ZZV 10

Zad 1.

Podsjetnik: D. J. familije krivulja dobivamo deriviranjem i eliminacijom konstanata. D. J. ortogonalnih trajektorija dobivamo zamjenom

$$\boxed{y' \to -\frac{1}{y'}}$$

1) $4x^2 + y^2 = k^2 \quad / \frac{d}{dx}$

$\overset{4}{\cancel{8}}x + \cancel{2}y y' = 0$

$4x - \frac{y}{y'} = 0$

$4x y' = y$

$\frac{dy}{y} = \frac{1}{4} \frac{dx}{x}$

$\ln y = \frac{1}{4}\ln x + \ln c$

$$\boxed{y = c \cdot \sqrt[4]{x}}$$

2) $y^2 = Ce^x + x + 1 \quad / \frac{d}{dx} \qquad 2yy' = Ce^x + 1 \implies y^2 = 2yy' + x$

$\implies y^2 = -\frac{2y}{y'} + x \qquad \frac{y^2 - x}{-2y} = \frac{1}{y'} \qquad y' = \frac{2y}{x - y^2}$

$\underbrace{(x - y^2)}_{Q} dy \underbrace{- 2y}_{P} dx = 0 \qquad Q'_x = 1 \qquad P'_y = -2$

$P'_y - Q'_x = -3 \implies \mu = \mu(y)$

$\ln \mu = -\int \frac{1}{P}\left[P'_y - Q'_x\right] dy = -\int \frac{1}{-2y}(-3)\, dy = -\frac{3}{2}\ln y$

$\mu = y^{-\frac{3}{2}}$

$\int_0^x -2y \cdot y^{-\frac{3}{2}}\, dx + \int_1^y (\cancel{0} - y^2) y^{-\frac{3}{2}}\, dy = C$

$\frac{-2x}{\sqrt{y}} - \int_1^y \sqrt{y}\, dy = c \quad / \cdot (-1)$

$$\boxed{\frac{2x}{\sqrt{y}} + \frac{2}{3} y^{\frac{3}{2}} = C_1}$$

3) $x^2 + 2y^2 = a^2 \quad / \frac{d}{dx} \qquad 2x + 4yy' = 0 \overset{y' \to -\frac{1}{y'}}{\implies} x - \frac{2y}{y'} = 0$

$xy' = 2y, \quad \frac{dy}{y} = 2\frac{dx}{x}, \quad \ln y = 2\ln x + \ln c \qquad \boxed{y = Cx^2}$

4) $y = \cos(x+c) \quad / \frac{d}{dx}$ $\qquad y^2 = 1 - \frac{1}{(y')^2}$

$y' = -\sin(x+c)$

$y^2 = \cos^2(x+c)$ $\qquad (y')^2 y^2 = (y')^2 - 1$

$y^2 = 1 - \sin^2(x+c)$ $\qquad (y')^2(y^2 - 1) = -1$

$y^2 = 1 - (y')^2$ $\qquad 1 = (1-y^2)(y')^2 \quad /\sqrt{\ }$

$y' \neq -\frac{1}{y'}$ $\qquad \pm 1 = \sqrt{1-y^2}\,\frac{dy}{dx}$

$$\pm dx = dy\sqrt{1-y^2} \quad /\int$$

$L = \pm\int dx = \pm x + C_1 \qquad D = \int\sqrt{1-y^2}\,dy$

$D = \int\frac{1-y^2}{\sqrt{1-y^2}}\,dy = (Ay+B)\sqrt{1-y^2} + \lambda\int\frac{dy}{\sqrt{1-y^2}} \quad /\frac{d}{dy}$

$\frac{1-y^2}{\sqrt{1-y^2}} = A\sqrt{1-y^2} + \frac{1}{2}\frac{Ay+B}{\sqrt{1-y^2}}\cdot(-2y) + \frac{\lambda}{\sqrt{1-y^2}} \quad /\cdot\sqrt{1-y^2}$

$1 - y^2 = A(1-y^2) - y(Ay+B) + \lambda$

$1 - y^2 = -y^2\cdot 2A - yB + A + \lambda$

$2A = 1 \qquad B = 0 \qquad \lambda + A = 1$

$A = \frac{1}{2} \qquad\qquad \lambda = \frac{1}{2}$

$D = \frac{1}{2}y\sqrt{1-y^2} + \frac{1}{2}\int\frac{dy}{\sqrt{1-y^2}} = \frac{1}{2}y\sqrt{1-y^2} + \frac{1}{2}\arcsin y$

$D = L \qquad \boxed{y\sqrt{1-y^2} + \arcsin y = \pm 2x + K}$

5) $y^2 + 2ax = a^2 \ (a>0)$ $\qquad x = x(y) \qquad y' = \frac{dy}{dx} = \frac{1}{\frac{dx}{dy}} = \frac{1}{x'}$

$\cancel{2}yy' + \cancel{2}a = 0$ $\qquad \frac{y}{x'} + 2x = yx'$

$\frac{y^2}{a} + 2x = a$

$(x')^2y - 2xx' - y = 0$

$\frac{y^{\cancel{2}}}{-\cancel{y}y'} + 2x = -yy'$ $\qquad x' = \frac{2x \pm\sqrt{4x^2+4y^2}}{2y}$

$y' \neq -\frac{1}{y'}$ $\qquad x' = \frac{x}{y} \pm\sqrt{1+\left(\frac{x}{y}\right)^2}$ $\qquad$ Homogene po x

$\frac{x}{y} = z \qquad x' = z + yz'$

$\cancel{z} + yz' = \cancel{z} \pm\sqrt{1+z^2}$

$yy' + 2x = \frac{y}{y'}$ $\qquad \frac{dz}{\sqrt{1+z^2}} = \pm\frac{dy}{y}$ $\qquad z + \sqrt{z^2+1} = Cy$

$\ln|z + \sqrt{z^2+1}| = \ln|Cy|$ $\qquad +\sqrt{x^2+y^2} = C\cdot y^2 - x \quad /^2$

$\cancel{x^2} + y^2 = C^2y^4 - 2Cy^2x + \cancel{x^2}$

$1 = C^2y^2 - 2Cx \quad /C^2$

$\boxed{y^2 - 2Kx = K^2}$

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6.) $x^2+y^2=2ay$

$x^2+(y-a)^2=a^2$

$\cancel{2}x+\cancel{2}(y-a)y'=0$

$x+yy'-ay'=0 \;/\cdot 2y$

$2xy+2y^2y'-2ayy'=0$

$2xy+2y^2y'-x^2y'-y^2y'=0$

$2xy+y^2y'-x^2y'=0$

$y' \downarrow -\frac{1}{y'}$

$2xy-\frac{y^2}{y'}+\frac{x^2}{y'}=0$

$2xyy'-y^2+x^2=0$

$y'=\frac{1}{2}\frac{y}{x}-\frac{1}{2}\frac{x}{y}$

Homogena $\frac{y}{x}=z \quad y'=z+xz'$

$z+xz'=\frac{1}{2}z-\frac{1}{2z}$

$xz'=-\frac{1}{2}z-\frac{1}{2z} \;/\cdot -2$

$-2x\,dz=\frac{z^2+1}{z}$

$\int\frac{2z}{z^2+1}dz=-\int\frac{dx}{x}$

$\ln|z^2+1|=-\ln x+\ln C$

$z^2+1=\frac{C}{x}$

$\frac{y^2}{x^2}+1=\frac{C}{x} \;/\cdot x^2$

$\boxed{y^2+x^2-Cx=0}$

7.) $x^2+(y-c)^2=c^2$

$2x+2(y-c)y'=0$

$x^2+y^2-2yc=0$

$c=\frac{x^2+y^2}{2y}$

$\cancel{2}x+\cancel{2}\left(y-\frac{x^2+y^2}{2y}\right)y'=0$

$x+\left(\frac{2y^2-x^2-y^2}{2y}\right)y'=0$

$2xy+(y^2-x^2)y'=0$

$y' \downarrow -\frac{1}{y'}$

$2xy+(x^2-y^2)\cdot\frac{1}{y'}=0$

$2xyy'=y^2-x^2$

$2y'=\frac{y}{x}-\frac{x}{y}$

$\boxed{y^2+x^2=Cx}$

8. $y=\mathrm{sh}(x+c)$ $\quad\boxed{\mathrm{ch}^2x-\mathrm{sh}^2x=1}$

$y'=\mathrm{ch}(x+c)$

$(y')^2-(y)^2=1$

$y'=\pm\sqrt{1+y^2}$

$y' \downarrow -\frac{1}{y'}$

$1=\pm y'\sqrt{1+y^2}$

$\pm\int dx=\int dy\sqrt{1+y^2} \qquad \pm x+C=D$ $\qquad$ kao u 4.

$D=\int\frac{1+y^2}{\sqrt{1+y^2}}dy=(Ay+B)\sqrt{1+y^2}+\lambda\int\frac{1}{\sqrt{1+y^2}}dy \quad \frac{d}{dy} \;/\cdot\sqrt{1+y^2}$

$1+y^2=A(1+y^2)+y(Ay+B)+\lambda$

$2A=1 \quad \boxed{B=0} \quad \lambda+A=1$

$A=\frac{1}{2} \qquad \lambda=\frac{1}{2}$

$D=\frac{1}{2}y\sqrt{1+y^2}+\frac{1}{2}\int\frac{dy}{\sqrt{1+y^2}}=\pm x+C_1$

$y\sqrt{1+y^2}+\mathrm{Arsh}(y)=\pm 2x+K$

9. $x^2+y^2=2ax$ $\boxed{x=x(y)}$

$2x+2yy'=2a \;/\cdot x$ $x^2-2xyx'=x^2$

$\cancel{x^2}+2xyy'=\cancel{x^2}+y^2$ $\frac{x}{y}-\frac{y}{x}=2x'$ $z=\frac{x}{y}$ $x'=z+yz'$

$y' \to \frac{1}{y'}$ $z-\frac{1}{z}=2z+2yz'$

$x^2-\frac{2xy}{y'}=y^2$ isto kao i 6 samo sa zamijenjenim

varijablama ...

