

Rješenja zadataka za vježbu iz Kompleksne analize

1. A) kružni vijenac polumjera 1 i $\sqrt{2}$ sa središtem u $S(1, -1)$
B) poluravnina $y < x$
C) parabola $x = \frac{1}{2}(y^2 - 1)$
2. A) pravac $y = \frac{4}{3}x + \frac{5}{6}$
B) parabola $y > \frac{1}{2}(x^2 - 1)$ (dio ravnine iznad parabole)
C) hiperbola $2y^2 - 2x^2 = 1$ (okrenuta po y -osi)
3. A) kružnica polumjera $\frac{\sqrt{2}}{2}$ sa središtem u $S(\frac{1}{2}, \frac{1}{2})$
B) elipsa $\frac{x^2}{7} + \frac{y^2}{16} = 1$
C) lijeva grana hiperbole $\frac{x^2}{4} - \frac{y^2}{5} > 1$ (dio ravnine lijevo od te grane)
4. A) pravac $y = 1 - x$
B) donja grana hiperbole $12y^2 - 4x^2 = 3$ (okrenuta po y -osi)
C) vijenac između elipsa $\frac{x^2}{4} + \frac{y^2}{3} > 1$ i $\frac{x^2}{16} + \frac{y^2}{15} < 1$
5. A) jedan obilazak u negativnom smjeru po kružnici polumjera 2 sa središtem u ishodištu
B) dva obilaska u pozitivnom smjeru po kružnici polumjera 1 sa središtem u $S(-1, 1)$
C) segment pravca $x = 1$, omeđen točkama $A(1, 1)$ i $B(1, -1)$, koji se prelazi dvaput, od A do B i natrag
6. A) $u(x, y) = e^{1-x} \cos y$, $v(x, y) = -e^{1-x} \sin y$
B) $u(x, y) = \frac{\sin x \cos x}{\operatorname{ch}^2 y - \sin^2 x}$, $v(x, y) = \frac{\operatorname{sh} y \operatorname{ch} y}{\operatorname{ch}^2 y - \sin^2 x}$
7. A) $u(x, y) = e^{x^2-y^2} \cos 2xy$, $v(x, y) = e^{x^2-y^2} \sin 2xy$
B) $u(x, y) = \operatorname{sh} x \cos y$, $v(x, y) = \operatorname{ch} x \sin y$
8. A) nije analitička
B) analitička, $w'(z) = \operatorname{sh}(z - 2)$
9. A) nije analitička
B) analitička, $w'(z) = e^z + ze^z$
10. A) nije analitička
B) analitička, $w'(z) = 3z^2$
11. A) nije harmonijska
B) harmonijska, $v(x, y) = 2xy + 2y + C$

