MAT 2 KPZ 1. VEKTORI

ORT

$$\boxed{\vec{a}_0 = \frac{\vec{a}}{|\vec{a}|}}$$

Linearna neovisnost

$$\boxed{\lambda_1 \vec{a}_1 + \dots + \lambda_n \vec{a}_n = \vec{0} \Rightarrow \lambda_1, \dots, \lambda_n = 0}$$

Skup svih linearnih komb.
npr. 3 vektora

$$L(\vec{a}_1, \vec{a}_2, \vec{a}_3) = \{ \lambda_1 \vec{a}_1 + \lambda_2 \vec{a}_2 + \lambda_3 \vec{a}_3 \, , \lambda_1, \lambda_2, \lambda_3 \in \mathbb{R} \}$$

$\vec{a}_1, \vec{a}_2, \vec{a}_3$ razapinju V^3 ako su linearno nezavisni

Radij vektor središta dužine $\overline{AB}$

$$\boxed{\vec{r}_S = \frac{\vec{r}_A + \vec{r}_B}{2}}$$

Radij vektor $T(x, y, z)$

$$\boxed{\vec{r}_T = x\vec{i} + y\vec{j} + z\vec{k}}$$

Duljina radij vektora

$$\boxed{|\vec{r}_T| = \sqrt{x^2 + y^2 + z^2}}$$

Vektor $\overrightarrow{AB}$

$$\boxed{\overrightarrow{AB} = (x_B - x_A)\vec{i} + (y_B - y_A)\vec{j} + (z_B - z_A)\vec{k}}$$

Koordinate ~~Radij vektor~~ središta dužine

$$\boxed{S\left(\frac{x_A + x_B}{2}, \frac{y_A + y_B}{2}, \frac{z_A + z_B}{2} \right)}$$

SKALARNI PRODUKT

ZADANO: $\vec{a}, \vec{b}$ i $\measuredangle(\vec{a}, \vec{b}) = \varphi$
„$\vec{a}$ in b" $(0 \leq \varphi \leq \pi)$

$$\boxed{\vec{a} \cdot \vec{b} = |\vec{a}| \cdot |\vec{b}| \cdot \cos\varphi}$$

$$\boxed{\vec{a} \perp \vec{b} \Rightarrow \varphi = 90^\circ \Rightarrow \vec{a} \cdot \vec{b} = 0}$$

Duljina vektora

$$\boxed{|\vec{a}| = \sqrt{\vec{a}^2}}$$

Skalarna projekcija „a na b"

$$\boxed{a_b = |\vec{a}| \cdot \cos\varphi = \vec{a} \cdot \vec{b}_0}$$

VEKTOR

$$\boxed{\vec{a}_b = \frac{\vec{a} \cdot \vec{b}}{|\vec{b}|^2} \cdot \vec{b}}$$

Konveksna kombinacija

$$\boxed{\overrightarrow{OT} = s \cdot (\overrightarrow{OA}) + (1 - s)\overrightarrow{OB} \quad 0 \leq s \leq 1}$$

$$\vec{i}\cdot\vec{i} = \vec{j}\cdot\vec{j} = \vec{k}\cdot\vec{k} = 1$$
$$\vec{i}\cdot\vec{j} = \vec{j}\cdot\vec{k} = \vec{k}\vec{i} = 0$$

RADIJ VEKTOR TEŽIŠTA JE ARITMETIČKA SREDINA RADIJ VEKTORA VRHOVA.
TROKUT: $\vec{r}_T = \frac{1}{3}(\vec{r}_A + \vec{r}_B + \vec{r}_C)$

Skalarni produkt u komponentama

$$\vec{a}\cdot\vec{b} = a_x b_x + a_y \cdot b_y + a_z \cdot b_z$$

Vektorski produkt 2 vektora

$\vec{a}\times\vec{b}$ „a ex b"

1) $|\vec{a}\times\vec{b}| = |\vec{a}|\cdot|\vec{b}|\cdot\sin\varphi$ (POVRŠINA PARALELOGRAMA)

