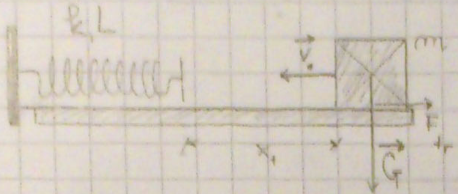
3. domaća zadaća iz Fizike 1

[redacted]

a)



$L = 1{,}5\,m$

$k = 100\,Nm^{-1}$

$x_1 = 50\,cm = 0{,}5\,m$

$m = 1\,kg$

$v_0 = 8\,m/s$

$\mu = 0{,}6$

$x_0 = ?$

Prema zakonu o očuvanju energije:

$$E_k = E_{el} + W_{tr}$$

$$\frac{mv_0^2}{2} = \frac{kx_0^2}{2} + \mu mg(x_1 + x_0) \quad \Big/ \cdot 2$$

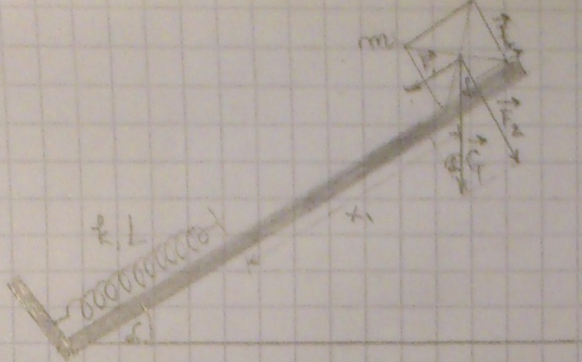
$$kx_0^2 + 2\mu mgx_0 + 2\mu mgx_1 - mv_0^2 = 0$$

$$x_0 = \frac{-2\mu mg \pm \sqrt{4\mu^2 m^2 g^2 - 4k(2\mu mgx_1 - mv_0^2)m}}{2k} =$$

$$= \frac{-2 \cdot 0{,}6 \cdot 1 \cdot 9{,}81 \pm \sqrt{4 \cdot 0{,}6^2 \cdot 1^2 \cdot 9{,}81^2 - 4 \cdot 100 \cdot (2 \cdot 0{,}6 \cdot 1 \cdot 9{,}81 \cdot 0{,}5 - 1 \cdot 8^2) \cdot 1}}{2 \cdot 100}\,m$$

$$= \frac{-11.772 \pm 152.919}{200}\,m$$

$$\Rightarrow \quad x_0 = 0.706\,m$$



α za $x_0' = 1{,}2x_0$?

$|\vec{F}_1| = |\vec{G}| \sin\alpha$

$|\vec{F}_2| = |\vec{G}| \cos\alpha$

Prema zakonu očuvanju energije:

$$E_k + E_p = E_{el} + W_{tr} + E_p'$$

$$\frac{mv_0^2}{2} + (x_1 + L)\, mg \sin\alpha = \frac{k(x_0')^2}{2} + \mu mg\cos\alpha\,(x_1 + x_0') + mg\sin\alpha\,(L - x_0')$$

$$mv_0^2 + 2\sin\alpha\,(x_1 + L)\, mg = (1{,}2)^2 k x_0^2 + 2\cos\alpha\,\mu mg(x_1 + 1{,}2x_0) + 2\sin\alpha\, mg(L - 1{,}2x_0)$$

$$2\sin\alpha\, mg(x_1 + \cancel{L} - \cancel{L} + 1{,}2x_0) - 2\cos\alpha\,\mu mg(x_1 + 1{,}2x_0) = (1{,}2)^2 k x_0^2 - mv_0^2$$

$$\sin\alpha = \frac{2\sin\frac{\alpha}{2}\cos\frac{\alpha}{2}}{\sin^2\frac{\alpha}{2} + \cos^2\frac{\alpha}{2}} = \frac{2\,tg\frac{\alpha}{2}}{tg^2\frac{\alpha}{2} + 1} \qquad \cos\alpha = \frac{\cos^2\frac{\alpha}{2} - \sin^2\frac{\alpha}{2}}{\sin^2\frac{\alpha}{2} + \cos^2\frac{\alpha}{2}} = \frac{1 - tg^2\frac{\alpha}{2}}{1 + tg^2\frac{\alpha}{2}}$$

$$\Rightarrow 4\,tg\frac{\alpha}{2}\, mg(x_1 + 1{,}2x_0) - 2\mu mg(x_1 + 1{,}2x_0) + 2\,tg^2\frac{\alpha}{2}\,\mu mg(x_1 + 1{,}2x_0) =$$

$$= (1{,}2)^2 k x_0^2 - mv_0^2 + tg^2\frac{\alpha}{2}\left[(1{,}2)^2 k x_0^2 - mv_0^2\right]$$

