6. zad.

A) $\sum_{n=1}^{\infty} n^n z^n$

$\frac{1}{R} = \lim_{n\to\infty} \sqrt[n]{|c_n|} = \lim_{n\to\infty} \sqrt[n]{|n^n|} = \lim_{n\to\infty} n = \infty$

$R = 0$

B) $\sum_{n=1}^{\infty} \cos(in)\, z^n$

$c_n = \cos in = \frac{e^{i\cdot in} + e^{-i\cdot in}}{2} = \frac{e^{-n}}{2} + \frac{e^{n}}{2}$

$\frac{1}{R} = \lim_{n\to\infty} \left|\frac{c_{n+1}}{c_n}\right| = \frac{\frac{e^{-n-1}+e^{n+1}}{2}}{\frac{e^{-n}+e^{n}}{2}} = \lim_{n\to\infty} \frac{e^{-n}\cdot e^{-1} + e^{n}\cdot e}{e^{-n} + e^{n}} \cdot \frac{e^{n}}{e^{n}}$

$\frac{1}{R} = \lim_{n\to\infty} \frac{e^{-1} + e^{2n}\cdot e}{1 + e^{2n}} \frac{: e^{2n}}{: e^{2n}} = \lim_{n\to\infty} \frac{e^{-2n}\cdot e^{-1} + e}{e^{-2n} + 1} = e$

$R = \frac{1}{e}$

7. zad.

A) $\sum_{n=1}^{\infty} \frac{n}{2^n} z^n$

$\frac{1}{R} = \lim_{n\to\infty} \left|\frac{c_{n+1}}{c_n}\right| = \lim_{n\to\infty} \frac{\frac{n+1}{2^{n+1}}}{\frac{n}{2^n}}$

$c_n = \frac{n}{2^n}$

$\frac{1}{R} = \lim_{n\to\infty} \frac{\frac{n+1}{2^n\cdot 2}}{\frac{n}{2^n}} = \lim_{n\to\infty} \frac{n+1}{2n} \frac{:n}{:n} = \lim_{n\to\infty} \frac{1+\frac{1}{n}}{2} = \frac{1}{2}$

$R = 2$

B) $\sum_{n=1}^{\infty} ch\frac{i}{n}\, z^n$ $\qquad c_n = ch\frac{i}{n} = \cos\frac{1}{n}$

$\frac{1}{R} = \lim_{n\to\infty} \sqrt[n]{|c_n|} = \lim_{n\to\infty} \left(\cos\frac{1}{n}\right)^{\frac{1}{n}} = 1$

$R = 1$

8. zad.

A) $\sum_{n=1}^{\infty} n!\, z^n \qquad c_n = n!$

$$\frac{1}{R} = \lim_{n\to\infty} \left|\frac{c_{n+1}}{c_n}\right| = \lim_{n\to\infty} \frac{(n+1)!}{n!} = \lim_{n\to\infty} \frac{\cancel{n!}\,(n+1)}{\cancel{n!}} = \infty$$

$R = 0$

B) $\sum_{n=1}^{\infty} \left(\frac{z}{in}\right)^n = \sum_{n=1}^{\infty} \frac{z^n}{(in)^n} \qquad c_n = \frac{1}{(in)^n}$

$$\frac{1}{R} = \lim_{n\to\infty} \sqrt[n]{|c_n|} = \frac{1^{\frac{1}{n}}}{in} = 0 \qquad R = \infty$$

9. zad.

A) $\sum_{n=1}^{\infty} \frac{(2n)!}{(n!)^2}\, z^n \qquad c_n = \frac{(2n)!}{(n!)^2}$

$$\frac{1}{R} = \lim_{n\to\infty} \frac{c_{n+1}}{c_n} = \lim_{n\to\infty} \frac{\frac{(2n+2)!}{((n+1)!)^2}}{\frac{(2n)!}{(n!)^2}} = \lim_{n\to\infty} \frac{\frac{\cancel{(2n)!}\,(2n+1)(2n+2)}{\cancel{n!}\,(n+1)\cdot\cancel{n!}\,(n+1)}}{\frac{\cancel{(2n)!}}{\cancel{n!^2}}}$$

$$\frac{1}{R} = \lim_{n\to\infty} \frac{(2n+1)\cdot 2\cancel{(n+1)}}{(n+1)\cancel{(n+1)}} = \lim_{n\to\infty} \frac{4n+2 \;:n}{n+1 \;:n} = \lim_{n\to\infty} \frac{4+2/n}{1+1/n} = 4$$