2) $\vec{a}\times\vec{b} \perp \vec{a}$ i $\vec{a}\times\vec{b} \perp \vec{b}$

$\boxed{\vec{a}\times\vec{b} = -(\vec{a}\times\vec{b})}$

→ U KOMPONENTAMA

$\vec{a} = a_x\vec{i} + a_y\vec{j} + a_z\vec{k}$
$\vec{b} = b_x\vec{i} + b_y\vec{j} + b_z\vec{k}$

$$\vec{a}\times\vec{b} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ a_x & a_y & a_z \\ b_x & b_y & b_z \end{vmatrix}$$

RAZVIJA SE PO PRVOM RETKU

Mješoviti produkt 3 vektora

$(\vec{a}\times\vec{b})\cdot\vec{c}$, volumen paralelepipeda

$$(\vec{a}\times\vec{b})\cdot\vec{c} = \begin{vmatrix} a_x & a_y & a_z \\ b_x & b_y & b_z \\ c_x & c_y & c_z \end{vmatrix}$$

1) $(\vec{a}\times\vec{b})\cdot\vec{c} = (\vec{b}\times\vec{c})\cdot\vec{a} = (\vec{c}\times\vec{a})\cdot\vec{b}$

Volumen paralelepipeda $= 0$
$\Rightarrow$ točke su u istoj ravnini

RAVNINA

Opći oblik jednadžbe

ZADANO: 1) $\vec{n} = A\vec{i} + B\vec{j} + C\vec{k}$ (normala) → vektor $\perp \pi$

2) $T_0(x_0, y_0, z_0)$ → točka u π

$$\boxed{\pi \equiv A(x-x_0) + B(y-y_0) + C(z-z_0) = 0}$$

$$\boxed{\pi \equiv Ax + By + Cz + D = 0}$$ opći oblik

$$\boxed{\pi_1 \parallel \pi_2 \Rightarrow \vec{n}_1 = \vec{n}_2}$$

Jednadžba pomoću 3 točke

$$\pi \equiv \begin{vmatrix} x-x_1 & y-y_1 & z-z_1 \\ x_2-x_1 & y_2-y_1 & z_2-z_1 \\ x_3-x_1 & y_3-y_1 & z_3-z_1 \end{vmatrix} = 0$$

Segmentni oblik

$$\pi \equiv \frac{x}{p} + \frac{y}{q} + \frac{z}{r} = 1$$

Udaljenost točke od ravnine

$$d = \frac{|A \cdot x_T + By_T + Cz_T + D|}{\sqrt{A^2+B^2+C^2}}$$

$$d(\pi_1, \pi_2) = d(T_1, \pi_2)$$
$$T_1 \in \pi_1$$

Kut između ravnina

$$\measuredangle(\pi_1, \pi_2) = \measuredangle(\vec{n}_1, \vec{n}_2)$$

Simetralna ravnina

$$\boxed{d(T, \pi_1) = d(T, \pi_2)}$$

$$(\pi_s)_{1,2} = \frac{A_1x + B_1y + C_1z}{\sqrt{A_1^2+B_1^2+C_1^2}} = \frac{\pm(A_2x+B_2y+C_2z)}{\sqrt{A_2^2+B_2^2+C_2^2}}$$

$D=0$ ako ravnina prolazi ishodištem

$C=0 \Leftrightarrow \pi \parallel Oz$

$B=0 \Leftrightarrow \pi \parallel Oy$

$A=0 \Leftrightarrow \pi \parallel Ox$

Pramen ravnina

$$A_1x + B_1y + C_1z + D_1 + \lambda(A_2x + B_2y + C_2z + D_2) = 0$$

PRAVAC

Parametarska jednadžba pravca

$$p \equiv \begin{cases} x = x_0 + \lambda l \\ y = y_0 + \lambda m \\ z = z_0 + \lambda n \end{cases}$$

IZVOD: $\overrightarrow{TT_0} = \lambda \cdot \vec{s}$

pravac određen točkom $T_0(x_0, y_0, z_0)$ i vektorom smjera $\vec{s} = l\vec{i} + m\vec{j} + n\cdot\vec{k}$