$x^2+y^2=Cy$ $\boxed{y=k(x^2+y^2)}$

10. $(2a-x)y^2=x^3 \;/\frac{d}{dx}$

$-y^2+2yy'(2a-x)=3x^2$

$-y^2+2yy'\cdot\frac{x^3}{y^2}=3x^2$

$y' \to \frac{1}{y'}$

$-y^2-\frac{2x^3}{yy'}=3x^2$

$x=x(y)$

$-y^2-2\frac{x^3}{y}x'=3x^2 \;/\cdot(-y)$

$y^3+2x^3x'=-3x^2y \;/:2x^3$

$x'=-\left(\frac{y}{x}\right)^3-3\frac{y}{x}$

$p=\frac{x}{y}$ $x'=p+yp'$

$p+yp'=-\frac{1}{p^3}-3\frac{1}{p}$

$=-\frac{1}{p}\left(\frac{1}{p^2}+1\right)$

$=-\frac{1}{p}\left(\frac{p^2+1}{p^2}\right)$

$\frac{p^2+1}{-p^3}$

$yp'=\frac{p^2+1+p^4}{-p^3}$

$\int\frac{-p^3}{p^4+p^2+1}\,dp=\int\frac{dy}{y}=\ln cx$

$\int\frac{p^3\,dp}{(p^2+\frac{1}{2})^2+\frac{3}{4}}=\left|\begin{array}{l}t=p^2+\frac{1}{2}\\ dt=2p\,dp\end{array}\right|$

$=\int\frac{p^2\cdot\frac{dt}{2p}}{t^2+\frac{3}{4}}=\frac{1}{2}\int\frac{(t-\frac{1}{2})\,dt}{t^2+\frac{3}{4}}$

$=\frac{1}{2}\left[\int\frac{t}{t^2+\frac{3}{4}}\,dt-\frac{1}{2}\int\frac{dt}{t^2+(\frac{\sqrt{3}}{2})^2}\right]$

$=\frac{1}{2}\left[\frac{1}{2}\int\frac{d(t^2+\frac{3}{4})}{t^2+\frac{3}{4}}-\frac{1}{2}\cdot\frac{2}{\sqrt{3}}\,\mathrm{arctg}\,t\cdot\frac{2}{\sqrt{3}}\right]$

$=\frac{1}{2}\left[\frac{1}{2}\ln(t^2+\frac{3}{4})-\frac{1}{\sqrt{3}}\,\mathrm{arctg}\,\frac{2t}{\sqrt{3}}\right]$

Pogledati u Dodatku na str. 22.

Zad 2.

1) $y^2(y-xy') = x^3y'$

$y^3 - y^2xy' = x^3y' \quad /:x^3$

$y' + \left(\frac{y}{x}\right)^2 y' = \left(\frac{y}{x}\right)^3$

$y'\left[1+\left(\frac{y}{x}\right)^2\right] = \left(\frac{y}{x}\right)^3$

$y' = \frac{\left(\frac{y}{x}\right)^3}{1+\left(\frac{y}{x}\right)^2}$

$\alpha = \frac{y}{x} \qquad y' = \alpha + x\alpha'$

$\alpha + x\alpha' = \frac{\alpha^3}{1+\alpha^2}$

$x\cdot\alpha' = \frac{\cancel{\alpha^3} - \alpha - \cancel{\alpha^3}}{1+\alpha^2}$

$\int \frac{1+\alpha^2}{\alpha}\, d\alpha = -\int \frac{dx}{x}$

$\int \left(\frac{1}{\alpha} + \alpha\right) d\alpha = -\int \frac{dx}{x}$

$\ln\alpha + \ln e^{\frac{1}{2}\alpha^2} = \ln\frac{C}{x}$

$\alpha e^{\frac{1}{2}\alpha^2} = \frac{C}{x}$

$\frac{y}{\cancel{x}} e^{\frac{1}{2}\left(\frac{y}{x}\right)^2} = \frac{C}{\cancel{x}}$

$$\boxed{y = Ce^{-\frac{1}{2}\left(\frac{y}{x}\right)^2}}$$

2) $y' = 2\left(\frac{y+2}{x+y-1}\right)^2 \qquad \begin{vmatrix}0 & 1\\ 1 & 1\end{vmatrix} = -1$ Sjeku se

$\left.\begin{matrix} y+2=0 \\ x+y-1=0 \end{matrix}\right\} \quad \begin{matrix} y=-2 \\ x-3=0 \quad x=3 \end{matrix}$

$\boxed{x=\alpha+3} \qquad \boxed{y=\beta-2} \qquad \frac{dy}{dx} = \frac{d\beta}{d\alpha}$

$\frac{d\beta}{d\alpha} = 2\left(\frac{\beta-2+2}{\alpha+3+\beta-2-1}\right)^2$

$\frac{d\beta}{d\alpha} = 2\left(\frac{\beta}{\alpha+\beta}\right)^2 = 2\left(\frac{\beta}{\beta\left(\frac{\alpha}{\beta}+1\right)}\right)^2$

$\beta' = \frac{2}{\left(\frac{\alpha}{\beta}+1\right)^2} \qquad p = \frac{\beta}{\alpha} \qquad \beta' = p + \alpha p'$

$p + \alpha p' = \frac{2}{\left(\frac{1}{p}+1\right)^2} = \frac{2}{\left(\frac{p+1}{p}\right)^2} = \frac{2p^2}{p^2+2p+1}$

$\alpha p' = \frac{\cancel{2p^2} - p^3 - \cancel{2p^2} - p}{(p+1)^2}$

$\frac{(p+1)^2}{p(p^2+1)}\, dp = -\frac{d\alpha}{\alpha}$

$\int\frac{dp}{p} + 2\int\frac{dp}{p^2+1} = -\int\frac{d\alpha}{\alpha}$

$\ln p + \ln e^{2\arctan p} = \ln\frac{C}{\alpha}$

$\frac{(p+1)^2}{p(p^2+1)} = \frac{A}{p} + \frac{Bp+C}{p^2+1}$

$p^2+2p+1 = Ap^2 + A + Bp^2 + Cp$

$A+B=1 \quad A=1 \quad C=2$

$B=0$

$= \frac{1}{p} + \frac{2}{p^2+1}$

$p\, e^{2\arctan p} = \frac{C}{\alpha}$

$\frac{\beta}{\cancel{\alpha}} e^{2\arctan\frac{\beta}{\alpha}} = \frac{C}{\cancel{\alpha}}$

$$\boxed{y+2 = Ce^{-2\arctan\frac{y+2}{x-3}}}$$

3) $y' = \frac{x-2y+5}{2x-y+4}$ $\begin{vmatrix}1 & -2\\ 2 & -1\end{vmatrix} = -1+4 \neq 0$ Sjeku se

$x-2y+5=0 \Rightarrow x=2y-5$

$2x-y+4=0$ $4y-10-y+4=0$

$\boxed{x=\alpha-1 \quad y=\beta+2}$ $3y=6$ $\boxed{y=2}$ $\boxed{x=-1}$

$\beta' = \frac{\alpha-1-2\beta-4+5}{2\alpha-2-\beta-2+4} = \frac{\alpha-2\beta}{2\alpha-\beta} = \frac{\alpha(1-2\frac{\beta}{\alpha})}{\alpha(2-\frac{\beta}{\alpha})}$

$p=\frac{\beta}{\alpha}$ $\beta' = p+\alpha p'$

$p+\alpha p' = \frac{1-2p}{2-p} \Rightarrow \alpha p' = \frac{1-2p-2p+p^2}{2-p} = \frac{p^2-4p+1}{2-p}$

$\int \frac{p-2}{p^2-4p+1}\,dp = -\int \frac{d\alpha}{\alpha}$ $\frac{p-2}{(p-2)^2-3}$ $\begin{vmatrix}p-2=r\\ dp=dr\end{vmatrix}$ $\frac{r}{r^2-3}$

$\int \frac{r}{r^2-3}\,dr = \frac{1}{2}\int \frac{d(r^2-3)}{r^2-3} = \frac{1}{2}\ln(r^2-3) = \frac{1}{2}\ln|p^2-4p+1|$

$\ln(p^2-4p+1) = \ln\left(\frac{c}{\alpha}\right)^2$

$p^2-4p+1 = \left(\frac{c}{\alpha}\right)^2$ $\left(\frac{\beta}{\alpha}\right)^2 - 4\frac{\beta}{\alpha}+1 = \frac{c^2}{\alpha^2}$ $/\alpha^2$

$\beta^2-4\alpha\beta+\alpha^2=c^2$

$\boxed{(y-2)^2-4(x+1)(y-2)+(x+1)^2=c^2}$

mislim da je ovo točno :)

