ZZV 12

Zad 1.

1) $yy'' + y'^2 = y'^3$ $\qquad y' = p, \ p = p(y)$

$$y'' = \frac{dy'}{dx} = \frac{dp}{dx} = \frac{dp}{dy}\frac{dy}{dx} = \boxed{p'p}$$

$$y \cdot p \cdot p' + p^2 = p^3$$

$p = 0$ $\qquad yp' + p = p^2 \Rightarrow \int \frac{dp}{p(p-1)} = \int \frac{dy}{y} = \ln c_1 y$

$\boxed{y = C}$

$$\int \frac{dp}{p(p-1)} = \int \frac{dp}{p-1} - \int \frac{dp}{p} = \ln \frac{p-1}{p}$$

$$\ln \frac{p-1}{p} = \ln c_1 y \Rightarrow 1 - \frac{1}{y'} = c_1 y \qquad x = x(y)$$

$$1 - x' = c_1 y \qquad \int dx = \int (1 - c_1 y)\, dx$$

$$\boxed{x = y - \frac{1}{2} c_1 y^2 + c_2}$$

2) $y'y'' = \frac{1}{y^2} - \frac{y'^3}{y}$ $\qquad y' = p, \ p = p(y) \Rightarrow y'' = p \cdot p'$

1)

$$p \cdot p \cdot p' = \frac{1}{y^2} - \frac{p^3}{y}$$

$$p^2 p' + \frac{1}{y} p^3 = \frac{1}{y^2} \quad / : p^2$$

$$p' + \frac{1}{y} p = \frac{1}{y^2} \frac{1}{p^2} \quad \text{Bernoulli}$$

$$p^2 p' + \frac{1}{y} p^3 = \frac{1}{y^2}$$

$$t = p^3 \qquad t' = 3p^2 p'$$

$$\frac{1}{3} t' + \frac{1}{y} t = \frac{1}{y^2}$$

$$t' + \underbrace{\frac{3}{y}}_{f(y)} t = \underbrace{\frac{3}{y^2}}_{g(y)}$$

$$\int f(y)\, dy = 3 \ln y$$

$$e^{-3\ln y} = \frac{1}{y^3}$$

$$e^{3\ln y} = y^3$$

2)

$$t = e^{-3\ln y} \cdot \left[\int \frac{3}{y^2} e^{+3\ln y} dx + C_1 \right]$$

$$= \frac{1}{y^3} \left[\int 3y\, dy + C_1 \right]$$

$$= \frac{1}{y^3} \left(\frac{3}{2} y^2 + C_1 \right)$$

$$= \frac{3}{2} \cdot \frac{1}{y} + \frac{1}{y^3} C_1$$

$$= \frac{1}{y^3} \left(\frac{3}{2} y^2 + C_1 \right)$$

$$y' = p = \sqrt[3]{t} = \frac{1}{y} \sqrt[3]{\frac{3}{2} y^2 + C_1}$$

$$\int \frac{y}{\sqrt[3]{\frac{3}{2} y^2 + C_1}}\, dy = \int dx = x + C_2$$

$$\int \frac{y}{\sqrt[3]{\frac{3}{2} y^2 + C_1}}\, dx = \left| \begin{array}{l} t^3 = \frac{3}{2} y^2 + C_1 \\ \cancel{3} t^2 dt = \cancel{3} y\, dy \end{array} \right| = \int \frac{\cancel{y} \cdot \frac{t^2 dt}{\cancel{y}}}{t} = \frac{1}{2} t^2$$

$$= \frac{1}{2} \left(\frac{3}{2} y^2 + C_1 \right)^{\frac{2}{3}}$$

$$\boxed{\left(\frac{3}{2} y^2 + C_1 \right)^{\frac{2}{3}} = 2x + C_2}$$

3) $y\sqrt{1+y'^2} = y'$ $\qquad y' = p = \frac{dy}{dx}$

1) $y\sqrt{1+p^2} = p \quad /^2$

$y^2(1+p^2) = p^2$

$p^2y^2 - p^2 = -y^2$

$p^2(y^2-1) = -y^2$

$p^2\frac{(1-y^2)}{y^2} = 1$

$p\cdot\frac{\sqrt{1-y^2}}{y} = \pm 1$

2) $\int\frac{1}{y}\sqrt{1-y^2}\,dy = \int \pm dx = \pm x + C_2$

$\int\frac{1}{y}\sqrt{1-y^2}\,dy = \left\| \begin{matrix} y = \sin t \\ dy = \cos t\, dt \end{matrix} \right.$

$= \int\frac{1}{\sin t}\cdot\cos t\cdot\cos t\,dt = \int\frac{\cos^2 t}{\sin t}\,dt$

$= \frac{1}{2}\int\frac{1}{\sin t} + \overbrace{\frac{1}{2}\int\frac{\cos 2t}{\sin t}\,dt}^{A}$

$\frac{1}{2}\ln(\tan\frac{t}{2}) = \frac{1}{2}\ln|\tan(\frac{1}{2}\arcsin y)|$

3) $A = \frac{1}{2}\int\frac{\cos 2t}{\sin t}\,dt = \frac{1}{2}\int\frac{1-2\sin^2 t}{\sin t}\,dt = \frac{1}{2}\int\frac{1}{\sin t}\,dt - \int \sin t\,dt$

$= \frac{1}{2}\ln|\tan(\frac{1}{2}\arcsin y)| + \underbrace{\cos(\arcsin y)}_{B}$

$B = \cos\arcsin y = \sqrt{1-\sin^2\arcsin y} = \sqrt{1-y^2}$

$\boxed{1\cdot\ln|\tan(\frac{1}{2}\arcsin y)| + \sqrt{1-y^2} + \cancel{C_1} = \pm x + C_2 \Rightarrow K}$

4) $y' = xy''^2 + y''^2$ $\qquad y' = p(x)$

1) $p = xp'^2 + p'^2$

$p = p'^2(x+1)$

$\frac{p}{x+1} = p'^2$

$\frac{1}{\sqrt{x+1}} = \frac{p'}{\sqrt{p}}$

$\int\frac{dp}{\sqrt{p}} = \pm\int\frac{dx}{\sqrt{x+1}}$

$\cancel{2}\sqrt{p} = \pm\cancel{2}\sqrt{x+1} + \cancel{2}C_1$

$p = x+1 \pm 2C_1\sqrt{(x+1)} + C_1^2$

2) $y = \int[x + (C_1^2+1) \pm 2C_1\sqrt{x+1}]\,dx$

$\boxed{= \frac{1}{2}x^2 + (C_1^2+1)x \pm \frac{4}{3}C_1(x+1)^{\frac{3}{2}} + C_2}$

