FIZ 1.1 ~~HMS~~ S SSAM.2

1MI 07/08

1) $F(t) = -k m e^{\beta t}$

$F(t=1) = 2F(t=0)$

$-k m e^{\beta} = 2 \cdot (-k m \cdot 1)$

$e^{\beta} = 2$

$\beta = \ln 2$

$m \cdot \frac{dv}{dt} = -k m e^{\beta t}$ → uvijek za silu promj. u vremenu

$dv = -k e^{\beta t} dt$

$v = -\frac{k}{\beta} e^{\beta t} + C$

$v_0 = -\frac{k}{\beta} e^{\beta \cdot 0} + C$

$C = v_0 + \frac{k}{\beta}$

$v(t) = -\frac{k}{\beta} e^{\beta t} + v_0 + \frac{k}{\beta}$

$0 = \frac{k}{\beta}\left(1 - e^{\beta t}\right) + v_0$

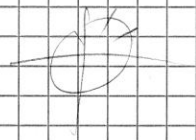
$\frac{k}{\beta}\left(1 - e^{\beta t}\right) = -v_0 \quad / \cdot \frac{\beta}{k}$

$1 - e^{\beta t} = -\frac{v_0 \beta}{k}$

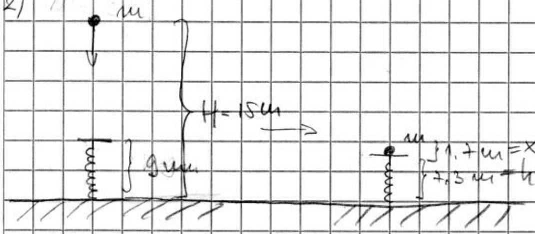
$1 + \frac{v_0 \beta}{k} = e^{\beta t} \quad / \ln$

$\ln\left(1 + \frac{v_0 \beta}{k}\right) = \beta t$

$t = \frac{\ln\left(1 + \frac{v_0 \beta}{k}\right)}{\beta} = 6,135\ s$



2)



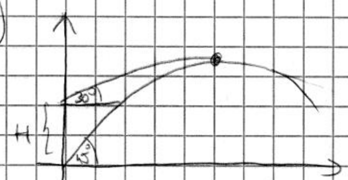
$F_e = -kx \qquad E_{EL} = \frac{kx^2}{2}$

$E_{p_1} + E_{k_1} = E_{p_2} + E_{k_2} + E_{EL} + Q$

$mgH + 0 = mgh + 0 + \frac{kx^2}{2} + Q$

$Q = mg(H - h) - \frac{kx^2}{2} = 246,33\ J$

3)



$v_{0_1} = 50\ m/s, \quad \beta_1 = 45°, \quad \beta_2 = 30°$

$H = ?$

$x = v_0 \cos\alpha\, t$

$x_1 = x_2$

$v_{0_1} \cos 45° \, t = v_{0_2} \cos 30° \, t$

$v_{0_2} = v_{0_1} \frac{\cos 45}{\cos 30°} = 50 \cdot \frac{\frac{\sqrt{2}}{2}}{\frac{\sqrt{3}}{2}} = 40,82\ m/s$

