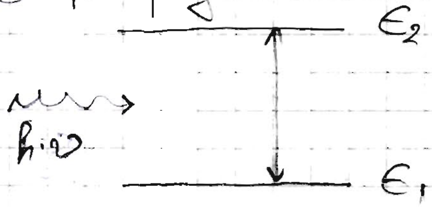
04. 03. 2010.

$$\Delta E \cdot \Delta t \geq h$$

$$-\frac{h^2}{2m}\Delta\psi + E_p\psi = E\psi$$

EMISIJA I APSORPCIJA SVJETLOSTI

1. apsorpcija



$E = h \cdot \nu$

Planckova konst.

$E_2 = E_1 + h \cdot \nu$

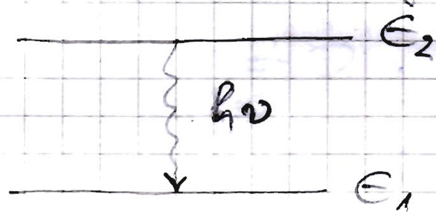
Vjerojatnost apsorpcije

$$\frac{dP_{12}^{aps.}}{dt} = B_{12}\, u(\nu)$$

B_{12} - Einsteinov koeficijent apsorpcije

$u(\nu)$ - gustoća energije zračenja

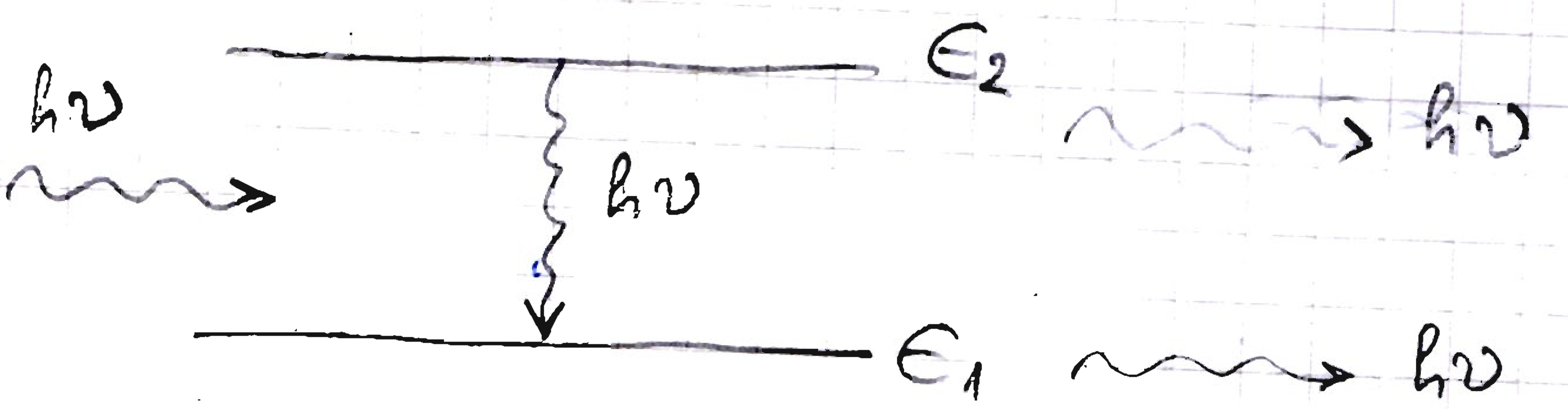
2. spontana emisija



$$\frac{dP_{21}^{spon.}}{dt} = A_{21}$$

A_{21} - Einsteinov koeficijent spontane emisije

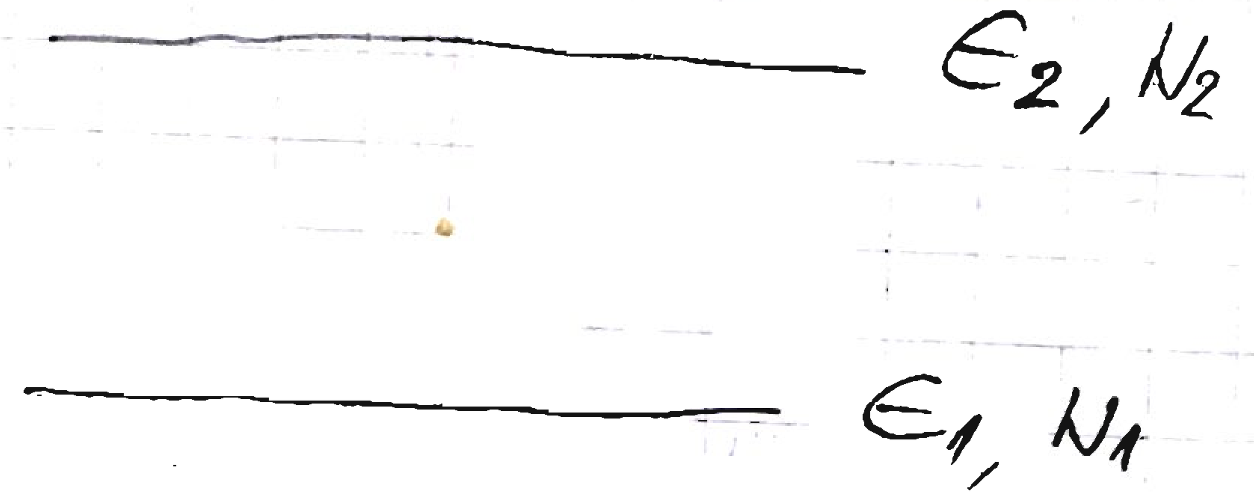
③ stimulirana emisija



$$\frac{dP_{21}^{st.\,em.}}{dt} = B_{21}\, u(\nu)$$

$B_{21}\, u(\nu)$ – Einsteinov koeficijent stimulirane emisije

---



①

$N_1 \rightarrow N_2$

$$E_1 \rightarrow E_2 \Longrightarrow N_1 B_{12}\, u(\nu)$$

$$E_2 \rightarrow E_1 \Longrightarrow N_2 \left(A_{21} + B_{21}\, u(\nu)\right).$$

ravnoteža

$$N_1 B_{12}\, u(\nu) = N_2 \left(A_{21} + B_{21}\, u(\nu)\right)$$

$$N_2 = N_1 e^{-\frac{h\nu}{kT}}$$ Boltzmanova raspodjela (uzimamo zdravo za gotov

$$N_1 B_{12}\, u(\nu) = N_1 e^{-\frac{h\nu}{kT}} \left(A_{21} + B_{21}\, u(\nu)\right)$$

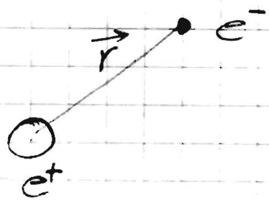
$$u(\nu) = \frac{A_{21}}{B_{12} e^{\frac{h\nu}{kT}} - B_{21}}$$

$$\boxed{u(\nu) = \frac{8\pi h\nu^3}{c^3} \cdot \frac{1}{e^{\frac{h\nu}{kT}} - 1}}$$ Planckov zakon