5.) $yy'' + y'^2 - y'^3 \ln y = 0$ $\qquad y' = p(y) \qquad y'' = p \cdot p'$

1) $y \cdot p \cdot p' + p^2 - p^3 \ln y = 0 \Rightarrow p = 0 \Rightarrow \boxed{y = C}$

$y p' + p - p^2 \ln y = 0$

$p' + \frac{1}{y} p = p^2 \ln y \frac{1}{y}$ Bernoulli

$\frac{p'}{p^2} + \frac{1}{y} \cdot \frac{1}{p} = \frac{1}{y} \ln y$

$t = \frac{1}{p} \qquad t' = -\frac{1}{p^2}$

$-t' + \frac{1}{y} t = \frac{1}{y} \ln y$ linear

4) $p = \frac{1}{1 + C_1 y + \ln y}$

$\int (1 + C_1 y + \ln y)\, dy = \int dx$

$y + \frac{1}{2} C_1 y^2 + \underbrace{\int \ln y\, dy}_{} = x + C_2$

$A = \int \ln y\, dy = \left| \begin{array}{ll} u = \ln y & du = \frac{1}{y} \\ dv = dy & v = y \end{array} \right|$

$= y \ln y - \int dx$

$= y \ln y - y$

$\cancel{y} + \underbrace{\frac{1}{2} C_1}_{C_1} y^2 + y \ln y - \cancel{y} = x + C_2$

$$\boxed{C_1 y^2 + y \ln y = x + C_2}$$

2) $-t' + \frac{1}{y} t = 0$

$\frac{dt}{t} = \frac{dy}{y}$

$\ln t = \ln y + \ln C_1$

$t_H = y \cdot C_1$

3) $t = C_1(y) \cdot y$

$-C_1' y - \cancel{C_1} + \cancel{C_1} = \frac{1}{y} \ln y$

$C_1' = -\frac{1}{y^2} \ln y$

$C_1(y) = -\int \frac{\ln y}{y^2} dx = \left| \begin{array}{ll} u = \ln y & du = \frac{1}{y} \\ dv = -\frac{1}{y^2} dy & v = \frac{1}{y} \end{array} \right|$

$= \frac{1}{y} \ln y - \int \frac{1}{y^2} = \frac{1}{y} \ln y + \frac{1}{y} + C_1$

$t = \ln y + 1 + C_1 \cdot y$

6) $yy'' = y'(2\sqrt{xy'} + y')$ $\quad y' = p(y) \quad y'' = p \cdot p'$

1) $y \cdot \cancel{p} \cdot p' = p(2\sqrt{xp} + p) \Rightarrow p = 0 \Rightarrow \boxed{y = C}$

$yp' = 2\sqrt{xp} + p$

$yp' - p = 2\sqrt{y} \cdot \sqrt{p} \quad / \sqrt{p} / y$

$\frac{p'}{\sqrt{p}} - \frac{1}{y} \cdot \sqrt{p} = \frac{2}{\sqrt{y}}$

$t = \sqrt{p} \quad t' = \frac{1}{2\sqrt{p}} \cdot p'$

$2t' - \frac{1}{y} t = \frac{2}{\sqrt{y}}$ $\quad$ linear

2) $2t' - \frac{1}{y} t = 0$

$\frac{dt}{t} = \frac{1}{2} \frac{dx}{x}$

$t = C_1 \cdot \sqrt{y}$

~~$2 \cdot C_1' \cdot \sqrt{y} + \frac{C_1}{\sqrt{y}} = \frac{C_1}{y} = \frac{2}{\sqrt{y}}$~~

$C_1' = \frac{1}{y} \quad C_1(y) = \ln y + C_1$

$\boxed{t = \sqrt{y} \ln y + C_1 \sqrt{y}}$

3)

$p = t^2 = y \ln^2 y + 2y C_1 \ln y + C_1^2 y$

$= y \cdot (\ln^2 y + 2 \ln y C_1 + C_1^2)$

$= y (\ln y + C_1)^2$

$\int \frac{dy}{y \cdot (\ln y + C_1)^2} = \int dx = x + C_2$

$\int \frac{d(\ln y + C_1)}{(\ln y + C_1)^2} = -\frac{1}{\ln y + C_1}$

$\Rightarrow \boxed{x + C_2 = -\frac{1}{\ln y + C_1}}$

7)

$yy' y'' = y'^3 + y'^2$ $\quad y' = p(y) \quad y'' = p \cdot p'$ $\quad \frac{dy}{dp} = \frac{1}{\frac{dp}{dy}} = \frac{1}{p'}$

$y \cdot \cancel{p} p p' = p^{\cancel{3}} + \cancel{p^2} p'^2 \quad p^2 = 0 \Rightarrow \boxed{y = C}$

$y p' = p + p'^2$

$p = yp' + p'^2$ $\quad \boxed{\text{Clairaut}}$

$p = y C_1 + C_1^2$

~~$\int \frac{dy}{C_1 y + C_1^2} = \int dx$~~

$\int \frac{dy}{y + C_1} = C_1 \int dx$

$\ln(y + C_1) = C_1 x + C_2$

$\boxed{y = e^{C_1 x + C_2} - C_1}$

8.) $xy''' = y'' - xy''$ $\quad y'' = p(x)$

1) $xp' = p - xp$

$xp' = p(1-x)$

$\int \frac{dp}{p} = \int\left(\frac{1}{x} - 1\right)dx$

$\ln p = \ln x - \ln e^x + \ln C$

$p = \frac{xC}{e^x}$

$\Rightarrow y'' = \frac{x \cdot C_1}{e^x}$ $\quad y' = C_1 \int \frac{x}{e^x} = C_1 \int x e^{-x} dx$

$= \left| \begin{matrix} t = -x \\ dt = -dx \end{matrix} \right| = +C_1 \int t \cdot e^t dt$

$= C_1 \int t e^t = \left| \begin{matrix} u = t & du = dt \\ dv = e^t & v = e^t \end{matrix} \right|$

