2 Funkcije kompleksne varijable

6. A)

$$w = e^{1-z} = e^{1-x-yi} = e^{1-x}e^{-iy} = e^{1-x}(\cos y - i\sin y)$$

$$\operatorname{Re} w = u(x,y) = e^{1-x}\cos y$$

$$\operatorname{Im} w = v(x,y) = -e^{1-x}\sin y$$

B)

$$w = \operatorname{tg} z = \frac{\sin z}{\cos z} = \frac{\sin(x+iy)}{\cos(x+iy)} = \frac{\sin x\cos(iy) + \cos x\sin(iy)}{\cos x\cos(iy) - \sin x\sin(iy)} = \begin{vmatrix} \cos(iy) = \operatorname{ch} y \\ \sin(iy) = i\operatorname{sh} y \end{vmatrix} =$$

$$= \frac{\sin x\operatorname{ch} y + i\cos x\operatorname{sh} y}{\cos x\operatorname{ch} y - i\sin x\operatorname{sh} y}\cdot\frac{\cos x\operatorname{ch} y + i\sin x\operatorname{sh} y}{\cos x\operatorname{ch} y + i\sin x\operatorname{sh} y} =$$

$$= \frac{\sin x\cos x\operatorname{ch}^2 y - \sin x\cos x\operatorname{sh}^2 y + i(\sin^2 x\operatorname{sh} y\operatorname{ch} y + \cos^2 x\operatorname{sh} y\operatorname{ch} y)}{\cos^2 x\operatorname{ch}^2 y + \sin^2 x\operatorname{sh}^2 y} = \begin{vmatrix} \operatorname{ch}^2 y - \operatorname{sh}^2 y = 1 \\ \sin^2 x + \cos^2 x = 1 \end{vmatrix} =$$

$$= \frac{\sin x\cos x + i\operatorname{sh} y\operatorname{ch} y}{\cos^2 x\operatorname{ch}^2 y + \sin^2 x\operatorname{sh}^2 y} = \frac{\sin x\cos x + i\operatorname{sh} y\operatorname{ch} y}{\operatorname{ch}^2 y - \sin^2 x}$$

$$\operatorname{Re} w = u(x,y) = \frac{\sin x\cos x}{\operatorname{ch}^2 y - \sin^2 x}$$

$$\operatorname{Im} w = v(x,y) = \frac{\operatorname{sh} y\operatorname{ch} y}{\operatorname{ch}^2 y - \sin^2 x}$$

7. A)

$$w = e^{z^2} = e^{(x+iy)^2} = e^{x^2-y^2+i2xy} = e^{x^2+y^2}(\cos(2xy) + i\sin(2xy))$$

$$\operatorname{Re} w = u(x,y) = e^{x^2+y^2}\cos(2xy)$$

$$\operatorname{Im} w = v(x,y) = e^{x^2+y^2}\sin(2xy)$$

B)

$$w = \operatorname{sh} z = \frac{e^z - e^{-z}}{2} = \frac{e^{x+iy} - e^{-x-iy}}{2} = \frac{e^x(\cos y + i\sin y) - e^{-x}(\cos y - i\sin y)}{2}$$

$$\operatorname{Re} w = u(x,y) = \frac{\cos y\,(e^x - e^{-x})}{2} = \operatorname{sh} x\cos y$$

$$\operatorname{Im} w = v(x,y) = \frac{\sin y\,(e^y + e^{-x})}{2} = \operatorname{ch} x\sin y$$

8. A)

$$w = z^2|z| = (x^2 - y^2 + i2xy)\sqrt{x^2 + y^2}$$

$$u(x,y) = (x^2 - y^2)\sqrt{x^2 + y^2}$$

$$v(x,y) = 2xy\sqrt{x^2 + y^2}$$

$$\frac{\partial u}{\partial x} = \frac{x(3x^2 + y^2)}{\sqrt{x^2 + y^2}}$$

$$\frac{\partial v}{\partial y} = \frac{2(x^3 + 2xy^2)}{\sqrt{x^2 + y^2}}$$

$$u'_x \neq v'_y$$

Nije analitička.

