Fizika 1

Grupa 1.03
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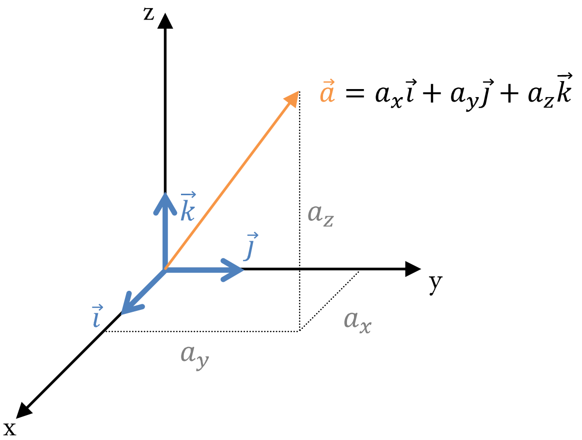
Predavanja 2010.

FIZIKALNE VELIČINE: -**SKALARI** (imaju samo iznos ~ temperatura, tlak, gustoća...)
-**VEKTORI** (imaju iznos i smjer ~ brzina, akceleracija, struja...)
-TENZORI

VEKTORI

PRAVOKUTNI KOORDINATNI SUSTAV (desno orijentirani)



$\vec{\imath}, \vec{\jmath}, \vec{k}$ - jedinični vektori
a_x, a_y, a_z - komponente vektora a

ZBRAJANJE VEKTORA

$$\vec{a} + \vec{b} = (a_x + b_x)\vec{\imath} + (a_y + b_y)\vec{\jmath} + (a_z + b_z)\vec{k}$$

MNOŽENJE VEKTORA SKALAROM

$$\lambda\vec{a} = \lambda a_x\vec{\imath} + \lambda a_y\vec{\jmath} + \lambda a_z\vec{k}$$

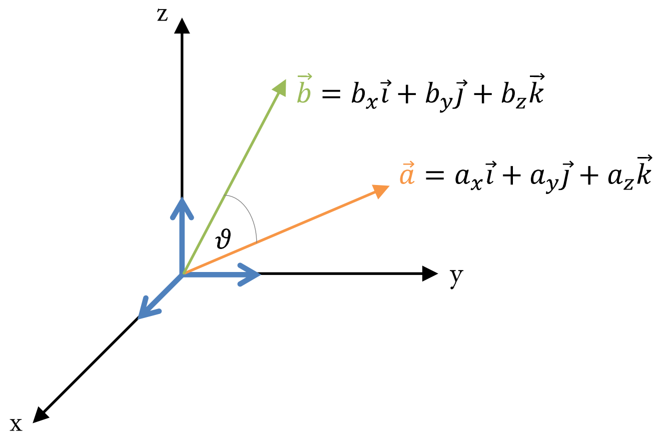
skalar

SKALARNO MNOŽENJE VEKTORA

$$\vec{a} \cdot \vec{b} \equiv \underbrace{a_x b_x + a_y b_y + a_z b_z}_{\text{skalar}}$$

$$\vec{a} \cdot \vec{a} = a_x^2 + a_y^2 + a_z^2$$

$$a = |\vec{a}| = \sqrt{\vec{a} \cdot \vec{a}} = \sqrt{a_x^2 + a_y^2 + a_z^2}$$



$$\vec{a} \cdot \vec{b} = |\vec{a}||\vec{b}| cos\, \vartheta$$

theta, kut među vektorima

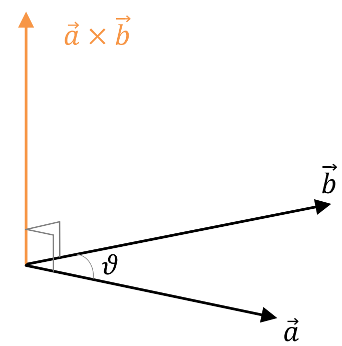
$$|\vec{a} + \vec{b}|^2 = |\vec{a}|^2 + 2|\vec{a}||\vec{b}| cos\, \vartheta + |\vec{b}|^2$$

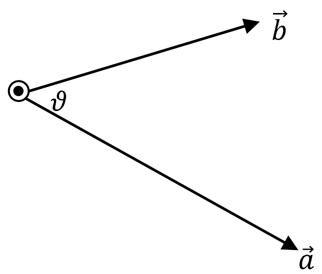
$$|\vec{a} + \vec{a}|^2 = 4|\vec{a}|^2 = |\vec{a}|^2 + 2|\vec{a}||\vec{a}| \underbrace{cos\, \vartheta}_{=1} + |\vec{a}|^2 \quad \blacksquare$$

$$|\vec{a} - \vec{a}|^2 = 0 = |\vec{a}|^2 + 2|\vec{a}||\vec{a}| \underbrace{cos\, \vartheta}_{=-1} + |\vec{a}|^2 \quad \blacksquare$$

VEKTORSKI PRODUKT

$$\vec{a} \times \vec{b} = \begin{vmatrix} \vec{\imath} & \vec{\jmath} & \vec{k} \\ a_x & a_y & a_z \\ b_x & b_y & b_z \end{vmatrix}$$





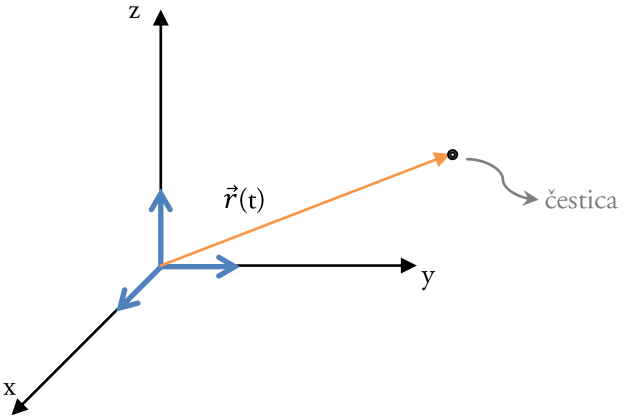
⊗ - „u ploču“

⊙ - prema nama

$$\vec{a} \times \vec{b} = -(\vec{b} \times \vec{a})$$

$$|\vec{a} \times \vec{b}| = |\vec{a}||\vec{b}| sin\, \vartheta$$

KINEMATIKA TOČKE



POLOŽAJ:

$$\vec{\boldsymbol{r}}(\boldsymbol{t}) = \boldsymbol{x}(\boldsymbol{t})\vec{\boldsymbol{\imath}} + \boldsymbol{y}(\boldsymbol{t})\vec{\boldsymbol{\jmath}} + \boldsymbol{z}(\boldsymbol{t})\vec{\boldsymbol{k}}$$

BRZINA:

$$\vec{\boldsymbol{v}}(\boldsymbol{t}) = \frac{\boldsymbol{d}}{\boldsymbol{dt}}\vec{\boldsymbol{r}}(\boldsymbol{t})$$

- vremenska derivacija vektora koji opisuje položaj

$$= \frac{d}{dt}x(t)\vec{\imath} + y(t)\vec{\jmath} + z(t)\vec{k} = \frac{dx(t)}{dt}\vec{\imath} + \frac{dy(t)}{dt}\vec{\jmath} + \frac{dz(t)}{dt}\vec{k}$$

$$= v_x(t)\vec{\imath} + v_y(t)\vec{\jmath} + v_z(t)\vec{k}$$

$$\vec{v} \equiv \dot{\vec{r}} \qquad \frac{d}{dt}$$

$$v_x\vec{\imath} + v_y\vec{\jmath} + v_z\vec{k} = \dot{x}\vec{\imath} + \dot{y}\vec{\jmath} + \dot{z}\vec{k}$$

INTEGRALNI ZAPIS:

$$\vec{v}(t) = \frac{d}{dt}\vec{r}(t) \quad /\cdot dt$$

$$d\vec{r}(t) = \vec{v}(t)dt \qquad \int_{t_0}^{t_1}$$

$$\int_{t_0}^{t_1} d\vec{r} = \int_{t_0}^{t_1} \vec{v}(t)\, dt$$

$$\vec{r}(t_1) - \vec{r}(t_0) = \int_{t_0}^{t_1} \vec{v}(t)\, dt$$

$t_1 \to t,\ t \to t'$

$$\vec{\boldsymbol{r}}(\boldsymbol{t}) - \vec{\boldsymbol{r}}(\boldsymbol{t_0}) = \int_{t_0}^{t} \vec{\boldsymbol{v}}(\boldsymbol{t'})\, \boldsymbol{dt'}$$

„početni uvjet“ brzina!