12. A) harmonijska, $v(x, y) = 2e^x \sin y + C$
 B) nije harmonijska
13. A) $f(z) = \frac{1}{z}$
 B) $f(z) = 2iz^2 + iz + C$
14. A) $f(z) = 2 \cos 2z + z$
 B) $f(z) = 2i \cos z - iz^2 + C$
15. A) $f(z) = 2 \sin z - z + C$
 B) $f(z) = 2 \operatorname{sh} z - z^2$
16. A) $|z| = 9e^{2k\pi}$, $k \in \mathbb{Z}$, $\arg z = -\ln 3$
 B) $|z| = \cos^2(\ln 3)$, $\arg z = 0$
 C) $|z| = 0$, $\arg z = \infty$ (neodređen)
17. A) $i(\frac{3\pi}{2} + 2k\pi)$, $k \in \mathbb{Z}$
 B) $e^{-(\frac{\pi}{2} + 2k\pi)(\frac{\pi}{2} + 2l\pi)}$, $k, l \in \mathbb{Z}$
 C) $k\pi + i \ln \sqrt{2}$, $k \in \mathbb{Z}$
18. A) $1 + 2k\pi i$, $k \in \mathbb{Z}$
 B) $e^{2k\pi}$, $k \in \mathbb{Z}$
 C) i
19. A) $i(\frac{3\pi}{2} + 2k\pi)$, $k \in \mathbb{Z}$
 B) $\frac{3\pi}{2} + 2k\pi$, $k \in \mathbb{Z}$
20. A) $i - e$
 B) $z_1 = 2k\pi - i \ln(-\pi + \sqrt{1 + \pi^2})$, $z_2 = \pi + 2k\pi - i \ln(\pi + \sqrt{1 + \pi^2})$, $k \in \mathbb{Z}$
21. A) $G^* = \{|w| < 1\}$
 B) $G^* = \{|w - \frac{1}{2}| > \frac{1}{2}, \operatorname{Re} w < 1\}$
22. A) $G^* = \{|w - 2| > 2\}$
 B) $G^* = \{|w| < 1, \operatorname{Im} w < 0\}$
23. A) $G^* = \{|w| > 1\}$
 B) $G^* = \{|w| > 1, \operatorname{Re} w + \operatorname{Im} w < 1, \operatorname{Re} w - \operatorname{Im} w < 1\}$
24. A) $G^* = \{\operatorname{Re} w < 0, \operatorname{Im} w > 0\}$
 B) $G^* = \{|w - \frac{2}{3}| > \frac{4}{3}, \operatorname{Re} w > -1\}$
25. A) $G^* = \{|w| < 1\}$
 B) $G^* = \{\operatorname{Re} w < 0, \operatorname{Im} w < 0\}$
26. A) $S(z) = \frac{z - i}{z + i}$
 B) $S(z) = 2i \frac{z - 1}{z + 1}$
27. A) $S(z) = 2z$

B) $S(z) = \frac{2iz}{z-2}$

U zadacima 28.-33. rješenje nije jednoznačno određeno (ovisi o izboru točaka i sl.).
Točnost svog rješenja možete provjeriti preslikavate li dobivenom funkcijom G u G^* .

- 34.** **A)** cijela ravnina s razrezom $(-\infty, 0]$
B) $\{\operatorname{Im} w > 0\}$
- 35.** **A)** prsten $\{1 < |w| < e\}$ s razrezom $[1, e]$
B) cijela ravnina s razrezima $(-\infty, -1]$ i $[1, \infty)$
- 36.** **A)** polukrug $\{|w| < 1, \operatorname{Im} w > 0\}$
B) cijela ravnina s razrezima $(-\infty, 0]$ i $[-i, i]$
- 37.** **A)** pruga $\{0 < \operatorname{Im} w < \pi\}$
B) $\{\operatorname{Re} w > 0\}$ s razrezom $[0, 1]$
- 38.** **A)** cijela ravnina s razrezima $[-1, 1]$ i $[0, +i\infty)$
B) cijela ravnina s razrezom $[0, +\infty)$
- 39.** **A)** $\{\operatorname{Im} w > 0\}$ s razrezom $[0, i]$
B) zraka $\{\arg w = \frac{\pi}{2}\}$
- 40.** **A)** $\{\operatorname{Re} w > 1 - \frac{(\operatorname{Im} w)^2}{4}\}$
B) kružnica $\{|w| = 1\}$ bez točke -1
- 41.** **A)** $ie^{1+i} + 1$
B) $4\pi i$
- 42.** **A)** $\sin 1 + i \operatorname{sh} 1$
B) $-\frac{8}{3}$
- 43.** **A)** $\pi \operatorname{ch} 1 + i(\operatorname{ch} 1 - \operatorname{sh} 1)$
B) $-16i$
- 44.** **A)** 0
B) πi
- 45.** **A)** $2\pi i$
B) $2\pi i \sin 1$
- 46.** **A)** 0
B) $\frac{1}{e}$
- 47.** **A)** 2
B) 1
- 48.** **A)** 0
B) ∞
- 49.** **A)** $\frac{1}{4}$
B) 1

