Informacijske mreže (34457) - *rješenja*

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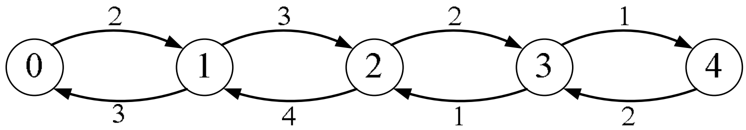
1. međuispit – 21. listopada 2009.

1. $\lambda = 15$, $\mu = 20$, $\rho = \lambda/\mu = 3/4$, red M/M/1 je stabilan.
(a) P{korisnik ne čeka} $= p_0 = 1 - \lambda/\mu = 1/4$; cijena/litra $= \alpha p_0 + 0.9\alpha(1 - p_0) = 7.4$ kn.
(b) Očekivani trošak po korisniku $= \sum_{n=1}^{\infty} np_n = \sum_{n=1}^{\infty} n\rho^n(1-\rho) = \dfrac{\rho}{1-\rho} = \dfrac{\lambda}{\mu-\lambda} = 3$ kn/korisnik

2. Pretpostavimo da je broj mogućih korisnika tako velik da ga možemo smatrati beskonačnim, te da je broj parkirnih mjesta dovoljno veliki da prihvati sva dolazeća vozila. Imamo red M/M/1.
(a) $\lambda = 5$, $\mu = 6$, $\rho = 5/6 < 1$(sustav je stabilan). $N_Q = \rho^2/(1 - \rho) = 4.17 \cong 4$.
(b) $p_0 + p_1 + ... + p_c \geq 0.8$ (c = broj parkirnih mjesta bez same praonice) $\Rightarrow (1-\rho) + (1-\rho)\rho + ... + (1 - \rho)\rho^c \geq 0.8 \Rightarrow 1 - \rho^{c+1} \geq 0.8 \Rightarrow \rho^{c+1} \leq 0.2$; logaritmiranjem obiju strana i sređivanjem dobivamo: $c \geq \log 0.2/(\log 5/6) - 1 = 7.8 \cong 8$ mjesta za parkiranje. Dakle, da bi za svako dolazeće vozilo u 80% vremena bilo mjesta, minimalni broj parkirnih mjesta je 8, tj. približno dvostruko od očekivane veličine reda čekanja N_Q.
(c) $p_0 = 1 - \rho \cong 0.17$, tj 17% vremena je praonica nezauzeta.
(d) $W = 1/[\mu(1 - \rho)] = 1/6[1 - (5/6)] = 1$ h $\Rightarrow$ predugo, uprava mora ubrzati posluživanje!
(e) $1 - p_0 = \rho = 0.833$.
(f) $p(n \geq 7) = \rho^7 = 0.279$.

3. (a) Markovljev lanac:



(b) JGR: $2p_0 = 3p_1$, $2p_0 + 4p_2 = 6p_1$, $3p_1 + p_3 = 6p_2$, $2p_2 + 2p_4 = 2p_3$, $p_3 = 2p_4$,
$p_0 + p_1 + p_2 + p_3 + p_4 = 1$;
(c) JLR: $2p_0 = 3p_1$, $3p_1 = 4p_2$, $2p_2 = p_3$, $p_3 = 2p_4$, $p_0 + p_1 + p_2 + p_3 + p_4 = 1$;
(d) $p_1 = (\lambda_0/\mu_1)p_0 = (2/3)\,p_0$, $p_2 = (\lambda_0\lambda_1/\mu_1\mu_2)p_0 = (1/2)\,p_0$, $p_3 = p_0$, $p_4 = (1/2)p_0$, $p_0 = 3/11$;
$p_0 = 3/11$, $p_1 = 2/11$, $p_2 = 3/22$, $p_3 = 3/11$, $p_4 = 3/22$;
(e) $N = p_1 + 2p_2 + 3p_3 = 20/11$; $N_Q = 0p_1 + 1p_2 + 2p_3 = 12/11$ [može i ovako: $N_Q = N - (1 - p_0)$] ;
(f) $E[\lambda] = \lambda_0 p_0 + \lambda_1 p_1 + \lambda_2 p_2 + \lambda_3 p_3 = 18/11$;
(g) $W = N/E[\lambda] = 10/9$, $W_Q = N_Q/E[\lambda] = 2/3$

4. Parametri sustava: $c = 4$, $\lambda = 1/2$ poziv/min, $1/\mu = 4$ min/poziv, $a = \lambda/\mu = 2$, $\rho = a/c = 1/2$.

(a) Prvo treba izračunati p_0 za sustav M/M/c:

$$p_0 = \left\{1 + 2 + \frac{2^2}{2!} + \frac{2^3}{3!} + \frac{2^4}{4!}\cdot\frac{1}{1-1/2}\right\}^{-1} = \frac{3}{23}$$

Vjerojatnost čekanja dobiva se pomoću Erlang-C formule [$E_{2,c}(a) \equiv C(c,a)$]:

$$C(c,a) = C(4,2) = P_Q = \frac{a^c}{c!}\frac{1}{1-\rho}p_0 = \frac{2^4}{4!}\frac{1}{1-1/2}\frac{3}{23} = \frac{4}{23} \simeq 0.17$$

(b) Vjerojatnost da paket čeka dulje od jedne minute:

$$
\begin{aligned}
P(W>1) &= 1-P(W\le 1)\\
&= C(c,a)\,e^{-c\mu(1-\rho)1} = \frac{4}{23}e^{-4(1/4)(1/2)1}\\
&= \frac{4}{23}e^{-1/2} \simeq 0.11
\end{aligned}
$$

(c) Erlang-B formula daje odgovor na pitanje o postotku preusmjerenih poziva [$E_{1,c}(a) \equiv B(c,a)$]:

$$B(4,2) = \frac{2^4/4!}{1+2+2^2/2!+2^3/3!+2^4/4!} = \frac{2}{21} \simeq 0.095 \quad \Rightarrow 9.5\%$$

5. Ponuđeni promet je $a = \lambda/\mu$ = 3 poziv/h × 1 h/poziv = 3 erl, c = 5. Vjerojatnost blokiranja je (Erlang-B):

$$B(5,3) = \frac{3^5/5!}{1+3+3^2/2!+3^3/3!+3^4/4!+3^5/5!} = \frac{81}{736} \simeq 0.11 \quad \Rightarrow 11\%$$

Korisne formule:

$\sum_{n=0}^{N} x^n = \frac{1-x^{N+1}}{1-x}$, za svaki $x \neq 1$; $\quad \sum_{n=0}^{\infty} x^n = \frac{1}{1-x}$, za $|x|<1$

Erlang-B: $B(c,a) = p_c = \frac{a^c}{c!}\left[\sum_{k=0}^{c}\frac{a^k}{k!}\right]^{-1}$, $a = \frac{\lambda}{\mu}$

Erlang-C: $C(c,a) = P_Q = \sum_{n=c}^{\infty} p_n = \frac{a^c}{(c-1)!(c-a)}p_0$, $p_0 = \left[\sum_{k=0}^{c-1}\frac{a^k}{k!} + \frac{a^c}{(c-1)!(c-a)}\right]^{-1}$, $0 \le a < c$

M/M/1: $T = \frac{N}{\lambda} = \frac{1}{\mu-\lambda}$, $N_Q = \lambda W = \frac{\rho^2}{1-\rho}$, $\rho = \lambda/c\mu = \lambda/\mu \ (c=1)$