AKCELERACIJA:

$$\boldsymbol{\vec{a}(t) \equiv \frac{d}{dt}\vec{v}(t)}$$

- derivacija brzine po vremenu

$$= \frac{d^2}{dt^2}\vec{v}(t)$$

$$\vec{a}(t) = a_x(t)\vec{\imath} + a_y(t)\vec{\jmath} + a_z(t)\vec{k}$$

$$= \frac{d}{dt}v_x(t)\vec{\imath} + \frac{d}{dt}v_y(t)\vec{\jmath} + \frac{d}{dt}v_z(t)\vec{k} = \frac{d^2x(t)}{dt^2}\vec{\imath} + \frac{d^2y(t)}{dt^2}\vec{\jmath} + \frac{dz(t)}{dt}\vec{k}$$

$$\vec{a} = a_x\vec{\imath} + a_y\vec{\jmath} + a_z\vec{k} = \dot{v}_x\vec{\imath} + \dot{v}_y\vec{\jmath} + \dot{v}_z\vec{k}$$

$$= \ddot{x}\vec{\imath} + \ddot{y}\vec{\jmath} + \ddot{z}\vec{k}$$

$$\vec{a}(t) = \frac{d}{dt}\vec{v}(t) \quad /\cdot dt$$

$$d\vec{v}(t) = \vec{a}(t)dt \qquad \int_{t_0}^{t_1}$$

$$\int_{t_0}^{t_1} d\vec{v} = \int_{t_0}^{t_1} \vec{a}(t)\, dt$$

$$\vec{v}(t_1) - \vec{v}(t_0) = \int_{t_0}^{t_1} \vec{a}(t)\, dt$$

$t_1 \rightarrow t,\ t \rightarrow t'$

$$\boldsymbol{\vec{v}(t) = v(t_0) + \int_{t_0}^{t} \vec{a}(t')\, dt'}$$

PRIMJER 1 – HARMONIČKO TITRANJE

položaj čestice na x-osi:

$x(t) = Acos\,(\omega t)$
$A, \omega \rightarrow konstante$
$v(t) = ?, a(t) = ?$

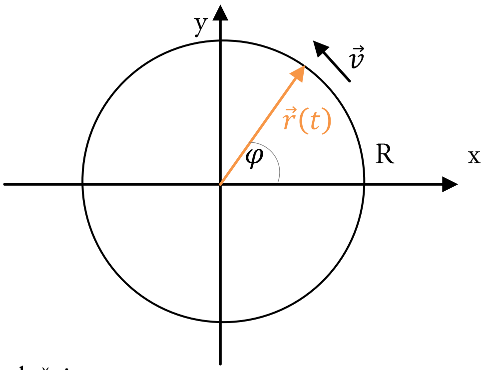
$x(t) = A \cdot cos\,(\omega t + \Phi)$

amplituda frekvencija fazni pomak

$$v(t) = \frac{d}{dt}x(t) = \frac{d}{dt}\big(Acos\,(\omega t + \Phi)\big) = \boldsymbol{-A\omega \cdot sin\,(\omega t + \Phi)}$$

$$a(t) = \frac{d}{dt}v(t) = \frac{d}{dt}\big(-A\omega \cdot sin\,(\omega t + \Phi)\big) = -\omega^2 A \cdot \cos(\omega t + \Phi)$$
$$= \boldsymbol{-\omega^2 x(t)}$$

PRIMJER 2 – GIBANJE PO KRUŽNICI POLUMJERA R BRZINOM STALNOG IZNOSA v



položaj:

$\vec{r}(t) = Rcos(\varphi)\vec{\imath} + Rsin(\varphi)\vec{\jmath}$

φ – raste linearno u vremenu

$\varphi = \omega t$

$$\vec{r}(t) = R\big(\vec{\imath}cos(\omega t) + \vec{\jmath}sin(\omega t)\big)$$

$$\vec{v}(t) = R\left(\vec{\imath}\big(-\omega sin(\omega t)\big) + \vec{\jmath}\big(\omega cos(\omega t)\big)\right) = -\omega R\big(\vec{\imath}sin(\omega t) - \vec{\jmath}cos(\omega t)\big)$$

$$\vec{a}(t) = -\omega^2 R\big(\vec{\imath}cos(\omega t) + \vec{\jmath}sin(\omega t)\big) = -\omega^2\vec{r}(t)$$

$$\omega T = 2\pi$$

$$v = \frac{2R\pi}{T} = \frac{2R\pi}{\frac{2\pi}{\omega}} = \omega R \rightarrow \ \omega = \frac{v}{R}$$

$$\begin{aligned}|\vec{v}| &= \sqrt{\vec{v}\cdot\vec{v}} = \sqrt{\big[-\omega R\big(\vec{\imath}sin(\omega t) - \vec{\jmath}cos(\omega t)\big)\big]\cdot\big[-\omega R\big(\vec{\imath}sin(\omega t) - \vec{\jmath}cos(\omega t)\big)\big]} \\ &= \sqrt{(-\omega R)^2\cdot\big(\vec{\imath}sin(\omega t) - \vec{\jmath}cos(\omega t)\big)\cdot\big(\vec{\imath}sin(\omega t) - \vec{\jmath}cos(\omega t)\big)} \\ &= \sqrt{(-\omega R)^2\cdot[(\vec{\imath}\cdot\vec{\imath})\sin^2(\omega t) - (\vec{\imath}\cdot\vec{\jmath})\,sin(\omega t)\cos(\omega t) - (\vec{\jmath}\cdot\vec{\imath})\,sin(\omega t)\cos(\omega t) + (\vec{\jmath}\cdot\vec{\jmath})\cos^2(\omega t)} \\ &= \omega R\sqrt{\sin^2(\omega t) + \cos^2(\omega t)} = \boldsymbol{\omega R} \quad \blacksquare\end{aligned}$$

EKSPERIMENT- SLOBODNI PAD

s/m	t/s			$\bar{t}/s$
0.2000	0.2019	0.2019		0.2019
0.8000	0.4037	0.4037		0.4037
1.8000	0.6062	0.6060	0.6066	0.6063

$s(t) = At^2$

$A = \frac{\sum_{i=1}^{N} S_i t_i^2}{\sum_{i=1}^{N} t_i}$

$s(t) = A't^n$

$g = \bar{g} \pm \sigma_g$ $\qquad\qquad$ $\boldsymbol{\bar{g} = \frac{2\bar{s}}{t^2}}$

$s = (1.8000 \pm 0.0005)$

$t = (0.6063 \pm 0.0001)$

ZADATAK – HORVAT – 1.1

-pješak; pola vremena hoda v_1=2 km/h, pola preostalog puta trči 7 km/h, a drugu polovicu preostalog puta v_3=5 km/h. Izračunaj srednju brzinu gibanja tog pješaka!