$R = \frac{1}{4}$

B) $\sum_{n=1}^{\infty} e^{i\pi/n} z^n, \qquad c_n = e^{\frac{i\pi}{n}}$

$$\frac{1}{R} = \lim_{n\to\infty} \sqrt[n]{|c_n|} = \lim_{n\to\infty} |e^{i\pi}|^{\frac{1}{n^2}} = 1$$

$R = 1$

10. zad.

A) $\sum_{n=1}^{\infty} \frac{n^n}{(2n)!} z^n$, $c_n = \frac{n^n}{(2n)!}$

$$\frac{1}{R} = \lim_{n\to\infty} \left|\frac{c_{n+1}}{c_n}\right| = \lim_{n\to\infty} \frac{\frac{(n+1)^{n+1}}{(2n+2)!}}{\frac{n^n}{(2n)!}} = \frac{\frac{(n+1)^n \cdot \cancel{(n+1)}}{\cancel{2n!} \cdot (2n+1)\, 2\cancel{(n+1)}}}{\frac{n^n}{\cancel{(2n)!}}}$$

$$\frac{1}{R} = \lim_{n\to\infty} \frac{(n+1)^n}{n^n (2n+1)} = \lim_{n\to\infty} \left(\frac{n+1}{n}\right)^n \cdot \frac{1}{2n+1} = \lim_{n\to\infty} \left(1+\frac{1}{n}\right)^n \cdot \frac{1}{2n+1}$$

$$\frac{1}{R} = \lim_{n\to\infty} \frac{1}{2n+1} = 0 \qquad R = \infty$$

B) $\sum_{n=1}^{\infty} e^{in} z^n$, $c_n = e^{in}$

$$\frac{1}{R} = \lim_{n\to\infty} \frac{c_{n+1}}{c_n} = \lim_{n\to\infty} \left|\frac{\cancel{e^{in}} \cdot e^{i}}{\cancel{e^{in}}}\right| = |e^{i}| = |\cos 1 + i \sin 1|$$

$$\frac{1}{R} = \sqrt{\cos^2 1 + \sin^2 1} = 1 \qquad R = 1$$

11. zad. Razvij u T. red. oko točke z_0 sljedeće funkcije:

A) $f(z) = \frac{1}{(1+z^3)^2} = (1+z^3)^{-2}$, $z_0 = 0$

$$f(z) = \sum_{n=0}^{\infty} \binom{-2}{n} z^{3n} = \sum_{n=0}^{\infty} (-1)^n \cdot (n+1) \cdot z^{3n}$$

$$\binom{-2}{n} = \frac{-2(-2-1)\dots(-2-n+1)}{n!} = (-1)^n \frac{1\cdot 2\cdot 3 \cdots (n+1)}{n!}$$

$$= (-1)^n \cdot \frac{(n+1)!}{n!} = (-1)^n \cdot \frac{\cancel{n!}\,(n+1)}{\cancel{n!}}$$

B) $f(z) = \sin(3z-1)$, $z_0 = -1$

$$f(z) = \sin(3(z+1-1)-1) = \sin(3(z+1)-4)$$

$$f(z) = \sin(3(z+1))\cos 4 - \cos(3(z+1))\sin 4$$

$$f(z) = \cos 4 \sum_{n=0}^{\infty} (-1)^n \frac{3^{2n+1}(z+1)^{2n+1}}{(2n+1)!} - \sin 4 \sum_{n=0}^{\infty} (-1)^n \frac{3^{2n}(z+1)^{2n}}{(2n)!}$$

12. zad.

