$

$t_{SA SIGNALOM} = t_1 + t_2 + t_3 = 459.38\ s$

$\Delta t = t_{SA SIGNALOM} - t_{BEZ SIGNALA} = 349.82\ s$

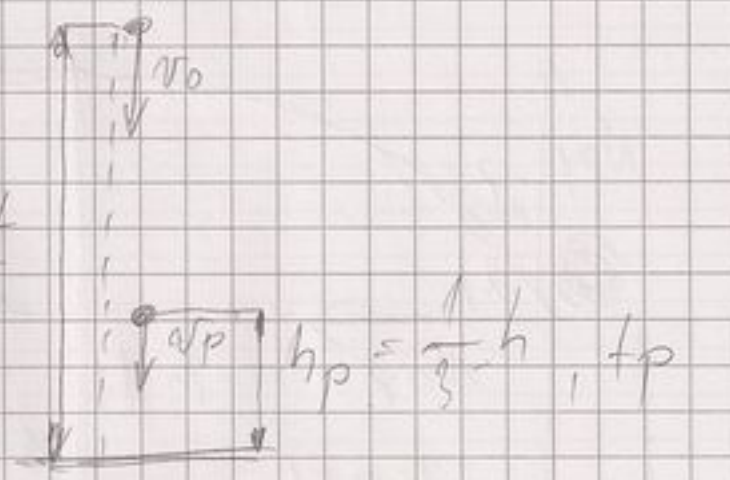
$= 5.83\ min$

4.) $v_0 = 4.5\ m/s$

$t_p = 2.5\ s$

$h_p = \frac{1}{3} h$

$t = ?$



$h = \frac{1}{2} g t^2 + v_0 t$

$h_p = \frac{1}{2} g t_p^2 + v_p t_p$

$\frac{1}{3} h = \frac{1}{2} g t_p^2 + v_p t_p \quad / \cdot 3$

$h = \frac{3}{2} g t_p^2 + 3 v_p t_p$

$h = \frac{3}{2} g t_p^2 + 3 t_p [v_0 + g(t - t_p)]$

$h = \frac{3}{2} g t_p^2 + 3 t_p \cdot v_0 + 3 t_p \cdot g (t - t_p)$

$v_p = v_0 + g \cdot (t - t_p)$

$$\frac{1}{2}gt^2 + v_0 t = \frac{3}{2}g t_p^2 + 3t_p v_0 + 3t_p \cdot g(t - t_p)$$

$$\frac{1}{2}gt^2 + v_0 t - \frac{3}{2}g t_p^2 - 3t_p v_0 - 3t_p \cdot g(t - t_p) = 0$$

$$\frac{1}{2}gt^2 + v_0 t - 3t_p \cdot g \cdot t - \frac{3}{2}g t_p^2 - 3t_p v_0 + 3t_p^2 \cdot g = 0$$

