BURIĆ 12.11.2009.

3. Laurentovi redovi

$$f(z)=\sum_{n=-\infty}^{\infty} c_n (z-z_0)^n$$

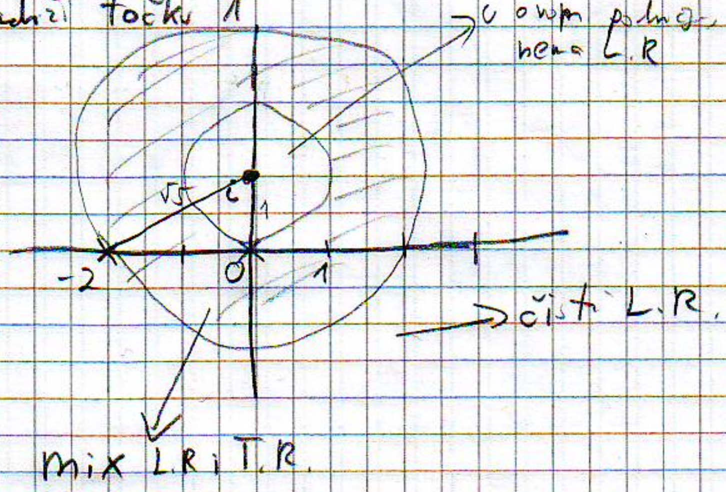
2. MI - 08

3) $z_0 = i$, L.R, područje sadrži točku 1

$$f(z)=\frac{2}{z^2+2z}=\frac{2}{z(z+2)}$$

$$=\frac{A}{z}+\frac{B}{z+2}$$

$$f(z)=\frac{1}{z}+\frac{-1}{z+2}$$



$$\frac{-1}{z+2}=\frac{-1}{z-i+2+i}=\frac{-1}{2+i}\cdot\frac{1}{1+\frac{z-i}{2+i}}=$$

PODRUČJE
- prije sing — T.R.
- poslije sing — L.R.

T.R

$$=-\frac{1}{2+i}\sum_{n=0}^{\infty}(-1)^n\frac{(z-i)^n}{(2+i)^n}$$

$$\frac{1}{z}=\frac{1}{z-i+i}=\frac{1}{z-i}\cdot\frac{1}{1+\frac{i}{z-i}}=\frac{1}{z-i}\cdot\sum_{n=0}^{\infty}(-1)^n\frac{i^n}{(z-i)^n}$$

L.R

$$\boxed{f(z)=\sum_{n=0}^{\infty}(-1)^n\frac{i^n}{(z-i)^{n+1}}-\sum_{n=0}^{\infty}(-1)^n\frac{(z-i)^n}{(2+i)^{n+1}}}$$

b) odred. područje konvergencije

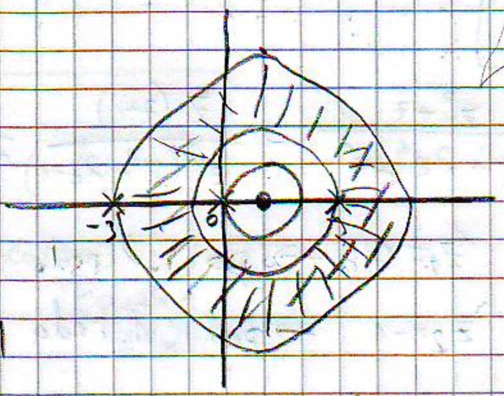
$1<|z-i|<\sqrt{5}$ (sa slike)

$$\frac{1}{(z+2)^2(z-1)^3}=$$

$$\frac{A}{z+2}+\frac{B}{(z+2)^2}+\frac{C}{(z-1)}+\frac{D}{(z-1)^2}+\frac{E}{(z-1)^3}$$

$\boxed{2B \; DZ1}$

$$f(z)=\frac{1}{(z^2-9)z^2} \qquad z_0=1 \qquad D=\{2<|z-1|<4\}$$



$$f(z)=\frac{1}{(z-3)(z+3)z^2}$$

$$=\frac{A}{z-3}+\frac{B}{z+3}+\frac{C}{z}+\frac{D}{z^2}$$

$$A(z+3)z^2+B(z-3)z^2+C(z^2-9)z+D(z^2-9)=1$$

$A=\frac{1}{54}$

$B=-\frac{1}{54}$

$C=0$

$D=-\frac{1}{9}$

$$f(z)=\frac{1}{54}\cdot\frac{1}{z-3}-\frac{1}{54}\cdot\frac{1}{z+3}-\frac{1}{9}\cdot\frac{1}{z^2}$$

