23.09.2009

AUD2_BURIĆ

38–74 str

2. FJA. KOMPL. VAR.

$\sin(1+i)$, $\ln(-1)$, $(i^{\pi})^{i}$, $\operatorname{arcth}(i)$

e^{z}, $T=2\pi i$, $e^{i+2\pi i} = e^{i} = \cos 1 + i\sin 1$

$$\boxed{\begin{aligned} \sin z &= \frac{1}{2i}\left(e^{iz}-e^{-iz}\right) \\ \cos z &= \frac{1}{2i}\left(e^{iz}+e^{-iz}\right) \end{aligned}}$$

DEF SIN I COS

$$\boxed{\begin{aligned} &\sin^2 z + \cos^2 z = 1 \\ &\sin(z_1 \pm z_2) \\ &\operatorname{sh} z = \frac{1}{2}\left(e^{z}-e^{-z}\right) \\ &\operatorname{ch} = \frac{1}{2}\left(e^{z}+e^{-z}\right) \end{aligned}}$$

VRIJEDI

$\sin(iz) = i \operatorname{sh} z$ STR 72 → SVE FJE TREBA ILI ZNAT ILI IZVEST NA MI

$\cos(iz) = \operatorname{ch} z$ IZVOD NEKE OD TIH FORMULA BIT ĆE NA MI (2BODA).

OBIČNI ln IZ SRED. ŠKS

$$\operatorname{Ln} z = \ln|z| + i(\arg z + 2k\pi)$$

GLAVNA VRIJ. LOG. FJE JE ZA $k=\emptyset$

$\cancel{\operatorname{Ln} z^2 = 2\operatorname{Ln} z}$ SVOJSTVA LOG. NE VRIJEDE

$\cancel{\operatorname{Ln} i^{i} = i\operatorname{Ln} i}$

NE VRIJEDI MNOŽ. POTENCIJA

$\cancel{\left(i^{2i}\right)^{1+i} = i^{2i(i+1)}}$

$$\boxed{z^{\alpha} = e^{\alpha \cdot \operatorname{Ln} z}}$$

→ OPĆA EKSPON. FJA

$$\boxed{e^{z_1} \cdot e^{z_2} = e^{z_1+z_2}}$$

VRIJ. SAMO ZA BAZU e (MOGUĆE U MI IZVOD) POGL. KNJIGU.

ZAD (3 BODA)

$$|tg(\pi+i)| = ?$$

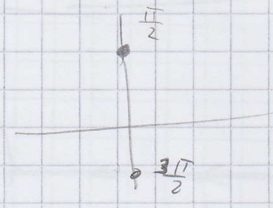
$$tg(\pi+i) = \frac{\sin(\pi+i)}{\cos(\pi+i)} = \frac{\overset{\emptyset}{\sin\pi}\cos i + \cos\pi \sin i}{\cos\pi \cos i - \underset{\emptyset}{\sin\pi \sin i}} = \frac{\sin i}{\cos i} = \frac{i\cdot sh1}{ch1} = i\cdot th1$$

$$|tg(\pi+i)| = |i\, th1| = \underline{\underline{th1}}$$

ZAD) $i + ch z = \emptyset$ RIJEŠI JED.

$ch z = -i \quad / Arch$ U KOMPL. OBIČAJ TRIG FJA PISATI S VELIKIM SLOVOM

$z = Arch(-i)$ $\boxed{Arch z = Ln(z + \sqrt{z^2-1})}$



$$z = Ln(-i + \sqrt{-1-1}) = Ln(-i + \sqrt{-2})$$

$$z = Ln(-i \pm i\sqrt{2}) = Ln(i(-1 \pm \sqrt{2}))$$

$$z = \ln|i(-1 \pm \sqrt{2})| + i(\arg(-1 \pm \sqrt{2}) + 2k\pi)$$

$$\left.\begin{array}{l} z_1 = \ln(-1+\sqrt{2}) + i\left(\frac{\pi}{2} + 2k\pi\right) \\ z_2 = \ln|-1-\sqrt{2}| + i\left(\frac{3\pi}{2} + 2k\pi\right) \end{array}\right\} Rj.$$

1.11.2008

2) $(i^i)^i$ MODUL I ARGUM. ODREDI (3 BODA)

$$i^i = e^{i Ln i} = e^{i(\overset{\emptyset}{\ln|i|} + i(\underbrace{\arg i}_{\frac{\pi}{2}} + 2k\pi))} = e^{\overset{i\cdot i}{-}(\frac{\pi}{2}+2k\pi)}$$

IMAMO VEĆ k PA KORISTI NEKO DRUGO SLOVO

$$\left(e^{-(\frac{\pi}{2}+2k\pi)}\right)^i = e^{i Ln(\underbrace{e^{-\frac{\pi}{2}-2k\pi}}_{arg=0})} = e^{i(\ln e^{-\frac{\pi}{2}-2k\pi} + i(0+2l\pi))}$$