$$4.905t^2 - 69.075t + 58.219 = 0$$

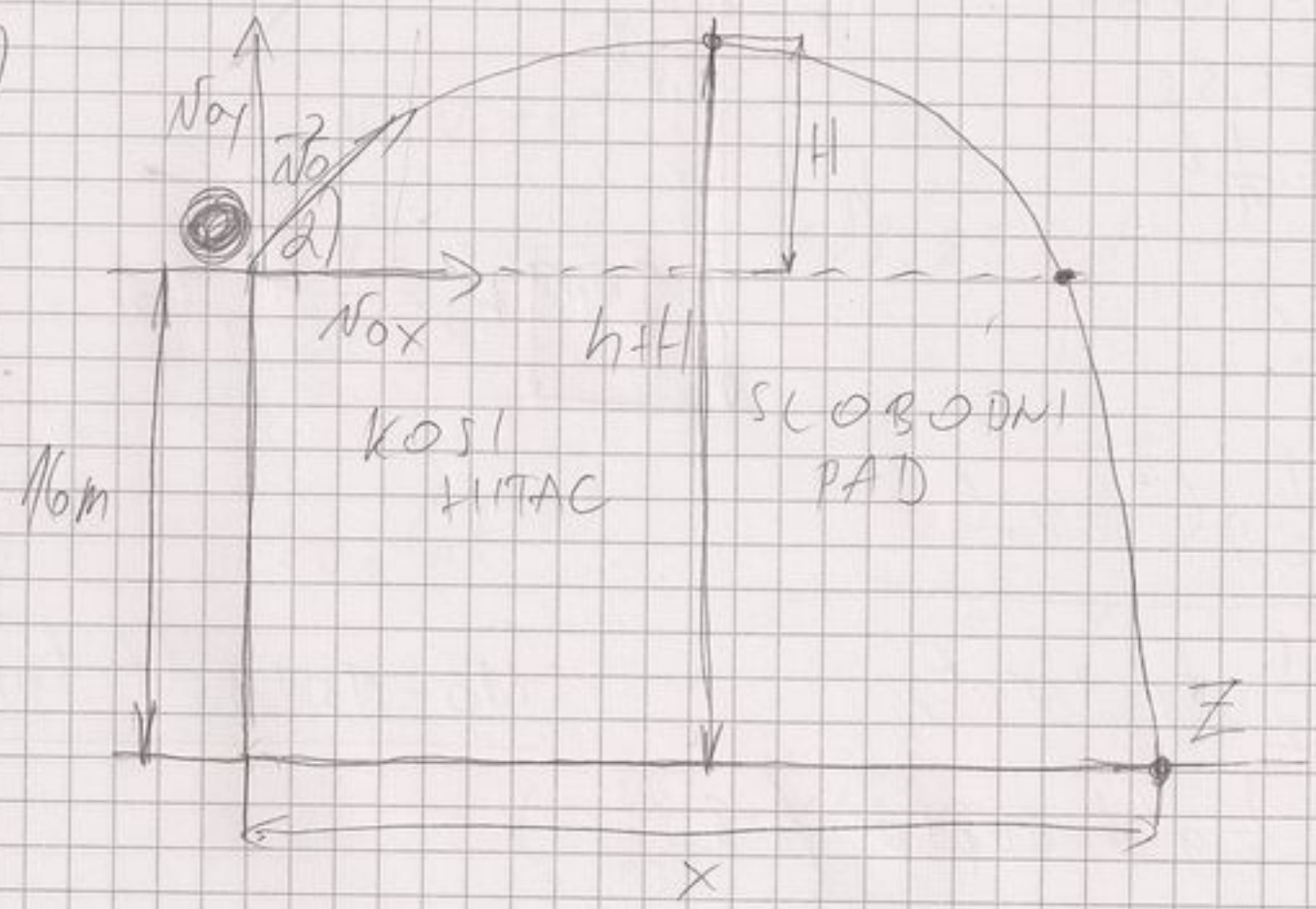
$t_1 = 13.182\ s$

$t_2 = 0.900\ s$ ~~$0.900\ s$~~

NEMOGUĆE JER JE $t_p = 2.5\ s$

$t_1 = t = 13.182\ s$

5.)