B)

$$w = \operatorname{ch}(z-2) = \frac{e^{z-2} + e^{-z+2}}{2} = \frac{e^{x-2+yi} + e^{-x+2-yi}}{2} = \frac{e^{x-2}(\cos y + i\sin y) + e^{-x+2}(\cos y - i\sin y)}{2} =$$

$$= \frac{\cos y\,(e^{x-2} + e^{-x+2})}{2} + i\frac{\sin y\,(e^{x-2} - e^{-x+2})}{2} = \operatorname{ch}(x-2)\cos y + i\operatorname{sh}(x-2)\sin y$$

$$u(x,y) = \operatorname{ch}(x-2)\cos y$$

$$v(x,y) = \operatorname{sh}(x-2)\sin y$$

$$\frac{\partial u}{\partial x} = \operatorname{sh}(x-2)\cos y$$

$$\frac{\partial v}{\partial y} = \operatorname{sh}(x-2)\cos y$$

$$\frac{\partial u}{\partial y} = -\operatorname{ch}(x-2)\sin y$$

$$\frac{\partial v}{\partial x} = \operatorname{ch}(x-2)\sin y$$

$$u'_x = v'_y \qquad\qquad u'_y = -v'_x$$

Zadovoljeni su Cauchy-Riemannovi uvjeti. Funkcija je analitička na cijelom području $\mathbb{C}$.

Kada deriviramo (kao što i inače deriviramo):

$$w'(z) = \operatorname{sh}(z-2)$$

Ili:

$$w'(z) = u'_x + iv'_x = \operatorname{sh}(x-2)\cos y + i\operatorname{ch}(x-2)\sin y = \cdots = \operatorname{sh}(z-2)$$

9. A)

$$w = \bar{z}\,\mathrm{Im}z = (x - iy)y = xy - iy^2$$

$$u(x, y) = xy$$

$$v(x, y) = -y^2$$

$$\frac{\partial u}{\partial x} = y$$

$$\frac{\partial v}{\partial y} = -2y$$

$$u_x' \neq v_y'$$

Nije analitička.

B)

$$w = ze^z = (x + iy)e^{x+iy} = e^x(x + iy)(\cos y + i \sin y) = e^x(x \cos y - y \sin y) + ie^x(x \sin y + y \cos y)$$

$$u(x, y) = e^x(x \cos y - y \sin y)$$

$$v(x, y) = e^x(x \sin y + y \cos y)$$

$$\frac{\partial u}{\partial x} = e^x[(x + 1) \cos y - y \sin y]$$

$$\frac{\partial v}{\partial y} = e^x[(x + 1) \cos y - y \sin y]$$

$$\frac{\partial u}{\partial y} = -e^x[(x + 1) \sin y + y \cos y]$$

$$\frac{\partial v}{\partial x} = e^x[(x + 1) \sin y + y \cos y]$$

$$u_x' = v_y' \qquad\qquad u_y' = -v_x'$$

Zadovoljeni su Cauchy-Riemannovi uvjeti. Funkcija je analitička na cijelom području $\mathbb{C}$.

Kada deriviramo (kao što i inače deriviramo):

$$w'(z) = e^z + ze^z = e^z(z + 1)$$

Ili:

$$w'(z) = u_x' + iv_x' = e^x[(x + 1) \cos y - y \sin y] + ie^x[(x + 1) \sin y + y \cos y] = \cdots = e^z(z + 1)$$

10. A)

$$w = |z|\mathrm{Im}z = y\sqrt{x^2 + y^2}$$

$$u(x,y) = y\sqrt{x^2 + y^2}$$

$$v(x,y) = 0$$

$$\frac{\partial u}{\partial x} = \frac{xy}{\sqrt{x^2 + y^2}}$$

$$\frac{\partial v}{\partial y} = 0$$

$$u'_x \neq v'_y$$

Nije analitička.

B)

$$w = z^3 = (x + iy)^3 = x^3 + i3x^2y - 3xy^2 - iy^3$$

$$u(x,y) = x^3 - 3xy^2$$

$$v(x,y) = 3x^2y - y^3$$

$$\frac{\partial u}{\partial x} = 3x^2 - 3y^2$$

$$\frac{\partial v}{\partial y} = 3x^2 - 3y^2$$

$$\frac{\partial u}{\partial y} = -6xy$$

$$\frac{\partial v}{\partial x} = 6xy$$

$$u'_x = v'_y \qquad\qquad u'_y = -v'_x$$

Zadovoljeni su Cauchy-Riemannovi uvjeti. Funkcija je analitička na cijelom području $\mathbb{C}$.

