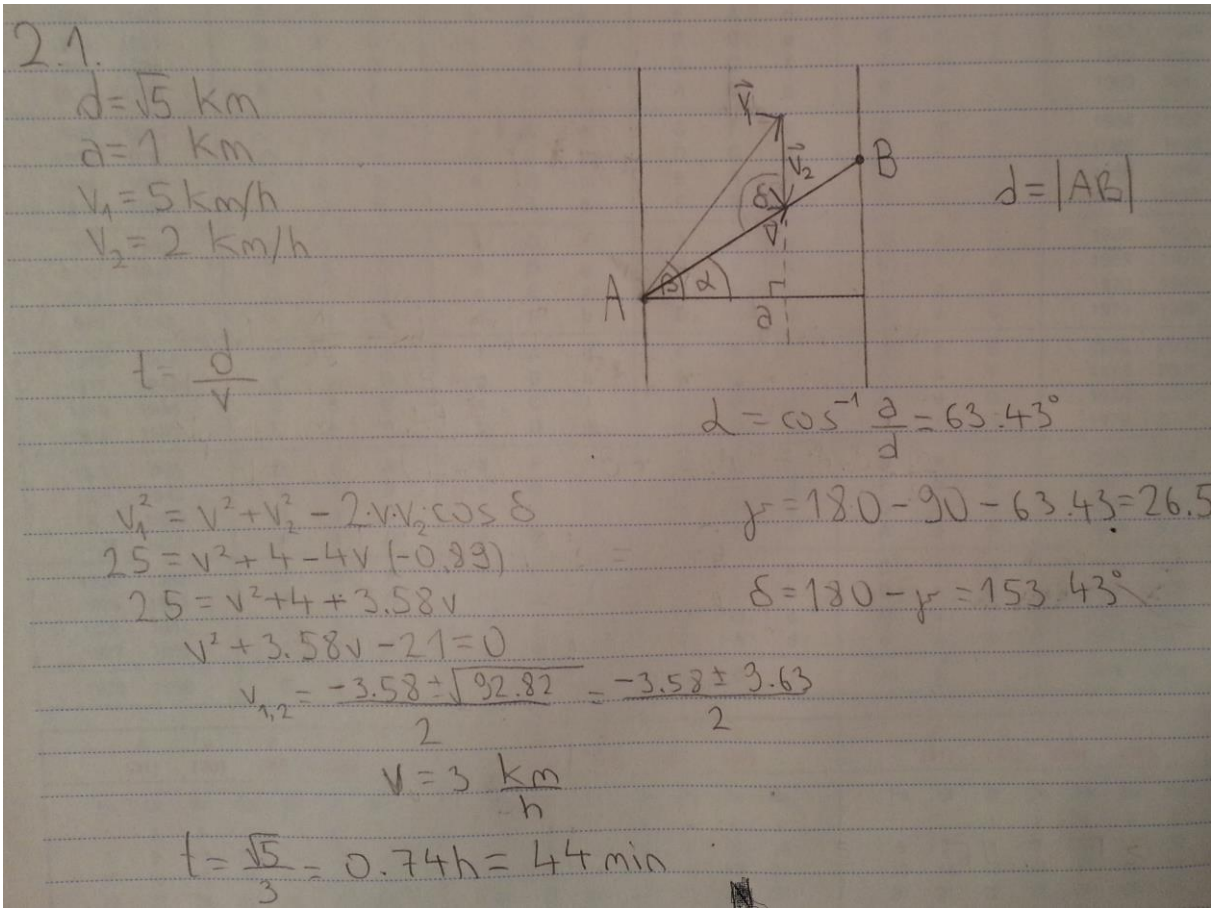
**Kulišić**

# 2.Kinematika čestice



2.2.