$\alpha = 65^\circ$

$h = 16\ m$

$v_0 = 10\ m/s$

$X = ?$

KOSI HITAC

$v_{0x} = v_0 \cos\alpha$

$v_{0y} = v_0 \sin\alpha - gt$

$t_H = \frac{v_0 \sin\alpha}{g} = 0.924\ s$

$H = \frac{v_0^2 \sin^2\alpha}{2g} = 4.186\ m$

H-Z = slobodni pad

$H+h = \frac{1}{2} g t^2 + v_0 t \quad , v_0 = 0\ m/s$

$t = \sqrt{\frac{2(h+H)}{g}} = 2.03\ s$

$t_{AB} = 2.06\ s$

$t_{uk} = t_H + t_{AB} = 2.95\ s$

TREBA NAM X DIMENZIJA PRIJEĐENE UDALJENOSTI, A NJU PRONAĐEMO X KOMPONENTA BRZINE

$x = v_{0x} \cdot t_{uk} = v_0 \cos\alpha \cdot t_{uk} = 12.47\ m$

6.) $v(x) = \frac{A}{B}(C+Dx)$, $A, B, C, D = \text{konst.}$

$$a = \frac{dv}{dt} = \frac{dv}{dx} \cdot \frac{dx}{dt} = \frac{A}{B} \cdot D \left(\frac{A}{B}(C+Dx) \right)$$

$$a(x) = \frac{A^2 D}{B^2}(C+Dx)$$

$$a\left(\frac{1}{D}\right) = \frac{A^2 D}{B^2}(C+1)$$

7.) $F = F_0 \left[1 - \left(\frac{2t-T}{T} \right)^2 \right]$ na $0 \le t \le T$

$$F = \frac{dv}{dt} \cdot m$$

$$\frac{dv}{dt} \cdot m = F_0 \left[1 - \left(\frac{2t-T}{T} \right)^2 \right] \quad / \cdot \frac{dt}{m}$$

$$dv = \frac{F_0}{m} \left[1 - \frac{4t^2 - 4tT + T^2}{T^2} \right] dt \quad \int$$

$$v = \int_0^T \frac{F_0}{m} dt - \frac{F_0}{mT^2} \int (4t^2 - 4tT + T^2) dt$$

$$= \frac{F_0}{m} t \Big|_0^T - \frac{F_0}{mT^2} \cdot 4 \cdot \frac{t^3}{3} \Big|_0^T + \frac{4F_0T}{mT^2} \cdot \frac{t^2}{2} \Big|_0^T - \frac{F_0 t T^2}{mT^2} \Big|_0^T$$

$$= \frac{F_0}{m} T - \frac{4}{3} \frac{F_0 T^3}{mT^2} + \frac{2F_0 T^3}{mT^2} - \frac{F_0}{m} T$$

$$= \frac{2}{3} \frac{F_0}{m} T$$

8) $G = 98.1\ N$, $m = 10\ kg$

$F = k(a - t)$ $a = 1\ s$ $k = 100\ Ns^{-1}$

$v(0) = 0.2\ m/s$

$$F = m\frac{dv}{dt}$$

$$dv = \frac{F}{m}dt$$

$$\int dv = \int \frac{k(a-t)}{m}dt = \int \frac{ka}{m}dt - \int \frac{kt}{m}dt$$

$$v(t) = \frac{ka}{m}t - \frac{k}{m}\cdot\frac{t^2}{2} + v_0$$

$$v(0) = 0.2 = v_0$$

$$\frac{ds}{dt} = \frac{ka}{m}t - \frac{k}{m}\cdot\frac{t^2}{2} + v_0 \quad /\cdot dt$$

$$ds = \left(\frac{ka}{m}t - \frac{k}{m}\cdot\frac{t^2}{2} + v_0\right)dt$$

$$s(t) = \frac{ka}{m}\cdot\frac{t^2}{2} - \frac{k}{m}\cdot\frac{t^3}{6} + v_0 t + s_0 \quad , s_0 = 0\ m$$

VRIJEME DO ZAUSTAVLJANJA?