50. A) ∞
 B) 1
51. A) $\sum_{n=0}^{\infty} (-1)^n (n+1) z^{3n}$
 B) $\cos 4 \sum_{n=0}^{\infty} (-1)^n \frac{3^{2n+1}}{(2n+1)!} (z+1)^{2n+1} - \sin 4 \sum_{n=0}^{\infty} (-1)^n \frac{3^{2n}}{(2n)!} (z+1)^{2n}$
52. A) $\frac{1}{3} \sum_{n=0}^{\infty} ((-1)^{n+1} - 2^{-(n+1)}) z^n$
 B) $\frac{\sqrt{2}}{2} \sum_{n=0}^{\infty} (-1)^n \frac{(z - \frac{\pi}{4})^{2n}}{(2n)!} - \frac{\sqrt{2}}{2} \sum_{n=0}^{\infty} (-1)^n \frac{(z - \frac{\pi}{4})^{2n+1}}{(2n+1)!}$
53. A) $\sum_{n=0}^{\infty} \binom{-\frac{1}{2}}{n} \frac{z^{2n+1}}{2n+1}$
 B) $\ln 4 - \sum_{n=1}^{\infty} \frac{(z+1)^n}{n 4^n}$
54. A) $\ln 2 - \sum_{n=1}^{\infty} \frac{1}{n} (2^{-n} + 1) z^n$
 B) $\frac{1}{2} - \frac{1}{2} \sum_{n=0}^{\infty} (-1)^n \frac{4^n}{(2n)!} z^{2n}$
55. A) $\sqrt{i} \sum_{n=0}^{\infty} \binom{\frac{1}{2}}{n} (-1)^n i^n z^n$
 B) $\frac{1}{2i} \sum_{n=0}^{\infty} \frac{1}{n!} ((1+i)^n - (1-i)^n) z^n$
56. A) $\frac{1}{5} \sum_{n=0}^{\infty} ((-1)^{n+1} - 4^{-(n+1)}) z^{2n+1}$
 B) $\frac{1}{2} + \frac{1}{2} \sum_{n=0}^{\infty} \frac{4^n}{(2n)!} z^{2n}$
57. A) $z = 0$ kratnosti 4, $z = 3i$ i $z = -3i$ kratnosti 1
 B) $z = i$ kratnosti 1, $z = 2k\pi i$ i $z = (2k+1)\pi i$, $k \in \mathbb{Z}$, kratnosti 1
58. A) $z = 0$ kratnosti 4, $z = k\pi$, $k \in \mathbb{Z} \setminus \{0\}$, kratnosti 1
 B) $z = 0$ kratnosti 1, $z = 2k\pi i$, $k \in \mathbb{Z} \setminus \{0\}$, kratnosti 2
59. A) $z = 0$ kratnosti 4
 B) $z = 0$ kratnosti 3
60. A) $z = 0$ kratnosti 5
 B) $z = 0$ kratnosti 1
61. A) $-\sum_{n=0}^{\infty} z^{2n}$