$\bar{v} =?$

$$\overbrace{\underbrace{- - - - - - - - -}_{s_1}\underbrace{- - - - - - - -}_{s_2}\underbrace{- - - - - - -}_{s_3}}^{s}$$

$s_1 = v_1 t_1 \qquad s_2 = v_2 t_2 \qquad s_3 = v_3 t_3$
$t_1 = t_2 + t_3$
$s_2 = s_3$

$\bar{v} = \frac{s}{t} = \frac{s_1 + s_2 + s_3}{t_1 + t_2 + t_3}$

$t_2 = \frac{v_3 t_3}{v_2}$

$t_3 = \frac{v_2 t_2}{v_3}$

$t_2 + t_3 = \frac{v_3 t_3}{v_2} + \frac{v_2 t_2}{v_3}$

$t_3 = t_1 - t_2 \rightarrow t_2 + t_1 - t_2 = \frac{v_3(t_1 - t_2)}{v_2} + \frac{v_2 t_2}{v_3}$

$\left(1 - \frac{v_3}{v_2}\right) t_1 = \left(\frac{v_2}{v_3} - \frac{v_3}{v_2}\right) t_2 \qquad /\cdot\, v_2 v_3 \qquad\qquad (\ldots)$

$t_3 = \frac{v_2}{v_2 + v_3} t_1$

$\bar{v} = \frac{s_1 + s_2 + s_3}{t_1 + t_2 + t_3} = \frac{v_1 t_1 + v_2 t_2 + v_3 t_3}{t_1 + t_2 + t_3} = \frac{v_1 t_1 + \frac{v_2 v_3 t_1}{v_2 + v_3} + \frac{v_2 v_3 t_1}{v_2 + v_3}}{2t_1} = \frac{v_1}{2} + \frac{v_2 v_3}{v_2 + v_3} = 1 + \frac{35}{12} = \frac{47}{12}$

ZADATAK – „ELEKTRIJADA“ – 1

Čestica se giba duž x-osi tako da joj se brzina mijenja kao $v(x) = \frac{A}{B}(1 + B(x)$ gdje su A i B konstante,a x je koordinata čestice. Odredi ubrzanje pri položaju $x = \frac{1}{B}$!

$$a(x) = \frac{d}{dt}v(x) = \frac{d}{dt}v\big(x(t)\big) = \frac{dv}{dx}\frac{dx}{dt} = \left[\frac{d}{dx}\left(\frac{A}{B}\big(1 + B(x)\big)\right)\right]\left[\frac{A}{B}\big(1 + B(x)\big)\right]$$

$$= \frac{A}{B}B\frac{A}{B}(1 + Bx) = \frac{A^2}{B}(1 + Bx)$$

$$a\left(\frac{1}{B}\right) = \frac{A^2}{B}\left(1 + B\frac{1}{B}\right) = \frac{2A^2}{B}$$

ZADATAK

Brzina čestice koja se giba duž x-osi je $v(x) = v_0 e^{-x/b}$. Pri tom su v_0 i b konstante. Odredi položaj kao funkciju vremena ako je poznato da je u početnom trenutku $t_0=0$, $x(t_0)=0$!

$$v = \frac{dx}{dt} = v_0 e^{-\left(\frac{x}{b}\right)} \qquad /\cdot e^{\left(\frac{x}{b}\right)}dt$$

$$e^{\frac{x}{b}}dx = v_0 e^{-\left(\frac{x}{b}\right)} \qquad \int_{"0"}^{"1"}$$

$$\int_{x_0}^{x_1} e^{\frac{x}{b}}\,dx = \int_{t_0}^{t_1} v_0 dt$$

$$be^{\frac{x}{b}}\Big|_{x_0}^{x_1} = v_0 t\Big|_{t_0}^{t_1}$$

$$b\left(e^{\frac{x_1}{b}} - e^{\frac{x_0}{b}}\right) = v_0(t_1 - t_0)$$

$$b\left(e^{\frac{x_1}{b}} - 1\right) = v_0 t_1$$

konačno stanje „poopćujemo“

$$t_1 \to t, x_1 \to x(t)$$

$$b\left(e^{\frac{x(t)}{b}} - 1\right) = v_0 t$$

$$e^{\frac{x(t)}{b}} = \frac{v_0 t}{b} + 1$$

$$x(t) = b \cdot ln\left(\frac{v_0 t}{b} + 1\right)$$

NEWTONOVI AKSIOMI

Čestica **m** – ima samo masu, nema ni oblik, ni dimenzije, ni orijentaciju

KOLIČINA GIBANJA

$\vec{\boldsymbol{p}} = \boldsymbol{m}\vec{\boldsymbol{v}}$

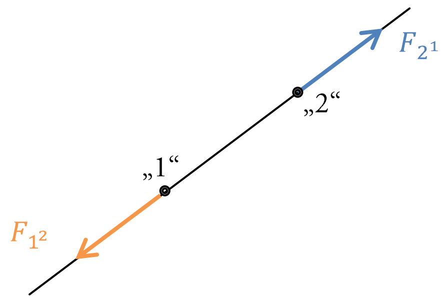
masa brzina

PRVI AKSIOM – Kad na česticu djeluje sila ona ostaje u stanju mirovanja ili jednolikog gibanja po pravcu
- vrijedi u inercijalnim sustavima

DRUGI AKSIOM – Vremenska promjena količine gibanja čestice je razmjerna sili koja na česticu djeluje

$$\frac{d\vec{p}}{dt} = \vec{F}$$

TREĆI AKSIOM – Ako tijelo/čestica djeluje na drugu česticu silom, tada druga čestica djeluje na prvu silom istog iznosa, ali suprotnog smjera. Te sile leže na istom pravcu (pravcu koji prolazi dvjema česticama).



NEWTONOVA JEDNADŽBA GIBANJA

$$\frac{d}{dt}\vec{p} = \frac{d}{dt}(m\vec{v}) = m\frac{d\vec{v}}{dt} = \boldsymbol{m\vec{a}} = \boldsymbol{\vec{F}}$$

$$\vec{F}(\vec{r},t) = \vec{F}_0 \rightarrow konst.$$

prvi korak: BRZINA

$$\frac{d\vec{p}}{dt} = \overrightarrow{F_0} \rightarrow \frac{d\vec{v}}{dt} = \frac{\overrightarrow{F_0}}{m} \qquad / \cdot dt$$

$$d\vec{v} = \frac{\overrightarrow{F_0}}{m}dt \qquad \int_{poč}^{kon}$$

$$\int_{poč}^{kon} d\vec{v} = \int_{poč}^{kon} \frac{\overrightarrow{F_0}}{m}dt$$

$$\vec{v}\,|_{poč}^{kon} = \frac{\overrightarrow{F_0}}{m}t\,|_{poč}^{kon}$$

$$\vec{v}_{kon} - \vec{v}_{poč} = \frac{\overrightarrow{F_0}}{m}(t_{kon} - t_{poč})$$

$$t_{poč} \rightarrow t_0, \qquad t_{kon} \rightarrow t$$

$$\vec{v}(t) - \vec{v}(t_0) = \frac{\overrightarrow{F_0}}{m}(t - t_0)$$

$$\vec{v}(t) = \frac{\overrightarrow{F_0}}{m}(t - t_0) + \vec{v}(t_0)$$

početni uvjet

drugi korak: POLOŽAJ

$$\frac{d\vec{r}}{dt} = \vec{v} \qquad / \cdot dt$$

$$d\vec{r} = \vec{v}dt$$

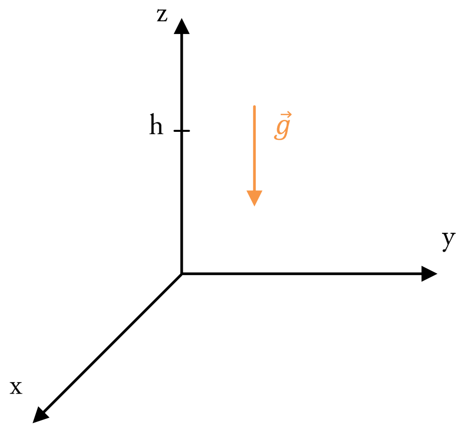
$$\int_{poč}^{kon} d\vec{r} = \int_{poč}^{kon} \vec{v}dt$$

$$\vec{r}(t_1) - \vec{r}(t_0) = \int_{t_0}^{t} \vec{v}(t')dt' = \int_{t_0}^{t} [\vec{v}(t_0) + \frac{\overrightarrow{F_0}}{m}(t' - t_0)]\,dt'$$

$$= \vec{v}(t_0)(t' - t_0) + \frac{\overrightarrow{F_0}}{2m}(t' - t_0)^2$$

$$\vec{r}(t) = \vec{r}(t_0) + \vec{v}(t_0)(t - t_0) + \frac{\overrightarrow{F_0}}{2m}(t - t_0)^2$$

PRIMJER – SLOBODNI PAD



sila $\vec{F_0} = -mg\vec{k}$

početni uvjeti u $t=t_0=0$

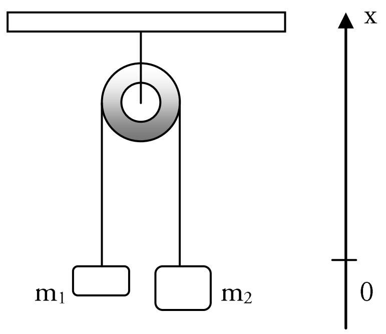
$\vec{v}(t_0) = \vec{v}(0) = \vec{v_0} = 0$