$\Rightarrow B_{12} = B_{21}$, tada imamo

$$\frac{A_{21}}{B_{21}} = \frac{8\pi h \nu^3}{c^3}$$

VJEROJATNOSTI PRIJELAZA



$\vec{p} = e \cdot \vec{r} \Rightarrow$ dipolni moment atoma

$\psi_i \rightarrow$ v.f

- srednja vrijednost dipolnog momenta : $\langle \vec{p} \rangle$

$$\langle \vec{p} \rangle = e \cdot \langle \vec{r} \rangle = e \cdot \int_V \psi_i^* \cdot \vec{r} \cdot \psi_i \, dV$$

$E_i\ (\psi_i) \longrightarrow E_k\ (\psi_k)$

- prijelazni dipolni moment : $\langle \vec{M}_{ik} \rangle$ (srednja vrijednost iz stanja i u stanje k)

$$\langle \vec{M}_{ik} \rangle = e \int_V \psi_i^* \vec{r}\, \psi_k \, dV$$

$A_{ik},\ B_{ik},\ B_{ki} \sim \langle \vec{M}_{ik} \rangle$

$\langle \vec{M}_{ik} \rangle \neq 0 \longrightarrow$ dozvoljeni

$\langle \vec{M}_{ik} \rangle \neq 0 \longrightarrow \Delta m = 0 \quad (\Delta m = \pm 1)$

$\Delta S = 0$ (L-S vezanje)

$\Delta J = \pm 1$ (j-j vezanje)

