

V DOMAĆA ZADACIA  
IZ LINEARNE ALGEBRE

① Neka je  $A$  realna matrica i neka je  
 $\langle Ax | y \rangle = \langle x | Ay \rangle \quad \forall x, y$ . Tada je  $A = A^T$

$$\Rightarrow \langle Ax | y \rangle - \langle x | Ay \rangle = 0$$

$$\langle x | A^T y \rangle - \langle x | Ay \rangle = 0$$

$$\langle x | (A^T - A)y \rangle = 0$$

$$\Rightarrow (A^T - A)y \perp x, \quad \underline{\forall x \in \mathbb{R}^n} \quad (A \in M_{n \times n}(\mathbb{R}), x \in \mathbb{R}^n, y \in \mathbb{R}^n)$$

$$\Rightarrow (A^T - A)y \perp \mathbb{R}^n \Rightarrow (A^T - A)y = 0 \quad \underline{\forall y \in \mathbb{R}^n}$$

$$\Rightarrow A^T - A = 0 \Rightarrow \underline{A^T = A}$$

② Neka je  $A$  kompleksna matrica. Tada vrijedi  
 $\langle Ax | y \rangle = \langle x | A^* y \rangle \quad \forall x, y$ .

$$\langle Ax | y \rangle = (Ax)^T \bar{y} = x^T A^T \bar{y} = x^T \overline{(\bar{A}^T y)} = \langle x | A^* y \rangle$$