$\vec{r}(t_0) = \vec{r_0} = h\vec{k}$

$\vec{v}(t) = -g\vec{k}t$

$\vec{r}(t) = (h - \frac{g}{2}t^2)\vec{k}$

PRIMJER – PADOSTROJ



$\vec{r_1}(t) = x_1(t)\vec{i}$

$\vec{r_2}(t) = x_2(t)\vec{i}$

$x_1(t) = -x_2(t)$

$\dot{x_1}(t) = -\dot{x_2}(t)$

$\ddot{x_1}(t) = -\ddot{x_2}(t)$

JEDNADŽBA GIBANJA

$m_1\ddot{x_1} = T - m_1g$

$m_2\ddot{x_2} = T - m_2g$

napetost niti

$m_1\ddot{x_1} - m_2\ddot{x_2} = (m_2 - m_1)g$

$\ddot{x_1} = -\ddot{x_2} = \frac{m_2 - m_1}{m_2 + m_1}g$

ZADATAK – PADOSTROJ

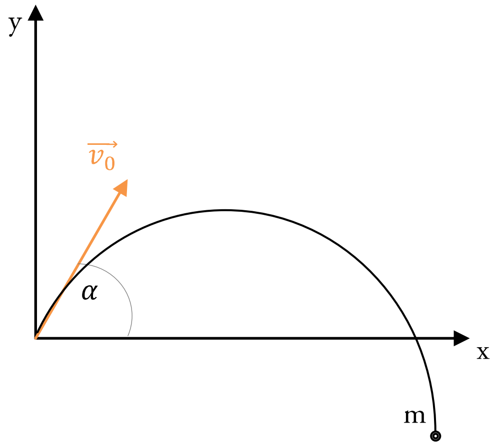
m_1=400 g
m_2=402 g
iz mirovanja
t_1-t_0=6.4 s
$x_1(t_1)$ –$x_1(t_0)$=0.5 m
izračunaj g !

$$x_1(t_1) - x_1(t_0) = \frac{a_x}{2}(t_1 - t_0)^2$$

$$x_1(t_1) - x_1(t_0) = \frac{a_x}{2}(t_1 - t_0)^2$$

$$a_x = \frac{m_2 - m_1}{m_2 + m_1} g \to g = \frac{m_2 + m_1}{m_2 - m_1} \cdot \frac{2[x_1(t_1) - x_1(t_0)]}{(t_1 - t_0)^2} = \frac{400 + 402}{402 - 400} \cdot \frac{(2 \cdot 0.5)}{(6.4)^2} = \mathbf{9.79} \frac{\boldsymbol{m}}{\boldsymbol{s^2}}$$

KOSI HITAC



početni uvjeti u $t=t_0=0$

$$\vec{v}(t_0) = \vec{v}(0) = \vec{v_0} = v_0(\cos\alpha\,\vec{i} + \sin\alpha\vec{j})$$
$$\vec{r}(t_0) = \vec{r}(0) = \vec{r_0} = 0$$

sila:

$$\vec{F_0} = -mg\vec{j}$$

$$\vec{v}(t) = \frac{\vec{F_0}}{m}(t - t_0) + \vec{v}(t_0)$$

$$\vec{r}(t) = \vec{r}(t_0) + \vec{v}(t_0)(t - t_0) + \frac{\vec{F_0}}{2m}(t - t_0)^2$$

uvrštavanje:

$$\to \vec{v}(t) = v_0(\cos\alpha\,\vec{i} + \sin\alpha\vec{j}) - gt\vec{j}$$
$$\to \vec{r}(t) = v_0(\cos\alpha\,\vec{i} + \sin\alpha\vec{j})t - \frac{g}{2}t^2\vec{j}$$

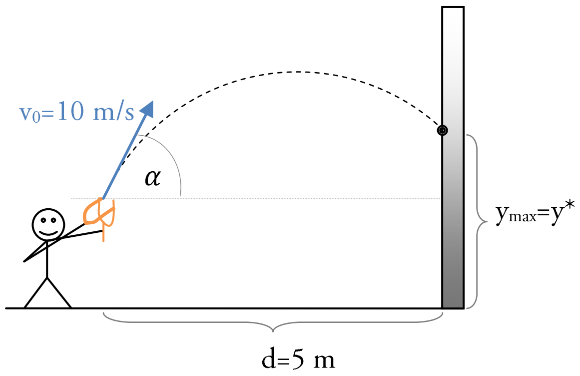
po komponentama:

$$v_x(t) = v_0\cos\alpha$$
$$v_y(t) = v_0\sin\alpha - gt$$
$$x(t) = v_0\cos\alpha \cdot t$$
$$y(t) = v_0\sin\alpha \cdot t - \frac{g}{2}t^2$$

$$y = v_0\sin\alpha \frac{x}{v_0 cos\alpha} - \frac{g}{2}\left(\frac{x}{v_0 cos\alpha}\right)^2 = x \cdot tg\,\alpha - \frac{g}{2}\frac{x^2}{v_0^2\cos^2\alpha}$$

$$\boldsymbol{y(x) = ux - \frac{gx^2}{2v_0^2}(1 + u^2)}$$ gdje je u=tg α

ZADATAK – DJEČAK S PRAĆKOM



Pod kojim kutem dječak treba iz praćke ispucati kamen brzinom 10 m/s ako stoji na 5 m od zida a da taj kamen pogodi što višu točku zida?

$y(x) = ux - \frac{gx^2}{2v_0^2}(1+u^2)$ gdje je u=tg α

tražimo max funkcije y*

$$0 = \frac{dy^*}{du} = \frac{d}{du}\left(ud - \frac{gd^2}{2v_0^2}(1+u)^2\right) = d - \frac{gd^2}{2v_0^2}2u = d - \frac{gd^2}{v_0^2}u$$

$$\rightarrow u = \frac{v_0^2}{gd} = \frac{100}{9.81 \cdot 5} = \frac{20}{9.81} = 2.038$$

$$arc\ tg\ u = 63.87°$$

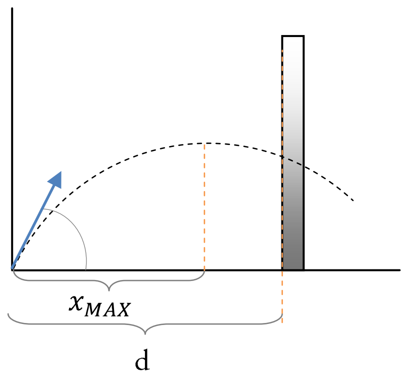
tražimo tjeme parabole

$$y(x) = u^*x - \frac{gx^2}{2v_0^2}(1+(u^*)^2) = \frac{v_0^2}{gd}x - \frac{gx^2}{2v_0^2}\left(1+\frac{v_0^4}{g^2d^2}\right) = \frac{v_0^2}{gd}x - \frac{gx^2}{2v_0^2} - \frac{v_0^2gx^2}{2g^2d^2} \qquad y = ax^2 + bx + x$$

$$x_{MAX} = -\frac{b}{2a} \rightarrow tjeme\ parabole = \frac{\frac{v_0^2}{gd}}{2\left(\frac{g}{2v_0^2}+\frac{v_0^2}{2gd^2}\right)} = \frac{\frac{v_0^2}{gd}}{\frac{2v_0^2}{gd}\left(\frac{g^2d}{2v_0^4}+\frac{1}{2d}\right)} = \frac{1}{2\left(\frac{g^2d}{2v_0^4}+\frac{1}{2d}\right)} = \frac{1}{\frac{g^2d}{v_0^4}+\frac{1}{d}}$$

$$= \frac{d}{1+\frac{g^2d^2}{v_0^4}}$$

$\boldsymbol{x_{MAX}} < d$ a to je ono što smo dokazivali, da će kamen dosegnuti najvišu točku zida ako mu je tjeme parabole po kojoj leti ispred zida ■



Rješavanje Newtonove jednadžbe gibanja za silu razmjernu brzini

$$\boldsymbol{\vec{F} = -\gamma \vec{v}}$$ → brzina

sila konst.