$\Delta L = \pm 1$

zadatak

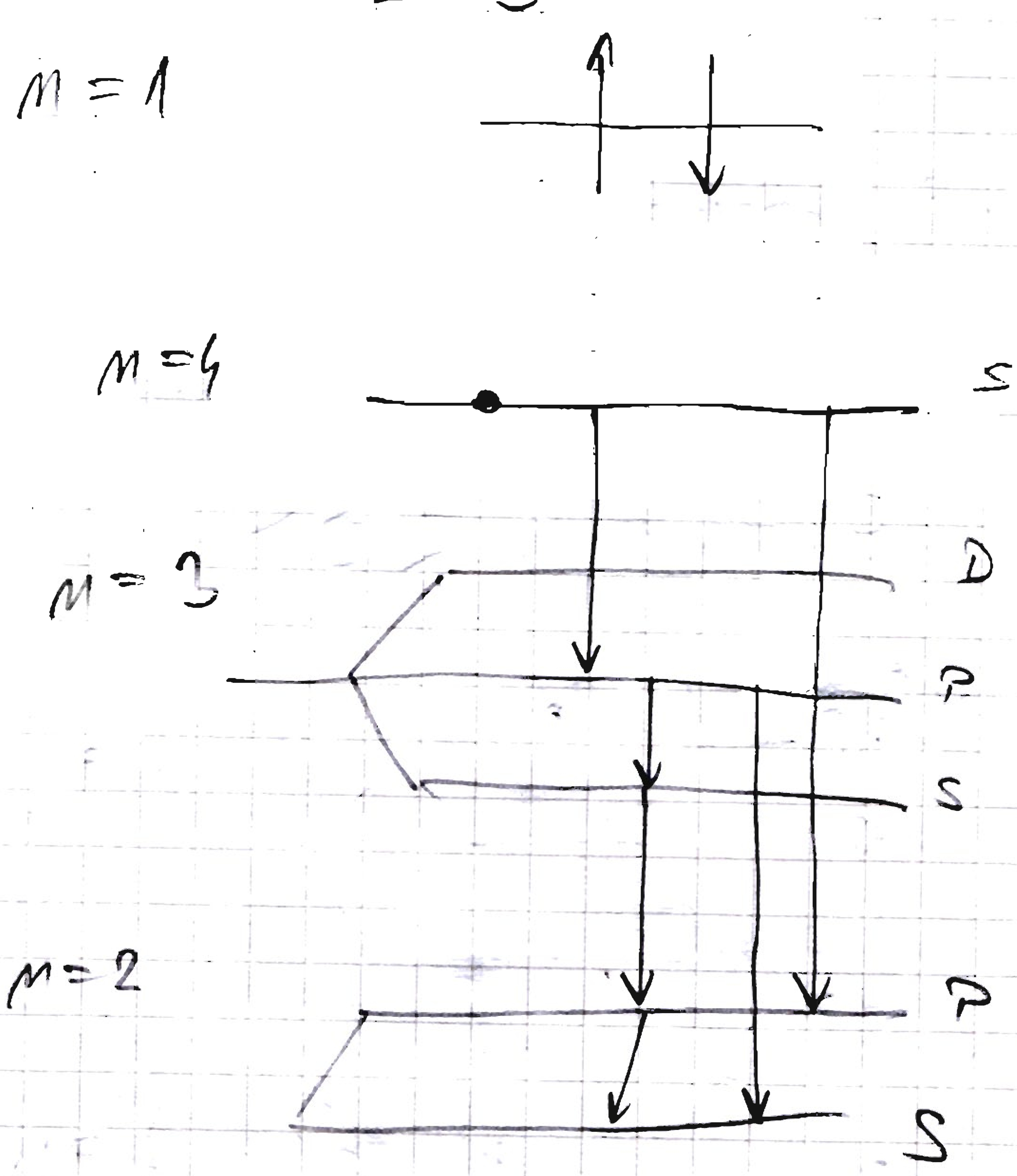
6. Li

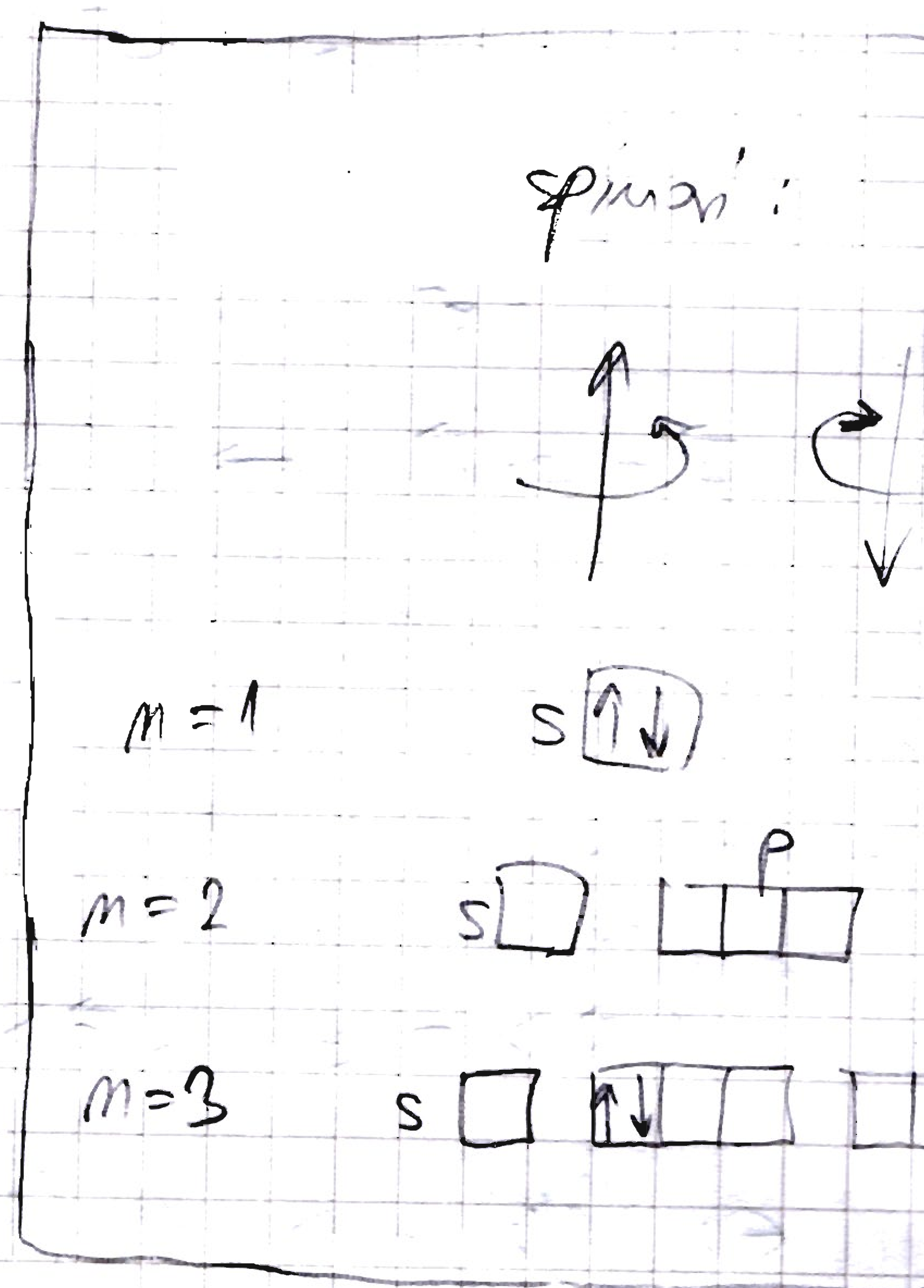
a) 4s   b) 4p

rješenje

Li → Z = 3

a)





$\Delta S = 0$

$\Delta L = \pm 1$

L	0	1	2
	S	P	D

dozvoljeni

S → P

P → D

zabranjeni

S → S

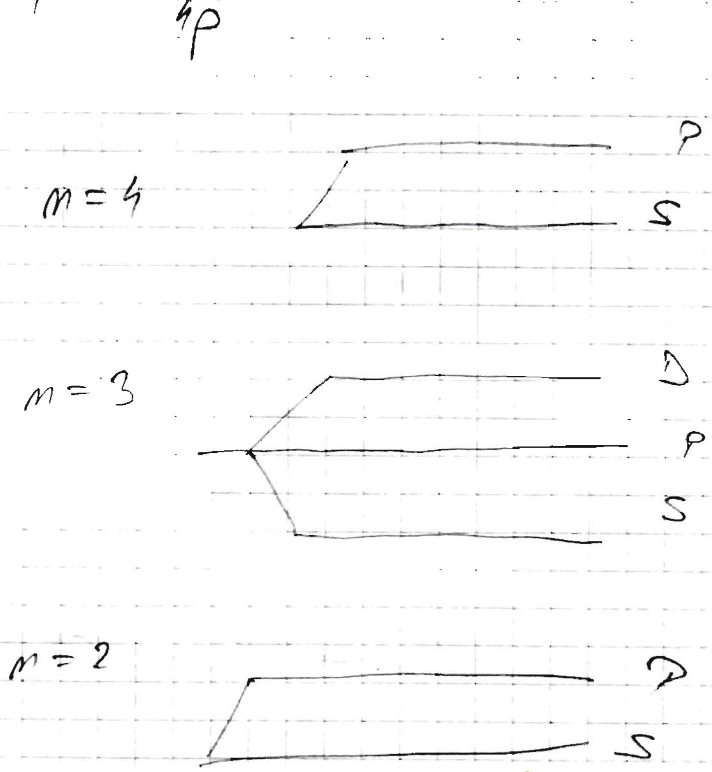
P → P

D → D

S → D

rješenje: 6 spektralnih linija

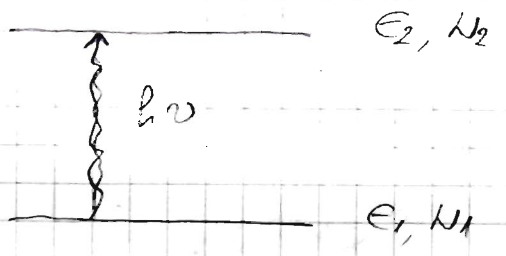
b)



rješenje: 12 spektralnih linija

11.03.2010.

VRIJEME ŽIVOTA POBUĐENOG STANJA



$$E_2 = E_1 + h\nu$$

dN_2 - promjena broja atoma

$$dN_2 = -A_{21} N_2 \, dt$$

minus jer je smanjenje broja atoma

$$\frac{dN_2}{N_2} = -A_{21} \, dt$$

$$N_2(t) = N_2(0) \, e^{-A_{21} t}$$

$$\boxed{\tau = \frac{1}{A_{21}}}$$

τ - srednje vrijeme života pobuđene čestice

τ - cca 10^{-8} s

- Ako radimo laser želimo da vrijeme života čestice bude što veće (trebamo dugoživuće čestice), τ cca 10^{-5} s.

(14.)

$\nu = 4{,}22 \cdot 10^{14}$ Hz

$\rho = 0{,}5$ Js m^{-3}

$\tau = 2$ ms

$A, B = ?$

$Q = ?$

$c = 3 \cdot 10^{8}$ m/s

$h = 6{,}626 \cdot 10^{-34}$ Js

$$A_{21} = \frac{1}{\tau} = \frac{1}{2 \cdot 10^{-3}} \; s^{-1}$$

$$B_{21} = \frac{A_{21} \cdot c^3}{8 \pi h \nu^3} = 6{,}7 \cdot 10^{15} \; J^{-1} s^{-2} m^{3}$$

$$Q = \frac{B_{21} \, u(\nu)}{A_{21}} = 10^{13}$$

Q - omjer stimuliranih i spontanih emisija (treba biti što veći)

15.

a) $T = 3000\ K$

b) $T = 300\ K$

$\lambda = 550\ nm$

$Q = ?$

$$Q = \frac{B_{21}\ u(\nu)}{A_{21}} = \frac{1}{e^{\frac{h\nu}{kT}} - 1}$$

$\boxed{\lambda \cdot \nu = c}$ ν - frekvencija

$\boxed{kT = E}$

$k = 1{,}38 \cdot 10^{-23}\ \frac{J}{K}$

$$Q = \frac{1}{e^{\frac{hc}{\lambda kT}} - 1}$$

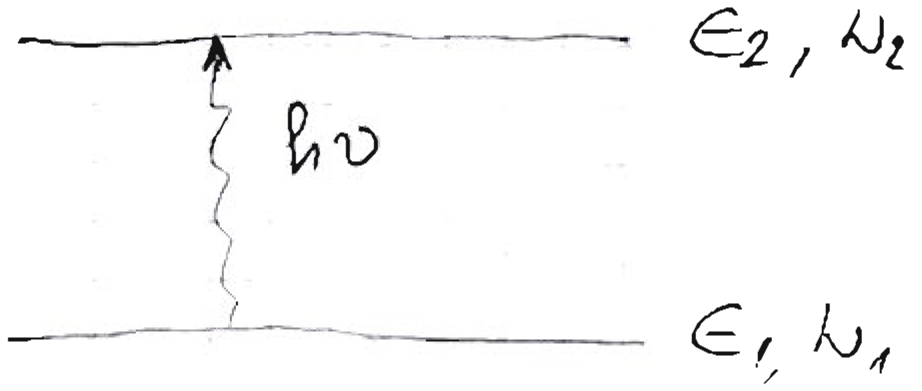
a) $T = 3000\ K$

$\underline{Q = 1{,}6 \cdot 10^{-4}}$

b) $T = 300\ K$

$\underline{Q = 1{,}22 \cdot 10^{-38}}$

$\Rightarrow$



① $N_1 \cdot B_{12}\, u(\nu) \cdot h\nu$ ⇒ apsorbirana energija

② $N_2 \cdot B_{21}\, u(\nu)\, h\nu$ ⇒ energija zračenja uslijed stimulirane emisije