Kada deriviramo (kao što i inače deriviramo):

$$w'(z) = 3z^2$$

Ili:

$$w'(z) = u'_x + iv'_x = 3x^2 - 3y^2 + i6xy = 3(x^2 + i2xy - y^2) = 3(x + iy)^2 = 3z^2$$

11. A)

$$v(x,y) = e^x \sin y + y^2$$

Provjerimo uvjet $\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} = 0$:

$$\frac{\partial v}{\partial x} = e^x \sin y \rightarrow \frac{\partial^2 v}{\partial x^2} = e^x \sin y$$

$$\frac{\partial v}{\partial y} = e^x \cos y + 2y \rightarrow \frac{\partial^2 v}{\partial y^2} = -e^x \sin y + 2$$

$$\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} = 2 \neq 0$$

Funkcija nije harmonijska.

B)

$$u(x,y) = x^2 + 2x - y^2$$

Provjerimo uvjet $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$:

$$\frac{\partial u}{\partial x} = 2x + 2 \rightarrow \frac{\partial^2 u}{\partial x^2} = 2$$

$$\frac{\partial u}{\partial y} = -2y \rightarrow \frac{\partial^2 u}{\partial y^2} = -2$$

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$$

Funkcija je harmonijska. Možemo odrediti pripadnu konjugiranu funkciju:

$$v(x,y) = \int_{x_0}^{x} \left(-\frac{\partial u(x,y)}{\partial y}\right) dx + \int_{y_0}^{y} \frac{\partial u(x_0,y)}{\partial x} dy + C$$

Odaberimo točku $(x_0, y_0) = (0,0)$.

$$v(x,y) = \int_0^x 2y dx + \int_0^y 2dy + C = 2yx\Big|_0^x + 2y\Big|_0^y + C = 2xy + 2y + C$$

12. A)

$$u(x,y)=2e^x\cos y$$

Provjerimo uvjet $\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}=0$:

$$\frac{\partial u}{\partial x}=2e^x\cos y\to\frac{\partial^2 u}{\partial x^2}=2e^x\cos y$$

$$\frac{\partial u}{\partial y}=-2e^x\sin y\to\frac{\partial^2 u}{\partial y^2}=-2e^x\cos y$$

$$\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}=0$$

Funkcija je harmonijska. Možemo odrediti pripadnu konjugiranu funkciju:

$$v(x,y)=\int_{x_0}^{x}\left(-\frac{\partial u(x,y)}{\partial y}\right)dx+\int_{y_0}^{y}\frac{\partial u(x_0,y)}{\partial x}dy+C$$

Odaberimo točku $(x_0,y_0)=(0,0)$.

$$v(x,y)=\int_0^x 2e^x\sin y\,dx+\int_0^y 2\cos y\,dy+C=2\sin y\,e^x\Big|_0^x+2\sin y\Big|_0^y+C=$$

$$=2e^x\sin y-2\sin y+2\sin y+C=2e^x\sin y+C$$

B)

$$v(x,y)=x^3+6x^2y-3xy^2-2y^3+y^2$$

Provjerimo uvjet $\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}=0$:

$$\frac{\partial v}{\partial x}=3x^2+12xy-3y^2\to\frac{\partial^2 v}{\partial x^2}=6x+12y$$

$$\frac{\partial v}{\partial y}=6x^2-6xy-6y^2+2y\to\frac{\partial^2 v}{\partial y^2}=-6x-12y+2$$

$$\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}=2\neq 0$$

Funkcija nije harmonijska.