A) $f(z) = \frac{1}{(z+1)(z-2)}$, $z_0 = 0$

$$\frac{1}{(z+1)(z-2)} = \frac{A}{z+1} + \frac{B}{z-2} = \frac{Az - 2A + Bz + B}{(z+1)(z-2)} = z(A+B) - 2A + B$$

$A + B = 0$ $\quad B = \frac{1}{3}$ $\quad A = -\frac{1}{3}$

$-2A + B = 1$

$3B = 1$

$$f(z) = -\frac{1}{3} \cdot \frac{1}{z+1} + \frac{1}{3} \cdot \frac{1}{z-2} = -\frac{1}{3} \cdot \frac{1}{1+z} + \frac{1}{3} \cdot \frac{1}{-2\left(1-\frac{z}{2}\right)}$$

$$f(z) = -\frac{1}{3} \sum_{n=0}^{\infty} (-1)^n z^n - \frac{1}{6} \sum \frac{z^n}{2^n} = -\frac{1}{3} \sum_{n=0}^{\infty} (-1)^n z^n - \frac{1}{3} \sum_{n=0}^{\infty} \frac{z^n}{2^{n+1}}$$

$$f(z) = \frac{1}{3} \sum_{n=0}^{\infty} (-1)^{n+1} \cdot z^n - \frac{1}{3} \sum_{n=0}^{\infty} \frac{z^n}{2^{n+1}} = \frac{1}{3} \sum_{n=0}^{\infty} \left((-1)^{n+1} - 2^{-(n+1)}\right) z^n$$

B) $f(z) = \cos z$, $z_0 = \frac{\pi}{4}$

$$f(z) = \cos\left(z - \frac{\pi}{4} + \frac{\pi}{4}\right) = \cos\left(z - \frac{\pi}{4}\right) \cdot \cos\frac{\pi}{4} - \sin\left(z - \frac{\pi}{4}\right) \cdot \sin\frac{\pi}{4}$$

$$= \frac{\sqrt{2}}{2} \sum_{n=0}^{\infty} (-1)^n \frac{(z - \pi/4)^{2n}}{(2n)!} - \frac{\sqrt{2}}{2} \sum_{n=0}^{\infty} (-1)^n \frac{(z - \pi/4)^{2n+1}}{(2n+1)!}$$

13. zad.

A) $f(z) = \operatorname{arsh} z \,,\quad z_0 = 0$

$$f'(z) = \frac{1}{\sqrt{1+z^2}} = (1+z)^{-\frac{1}{2}}$$

$$f'(z) = \sum_{n=0}^{\infty} \binom{-\frac{1}{2}}{n} z^{2n} \quad / \int$$

$$f(z) = \sum_{n=0}^{\infty} \binom{-\frac{1}{2}}{n} \frac{z^{2n+1}}{2n+1}$$

B) $f(z) = \ln(3-z) \,,\quad z_0 = -1$

$$f(z) = \ln(3-(z+1-1)) = \ln(4-(z+1)) = \ln(4(1-(z+1)/4))$$

$$= \ln 4 + \ln(1-(z+1)/4)$$

$$f(z) = \ln(4) + \sum (-1)^{n-1} \cdot \frac{1}{n} \left(\frac{-(z+1)}{4}\right)^n$$

$$f(z) = \ln(4) + \sum_{n=1}^{\infty} (-1)^{n-1} \frac{1}{n} \cdot (-1)^n \frac{(z+1)^n}{4^n}$$

$$f(z) = \ln(4) + \sum_{n=1}^{\infty} (-1)^{2n-1} \cdot \frac{(z+1)^n}{n \cdot 4^n} = \ln(4) - \sum_{n=1}^{\infty} \frac{(z+1)^n}{n \cdot 4^n}$$

14. zad.

A) $f(z) = \ln(z^2 - 3z + 2)$ $\qquad z^2 - 3z + 2 = 0$

$$z_{1,2} = \frac{3 \pm \sqrt{9-8}}{2} = \frac{3 \pm 1}{2}$$

$$z_1 = 2 \quad z_2 = 1$$

$$f(z) = \ln((z-2)(z-1))$$

$$= \ln(z-2) + \ln(z-1)$$

$$= \ln\left(-2\left(1-\frac{z}{2}\right)\right) + \ln(-1(1-z))$$

$$= \ln 2 + \ln\left(1-\frac{z}{2}\right) + \ln(-1) + \ln(1-z)$$

$$= \ln 2 + \ln\left(1-\frac{z}{2}\right) + \ln(1-z)$$

$$= \ln 2 + \sum (-1)^{n-1} \cdot \frac{1}{n}\left(\frac{-z}{2}\right)^n + \sum (-1)^{n-1} \cdot \frac{1}{n}(-z)^n$$