$v(t) = 0\ m/s$ $$0 = \frac{ka}{m}t - \frac{k}{m}\frac{t^2}{2} + v_0$$

$$\frac{k}{2m}\cdot t^2 - \frac{ka}{m}t - v_0 = 0$$

$$5t^2 - 10t - 0.2 = 0$$

$$t_1 = 2.02\ s \quad t_2 = ~~-0.02\ s~~$$

$t = 2.02\ s$

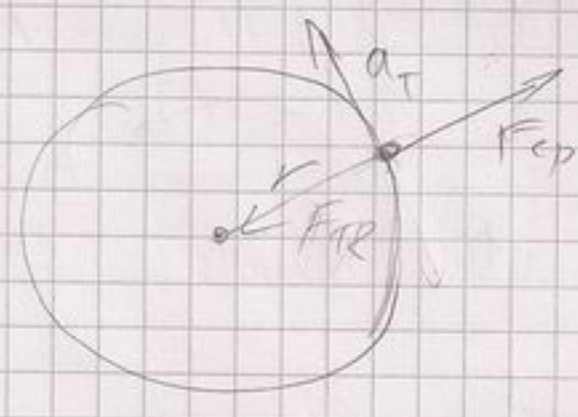
$s(2.02) = 7.069\ m$

9.) $a_T = 0.7\ m/s^2$

$R = 84\ m, r = 42\ m$

$\mu = 0.25$

$v_0 = 0\ m/s$



$s = ?$

Da ne bi klizio mora vrijediti

$$F_{TR} \geq F_{cp}$$

$F_{cp} = m \cdot a = m \cdot \omega^2 r$ $\quad a = \omega^2 R$

$F_{TR} = \mu m g$

$F_{TR} = F_{cp}$

$\mu m g = m \omega^2 r$

$\omega^2 = \frac{\mu g}{r}$

$\omega = \sqrt{\frac{\mu g}{r}} = 0.2416\ rad/s$

$\alpha = \frac{a_T}{r} = 0.0167\ rad/s^2$

$\omega = \alpha t$

$t = \frac{\omega}{\alpha} = 14.5\ s$

$s = \frac{1}{2} a t^2 + v_0 t$

$= 73.59\ m$

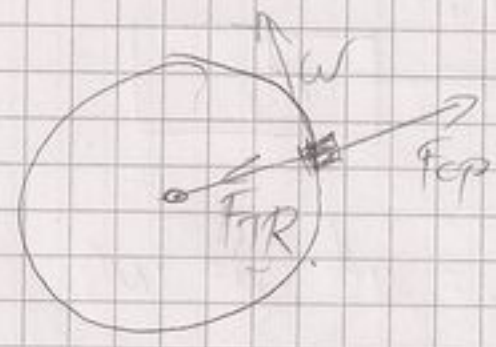
10.) $m = 44g$

$F_{MAX} = 98.1\ N$

$l = 0.3\ m$

$\mu_s = 0.6$

$\omega_{MAX} = ?$



$$F_{MAX} = F_{cp} - F_g$$

$$F_{MAX} = m\omega^2 r - \mu m g$$

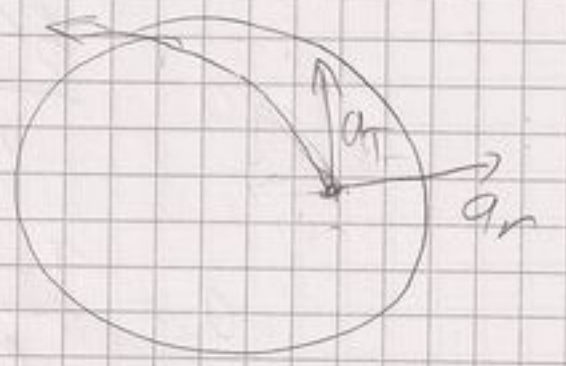
$$\omega_1 = \sqrt{\frac{F_{MAX} + \mu m g}{mr}} = 10.07\ rad/s$$