$\boxed{2R}$ $$\frac{1}{z-3}=\frac{1}{z-1-2}=\frac{1}{z-1}\cdot\frac{1}{1-\frac{2}{z-1}}=\sum_{n=0}^{\infty}\frac{2^n}{(z-1)^{n+1}}$$

$\boxed{T.R}$ $$\frac{1}{z+3}=\frac{1}{z-1+4}=\frac{1}{4}\cdot\frac{1}{1+\frac{(z-1)}{4}}=\sum_{n=0}^{\infty}(-1)^n\frac{(z-1)^n}{4^{n+1}}$$

$\boxed{Ln}$ $$\frac{1}{z^2}=\frac{1}{(z-1+1)^2}=\frac{1}{(z-1)^2}\cdot\frac{1}{(1-\frac{1}{z-1})^2}=\frac{1}{(z-1)^2}\cdot\sum(-1)^n\frac{(n+1)}{(z-1)^n}$$

$$\boxed{\begin{gathered}\frac{1}{1-x}=\sum x^n \\ \frac{1}{(1-x)^2}=\sum_{n=0}^{\infty}(n+1)x^n\end{gathered}}$$

4. Singulariteti 48 str.

– Klasif. singulariteta po lim / L.R. ...

$\boxed{17A \quad DZ5}$

$$f(z) = \frac{z^2+z}{z^5+2z^4+z^3} = \frac{z(z+1)}{z^3(z^2+2z+1)} = \frac{z(\cancel{z+1})}{z^{\cancel{3}2}(z+1)^{\cancel{2}1}}$$

$z_1 = 0 \rightarrow$ pol 2. reda

$z_2 = -1 \rightarrow$ pol 1. reda

$\boxed{9A \quad DZ6}$

$$f(z) = \exp\frac{z}{1-z} = e^{\frac{z}{1-z}}$$

$z = 1$, singularitet, bitan

$\lim\limits_{z\to 1} f(z) = \lim\limits_{z\to 1} e^{\frac{z}{1-z}} = e^{\infty} \Rightarrow$ nije definirano

$\boxed{L'H = \frac{d(B)}{d(N)}}$

$\boxed{8A \quad DZ6}$

$$f(z) = z^2 \operatorname{ctg} z = z^2\frac{\cos z}{\sin z}$$

$\sin z = 0$

$z = \operatorname{Arcsin} 0$

$\boxed{z = k\pi \text{ , singulariteti , pol 1. reda, } k \neq 0}$

za $k \neq 0$

$$\lim_{z\to k\pi} \frac{z^2\cos z}{\sin z} = \frac{\text{const}}{0} = \infty$$

$k=0$

$$\lim_{z\to 0} \frac{z^2\cos z}{\sin z} = \left(\frac{0}{0}\right) L'H = \lim_{z\to 0} \frac{2z\cos z - z^2\sin z}{\cos z} = \frac{0}{1} = 0$$

$\boxed{z = 0 \text{ , uklonjiv}}$

$\boxed{\text{Primjer 1}}$

$\boxed{\text{ZI - 2008. 7}}$

Odredi nultočke i singularitete → ne može biti 0

$$f(z) = \frac{(1-e^z)\, e^{\frac{1}{z+2}}\, (z^2-1)}{(1-\cos z)(z-1)^2}$$

$\boxed{\text{NULTOČKE}}$

- $1-e^z=0$

$e^z=1$

$z=\ln 1$

$z=i2k\pi$

- $z^2-1=0$

$z_{1,2}=\pm 1$

$\boxed{\text{SING}}$

- $1-\cos z=0$

$\cos z=1$

$z=2k\pi$

- $z=1$

- $z=-2$

→ $z=1$ → pol 1. reda

$z=-1$ → nultočka kratnosti 1

$z=-2$ → bitni sing

$z=i2k\pi$ → nultočka kratnosti 1, $k\neq 0$

$z=2k\pi$ → polovi 2. reda

$z=0$ → pol 1. reda

$$\left.\begin{aligned} g(z)&=1-\cos z \\ g'(z)&=\sin z \end{aligned}\right\} g(2k\pi)=g'(2k\pi)=0$$

$g''(z)=\cos z$