$= C_1 \cdot (t e^t - e^t) = C_1 (e^{-x} - x e^{-x}) + C_2$

$y = \int \left[C_1 e^{-x} - C_1 x e^{-x} + C_2 \right] dx$

$= + C_1 e^{-x} + C_1 x e^{-x} + C_1 e^{-x} + C_2 x + C_3$

$= \boxed{C_1 (x e^{-x} + 2 e^{-x}) + C_2 x + C_3}$

Zad 2

1) $yy'' + y'^2 = 1$ $\quad y' = p(y)$ $\quad y'' = p \cdot p'$

1) $y \cdot p \cdot p' + p^2 = 1$

$p \cdot p' + \frac{1}{y} p^2 = \frac{1}{y}$ Bernoulli

$t = p^2$ $\quad t' = 2pp'$

$\frac{1}{2} t' + \frac{1}{y} t = \frac{1}{y}$ linear

2) $t' + \frac{2}{y} t = 0$

$\frac{dt}{t} = -\frac{2}{y} dy$

$\ln t = \ln \frac{C_1}{y^2}$ $\quad t = \frac{C_1}{y^2}$

$\frac{1}{2} \cdot \frac{C_1'}{y^2} \cancel{- \frac{C_1}{y^3}} \cancel{+ \frac{C_1}{y^3}} = \frac{1}{y}$

$C_1' = 2y$ $\quad C_1(y) = y^2 + C_1$

$t = 1 + \frac{1}{y^2} C_1 = \frac{1}{y^2}(C_1 + y^2)$

3) $p = \sqrt{t} = \pm \frac{1}{y} \sqrt{y^2 + C_1}$

$\int \frac{y}{\sqrt{y^2 + C_1}} dy = \pm \int dx$

$\int d(\sqrt{y^2 + C_1}) = \pm \int dx$

$\boxed{\sqrt{y^2 + C_1} = \pm x + C_2}$

2) $yy'' - y'^2 = y^2 y'$ $\quad y' = p(y) \quad y'' = p \cdot p'$

1) $y \cancel{p} p' - p^2 = y^2 \cancel{p} \Rightarrow p = 0 \quad \boxed{y = C}$

$p'y - p = y^2 \quad / : y$

$p' - \frac{1}{y} p = y$ linearna

$p' - \frac{1}{y} p = 0$

$\frac{dp}{p} = \frac{dy}{y}$

$p = C_1 y$

2) $C_1' y + \cancel{C_1} - C_1 = y$

$C_1' = 1$

$C_1 = y + C_1$

$p = y(y + C_1)$

$C_1 \rightarrow 2C_1$

$p = y(y + 2C_1)$

3) $\int \frac{dy}{y(y+2C_1)} = \int dx$

$\int \frac{d(y+C_1)}{(y+C_1)^2 - C_1^2} = x + C_2$

$\frac{1}{2C_1} \ln \left| \frac{y + C_1 - C_1}{y + C_1 + C_1} \right| = x + C_2$

$$\boxed{\frac{1}{C_1} \ln \left| \frac{y}{y + C_1} \right| = x + C_2}$$

3) $xyy'' - xy'^2 = yy'$ $\quad$ Homogena u y, y', y''

$\boxed{y = e^{\int z \, dx} \quad z = z(x)}$ $\quad y' = z e^{\int z dx} = \boxed{yz}$

$y'' = y'z + z'y = \boxed{yz^2 + z'y}$

$x \cdot y(yz^2 + z'y) - x \cdot y^2 z^2 = y \cdot y \cdot z \quad / : y^2$

$\cancel{xz^2} + xz' - \cancel{xz^2} = z$

$\frac{dz}{z} = \frac{dx}{x} \quad z = C_1 x \quad y = e^{\int C_1 x \, dx} = e^{\frac{1}{2} C_1 x^2 + C_1 \cdot C_2}$

$e^{C_1 C_2} = C_2$

$$= \boxed{C_2 e^{C_1 x^2}}$$

4) $yy'' - y'^2 = y^2 \ln y$ $\quad y' = p(y) \quad y'' = p \cdot p'$

1) $y \cdot p \cdot p' - p^2 = y^2 \ln y$

$p \cdot p' - \frac{1}{y} p^2 = y \ln y$ Bernoulli

$t = p^2 \quad t' = 2pp'$

$\frac{1}{2} t' - \frac{1}{y} t = y \ln y$ LDJ

2) $\frac{1}{2} t' - \frac{1}{y} t = 0$

$\frac{dt}{t} = 2 \frac{dy}{y}$

$t = y^2 \cdot C_1$

$\frac{1}{2} C_1' y^2 + \cancel{C_1 y} - \cancel{C_1 y} = y \ln y$

$C_1' = 2 \cdot \frac{\ln y}{y}$

$C_1(y) = \ln^2 y + C_1$

$t = y^2 (\ln^2 y + C_1)$

3) $p = \pm y \sqrt{\ln^2 y + C_1}$

$\int \frac{dy}{y \sqrt{\ln^2 y + C_1}} = \pm \int dx$

$\int \frac{d(\ln y)}{\sqrt{\ln^2 y + C_1}} = \pm x + C_2$

$$\boxed{\ln \left| \ln y + \sqrt{\ln^2 y + C_1} \right| = \pm x + C_2}$$

5) $yy'' + y'^2 - \frac{1}{x}yy' = 0$ Homogena po y, y', y''

$y = e^{\int z(x)\,dx}$ $\quad y' = y\cdot z \quad y'' = yz^2 + z'y$

1)

$\cancel{y}\cdot(\cancel{y}z^2 + z'\cancel{y}) + \cancel{y^2}z^2 - \frac{1}{x}\cancel{y\cdot y}\,z = 0$

$z^2 + z' + z^2 - \frac{z}{x} = 0$

$z' - \frac{1}{x}z = -2z^2 \quad /: -z^2$

$-\frac{z'}{z^2} + \frac{1}{x}\cdot\frac{1}{z} = 2$ Bernoulli

$t = \frac{1}{z} \quad t' = -\frac{1}{z^2}z'$

$t' + \frac{1}{x}t = 2$ Linearna

2) $t' + \frac{1}{x}t = 0$

$\frac{dt}{t} = -\frac{dx}{x}$

$t = \frac{C_1}{x}$

$\frac{C_1'}{x} - \cancel{\frac{C_1}{x^2}} + \cancel{\frac{C_1}{x^2}} = 2$

$C_1' = 2x$

$C_1(x) = x^2 + C_1$

$t = \frac{1}{x}(x^2 + C_1)$

3) $z = \frac{1}{t} = \frac{x}{x^2 + C_1}$

$\int z(x)\,dx = \frac{1}{2}\int\frac{d(x^2+C_1)}{x^2+C_1} = \frac{1}{2}\ln(x^2+C_1) = \ln(\sqrt{x^2+C_1}) + \ln C_2$