Kanonska jednadžba

$$p \equiv \frac{x - x_0}{l} = \frac{y - y_0}{m} = \frac{z - z_0}{n} \quad (= \lambda)$$

Kroz dvije točke

$$p \equiv \frac{x - x_1}{x_2 - x_1} = \frac{y - y_1}{y_2 - y_1} = \frac{z - z_1}{z_2 - z_1}$$

RAVNINA & PRAVAC

Sjecište pravca i ravnine

$$\left.\begin{array}{ll} (1) & Ax + By + Cz + D = 0 \\ (2) & x = x_0 + t \cdot l \\ (3) & y = y_0 + t \cdot m \\ (4) & z = z_0 + t \cdot n \end{array}\right\} \text{rješimo sustav}$$

Udaljenost točke i pravca

$$d = \frac{|\overrightarrow{T_0T} \times \vec{s}|}{|\vec{s}|}$$

Presjek 2 ravnine

$$p \equiv \pi_1 \cap \pi_2$$
$$\vec{s} = \vec{n}_1 \times \vec{n}_2$$

Uvjet komplanarnosti 2 prav

$$V_{par} = (\vec{s}_1 \times \vec{s}_2) \cdot \overrightarrow{T_1T_2} =$$

Najkraća udaljenost mimosmjernih pravac

Naći točke A i B;
odrediti $\overrightarrow{AB}$;
poslužiti se $\vec{s}$

$$\left.\begin{array}{l} \overrightarrow{AB} \cdot \vec{s}_1 = 0 \text{ i} \\ \overrightarrow{AB} \cdot \vec{s}_2 = 0 \end{array}\right\} \begin{array}{l} \overrightarrow{AB} \perp \\ \overrightarrow{AB} \perp \end{array}$$

Kut između ravnine i pravca

$$\varphi = 90^\circ - \arccos\left(\frac{\vec{s} \cdot \vec{n}}{|\vec{s}| \cdot |\vec{n}|}\right)$$

Udaljenost mimosmjernih prava

$$d = \frac{|\overrightarrow{T_1T_2} \cdot (\vec{c}_1 \times \vec{c}_2)|}{|\vec{c}_1 \times \vec{c}_2|}$$

FUNKCIJE VIŠE VARIJABLI

Euklidski prostor $\mathbb{R}^n$

$$\mathbb{R}^n = \{(x_1, \dots, x_n) \mid x_i \in \mathbb{R}\}$$

$$\vec{x} = (x_1, \dots, x_n)$$

Norma (Euklidska norma)

- poopćenje apsolutne vrijednosti

$$\vec{x} = (x_1, \dots, x_n) \longmapsto \|\vec{x}\| = \sqrt{x_1^2 + \dots + x_n^2}$$

Prostorna krivulja

$$\mathcal{C} \equiv \begin{cases} x = x(t) \\ y = y(t) \\ z = z(t) \end{cases} \quad t \in T$$

Vektorska funkcija

$$\vec{r}(t) = x(t)\vec{i} + y(t)\vec{j} + z(t)\vec{k}$$

$$t \longmapsto \vec{r}(t)$$

OPĆENITO

$$t \longmapsto f(t) = (x_1(t), \dots, x_n(t))$$

$$\vec{r}(t) = x_1(t)\cdot\vec{e}_1 + \dots + x_n(t)\cdot\vec{e}_n$$

Derivacija vektorske funkcije

$$r'(t) = x'(t)\vec{i} + y'(t)\vec{j} + z'(t)\vec{k}$$

Jednadžba prostorne tangente

$$t \equiv \frac{x - x_0}{x'(t_0)} = \frac{y - y_0}{y'(t_0)} = \frac{z - z_0}{z'(t_0)}$$

Oskularna ravnina

$$\vec{n} = \vec{v} \times \vec{a}$$

Realna funkcija

$$f : D_f \longmapsto \mathbb{R}$$

$$D_f \subseteq \mathbb{R}^n$$

$$\vec{x} = (x_1, \dots, x_n) \longmapsto f(\vec{x}) = f(x_1, \dots, x_n) \in \mathbb{R}$$

$$n = 2 \Rightarrow u = f(x_1, x_2) \quad \text{tj.} \quad \boxed{z = f(x, y)}$$

MAT 2 [1. KPZ, MI] gradivo

Plohe drugog reda - (varijable na kvadrat)