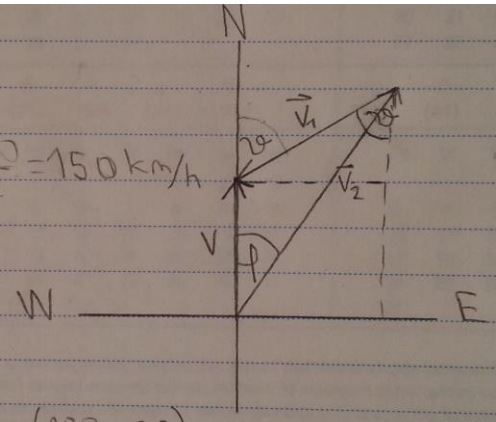
$t = 2\,h$

$d = 300\,km$ $\quad v = \frac{300}{2} = 150\,km/h$

$\vartheta = 30°$

$v_1 = 27\,km/h$

$\vec{v}_2, \varphi = ?$




$v_2^2 = v_1^2 + v^2 - 2vv_1\cos(180 - \vartheta)$

$v_2 = \sqrt{30243.8} = 173.91\,km/h = 48.3\,\frac{m}{s}$

$v_1^2 = v^2 + v_2^2 - 2vv_2\cos\varphi \Rightarrow \cos\varphi = \frac{v^2 + v_2^2 - v_1^2}{2vv_2} = 0,9969848$

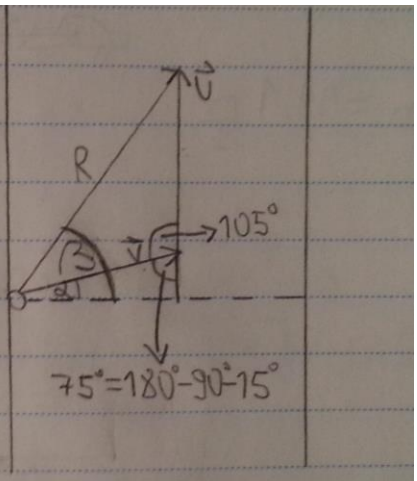
$\varphi = 4.45°$

2.3.

$v = 1\,\frac{m}{s}$

$u = 1.5\,\frac{m}{s}$

$\alpha = 15°$




$R^2 = u^2 + v^2 - 2uv\cos(105°)$

$R = 2\,\frac{m}{s}$

$u^2 = R^2 + v^2 - 2Rv\cos\beta$

$\gamma = \cos^{-1}\left(\frac{R^2 + v^2 - u^2}{2R}\right) = 46.57°$

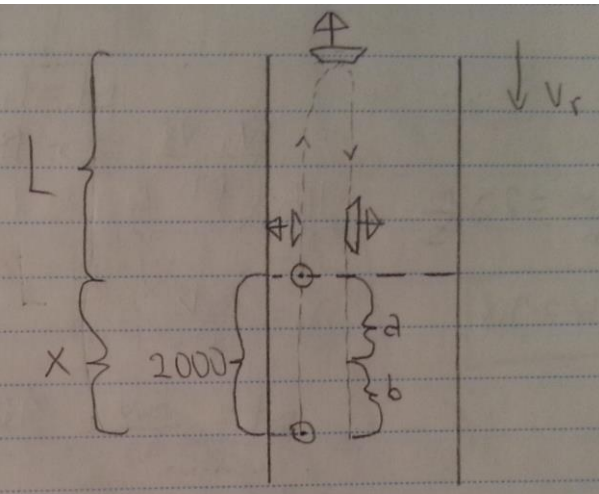
$\beta = \alpha + \gamma = 61.57°$

2.4.

$t_1 = 30\,min$

$X = 2000\,m$

$v_r = ?$




$L = (v_c - v_r)\,t_1$

$a = v_r \cdot t_1$

$L + X = (v_c + v_r)\,t_2$

$b = v_r \cdot t_2$

$t_1 = t_2 = 30\,min$

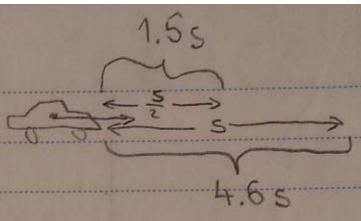
$$-\begin{cases} L + X = (v_c + v_r)\,t_2 \\ L = (v_c - v_r)\,t_1 \end{cases}$$

$X = v_r t + v_r t$

$2000 = v_r (1800 \cdot 2)$

$v_r = 0.56\ \frac{m}{s}$

2.5.




