19.03.2010.

Fizika lasera

Oblik i širina spektralne linije

- jezgra linije i krila linije

- zbog relacija neodređenosti je širina linije različita

(1.) Prirodna širina linije - Lorentzov oblik

$$I(\omega-\omega_0) = I_0 \frac{(\gamma/2)^2}{(\omega-\omega_0)^2 + (\gamma/2)^2}$$

$$\Delta E = \Delta \nu \cdot h = \frac{\hbar}{\Delta t} \qquad \Delta \nu = \frac{1}{2\pi \cdot \Delta t}$$

(2) Širenje linija zbog gibanja - Dopplerov oblik

$$I(\omega) = I_0 \cdot e^{-a\frac{(\omega-\omega_0)^2}{\omega_0^2}}$$

$$a = \frac{Mc^2}{2kT} \qquad \Delta\omega = 2\omega_0 \sqrt{\frac{\ln 2}{a}}$$

$$\Delta\lambda = \frac{hc \frac{\hbar}{\Delta t}}{\left(\frac{hc}{\lambda}\right)^2} = \frac{\frac{h}{2\pi\Delta t}}{\frac{h^2c}{\lambda^2}}$$

$$\Delta\lambda = \frac{\lambda^2}{2\pi c \Delta t} \checkmark$$

6. Zadaci

(1.) $\lambda = 632{,}8\ nm$

$\Delta t = 10^{-8}\ s$

$\Delta\lambda = ?$

$\Delta\nu$

$$\Delta E \cdot \Delta t \approx \hbar \qquad \hbar = \frac{h}{2\pi}$$

$$E = h \cdot \nu$$

$$\Delta E = h \cdot \Delta\nu$$

$$\cancel{h} \cdot \Delta\nu \cdot \Delta t = \frac{\cancel{h}}{2\pi}$$

$$\Rightarrow \Delta\nu = \frac{1}{2\pi\Delta t}$$

$$\nu \cdot \lambda = c$$

$$E = h \cdot \frac{c}{\lambda} \qquad \lambda = \frac{hc}{E}$$

$$d\lambda = hc\left(-\frac{dE}{E^2}\right)$$

apsolutna vrijednost $|d\lambda| \approx \Delta\lambda$

$dE = \Delta E$

$$\Delta\lambda = hc \frac{\Delta E}{E^2} = \frac{hc}{\left(h\frac{c}{\lambda}\right)^2} \cdot \frac{\hbar}{\Delta t} = \frac{\lambda^2}{2\pi c \Delta t}$$

$$\hbar = \frac{h}{2\pi}$$

$$= 2{,}12 \cdot 10^{-14}\ m \ //$$

$c = 3 \cdot 10^8\ m/s$

a) vidljivi dio spektra :

Na $\lambda = 589,1$ nm

$\Delta t = 16$ ns

$\Delta\nu = ?$

$$\Delta\nu = \frac{1}{2\pi\Delta t} = 9,95 \cdot 10^{6} \ s^{-1}$$

b) IR $\Delta t = 10^{-3}$ s (vibracijski spektar)

vrijeme života

toplina → od titranja

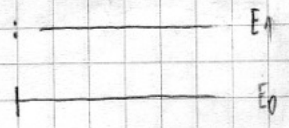
$$\Delta\nu = \frac{1}{2\pi\Delta t} = 159,2 \ s^{-1}$$ $\approx 10^{4}$ puta manje od a)

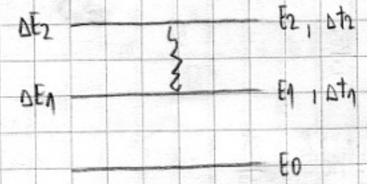
c) UV područje

$\Delta t = 8,23$ s

$\Delta\nu = 0,02 \ s^{-1}$

što su kraće valne duljine, širine su manje !





$\Delta E = \Delta E_1 + \Delta E_2$

$$\Delta\nu = \frac{1}{2\pi\Delta t_1} + \frac{1}{2\pi\Delta t_2}$$

prijelazi između dva pobuđena stanja

$\Delta t_1 = 1{,}2 \cdot 10^{-8}$ s

$\Delta t_2 = 2 \cdot 10^{-8}$ s

$\Delta \lambda = ?$

$\lambda \cdot \nu = c$

$\lambda = \frac{c}{\nu}$

$d\lambda = c \cdot \frac{d\nu}{\nu^2}$

$d\lambda \approx \Delta\lambda$

$|d\nu| \approx \Delta\nu$

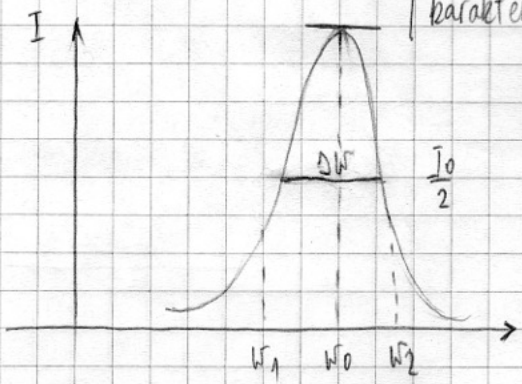
$$\Delta\lambda = c \cdot \frac{\Delta\nu}{\left(\frac{c}{\lambda}\right)^2}$$

$$\Delta\lambda = \frac{\lambda^2}{2\pi c}\left(\frac{1}{\Delta t_1} + \frac{1}{\Delta t_2}\right)$$