13. A)

$$u(x,y)=\frac{x}{x^2+y^2}$$

$$f(\pi)=\frac{1}{\pi}$$

$$f(z)=?$$

Provjerimo prvo je li funkcija $u(x,y)$ harmonijska:

$$\frac{\partial u}{\partial x}=\frac{y^2-x^2}{(x^2+y^2)^2}\rightarrow\frac{\partial^2 u}{\partial x^2}=\frac{2x(x^2-3y^2)}{(x^2+y^2)^3}$$

$$\frac{\partial u}{\partial y}=-\frac{2xy}{(x^2+y^2)^2}\rightarrow\frac{\partial^2 u}{\partial y^2}=-\frac{2x(x^2-3y^2)}{(x^2+y^2)^3}$$

$$\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}=0$$

Funkcija je harmonijska. Možemo odrediti pripadnu konjugiranu funkciju:

$$v(x,y)=\int_{x_0}^{x}\left(-\frac{\partial u(x,y)}{\partial y}\right)dx+\int_{y_0}^{y}\frac{\partial u(x_0,y)}{\partial x}dy+C$$

Odaberimo točku $(x_0,y_0)=(0,1)$.

$$v(x,y)=\int_0^x\frac{2xy}{(x^2+y^2)^2}dx+\int_1^y\frac{y^2-x_0^2}{(x_0^2+y^2)^2}dy+C=y\int_0^x\frac{2xdx}{(x^2+y^2)^2}+\int_1^y\frac{1}{y^2}dy+C=$$

$$=\begin{vmatrix}x^2+y^2=t\\2xdx=dt\end{vmatrix}=y\int_{y^2}^{x^2+y^2}\frac{dt}{t^2}-\frac{1}{y}\bigg|_1^y+C=-y\frac{1}{t}\bigg|_{y^2}^{x^2+y^2}-\frac{1}{y}+1+C=-\frac{y}{x^2+y^2}+\frac{1}{y}-\frac{1}{y}+1+C=$$

$$=-\frac{y}{x^2+y^2}+K$$

$$f(z)=f(x,y)=\frac{x}{x^2+y^2}+i\left(K-\frac{y}{x^2+y^2}\right)$$

Imamo uvjet $f(\pi)=\frac{1}{\pi}$. To znači da je $z=\pi=x+iy\rightarrow x=\pi,\ y=0$.

$$f(\pi,0)=\frac{1}{\pi}+iK=\frac{1}{\pi}\rightarrow K=0$$

Konačno je

$$f(z)=\frac{x-iy}{x^2+y^2}=\frac{\bar{z}}{|z|^2}=\frac{\bar{z}}{z\cdot\bar{z}}=\frac{1}{z}$$

B)

$$v(x,y) = 2x^2 - 2y^2 + x$$

$$f(z) = ?$$

Provjerimo prvo je li funkcija $v(x,y)$ harmonijska:

$$\frac{\partial v}{\partial x} = 4x + 1 \rightarrow \frac{\partial^2 v}{\partial x^2} = 4$$

$$\frac{\partial v}{\partial y} = -4y \rightarrow \frac{\partial^2 v}{\partial y^2} = -4$$

$$\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} = 0$$

Funkcija je harmonijska. Možemo odrediti pripadnu konjugiranu funkciju:

$$u(x,y) = \int_{x_0}^{x} \frac{\partial v(x,y)}{\partial y} dx + \int_{y_0}^{y} \left(-\frac{\partial v(x_0,y)}{\partial x} \right) dy + C$$

Odaberimo točku $(x_0, y_0) = (0,0)$.

$$u(x,y) = -4y \int_0^x dx - \int_0^y (4x_0 + 1) dy + C = -4yx \Big|_0^x - y \Big|_0^y + C = -4xy - y + C$$

$$f(z) = f(x,y) = -4xy - y + C + i(2x^2 - 2y^2 + x) = -4xy - y + i2x^2 - i2y^2 + ix + C =$$

$$= i^2 4xy + 2i(x^2 - y^2) + i^2 y + ix + C = 2i(x^2 - y^2 + i2xy) + i(x + iy) + C = 2iz^2 + iz + C$$

14. A)

$$v(x,y) = -2\sin(2x)\,\mathrm{sh}(2y) + y$$

$$f(0) = 2$$

$$f(z) = ?$$

Provjerimo prvo je li funkcija $v(x,y)$ harmonijska:

$$\frac{\partial v}{\partial x} = -4\cos(2x)\,\mathrm{sh}(2y) \rightarrow \frac{\partial^2 v}{\partial x^2} = 8\sin(2x)\,\mathrm{sh}(2y)$$

$$\frac{\partial v}{\partial y} = -4\sin(2x)\,\mathrm{ch}(2y) + 1 \rightarrow \frac{\partial^2 v}{\partial y^2} = -8\sin(2x)\,\mathrm{sh}(2y)$$

$$\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} = 0$$

Funkcija je harmonijska. Možemo odrediti pripadnu konjugiranu funkciju:

$$u(x,y) = \int_{x_0}^{x} \frac{\partial v(x,y)}{\partial y} dx + \int_{y_0}^{y} \left(-\frac{\partial v(x_0,y)}{\partial x} \right) dy + C$$