$$tg^2\frac{\alpha}{2}\left[2\mu mg(x_1 + 1{,}2x_0) - (1{,}2)^2 k x_0^2 + mv_0^2\right] + 4\,tg\frac{\alpha}{2}\, mg(x_1 + 1{,}2x_0)$$

$$- 2\mu mg(x_1 + 1{,}2x_0) - (1{,}2)^2 k x_0^2 + mv_0^2 = 0$$

$$8.084\, tg^2\frac{\alpha}{2} + 52.864\, tg\frac{\alpha}{2} - 23.634 = 0$$

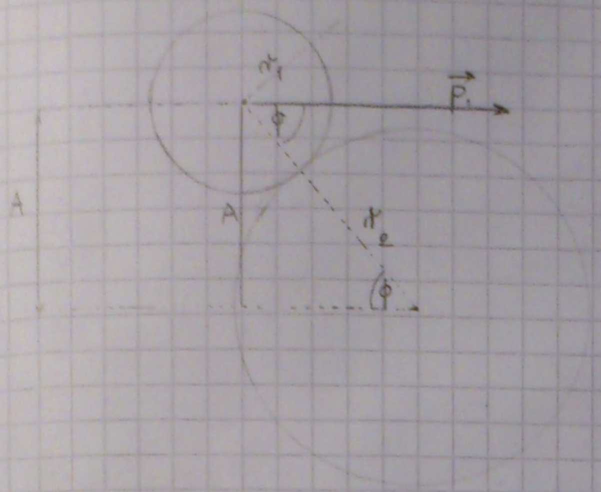
$$\Rightarrow tg\frac{\alpha}{2} = 0.420 \text{ ili } 6.958$$

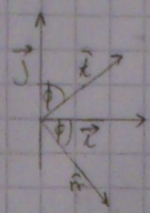
$$\Rightarrow tg\frac{\alpha}{2} = 0{,}420$$

$$\frac{\alpha}{2} = 22.782\,, \quad \alpha = 45.565^\circ$$

2

$$\boxed{\sin\phi = \frac{A}{r_1+r_2}}$$





$\hat{t} = \sin\phi\,\vec{i} + \cos\phi\,\vec{j}$

$\hat{n} = \cos\phi\,\vec{i} - \sin\phi\,\vec{j}$

$\hat{n}$ i $\hat{t}$ su jedinični vektori

$\hat{n}$ - u smjeru kuta ϕ,

$\hat{t} \perp \hat{n}$.

Prije sraza:

$\vec{p}_1 = m\vec{v}_1 = m_1 v_1 \vec{i}$

$\vec{p}_2 = 0$

$$\begin{cases} \hat{t} = \sin\phi\,\vec{i} + \cos\phi\,\vec{j} \\ \hat{n} = \cos\phi\,\vec{i} - \sin\phi\,\vec{j} \end{cases}$$

$$\Rightarrow \vec{i} = \frac{\hat{t} - \cos\phi\,\vec{j}}{\sin\phi},$$

$$\vec{j} = \frac{\cos\phi\,\vec{i} - \hat{n}}{\sin\phi}$$

$$* \quad \frac{1}{1+ctg^2\phi} = \frac{1}{1+\frac{\cos^2\phi}{\sin^2\phi}} = \frac{\sin^2\phi}{1} = \sin^2\phi$$

$$\vec{i} = \frac{\hat{t}}{\sin\phi} - ctg\phi\left(ctg\phi\,\vec{i} - \frac{\hat{n}}{\sin\phi}\right)$$

$$* \quad (1+ctg^2\phi)\vec{i} = \frac{\hat{t}}{\sin\phi} + \frac{ctg\phi\,\hat{n}}{\sin\phi} \quad \Big/ \cdot \frac{1}{1+ctg^2\phi}$$

$$\Rightarrow \vec{i} = \sin\phi\,\hat{t} + \cos\phi\,\hat{n}$$

$$\vec{p}_1 = m v_1 \vec{i} = m_1 v_1 \sin\phi\,\hat{t} + m_1 v_1 \cos\phi\,\hat{n} =$$

$$\vec{p}_2 = 0. = 0\,\hat{t} + 0\,\hat{n}$$

ne mijenja se nakon sraza — mijenja se nakon sraza

$$v_1 \cos\phi = v_{1,n}$$

$$v'_{1,n} = \frac{v_{1,n}(m_1 - m_2)}{m_1 + m_2} = \frac{v_1 \cos\phi\,(m_1 - m_2)}{m_1 + m_2}$$

$$v'_{2,n} = \frac{2m_1 v_{1,n}}{m_1+m_2} = \frac{2m_1 v_1 \cos\phi}{m_1+m_2}$$

$\underbrace{\text{pri §}}$

$$\Rightarrow \vec{p_1} = m_1\vec{v_1} = m_1 v_1 \sin\phi\,\hat{t} + m_1 v_1 \cos\phi\,\hat{n}$$