4) $xy' = y\ln\frac{y}{x}$ $p=\frac{y}{x}$ $y'=p+xp'$

$y' = \frac{y}{x}\ln\frac{y}{x}$

$\int \frac{dp}{p(\ln p-1)} = \int \frac{d(\ln p-1)}{\ln p-1} = \ln\ln(p-1)$

$p+xp' = p\ln p$

$\ln\ln p-1 = \ln cx$

$xp' = p(\ln p-1)$

$\ln p-1 = cx$

$\frac{dp}{p(\ln p-1)} = \frac{dx}{x}$

$p-1 = e^{cx}$

$\frac{y}{x}-1 = e^{cx}$

$\boxed{y = x\left[e^{cx}+1\right]}$

Zad 3.

linearna: I) var C, II) direktno

1) $y' - \frac{y}{x} = x\sin x$

I) $y' - \frac{y}{x} = 0$

$\frac{dy}{y} = \frac{dx}{x}$

$y_H = Cx$

$y = C(x)\cdot x$

$C'x + \cancel{C} - \cancel{C} = x\sin x$

$C' = \sin x$

$C = -\cos x + C_1$

$\boxed{y = C_1 x - x\cos x}$

II) $f(x) = -\frac{1}{x}$ $\quad g(x) = x\sin x$

$\int f(x)dx = -\ln x$

$e^{-\int f(x)dx} = x$

$e^{\int f(x)dx} = \frac{1}{x}$

$y = x\cdot\left[\int \frac{1}{x}\, x\cdot \sin x\, dx + C_1\right]$

$\boxed{y = -x\cos x + x\cdot C_1}$

2) $y = xy' + y'\ln y$

$y = \frac{x}{x'} + \frac{\ln y}{x'}$

$x' - \frac{1}{y}x = \frac{\ln y}{y}$

$C'y + \cancel{C} - \cancel{C} = \frac{\ln y}{y}$

$C' = \frac{\ln y}{y^2}$

$x = x(y)$

$x' - \frac{x}{y} = 0$

$\frac{dx}{x} = \frac{dy}{y}$

$x = Cy$

$C(y) = \int \frac{1}{y^2}\ln y\, dy$

$= \left| \begin{matrix} u = \ln y & du = \frac{1}{y} \\ dv = \frac{1}{y^2}dx & v = -\frac{1}{y} \end{matrix} \right.$

$= -\frac{1}{y}\ln y + \int \frac{1}{y^2}dy$

$= -\frac{1}{y}\ln y - \frac{1}{y} + C_1$

$\boxed{x = C_1 y - \ln y - 1}$

3) $dx + (e^y - x)dy = 0$

$P'_y = 0 \quad Q'_x = -1$

$\ln\mu = -\int \frac{1}{1}[0 - (-1)]dy$

$\ln\mu = -\int dy$

$\ln\mu = -y$

$\mu = e^{-y}$

$\int_0^x e^{-y}dx + \int_0^y (1 - \cancel{x e^{-y}})dy = C$

$\boxed{xe^{-y} + y = C}$

4.) $(x+1)(xy'-1)=y^2$

Bernoullijeva D.J.

$$(x+1)\,y\,y' - (x+1) = y^2$$

$$y y' - 1 = \frac{y^2}{x+1}$$

$$y' - \frac{1}{y} = \frac{y}{x+1}$$

$$y' - \frac{1}{(x+1)}\,y = 1 \cdot y^{-1} \quad / \cdot y^{-1}$$

$f(x)$ $g(x)$

$$\frac{y'}{y^{-1}} + y^{1-(-1)} \cdot \frac{-1}{x+1} = 1$$

$$z = y^{1-(-1)} = y^2$$

$$z' = 2yy' \qquad y \cdot y' = \frac{1}{2} z'$$

$$\frac{1}{2} z' - \frac{1}{(x+1)}\, z = 1 \quad / \cdot 2$$

$$z' - \frac{2}{x+1}\, z = 2$$

$f(x)$ $g(x)$

Linearna po z i z'

$$\int f(x)\,dx = -2\int \frac{dx}{x+1} = -2\ln(x+1)$$

$$e^{-\int f(x)dx} = (x+1)^2$$

$$e^{\int f(x)dx} = \frac{1}{(x+1)^2}$$

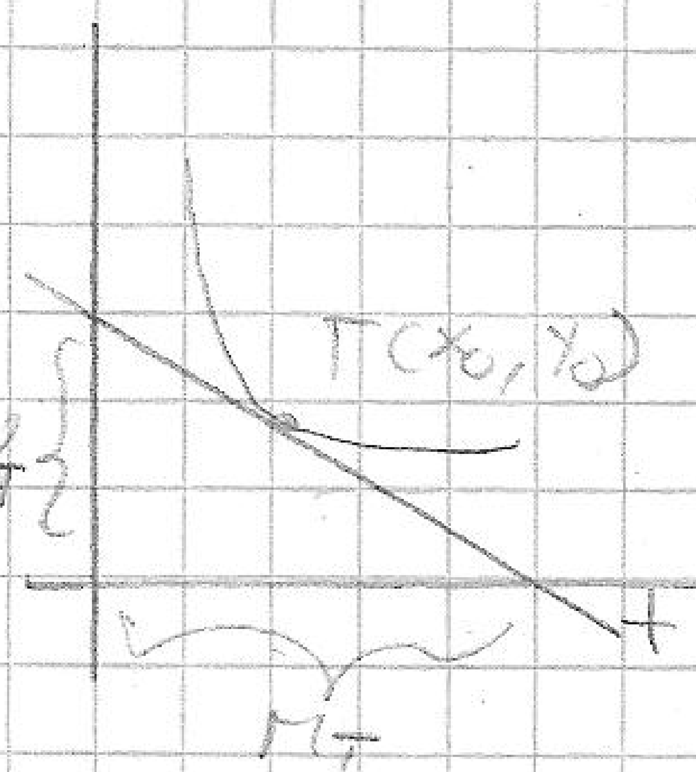
$$z = (x+1)^2 \left[\int \frac{2}{(x+1)^2} + C_1 \right]$$

$$z = (x+1)^2 \left[-\frac{2}{x+1} + C_1 \right] = -2(x+1) + (x+1)^2 C_1$$

$$\boxed{y^2 = C(x+1)^2 - 2(x+1)}$$

U nekim od sljedećih zadataka koristit ćemo odsječke tangente i normale na koordinatnim osima. Sada ćemo izvesti formule za njih i koristiti ih u zadacima poslije.

tangenta:



$t \ldots \quad y - y_0 = y_0'(x - x_0)$

$M_T \ldots \quad 0 - y_0 = y_0'(M_T - x_0)$

$-y_0 = y_0' M_T - y_0' x_0$

$$M_T = x_0 - \frac{y_0}{y_0'}$$

$N_T \ldots \quad N_T - y_0 = y_0'(-x_0)$

$N_T = y_0 - y_0' x_0$

normala

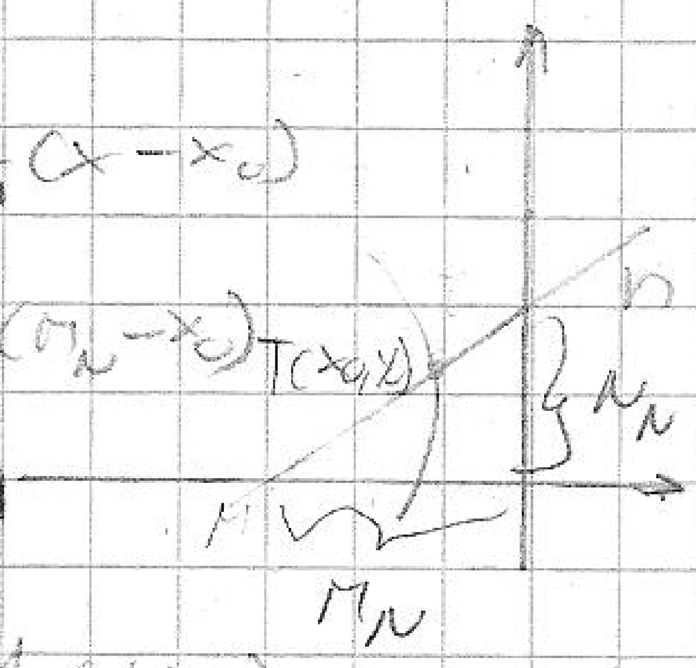
$n \ldots \quad y - y_0 = -\frac{1}{y_0'}(x - x_0)$

$M_N \ldots \quad 0 - y_0 = -\frac{1}{y_0'}(M_N - x_0)$

$M_N = x_0 + y_0 y_0'$

$N_N \ldots \quad N_N - y_0 = -\frac{1}{y_0'}(-x_0)$

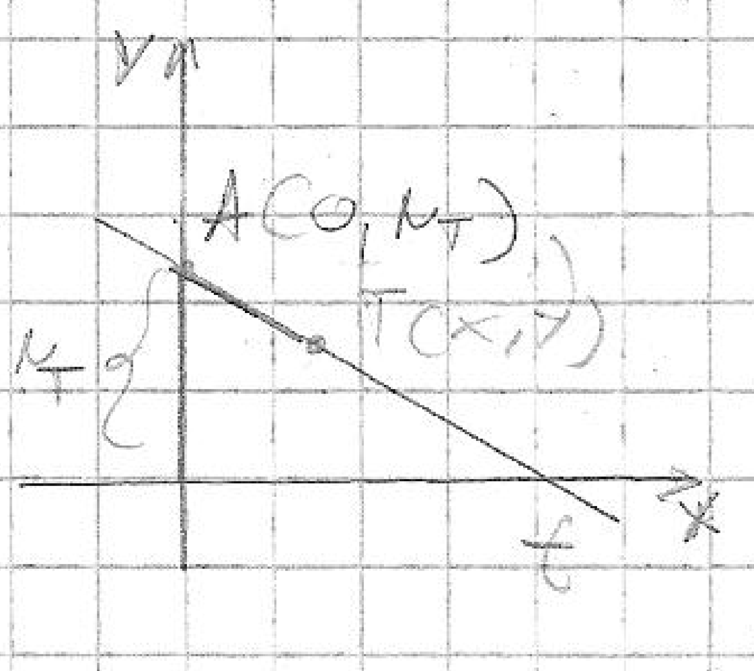
$$N_N = y_0 + \frac{x_0}{y_0'}$$



Zamjenama: $x_0 \downarrow x$, $y_0 \downarrow y$, $y_0' \downarrow y'$ radi lakšeg zapisa konačno dobivamo:

$$\boxed{\begin{aligned} M_T &= x - \frac{y}{y'} & M_N &= x + yy' \\ N_T &= y - y'x & N_N &= y + \frac{x}{y'} \end{aligned}}$$

Zad 4.