$$B_{21} = B_{12} = B$$

$$\frac{du(\nu)}{dt} = B\, u(\nu)\, h\nu (N_2 - N_1)$$ → ② - ①

$$\frac{du(\nu)}{dt} = ?$$

$$\frac{d}{dt} = \frac{d}{dx}\underbrace{\frac{dx}{dt}}_{c} = c\frac{d}{dx}$$

$$c\,\frac{du(\nu)}{dx} = B\, u(\nu)\, h\nu (N_2 - N_1)$$

$$\frac{du(\nu)}{u(\nu)} = \frac{h\nu}{c} B (N_2 - N_1)\, dx = -\alpha\, dx$$

$$\boxed{u(\nu) = u_0 \cdot e^{-\alpha x}}$$

$-\alpha$ ⇒ koeficijent apsorpcije

$$-\alpha = \frac{h\nu}{c} B (N_2 - N_1)$$

- za $N_1 > N_2 \Rightarrow \alpha > 0$

$u(x) = u_0 e^{-\alpha x}$ => da bi imali pojačanje izraz u eksponentu mora biti pozitivan, tj. α mora biti negativan

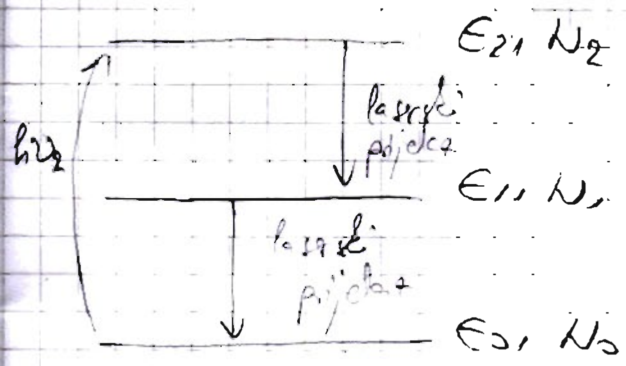
- kad bi α bio veći od 0 imali bi više apsorpcije od emisije
- za $N_2 > N_1 \Rightarrow \underline{\alpha < 0}$

$u(x) = u_0 e^{\alpha x}$ => pojačanje

$N_2 > N_1$ => INVERZIJA NASELJENOSTI /

- $N_2 > N_1$ više pobuđeno stanje

Kako postići inverziju naseljenosti?

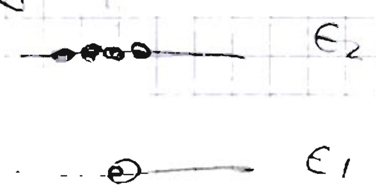


trebamo naštimati koji je kratko živući a koji dugoživući

$h\nu_2 = E_2 - E_0$

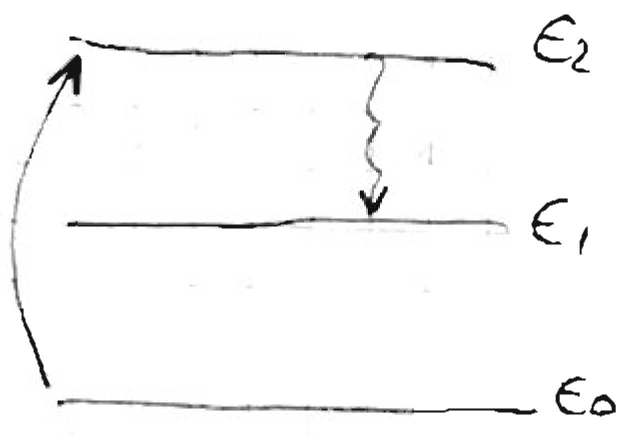
1. OPTIČKO PUMPANJE - prijelaz kod kojeg elektromagnetskim zračenjem izvana pobudimo atome (npr. bljeskalica)

- najčešće ga imamo kod lasera čvrstog stanja
- kad imamo inverziju naseljenosti možemo pojačati zračenje
- kod inverzije atomi se nalaze na višoj energetskoj razini.



inverzija naseljenosti

(2) POBUDA ELEKTRONIMA

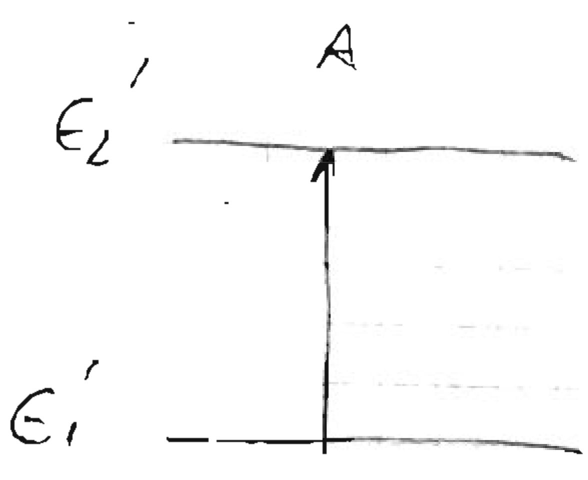


elektron $E_e = E_2 - E_0$

(3) SRAZ 2. REDA

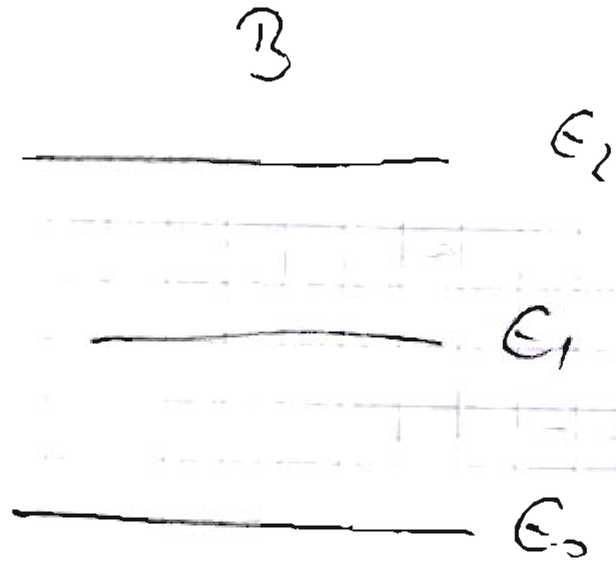
- He – Ne laser
- laser u sebi ima dva plina koji imaju bliske energijske nivoe

He – Ne laser
A B



$A + e \to A^*$

pobuđeni atom



$A^* + B = B^* + A$

- elektroni pobuđuju atome helija, atomi helija se sudaraju s atomima neona
- atomi helija se vraćaju u početno stanje, a atomi neona se pobuđuju
- helij se koristi samo za pobudu, a neon se koristi za emisiju

