Kantorov skup:

$x \in C \Leftrightarrow x = 0.a_1 a_2 a_3 ...|_3$ - barem jedan zapis ne sadrži 1, $a_k \in \{0,2\}$, $a_k \notin \{1\}$

HIPOTEZA:

$(0.0\bar{2})_3 = (0.1\bar{0})$

DOKAZ:

$(0.a_1 a_2 a_{3,} .., a_{k-1} 1\bar{0})_3 = (0.a_1 a_2 a_{3,} .., a_{k-1} 0\bar{2})_3$

$$a_1\left(\frac{1}{3}\right) + a_2\left(\frac{1}{3^2}\right) + ... + a_{k-1}\left(\frac{1}{3^{k-1}}\right) + \left(\frac{1}{3^k}\right) = a_1\left(\frac{1}{3}\right) + a_2\left(\frac{1}{3^2}\right) + ... + a_{k-1}\left(\frac{1}{3^{k-1}}\right) + 0\left(\frac{1}{3^k}\right) + 2*\left(\frac{1}{3^{k+1}}\right)...$$

$$\left(\frac{1}{3^k}\right) = 2\cdot \sum_{i=k+1}^{\infty}\left(\frac{1}{3^{k+1}}\right) = \frac{2}{3}\sum_{i=k}^{\infty}\left(\frac{1}{3^k}\right) = \frac{\frac{2}{3}\cdot\left(\frac{1}{3}\right)^k}{1-\frac{1}{3}} = \frac{\frac{2}{3}\cdot\left(\frac{1}{3}\right)^k}{\frac{2}{3}} = \left(\frac{1}{3}\right)^k$$

Stabilnost i bifurkacije

Fiksne točke:

$$f_r(x*) = x*$$

Stabilne fiksne točke ispunjavaju uvijet:

$$\left|\frac{d f_r(x*)}{dx}\right| < 1$$

Bifurkacija nastupa:

(1) $f_r(x*) = x*$

(2) $\frac{df_r}{dx}(r*) < 1$

(3) $\frac{\partial f_r}{\partial r}(r*, x*) \neq 0$

Ukoliko je $\frac{\partial f_r}{\partial r}(r*, x*) = 0$ tada provjeravamo sljedece dvije tvrdnje:

(3.1) $\frac{df_r^3}{dx^3}(x*, r*) \neq 0$

(3.2) $\frac{\partial f_r}{\partial r}\left(\frac{df_r^2}{dx^2}\right)(x*, r*) \neq 0$

Sličnost

Općenito sličnost je neka trasformacija $S : \mathbb{R}^2 \rightarrow \mathbb{R}^3$

Transformacija $S(x,y)$

$(x_{1,}x_2)=S_2(y_{1,}y_2)$ je sličnost ako vrijedi:
$\|S(x)-S(y)\|=k\cdot\|x-y\|$
$\|S(x_{1,}x_2)-S(y_{1,}y_2)\|=k\cdot\|(x_{1,}x_2)-(y_{1,}y_2)\|$

$k=\frac{\|Sx-Sy\|}{\|x-y\|}$ - skalirana udaljenost između bilo koje dvije točke

TRANSLACIJA
$T_{a,b}(x)=T_{a,b}(x_{1,}x_2)=(x_1+a,x_2+b),\forall(x)\in\mathbb{R}^2$
D.Z.
$\|S(x)-S(y)\|=\|(x_1+a,x_2+b)-(y_1+a,x_2+b)\|=$
$\sqrt{(x_1+a-y_1-a)^2+(x_2+b-y_2-b)^2}=\sqrt{(x_1-y_1)^2+(x_2-y_2)^2}=1\|x-y\|$

HOMOTETIJA
$H_\alpha(x)=H_\alpha(x_{1,}x_2)=(x_1\alpha,x_2\alpha),\forall X\in\mathbb{R}^2$
D.Z.
$\|S(x)-S(y)\|=\|(x_1+a,x_2+b)-(y_1+a,x_2+b)\|=$
$\sqrt{\alpha^2(x_1-y_1)^2+\alpha^2(x_2-y_2)^2}$

ROTACIJA
$R_\theta(x)=R_\theta(x_{1,}x_2)=(x_1\cos\theta-x_2\sin\theta,x_1\sin\theta+x_2\cos\theta),\forall x\in\mathbb{R}^2,\theta\in[0,2\pi]$
D.Z.
$\|S(x)-S(y)\|=\|(x_1\cos\theta-x_2\sin\theta,x_1\sin\theta+x_2\cos\theta)-(y_1\cos\theta-y_2\sin\theta,y_1\sin\theta+y_2\cos\theta)\|=$
$=\sqrt{(x_1\cos\theta-x_2\sin\theta-y_1\cos\theta-y_2\sin\theta)^2+(x_1\sin\theta+x_2\cos\theta-y_1\sin\theta-y_2\cos\theta)^2}=$
$=\sqrt{(x_1\cos\theta-x_2\sin\theta-y_1\cos\theta-y_2\sin\theta)^2+(x_1\sin\theta+x_2\cos\theta-y_1\sin\theta-y_2\cos\theta)^2}=$
$=\sqrt{(x_1-y1)^2\cos^2\theta+(x_2-y2)^2\sin^2\theta-\cancel{2\sin\theta\cos\theta(x_1-y_1)(x_2-y_2)}+}$
$\overline{(x_1-y1)^2\sin^2\theta+(x_2-y2)^2\cos^2\theta+\cancel{2\sin\theta\cos\theta(x_1-y_1)(x_2-y_2)}}=$
$=\sqrt{(x_1-y_1)^2+(x_2-y_2)^2}=1\|x-y\|$

KOMPOZICIJA TRANSFORMACIJA
$S_1\circ S_2(x)$
D.Z.
$\|S_2(S_1(x_{1,}x_2))-S_2(S_1(y_{1,}y_2))\|=k_1\|S_1(x_{1,}x_2)-S_1(y_{1,}y_2)\|=k_1k_2\|(x_{1,}x_2)-(y_{1,}y_2)\|$

Fraktali

Jednogrbasti iterator

1) $x_{max}\in[0,1]$

2) $f \uparrow (0, x_{may}), \frac{df}{dx} > 0$

3) $f \downarrow (0, x_{may}), \frac{df}{dx} < 0$

4) f je neprakinut

Shift operator

$\Sigma_2 = \{s = s_1 s_2 s_3 ..., s_k \in \{0,1\}\}$

$\delta : \Sigma_2 \rightarrow \Sigma_2$, $\delta(t_0 t_{1,} t_{2,} ...) = t_{1,} t_{2,} ...$

$\|s, t,\| = \sum_{i=o}^{\infty} \frac{(s_i - t_i)}{2^i}$