$|N_T| = d(T, A)$

$|y - y'x| = \sqrt{(x-0)^2 + (y-(y-y'x))^2}$

$|y - y'x| = \sqrt{x^2 + y'^2x^2} \quad /^2$

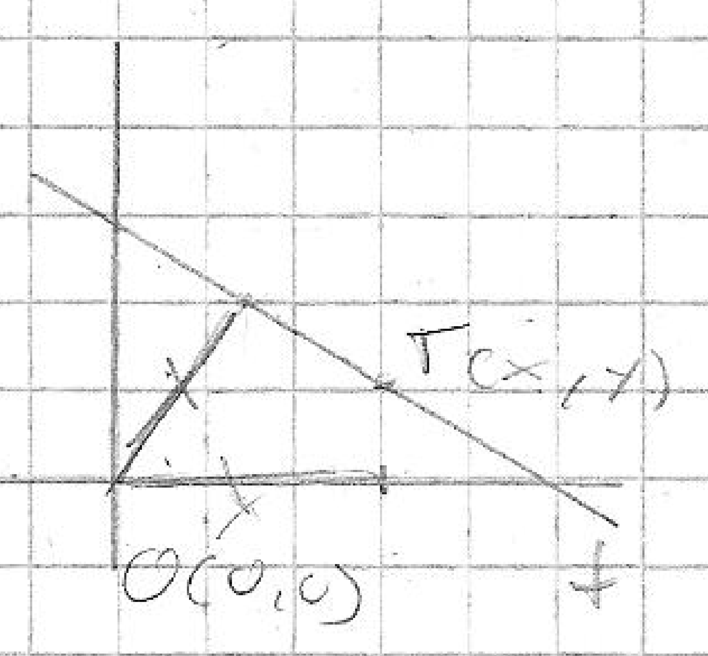
$y^2 - 2xyy' + \cancel{(y')^2x^2} = x^2 + \cancel{y'^2x^2}$

$2xyy' = y^2 - x^2 \quad /:2xy$

$y' = \frac{1}{2}\frac{y}{x} - \frac{1}{2}\frac{x}{y}$ Homogena, isto kao Zad 1: 6/2 na str. 3.

$$\boxed{y^2 + x^2 = Cx}$$

Zad 5



t... $y - y_0 = y_0'(x - x_0) \overset{\text{zamjena}}{\Downarrow} \beta - y = y'(\alpha - x)$

$\underbrace{1}_{A}\cdot\beta \underbrace{-y'}_{B}\alpha + \underbrace{y'x - y}_{C}$

$d(T, O) = x$

$\frac{|1\cdot 0 - y'\cdot 0 + y'x - y|}{\sqrt{1^2 + (-y')^2}} = x$

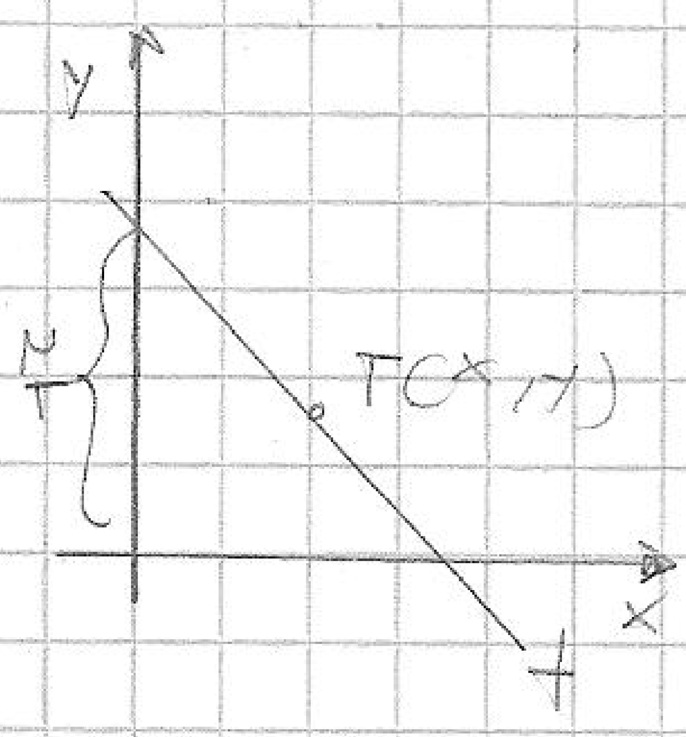
$|y'x - y| = x\cdot\sqrt{1 + (y')^2} \quad /^2$

$\cancel{(y'x)^2} - 2xyy' + y^2 = x^2 + \cancel{(xy')^2}$

$2xyy' = y^2 - x^2$ kao i Zad 4.

$$\boxed{y^2 + x^2 = Cx}$$

Zad 6.



Komentar: $|x \cdot y|$ nije potreban jer μ_T^2 je uvijek ≥ 0

$$|x \cdot y| = (\mu_T)^2$$

$$|x \cdot y| = (y - y'x)^2$$

$$y - y'x = \pm\sqrt{x \cdot y}$$

$$y'x = y \pm \sqrt{x \cdot y}$$

$$y' = \frac{y}{x} \pm \sqrt{\frac{y}{x}}$$

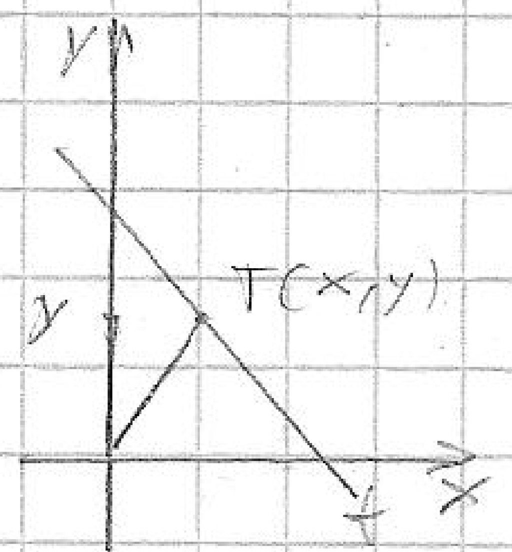
Homogena: $p = \frac{y}{x}$ $y' = p + p'x$

$$\cancel{p} + p'x = \cancel{p} \pm \sqrt{p}$$

$$\int \frac{dp}{\sqrt{p}} = \pm\int \frac{dx}{x} \qquad 2\sqrt{p} = \pm(\ln x + \ln C)$$

$$\boxed{\pm 2\sqrt{\frac{y}{x}} = \ln Cx}$$

Zad 7.



pogledati Zad 5. ...

$$|y'x - y| = y\sqrt{1 + (y')^2} \quad /^2$$

$$y'^2 x^2 - 2xyy' + \cancel{y^2} = \cancel{y^2} + y^2 (y')^2 \quad /: y'$$

$y' = 0$
$\boxed{y = C}$ → singularno rješenje

$$y'(x^2 - y^2) = 2xy$$

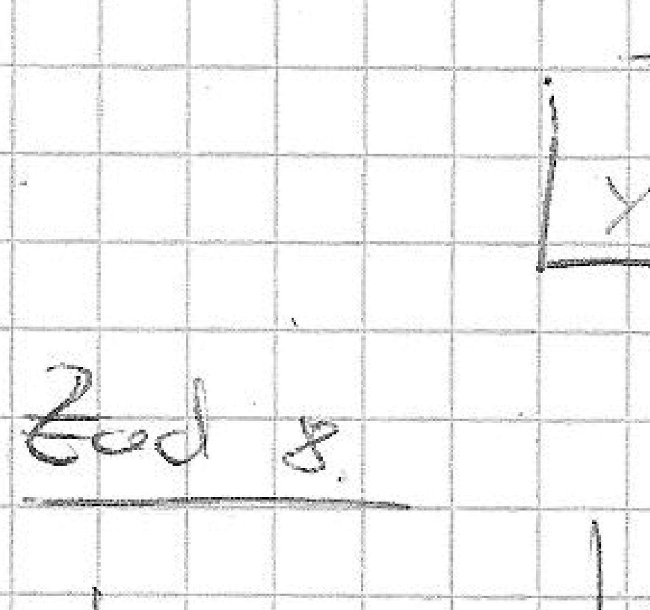
$$y' = \frac{2xy}{x^2 - y^2} \quad \ldots \quad x = x(y)$$

$$\frac{1}{x'} = \frac{2xy}{x^2 - y^2} \qquad x' = \frac{1}{2}\frac{x}{y} - \frac{1}{2}\frac{y}{x}, \text{ kao i Zad 1. = 9}$$

na str. 4.

