

29.09.2009

BURIC - PREDAVANJE

• trig FSE

$$\sin z = \frac{1}{2i} (e^{iz} - e^{-iz})$$

$$\cos z = \frac{1}{2} (e^{iz} + e^{-iz})$$

$$\tanh z = \frac{\sinh z}{\cosh z}$$

$$\coth z = \frac{\cosh z}{\sinh z}$$

• hiperb. FSE

$$\sinh z = \frac{1}{2} (e^z - e^{-z})$$

$$\cosh z = \frac{1}{2} (e^z + e^{-z})$$

prim

$$\cosh i = \cos 1 \approx 1,54$$

$$\sin iz = i \cdot \sinh z$$

$$\cos iz = \cosh z$$



DEF

$$w = \operatorname{Arcsin} z \iff z = \sin w$$

42 STR 12 VOD JEONE OD FORMULA

1M1 2008

3) a) $w = \operatorname{Arccos} z$

$$z = \cos w$$

$$z = \frac{1}{2} (e^w + e^{-w})$$

$$-e^{iw} + 2z - e^{-iw} = 0 / e^{iw}$$

$$-e^{2iw} + 2z e^{iw} - 1 = 0$$

$$t = e^{iw}$$

$$e^{iw} = \frac{-2z \pm \sqrt{4z^2 - 4}}{-2}$$

$$e^{iw} = z \pm \sqrt{z^2 - 1} / \operatorname{Ln}$$

$$iw = \operatorname{Ln} (z \pm \sqrt{z^2 - 1}) / \frac{1}{i}$$

$$w = -i \operatorname{Ln} (z \pm \sqrt{z^2 - 1})$$

b) $\operatorname{Arccos}(2i) = ?$

$$= -i \operatorname{Ln} (2i + \sqrt{-4 - 1}) = -i \operatorname{Ln} (2i \pm i\sqrt{5})$$

$$= -i \operatorname{Ln} (i(2 \pm \sqrt{5}))$$

$$\operatorname{Ln} z = \ln|z| + i(\arg z + 2k\pi)$$

$$w_1 = -i (\ln|2 + \sqrt{5}| + i(\frac{\pi}{2} + 2k\pi))$$

$$w_2 = -i (\ln|2 - \sqrt{5}| + i(\frac{3\pi}{2} + 2k\pi))$$

VRATA
SE INAC
NE RIJE
PODIZU SE

1. OPCA EKSPON. FJA

$$z^d := e^{d \cdot \operatorname{Lm} z}$$

~~$$(z^d)^n = z^{d \cdot n}$$~~

prin

$$i^{\operatorname{Lm} i}$$

$$= e^{\operatorname{Lm} i \cdot \operatorname{Lm} i} = e^{i(\frac{\pi}{2} + 2k\pi) \cdot i(\frac{\pi}{2} + 2l\pi)}$$

$$= e^{-\left(\frac{\pi}{2} + 2k\pi\right)\left(\frac{\pi}{2} + 2l\pi\right)}$$

modul $|i^{\operatorname{Lm} i}| = e^{-\left(\frac{\pi}{2} + 2k\pi\right)\left(\frac{\pi}{2} + 2l\pi\right)}$

$$, \arg i^{\operatorname{Lm} i} = \phi$$

6

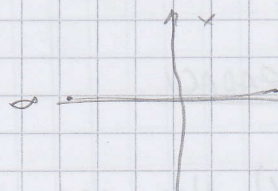
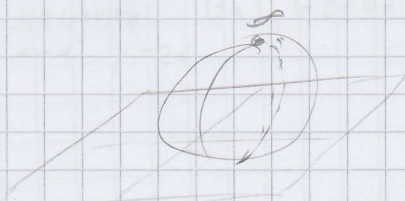
MÖBIUSOVA TRANSFORMACIJA

$$S: \bar{\mathbb{C}} \rightarrow \bar{\mathbb{C}}$$

$$\bar{\mathbb{C}} = \mathbb{C} \cup \{\infty\}$$

JEDNA JEDINA \leftrightarrow

NEMA $\pm \infty$



$$S(z) = \frac{az+b}{cz+d}, \quad a, b, c, d \in \mathbb{C}, \quad ad \neq bc$$

Prim. $S(z) = \frac{iz+1}{z}$

$$S(z) = \frac{z+i}{z+z}$$

ANALITIČKA \rightarrow E ODRV.

$$S(z) = \frac{1}{z+i}$$

KONFORMNO PRESLIK.

$$S'(z) = \frac{ad-bc}{(cz+d)^2} \neq 0$$

$$ad-bc \neq 0$$

BOD BESKON.