11.) $r = 0.8\ m$

$s = (0.1t^3)\ m$

$v = 3\ m/s$

$a = ?$



$$v = \frac{ds}{dt} = 0.3t^2$$

$$3 = 0.3t^2$$

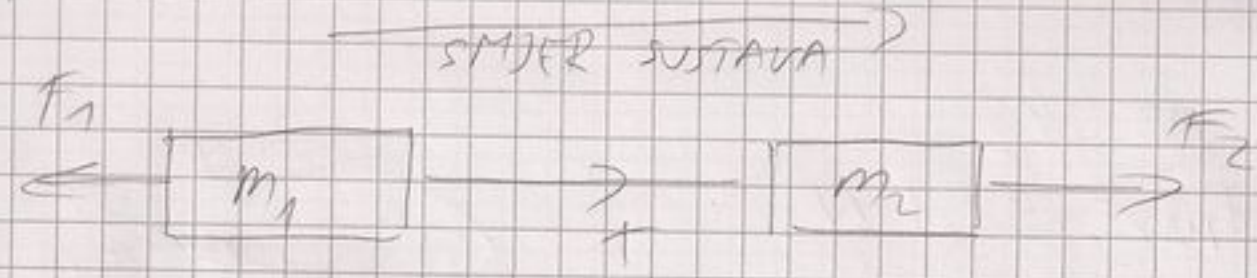
$$t = \sqrt{10}\ s = 3.16\ s$$

$$a_T = \frac{dv}{dt} = 0.6t = 1.9\ m/s^2$$

$$a_r = \frac{v^2}{r} = 11.25\ m/s^2$$

$$a = \sqrt{a_r^2 + a_T^2} = 11.41\ m/s^2$$

12.)



$F_1 = at$ $a = 4\,N/s$ ZORNI JE ZABORAVIO MASE PREPISATI IZ TEKSTA ZADATKA

$F_2 = bt$ $b = 8\,N/s$

$T = 20\,N$

$t = ?$

$F = F_2 - T$

$m_2 a = F_2 - T = bt - T$

$m_1 a = T - F_1 \quad / \; T - at$

$a = \frac{bt - T}{m_2}$

$a = \frac{T - at}{m_1}$

$\frac{T - at}{m_1} = \frac{bt - T}{m_2}$

$\frac{T}{m_1} - \frac{at}{m_1} = \frac{bt}{m_2} - \frac{T}{m_2}$

$\frac{T}{m_1} + \frac{T}{m_2} = \frac{bt}{m_2} + \frac{at}{m_1}$

$\frac{T}{m_1} + \frac{T}{m_2} = \left(\frac{b}{m_2} + \frac{a}{m_1}\right) t$

$t = \frac{T\left(\frac{1}{m_1} + \frac{1}{m_2}\right)}{\frac{b}{m_2} + \frac{a}{m_1}} = T \frac{\frac{m_2 + m_1}{m_1 m_2}}{\frac{bm_1 + am_2}{m_2 m_1}}$