$$\boxed{y = C(x^2 + y^2)}$$

Zad 8.



$$|\mu_T| = 2\sqrt{x^2 + y^2}$$

$$|y - y'x| = 2\sqrt{x^2 + y^2}$$

$$y - y'x = \pm 2\sqrt{x^2 + y^2}$$

$$y'x = y \pm 2\sqrt{x^2 + y^2}$$

$$y' = \frac{y}{x} \pm 2\sqrt{1 + \left(\frac{y}{x}\right)^2}$$

$$p = \frac{y}{x}, \quad y' = \cancel{p} + p'x = \cancel{p} \pm 2\sqrt{1 + p^2}$$

$$\int \frac{dp}{\sqrt{1 + p^2}} = \frac{\pm 2dx}{x}$$

$$\ln|p + \sqrt{p^2 + 1}| = \pm 2\ln x + \ln C$$

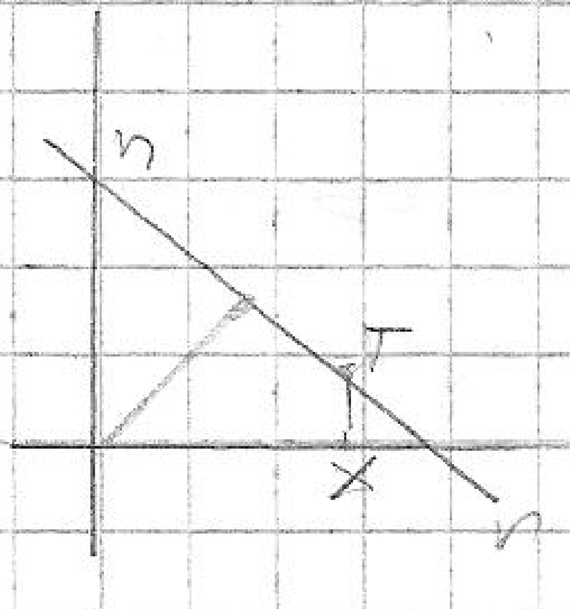
$$p + \sqrt{p^2 + 1} = x^2 C \qquad p + \sqrt{p^2 + 1} = \frac{C}{x^2}$$

$$\frac{y}{x} + \frac{1}{x}\sqrt{x^2 + y^2} = x^2 C \qquad \frac{y}{x} + \frac{1}{x}\sqrt{x^2 + y^2} = \frac{C}{x^2}$$

$$\boxed{y + \sqrt{x^2 + y^2} = x^3 C} \qquad \boxed{xy + x\sqrt{x^2 + y^2} = C}$$

Zad 9. $d(n, O)$ $n \ldots$ $y - y_0 = -\frac{1}{y_0'}(x - x_0)$ $\beta - y = -\frac{1}{y'}(\alpha - x)$

$$\underbrace{y'}_{A}\beta + \underbrace{1}_{B}\alpha \underbrace{- x - yy'}_{C} = 0$$



$d(O, N) = x$

$$\frac{|0 \cdot y' + 0 \cdot 1 - x - yy'|}{\sqrt{1 + y'^2}} = x$$

$y' = 0 \Rightarrow \boxed{y = C}$

trivijalno rješenje

$|x + yy'| = x\sqrt{1 + y'^2}$ $/^2$

$\cancel{x^2} + 2xyy' + y^2 y'^2 = \cancel{x^2} + y'^2 x^2$ $/ : y'$

$2xy = y'(x^2 - y^2)$

$y' = \frac{2xy}{x^2 - y^2}$ $\quad x = x(y)$

$x' = \frac{x^2 - y^2}{2xy} = \frac{1}{2}\frac{x}{y} - \frac{1}{2}\frac{y}{x}$

$p = \frac{x}{y}$ $\quad x' = p + yp'$

$p + yp' = \frac{1}{2}p - \frac{1}{2p}$

$yp' = -\frac{1}{2}\left[p + \frac{1}{p}\right] = -\frac{1}{2}\left[\frac{p^2 + 1}{p}\right]$

$\int \frac{2p}{p^2 + 1}\,dp = -\int \frac{dy}{y}$

$\ln(p^2 + 1) = \ln\frac{C}{y}$

$p^2 + 1 = \frac{C}{y}$

$\frac{x^2}{y^2} + 1 = \frac{C}{y}$ $/ \cdot y^2$

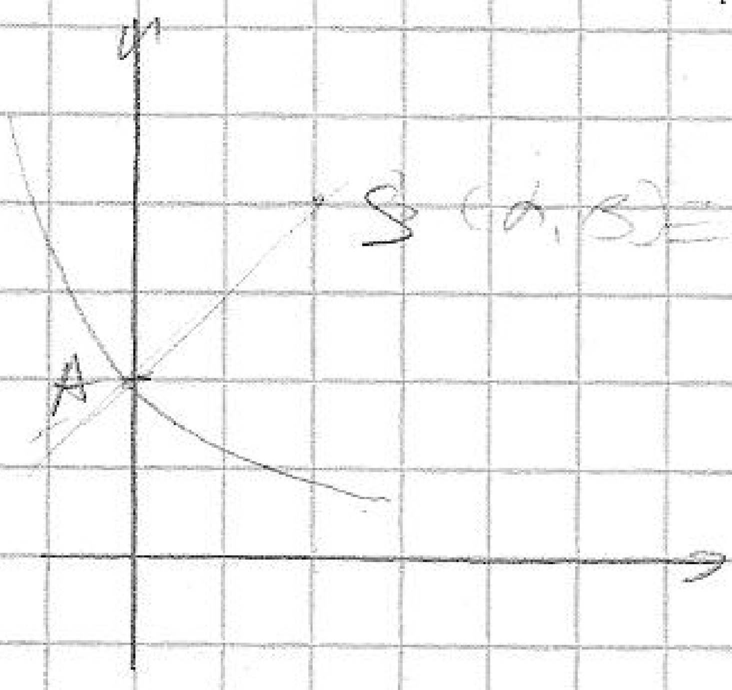
$x^2 + y^2 = Cy$ $/ : C$

$\boxed{k(x^2 + y^2) = y}$

Zad 10

identičan zadatku 6

Zad 11.



$n.\ \beta - y = -\frac{1}{y'}(\alpha - x)$

$S(\alpha,\beta) = (1,2) \quad 2 - y = -\frac{1}{y'}(1-x)$

$y'(2-y) = x-1$

$y' = \frac{x-1}{-y+2}$

$x = \alpha + 1$

$y = \beta + 2$

$y' = \beta'$

$\beta' = \frac{\alpha}{-\beta} = -\frac{\alpha}{\beta}$

$p = \frac{\beta}{\alpha} \quad \beta' = p + \alpha p'$

1)

$p + \alpha p' = -\frac{1}{p}$

$\alpha p' = -\left[\frac{1+p^2}{p}\right]$

$\int \frac{p}{1+p^2}\,dp = -\int \frac{d\alpha}{\alpha}$

$\int \frac{d(1+p^2)}{1+p^2} = -2\int \frac{d\alpha}{\alpha}$

$\ln(1+p^2) = -2\ln\alpha + \ln C$

$1+p^2 = \frac{C}{\alpha^2}$

$1 + \frac{\beta^2}{\alpha^2} = \frac{C}{\alpha^2} \qquad \alpha^2 + \beta^2 = C$

$(x-1)^2 + (y-2)^2 = C$

2)

$(x-1)^2 + (y-2)^2 = C \ / A$

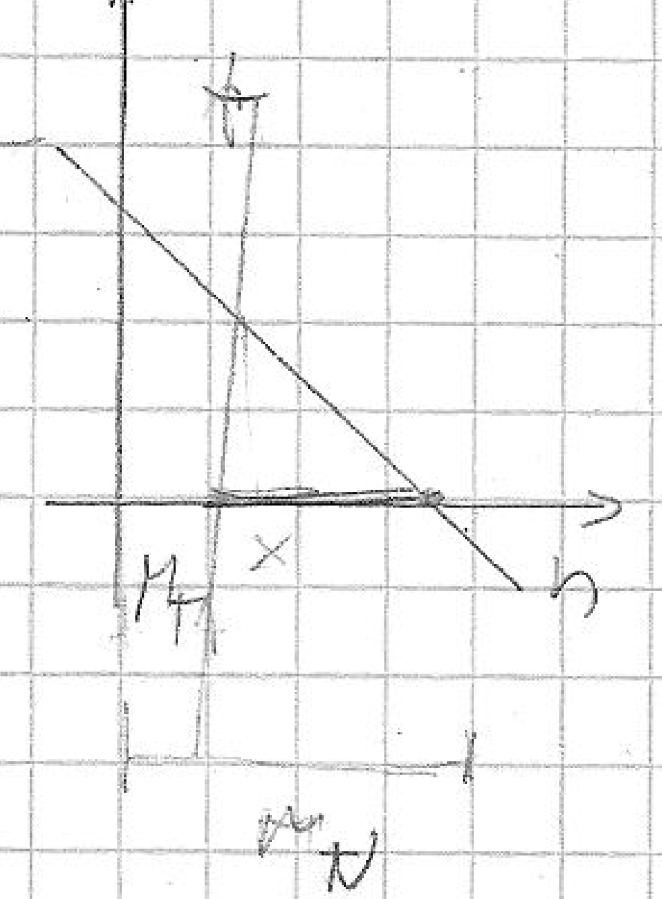
$(0-1)^2 + (1-2)^2 = C$

$1 + 1 = C$

$C = 2$

$$\boxed{(x-1)^2 + (y-2)^2 = 2}$$

Zad 12



$M_N - M_T = 2x$

$\cancel{x} + yy' - \cancel{x} + \frac{y}{y'} = 2x$

$y(y')^2 - 2xy' + y = 0$

$y' = \frac{2x \pm \sqrt{4x^2 - 4y^2}}{2y}$

$y' = \frac{x}{y} \pm \sqrt{\left(\frac{x}{y}\right)^2 - 1}$

$p = \frac{y}{x}$

$y' = p + xp'$

$p + xp' = \frac{1}{p} \pm \sqrt{p^2 - 1}$

$xp' = \frac{1}{p} - p \pm \sqrt{p^2-1}$

$xp' = \frac{1-p^2}{p} \pm \sqrt{p^2-1}$

Na starijem modulu je mi se ne da komplicirat. :)