$\Delta\lambda = 2 \cdot 10^{-14}$ m

(8) $$I(\omega) = I_0 \frac{(\gamma/2)^2}{(\omega-\omega_0)^2 + (\gamma/2)^2}$$

konstanta prigušenja (karakteristika materijala)



$\Delta\omega = \omega_2 - \omega_1$

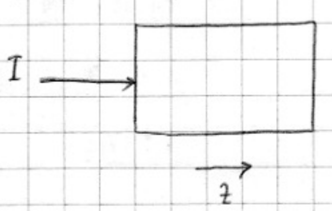
$$\Rightarrow \frac{1}{2} I_0 = I_0 \frac{(\gamma/2)^2}{(\omega-\omega_0)^2 + (\gamma/2)^2}$$

$$(\omega-\omega_0)^2 + \left(\frac{\gamma}{2}\right)^2 = 2\left(\frac{\gamma}{2}\right)^2$$

$$(\omega-\omega_0)^2 = \left(\frac{\gamma}{2}\right)^2$$

$$\omega_{1/2} = \omega_0 \pm \frac{\gamma}{2} \Rightarrow \Delta\omega = \omega_2 - \omega_1 = \gamma$$

apsorpcija



smjer širenja ravnog vala

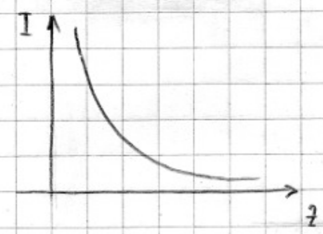
$$dI = -\alpha I \, dz$$

koeficijent apsorpcije

$$\frac{dI}{I} = -\alpha \, dz$$

$$I = I_0 e^{-\alpha(\omega)\cdot z}$$

Beerov zakon



- vibracije se događaju samo ako imamo neku periodičnu silu (titranje)

titranje

$F = q \cdot E$ električno polje

$F = qE_0 e^{i\omega t}$ (periodično el. polje)

$$m\frac{d^2x}{dt^2} + b\frac{dx}{dt} + kx = qE_0 e^{i\omega t}$$

$$x(t) = \frac{qE_0 e^{i\omega t}}{m(\omega_0^2 - \omega + i\gamma\omega)}$$

$\gamma = \frac{b}{m}$ $\omega_0 = \frac{k}{m}$

indeks loma sredstva

$$\vec{P} = \varepsilon_0(\varepsilon_r - 1)\vec{E} = \varepsilon_0 \chi_e \vec{E}$$

$n = \sqrt{\varepsilon_r}$

relativna dielektrična konstanta

$p = q \cdot x$

$P = N \cdot p = N \cdot q \cdot x$

polarizacija

$$P = N \cdot q \cdot \frac{qE_0 e^{i\omega t}}{m(\omega_0^2 - \omega + i\gamma\omega)} = \varepsilon_0(n^2 - 1)E_0 e^{i\omega t}$$

$$n^2 = 1 + \frac{Nq^2}{\varepsilon_0 m(\omega_0^2 - \omega + i\gamma\omega)}$$

$$n^2 - 1 = (n-1)(n+1) \approx 2(n-1)$$

$$n \approx 1$$

$$n = 1 + \frac{Nq^2}{2\varepsilon_0 m (\omega_0^2 - \omega + i\gamma\omega)}$$

$$n = n' - i\mathcal{K}$$

EM val: $E = E_0 e^{i(\underbrace{\omega t - kz})}$

$$k = |\vec{k}|$$

$$k = \frac{2\pi}{\lambda}$$

$$\lambda = \frac{\lambda_0}{n}$$

$$\frac{2\pi n}{\lambda_0} = k$$

$$k = k_0 \cdot n$$

$$E = E_0 e^{i[\omega t - k_0(n' - i\mathcal{K})z]} = \underbrace{E_0 e^{-k_0 \mathcal{K} z}}_{\text{apsorpcija}} \cdot \underbrace{e^{i(\omega t - k_0 n' z)}}_{\text{disperzija}}$$

$$I \sim E^2 \Rightarrow I = I_0 e^{-2k_0\mathcal{K}t} = I_0 e^{-\alpha(\omega)z}$$