$$t = T \cdot \frac{m_2 + m_1}{5m_1 + 9m_2}$$

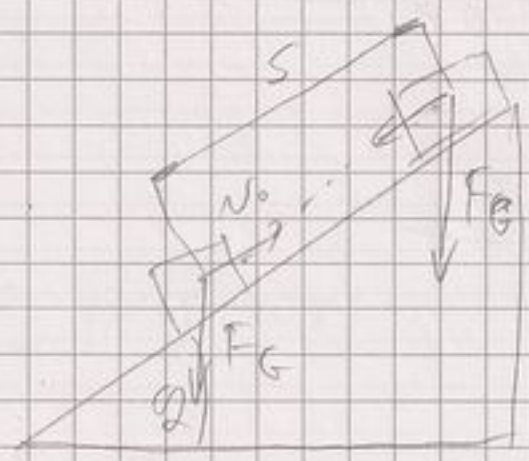
13) $m = 0.35\ kg$

$\alpha = 35°$

$v_0 = 3.5\ m/s$

$\mu = 0.35$

$E_k = ?$



PRVNA CAST

$ma = mg\sin\alpha + \mu r_4 \cos\alpha$

$a = g(\sin\alpha + \mu\cos\alpha)$

$= 8.44\ m/s^2$

$v_4 = 0\ m/s$

$v_0^2 = -2as$

$s = \frac{v_0^2}{-2a} = 0.72\ m$

DRUHA CAST

$ma = mg\sin\alpha - \mu mg\cos\alpha$

$a = g(\sin\alpha - \mu\cos\alpha)$

$= 2.81\ m/s^2$

$v_4 = \sqrt{2as} = 2.01\ m/s$

$E_4 = \frac{mv^2}{2} = 0.709\ J$

14.) $m = 5\ kg$

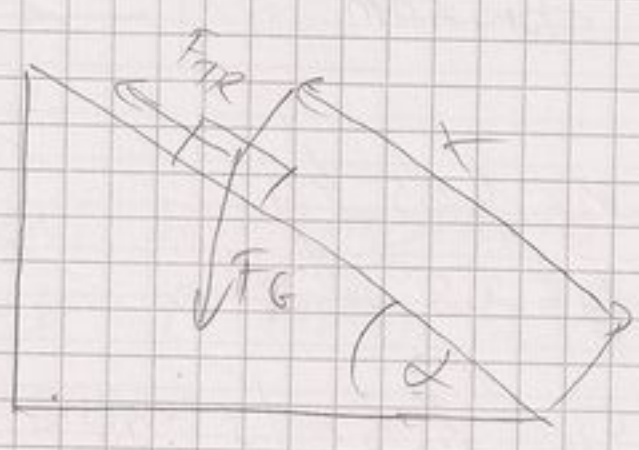
$v_0 = 4\ m/s$

$x = 4.8\ m$

$\alpha = 37°$

$F_{TR} = 8\ N$

$E_k = ?$



$ma = mg\sin\alpha - F_{TR}$

$a = g\sin\alpha - \frac{F_{TR}}{m}$

$a = 4.3\ m/s^2$

$v_k^2 = 2ax + v_0^2$

$v_k = \sqrt{2ax + v_0^2} = 7.57\ m/s$

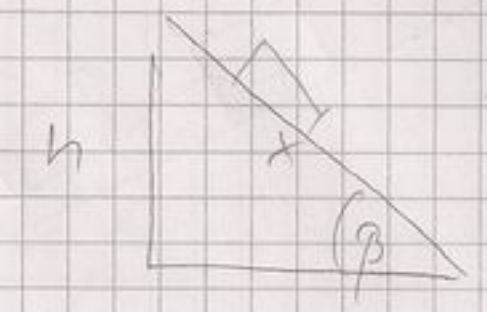
$E_k = \frac{mv^2}{2} = 143.29\ J$

15.) $h = 2.5\,m$

$\alpha = 50°$

$\beta = 45°$

$\mu_k = 0.85\mu_s$



$$mg\sin\beta = \mu_s \cdot mg\cdot\cos\beta$$

$$\mu_s = \tan\beta = 1$$

$$\mu_D = 0.85$$

$$ma = mg\sin\alpha - \mu_D mg\cos\alpha$$

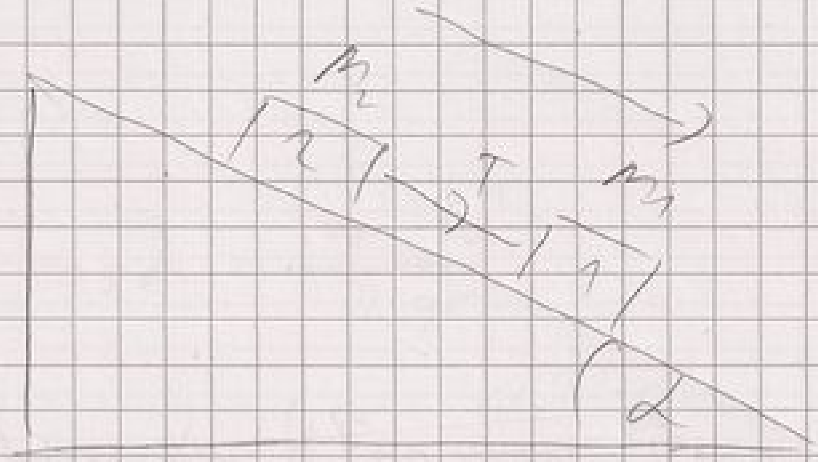
$$a = g(\sin\alpha - \mu_D\cos\alpha) = 2.15\ m/s^2$$

$$x = \frac{h}{\sin\alpha} = 3.26\ m$$

$$x = \frac{1}{2}at^2$$

$$t = \sqrt{\frac{2x}{a}} = 1.74\ s$$

16.)