$\int \frac{dp}{\frac{1-p^2}{p} \pm \sqrt{p^2-1}} = \int \frac{dx}{x}$

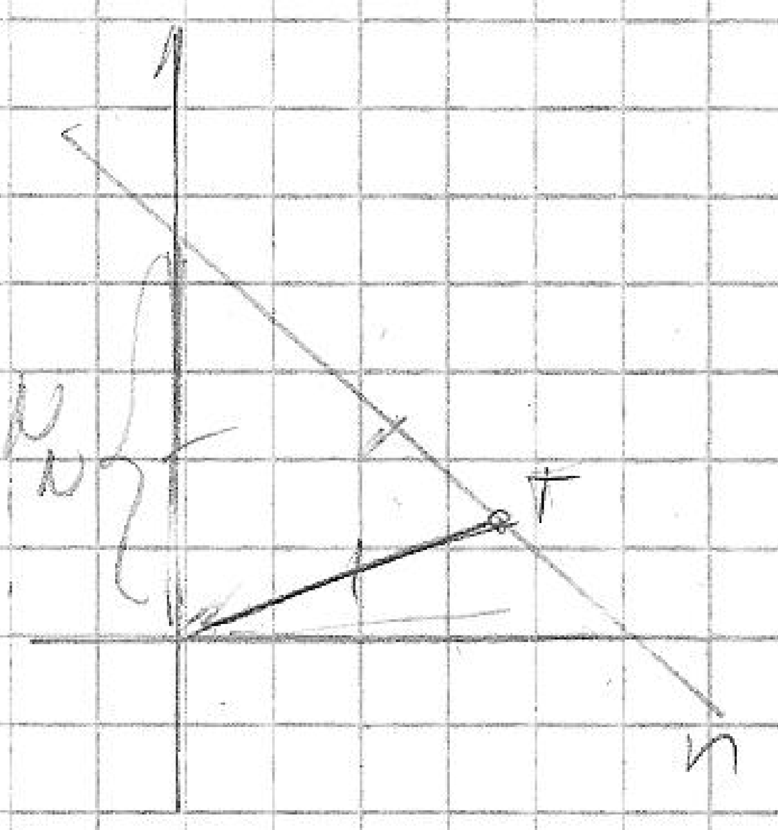
$\left| \begin{matrix} p^2 - 1 = t^2 \\ 2p\,dp = 2t\,dt \\ dt = p\,dp \end{matrix} \right| = \int \frac{dt}{p\left[\frac{-t^2}{p} \pm t\right]}$

$= \int \frac{t\,dt}{t\left[\frac{-t}{p} \pm 1\right]} = \int \frac{dt}{-t \pm p}$

$\int \frac{dt}{-t \pm \sqrt{t^2+1}}$

Pogledati rješenje u Dodatku na str. 23.

Zad 13.



$|N_n| = d(T,O)$

$\left|y + \frac{x}{y'}\right| = \sqrt{x^2+y^2} \quad / \cdot |y'|$

$|yy' + x| = |y'|\sqrt{x^2+y^2} \quad /^2$

$\cancel{y^2y'^2} + 2xyy' + x^2 = y'^2x^2 + \cancel{y'^2y^2}$

$y'^2x^2 - 2xyy' - x^2 = 0 \quad / : x^2$

$y'^2 - 2\frac{y}{x}y' - 1 = 0$

$y' = \frac{2\frac{y}{x} \pm \sqrt{4\left(\frac{y}{x}\right)^2+4}}{2} = \frac{y}{x} \pm \sqrt{\left(\frac{y}{x}\right)^2+1}$

$p = \frac{y}{x} \qquad y' = \cancel{p} + xp' = \cancel{p} \pm \sqrt{p^2+1}$

$\int \frac{dp}{\sqrt{p^2+1}} = \pm \int \frac{dx}{x}$

$\ln\left|p + \sqrt{p^2+1}\right| = \pm \ln x + \ln C$

$\frac{y}{x} + \sqrt{\left(\frac{y}{x}\right)^2+1} = xC \quad / \cdot x \qquad\qquad \frac{y}{x} + \sqrt{\left(\frac{y}{x}\right)^2+1} = \frac{C}{x} \quad / \cdot x$

$y + \sqrt{y^2+x^2} = x^2C \qquad\qquad y + \sqrt{y^2+x^2} = C$

$x^4C^2 - 2Cx^2y + \cancel{y^2} = \cancel{y^2} + x^2 \qquad\qquad \cancel{y^2} + x^2 = C^2 - 2Cy + \cancel{y^2}$

$x^4C^2 - x^2[2Cy+1] = 0 \quad / : x^2 \qquad\qquad 2Cy = C^2 - x^2$

$x^2C^2 - 2Cy - 1 = 0 \qquad\qquad y = \frac{1}{2}C - \frac{1}{2C}x^2$

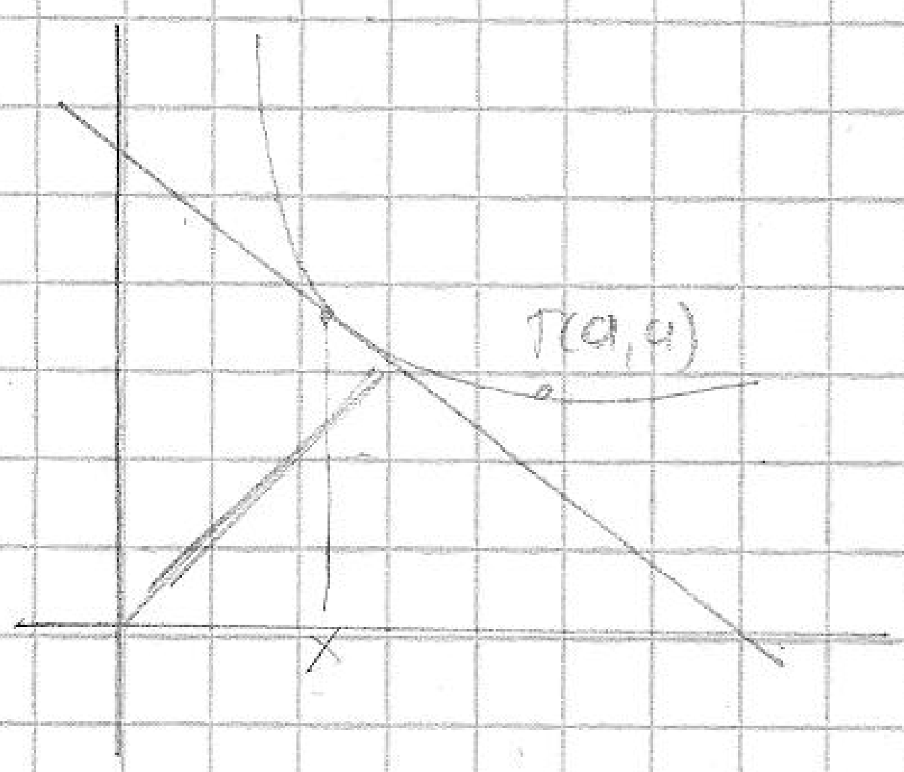
$2Cy = x^2C^2 - 1 \quad / : 2C \qquad\qquad \frac{1}{2}C = -\frac{1}{2k}$

$\boxed{y = \frac{1}{2}Cx^2 - \frac{1}{2C}} \qquad\qquad C = -\frac{1}{k}$

$y = -\frac{1}{2k} + \frac{1}{2}kx^2$

Rješenja su ekvivalentna

Zad 14.