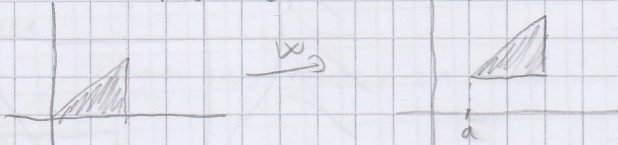
DOLGAZ $S(z)$ JE KONFORMNA ZA $\forall z \in \bar{\mathbb{C}}$

RASTAV M.T. NA OSNOVNI GEOM. PRESLIKAVANJA

1) TRANSLACIJA

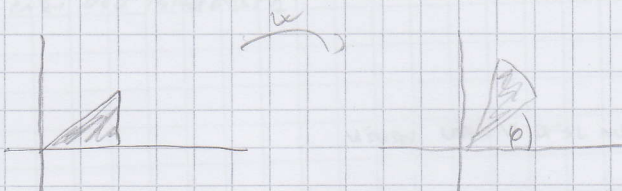
$$w = z + a$$

NAGI SLIKU MÖ TR.



2) ROTACIJA

$$w = e^{i\varphi} \cdot z$$



ZA KUT φ OKO ISHOD.

3) HOMOTETIJA

$$w = k \cdot z, \quad k \in \mathbb{R}^+$$

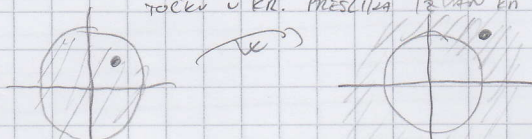


DILATACIJA ($k > 1$)

KONTRAKCIJA ($k < 1$)

4) INVERZIJA

$$w = \frac{1}{z}$$



TOČKA U KR. PRESLIKUJE IZVAN KR.

M.T. KOMPOZICIJA TRANSFORMACIJA

M. t. PRESLIKAVA KRUŽNICE U KRUŽNICE (PRAVCI SU BESK. KRUŽ.)

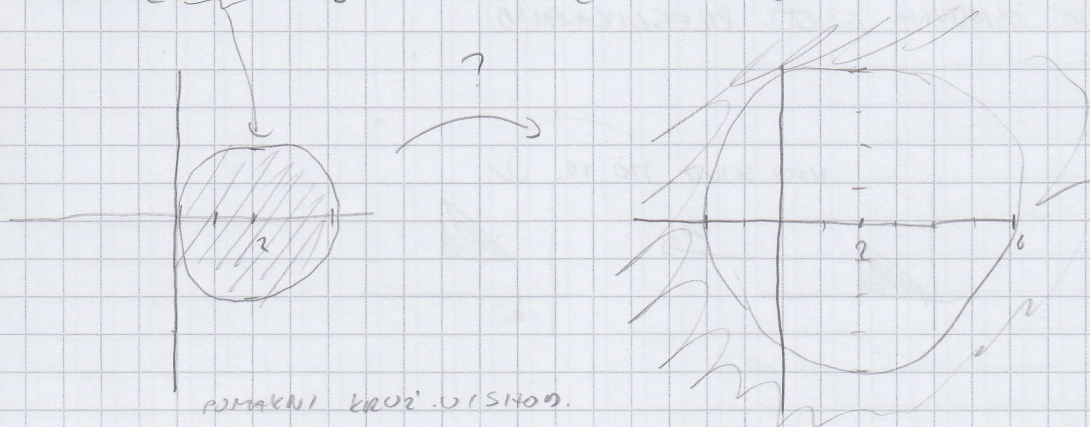
ZADACI

- 1.) NABI M. t. KOJA PRESLIKAVA G U G^*
 - a.) PREKO KOMPOZICIJA ELEM. TRANSF.
 - b.) PREKO FORMULE
- 2.) PRESLIKAJ M. t. NEKO PODRUČJE G .
- 3.) PRESLIKAJ PODRUČJE G NEKOM OPĆENITOM FJOM.
- 4.) PRESLIKAJ KONVOZICIJOM FJOM IZ 2.) I 3.)

ZADATAK

- 1.) ODN. M. t. KOJA PRESLIKAVA G U G^*

$$G = \{ |z-2| < 2 \} \cup G^* = \{ |z-2| > 4 \}$$



POMAKNI KRUŽ. U IŠTOM.