$y = v_y t - \frac{gt^2}{2}$

$y_1 = y_2$

$v_{0_1} t - \frac{gt^2}{2} = v_{0_2} t - \frac{gt^2}{2} + H$

$H = v_{0_1} \sin 45° - v_{0_2} \sin 30° = 7,47\ m$

NMI 08/09

1) $v_k = 8\ m/s$

$a(t) = 2\sqrt{t}$

$\frac{dv}{dt} = 2\sqrt{t}$

$dv = 2\sqrt{t}\,dt$

$v(t) = 2 \cdot \frac{2}{3} t^{3/2} + C_1$

$v(0) = 0$

$0 = 2 \cdot \frac{2}{3} 0^{3/2} + C_1$

$C_1 = 0$

$v(t) = \frac{4}{3} t^{3/2}$

$\frac{ds}{dt} = \frac{4}{3} t^{3/2}$

$ds = \frac{4}{3} t^{3/2}\,dt$

$s(t) = \frac{4}{3} \frac{2}{5} t^{\frac{5}{2}} + C_2$

$C_2 = 0$

$s(t) = \frac{8}{15} t^{5/2}$

$s_a = s_k$

$\frac{8}{15} t^{5/2} = v_k t$

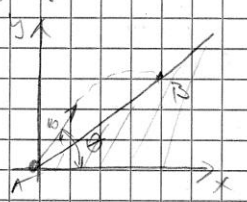
$t^{3/2} = \frac{15}{8} v_k$

$t = 6{,}571\ s$

$v(6{,}571) = 22{,}5\ m/s$

$s(6{,}571) = 59{,}2\ m$

2) $y = kx$



$d = \sqrt{x^2 + y^2} = \sqrt{x^2 + k^2 x^2} = x\sqrt{1 + k^2}$

$y(x) = x\,tg\theta - \frac{g x^2}{2 v_0^2}(1 + tg^2\theta) = kx$

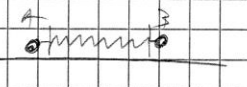
$tg\theta - k = \frac{g x (1 + tg^2\theta)}{2 v_0^2}$

$2 v_0^2 (tg\theta - k) = x\, g (1 + tg^2\theta)$

$x = \frac{2 v_0^2 (tg\theta - k)}{g(1 + tg^2\theta)}$

$d = \frac{2 v_0^2 (tg\theta - k)}{g(1 + tg^2\theta)} \sqrt{1 + k^2}$

3) $m_A = 2 m_B$



$m_A v_A + m_B v_B = m_A v_A' + m_B v_B'$

$0 + 0 = m_B v_B + m_A v_A$ → jes se "odbija"

$m_B v_B = m_A v_A$

$m_B v_B = 2 m_B v_A$

$v_B = 2 v_A$

$\frac{m_A v_A^2}{2} + \frac{m_B v_B^2}{2} = 60\ J$

$\frac{m_A v_A^2}{2} + \frac{m_A}{2} \cdot \frac{4 v_A^2}{2} = 60$

$\frac{m_A v_A^2}{2}(1 + 2) = 60$

$\frac{m_A v_A^2}{2} = 20\ J$

$\frac{m_A v_A^2}{2} = 40\ J$

1. B1. 08/09.

1) $v_0 = 20\ km/h = 5.55\ m/s$

$A = 0.05\ m/s^2$, $\Delta s = 10\ m$

$s_1 - s_0 = 10\ m$

$v_1 - v_0 = 0.05$

$v_1 = 0.05 + 5.55 = 5.6055$

$v_1 = v_0 + at$

$s = v_0 t + \frac{at^2}{2}$

$v_1^2 - v_0^2 = 2as$

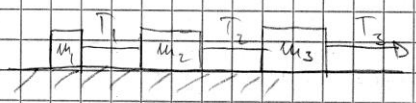
$a = \frac{v_1^2 - v_0^2}{2s} = 0,028\ m/s^2$

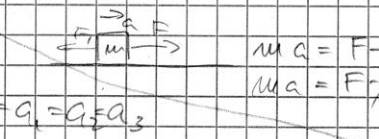
$v(t) = v_0 + at = 10 + 0,028t$

$s(t) = v_0 t + \frac{at^2}{2} = 10t + 0,014t^2$

2) F_N, T_1 = ? F_{N1}, T_2 = ?

$T_3 = 24\ N$, $m_2 / m_1 = 2$, $m_3 / m_1 = 3$





$ma = F - Ft_1 - F_1$

$ma = F - \mu m g + T_1$

$a = a_1 = a_2 = a_3$

$m_3 a = T_3 - T_2 - \mu m_3 g$

$m_2 a = T_2 - T_1 - \mu m_2 g$

$m_1 a = T_1 - \mu m_1 g$

$\Rightarrow \begin{cases} m_3 a = 24 - T_2 - \mu m_3 g \\ m_2 a = T_2 - T_1 - \mu m_2 g \\ m_1 a = T_1 - \mu m_1 g \end{cases}$ $\quad m_2 = 2m_1$, $m_3 = 3m_1$

$(m_1 + m_2 + m_3)a = 24 - \mu g(m_1 + m_2 + m_3)$

$6m_1 a = 24 - \mu g \cdot 6m_1$

$m_1 a = 4 - \mu g m_1$

$m_1 a + \mu m_1 g = 4$

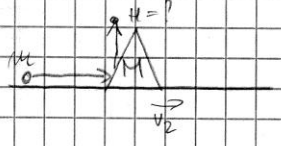
$\Rightarrow T_1 = m_1 a + \mu m_1 g$

$T_1 = 4$

$T_2 = m_2 a + \mu m_2 g + T_1 = 2m_1 a + 2\mu m_1 g + T = 2(m_1 a + \mu m_1 g) + T =$