$$\vec{p_2} = 0$$

nakon

$$\vec{p_1'} = m_1 v_1 \sin\phi\,\hat{t} + \frac{m_1^2 v_1 \cos\phi(m_1-m_2)}{m_1+m_2}\hat{n}$$

$$\vec{p_2'} = 0\hat{t} + \frac{2m_1 m_2 v_1 \cos\phi}{m_1+m_2}\hat{n}$$

$$\vec{p_1'} = m_1 v_1 \sin^2\phi\,\vec{i} + m_1 v_1 \sin\phi\cos\phi\,\vec{j} + \frac{m_1 v_1 \cos^2\phi}{m_1+m_2}(m_1-m_2)\vec{i} - \frac{m_1 v_1 \sin\phi\cos\phi(m_1-m_2)}{m_1+m_2}\vec{j}$$

$$\vec{p_1'} = \left(p_1 \sin^2\phi + \frac{(m_1-m_2)}{m_1+m_2} p_1 \cos^2\phi\right)\vec{i} + \left(\frac{1}{2}p_1 \sin 2\phi - \frac{\sin 2\phi}{2(m_1+m_2)}(m_1-m_2)p_1\right)\vec{j}$$

$$\vec{p_2'} = p_1\left(\frac{2m_2\cos\phi}{m_1+m_2}\right)\left(\cos\phi\,\vec{i} - \sin\phi\,\vec{j}\right)$$

$$\Rightarrow \vec{p'_{1,i}} = \left(m_1 v_1 \sin^2\phi + \frac{m_1-m_2}{m_1+m_2} m_1 v_1 \cos^2\phi\right)\vec{i} = \left(\sin^2\phi + \frac{m_1-m_2}{m_1+m_2}\cos^2\phi\right)p_1\vec{i}$$

$$\vec{p'_{2,i}} = \frac{2m_2\cos^2\phi}{m_1+m_2} m_1 v_1 \vec{i} = \frac{2m_2\cos^2\phi}{m_1+m_2} p_1 \vec{i}$$

slučaj (a)

$A = R$

$r_1 = R$

$m_2 = 2m_1$

$$\sin\phi = \frac{A}{r_1+r_2} = \frac{R}{R+r_2} \qquad \cos^2\phi = 1 - \frac{R^2}{(R+r_2)^2} = \frac{r_2(2R+r_2)}{(R+r_2)^2}$$

$$\underbrace{\vec{p'_{1,i}}} = \left(\sin^2\phi + \frac{m_1-2m_1}{m_1+2m_1}\cos^2\phi\right)p_1\vec{i} =$$

$$= \left(1 + \frac{-m_1-m_1-2m_1}{3m_1}\cos^2\phi\right)p_1\vec{i} =$$

$$= \left(1 - \frac{4}{3}\cos^2\phi\right)p_1\vec{i}$$

$$\overrightarrow{p'_{2,i}} = \frac{2m_2\cos^2\phi}{m_1+m_2}\, p_1\vec{i} = \frac{4m_1}{3m_1}\cos^2\phi\, p_1\vec{i} = \frac{4}{3}\cos^2\phi\, p_1\vec{i}$$

$$\bullet \quad \cos^2\phi = \frac{r_2(R+R+r_2)}{(R+r_2)^2} = \frac{r_2\left(R+\frac{R}{\sin\phi}\right)}{\frac{R^2}{\sin^2\phi}} = \frac{r_2(\sin\phi+1)}{R\sin\phi}$$

slučaj (b)

$A = 2R$

$r_2 = 2R$

$m_2 = 4m_1$

$$\sin\phi = \frac{A}{r_1+r_2} = \frac{2R}{r_1+2R} \qquad \cos^2\phi = 1-\sin^2\phi = \frac{r_1(r_1+4R)}{(r_1+2R)^2}$$

$$\cos^2\phi = \frac{r_1\left(2R+\frac{2R}{\sin\phi}\right)}{\frac{4R^2}{\sin^2\phi}} = \frac{r_1(\sin\phi+1)}{2R\sin\phi}$$

$$\overrightarrow{p'_{1,i}} = \left(1-\cos^2\phi+\frac{m_1-m_2}{m_1+m_2}\cos^2\phi\right)p_1\vec{i} =$$

$$= \left(1+\frac{\cancel{m_1}-m_2-\cancel{m_1}-m_2}{m_1+m_2}\cos^2\phi\right)p_1\vec{i} = \left(1-\frac{8}{5}\cos^2\phi\right)p_1\vec{i}$$

$$\overrightarrow{p'_{2,i}} = \frac{2m_2\cos^2\phi}{m_1+m_2}\, p_1\vec{i} = \frac{8}{5}\cos^2\phi\, p_1\vec{i}$$