$y = e^{\ln(\sqrt{x^2+C_1}) + \ln C_2} = \boxed{C_2\cdot\sqrt{x^2+C_1}}$

6) $xy'' - y' - x\sin\frac{y'}{x} = 0$

$\frac{y'}{x} = \arcsin[p(x)]$

$y' = x\cdot\arcsin p$

$y'' = \arcsin p + x\cdot\frac{1}{\sqrt{1-p^2}}\,p'$

1)

$\cancel{x\arcsin p} + \frac{x^2p'}{\sqrt{1-p^2}} - \cancel{x\arcsin p} - x\cdot p = 0$

$\frac{x^{\cancel{2}}p'}{\sqrt{1-p^2}} - \cancel{x}\cdot p = 0 \quad xp' = p\sqrt{1-p^2}$

$\underbrace{\int\frac{dp}{p\sqrt{1-p^2}}}_{A} = \int\frac{dx}{x} = \ln x\cdot C_1$

2) A $\left|\begin{matrix} p = \sin t \\ dp = \cos t\,dt\end{matrix}\right.$

$= \int\frac{\cos t\,dt}{\sin t\cdot\cos t} = \ln|\tan\frac{t}{2}|$

3)

$\cancel{\ln}\tan\frac{t}{2} = \cancel{\ln}x\cdot C_1$

$\frac{t}{2} = \arctan x\cdot C_1$

$\frac{1}{2}\arcsin\frac{y'}{x} = \arctan x\cdot C_1$

$y' = x\cdot\underbrace{\sin 2\arctan(x\cdot C_1)}_{B}$

4) $B = \sin(2\cdot\arctan(x\cdot C_1))$

$\alpha = \tan\left[\frac{1}{2}\cdot 2\cdot\arctan(x\cdot C_1)\right] \quad \sin\alpha = \frac{2\alpha}{1+\alpha^2}$

$\alpha = x\cdot C_1$

$B = \frac{2\cdot x\cdot C_1}{1+x^2C_1^2}$

$\boxed{y' = \frac{2x^2C_1}{1+x^2C_1^2}}$

6) $xy'' - y' - x \cdot \sin\frac{y'}{x} = 0$ $\quad \frac{y'}{x} = \arcsin p \quad (p = p(x))$

$$y' = x \cdot \arcsin p$$

$$y'' = \arcsin p + \frac{x}{\sqrt{1-p^2}} \cdot p'$$

$$\cancel{x \arcsin p} + \frac{x^{\cancel{2}} p'}{\sqrt{1-p^2}} - \cancel{x \arcsin p} - \cancel{x} \cdot p = 0 \qquad \boxed{x = 0} \text{ trivijalno rješenje}$$

$$\frac{x p'}{\sqrt{1-p^2}} = p \Rightarrow x p' = p\sqrt{1-p^2} \Rightarrow \int \frac{dp}{p\sqrt{1-p^2}} = \int \frac{dx}{x} = \ln x \cdot c_1$$

$$\int \frac{dp}{p\sqrt{1-p^2}} = \left| \begin{array}{l} p = \sin\alpha \\ dp = \cos\alpha \, d\alpha \end{array} \right. = \int \frac{\cancel{\cos\alpha} \, d\alpha}{\sin\alpha \cdot \cancel{\cos\alpha}} = \ln\left|\tan\frac{\alpha}{2}\right|$$

$$= \ln\left|\tan\left(\frac{1}{2}\arcsin p\right)\right|$$

$$\cancel{\ln}\left|\tan\left(\frac{1}{2}\arcsin p\right)\right| = \cancel{\ln}\, c_1 x$$

$$\tan\left(\frac{1}{2}\frac{y'}{x}\right) = c_1 \cdot x \quad / \arctan$$

$$\frac{1}{2}\frac{y'}{x} = \arctan c_1 \cdot x \Rightarrow \boxed{y' = 2x \cdot \arctan c_1 \cdot x}$$

$$y = 2\int x \cdot \arctan c_1 \cdot x = \left| \begin{array}{ll} u = \arctan c_1 \cdot x & du = \frac{1}{1+c_1^2 x^2} c_1 \\ dv = 2x\,dx & v = x^2 \end{array} \right.$$

$$= x^2 \arctan c_1 \cdot x - \underbrace{\int \frac{x^2 c_1}{1+c_1^2 x^2}}_{A} + c_2$$

$x^2 c_1 : x^2 c_1^2 + 1 = \frac{1}{c_1} - \frac{1}{c_1(x^2 c_1^2 + 1)}$

$x^2 c_1 + \frac{1}{c_1}$

$- \frac{1}{c_1}$

$$A = \frac{1}{c_1}\int\left[1 - \frac{1}{(xc_1)^2+1}\right]dx = \frac{1}{c_1} \cdot x - \frac{1}{c_1^2}\arctan x \cdot c_1$$

$$\boxed{y = x^2 \arctan c_1 x - \frac{1}{c_1}x + \frac{1}{c_1^2}\arctan c_1 x + c_2}$$

2) $y'' + y'^2 = e^{-y}$ $\quad y' = p(y) \quad y'' = p\cdot p'$

(1) $p\cdot p' + p^2 = e^{-y}$ Bernulli

$t = p^2 \quad t' = 2pp'$

$\frac{1}{2}t' + t = e^{-y}$ Linearna

(2) $\frac{1}{2}t' + t = 0$

$\frac{dt}{t} = -2dy$

$\ln t = \ln e^{-2y}\cdot c_1$

$t = c_1 e^{-2y}$

$\frac{1}{2}c_1' e^{-2y} - \cancel{c_1 e^{-2y}} + \cancel{c_1 e^{-2y}} = e^{-y}$

$\frac{1}{2}c_1' = e^{y}$

$c_1' = 2e^{y}$

$c_1(y) = 2e^{y} + c_1$

$t = c_1 e^{-2y} + 2e^{-y}$

$= e^{-2y}(c_1 + 2e^{y})$

(3) $p = \pm\sqrt{t} = \pm e^{-y}\sqrt{c_1 + 2e^{y}}$

$\int \frac{e^{y}\,dy}{\sqrt{2e^{y}+c_1}} = \pm\int dx \quad /\cdot 2$

$\frac{1}{2}\int \frac{d(2e^{y}+c_1)}{\sqrt{2e^{y}+c_1}} = \pm x + c_2$

$\frac{1}{2}\cdot 2\sqrt{2e^{y}+c_1} = \pm x + c_2 \quad /^2$

$(2e^{y}+c_1) = (x+c_2)^2$

$$\boxed{2(e^{y}+c_1) = (x+c_2)^2}$$

8) $xy'(yy'' - y'^2) - yy'^2 = x^4y^3$ Homogena po y, y', y''