$m_1 = 8\,kg$

$m_2 = 20\,kg$

$\mu_1 = 0.2$

$\mu_2 = 0.3$

$T = ?$

$$m_1 a = m_1 g \sin\alpha - \mu_1 m_1 g \cos\alpha - T$$
$$m_2 a = m_2 g \sin\alpha - \mu_2 m_2 g \cos\alpha + T$$

$a = 0$ JER JE KONSTANTNA BRZINA

$$0 = m_1 g \sin\alpha - \mu_1 m_1 g \cos\alpha - T$$
$$0 = m_2 g \sin\alpha - \mu_2 m_2 g \cos\alpha + T \quad \Big/ +$$

$$m_1 g \sin\alpha + m_2 g \sin\alpha = \mu_1 m_1 g \cos\alpha + \mu_2 m_2 g \cos\alpha$$

$$\sin\alpha\,(m_1 g + m_2 g) = \cos\alpha\,(\mu_1 m_1 g + \mu_2 m_2 g) \Big/ : \cos\alpha$$

$$\mathrm{tg}\,\alpha = \frac{\mu_1 m_1 g + \mu_2 m_2 g}{m_1 g + m_2 g}$$

$$= 0.271$$

$$\alpha = 15.18$$

$$T = m_1 g \sin\alpha - \mu_1 m_1 g \cos\alpha = 5.402\ N$$

17.) $F_1 = F_2 = F$

$F_3 = 2F$

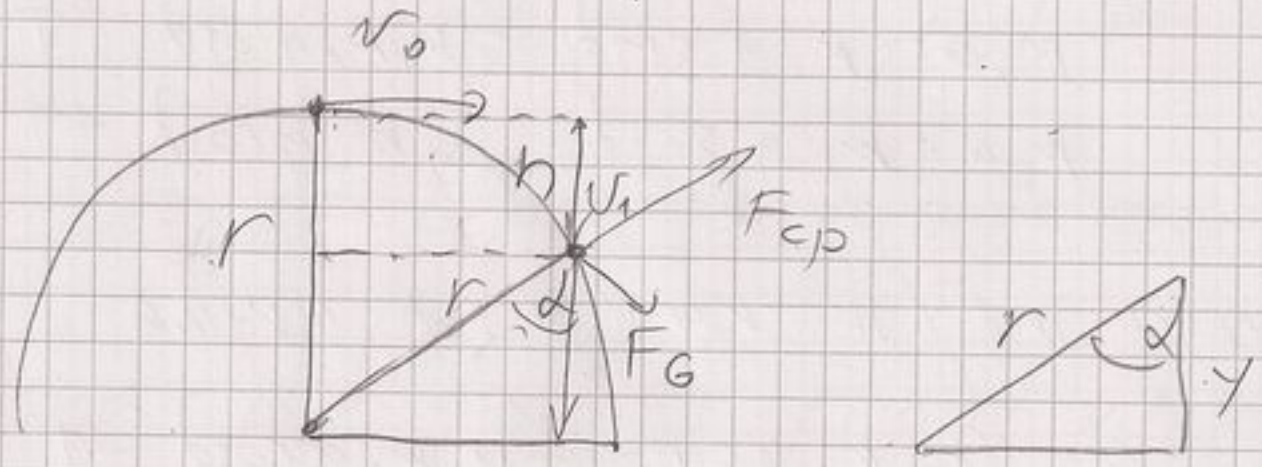
$\vec{F_1} = F(\cos 30° \vec{i} + \sin 30 \vec{j}) = F\left(\frac{\sqrt{3}}{2}\vec{i} + \frac{1}{2}\vec{j}\right)$