OBLIK I ŠIRINA SPEKTRALNE LINIJE

emisija

1. atomi miruju (ne gibaju se znatno)
 - u čvrstim tijelima
 - Prirodna ili LORENTZOVA širina linije

2. atomi u gibanju
 - Dopplerova ili Gaussova širina linije

(1.) Prirodna širina linije

ΔE - neodređenost u energiji
Δt - -||- u vremenu mirovanja

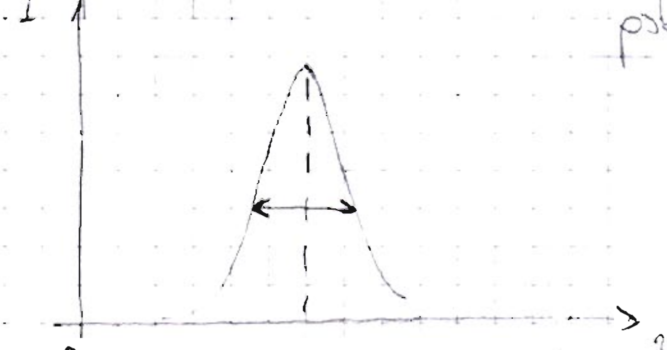
$$\Delta E \cdot \Delta t \approx \hbar \qquad \hbar = \frac{h}{2\pi}$$

$$E = h\nu$$

$$\Delta E = h \cdot \Delta \nu$$ $\Delta \nu$ - neodređenost u frekvenciji

$$h \cdot \Delta\nu \cdot \Delta t = \frac{h}{2\pi}$$

$$\Delta \nu = \frac{1}{2\pi \Delta t}$$ → aproksimativno vrijeme života pobuđenog stanja



- ukoliko ne možemo dobiti užu liniju od ovoga

što predviđaju zakoni neodređenosti
- ukad ne možemo dobiti diskretni (monokromatski) spektar

- gibanje elektrona oko jezgre možemo opisati pomoću prigušenog harmoničkog titranja

- prigušeno titranje :

$$ma = m\frac{d^2x}{dt^2} + kx = 0 \Rightarrow \text{idealni harm. oscilator}$$

elastična sila

$$F_{tr} = -bv = -b\frac{dx}{dt} \Rightarrow \text{prigušenje zbog trenja}$$

$$m\frac{d^2x}{dt^2} + b\frac{dx}{dt} + kx = 0$$

$$\frac{d^2x}{dt^2} + \underbrace{\frac{b}{m}}_{\gamma}\frac{dx}{dt} + \underbrace{\frac{k}{m}}_{\omega_0^2}x = 0 \Rightarrow \text{prigušeni oscilator}$$

γ - koeficijent prigušenja

ω_0^2 vlastita frekv.

- prigušenje tražimo u obliku $x(t) = e^{\lambda t}$

$$x(t) = e^{\lambda t}$$

$$\lambda^2 + \gamma\lambda + \omega_0^2 = 0$$

$$\lambda_{1,2} = \frac{1}{2}\left(-\gamma \pm \sqrt{\gamma^2 - 4\omega_0^2}\right)$$

- slabo prigušenje :

$$\gamma^2 < 4\omega_0^2$$

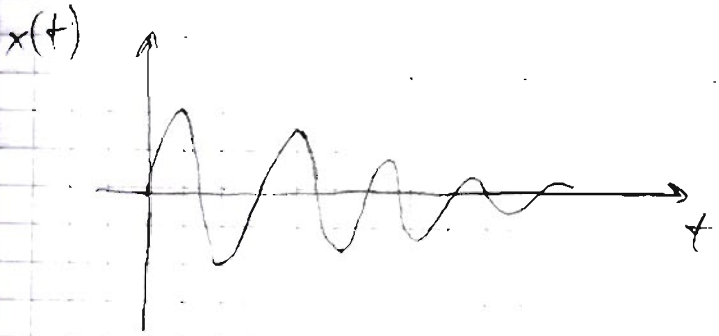
$$\sqrt{\gamma^2 - 4\omega_0^2} = i\omega$$

$$\lambda_{1,2} = \frac{1}{2}\left(-\gamma \pm i\omega\right)$$

$$x(t) = c_1 e^{\lambda_1 t} + c_2 e^{\lambda_2 t}$$

$$x(t) = e^{-\frac{\gamma}{2}t}\left[c_1 e^{i\omega t} + c_2 e^{-i\omega t}\right]$$

$$x(t) = x_0 \, e^{-\frac{\gamma}{2}t} \cos \omega t$$



$$A(\omega) = \frac{1}{\sqrt{2\pi}} \int_0^\infty x(t)\, e^{-i\omega t}\, dt$$

$$A(\omega) \sim E(\omega)$$

$I \Rightarrow$ intenzitet svjetlosnog vala

$$I \sim E^2$$

$$I(\omega) \sim A(\omega)\, A^*(\omega)$$

$$I(\omega) = \frac{c}{(\omega - \omega_0)^2 + \left(\frac{\gamma}{2}\right)^2}$$

ω_0 – vlastita frekv.

ω – frekv. prigušenih titraja

(1.) $I_0 = \int_0^\infty I(\omega)\,d\omega$ $\qquad c = \frac{I_0 \Gamma}{2\pi}$