$$\Rightarrow \alpha = 2k_0\mathcal{K}$$

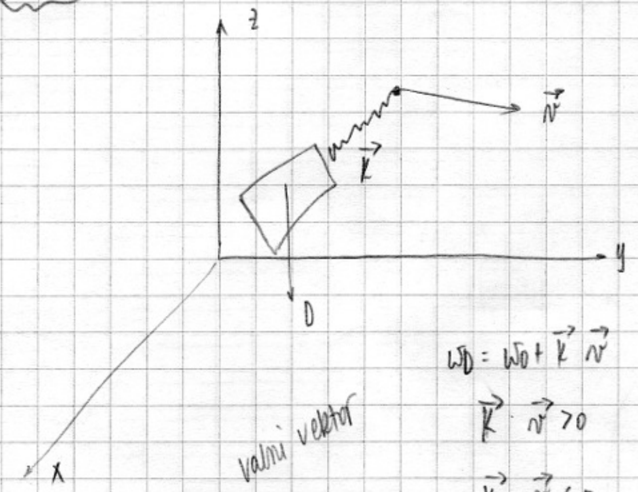
$$\alpha = \frac{4\pi\mathcal{K}}{\lambda_0}$$

$$\alpha = \frac{Nq^2\omega_0}{c\varepsilon_0 m} \frac{\gamma\omega}{(\omega_0 - \omega)^2 + \gamma^2\omega^2} \qquad |\omega_0 - \omega| \ll \omega_0$$

$$\alpha(\omega) = \frac{Ne^2\pi}{4\varepsilon_0 mc} \frac{\frac{\gamma}{2\pi}}{(\omega_0 - \omega)^2 + \left(\frac{\gamma}{2}\right)^2}$$

Lorentzov oblik

Dopplerova širina



$$\omega_D = \omega_0 + \vec{k}\,\vec{v}$$

$\vec{k}\cdot\vec{v} > 0$ približen

$\vec{k}\cdot\vec{v} < 0$ udaljen

valni vektor

$$n_j(v)\,dv \sim e^{-\frac{E}{kT}} = e^{-\frac{mv^2}{2kT}}$$

Maxwell

$$v_p = \sqrt{\frac{2kT}{m}}$$ najvjerojatnije

$$n_j(v)\,dv \sim e^{-\left(\frac{v}{v_p}\right)^2}$$

$\vec{k} \parallel \vec{v}$

$$\omega - \omega_0 = k\cdot v = \frac{v}{c}\,\omega_0$$

$$k = \frac{2\pi}{\lambda} = \frac{2\pi\nu}{c} = \frac{\omega_0}{c}$$

$$n_j(v)\,dv \sim e^{-\left[\frac{1}{v_p}\cdot\frac{(\omega-\omega_0)c}{\omega_0}\right]^2}$$

$$I(\omega) = I_0\, e^{-\left[\frac{c(\omega-\omega_0)}{\omega_0 v_p}\right]^2}$$

Gama

Zadaci

(10.) $I(\omega) = I_0 \, e^{-a \frac{(\omega-\omega_0)^2}{\omega_0^2}}$

$$a = \frac{c^2}{v_p^2} = \frac{c^2 m}{2kT}$$

Boltzmanova konstanta

$$I(\omega) = \frac{I_0}{2} \Rightarrow \frac{\cancel{I_0}}{2} = \cancel{I_0} e^{-a \frac{(\omega-\omega_0)^2}{\omega_0^2}} \quad \Big/ \ln$$

$$(\omega-\omega_0)^2 = \frac{\omega_0^2}{a} \ln 2$$

$$\omega_{1/2} = \omega_0 \pm \omega_0 \sqrt{\frac{\ln 2}{a}}$$

$$\Delta\omega = \omega_2 - \omega_1 = 2\omega_0 \sqrt{\frac{\ln 2}{a}}$$

(12) $\Delta t = 1{,}5 \cdot 10^{-7}\,s$

$\frac{\Delta\lambda_D}{\Delta\lambda} = ?$

$\lambda = 253{,}65\ nm$

$T = 300\,K$

$M = 200{,}59$

$$\Delta\lambda = \frac{\lambda^2}{2\pi c \Delta t}$$

$$\Delta\lambda_D = \frac{\lambda^2 \Delta\nu_D}{c} \qquad \frac{\Delta\omega_D}{2\pi}$$

$$\omega = 2\pi\nu$$

$$\Delta\omega = 2\pi \Delta\nu$$

$$\frac{\Delta\lambda_D}{\Delta\lambda} = \frac{\frac{\lambda^2 \Delta\nu_D}{c}}{\frac{\lambda^2}{2\pi c \Delta t}} = \Delta t \cdot \Delta\omega_D = \Delta t \cdot 2\omega_0 \sqrt{\frac{\ln 2}{a}}$$

$$2\pi \Delta\nu = \frac{2\pi c}{\lambda}$$

$$a = \frac{mc^2}{2kT}$$

$$m = M \cdot u$$

$$a = \frac{200{,}59 \cdot 1{,}67 \cdot 10^{-27} \cdot (3 \cdot 10^8)^2}{2 \cdot 1{,}38 \cdot 10^{-23} \cdot 300}$$