uvrštavanje:

$$\frac{d\vec{p}}{dt} = \vec{F}$$

$$m \cdot \dot{\vec{v}} = -\gamma \vec{v}$$

x komponenta –

$$m \cdot \dot{v_x} = -\gamma v_x$$

$$m\frac{dv_x}{dt} = -\gamma v_x \qquad /\cdot \frac{dt}{mv_x}$$

$$\frac{dv_x}{v_x} = -\frac{dt\gamma}{m} \qquad \int_{poč}^{kon}$$

$$\ln v_x(kon) - \ln v_x(poč) = -\frac{\gamma}{m}\big(t(kon) - t(poč)\big)$$

v(t) $\quad$ v(t_0) $\quad$ t $\quad$ t_0

$$\boldsymbol{v_x(t) = v_x(t_0) \cdot e^{-\frac{\gamma}{m}(t-t_0)}}$$

položaj:

$$\vec{v} = \frac{d\vec{r}}{dt}$$

$$d\vec{r} = \vec{v}dt$$

$$d\vec{r} = \vec{v}_0 e^{-\frac{\gamma}{m} t}\, dt \qquad \int_{poč}^{kon}$$

$$\vec{r}_k - \vec{r}_p = \vec{v}_0\left(-\frac{m}{\gamma}\right) e^{-\frac{\gamma}{m}t}\ \Big|_{poč}^{kon}$$

r(t) $\quad$ r_0

$$\vec{r}(t) = \vec{r}_0 + \overrightarrow{v_0}\frac{m}{\gamma}\left(1 - e^{-\frac{\gamma}{m}t}\right)$$

$$\vec{r}_\infty = \lim_{t\to\infty} r(t) = \vec{r}_0 + \overrightarrow{v_0}\frac{m}{\gamma}(1-0) = \vec{r}_0 + \overrightarrow{v_0}\frac{m}{\gamma}$$

Rješavanje Newtonove jednadžbe gibanja za konstantnu silu uz otpor razmjeran brzini ($\alpha\vec{v}$)

brzina:

$$m\dot{\vec{v}} = \vec{F_0} - \gamma\vec{v}$$

$$m\frac{d\vec{v_x}}{dt} = \vec{F_0}_X - \gamma\vec{v_x}$$

$$\frac{d\vec{v_x}}{\frac{\vec{F_0}_X}{m} - \frac{\gamma\vec{v_x}}{m}} = dt \qquad \int_{poč}^{kon} \qquad\qquad (\dots)$$

$$\vec{v}(t) = \vec{v_0}e^{-\frac{\gamma}{m}(t-t_0)} + \frac{\vec{F_0}}{\gamma}\left(1 - e^{-\frac{\gamma}{m}(t-t_0)}\right)$$

položaj:

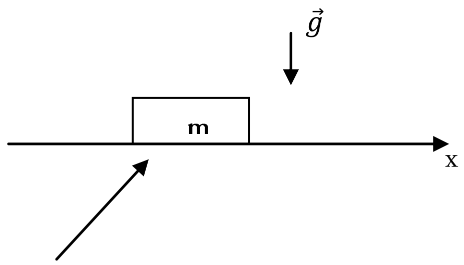
$$\vec{r} = \vec{v}dt \qquad \int_{poč}^{kon}$$

$$\vec{r}(t) = \vec{r_0} + \int_0^t \vec{v}(t')dt' = \qquad\qquad (\dots)$$

$$\vec{r}(t) = \vec{r_0} + \frac{\vec{F_0}}{\gamma}(t - t_0) + \left(\vec{v_0} - \frac{\vec{F_0}}{\gamma}\right)\frac{m}{\gamma}\left(1 - e^{-\frac{\gamma}{m}(t-t_0)}\right)$$

Trenje

- na vodoravnoj podlozi miruje tijelo mase m

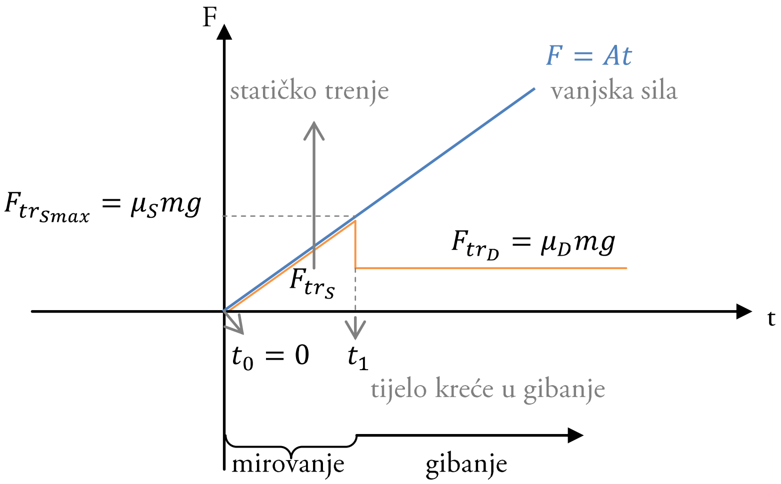


$\mu_D < \mu_S$

$t = t_0 = 0$

$F = At$

$x(t) =? \; za \; t > t_0$



mirovanje

$t_0 < t < t\_1$

$\mu_S mg = At_1 \rightarrow \boldsymbol{t_1} = \frac{\boldsymbol{\mu_S mg}}{\boldsymbol{A}}$

$F_{tr_{Smax}}$

početni uvjeti: $t = t_1 = \frac{\mu_S mg}{A}, x_1 = 0, v_1 = 0$

jednadžba gibanja za t>t_1

$$m\dot{\vec{v}} = -\mu_D mg + At$$

$$\frac{dv}{dt} = -\mu_D g + \frac{A}{m}t$$

$$dv = \left(-\mu_D g + \frac{A}{m}t\right)dt \qquad \int_{poč}^{kon}$$

$$\int_{poč}^{kon} dv = \int_{poč}^{kon} \left(-\mu_D g + \frac{A}{m}t\right)dt$$

$$\vec{v}_K - \vec{v}_P = \int_{t_1}^{t} \left(-\mu_D g + \frac{A}{m}t'\right)dt'$$

$$\vec{v}(t) - \underbrace{\vec{v}(t_1)}_{=0} = -\mu_D g(t - t_1) + \frac{A}{2m}(t - t_1)^2$$

$$\vec{v}(t) = -\mu_D g(t - t_1) + \frac{A}{2m}(t - t_1)^2$$

$$x(t) = -\frac{\mu_D g}{2}(t - t_1)^2 + \frac{A}{6m}(t - t_1)^3$$

Veza između promjene količine gibanja i impulsa sile

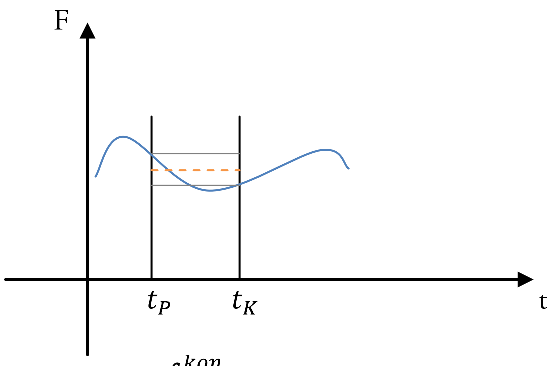
$\frac{d\vec{p}}{dt} = \vec{F} \rightarrow vremenska\ derivacija\ količine\ gibanja$

$$d\vec{p} = \vec{F}dt \qquad \int_{poč}^{kon}$$

$$\overrightarrow{p_K} - \overrightarrow{p_P} = \int_{poč}^{kon} \vec{F}dt$$

$\Delta\vec{p} = \vec{F}\Delta t$ ⟶ impuls sile

promjena količine gibanja

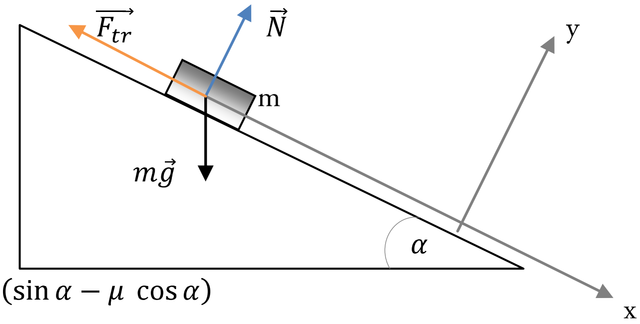


$$\int_{poč}^{kon} \vec{F}dt \simeq \vec{F}\Delta t$$

PRIMJER – KOSINA

$\alpha, \mu_D = \mu$

$a = ?$



$m\ddot{y} = -mg\cos\alpha + N$
$N = mg\cos\alpha$
$m\ddot{x} = mg\sin\alpha - \mu N = mg\sin\alpha - \mu\, mg\cos\alpha = mg(\sin\alpha - \mu\ \cos\alpha)$

kružno gibanje:

$$\boldsymbol{F_C = \frac{mv^2}{R} = m\omega^2 R}$$

-centripetalna sila

položaj:

$\vec{r} = R(\cos\Phi\,\vec{\imath} + \sin\Phi\,\vec{\jmath}), \quad \Phi = \Phi(\mathrm{t})$

$\vec{v} = \frac{d\vec{r}}{dt} = R\left(-\dot{\Phi}\sin\Phi\,\vec{\imath} + \dot{\Phi}\cos\Phi\,\vec{\jmath}\right) = R\dot{\Phi}\,(-\sin\Phi\,\vec{\imath} + \cos\Phi\,\vec{\jmath})$

$\vec{a} = R\ddot{\Phi}(-\sin\Phi\,\vec{\imath} + \cos\Phi\,\vec{\jmath}\,) + R\dot{\Phi}\left(-\cos\Phi\dot{\Phi}\,\vec{\imath} + \sin\Phi\dot{\Phi}\,\vec{\jmath}\right)$

$$= \boldsymbol{\frac{\ddot{\Phi}}{\dot{\Phi}}\vec{v} - \dot{\Phi}^2\vec{r}}$$

komponenta tangencijalna na putanju i komponenta okomita na putanju

važno!
$\vec{r} \perp \vec{v} \rightarrow (\vec{r} \cdot \vec{v}) = 0$

$\vec{r} \cdot \vec{v} = r_x v_x + r_y v_y + r_z v_z = -R\cos\Phi\, R\dot{\Phi}\sin\Phi + \ R\sin\Phi\, R\dot{\Phi}\cos\Phi = 0$

$\vec{a} = \frac{\ddot{\Phi}}{\dot{\Phi}} v\hat{\vec{v}} - \dot{\Phi}^2 \mathrm{R}\hat{\vec{r}}\,, \qquad v = R\dot{\Phi} \rightarrow\ \vec{a} = R\ddot{\Phi}\hat{\vec{v}} - R\dot{\Phi}^2\hat{\vec{r}}\ \equiv \vec{a}_{tang} - \vec{a}_{rad}$

$\alpha = \ddot{\Phi} \rightarrow$ kutna akceleracija
$\omega = \dot{\Phi} \rightarrow$ kutna brzina

NEWTONOVA JEDNADŽBA

$\vec{F} = m\vec{a} = \vec{F}_{tang} + \vec{F}_{rad}$

$\hat{\vec{v}}\, m\alpha R \qquad -\hat{\vec{r}}\, m\omega^2 R = -\hat{\vec{r}}\frac{v^2}{R} m$

RAD

Sila $\vec{F}$ djeluje duž puta $d\vec{s}$

diferencijal obavljenog rada:

$$dW = \vec{F} d\vec{s}$$

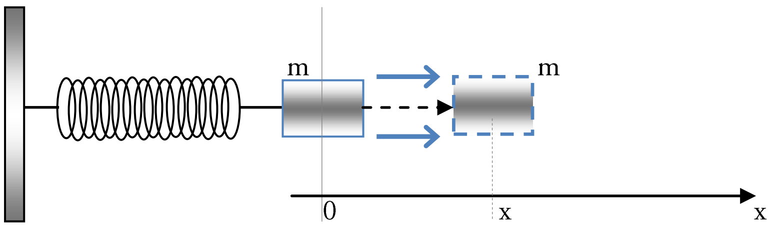
primjeri: „W=Fs“

-sila paralelna s putanjom
-sila je stalna

0

$$W = \int \vec{F} d\vec{s} = \int F \cos\vartheta \; ds = \int F \, ds = F \int ds = F \cdot s$$

PRIMJER – OPRUGA

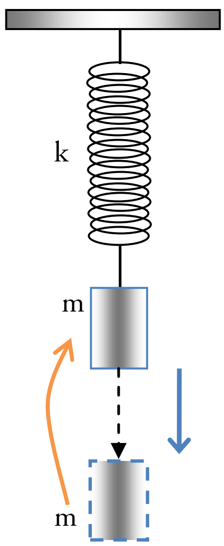


$\vec{\boldsymbol{F}} = -\boldsymbol{kx}$ pomak koji ćemo napraviti

konstanta opruge

rad pri rastezanju opruge za x:

$$W = \int \vec{F} d\vec{s} = \int_0^x (kx')dx' = \frac{kx^2}{2}$$



$$W = \int \vec{F} d\vec{s} = \int_0^x (-kx')(-dx') = \frac{kx^2}{2} \;\blacksquare$$

KINETIČKA ENERGIJA

Neka sila $\vec{F}$ djeluje na tijelo mase m, gibanje duž x-osi

$$\vec{F} = \frac{d\vec{p}}{dt} = m\frac{d\vec{v}}{dt} = m\frac{d\vec{v}}{dx}\frac{dx}{dt} = m\vec{v}\frac{d\vec{v}}{dx}$$

$$W = \int \vec{F} dx = \int mv\frac{dv}{dx}dx = \int mv\, dv = \frac{mv^2}{2} \equiv E_{KIN}$$

$$W_{1\to2} = \int_1^2 F\, dx = \frac{mv_2^2}{2} - \frac{mv_1^2}{2}$$

TEOREM O RADU I KINETIČKOJ ENERGIJI:

(obavljen rad)=(promjena kinetičke energije)
$\Delta W = \Delta E_{KIN}$

SNAGA

mjera rada obavljenog u jedinici vremena
$$P = \frac{dW}{dt} = \frac{d}{dt}\vec{F}d\vec{s} = \vec{F}\frac{d\vec{s}}{dt} = \vec{F}\vec{v}$$

PRIMJER

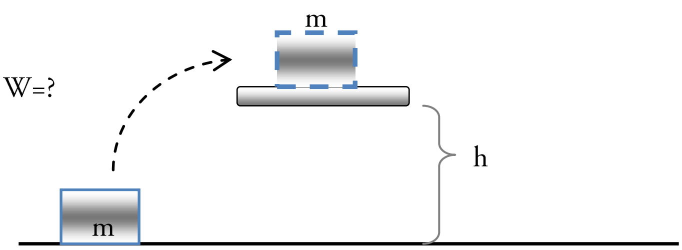
auto kreće iz mirovanja ($\vec{v}_0 = 0$); $m = 1T = 1000\ kg$; za vrijeme $\Delta t = t_1 - t_0 = 15\ s$ postiže vrzinu $v_1 = 100\frac{km}{h} = 27.777\frac{m}{s}$; kolika je prosječna snaga?

$$\bar{P} = \frac{\Delta W}{\Delta t} = \frac{\Delta E_{KIN}}{\Delta t} = \frac{\frac{m}{2}(v_1^2 - v_0^2)}{\Delta t} = \frac{\frac{m}{2}v_1^2}{\Delta t} = \frac{500 \cdot 771.6}{15} = 25.72\ kW$$

POTENCIJALNA ENERGIJA

PRIMJER – HOMOGENO POLJE GRAVITACIJSKE SILE

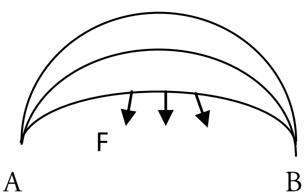
W=?



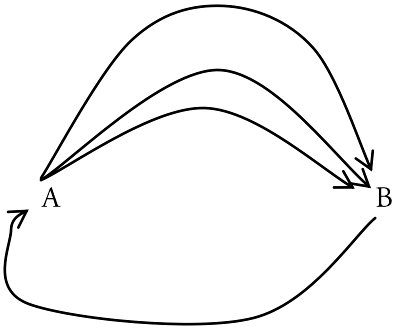
$$W_{A\to B} = \int_A^B \vec{F}\, d\vec{s}$$

sve ovisi o izboru putanje A – B

$$W = mgh = mg\sin\alpha\frac{h}{\sin\alpha}$$



Sila je „konzervativna“!