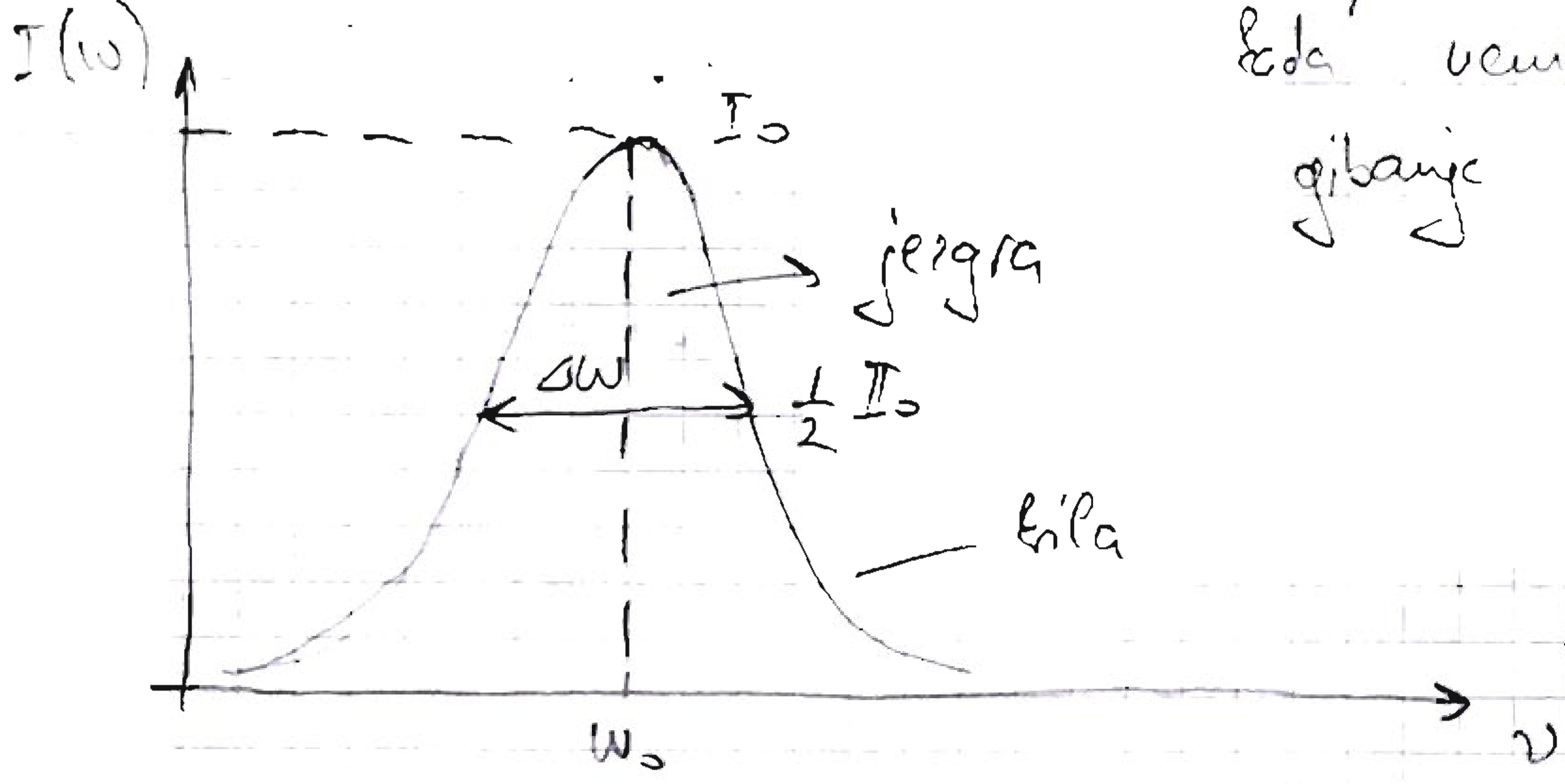
$$I(\omega) = \frac{I_0 \frac{\Gamma}{2\pi}}{(\omega-\omega_0)^2 + \left(\frac{\Gamma}{2}\right)^2}$$

(2.) $I_0 = I(\omega_0)$

$c = I_0 \left(\frac{\Gamma}{2}\right)^2$

$$I(\omega) = I_0 \frac{\left(\frac{\Gamma}{2}\right)^2}{(\omega-\omega_0)^2 + \left(\frac{\Gamma}{2}\right)^2}$$

→ Lorentzov oblik linije. Vrijedi kod emisije zračenja kada nemamo previše gibanje atoma.



$\Delta\omega$ – širina ili poluširina

18.02.2010.

zadatak 1.

$\lambda = 632,8$ nm

$\Delta t = 10^{-8}$ s

$h = 6,626 \cdot 10^{-34}$ Js

$c = 3 \cdot 10^{8}$ m/s

$\Delta E \cdot \Delta t = \hbar = \frac{h}{2\pi}$

$E = h \cdot \nu$

$\Delta E = h \cdot \Delta \nu$

$\Delta \nu, \Delta \lambda = ?$

$\lambda = \frac{c}{\nu} \Rightarrow \nu = \frac{c}{\lambda}$

$E = \frac{hc}{\lambda} \Rightarrow \lambda = \frac{hc}{E}$

$d\lambda = h \cdot c \left(- \frac{dE}{E^2} \right)$

$|d\lambda| = \Delta \lambda$

$dE = \Delta E$

$\Delta \lambda = hc \, \frac{\Delta E}{E^2}$

$= \cancel{hc} \cdot \frac{\frac{h}{2\pi \Delta t}}{\frac{\cancel{h^2} c^2}{\lambda^2}} = \frac{\lambda^2}{2\pi c \Delta t} = 2,12 \cdot 10^{-14}$ m

zadatak 2.

a) $\lambda = 589,1$ nm

$\Delta t = 16$ ns

$\Delta \nu = ?$

$E = h \cdot \Delta \nu - \frac{\hbar}{\Delta t}$

$\lambda \cdot \nu = c$

$\Delta \nu = \frac{\frac{\cancel{h}}{2\pi}}{\cancel{h} \cdot \Delta t} = \frac{1}{2\pi \Delta t}$

$\Delta \nu = 9,95 \cdot 10^{6}$ s^{-1}

b) IR (infra crveno područje)

$\Delta t = 10^{-3}$ s

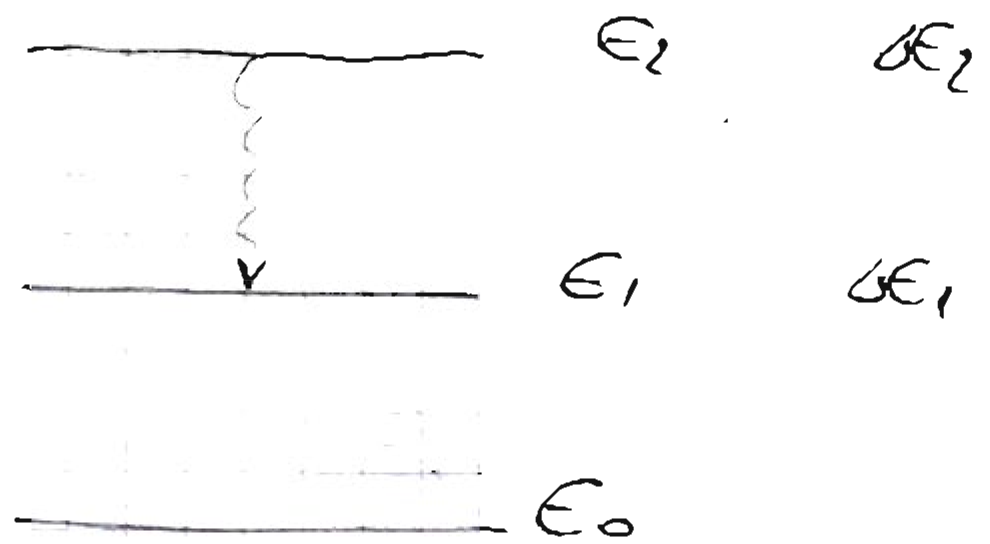
$$\Delta\nu = \frac{1}{2\pi\Delta t} = 159{,}2\ \underline{s^{-1}}$$

c) UV

$\Delta t = 8{,}23$ s

$\Delta\nu = 0{,}02\ \underline{s^{-1}}$

<u>4. i 5. zadatak</u>



Pri pojedinoj neodređenosti energetskih razina (ΔE_1 i ΔE_2) se zbrajaju

$$\Delta\nu = \Delta\nu_1 + \Delta\nu_2$$

$$\Delta\nu = \frac{1}{2\pi\Delta t_1} + \frac{1}{2\pi\Delta t_2}$$

<u>5.</u>

$\lambda = 532$ nm

$\Delta t_1 = 1{,}2 \cdot 10^{-8}$ s

$\Delta t_2 = 2 \cdot 10^{-8}$ s

$\Delta\lambda = ?$

$\nu \cdot \lambda = c$

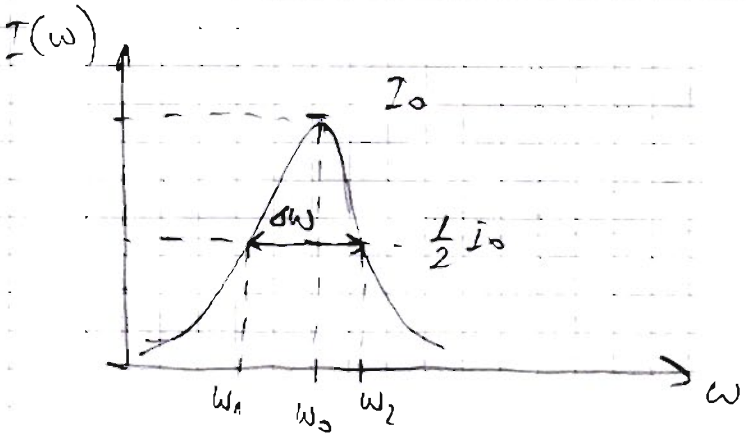
$\nu = \frac{c}{\lambda}$

$$d\nu = c \cdot \frac{-d\lambda}{\lambda^2} \rightarrow \Delta\lambda = \frac{\lambda^2 \Delta\nu}{c}$$