$y = e^{\int z\,dx} \quad y' = yz \quad y'' = yz^2 + yz'$

(1)

$x\cdot \cancel{y}z(\cancel{y}\cancel{y}(z^2+z') - \cancel{y}\cancel{y}z^2) - \cancel{y}\cancel{y^2}z^2 = x^4\cancel{y^3}$

$xz(\cancel{z^2} + z' - \cancel{z^2}) - z^2 = x^4$

$xzz' - z^2 = x^4 \quad /:x$

$zz' - \frac{1}{x}z^2 = x^3$ Bernulli

$t = z^2 \quad t' = 2zz'$

$\frac{1}{2}t' - \frac{1}{x}t = x^3$ Linearna

(2) $\frac{1}{2}t' - \frac{1}{x}t = 0$

$\frac{dt}{t} = 2\frac{dx}{x}$

$t = x^2 c_1$

$\frac{1}{2}c_1' x^2 + \cancel{xc_1} - \cancel{xc_1} = x^3$

$c_1' = 2\cdot x$

$c_1(x) = x^2 + c_1$

$t = x^2(x^2+c_1)$

(3)

$z = \pm\sqrt{t} = \pm x\sqrt{x^2+c_1}$

$\int z\,dx = \pm\int x\sqrt{x^2+c_1}\,dx = \pm\frac{1}{2}\int\sqrt{x^2+c_1}\,d(x^2+c_1) = \pm\frac{1}{2}\cdot\frac{2}{3}(x^2+c_1)^{\frac{3}{2}} + \ln c_2$

$$\boxed{y = c_2 e^{\pm\frac{1}{3}(x^2+c_1)^{\frac{3}{2}}}}$$

Zad 3

$y' = p(x)$

1) $x^2 y'' = y'^2$

$x^2 p' = p^2 \qquad \int \frac{dp}{p^2} = \int \frac{dx}{x^2} \quad \Rightarrow + \frac{1}{p} = + \frac{1}{x} + C_1$

$p = \frac{x}{1 + C_1 x}$

$y = \int \frac{x}{C_1 x + 1} dx = \frac{1}{C_1} \int \frac{C_1 x + 1 - 1}{C_1 x + 1} dx = \frac{1}{C_1} \int \left(1 - \frac{1}{C_1 x + 1}\right) dx$

$= \frac{1}{C_1} \cdot x - \frac{1}{C_1^2} \ln|C_1 x + 1| \quad / \cdot C_1^2$

$\boxed{C_1^2 y = C_1 x - \ln|C_1 x + 1| + C_2}$

2) $2x y' y'' = y'^2 - 1 \qquad y' = p(x)$

1. $2x p p' = p^2 - 1 \quad \Rightarrow \boxed{p = \pm 1 \quad y = \pm x + C}$

$p p' - \frac{1}{2x} p^2 = -\frac{1}{2x}$ Bernoulli

$t = p^2 \qquad t' = 2 p p'$

$\cancel{\frac{1}{2}} t' - \cancel{\frac{1}{2}} \frac{1}{x} \cdot t = -\cancel{\frac{1}{2}} \cdot \frac{1}{x}$ linearny

2. $t' - \frac{1}{x} \cdot t = 0$

$\int \frac{dt}{t} = \int \frac{dx}{x}$

$t = C_1 x$

$C_1' x + \cancel{C_1} - \cancel{C_1} = -\frac{1}{x}$

$C_1' = -\frac{1}{x^2}$

$C_1(x) = \frac{1}{x} + C_1$

$t = 1 + x \cdot C_1$

3. $p = \pm \sqrt{C_1 x + 1}$

$\frac{dy}{dx} = \pm \sqrt{C_1 x + 1}$

$y = \pm \int \sqrt{C_1 x + 1} + C_2$

$y = \pm \frac{2}{3} (C_1 x + 1)^{\frac{3}{2}} + C_2$

$3(y - C_2) = \pm 2 (C_1 x + 1)^{\frac{3}{2}} \quad /^2$

$\boxed{9(y - C_2)^2 = 4 (C_1 x + 1)^3}$

3) $y'^2 + 2yy'' = 0 \quad \Rightarrow \quad y' = p(y) \quad y'' = p \cdot p'$

$\cancel{p}^2 + 2y\cancel{p} \cdot p' = 0 \Rightarrow p = 0 \Rightarrow \boxed{y = C}$

$2yp' = -p$

$\frac{dp}{p} = -\frac{1}{2}\frac{dy}{y} \Rightarrow \ln p = \ln \frac{C_1}{\sqrt{y}} \Rightarrow p = \frac{C_1}{\sqrt{y}}$

$\int \sqrt{y}\, dy = \int C_1 x$

$\frac{2}{3} y^{\frac{3}{2}} = C_1 x + C_2 \quad / \cdot \frac{3}{2}$

$y^{\frac{3}{2}} = C_1 x + C_2 \quad /^2$

$\boxed{y^3 = \left(C_1 x + C_2\right)^2}$

4) $y'' = 2yy' \qquad y' = p(y) \quad y'' = p \cdot p'$

$\cancel{p} \cdot p' = 2y\cancel{p} \Rightarrow p = 0 \quad \boxed{y = C}$

$dp = 2y\,dy$

$p = y^2 + C_1^2 \Rightarrow \int \frac{dy}{y^2 + C_1^2} = \int dx \Rightarrow \frac{1}{C_1} \cdot \arctan \frac{y}{C_1} = x + \frac{C_2}{C_1}$

$\boxed{y = C_1 \tan(C_1 x + C_2)}$

5) $yy'' + 1 = y'^2, \quad y' = p(y) \quad y'' = p \cdot p'$

1. $y \cdot p\, p' + 1 = p^2$

$p \cdot p' - \frac{1}{y} p^2 = -\frac{1}{y}$ Bernoulli

$t = p^2 \quad t' = 2pp'$

$\frac{1}{2} t' - \frac{1}{y} t = -\frac{1}{y}$ Lineарна

$\frac{1}{2} t' - \frac{1}{y} t = 0$

$\frac{dt}{t} = 2\frac{dy}{y}$

$t = C_1 y^2$

$\frac{1}{2} C_1' y^2 + \cancel{C_1 y} - \cancel{C_1 y} = -\frac{1}{y}$

$\boxed{C_1' = -\frac{2}{y^3}}$

2. $C_1(y) = \int -\frac{2}{y^3} = \frac{1}{y^2} + C_1^2$

$\boxed{t = 1 + y^2 C_1^2}$

$p = \pm\sqrt{t} = \pm\sqrt{C_1^2 y^2 + 1}$

$\int \frac{dy}{\sqrt{1 + C_1^2 y^2}} = \pm\int dx = \pm\left(x + \frac{C_2}{C_1}\right)$

$\mathrm{Arsh}(C_1 y) \cdot \frac{1}{C_1} = \pm\left(x + \frac{C_2}{C_1}\right)$

$\mathrm{Arsh}(C_1 y) = \pm(C_1 x + C_2)$

$\boxed{C_1 y = \pm\, \mathrm{sh}(C_1 x + C_2)}$

6) $y''^2 + y' = x y''$ $y' = p(x)$

$p'^2 + p = x p'$

$p = x p' - p'^2$ Clairaut !