$= 2 \cdot 4 + 4 = 8 + 4 = 12\ N$

3)



$H = 100\ m$

$m_1 v_1 = H v_2$

$m_1 v_1 = 100 m_1$ $\quad (v_2 = 1\ m/s)$

$v_1 = 100\ m/s$

$\frac{m v_1^2}{2} = mgH + \frac{m v_2^2}{2}$

$\frac{m v_1^2}{2} = \frac{m v_1'^2}{2} + \frac{m v_2'^2}{2}$

$\frac{100^2}{2} = gH + \frac{1}{2}$

$H = \frac{9999}{2} \cdot \frac{1}{9.81} = 504,58$

TPM1 08/09

1)

$m \cdot a = mg(\sin\alpha - \mu\cos\alpha)$

$m \cdot a = 0$

$0 = mg(\sin\alpha - \mu\cos\alpha)$

$\sin\alpha = \mu\cos\alpha$

$\mu_s = \operatorname{tg}\alpha$

$V = \frac{Ah}{3} = \frac{R^2\pi \cdot h}{3}$

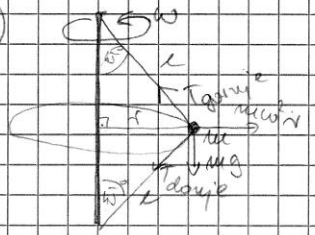
$\operatorname{tg}\alpha = \frac{h}{R}$

$\mu_s = \frac{h}{R}$

$h = \mu_s R$

$V = \frac{1}{3} R^3 \pi \mu_s$

2)

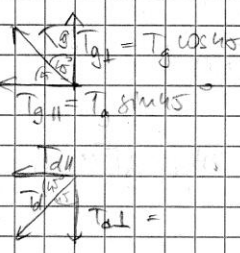


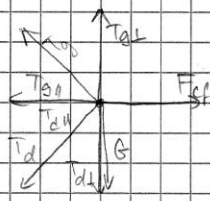
$\omega = \frac{4}{3}\pi$

$t = 0.9\ m$

$l^2 = 2r^2$

$r = \frac{l}{\sqrt{2}}$





$T_{g\perp}\vec{j} - T_{d\perp}\vec{j} - G\vec{j} = 0$

$T_{g\perp} - T_{d\perp} = G$

$T_g \cos 45 - T_d \cos 45 = mg$

$T_{g\parallel} + T_{d\parallel} = F_{cp}$

$T_g \sin 45 + T_d \sin 45 = m$

$\frac{\sqrt{2}}{2} T_g - \frac{\sqrt{2}}{2} T_d = mg$

$T_g\left(\frac{\sqrt{2}}{2} T_g + \frac{\sqrt{2}}{2} T_d = m\omega^2 r\right)$

$\sqrt{2}\, T_g = m(g + \omega^2 r)$

$T_g = \frac{100\left(9{,}81 + \frac{16}{9\pi^2} \cdot \frac{0{,}9}{\sqrt{2}}\right)}{\sqrt{2}} = 1{,}428\ N$

$T_d = \left(m\omega^2 r - \frac{\sqrt{2}}{2} T_g\right) \frac{2}{\sqrt{2}} = 0{,}09\ N$

3) $m_1 = 0{,}20\ kg$

$m_1 v_1 + m_2 v_2 = -m_1 v_1' + m_2 v_2'$ $\quad v_2 = 0$

$m_1 v_1 = -m_1 v_1' + m_2 v_2'$

$\frac{m_1 v_1^2}{2} + 0 = \frac{m_1 v_1'^2}{2} + \frac{m_2 v_2'^2}{2} \quad / \cdot 2$