$v_0 = 40\ \frac{km}{h} = 11.1\ \frac{m}{s}$

$t_1 = 4.6\,s$

$t_2 = 1.5\,s$

$s = v_0 t_1 + \frac{a t_1^2}{2}$

$\frac{s}{2} = v_0 t_2 + \frac{a t_2^2}{2} \mid \cdot 2$

$v_0 t_1 + \frac{a t_1^2}{2} = 2 v_0 t_2 + a t_2^2$

$51.06 + 10.58\,a = 33.3 + 2.25\,a$

$a = -2.13\ \frac{m}{s^2}$

2.6. $t_0 = 0\ s$

$v_0 = 0\ \frac{m}{s}$

$a_1 = 2\ \frac{m}{s^2}$

$v_1 = 72\ \frac{km}{h} = 20\ \frac{m}{s}$

$a_2 = -3\ \frac{m}{s^2}$

$v_2 = 36\ km/h = 10$

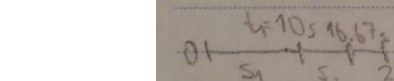
$t = 20\ s$

$\Delta t = t_1 - t_0 = t_1$

$2 = \frac{v_1 - v_0}{\Delta t} \Rightarrow \Delta t = \frac{20}{2} = 10\ s = t_1$

$a_2 = \frac{\Delta v}{\Delta t}$

$\Delta t = \frac{\Delta v}{a_2} = \frac{10 - 20}{-3} = 3.33$




$t_3 - t_2 = 3.33$

$20 - t_2 = 3.33$

$t_2 = 16.67$

$t_2 = 20 - t_1 - t_2 =$

$S_1 = \frac{a t_1^2}{2} = 100\ m$

$S_2 = v_1 \cdot t_2$

$S_2 = 20 \cdot (16.67 - 10) = 133.4\ m$

$S_3 = \frac{a_2 (20 - 16.67)^2}{2} = 16.63\ m$

$S = S_1 + S_2 + S_3 = 250\ m$

2.9.

$h = 15\ m$

$v_0 = 10\ m/s$

$t_p, v_k, t_{sp} = ?$

$h = v_0 t_p + \frac{g t_p^2}{2}$

$15 = 10 t_p + 4.905 t_p^2$

$4.905 t_p^2 + 10 t_p - 15 = 0$

$t_{p_{1,2}} = \frac{-10 \pm \sqrt{100 + 294.3}}{3.81} = \frac{-10 \pm 19.85}{3.81}$

$t_p \approx 1\ s$

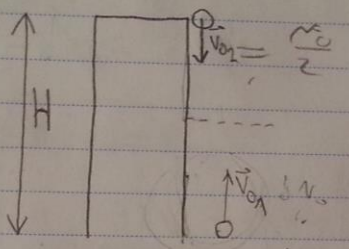
2.10.

$v_0 = v_{01} = 10\ \frac{m}{s}$

$H = 21\ m$

$v_{02} = \frac{v_{01}}{2} = 5\ \frac{m}{s}$

$h_{sus.}, v_{0min} = ?$




$h_2 = H - (v_{02}t + \frac{gt^2}{2})$

$h_1 = v_{01}t - \frac{gt^2}{2}$

$h_1 = h_2 = h_{sus}$

① $H - v_{02}t - \frac{gt^2}{2} = v_{01}t - \frac{gt^2}{2}$

$21 - 10t = v_{01}t$

$21 - 5t = 10t$

$t = 1.4\ s$

$h_{sus} = h_1 = h_2 = 14 - \frac{9.81 \cdot 1.4^2}{2} = 4.39\ m$

tijelo 1 vrijeme leta $t_{L1} = \frac{2v_{01}}{g}$

vrijeme susreta tijela 1 i 2 → iz jednadžbe ① $v_{02} = \frac{v_0}{2}, v_{01} = v_0$

$H - \frac{v_0}{2}t = v_0 t$

$H = \frac{3}{2}t(v_0 + v_0)$

$H = \frac{3}{2}v_0 t^2 \Rightarrow t_{susreta} = \frac{2H}{3v_0}$

uvjet $\Rightarrow t_{susreta} < t_{L1}$

$\frac{2H}{3v_0} < \frac{2v_{01}}{g}$ ← $v_{01} = v_0$

$6v_0^2 > 412.02 \Rightarrow v_0 > 8.29\ \frac{m}{s}$

2.11.