$$d(O,t) \qquad t \ldots \beta - y = y'(\alpha - x)$$

$$\underbrace{1 \cdot \beta}_{A} \underbrace{- y' \cdot \alpha}_{B} \underbrace{- y + xy'}_{C}$$

$$\frac{|0 + 0 + xy' - y|}{\sqrt{1 + y'^2}} = x$$

$$|xy' - y| = x\sqrt{1 + y'^2} \quad /^2$$

$$\cancel{x^2 y'^2} - 2xyy' + y^2 = x^2 + \cancel{x^2 y'^2}$$

$$2xyy' = y^2 - x^2$$

$$y' = \frac{1}{2}\frac{y}{x} - \frac{1}{2}\frac{x}{y} \qquad p = \frac{y}{x} \qquad y' = p + xp'$$

$$p + xp' = \frac{1}{2}p - \frac{1}{2} \cdot \frac{1}{p}$$

$$xp' = -\frac{1}{2}\left[p + \frac{1}{p}\right] = -\frac{1}{2}\left[\frac{p^2+1}{p}\right]$$

$$\int \frac{2p\,dp}{1+p^2} = -\int \frac{dx}{x} \Rightarrow \ln(1+p^2) = \ln\frac{c}{x}$$

$$1 + p^2 = \frac{c}{x}$$

$$1 + \frac{y^2}{x^2} = \frac{c}{x} \quad / \cdot x^2$$

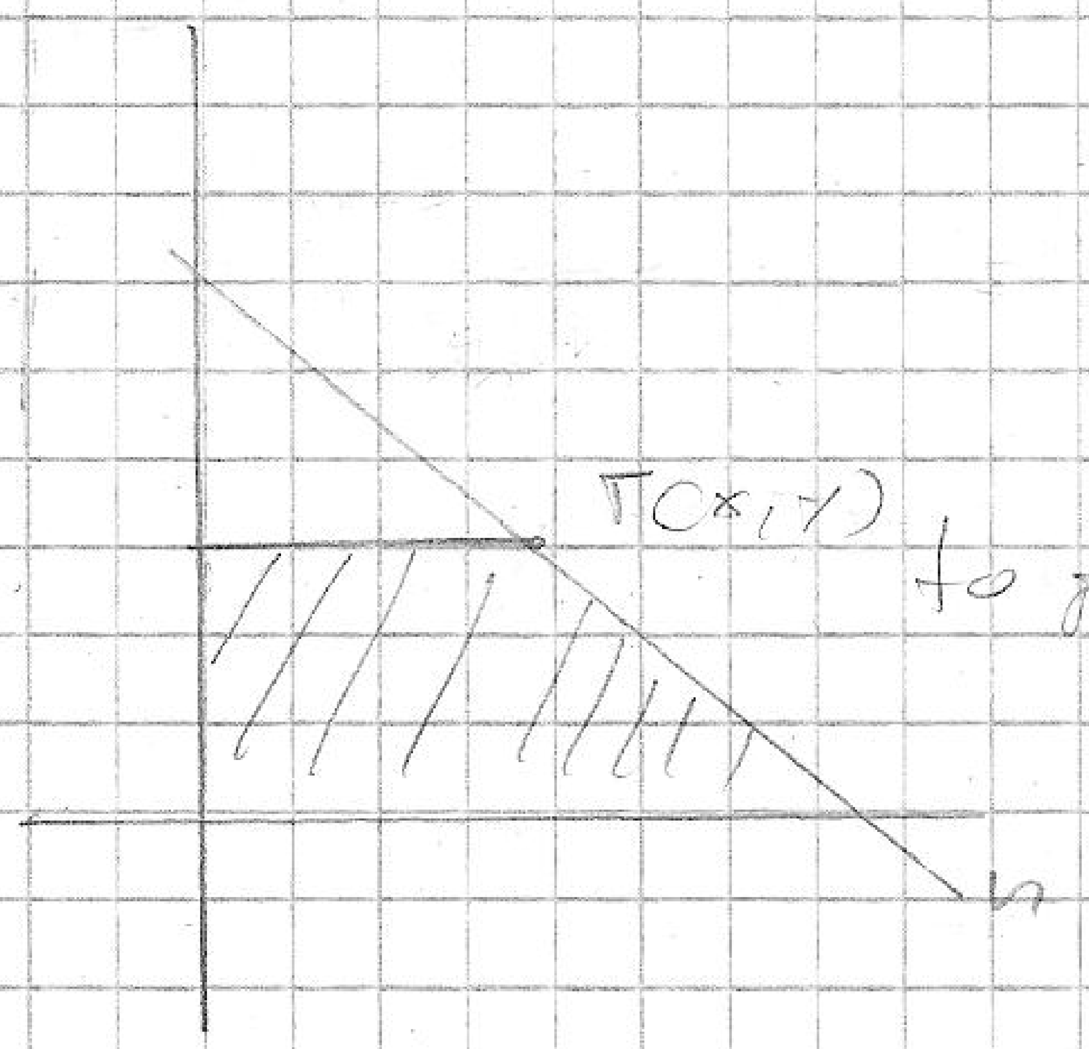
$$x^2 + y^2 = c \cdot x \quad / \cdot T \qquad a^2 + a^2 = c \cdot a \Rightarrow c = 2a$$

$$\boxed{x^2 + y^2 = 2ax}$$

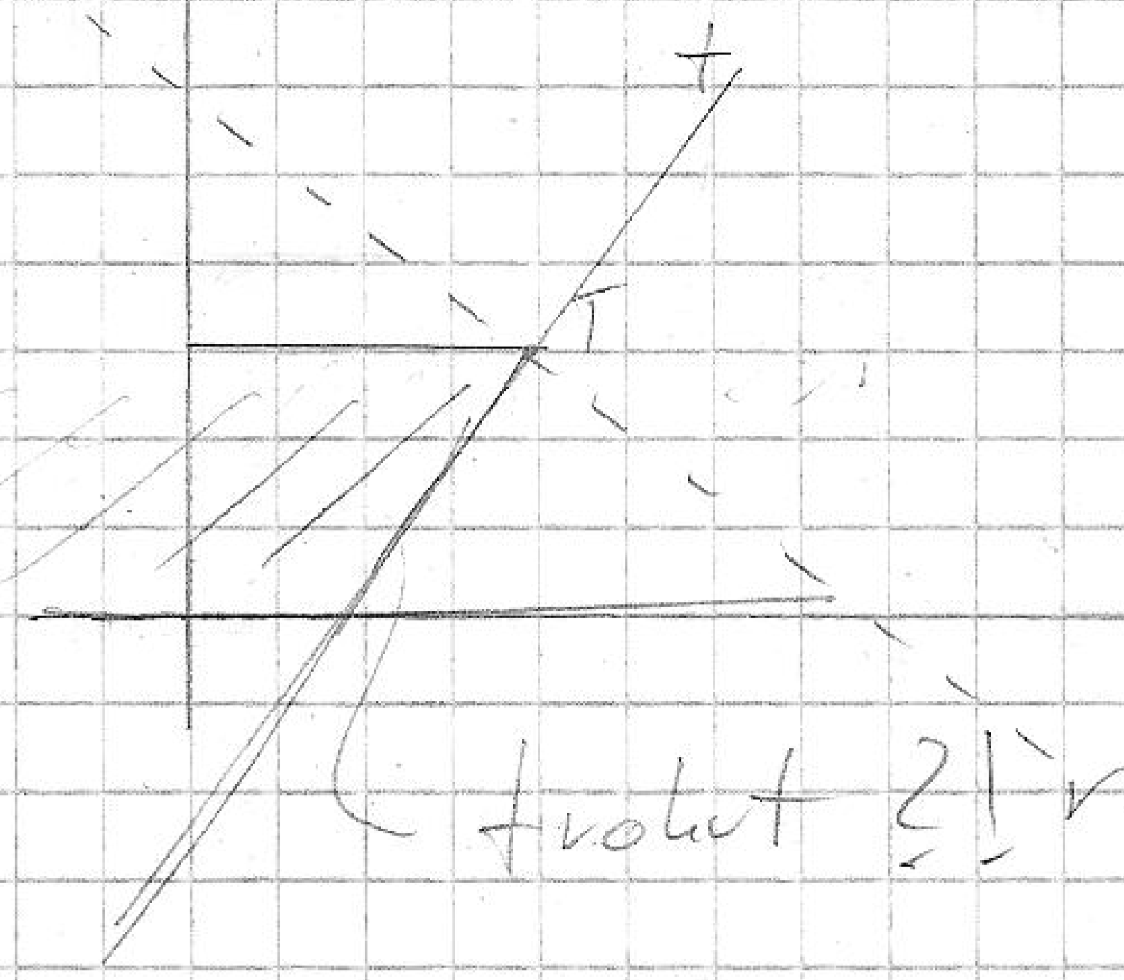
14

Zad 8

trapez



trokut



Neću se baviti pogađanjem što je zapravo trebalo pisati u zadatku da se dobije D.J: $2xyy' + x^2 - 2y^2 = 0$

Ali ću je riješiti

$2xyy' + x^2 - 2y^2 = 0$ $\qquad p = \frac{y}{x} \quad y' = p + xp'$

$y' = -\frac{1}{2}\frac{x}{y} + \frac{y}{x}$

$\cancel{p} + xp' = -\frac{1}{2p} + \cancel{p}$

$\int 2p\,dp = -\int \frac{dx}{x}$

$p^2 = \ln\frac{C}{x}$

$\frac{y^2}{x^2} = \ln\frac{C}{x}$

$\boxed{y^2 = x^2 \ln\frac{C}{x}}$

Zad 16.

Kao u zadatku 11.