DA JE NEG BROJ BI π
POZ $\emptyset$

$$= e^{i(-\frac{\pi}{2}-2k\pi)-2l\pi} =$$

$$= e^{-2l\pi} \cdot e^{i(-\frac{\pi}{2}) - 2k\pi\cdot i}$$

NEBITNO JER SE e PERIODIČNA

$\boxed{z = r e^{i\varphi}}$

$\Rightarrow$ $r = e^{-2l\pi}$, $\varphi = \frac{3\pi}{2}$ Rj:

ANALITIČNOST

$f(z) = f(x,y) = u(x,y) + i \cdot v(x,y)$

$z = x + iy$

$(z^2)' = 2z$

$\cancel{((|z|^i)' = ?)}$

C-R: $\boxed{\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y}}$ & $\boxed{\frac{\partial u}{\partial y} = -\frac{\partial v}{\partial x}}$

$\boxed{\bar{z} \quad \text{ILI} \quad |z| \quad \text{NIJE ANALITIČNO}}$

u i v HARMONIJSKE
($\Delta u = 0$, $\Delta v = 0$)

$\Delta f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2}$ → SIGURNO NA MI (HARMONIJSKE ?)

1MI.08

4) $f(z) = \sin z^2$ DOKAŽI DA JE ANALITIČKA

$f(x,y) = \sin(x+iy)^2 = \sin(x^2 - y^2 + 2xyi)$

$= \sin(x^2-y^2)\cos(2xyi) + \cos(x^2-y^2)\sin(2xyi)$

$= \underbrace{\sin(x^2-y^2)\,\mathrm{ch}\,2xy}_{u(x,y)} + i\underbrace{\cos(x^2-y^2)\,\mathrm{sh}\,2xy}_{v(x,y)}$

$\left.\begin{aligned} \frac{\partial u}{\partial x} &= \cos(x^2-y^2)\cdot 2x \cdot \mathrm{ch}\,2xy + \sin(x^2-y^2)\,\mathrm{sh}\,2xy \cdot 2y \\ \frac{\partial v}{y} &= -\sin(x^2-y^2)(-2y)\,\mathrm{sh}(2xy) + \cos(x^2-y^2)\,\mathrm{ch}(2xy)\cdot 2x \end{aligned}\right\} = \checkmark$

SIGURNO NA MI

ZADAN Re NAĐI Im

Z1-6) $v(x,y) = y\cos x\,\mathrm{ch}\,y - x\sin x\,\mathrm{sh}\,y$

1 kor. $\Delta v = ?$

$\frac{\partial v}{\partial x} = -\sin x\, y\,\mathrm{ch}\,y - \sin x\,\mathrm{sh}\,y - x\cos x\,\mathrm{sh}\,y$

$\frac{\partial^2 v}{\partial x^2} + \frac{\partial v^2}{\partial y^2} = \emptyset$ O.K. POSTOJI $u(x,y)$ t.d. je f ANALITIČKA

$\frac{\partial v}{\partial y} = \cos x\,\mathrm{ch}\,y + y\cos x\,\mathrm{sh}\,y - x\sin x\,\mathrm{ch}\,y$

2.kor. $\frac{\partial u}{\partial y} = -\frac{\partial v}{\partial x} = \sin x \, y \, ch y + \sin x \, sh y + x \cos x \, sh y \quad / \int dy$

$u(x,y) = \cancel{\sin x \, ch y} + x \cos x \, ch y + \sin x \int y \, sh y - \cancel{\sin x \, ch y} + \varphi(x)$

$\int \underset{u}{y} \, \underset{dv}{ch y \, dy} = \left| \begin{matrix} u = y & dv = ch y \, dy \\ du = dy & v = sh y \end{matrix} \right| = uv - \int v \, du = y \, sh y - ch y$

$u(x,y) = x \cos x \, ch y + \sin x \, y \, sh y + \varphi(x)$

3. korak $\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y} \rightarrow \cos x \, ch y - x \sin x \, ch y + \cos x \, y \, sh y + \varphi'(x) \overset{\text{IZJEDN.}}{=} \frac{\partial v}{\partial y}$

$\varphi'(x) = 0 \Rightarrow \varphi(x) = C$

$\boxed{u(x,y) = x \cos x \, ch y + \sin x \, y \, sh y + C}$ Rj.

KOMP. BROJ   Re → x, Im → y

POČ. UVJET $f(\pi) = 0$

$f(x,y) = u(x,y) + i \, v(x,y)$

$x = \pi, \quad y = 0$

$f(\pi, 0) = -\pi + C + i \cdot 0 = 0 \Rightarrow \boxed{C = \pi}$

$f(x,y) = x \cos x \, ch y + \sin x \, y \, sh y + \pi + i \, y \cos x \, ch y - i \, x \sin x \, sh y$

PRIKAŽI RJ. U EKSPLICITNOM OBLIKU $f(z)$ IZLUČI DA DOBIŠ $x+iy$

$f(z) = ?$

$f(x,y) = \cos x \, ch y \, (x+iy) - i \sin x \, sh y \, (x+iy) + \pi$
$= (x+iy) \left[\cos x \cos iy - \sin x \sin iy\right] + \pi$
$= (x+iy) \cos(x+iy) + \pi$

$\boxed{f(z) = z \cos z + \pi}$ Rj...