- B) $-\frac{1}{9} \sum_{n=0}^{\infty} (-1)^n \frac{n+1}{(z-1)^{n+2}} - \frac{1}{27} \sum_{n=0}^{\infty} \left(\frac{1}{2^{n+2}} + \frac{(-1)^n}{2^{2n+3}} \right) (z-1)^n$
62. A) $\sum_{n=0}^{\infty} \frac{1}{z^{2n+2}}$
 B) $-\frac{1}{9} \sum_{n=0}^{\infty} (-1)^n \frac{n+1}{(z-1)^{n+2}} + \frac{1}{27} \sum_{n=0}^{\infty} \frac{2^{n-1}}{(z-1)^{n+1}} - \frac{1}{27} \sum_{n=0}^{\infty} (-1)^n \frac{(z-1)^n}{2^{2n+3}}$
63. A) $\sum_{n=0}^{\infty} (-1)^n \frac{(z-1)^{n-1}}{2^{n+1}}$
 B) $-\sum_{n=0}^{\infty} \frac{z^n}{3^{n+1}} - \sum_{n=0}^{\infty} \frac{2^n}{z^{n+1}}$
64. A) $\sum_{n=0}^{\infty} (-1)^n \frac{2^n}{(z-1)^{n+2}}$
 B) $\sum_{n=0}^{\infty} \frac{3^n - 2^n}{z^{n+1}}$
65. A) $\frac{1}{2} \sum_{n=0}^{\infty} \frac{(-1)^n}{(z-2)^{n+1}} - \frac{1}{2} \sum_{n=0}^{\infty} (-1)^n \frac{(z-2)^n}{3^{n+1}}$
 B) $\sum_{n=0}^{\infty} (-1)^n \frac{(z-i)^{n-1}}{(2i)^{n+1}}$
66. A) $z=0$ pol 1. reda, $z=i$ i $z=-i$ polovi 2. reda
 B) $z=0$ bitni singularitet
67. A) $z=0$ pol 2. reda, $z=-1$ pol 1. reda
 B) $z=0$ uklonjiv singularitet, $z=k\pi$, $k \in \mathbb{Z} \setminus \{0\}$, polovi 1. reda
68. A) $z=0$ uklonjiv singularitet, $z=k\pi$, $k \in \mathbb{Z} \setminus \{0\}$, polovi 1. reda
 B) $z=0$ pol 1. reda, $z=-3i$ pol 2. reda, $z=3i$ pol 3. reda
69. A) $z=1$ bitni singularitet
 B) $z_k = \cos \frac{\pi(2k+1)}{6} + i \sin \frac{\pi(2k+1)}{6}$, $k=0, \dots, 5$, polovi 1. reda
70. A) $\text{Res}(f, 0) = 1$, $\text{Res}(f, i) = -\frac{1}{2}$, $\text{Res}(f, -i) = -\frac{1}{2}$
 B) $\text{Res}(f, 0) = -\frac{1}{2}$, $\text{Res}(f, 1) = 1 - \cos 1$
71. A) $\text{Res}(f, 0) = 1$, $\text{Res}(f, 1) = -\frac{1}{2}$, $\text{Res}(f, -1) = -\frac{1}{2}$
 B) $\text{Res}(f, 1) = 0$
72. A) $\text{Res}(f, 1) = \frac{1}{4}$, $\text{Res}(f, -1) = -\frac{1}{4}$, $\text{Res}(f, i) = -\frac{i}{4}$, $\text{Res}(f, -i) = \frac{i}{4}$
 B) $\text{Res}(f, 0) = \frac{9}{2}$
73. A) $-\frac{4\pi i}{9}$
 B) $-2\pi i \cos 1$
74. A) πi
 B) $2\pi i$

75. A) $\frac{2}{3}\pi e^{2i}$
 B) $4\pi i(\cos 1 - \sin 1)$
76. A) $2\pi i \sin 1$
 B) $-12i$
77. A) $\frac{2\pi}{3}$
 B) $\frac{\pi}{5}(\cos 1 - e^{-2})$
78. A) $\frac{\pi}{2}$
 B) $\frac{\pi}{4}(1 - e^{-6})$
79. A) $\frac{\pi}{3}$
 B) $\frac{\pi}{3}(3 \cos 1 + \sin 1)e^{-3}$
80. A) $\pi\sqrt{2}$
 B) $\frac{\pi}{3}(\cos 1 - 3 \sin 1)e^{-3}$
81. A) $\Gamma(\frac{4}{3})$
 B) π
82. A) $\Gamma(\frac{7}{6})$
 B) $\frac{2\pi}{3\sqrt{3}}$
83. A) $\frac{\sqrt{\pi}}{4}$
 B) $\frac{\sqrt{\pi}}{2} \frac{\Gamma(\frac{1}{4})}{\Gamma(\frac{3}{4})}$
84. A) $\frac{1}{3}\Gamma(\frac{2}{3})$
 B) $\frac{\pi}{\sqrt{2}}$
85. A) $\sqrt{\pi} \frac{\Gamma(\frac{5}{4})}{\Gamma(\frac{3}{4})}$
 B) $\frac{\pi}{2\sqrt{2}}$
86. Dokaz.
87. Dokaz.