$|d\nu| = \Delta\nu$

$|d\lambda| = \Delta\lambda$

$$\delta\lambda = \frac{\lambda^2}{c}\left(\frac{1}{2\pi\,\delta t_1} + \frac{1}{2\pi\,\delta t_2}\right) = 2\cdot 10^{-14}\ m$$



$$\delta\omega = \omega_2 - \omega_1$$

Lorentz:

$$I(\omega) = I_0 \frac{\left(\frac{\gamma}{2}\right)^2}{(\omega-\omega_0)^2 + \left(\frac{\gamma}{2}\right)^2}$$

γ - konstanta prigušenja

$$I(\omega) = \frac{1}{2}\cancel{I_0} = \cancel{I_0}\frac{\left(\frac{\gamma}{2}\right)^2}{(\omega-\omega_0)^2 + \left(\frac{\gamma}{2}\right)^2}$$

$$(\omega-\omega_0)^2 + \left(\frac{\gamma}{2}\right)^2 = 2\left(\frac{\gamma}{2}\right)^2$$

$$(\omega-\omega_0)^2 = \left(\frac{\gamma}{2}\right)^2$$

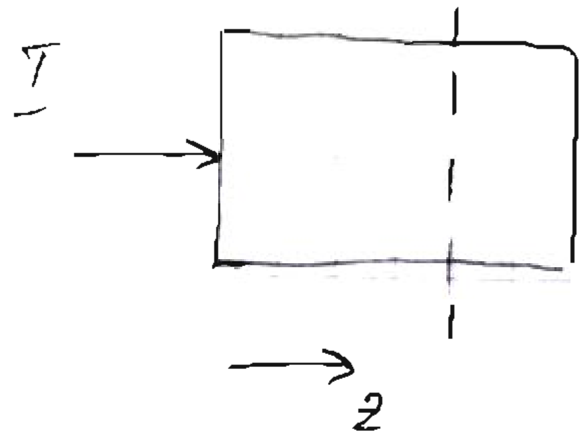
$$\omega_{1,2} = \omega_0 \pm \frac{\gamma}{2}$$

$$\boxed{\delta\omega = \omega_2 - \omega_1 = \gamma}$$

→

PRIRODNA ŠIRINA LINIJE KOD APSORPCIJE

Elektro-magnetsko zračenje (intenzitet tog zračenja je I)



$$dI = -\alpha \cdot I \cdot dz$$

$$\boxed{I = I_0 \cdot e^{-\alpha(\omega)\cdot z}}$$ Beerov zakon

z - debljina materijala

$$E = E_0 \cdot e^{i\omega t}$$

$$F = e \cdot E$$

$$m\frac{d^2x}{dt^2} + b\frac{dx}{dt} + kx = qE_0 e^{i\omega t}$$

$$x(t) = \frac{qE_0 e^{i\omega t}}{m(\omega_0^2 - \omega^2 + i\gamma\omega)}$$

$$\gamma = \frac{b}{m}$$

$$\omega_0^2 = \frac{k}{m}$$

- kad dovodimo izvana elektromagnetsko zračenje možemo primijeniti model prisilnog oscilatora

$$x(t) = x$$

p - dipolni moment

$$p = q \cdot x$$

$$P = N \cdot p$$ N - broj atoma

$\mathcal{P}$ - polarizacija materijala

$$\mathcal{P} = N \cdot p = N \cdot q \cdot x$$

$$\vec{\mathcal{P}} = \varepsilon_0 (\varepsilon_r - 1) \vec{E} = \varepsilon_0 \underbrace{\chi_e}_{\Downarrow} \vec{E}$$

susceptibilnost

$n = \sqrt{\varepsilon_r}$ $\qquad$ n - indeks loma materijala

$$\mathcal{P} = N \cdot q \cdot x(t) = N \cdot q \cdot \frac{q \cdot \cancel{E_0 e^{i\omega t}}}{m(\omega_0^2 - \omega^2 + i\gamma\omega)} =$$

$$= \varepsilon_0 (n^2 - 1) \cancel{E_0 e^{i\omega t}}$$

$$n^2 = 1 + \frac{N q^2}{\varepsilon_0 m (\omega_0^2 - \omega^2 + i\gamma\omega)}$$

$$n^2 - 1 = (n+1)(n-1) = 2(n-1)$$

$$n = 1 + \frac{N \cdot q^2}{2 \varepsilon_0 m (\omega_0^2 - \omega^2 + i\gamma\omega)}$$

$$n = n' - iK'$$

$$E = E_0 e^{i(\omega t - kz)} \xrightarrow{\text{(Imaginarni zapis)}} \sin(\omega t - kz)$$

$k = \frac{2\pi}{\lambda}$ $\qquad$ k - valni broj

$$\lambda = \frac{\lambda_0}{n}$$

$$k = \frac{2\pi n}{\lambda_0} = k_0 \cdot n$$

$$E = E_0 e^{i(\omega t - k_0 (n' - iK) z)}$$

$\Rightarrow$

$$E = E_0 \underbrace{e^{-k_0 \mathcal{K} z}} e^{i(\omega t - k_0 n' z)}$$

apsorpcija vala (eksponencijalno opadanje amplitude)

disperzija (raspršenje vala)

- za apsorpciju vala javlja se $\mathcal{K}$ (imaginarni dio indeksa loma sredstva)

$$I \sim E^2$$

$$I = I_0 e^{-2k_0 \mathcal{K} z}$$

$$I = I_0 e^{-\alpha(\omega) z}$$

$$\alpha(\omega) = 2k_0 \mathcal{K} = \frac{N q^2 \omega_0}{c \cdot \varepsilon_0 \cdot m} \cdot \frac{\gamma \omega}{(\omega_0^2 - \omega^2) + \gamma^2 \omega^2}$$