(A) Ravnina $\pi \equiv A(x-x_0) + B(y-y_0) + C(z-z_0) = 0$
(ploha prvog reda)

(B) Cilindrične (valjkaste)

PREPOZNAJ
$x^2+y^2=R^2$ $\quad \frac{x^2}{a^2}+\frac{y^2}{b^2}=1$
$z=y^2$

→ izvodnice paralelne s Oz
→ jednadžba izvedbene krivulje u XOY
je $f(x,y)=0$

$\boxed{F(x,y)=0}$ (analogno za $F(x,z)$; $F(y,z)$)

LJUBO PROTIP: nema varijable „z"

(C) Stožaste (konusne)

PREPOZNAJ:
$z=\sqrt{x^2+y^2}$ $\quad z^2=\frac{1}{R^2}(x^2+y^2)$
$x^2=z^2+y^2$ $\quad z^2=\frac{x^2}{a^2}+\frac{y^2}{b^2}$

→ izvodnice prolaze kroz vrh V
i izvedbenu krivulju $\mathcal{C}$

IZVEDBENA KRIVULJA → STOŽASTA PLOHA

u jednadžbi izvedbene krivulje:

$\begin{cases} F(x,y) \\ z=c \end{cases}$; načine se zamjene $\boxed{x \to \frac{x\cdot c}{z}}$, $\boxed{y \to \frac{y\cdot c}{z}}$

(D) ROTACIJSKA TIJELA

→ $y=f(x)$: neprekinuta funkcija
→ rotacijom oko Ox nastaje rotaciona ploha
→ uvodimo novu koordinatnu os Oz

KRIVULJA	OS ROTACIJE	JED. ROT. PLOHE
$y=f(x)$	Ox	$\sqrt{y^2+z^2}=f(x)$
	Oy	$y=f(\sqrt{x^2+z^2})$

OS OKO KOJE SE NE ROTIRA (se zamijeni)
→ $\sqrt{(\text{ta os})^2+(\text{nova os})^2}$

Analogno za $y=f(z)$, $z=f(y)$ etc.

(E) Sedlasta ploha

→ hiperbolički paraboloid

$$\boxed{z = x^2 - y^2}$$

Vrste (dodatno):

Sfera

$$\boxed{x^2 + y^2 + z^2 = R^2}$$

→ općenito: elipsoid $\boxed{\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1}$

Rotacioni paraboloid

$$\boxed{z = k(x^2 + y^2)}$$

→ općenitije: eliptični paraboloid $\boxed{z = \frac{x^2}{a^2} + \frac{y^2}{b^2}}$

Rotacioni stožac

$$\boxed{z^2 = x^2 + y^2}$$

Opće

$$\boxed{z^2 = k(x^2 + y^2)}$$

eliptički stožac

$$\boxed{z^2 = \frac{x^2}{a^2} + \frac{y^2}{b^2}}$$

Limes i neprekinutost funkcija više varijabli

LIMES NIZA

$$\vec{x}_0 = \lim_{k\to\infty} (\vec{x}_k) :\Leftrightarrow (\forall \varepsilon > 0)(\exists k_0 \in \mathbb{N}) \left[k > k_0 \Rightarrow \|\vec{x}_k - \vec{x}_0\| < \varepsilon \right]$$

LIMES FUNKCIJE n VARIJABLI

$$\lim_{\vec{x}\to\vec{x}_0} f(\vec{x}) = L :\Leftrightarrow (\forall \vec{x}_k)\left[\left(\{\vec{x}_k\} \subseteq D(f) \setminus \{\vec{x}_0\} \wedge \lim_{k\to\infty} \vec{x}_k = \vec{x}_0\right) \Rightarrow \lim_{k\to\infty} f(\vec{x}_k) = L\right]$$