$\vec{F_2} = F(\cos 135 \vec{i} + \sin 135 \vec{j}) = F\left(-\frac{\sqrt{2}}{2}\vec{i} + \frac{\sqrt{2}}{2}\vec{j}\right)$

$\vec{F_3} = 2F(\cos 270 \vec{i} + \sin 270 \vec{j}) = 2F(-\vec{j}) = F(-2\vec{j})$

$F_{ük} = F_1 + F_2 + F_3 = F(0.159\vec{i} - 0.793\vec{j})$

18.)



$v_0 = 2.5\ m/s$

$r = 30\ m$

$s = ?$

Z.O.E.

$\cos\alpha = \frac{y}{r}$

$y = \cos\alpha \cdot r$

$h = r - y$

$= r - r\cos\alpha$

$= r(1 - \cos\alpha)$

$\frac{mv_0^2}{2} + mgh_1 = \frac{mv_1^2}{2} + mgh_2 \quad , h_2 = 0$

$\frac{v_0^2}{2} + g \cdot (r(1-\cos\alpha)) = \frac{v_1^2}{2}$

$gr(1-\cos\alpha) = \frac{v_1^2 - v_0^2}{2}$

$$1-\cos\alpha = \frac{v_1^2 - v_0^2}{2gr}$$

$$\cos\alpha = 1 - \frac{v_1^2 - v_0^2}{2gr}$$

DA BI SE ODVOJIO MORA VRIJEDITI

$$F_{cp} = F_G$$

$$\frac{mv_1^2}{r} = mg\cos\alpha$$

$$v_1^2 = rg\cos\alpha$$

$$\cos\alpha = 1 - \frac{rg\cos\alpha}{2gr} + \frac{v_0^2}{2gr}$$

$$\frac{3}{2}\cos\alpha = 1 + \frac{v_0^2}{2gr}$$

$$\cos\alpha = \frac{2}{3} + \frac{v_0^2}{3gr}$$

$$\alpha = 47.64^\circ$$

$$s = 2r\pi \cdot \frac{\alpha}{360}$$

$$= 24.93\ m$$

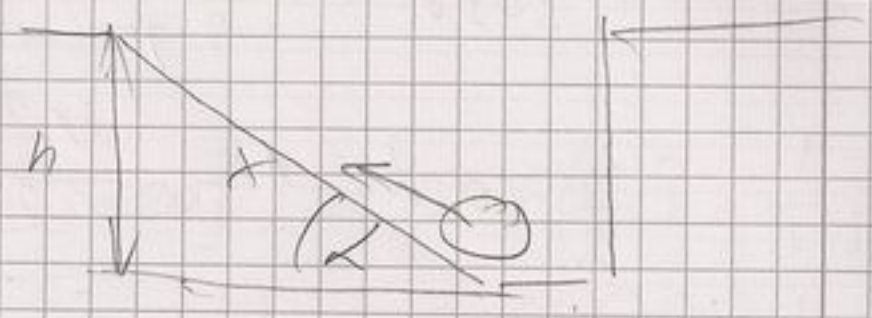
19.) $m = 20 \cdot 10^3 kg$

$h = 15 m$

$\alpha = 30°$

$\mu = 0.25$

$W = ?$



$$\sin\alpha = \frac{h}{x}$$

$$x = \frac{h}{\sin\alpha} = 30 m$$

$$F = mg\sin\alpha + \mu mg\cos\alpha$$

$$= mg(\sin\alpha + \mu\cos\alpha)$$

$$= 140\,578.54 N$$

$$W = F \cdot s = 4\,217\,356.38 J$$

$$= 4.22 MJ$$