O.R. $p = x C_1 - C_1^2$

$\int dy = \int (x C_1 - C_1^2) dx$

$\boxed{y = \frac{1}{2} C_1 x^2 - C_1^2 x + C_2}$

S.R. $p' = t$

$x = 2t$ $t = \frac{1}{2}x$

$p = xt - t^2$

$= \frac{1}{2}x^2 - \frac{1}{4}x^2 = \frac{1}{4}x^2$

$dy = (\frac{1}{4}x^2 dx)$

$\boxed{y = \frac{1}{12}x^3 + C}$

7) $y'' + y'^2 = 2e^{-y}$ $\qquad y' = p(y) \quad y'' = p \cdot p'$

(1) $p \cdot p' + p^2 = 2e^{-y}$ Bernoulli

$t = p^2 \quad t' = 2pp'$

$\frac{1}{2} t' + t = 2e^{-y}$ Lineara

(2) $\frac{1}{2} t' + t = 0$

$\frac{dt}{t} = -2dy$

$\ln t = -2y + \ln C_1$

$t = C_1 e^{-2y}$

$\frac{1}{2} C_1' e^{-2y} - \cancel{C_1 e^{-2y}} + \cancel{C_1 e^{-2y}} = 2e^{-y}$

$C_1' = 4e^{y}$

$C_1(y) = 4e^{y} + C_1$

$t = e^{-2y}(4e^{y} + C_1)$

(3) $p = \pm\sqrt{t} = e^{-y}\sqrt{4e^{y} + C_1}$

$\int \frac{e^{y}}{\sqrt{4e^{y} + C_1}} dy = \pm \int dx$

$\frac{1}{4} \int \frac{d(4e^{y} + C_1)}{\sqrt{4e^{y} + C_1}} = \pm (x + C_2)$

$\frac{1}{4} \cdot 2 \sqrt{4e^{y} + C_1} = \pm (x + C_2) \quad /^2$

$\frac{1}{4}(4e^{y} + C_1) = \pm (x + C_2)^2$

$$\boxed{e^{y} + C_1 = \pm (x + C_2)^2}$$

8) $y''' \cdot y'^2 = y''^2$ $\qquad y' = p(y) \quad y'' = \frac{dy'}{dx} = \frac{dp}{dy} \cdot \frac{dy}{dx} = p \cdot p'$

$y''' = \frac{dy''}{dx} = \frac{dy'}{dy} \cdot \frac{dy}{dx} = p\left[p'^2 + pp''\right]$

$p\left[p'^2 + pp''\right] \cdot \cancel{p^2} = \cancel{p^2} p'^2$

$\Rightarrow p = 0, \; p' = 0$

$\Rightarrow y'' = 0 \quad \boxed{y = C_1 x + C_2}$

(1)

$\cancel{pp'^2 + p^2 p'' = p'^2}$

$\cancel{pt^2 + p^2 t t' = t^2}$

$\cancel{pt + p^2 t' = t}$

$\cancel{t(p-1) = p^2 t'}$

$\cancel{\frac{dp}{dx}\left(\frac{dy}{dx} - 1\right) = \frac{dx^2}{dx^2} \cdot \frac{dx\,dx}{dx\,dx}}$

$\cancel{\frac{1}{dx} - \frac{1}{dy} = \frac{1}{dx}}$

$\cancel{\frac{2}{dx} = \frac{1}{dy} \quad \frac{dy}{dx} = \frac{1}{2} \quad y = \frac{1}{2}x}$

$\cancel{p' = t(p) \quad \frac{dp}{dy} = t \quad dp = t\,dy}$

$\cancel{dy = \frac{1}{t} dp}$

$\cancel{t = p' \quad t \cdot t' = p'}$

$\cancel{t' = \frac{p'}{t} = \frac{\frac{dp}{dx}}{\frac{dy}{dx}} = \frac{dx\,dp}{dy\,dy}}$

Ne znam :)

Zad 14

1)

$$y'y''' + 3y'y'' = 0 \quad \Rightarrow y''=0 \quad \boxed{y = c_1 x + c_2}$$

1. $\frac{d}{dx}\left(yy'' + (y')^2\right) = 0$

$$yy'' + y'^2 = c_1$$

$$\frac{d}{dx}\left(yy' - c_1 x\right) = 0$$

$$yy' - c_1 x = c_2$$

$$y\,dy = [c_1 x + c_2]\,dx$$

2. $\int y\,dy = \int [c_1 x + c_2]\,dx$

$$\frac{1}{2} y^2 = \frac{1}{2} c_1 x^2 + c_2 x + c_3$$

$$\boxed{y^2 = c_1 x^2 + c_2 x + c_3}$$

2)

$$y'y''' = 2y''^2 \quad y''=0\ ? \Rightarrow y'''=0 \quad y'\cdot 0 = 2\cdot 0 \text{ ok}$$

$$\boxed{y = c_1 x + c_2}$$

1. $\frac{y'''}{y''} = 2\frac{y''}{y'}$

$$\frac{d}{dx}\left[\ln y'' - 2\ln y'\right] = 0$$

$$\ln y'' - 2\ln y' = \ln c_1$$

$$y'' = c_1 y'^2$$

2. $\frac{y''}{y'} = c_1 y'$

$$\frac{d}{dx}\left[\ln y' - c_1 y\right] = 0$$

$$\ln y' - c_1 y = c_2$$

$$y' = e^{c_1 y + c_2}$$

$$\int e^{-c_1 y - c_2}\,dy = \int dx$$

$$-\frac{1}{c_1} e^{-c_1 y - c_2} = x + c_3 \qquad -c_1 \to c_1$$

$$c_1 \cdot c_3 \to c_3$$

$$c_1 y - c_2 = \ln|c_1 x + c_3|$$

$$\boxed{c_1 y = \ln|c_1 x + c_3| + c_2}$$

3) $y'' = xy'' + y + 1$

1. $\frac{d}{dx} y' = \frac{d}{dx}\left[xy + x \right]$

$y' = xy + x + c_1$ lineare

$\underbrace{y' - xy}_{f(x)} = \underbrace{x + c_1}_{g(x)}$

$y = -1$

2. $\int f(x)\,dx = \frac{1}{2}x^2$

$y = e^{+\frac{1}{2}x^2}\left[\int (x + c_1)\, e^{-\frac{1}{2}x^2}\,dx + c_2\right]$