$d = v_2' t_{padanja}$

$2d = v_1' t_{pad.}$

$2v_2' + pod = v_1' + t_{pad}$

$v_1' = 2v_2'$

$m_2(m_2 - 5m_1) = 0$

$m_1 = 0$ $\quad m_2 = 5m_1 = 5 \cdot 0{,}2 = 1kg$

$m_1 v_1 = -2m_1 v_2' + m$

$m_1 v_1^2 = 4m_1 v_2'^2 + m$

$v_1 = -2v_2' + \frac{m_2}{m_1} v_2$

$m_1 \left(\frac{m_2 - 2m_1}{m_1}\right)^2 = m$

$m_2^2 - 4m_1 m_2 + 4m_1^2 = 4m$

$m_2^2 - 4m_1 m_2 + 4m_1^2 = 4m$

$m_2^2 - 5m_1 m_2 = 0$

1 MI 09/10

1) $\vec{r}(t) = R[\cos\omega t\,\vec{i} + (at - \sin\omega t)\vec{j}]$

$R = 50\,mm = 0.05\,m$ $\omega = \frac{\pi}{7}\,rad\,s^{-1}$ $a = 0.1\,s^{-1}$

u $t = 6$? v i a

$v = \frac{dr}{dt} = R[\cos\omega t\,\vec{i} + (at - \sin\omega t)\vec{j}]'$

$= R[-\omega\sin\omega t\,\vec{i} + (a - \omega\cos\omega t)\vec{j}]$

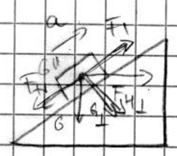
$a = \frac{dv}{dt} = R[-\omega^2\cos\omega t\,\vec{i} + \omega^2\sin\omega t\,\vec{j}]$

$v(t=6) = -0.009736\,\vec{i} + 0.025\,\vec{j}$

$a(t=6) = 0.0090737\,\vec{i}$

$\vec{a}\cdot\vec{v} = |\vec{a}||\vec{v}|\cos\alpha$

$\cos\alpha = \frac{\vec{a}\cdot\vec{v}}{|\vec{a}||\vec{v}|} \rightarrow \alpha = 86°$

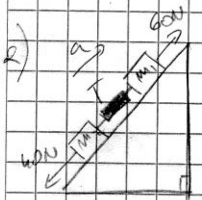


$F_{tr} \neq \mu mg\cos\alpha$

$F_{tr} = \mu(mg\cos\alpha + F_H\sin\alpha)$

$ma = F_1 + F_H\cos\alpha - G\sin\alpha - F_{tr}$

2)



$m_1 a = 60 - T - \mu m_1 g\cos\alpha - m_1 g\sin\alpha$

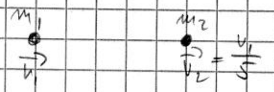
$+ \; m_2 a = T - 40 - \mu m_2 g\cos\alpha - m_2 g\sin\alpha$

$(m_1 + m_2)a = 20 - \mu g\cos\alpha(m_1 + m_2) - g\sin\alpha(m_1 + m_2)$

$a = 0.705\,m/s^2$

$T = 45\,N$

3)



$m_1 v_1 + m_2 v_2 = m_1 v_1' + m_2 v_2'$ ($v_1' = 0$)

$m_1 v_1 + m_2\frac{v_1}{5} = m_2 v_2'$

$v_2' = \frac{m_1 v_1 + m_2\frac{v_1}{5}}{m_2}$

$\frac{m_1 v_1^2}{2} + \frac{m_2 v_2^2}{2} = \frac{m_2 v_2'^2}{2}$

$m_1 v_1^2 + m_2 v_2^2 = m_2\frac{\left(\frac{5m_1 v_1 + m_2 v_1}{5}\right)^2}{m_2^2}$

$m_1 v_1^2 + m_2\frac{v_1^2}{25} = \frac{25m_1^2 v_1^2 + 10m_1 m_2 v_1^2 + m_2^2 v_2^2}{25 m_2}$

$\frac{v_1^2(25m_1 + m_2)}{25} = \frac{25m_1^2 m_2 v_1^2 + 10m_1 m_2^2 v_1^2 + m_2^3 v_1^2}{25}$

$v_1^2(25m_1 + m_2) = v_1^2(25m_1^2 m_2 + 10m_1 m_2^2 + m_2^3)$

$\frac{m_1}{m_2} = \frac{3}{5}$

1. NASS

2) $v_k = 80\ km/h = 22{,}22\ m/s$

$m = 1\ t = 1000\ kg$

$s = 100\ m$

$\mu = 0{,}2$

$v_k^2 - v_0^2 = 2as$ → JEDN. UBRZ.

$v_k^2 = 2as$

$a = \frac{v_k^2}{2s} = 2{,}4686\ m/s^2$

$P = \frac{W}{t} = \frac{F \cdot s}{t} = F_m \cdot v = 4430{,}6 \cdot 22{,}22 = 98{,}46\ kW$

$F_m = F_H + F_a = \mu mg + ma = 4430{,}6$