NEPREKINUTOST

$f(\vec{x})$ je neprekinuta u $\vec{x}_0$ $\Longleftrightarrow$ ako ima limes i iznosi $\boxed{\lim_{\vec{x}\to\vec{x}_0} f(\vec{x}) = f(\vec{x}_0)}$

(5) Ispitivanje neprekinutosti u $T_0(x_0, y_0)$

→ odabiremo više smjerova dolaska u ~~[illegible]~~ točku T_0

npr. $x = 0$, $x = y$

→ prelaskom na polarne koordinate

$$\boxed{z = f(x, y) = f(r\cos\varphi, r\sin\varphi)}$$

Gradijent funkcije

$$\boxed{\operatorname{grad} f := \frac{\partial f}{\partial x_1}\vec{e}_1 + \dots + \frac{\partial f}{\partial x_n}\vec{e}_n}$$

$$\operatorname{grad} f(\vec{x}) = \nabla f(\vec{x})$$

NABLA

$$\nabla = \frac{\partial f}{\partial x_1}\vec{e}_1 + \dots + \frac{\partial f}{\partial x_n}\vec{e}_n$$

$n = 3$ SKALARNO POLJE

$u = f(x, y, z)$

$$\nabla u = \frac{\partial f}{\partial x}\vec{i} + \frac{\partial f}{\partial y}\vec{j} + \frac{\partial f}{\partial z}\cdot\vec{k}$$

Parcijalne derivacije

$z = f(x, y)$ neprekinuta na $M \subseteq \mathbb{R}$, $T_0(x_0, y_0) \in M$

$\varphi_1(x) = f(x, y_0)$ i $\varphi_2(y) = f(x_0, y)$ su f-je jedne varijable, $\varphi_1'(x_0)$ i $\varphi_2'(y_0)$ su parcijalne derivacije f po x_0, y_0

$$\boxed{\left(\frac{\partial f}{\partial x}\right)_0 = f_x'(x_0, y_0) := \varphi_1'(x_0) = \lim_{\Delta x \to 0} \frac{f(x_0 + \Delta x, y_0) - f(x_0, y_0)}{\Delta x}}$$

GRADIJENT POKAZUJE SMJER NAJBRŽE PROMJENE FUNKCIJE $F(x, y, z)$ U $T(x_0, y_0, z_0)$

Jednadžba tangencijalne ravnine (π_T)

z - eksplicitno $z = f(x, y)$; $T(x_0, y_0, z_0)$

$$\pi_T \quad \boxed{z - z_0 = \left(\frac{\partial f}{\partial x}\right)(x - x_0) + \left(\frac{\partial f}{\partial y}\right)(y - y_0)}$$

z - implicitno $F(x, y, z) = 0$; $T_0(x_0, y_0, z_0)$

$$\pi_T \equiv \boxed{\left(\frac{\partial F}{\partial x}\right)_0 (x - x_0) + \left(\frac{\partial F}{\partial y}\right)_0 (y - y_0) + \left(\frac{\partial F}{\partial z}\right)_0 (z - z_0) = 0}$$

Diferencijabilnost $f(\vec{x})$

$$\boxed{\lim_{\vec{h} \to 0} \frac{o(\vec{h})}{\|\vec{h}\|} = 0}$$

(1) postoji grad f u točki $\vec{x}$

(2) za svaki $\vec{h}$, promjenu $\Delta f(\vec{x})$, moguće prikazati: $\boxed{f(\vec{x} + \vec{h}) - f(\vec{x}) = \nabla f(\vec{x}) \cdot \vec{h} + o(\vec{h})}$

PRVI DIFERENCIJAL $df(\vec{x}, \vec{h})$

PRVI DIFEREN.

$$df = \nabla f(\vec{x}) \cdot \vec{h} = \frac{\partial f}{\partial x_1} \cdot h_1 + \dots + \frac{\partial f}{\partial x_n} h_n$$

skalarni produkt u ortonormiranoj bazi

Nužni i dovoljni uvjeti za diferencijabilnost

$u = f(\vec{x})$, definirana na $M \subseteq \mathbb{R}^n$ i $T_0(x_1, \dots, x_n) \in M$. Ako u svakoj točki u M postoje parcijalne der. po svim varijablama i ako su sve te der. neprekinute u T_0.