$y = e^{\frac{1}{2}x^2}\left[\underbrace{\int x e^{-\frac{1}{2}x^2}\,dx}_{-e^{-\frac{1}{2}x^2}} + c_1 \int e^{-\frac{1}{2}x^2}\,dx + c_2\right]$

$$\boxed{y = c_2 e^{\frac{1}{2}x^2} - 1 + c_1 \int e^{-\frac{1}{2}x^2}}$$

tutto risolto

4) $y y'' - y'^2 = y^2 y'$ $\qquad y' = 0 \Rightarrow \boxed{y = c}$

1. $\frac{y y'' - y'^2}{y^2} = y'$

$\frac{d}{dx}\left(\frac{y'}{y}\right) = \frac{d}{dx} y$

$\frac{y'}{y} = y + 2c_1$

2. $\int \frac{dy}{y(y + 2c_1)} = \int dx = x + c_2$

$\int \frac{dy}{y(y + 2c_1)} = \int \frac{d(x + c_1)}{(y + c_1)^2 - c_1^2}$

$= \frac{1}{2c_1} \ln \left| \frac{y + c_1 - c_1}{y + c_1 + c_1} \right|$ $\qquad 2c_1 \to c_1$

$= \frac{1}{c_1} \ln \left| \frac{y}{y + c_1} \right| = x + c_2$ $\qquad c_2 c_1 \to c_2$

$$\boxed{\ln \left| \frac{y}{y + c_1} \right| = c_1 x + c_2}$$

Zad 5.

1) $y'' + 4y' + 5y = 0 \Rightarrow r^2 + 4r + 5 = 0 \quad r = \frac{-4 \pm \sqrt{16-20}}{2}$

$r = -2 \pm i$

$$\boxed{y = e^{-2x}\left[C_1 \cos x + C_2 \sin x\right]}$$

2.) $y'' + 4y' - 5y = 0 \Rightarrow r^2 + 4r - 5 = 0 \quad r = \frac{-4 \pm \sqrt{16+20}}{2}$

$= -2 \pm 3 \quad r_1 = -5,\ r_2 = 1$

$$\boxed{y = C_1 e^{-5x} + C_2 e^{x}}$$

3.) $y'' + 6y' = 0 \Rightarrow r^2 + 6r = 0 \quad r(r+6) = 0 \quad r_1 = 0 \quad r_2 = -6$

$$\boxed{y = C_1 + C_2 e^{-6x}}$$

4.) $y'' + 6y = 0 \Rightarrow r^2 + 6 = 0 \quad r = \pm\sqrt{6}\, i$

$$\boxed{y = C_1 \cos\sqrt{6}x + C_2 \sin\sqrt{6}x}$$

Zad 6

$2y' + (y'^2 - 6x)y'' = 0 \qquad y(2) = 0 \quad y'(2) = 2$

$y' = p(x) \qquad 2p + (p^2 - 6x)p' = 0$

$\underbrace{2p}_{P} \cdot dx + \underbrace{(p^2 - 6x)}_{Q}\, dp = 0$

$P'_p = 2$

$Q'_x = -6$

$P'_p - Q'_x = 8$

① $\ln\mu(p) = -\int \frac{1}{P}\left[P'_p - Q'_x\right] = -\int \frac{8}{2p}$

$\ln\mu = -4\ln p \Rightarrow \mu = \frac{1}{p^4}$

$\int_0^x \frac{2}{p^3}\,dx + \int_1^p \left(\frac{1}{p^2} - \cancel{\frac{6x}{p^4}}\right)dp = C_1$

$\frac{2}{p^3}x - \frac{1}{p} + 1 = C_1 \qquad p(2) = 2$

$\cancel{\frac{2\cdot 2}{2^3}} - \frac{1}{2} + 1 = C_1$

$\boxed{C_1 = 1}$

② $\frac{2x}{p^3} - \frac{1}{p} \cancel{+1} = \cancel{1} \quad / \cdot p^3$

$2x - p^2 = 0$

$p = \pm\sqrt{2x}$

$\int dy = \pm\sqrt{2}\int\sqrt{x}\,dx + C_2$

$y = \pm\left(\sqrt{2}\cdot\frac{2}{3}x^{\frac{3}{2}} + C_2\right)$

$0 = \pm\left(\underbrace{\sqrt{2}\,\frac{2}{3}\sqrt{8}}_{=\frac{8}{3}} + C_2\right) \Rightarrow C_2 = \mp\frac{8}{3}$

$$\boxed{y = \pm\frac{2\sqrt{2}}{3}x^{\frac{3}{2}} \mp \frac{8}{3}}$$

Zad 7 $y''(1-2yy') = y'^3$ $y' = p(y)$, $y(3)=2$ $y'(3)=1$

$y'' = p\cdot p'$

$\cancel{p}\cdot p'(1-2yp) = p^{\cancel{3}2} \Rightarrow p = 0 \Rightarrow \cancel{y=C}$

$\underbrace{dp(1-2yp)}_{A} \underbrace{- p^2}_{B} dy = 0$ $A'_y = -2p$ $B'_p = -2p$

Egzaktna

$\int_0^p (1-2yp)\,dp + \int_0^y (-\cancel{p^2})\,dy = C_1$

$p - yp^2 = C_1$ $y(3)=2$ $y'(3)=1$

$y' - yy'^2 = C_1$ $/x=3$ $1-2\cdot 1 = C_1 \Rightarrow C_1 = -1$

$p - yp^2 = -1$

$p^2 y - p - 1 = 0 \Rightarrow p_{1,2} = \frac{1\pm\sqrt{1+4y}}{2y}$

$\frac{dy}{dx} = \frac{1\pm\sqrt{1+4y}}{2y}$ $\underbrace{\int \frac{2y}{1\pm\sqrt{1+4y}}\,dy}_{I} = \int dx = x + C_2$