Odaberimo točku $(x_0, y_0) = (0,0)$.

$$u(x,y) = \int_0^x (-4\sin(2x)\operatorname{ch}(2y) + 1) dx + \int_0^y 4\cos(2x_0)\operatorname{sh}(2y)\, dy + C =$$

$$= -4\operatorname{ch}(2y) \left[-\frac{\cos(2x)}{2} \right] \Bigg|_0^x + x \Bigg|_0^x + 4 \left[\frac{\operatorname{ch}(2y)}{2} \right] \Bigg|_0^y + C =$$

$$= -4\operatorname{ch}(2y) \frac{1-\cos(2x)}{2} + x + 4\frac{\operatorname{ch}(2y)-1}{2} + C =$$

$$= -2\operatorname{ch}(2y)\,(1-\cos(2x)) + x + 2(\operatorname{ch}(2y)-1) + C =$$

$$2\cos(2x)\operatorname{ch}(2y) + x + K$$

$$f(z) = f(x,y) = 2\cos(2x)\operatorname{ch}(2y) + x + K + i(-2\sin(2x)\operatorname{sh}(2y) + y)$$

$$f(0,0) = 2 + K = 2 \rightarrow K = 0$$

$$f(z) = f(x,y) = 2\cos(2x)\operatorname{ch}(2y) + x + i(-2\sin(2x)\operatorname{sh}(2y) + y) =$$

$$= 2\cos(2x)\operatorname{ch}(2y) - 2i\sin(2x)\operatorname{sh}(2y) + x + iy = 2\cos(2x)\cos(2iy) - 2\sin(2x)\sin(2iy) + z =$$

$$= 2\cos(2x + 2iy) + z = 2\cos(2z) + z$$

B)

$$v(x,y) = 2\cos x \operatorname{ch} y - x^2 + y^2$$

$$f(z) = ?$$

Provjerimo prvo je li funkcija $v(x,y)$ harmonijska:

$$\frac{\partial v}{\partial x} = -2\sin x \operatorname{ch} y - 2x \rightarrow \frac{\partial^2 v}{\partial x^2} = -2\cos x \operatorname{ch} y - 2$$

$$\frac{\partial v}{\partial y} = 2\cos x \operatorname{sh} y + 2y \rightarrow \frac{\partial^2 v}{\partial y^2} = 2\cos x \operatorname{ch} y + 2$$

$$\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} = 0$$

Funkcija je harmonijska. Možemo odrediti pripadnu konjugiranu funkciju:

$$u(x,y)=\int_{x_0}^{x}\frac{\partial v(x,y)}{\partial y}dx+\int_{y_0}^{y}\left(-\frac{\partial v(x_0,y)}{\partial x}\right)dy+C$$

Odaberimo točku $(x_0,y_0)=(0,0)$.

$$u(x,y)=\int_{0}^{x}(2\cos x\operatorname{sh}y+2y)dx+\int_{0}^{y}(2\sin x_0\operatorname{ch}y+2x_0)dy+C=$$

$$=2\operatorname{sh}y\sin x\Big|_0^x+2yx\Big|_0^x+C=2\sin x\operatorname{sh}y+2xy+C$$

$$f(z)=f(x,y)=2\sin x\operatorname{sh}y+2xy+C+i(2\cos x\operatorname{ch}y-x^2+y^2)=$$

$$=2\sin x\operatorname{sh}y+2i\cos x\operatorname{ch}y+2xy+i(-x^2+y^2)+C=$$

$$=-i^2 2\sin x\operatorname{sh}y+2i\cos x\operatorname{ch}y-i(x^2-y^2+i2xy)+C=2i(\cos x\operatorname{ch}y-i\sin x\operatorname{sh}y)-iz^2+C=$$

$$=2i(\cos x\cos(iy)-\sin x\sin(iy))-iz^2+C=2i\cos(x+iy)-iz^2+C=2i\cos(z)-iz^2+C$$

