16. zad. Razvij u Maclaurinov red sljedeće funkcije:

A) $f(z)=\frac{z}{(z^2+1)(z^2-4)}$

$$\frac{z}{(z^2+1)(z^2-4)}=\frac{Az+B}{(z^2+1)}+\frac{Cz+D}{(z^2-4)}=\frac{(Az+B)(z^2-4)+(Cz+D)(z^2+1)}{(z^2+1)(z^2-4)}$$

$$=\frac{Az^3-4Az+Bz^2-4B+Cz^3+Cz+Dz^2+D}{(z^2+1)(z^2-4)}$$

$$=\frac{z^3(A+C)+z^2(B+D)+z(-4A+C)-4B+D}{}$$

$A+C=0 \qquad A=-C$

$B+D=0 \qquad 4C+C=1 \qquad C=\frac{1}{5} \qquad A=-\frac{1}{5}$

$-4A+C=1 \qquad B=-D \qquad 4D+D=0$

$-4B+D=0 \qquad D=0 \qquad B=0$

$$f(z)=-\frac{1}{5}\cdot\frac{z}{z^2+1}+\frac{1}{5}\cdot\frac{z}{z^2-4}=-\frac{z}{5}\cdot\frac{1}{1+z^2}+\frac{z}{5}\cdot\frac{1}{-4\left(1-\frac{z^2}{4}\right)}$$

$$f(z)=-\frac{z}{5}\cdot\frac{1}{1+z^2}-\frac{z}{20}\cdot\frac{1}{1-\frac{z^2}{4}}=-\frac{z}{5}\cdot\sum_{n=0}^{\infty}(-1)^n z^{2n}-\frac{z}{20}\sum_{n=0}^{\infty}\frac{z^{2n}}{4^n}$$

$$f(z)=\frac{1}{5}\sum_{n=0}^{\infty}(-1)^n\cdot(-1)\cdot z^{2n}\cdot z-\frac{1}{5}\sum_{n=0}^{\infty}\frac{z^{2n}\cdot z}{4^n\cdot 4}$$

$$f(z)=\frac{1}{5}\left[\sum_{n=0}^{\infty}(-1)^{n+1}\cdot z^{2n+1}-\sum_{n=0}^{\infty}\frac{z^{2n+1}}{4^{n+1}}\right]=\frac{1}{5}\sum_{n=0}^{\infty}\left((-1)^{n+1}-4^{-(n+1)}\right)\cdot z^{2n+1}$$

B) $$f(z)=\mathrm{ch}^2 z=\frac{1+\mathrm{ch}\,2z}{2}=\frac{1}{2}+\frac{1}{2}\mathrm{ch}\,2z=\frac{1}{2}+\frac{1}{2}\sum_{n=0}^{\infty}\frac{(2z)^{2n}}{(2n)!}$$

$$=\frac{1}{2}+\frac{1}{2}\sum_{n=0}^{\infty}\frac{4^n\cdot z^{2n}}{(2n)!}$$