Diferencijabilnost i neprekinutost

$f: D_f \mapsto \mathbb{R}$; $D_f \subseteq \mathbb{R}^n$

Ako je f diferencijabilna u $\vec{x}$, onda je f i neprekinuta u $\vec{x}$

Nalaženje 1. diferencijala

$$\boxed{du = \sum_{i=1}^{n} \frac{\partial u}{\partial x_i} dx_i}$$

$z = f(x, y)$ $\boxed{n=2}$

$$dz = \frac{\partial z}{\partial x} dx + \frac{\partial z}{\partial y} dy$$

Primjena diferencijala

za $\|\vec{h}\| \ll$ (malen!)

$$f(\vec{x} + \vec{h}) \approx f(\vec{x}) + d f(\vec{x}, \vec{h})$$

$$\boxed{f(\vec{x} + \vec{h}) \approx f(\vec{x}) + \sum_{i=1}^{n} \frac{\partial f}{\partial x_i} \cdot \Delta x_i}$$

$\boxed{n=2}$

$$\boxed{z(x+\Delta x, y+\Delta y) = z(x,y) + \frac{\partial z}{\partial x} \cdot \Delta x + \frac{\partial z}{\partial y} \cdot \Delta y}$$

Derivacija složene funkcije

$v = f(u_1, \dots, u_n)$

$u_i = \varphi_i(x_i, \dots, x_m)$

$$\boxed{\frac{\partial v}{\partial x_i} = \sum_{j=1}^{m} \frac{\partial v}{\partial u_j} \cdot \frac{\partial u_j}{\partial x_i}}$$

$v = f(u_1, \dots, u_m)$

$u_i = \varphi_1(t)$ $i = 1, \dots, m$

$$\boxed{\frac{dv}{dt} = \sum_{j=1}^{m} \frac{\partial v}{\partial u_j} \cdot \frac{du_j}{dt}}$$

Usmjerena derivacija

$$\boxed{\frac{\partial f}{\partial \vec{h}} := \lim_{t \to 0} \frac{f(\vec{x} + t \cdot \vec{h}_0) - f(\vec{x})}{t}} \quad \left(\vec{h}_0 = \frac{\vec{h}}{\|\vec{h}\|}\right)$$

$$\boxed{\frac{\partial f(\vec{h})}{\partial \vec{h}} = \vec{h}_0 \cdot \operatorname{grad} f(\vec{x})}$$

6 Najveća vrijednost usmjerene derivacije

$\vec{n} = \text{grad} f(\vec{x})$; $\vec{n} \perp$ nivo plohu kroz $\vec{x}$

$$-\text{grad} f \leq \frac{\partial f}{\partial \vec{s}} \leq \text{grad} f$$

Derivacija implicitno zadane funkcije

$$\frac{dy}{dx}(x_0) = -\frac{\frac{\partial f}{\partial x}(x_0,y_0)}{\frac{\partial f}{\partial y}(x_0,y_0)}$$

$$\frac{\partial z}{\partial x} = -\frac{\frac{\partial F}{\partial x}}{\frac{\partial F}{\partial z}}$$

Gradijent lin. funkcije

$Df \subseteq \mathbb{R}^n$ je omeđen i zatvoren skup onda afina f-ja (npr. $ax+by+cz+d$) $f: Df \mapsto \mathbb{R}$ $f(\vec{x}) = \vec{a} \cdot \vec{x} + d$ postiže min i MAX na rubu skupa Df.