15. A)

$$u(x,y)=2\sin x\operatorname{ch}y-x$$

$$f(z)=?$$

Provjerimo prvo je li funkcija $u(x,y)$ harmonijska:

$$\frac{\partial u}{\partial x}=2\cos x\operatorname{ch}y-1\rightarrow\frac{\partial^2 u}{\partial x^2}=-2\sin x\operatorname{ch}y$$

$$\frac{\partial u}{\partial y}=2\sin x\operatorname{sh}y\rightarrow\frac{\partial^2 u}{\partial y^2}=2\sin x\operatorname{ch}y$$

$$\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}=0$$

Funkcija je harmonijska. Možemo odrediti pripadnu konjugiranu funkciju:

$$v(x,y)=\int_{x_0}^{x}\left(-\frac{\partial u(x,y)}{\partial y}\right)dx+\int_{y_0}^{y}\frac{\partial u(x_0,y)}{\partial x}dy+C$$

Odaberimo točku $(x_0,y_0)=(0,1)$.

$$v(x,y)=\int_{0}^{x}(-2\sin x\operatorname{sh}y)dx+\int_{1}^{y}(2\cos x_0\operatorname{ch}y-1)dy+C=2\operatorname{sh}y\cos x\Big|_0^x+(2\operatorname{sh}y-y)\Big|_0^y+C=$$

$$=2\cos x\operatorname{sh}y-2\operatorname{sh}y+2\operatorname{sh}y-y+C=\cos x\operatorname{sh}y-y+C$$

$$f(z) = f(x,y) = 2\sin x \operatorname{ch} y - x + i(2\cos x \operatorname{sh} y - y + C) =$$

$$= 2\sin x \operatorname{ch} y + 2i\cos x \operatorname{sh} y - x - iy + iC = 2\sin x \cos(iy) + 2\cos x \sin(iy) - (x + iy) + K =$$

$$= 2\sin(x + iy) - z + K = 2\sin z - z + K$$

B)

$$v(x,y) = 2(\operatorname{ch} x \sin y - xy)$$

$$f(0) = 0$$

$$f(z) = ?$$

Provjerimo prvo je li funkcija $v(x,y)$ harmonijska:

$$\frac{\partial v}{\partial x} = 2(\operatorname{sh} x \sin y - y) \rightarrow \frac{\partial^2 v}{\partial x^2} = 2\operatorname{ch} x \sin y$$

$$\frac{\partial v}{\partial y} = 2(\operatorname{ch} x \cos y - x) \rightarrow \frac{\partial^2 v}{\partial y^2} = -2\operatorname{ch} x \sin y$$

$$\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} = 0$$

Funkcija je harmonijska. Možemo odrediti pripadnu konjugiranu funkciju:

$$u(x,y) = \int_{x_0}^{x} \frac{\partial v(x,y)}{\partial y} dx + \int_{y_0}^{y} \left(-\frac{\partial v(x_0,y)}{\partial x}\right) dy + C$$

Odaberimo točku $(x_0, y_0) = (0,0)$.

$$u(x,y) = \int_{0}^{x} 2(\operatorname{ch} x \cos y - x) dx - \int_{0}^{y} 2(\operatorname{sh} x_0 \sin y - y) dy + C =$$

$$= 2\cos y \operatorname{sh} x \Big|_0^x - x^2 \Big|_0^x + y^2 \Big|_0^y + C = 2\operatorname{sh} x \cos y - x^2 + y^2 + C$$

$$f(z) = f(x,y) = 2\operatorname{sh} x \cos y - x^2 + y^2 + C + 2i(\operatorname{ch} x \sin y - xy)$$

$$f(0,0) = C = 0$$

$$f(z) = f(x,y) = 2\operatorname{sh} x \cos y - x^2 + y^2 + 2i(\operatorname{ch} x \sin y - xy)$$

$$= 2\operatorname{sh} x \cos y + 2i\operatorname{ch} x \sin y - (x^2 - y^2 + 2ixy) = 2\operatorname{sh} x \operatorname{ch}(iy) + 2\operatorname{ch} x \operatorname{sh}(iy) - z^2 =$$

$$= 2\operatorname{sh}(x + iy) - z^2 = 2\operatorname{sh} z - z^2$$