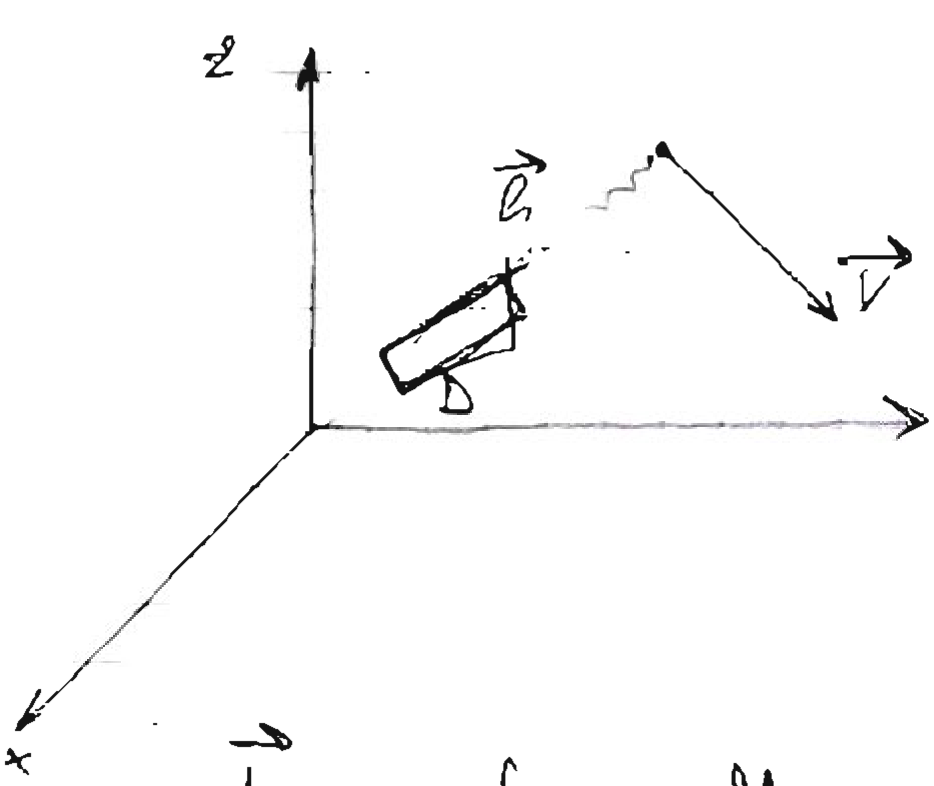
— $\underline{\omega_0 - \omega \ll \omega_0}$:

$q = e$

$$\boxed{\alpha(\omega) = \frac{N e^2 \pi}{4 \varepsilon_0 m c} \frac{\frac{\gamma}{2\pi}}{(\omega_0 - \omega)^2 + \left(\frac{\gamma}{2}\right)^2}}$$

Lorentzov oblik linije

DOPPLEROVA ŠIRINA LINIJE



$\vec{k}$ - valni vektor

$$|\vec{k}| = k = \frac{2\pi}{\lambda}$$

$$\omega_D = \omega_0 + \vec{k}\cdot\vec{v}$$

$\vec{k}\cdot\vec{v} > 0$ približavanje molekule prema elektronu

$\vec{k}\cdot\vec{v} < 0$ udaljavanje molekule od elektrona

Maxwellova raspodjela:

$$n_i(v)\,dv \approx e^{-\frac{mv^2}{2kT}} = e^{-\left(\frac{v}{v_p}\right)^2}$$

$$v_p = \sqrt{\frac{2kT}{m}}$$

$\vec{k} \parallel \vec{v}$ $\qquad \omega - \omega_0 = k\cdot v = \frac{\omega_0}{c}\cdot v$

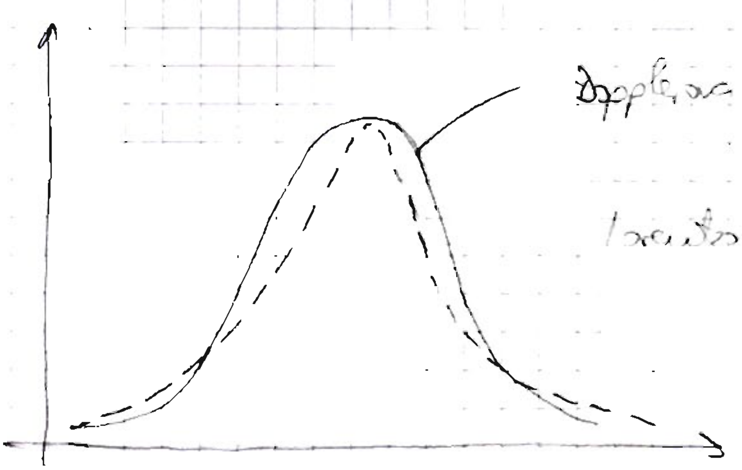
$$k = \frac{2\pi}{\lambda} = \frac{2\pi\nu}{c} = \frac{\omega_0}{c} \Rightarrow \omega - \omega_0 = \frac{\omega_0 v}{c}$$

$$n_i(v)\,dv = e^{-\left[\frac{1}{v_p}\cdot\frac{(\omega-\omega_0)\cdot c}{\omega_0}\right]^2}$$

$$\boxed{I(\omega) = I_0\, e^{-\left[\frac{c(\omega-\omega_0)}{\omega_0 v_p}\right]^2}}$$

Dopplerov oblik linije (Gaussov)

- matematički oblik: Gauss



Dopplerova (Gaussova) širina linije

Lorentzova širina linije

(11.)

$$I(\omega) = I_0 \, e^{-a\frac{(\omega-\omega_0)^2}{\omega_0^2}}$$

$$a = \frac{c^2}{v_p^2} = \frac{c^2 M}{2kT}$$

$$I(\omega) = \frac{I_0}{2}$$

$$\frac{\cancel{I_0}}{2} = \cancel{I_0}\, e^{-a\frac{(\omega-\omega_0)^2}{\omega_0^2}}$$

$$e^{-a\frac{(\omega-\omega_0)^2}{\omega_0^2}} = \frac{1}{2} \Big/ \ln$$

$$-a\frac{(\omega-\omega_0)^2}{\omega_0^2} = \ln\frac{1}{2}$$

$$(\omega-\omega_0)^2 = \frac{\omega_0^2}{a}\ln 2$$

$$\omega_{1,2} = \omega_0 \pm \omega_0\sqrt{\frac{\ln 2}{a}}$$

$$\Delta\omega = \omega_2 - \omega_1$$

$$\Delta\omega = 2\omega_0\sqrt{\frac{\ln 2}{a}}$$

(12.)

$$\Delta t = 1{,}5 \cdot 10^{-7}\, s$$

$$\frac{\Delta\lambda_D}{\Delta\lambda} = ?$$

$\lambda = 253{,}65$ nm

$T = 300$ K

$M = 200{,}59$

$$\Delta\lambda = \frac{\lambda^2}{2\pi c\, \Delta t} \quad \rightarrow \text{prirodna}$$

$$\omega = 2\pi\nu$$

$$\Delta\lambda_D = \frac{\lambda^2 \Delta\nu_D}{c}$$

$$\Delta\nu_D = \frac{\Delta\omega_D}{2\pi}$$

$$\frac{\Delta\lambda_D}{\Delta\lambda} = \omega_D \cdot \Delta t = \Delta t \cdot 2 \cdot \omega_0 \sqrt{\frac{\ln 2}{a}}$$

$$\omega_0 = \frac{2\pi c}{\lambda}$$

$$a = \frac{c^2 \cdot m}{2kT}$$

$m = M \cdot u$ u - atomska jedinica mase

$m = 200{,}59 \cdot 1{,}67 \cdot 10^{-23}$

$$\frac{\Delta\lambda_D}{\Delta\lambda} = \underline{973}$$