$I = \int \frac{2y}{1\pm\sqrt{1+4y}}\,dy = \frac{1}{2}\int \frac{4y}{1\pm\sqrt{1+4y}}\,dy = \left| \begin{matrix} t^2 = 4y+1 \\ 2t\,dt = 4dy \end{matrix} \right.$

$= \frac{1}{2}\int \frac{(t^2-1)}{1\pm t}\cdot\frac{1}{2}t\,dt = \frac{1}{4}\int \frac{t(t^2-1)}{1\pm t}\,dt = \frac{1}{4}\int \frac{t(t+1)(t-1)}{1\pm t}$

$I^+ = \frac{1}{4}\int t(t-1)\,dt = \frac{1}{4}\left[\frac{1}{3}t^3 - \frac{1}{2}t^2\right]$ $t^2 = 4y+1$

$I^- = \frac{1}{4}\int \frac{t(t-1)(t+1)}{-(t-1)} = -\frac{1}{4}\left[\frac{1}{3}t^3 + \frac{1}{2}t^2\right]$ $t(3) = \sqrt{4\cdot 2+1} = \boxed{3}$

$I^{(+)}: \frac{1}{4}\left[\frac{1}{\cancel{3}}3^{\cancel{3}2} - \frac{1}{2}\cdot 3^2\right] = 3 + C_+$ $C_+ = \frac{1}{4}\cdot\frac{1}{2}3^2 - 3 = \boxed{-\frac{15}{8}}$

$$\boxed{x - \frac{15}{8} = \frac{1}{12}(1+4y)^{\frac{3}{2}} - \frac{1}{8}(1+4y)}$$

$(-)$: $-\frac{1}{4}\left[\frac{1}{\cancel{3}}3^{\cancel{3}2} + \frac{1}{2}3^2\right] = 3 + C_- \Rightarrow C_- = -\frac{1}{4}\cdot\frac{3}{2}\cdot 3^2 - 3 = \boxed{-\frac{51}{8}}$

$$\boxed{x - \frac{51}{8} = -\frac{1}{12}(1+4y)^{\frac{3}{2}} - \frac{1}{8}(1+4y)}$$

Zad 8 $\quad y^2y'' + 2yy'^2 - y' = 0 \qquad y(0)=2 \quad y'(0)=1$

$c=2 \quad c'=1$

$y' = p(y) \quad y'' = pp'$

$y^2 \cancel{p}p' + 2yp^{\cancel{2}} - \cancel{y'} = 0 \quad \Rightarrow p=c \Rightarrow y' \neq c$

$y^2p' + 2yp - 1 = 0$

$\underbrace{y^2}_{A}dp + \underbrace{(2yp-1)}_{B}dy = 0 \qquad A'_y = 2y \quad B'_p = 2y$

Egzaktna

$\int_0^p y^2 dp + \int_0^y (2yp-1)\,dy = C_1$

$py^2 - y = C_1 \qquad y(0)=2 \quad y'(0)=1$

$1\cdot 2^2 - 2 = C_1 \Rightarrow C_1 = 2$

$py^2 - y = 2$

$y' = \frac{2+y}{y^2} \Rightarrow \underbrace{\int \frac{y^2}{2+y}\,dy}_{I} = \int dx = x + C_2$

$I = \int \frac{y^2}{2+y}\,dy$

$y^2 : y+2 = y - 2 + \frac{4}{y+2}$

y^2+2y

$-2y$

$-2y-4$

$+4$

$I = \int \left(y-2+\frac{4}{y+2}\right)dy = \frac{1}{2}y^2 - 2y + 4\ln|y+2| = x + C_2, \quad x=0$

$\frac{1}{2}\cdot 2^2 - 2\cdot 2 + 4\ln 4 = C_2$

$2 - 4 + 4\ln 4 = C_2 \qquad \boxed{C_2 = -2 + 4\ln 4}$

$$\boxed{x = \frac{1}{2}y^2 - 2y + 4\ln|y+2| + 2 - 4\ln 4}$$

Zad 9 $\quad y'' + \lambda y = 0 \quad y'(0) = y'(\pi) = 0 \quad y \neq 0$

linearna: $\Rightarrow r^2 + \lambda = 0$

$r = \pm\sqrt{\lambda}\, i$

$y = C_1 \cos[\sqrt{\lambda} x] + C_2 \sin[\sqrt{\lambda} x]$

$y' = -\sqrt{\lambda} C_1 \sin(\sqrt{\lambda} x) + \sqrt{\lambda} C_2 \cos(\sqrt{\lambda} x)$

$x = 0$

$0 = -\sqrt{\lambda} \cancel{C_1} \cdot 0 + \sqrt{\lambda} \cdot C_2 \cdot 1$

$\sqrt{\lambda} \cdot C_2 = 0 \quad \Rightarrow \lambda = 0,\ C_2 = 0 \Rightarrow C_1 \neq 0$

$x = \pi$

$0 = -\sqrt{\lambda} \cdot C_1 \cdot \sin(\sqrt{\lambda} \cdot \pi) + \cancel{\sqrt{\lambda} \cdot C_2 \cdot \cos(\sqrt{\lambda} \cdot \pi)}$

$0 = \cancel{-\sqrt{\lambda}\, C_1} \cdot \sin(\sqrt{\lambda} \cdot \pi) \quad \sin(\sqrt{\lambda} \cdot \pi) = 0$

$\sqrt{\lambda} \cdot \cancel{\pi} = k \cdot \cancel{\pi}$

$\boxed{\lambda = k^2}$

Zad 10 $\quad xy'' + 2y' + xy = 0 \quad y_1 = \frac{\sin x}{x}$

$y'' + \underbrace{\frac{2}{x}}_{f(x)} y' + \underbrace{1}_{g(x)} y = 0$

Iz-od gotove formule na str 29. obrazl.

$y_2 = y_1 \int \frac{1}{y_1^2} e^{-\int f(x)dx} dx$

$-\int f(x)dx = -2\ln x$

$y_2 = \frac{\sin x}{x} \int \frac{\cancel{x^2}}{\sin^2 x} \cdot \frac{1}{\cancel{x^2}} dx$

$= \frac{\cancel{\sin x}}{x} \cdot \frac{-\cos x}{\cancel{\sin x}} = -\frac{\cos x}{x}$

$y = C_1 \cdot y_1 + C_2 \cdot y_2 \quad \boxed{y = C_1 \cdot \frac{\sin x}{x} + C_2 \cdot \frac{\cos x}{x}}$