AUD 4 → BURIĆ

05.11.2009.

D152

14h

$e^z = \sum \frac{z^n}{n!}$, sin, cos

$$\left.\begin{aligned} (1+z)^\alpha &= \sum_{n=0} \binom{\alpha}{n} z^n \\ \frac{1}{1-z} &= \sum z^n \end{aligned}\right\} |z|<1$$ OKO TOČKE $\emptyset$

$f(z) = \sum c_n (z-z_0)^n$

IMA MJESTA KOD ŽELJKE

DZ3

11. B) $z_0 = -1$

$f(z) = \sin(3z-1)$

$\sin z = \sum (-1)^k \frac{z^{2k+1}}{(2k+1)!}$

NAMJESTIMO Z OKO z_0

$$f(z) = \sin(3(z+1-1)-1) = \sin(3(z+1)-4)$$
$$= \sin 3(z+1)\cos 4 - \cos 3(z+1)\sin 4$$
$$= \cos 4 \sum_{k=0} (-1)^k \frac{3^{2k+1}(z+1)^{2k+1}}{(2k+1)!} - \sin 4 \sum_{k=0} (-1)^k \frac{3^{2k}(z+1)^{2k}}{(2k)!}$$ KRAJ.

13 A) $f(z) = \operatorname{arsh} z$, $z_0 = \emptyset$

$$f'(z) = \frac{1}{\sqrt{1+z^2}} \overset{\text{BINOMNA FORMULA}}{=} (1+z^2)^{-\frac{1}{2}} = \sum_{n=0} \binom{-\frac{1}{2}}{n} z^{2n} \quad / \int dz$$

$$f(z) = \sum \binom{-\frac{1}{2}}{n} \frac{z^{2n+1}}{2n+1}$$ KRAJ.

NULTOČKE ... DEF. MOGUĆE NA 2MI 30 STR

$\boxed{17.a}$ $f(z) = z^6 + 9z^4$

$f(z) = 0$, NAĐEMO SVE NULTOČKE

$z^4(z^2+9) = 0$

$z^4(z+3i)(z-3i) = 0$

$z_1 = 0$, $z_2 = 3i$, $z_3 = -3i$

KRATNOST 4 , 1 , 1

SUMA KRATNOSTI MORA BITI JEDNAKA NAJV. POTENCIJI.

$k_1 = 4$ $\quad k_2 = 1$ $\quad k_3 = 1$

$\boxed{19.A}$ $f(z) = 2(\operatorname{ch} z - 1) - z^2$, $z=0$ ← KRATNOST NULTOČKE

$f(0) = 0$ ✓

$f'(z) = 2\operatorname{sh} z - 2z \rightarrow f'(0) = 0$ ✓

$f''(z) = 2\operatorname{ch} z - 2 \rightarrow f''(0) = 0$ ✓

$f'''(z) = 2\operatorname{sh} z \rightarrow f'''(0) = 0$ ✓

$f^{IV}(z) = 2\operatorname{ch} z \rightarrow f^{IV}(0) = 2 \neq 0$ → KRATNOST 4

$\boxed{20.A}$ $f(z) = z^2(e^{z^3} - 1)$ KRATNOST NULTOČKE $z=0$

PREKO TAYLORA

$$= z^2\left(\sum_{n=0}^{\infty} \frac{z^{3n}}{n!} - 1\right) = z^2\left(\cancel{1} + \frac{z^3}{1} + \frac{z^6}{2!} + \frac{z^9}{3!} + \dots \cancel{-1}\right) = z^5\left(1 + \frac{z^3}{2!} + \frac{z^6}{3!} + \dots\right)$$

KRATNOST

$k = 5$

ZAD $f(z) = \sum c_n (z-z_0)^n$

18. b) $f(z) = \frac{1-\operatorname{ch} z}{z}$

$f(z) = 0$

$1 - \operatorname{ch} z = 0$

$\operatorname{ch} z = 1 \quad / \operatorname{Arch}$

$z = \operatorname{Arch} 1$

$\operatorname{Arch} z = \operatorname{Ln}(z + \sqrt{z^2-1}) \quad \operatorname{ch} z = 1 \quad / \operatorname{Arch}$

$z = \operatorname{Arch} 1 = \operatorname{Ln} 1 = i 2k\pi$

$z = i2k\pi$ → NULTOČKE (BESK. NJIH)

BROJNIK

$b(z) = 1 - \operatorname{ch} z \quad \to \quad b(2k\pi i) = \emptyset$

$b'(z) = -\operatorname{sh} z \quad \to \quad b'(2k\pi i) = -\operatorname{sh}(2k\pi i) = -i \underbrace{\sin(2k\pi)}_{\emptyset} = \emptyset$

$b''(z) = -\operatorname{ch} z \quad \to \quad b''(2k\pi i) = -\operatorname{ch}(2k\pi i) = -\cos(2k\pi) = -1 \neq 0$

$z = 2k\pi i$, KRATNOST 2, $k \in \mathbb{Z} \setminus \{0\}$ → PROVJ. NAZIVNIK

$z = \emptyset$, KRATNOST 1 $\quad (2 - 1 = 1)$

KRATN. $z=0$ U BROJNIKU MINUS

KRATNOST $z=0$ U NAZIVNIKU