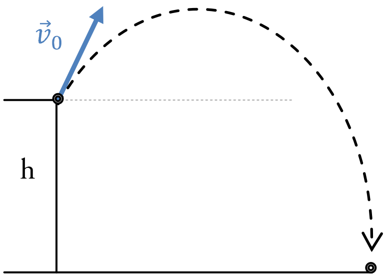


$$W_{B\to A} = -W_{A\to B}$$

$$\oint \vec{F}\, d\vec{s} = 0$$

integral zatvorene putanje

Zadatak



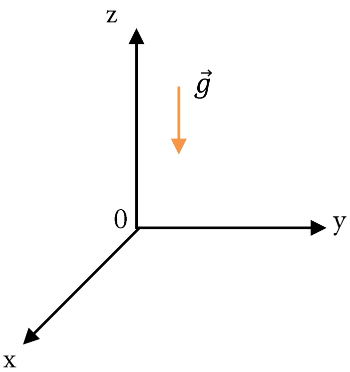
-**potencijalna en.** – skalarna funkcija u polju konzervativne sile; njena vrijednost je jednaka radu potrebnom da česticu dovedemo u neku točku

$$U(B) - U(A) = -\int_A^B \vec{F}\, d\vec{s}$$

$$U(B) = U(A) - \underbrace{\int_A^B \vec{F}\, d\vec{s}}_{\text{rad } A \to B}$$

$$U(\vec{r}) = -\int_{\vec{r}_0}^{\vec{r}} \vec{F}\, d\vec{r}'$$

Primjer – homogena gravitacija



$$\vec{F} = -mg\vec{k}$$

$$U(\vec{r}) = -\int_{\vec{r}_0}^{\vec{r}} (-mg\vec{k}) d\vec{r} = -\int_{\vec{r}_0}^{\vec{r}} (-mg) dz$$

$$= mg \int_{\vec{r}_0}^{\vec{r}} dz = \boldsymbol{mgz}$$

=0

$\vec{F}(\vec{r})$ … *konzervativna sila*
$U(\vec{r})$ … *potencijalna en.*

$$U(\vec{r}) = U(\vec{r}_0) - \int_{\vec{r}_0}^{\vec{r}} \vec{F}(\vec{r}') d\vec{r}'$$

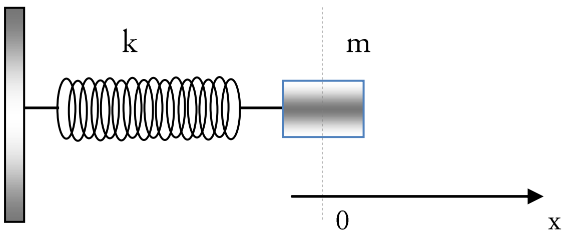
$$\vec{F} = -\frac{\partial U}{\partial x}\vec{\imath} - \frac{\partial U}{\partial y}\vec{\jmath} - \frac{\partial U}{\partial z}\vec{k} = -\frac{\partial U}{\partial \vec{r}} = -\vec{\nabla}\mathrm{U}$$

Primjer

$U = mgz$

$$\vec{F} = -\frac{\partial U}{\partial x}\vec{\imath} - \frac{\partial U}{\partial y}\vec{\jmath} - \frac{\partial U}{\partial z}\vec{k} = -(mg)\vec{k}$$

Primjer – opruga



$$\vec{F} = -kx$$

$$U(x) = U(0) - \int_0^x \vec{F}(x')\,dx' = \frac{1}{2}kx^2$$

$$F(x) = -\frac{\partial}{\partial x}U(x) = -\frac{\partial}{\partial x}\frac{1}{2}kx^2 = -kx$$

Primjer

$U = -\frac{k}{x}$

sila:

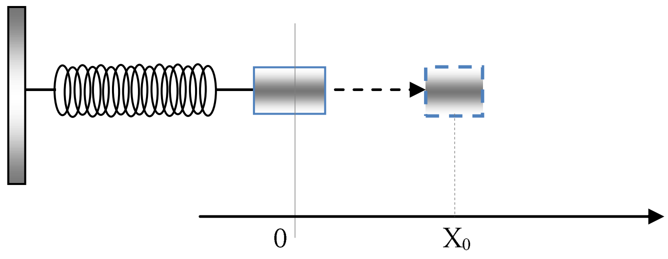
$$F(x) = -\frac{\partial U}{\partial x} = -\frac{\partial}{\partial x}\left(-\frac{k}{x}\right) = -\frac{k}{x^2}$$

Zakon očuvanja mehaničke energije

$\Delta W_{KONZ} = \Delta E_{KIN} = -\Delta(U)$

$\rightarrow \Delta(E_{KIN} + U) = 0$

Primjer



početni otklon $x=x_0$; $v=v_0=0$
brzina pri $x=0=?$

$$E = E_{KIN} + E_{POT} = \frac{1}{2}mv^2 + \frac{1}{2}kx^2$$
$$\Delta E = 0$$

$$E_0 = \frac{1}{2}mv_0^2 + \frac{1}{2}kx_0^2 = \frac{1}{2}kx_0^2$$

$$E_1 = \frac{1}{2}mv_1^2 + \frac{1}{2}kx_1^2 = \frac{1}{2}mv_1^2$$

$$E = E_0 = E_1$$

$$\frac{1}{2}kx_0^2 = \frac{1}{2}mv_1^2$$

$$v_1 = x_0\sqrt{\frac{k}{m}}$$

Zakon očuvanja količine gibanja sustava čestica

$$\underline{\vec{P}} = \sum_{i=1}^{N} \overrightarrow{P_i}$$

$$\frac{d}{dt}\underline{\vec{P}} = \frac{d}{dt}\sum_{i=1}^{N} \overrightarrow{P_i} = \sum_{i=1}^{N} \frac{d\overrightarrow{P_i}}{dt} = \sum \overrightarrow{F_i}$$

$$\vec{F_i} = \vec{F}_{iEXT} + \sum_{j\neq i} \vec{F}_{ij}$$

sila kojom j-ta čestica djeluje na i-tu

vanjska sila na i-tu česticu

$$\vec{F}_{ij} = -\vec{F}_{ji}$$

$$\frac{d}{dt}\vec{P} = (\dots) = \sum_i \left(\vec{F}_{i\,EXT} + \sum_{j\neq i} \vec{F}_{ij}\right) = \sum_i \vec{F}_{i\,EXT} + \sum_i \sum_{j\neq i} \vec{F}_{ij} = \sum_i \vec{F}_{i\,EXT}$$

ako su $\vec{F}_{i\,EXT}$=0 P=konst

ENERGIJA

$$E = \frac{mv_0^2}{2} = mgh + F_{TR}\frac{h}{\sin\alpha} = \frac{mv_2^2}{2} + 2F_{TR}\frac{h}{\sin\alpha}$$

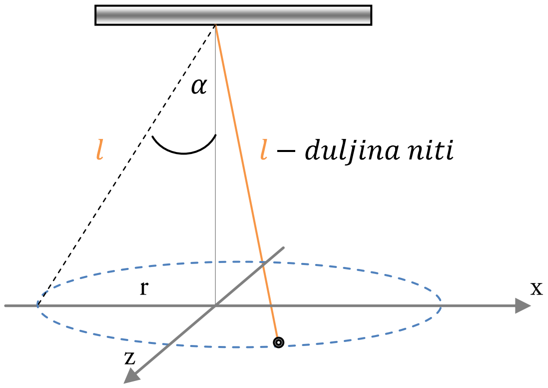
$$h = \frac{mv_0^2}{2}\left(mg + \frac{F_{TR}}{\sin\alpha}\right)^{-1}$$

$$\frac{mv_2^2}{2} = \frac{mv_0^2}{2} - \frac{2F_{TR}}{\sin\alpha}h = \frac{mv_0^2}{2} - \frac{2F_{TR}}{\sin\alpha}\frac{mv_0^2}{2}\left(mg + \frac{F_{TR}}{\sin\alpha}\right)^{-1}$$