Gore

$v_0 = 100\ \frac{m}{s}$

$a = 9.8\ \frac{m}{s^2} = kv^2$

$k = 0.001\ m^{-1}$

$v_1,\ t = ?$

$$a = \frac{dv}{dt} = -g - kv^2$$

$$\frac{dv}{g + kv^2} = -dt \quad \int$$

$$\int_{v_0}^{0} \frac{dv}{g + kv^2} = -\int_{0}^{t_1} dt$$

$$\frac{-\arctg\left(\sqrt{\frac{k}{g}}\, v_0\right)}{\sqrt{gk}} = -t_1 \Rightarrow t_1 = \frac{\arctg\left(\sqrt{\frac{k}{g}}\, v_0\right)}{\sqrt{gk}}$$

$$t_1 = 7.38\ s$$

$$v = \sqrt{\frac{g}{k}}\, tg\left(t\sqrt{gk}\right) \quad / \frac{dx}{dt}$$

$$x(t) = \sqrt{\frac{g}{k}} \cdot \frac{-\ln\left(\cos\left(\sqrt{gk}\cdot t\right)\right)}{\sqrt{gk}} = -\frac{\ln\left(\cos\left(\sqrt{gk}\, t\right)\right)}{k} = 351.38\ m$$

dolje

$$a = \frac{dv}{dt} = g - kv^2$$

$$\frac{dv}{g - kv^2} = dt \Rightarrow t = \frac{\operatorname{arctanh}\left(\sqrt{\frac{k}{g}}\cdot v\right)}{\sqrt{gk}}$$

$$v = \sqrt{\frac{g}{k}}\, \tanh\left(t\sqrt{gk}\right) \quad / \frac{dx}{dt}$$

$$x = \frac{\ln\left(\cosh\left(\sqrt{gk}\, t\right)\right)}{k} \Rightarrow t_2 = \frac{1}{\sqrt{gk}} \operatorname{arcosh}\left(e^{kx}\right) = 3.37\ s$$

$$t_{ukupno} = t_1 + t_2$$

$$v = \sqrt{\frac{g}{k}}\, \tanh\left(t_{ukupno}\sqrt{gk}\right) = 70.37\ \frac{m}{s}$$

2.12

$H = 200\,m$

$t = 3.8\,s$

$v_k = 4.8\,\frac{m}{s}$

$a = g - kv$

$k, v_0 = ?$

$a = \frac{dv}{dt}$

$\frac{dv}{dt} = g - kv$

$\frac{dv}{g-kv} = dt \;/\; \int$

$\int \frac{1}{g-kv}\,dv = \int dt$

① $-\frac{\ln|g-kv|}{k} = t + c$

Računanje c:

$-\ln|g-kv| = -kt - kc \xrightarrow{t=0,\,v=v_0} \ln|g-kv_0| = -kc \Rightarrow c = -\frac{\ln|g-kv_0|}{k}$

Povratak u ①:

$-\frac{1}{k}\ln|g-kv| = t - \frac{1}{k}\ln|g-kv_0|$

$\ln\left|\frac{g-kv_0}{g-kv}\right| = t\cdot k \;/\cdot(-1)$

$\ln\left|\frac{g-kv}{g-kv_0}\right| = -kt \;/\cdot e$

$g - kv = e^{-kt}\cdot(g-kv_0)$

$kv = g - e^{-kt}(g-kv_0)$

② $v = \frac{g}{k} - \left(\frac{g}{k} - v_0\right)e^{-kt}$

$v = v_0 e^{-kt} + \frac{g}{k}(1-e^{-kt}) \;/\int$

$\int ds = \int \left(v_0 e^{-kt} + \frac{g}{k}(1-e^{-kt})\right)dt$

$s = -\frac{v_0}{k}e^{-kt} + \frac{g}{k}t - \frac{g}{k}\left(-\frac{1}{k}\right)e^{-kt} + C_2$

③ $s(t) = \frac{1}{k}\left(\frac{g}{k} - v_0\right)e^{-kt} + \frac{g}{k}t + C_2$

računam $C_2$:

$t = 0,\ s = 0$

$0 = \frac{g}{k^2} - \frac{v_0}{k} + C_2 \Rightarrow C_2 = \frac{v_0}{k} - \frac{g}{k^2}$

Povratak u ③:

④ $s = \frac{1}{k}\left(\frac{g}{k} - v_0\right)e^{-kt} + \frac{g}{k}t + \frac{v_0}{k} - \frac{g}{k^2}$

② i ④

$s=200, t=38, N=4.8$:

④ $200 = \frac{1}{k}\left(\frac{g}{k} - N_0\right)e^{-k\cdot 38} + \frac{g}{k}\cdot 38 + \frac{N_0}{k} - \frac{g}{k^2}$

② $4.8 = \frac{g}{k} - \left(\frac{g}{k} - N_0\right)e^{-k\cdot 38} \Rightarrow \left(\frac{g}{k} - N_0\right)e^{-k\cdot 38} = \frac{g}{k} - 4.8$

$$200 = \frac{1}{k}\left(\frac{g}{k} - 4.8\right) + \frac{g}{k}\cdot 38 + \frac{N_0}{k} - \frac{g}{k^2}$$

$$200 = -\frac{4.8}{k} + \frac{38\cdot g}{k} + \frac{N_0}{k} \quad / \cdot k$$

⑤ $\boxed{200k = -4.8 + 38\cdot g + N_0}$

Kombinacija ② i ⑤ ②malo sredimo $N_0 = N - \frac{g}{k}(1 - e^{-kt})$ / $e^{-kt}$

$N = 4.8, t = 38, s = 200$

$$200k = -4.8 + 38\cdot g + \frac{4.8 - \frac{g}{k}(1 - e^{-k\cdot 38})}{e^{-k\cdot 38}}$$

$$\frac{4.8 - \frac{g}{k}(1 - e^{-k\cdot 38})}{e^{-k\cdot 38}} + 367.38 - 200k = 0$$

Ovu jebadžbu riješimo metodom pokušaja i pogreške (kao što je navedeno u zbirci na stranici 31.) pa dobivamo (ja sam računao sa Wolfram|Alpha):

~~$k_1 \approx -8.36\cdot 10^{-20}$~~

$\boxed{k_2 \approx 2.04\ s^{-1}}$

vratimo se u ⑤

$\boxed{N_0 = 200k + 4.9 - 38\cdot g = 40.02\ \frac{m}{s}}$

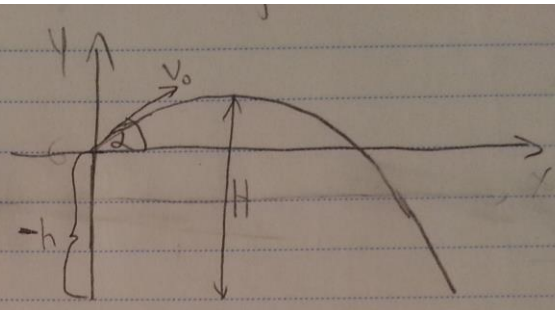
2.13.

$v_0 = 10 \frac{m}{s}$

$\alpha = 6.5^\circ$

$h = 16\,m$

$X, H = ?$




$$y = x \operatorname{tg}\alpha - \frac{g x^2}{2 v_0^2 \cos^2\alpha}$$

$$-16 = x \cdot 2.14 - 0.275 x^2$$

$$0.275 x^2 - 2.14 x - 16 = 0$$

$$x_{1,2} = \frac{2.14 \pm \sqrt{21.86}}{2 \cdot 0.275} = \frac{2.14 \pm 4.71}{2 \cdot 0.275}$$

$x_1 = 12.45\,m$

~~$x_2 = -4.67\,m$~~

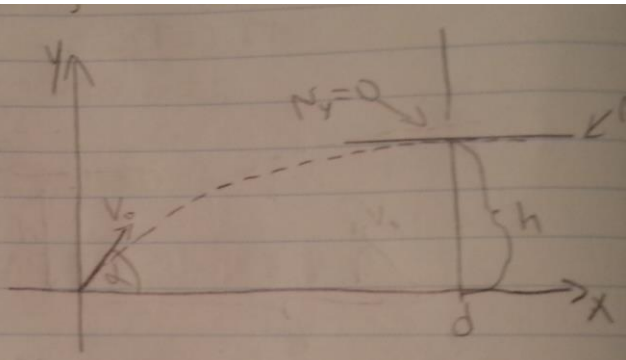
$$H = h + \frac{v_0^2 \sin^2\alpha}{2g} = 16 + 4.19 = 20.19\,m$$

2.14.