Integrali ovisni o parametru

$$I'(\alpha) = \frac{d}{d\alpha}\left(\int_{\varphi(\alpha)}^{\psi(\alpha)} f(x,\alpha)\,dx\right) = f[\psi(\alpha),\alpha]\cdot\psi'(\alpha) - f[\varphi(\alpha),\alpha]\cdot\varphi'(\alpha) + \int_{\varphi(\alpha)}^{\psi(\alpha)} \frac{\partial}{\partial\alpha} f(x,\alpha)\,dx$$

TATA FORMULA

$$\frac{d}{d\alpha}\int_a^b f(x,\alpha)\,dx = \int_a^b \frac{\partial}{\partial\alpha} f(x,\alpha)\,dx$$

NEKI STAVAK

1) $\text{grad} f \equiv \vec{0} \Rightarrow f = \text{const.}$

2) $\text{grad} f = \text{grad} g \Rightarrow$ g i f se razlikuju za c

Lagrangeov teorem srednje vrijednosti

$$f(\vec{b}) - f(\vec{a}) = \nabla f(\vec{c}) \cdot (\vec{b} - \vec{a})$$

$f: \mathcal{U} \mapsto \mathbb{R}$ $\mathcal{U} \in \mathbb{R}^n$

$\vec{a}, \vec{b} \in \mathbb{R}$; spojnica$(\vec{a}, \vec{b}) \in \mathbb{R}$

U slučaju $n=2$ $\mathcal{U}$ mora biti konveksan skup

Schwarzov teorem

$z = (x,y)$; $Df = M$; $T_0 \in M$

Ako u nekoj okolini od T_0 postoje $f'_x, f'_y, f''_{xy}, f''_{yx}$ i f''_{xy} i f''_{xy} su neprekinute.

$$\boxed{f''_{xy} = f''_{yx}}$$

VIŠE PARCIJALNE DERIVACIJE NE OVISE O REDOSLIJEDU DERIVIRANJA.

Drugi diferencijal

$$d^2 z = d(dz)$$

$$dz = \frac{\partial z}{\partial x} dx + \frac{\partial z}{\partial y} dy$$

$$d^2 z = z''_{xx} (dx)^2 + 2 \cdot z''_{xy}\, dx\, dy + z''_{yy} (dy)^2$$

Treći diferencijal

$$d^3 z = z'''_{x} (dx)^3 + 3 \cdot z'''_{xxy} (dx)^2 dy + 3 \cdot z'''_{yyx} (dx)(dy)^2 + z'''_{yyy} (dy)^3$$

Taylorova formula za f-je dvije varijable

$f(x,y)$ definirana na $\mathcal{U} \in \mathbb{R}^2$;

funkcija klase C^{n+1};

$(x_0, y_0) \in \mathcal{U}$;

$$f(x,y) = f(x_0,y_0) + \frac{1}{1!}\left[(f'_x)_0 (x-x_0) + (f'_y)_0 (y-y_0)\right]$$
$$+ \frac{1}{2!}\left[(f''_{xx})_0 (x-x_0)^2 + 2 (f''_{xy})_0 (x-x_0)(y-y_0) + (f''_{yy})_0 (y-y_0)^2\right]$$
$$+ \dots + \frac{1}{n!}\left[(x-x_0)\frac{\partial}{\partial x} + (y-y_0)\frac{\partial}{\partial y}\right]^{[n]} f(x_0,y_0) + R_n(x,y)$$

Lagrangeov ostatak → sadrži derivacije $n+1$ reda u T_c

$$R_n = \frac{1}{(n+1)!}\left[(x-x_0)\frac{\partial}{\partial x} + (y-y_0)\frac{\partial}{\partial y}\right]^{[n+1]} f(T_c)$$

T_c ... točka na spojnici $\overline{TT_0}$

7 Kvadratna forma

- homogena kvad. f-ja 2 realnih varijabli

$Q : \mathbb{R}^2 \longmapsto \mathbb{R}$

$$\boxed{Q(h,k) = a \cdot h^2 + 2 \cdot b \cdot h \cdot k + c \cdot k^2}$$

$A = \begin{bmatrix} a & b \\ b & c \end{bmatrix}$; $M_1 = a$; $M_2 = ac - b^2$

Sylvesterov teorem

M_1	M_2	$Q(h,k)$
+	+	POZITIVNO DEF.
−	+	NEGATIVNO DEF.
/	−	INDEFINITNA

$\boxed{n=3}$

$$\boxed{Q(h,k,l) = ah^2 + bk^2 + cl^2 + 2dhk + 2ehl + 2fkl}$$

$A = \begin{bmatrix} a & d & e \\ d & b & f \\ e & f & c \end{bmatrix}$; $M_1 = a$; $M_2 = ab - d^2$