$$S_1(z) = z-2$$

$$S_2(z) = \frac{1}{2}z \rightarrow \text{STAVI NA JEDINIČNU KRUŽN.}$$

$$S_3(z) = \frac{1}{z} \rightarrow \text{INVERZ}$$

$$S_4(z) = 4 \cdot z \rightarrow \text{HOMOTETIJA (VEĆAJ NA POL. 4)}$$

$$S_5(z) = z+2 \rightarrow \text{TRANSLACIJA U DESNO ZA 2}$$

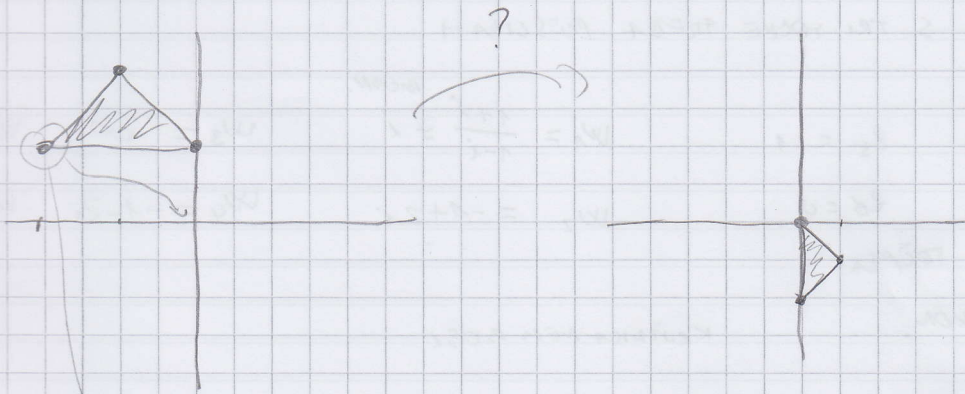
$$S(z) = S_5 \circ S_4 \circ S_3 \circ S_2 \circ S_1(z)$$

REDOSLED

1. INVERZIJA
2. HOMOTETIJA
3. ROTACIJA
4. TRANSLACIJA

$$S(z) = 4 \cdot \frac{1}{z-2} + 2$$

141-2008) 6) ODR. M. E. IČOJA JE PROJEKCIJA S VRIJEDNOSTIMA $(3i)$, $(-2+4i)$, $(-4+2i)$ NISLI U TROKUTU SVRHI 0 , $1-i$, $-2i$



$$S_1(z) = z + 4 - 2i$$

TRANSLACIJA

$$S_2(z) = \frac{1}{2}z$$

SKALIRANJE

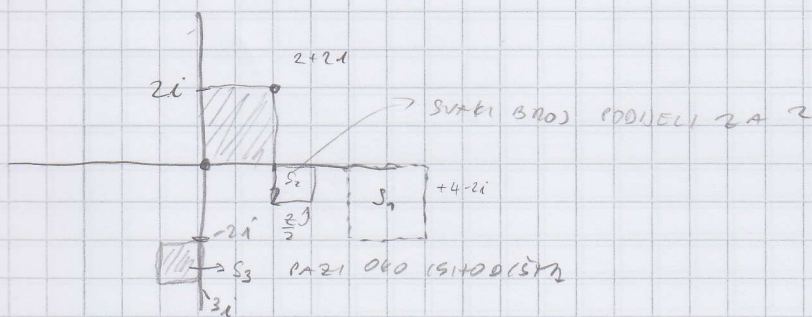
$$S_3(z) = z \cdot e^{-i\frac{\pi}{2}}$$

$$S(z) = S_3 \circ S_2 \circ S_1(z)$$

$$= \frac{1}{2}(z + 4 - 2i) \cdot e^{-i\frac{\pi}{2}} = \cos(-\frac{\pi}{2}) + i\sin(-\frac{\pi}{2})$$

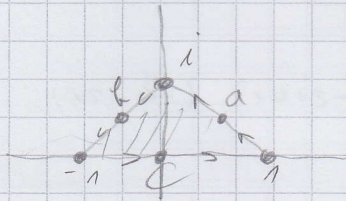
$$S(z) = -\frac{2i}{2} - 2i - 1$$

6.) DODIV. M. E. TRANS. MAPI SRIKOV KVARATO S VRIJEDNOSTIMA 0 , 2 , $2+2i$



22.)
ZAD

PRESL. TROKUT. S VRH -1, 1, i S JOM $w = \frac{z+i}{z-i}$



NE PŠE DA JE MÖB. TRAN.

1. POGLERAJ DAL JE P1.4

SVAKI SEGMENT POSRBNO S TRI TOČKE TREBA PRESLIKATI

$$\begin{aligned} z_1 &= 1 & z_3 &= i & z_5 &= -1 \\ z_2 &= \frac{1}{2} + i\frac{1}{2} & z_4 &= -\frac{1}{2} - i\frac{1}{2} & z_6 &= 0 \end{aligned}$$

$$w_1 = \frac{1+i}{1-i} = i \quad \text{BACION.}$$

$$w_3 = \infty$$

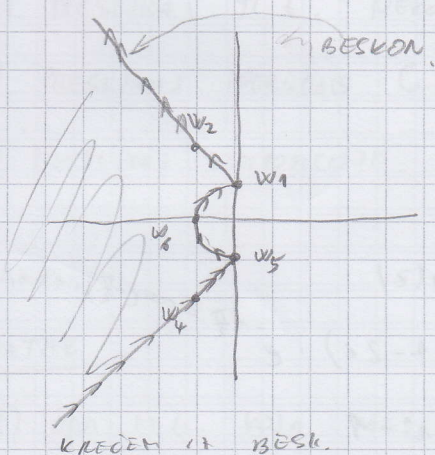
$$w_5 = -$$

$$w_2 = -1 + 2i$$

$$w_4 = -1 - 2i$$

$$w_6 = -$$

BITAN JE REDOSLJED TOČAKA



KREĆENJE BESKON.

DEŠNA RUKA