① Neka je A realna matrica i neka je
$$\langle Ax \mid y \rangle = \langle x \mid Ay \rangle \qquad \forall x, y. \quad \text{Tada je} \quad A = A^\tau$$

$$\Rightarrow \quad \langle Ax \mid y \rangle - \langle x \mid Ay \rangle = 0$$
$$\langle x \mid A^\tau y \rangle - \langle x \mid Ay \rangle = 0$$
$$\langle x \mid (A^\tau - A) y \rangle = 0$$

$$\Rightarrow (A^\tau - A) y \perp x \quad , \quad \forall x \in \mathbb{R}^n \qquad \left(A \in M_{m \times n}(\mathbb{R}), \; x \in \mathbb{R}^n, \, y \in \mathbb{R}^m \right)$$

$$\Rightarrow (A^\tau - A) y \perp \mathbb{R}^n \quad \Rightarrow \quad (A^\tau - A) y = 0 \qquad \forall y \in \mathbb{R}^m$$

$$\Rightarrow \quad A^\tau - A = 0 \quad \Rightarrow \quad \underline{A^\tau = A}$$

② Neka je A kompleksna matrica. Tada vrijedi
$$\langle Ax \mid y \rangle = \langle x \mid A^* y \rangle \qquad \forall x, y.$$

$$\langle Ax \mid y \rangle = (Ax)^\tau \, \overline{y} = x^\tau A^\tau \, \overline{y} = x^\tau \overline{\left(\overline{A^\tau} \, y \right)} = \langle x \mid A^* y \rangle$$