$M_3 = \begin{vmatrix} a & d & e \\ d & b & f \\ e & f & c \end{vmatrix}$

M_1	M_2	M_3	$Q(h,k,l)$
+	+	+	POZITIVNO DEF.
−	+	−	NEGATIVNO DEF.
SVI OSTALI			INDEFINITNA

Matrica kvadratne forme od d^2f se naziva <u>Hesseova matrica</u> i njena definitnost se ispituje <u>Sylvesterovim t.</u>

Definitnost kvad. forme

ⓐ POZITIVNO DEFINITNA

$Q(h,k) > 0$; $\forall (h,k) \neq (0,0)$

ⓑ NEGATIVNO DEF.

$Q(h,k) < 0$; $\forall (h,k) \neq (0,0)$

ⓒ INDEFINITNA FORMA

$Q(h,k) \gtrless 0$; + i − vrijednosti

Nužni uvjet za lokalni eks.

$$\boxed{\nabla f(\vec{a}) = \vec{0}}$$

$$\boxed{\frac{\partial f}{\partial x_1} = 0, \ldots, \frac{\partial f}{\partial x_n} = 0}$$

Ima n jed. i n rješenja: STACIONARNE TOČKE

1) Maksimum
2) Minimum
3) sedlasta točka

Dostatni uvjeti za lokalni ekstrem

$\mathcal{U} \subseteq \mathbb{R}^n$; $f : \mathcal{U} \longmapsto \mathbb{R}$; f je klase C^2; $T_0 \in \mathcal{U}$; $\nabla f(T_0) = \vec{0}$

Ⓐ $d^2f(T_0)$ je POZITIVNO DEFINITNA u T_0 je <u>min</u>

Ⓑ $d^2f(T_0)$ je NEGATIVNO DEFINITNA u T_0 je <u>MAX</u>

Ⓒ $d^2f(T_0)$ je INDEFINITNA u T_0 $\boxed{\text{NIJE}}$ ekstrem već SEDLASTA TOČKA

Uvjetni (vezani) ekstremi

$\boxed{u = f(\vec{x})}$; $u : S \longmapsto \mathbb{R}$

$S \subseteq D(f)$; S je definiran s $\boxed{\varphi(\vec{x}) = 0}$ uvjet.

Metoda Lagrangeovih multiplikatora

$z = F(x, y)$; x, y nezavisne varijable vezane uvjetom $\varphi(x, y) = 0$.

$F(x, y)$ postiže vezani ekstrem u $(x, y) = (x_0, y_0)$ $\iff$ ako takav ekstrem u točki $T(x_0, y_0)$ postiže Lagrangeova funkcija.

Lagrangeova funkcija

$$\boxed{L(x, y; \lambda) := f(x, y) + \lambda \cdot \varphi(x, y)}$$

Lagrangeov multiplikator

$\boxed{\begin{matrix}(x_0, y_0) \\ \lambda = \lambda_0\end{matrix}}$ se određuju iz uvjeta:

$$\boxed{\begin{aligned} L'_x &= F'_x + \lambda \cdot \varphi_x'(x, y) \\ L'_y &= F'_y + \lambda \, \varphi_y'(x, y) \\ L'_\lambda &= \varphi(x, y) \end{aligned}}$$

Radi ustanovljavanja predznaka treba gledati $d^2L(x_0, y_0)$ ALI i uvjet

$\varphi'(x, y) = 0$

$$\boxed{\frac{\partial \varphi}{\partial x} dx + \frac{\partial \varphi}{\partial y} dy = 0}$$

BITNO

$$\begin{cases} d^2L(S) = L''_{xx}(dx)^2 + 2 \cdot L''_{xy}(dx\,dy) + L''_{yy}(dy)^2 \\ \dfrac{\partial \varphi}{\partial x} dx + \dfrac{\partial \varphi}{\partial y} dy = 0 \end{cases}$$