$d = 100\,m \quad \beta = 0$

$h = 50\,m$

$d, v_0 = ?$




$$h = v_{y_0} t + \frac{1}{2} g t^2 \qquad v_{y_0} = g t \Rightarrow t = \frac{v_{y_0}}{g}$$

$$v_y = v_{y_0} - g t^2$$

$$h = \frac{v_{y_0}^2}{g} - \frac{1}{2} g \frac{v_{y_0}^2}{g^2} = \frac{v_{y_0}^2}{g}\left(1 - \frac{1}{2}\right) = \frac{v_{y_0}^2}{2g}$$

$$v_{y_0} = \sqrt{2gh} = 31.32 \frac{m}{s}$$

$$d = v_{x_0} t \Rightarrow v_{x_0} = \frac{d}{\frac{v_{y_0}}{g}} = \frac{g d}{v_{y_0}} = 31.32 \frac{m}{s}$$

2.1.4.6

$$v_0 = \sqrt{v_{x0}^2 + v_{y0}^2} = 44{,}3 \frac{м}{с}$$

$$tg\,\varphi = \frac{v_{y0}}{v_{x0}} \Rightarrow \varphi = 45°$$

2.15.

$\alpha = 53°$

$\vec{v}_0 = 100\ m/s = 100\cos 53°\,\hat{x} + 100\sin 53°\,\hat{y} = (60.18\,\hat{x} + 79.86\,\hat{y})\ m/s$

$\vec{a}_0 = 30\ m/s = (18.05\,\hat{x} + 23.96\,\hat{y})\ m/s^2$

$t_1 = 3\ s$

---

$a = \frac{dv}{dt} = 30(\cos 53\,\hat{x} + \sin 53\,\hat{y})$

$dv = 30(\cos 53\,\hat{x} + \sin 53\,\hat{y})\,dt \quad / \int$ ($v_0$)

$v(t) = 30(\cos 53\,\hat{x} + \sin 53\,\hat{y})\,t + 100(\cos 53\,\hat{x} + \sin 53\,\hat{y})$

$\frac{ds}{dt} = \ldots . \quad / \cdot dt \ / \int$

$s(t) = 15(\cos 53\,\hat{x} + \sin 53\,\hat{y})\,t^2 + 100(\cos 53\,\hat{x} + \sin 53\,\hat{y}) + 0$ ($s_0$)

$s(3) = 81.25\,\hat{x} + 107.81\,\hat{y} + 180.544\,\hat{x} + 239.6\,\hat{y}$

$= (261.8\,\hat{x} + 347.41\,\hat{y})\ m$ = movi $s_0$

$v(3) = (114.34\,\hat{x} + 151.74\,\hat{y})\ m/s$ = movi $v_0$

movi $a_0 = 0$

---

$a = g$

$v(t) = v_0 - gt$

$s(t) = s_0 + v_0 t - \frac{g}{2}t^2$

$s_x(t) = s_x^\circ + v_x^\circ t$

$s_x(33.07) = 261.8 + 3781.22$

$= 4043\ m$

$s_y(t) = s_y^\circ + v_y^\circ t - \frac{g}{2}t^2 = 0$

$\frac{g}{2}t^2 - v_y^\circ t - s_y^\circ = 0 \quad / \cdot \frac{2}{g}$

$t^2 - \frac{2v_y^\circ t}{g} - \frac{2s_y^\circ}{g} = 0 \qquad g = 9.81$

$t^2 - 30.93\,t - 70.83 = 0$

$t_{1,2} = \frac{30.93 \pm 35.21}{2}$

$t_1 = -2.14\ s \qquad t_2 = 33.07\ s$