$$\frac{v_2^2}{v_0^2} = 1 - \frac{2\mu mg\cos\alpha}{\sin\alpha}\left(mg + \frac{\mu mg\cos\alpha}{\sin\alpha}\right)^{-1} = 1 - \frac{2\mu}{\tan\alpha + \mu} = \frac{\tan\alpha - \mu}{\tan\alpha + \mu}$$

$$v_2 = v_0\sqrt{\frac{\tan\alpha - \mu}{\tan\alpha + \mu}}$$

PRIMJER – STOŽASTO NJIHALO



općenito:

$$m\vec{a} = m(\ddot{x}\vec{i} + \ddot{y}\vec{j} + \ddot{z}\vec{k}) = \vec{F} + m\vec{g} = -m\omega\vec{r}$$

u trenutku z=0:

$$\vec{T} + mg = T(\sin\alpha\,\vec{i} + \cos\alpha\,\vec{j}) - mg\vec{j} = (T\sin\alpha)\vec{i} + (T\cos\alpha)\vec{j} - m\omega^2\vec{r} = -m\omega^2 R(-\vec{i}) = m\omega^2 R\vec{i}$$

$$T\cos\alpha = mg$$
$$T\sin\alpha = m\omega^2 R$$

$$\tan\alpha = \frac{\omega^2 R}{g} \to \omega^2 = \frac{g}{R}\tan\alpha = \frac{g}{l\sin\alpha}\cdot\frac{\sin\alpha}{\cos\alpha} = \frac{g}{l}\cdot\frac{1}{\cos\alpha}$$

Zadatak – raketa

Ako raketa kreće iz mirovanja, kolika će biti njena brzina u trenutku u kojem će se njena masa smanjiti na polovicu početne mase?

$$M \to \frac{1}{2}M$$

ukupna količina gibanja - konstantna

$$\underline{\vec{P}} = 0$$

$$dpg = -dpr$$

$$dmv_g = -mdv_r$$

$$v_g \frac{dm}{m} = -dv_r \qquad \int_{poč}^{kon}$$

$$v_g \ln m \ \Big|_{poč}^{kon} = -v_r \ \Big|_{poč}^{kon}$$

$$v_g \ln \frac{\frac{1}{2}M}{M} = v_{poč} - v_{kon}$$

$$v_g \ln \frac{1}{2} = -v_{kon}$$

$$|v_{kon}| = v_g \ln 2$$

Zakon gibanja središta mase

$$\underline{\vec{P}} = \sum_i \vec{P}_i = \sum_i m_i \vec{v}_i$$

$$\frac{\mathrm{d}}{\mathrm{dt}} \underline{\vec{P}} = \sum_i \vec{F}_{i_{EXT}} = \vec{F}_{EXT}$$

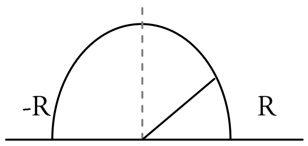
definicija središta mase:

$$\vec{R} = \frac{\sum_i m_i \vec{r}_i}{\sum_i m_i} = \frac{1}{M} \sum_i m_i \vec{r}_i$$

$$\vec{R} = \frac{1}{M} \int \vec{r} g \, dV = \frac{1}{M} \int \vec{r} \vec{v} \, da = \frac{1}{M} \int \vec{r} \lambda \, dl$$

Primjer

žica u svinuta u polukrug



$\vec{R}$ =?

$$Y = \frac{1}{M}\int y\lambda\, dl = \frac{1}{M}\int y\frac{M}{R\pi}\, dl$$

$$\lambda = \frac{M}{R\pi}, \qquad dl = R\, d\Phi, \qquad \mathrm{y} = \mathrm{R}\sin\Phi$$

$$Y = \frac{1}{M}\int_0^{\pi}(\mathrm{R}\sin\Phi)\frac{M}{R\pi}\, R\, d\Phi = \frac{\mathrm{R}}{\pi}\int_0^{\pi}\sin\Phi\, \mathrm{d}\Phi = \frac{R}{\pi}(-\cos\Phi)\,|_0^{\pi} = \frac{2R}{\pi}$$

Primjer

$$\vec{V} = \dot{\vec{R}} = \frac{1}{M}\sum_i m_i\dot{\vec{r}_i} = \frac{1}{M}\sum_i \vec{P}_i = \frac{\vec{P}}{M}$$

$$\underline{\vec{P}} = M\vec{V}$$

$$\underline{\dot{\vec{P}}} = M\dot{\vec{V}} = \vec{F}_{EXT}$$

- jednadžba gibanja za središte mase sustava čestica

Sudari 2 čestice

u svim sudarima očuvana je količina gibanja

$$\underline{\vec{P}} = \sum_i \vec{P}_i = \sum_i \vec{P}_i{}' \rightarrow m_1\overrightarrow{v_1} + m_2\overrightarrow{v_2} = m_1\overrightarrow{v_1}' + m_2\overrightarrow{v_2}'$$

SAVRŠENO ELASTIČNI SUDAR:

$$E_{KIN} = \sum_i \frac{m_i \vec{v}_i^2}{2} = \sum_i \frac{m_i \vec{v'}_i^2}{2}$$

2 čestice:

$$m_1 \overrightarrow{v_1} + m_2 \overrightarrow{v_2} = m_1 \overrightarrow{v_1}' + m_2 \overrightarrow{v_2}'$$

$$m_1 \overrightarrow{v_1}^2 + m_2 \overrightarrow{v_2}^2 = m_1 \overrightarrow{v_1}'^2 + m_2 \overrightarrow{v_2}'^2$$

pojednostavljeno u 1D:

$$m_1 v_{1X} + m_2 v_{2X} = m_1 v_{1X}' + m_2 v_{2X}'$$

$$m_1 v_{1X}^2 + m_2 v_{2X}^2 = m_1 v_{1X}'^2 + m_2 v_{2X}'^2$$

zadano:

$m_1, m_2; v_{1x}, v_{2x};$ $\quad v_{1x}', v_{2x}' = ?$

$$v_{1x}' = \begin{cases} v_{1x} & - nema\ sudara \\ \frac{m_1 - m_2}{m_1 + m_2} v_{1x} + \frac{2m_2}{m_1 + m_2} v_{2x} \end{cases}$$

$$v_{2x}' = \begin{cases} v_{2x} & - nema\ sudara \\ \frac{m_2 - m_1}{m_1 + m_2} v_{2x} + \frac{2m_1}{m_1 + m_2} v_{1x} \end{cases}$$

općenit (ne/?)elastični sudar:

$$m_1 \overrightarrow{v_1} + m_2 \overrightarrow{v_2} = m_1 \overrightarrow{v_1}' + m_2 \overrightarrow{v_2}'$$

$$m_1 \overrightarrow{v_1}^2 + m_2 \overrightarrow{v_2}^2 = m_1 \overrightarrow{v_1}'^2 + m_2 \overrightarrow{v_2}'^2 + 2Q$$

savršeno ne-elastični sudar

$$Q \to Q_{MAX}$$

$$\overrightarrow{v_1}' = \overrightarrow{v_2}' = \frac{m_1 \overrightarrow{v_1} + m_2 \overrightarrow{v_2}}{m_1 + m_2}$$

Zadatak

na križanju se događa savršeni ne-elastični sudar, $m_1 = 1800\ kg$ nalijeće s $\overrightarrow{v_1} = 60 \frac{km}{h}$ gibajući se na istok. $m_2 = 2400\ kg; \overrightarrow{v_2} = 30 \frac{km}{h}$ i giba se u smjeru sjevera. Kolikom će se brzinom nakon sudara gibati „olupina“?

$$\vec{v}' = \frac{m_1 \overrightarrow{v_1} + m_2 \overrightarrow{v_2}}{m_1 + m_2} = \frac{m_1 \overrightarrow{v_1}\, \vec{\imath} + m_2 \overrightarrow{v_2}\, \vec{\jmath}}{m_1 + m_2} = \frac{\overrightarrow{v_1}\, \vec{\imath} + \frac{4}{3} \overrightarrow{v_2}\, \vec{\jmath}}{\frac{7}{3}} = 25.71 \vec{\imath} + 17.14\, \vec{\jmath} \to v = 30.9 \frac{km}{h}$$

zaključno sa predavanjem 24. 03. '10.

-prvi ciklus!