$y = \arctan(Cx)$ $\qquad \underline{\tan y = Cx}$

$$y' = \frac{1}{1+C^2x^2}\cdot C \qquad\qquad y' = -\frac{x(1+\tan^2 y)}{\tan y}$$

$$y' = \frac{\tan y}{x\cdot(1+\tan^2 y)} \qquad \int \frac{\tan y}{1+\tan^2 y}\,dy = \int x\,dx = -\frac{1}{2}x^2 + C$$

$$y' \rightarrow -\frac{1}{y'} \qquad \int \frac{\frac{\sin y}{\cos y}}{\frac{1}{\cos^2 y}}\,dy = \int \sin y \cos y\,dy = \int \sin y\, d(\sin y)$$

$$= \frac{1}{2}\sin^2 y$$

$$\frac{1}{2}\sin^2 y = -\frac{1}{2}x^2 + C \qquad / A(1,0)$$

$$0 = -\frac{1}{2} + C \Rightarrow C = \frac{1}{2}$$

$$\boxed{\sin^2 y + x^2 = 1}$$

Zad 17.

$T(2, \frac{\pi}{2})$

$$y = \operatorname{arccot}(Cx) \qquad y' = \frac{1+\cot^2 y}{\frac{\cot y}{x}} = \frac{x\cdot\frac{1}{\sin^2 y}}{\frac{\cos y}{\sin y}} = x\cdot\frac{1}{\sin y\cdot\cos y}$$

$$y' = -\frac{1}{1+C^2x^2}\cdot C \qquad \int \sin y\cdot\cos y\,dy = \int x\,dx$$

$$y' \rightarrow -\frac{1}{y'} \qquad \int \frac{1}{2}\sin 2y\,dy = \frac{1}{2}x^2 + \frac{1}{2}C$$

$$y' = \frac{1+C^2x^2}{C} \qquad -\frac{1}{2}\cos 2y = x^2 + C \qquad / T$$

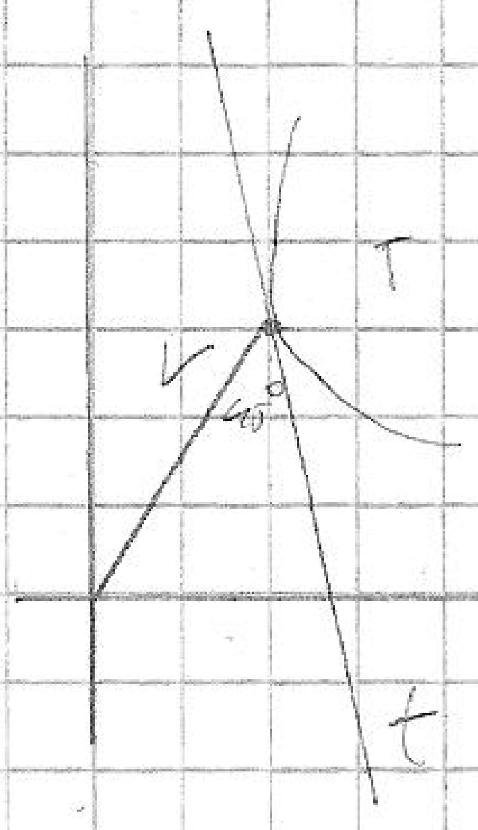
$$-\frac{1}{2}\cdot -1 = 4 + C$$

$$Cx = \cot y \qquad C = -4 + \frac{1}{2} = -\frac{7}{2}$$

$$-\frac{1}{2}\cos 2y = x^2 - \frac{7}{2} \qquad / \cdot -2$$

$$\boxed{\cos 2y = -2x^2 + 7}$$

Zad 18.



$t: \ \beta - y = y'(\alpha - x)$

$$\beta = \underset{k_1}{y'}\alpha - xy' + y$$

$v_2: \ \beta - 0 = \frac{y-0}{x-0}(\alpha - 0) \qquad \beta = \underset{k_2}{\frac{y}{x}}\cdot\alpha - y$

$$\tan 45^\circ = \left|\frac{k_2-k_1}{1+k_1k_2}\right| = \left|\frac{\frac{y}{x}-y'}{1+\frac{y}{x}y'}\right|$$

$$\left|1+\frac{y}{x}y'\right| = \left|\frac{y}{x}-y'\right|$$

$$1+\frac{y}{x}y' = \frac{y}{x}-y' \qquad\qquad 1+\frac{y}{x}y' = -\frac{y}{x}+y'$$

$$y'\left(\frac{y}{x}+1\right) = \frac{y}{x}-1 \qquad\qquad y'\left(\frac{y}{x}-1\right) = -\frac{y}{x}-1$$

$$y' = \frac{\frac{y}{x}-1}{\frac{y}{x}+1} \qquad\qquad y' = \frac{\frac{y}{x}+1}{1-\frac{y}{x}}$$

$$p = \frac{y}{x} \quad y' = p + xp'$$

$$p+xp' = \frac{p-1}{p+1} \qquad\qquad p+xp' = \frac{p+1}{1-p}$$

$$xp' = \frac{\cancel{p}-1-p^2-\cancel{p}}{p+1} \qquad\qquad xp' = \frac{p+1-\cancel{p}+p^2}{1-p}$$

$$xp' = -\frac{p^2+1}{p+1} \qquad\qquad xp' = \frac{p^2+1}{1-p}$$

$$\int\frac{p+1}{p^2+1}\,dp = -\int\frac{dx}{x} = -\ln x + \ln c \qquad \int\frac{1-p}{p^2+1}\,dp = \int\frac{dx}{x} = \ln x + \ln c$$

$$\frac{1}{2}\int\frac{d(p^2+1)}{p^2+1}\,dp + \int\frac{dp}{p^2+1} = \ln\frac{c}{x} \qquad \int\frac{dp}{p^2+1} - \frac{1}{2}\int\frac{d(p^2+1)}{p^2+1} = \ln c\cdot x$$

$$\frac{1}{2}\ln(p^2+1) + \arctan p = \ln\frac{c}{x} \qquad \arctan p - \frac{1}{2}\ln(p^2+1) = \ln c\cdot x$$

$$\arctan\frac{y}{x} = \ln\frac{c}{x\sqrt{\frac{y^2}{x^2}+1}} \qquad \arctan\frac{y}{x} = \ln c\cdot x\sqrt{\frac{y^2}{x^2}+1}$$

$$\boxed{\arctan\frac{y}{x} = \ln\frac{c}{\sqrt{x^2+y^2}}} \qquad \boxed{\arctan\frac{y}{x} = \ln c\sqrt{x^2+y^2}}$$

$$y = r\cdot\sin\varphi, \quad x = r\cdot\cos\varphi$$

$$\varphi = \ln\frac{c}{r} \qquad\qquad \varphi = \ln c\cdot r$$

$$r = ce^{-\varphi} \qquad\qquad r = \frac{1}{c}e^{\varphi}$$

Zad 19 Dva su rješenja, drugo sami za vježbu :)

$\alpha = 45^\circ$

$y^2 = 4ax$ 1) $y' = \dfrac{y' - \tan\alpha}{1 + y'\tan\alpha}$ 2) $y' = \dfrac{y' + \tan\alpha}{1 - y'\tan\alpha}$

$2yy' = 4a$

$2yy' = \dfrac{y^2}{x}$ $\quad y' = \dfrac{y'-1}{1+y'}$

$2y' = \dfrac{y}{x}$

$\dfrac{y'-1}{1+y'} = \dfrac{1}{2}\dfrac{y}{x}$ $\quad 2(y'-1)x = (1+y')y$

$y'(2x - y) = y + 2x$

$y' = \dfrac{y+2x}{2x-y} = \dfrac{x(2+\frac{y}{x})}{x(2-\frac{y}{x})}$

$p = \dfrac{y}{x}$ $\quad y' = p + xp'$

$p + xp' = \dfrac{2+p}{2-p} \Rightarrow xp' = \dfrac{2+p-2p+p^2}{2-p} = \dfrac{p^2-p+2}{2-p}$

$\displaystyle\int \frac{2-p}{p^2-p+2}\,dp = \int\frac{dx}{x} = \ln Cx$

$\displaystyle\int \frac{2-p}{(p-\frac{1}{2})^2 + \frac{7}{4}}\,dp = \left| \begin{matrix} p - \frac{1}{2} = t \\ dp = dt \end{matrix}\right| = \int \frac{\frac{3}{2} - t}{t^2 + \frac{7}{4}}\,dt =$

$\displaystyle= \frac{3}{2}\int\frac{dt}{t^2 + (\frac{\sqrt7}{2})^2} - \frac{1}{2}\int\frac{d(t^2+\frac{7}{4})}{t^2+\frac{7}{4}} = \frac{3}{2}\cdot\frac{2}{\sqrt7}\arctan t\frac{2}{\sqrt7} - \frac{1}{2}\ln(t^2+\frac{7}{4})$

$\displaystyle= \frac{3}{\sqrt7}\arctan\left[\left(\frac{y}{x}-\frac{1}{2}\right)\frac{2}{\sqrt7}\right] - \frac{1}{2}\ln\left(\left(\frac{y}{x}\right)^2 - \frac{y}{x} + 2\right)$

$$\Rightarrow \boxed{\frac{3}{\sqrt7}\arctan\frac{2\frac{y}{x}-1}{\sqrt7} = \ln C\cdot x\sqrt{\left(\frac{y}{x}\right)^2 - \frac{y}{x} + 2}}$$

Zad 20 Pogledati Zad 18. na str. 17., urađen samo jedan slučaj, ali zaključak :)

1. a $\tan A = \left|\dfrac{\frac{y}{x} - y'}{1 + \frac{y}{x}y'}\right|$ $\quad p + xp' = \dfrac{p-a}{1+ap} \Rightarrow xp' = \dfrac{\cancel{p} - a - \cancel{p} - ap^2}{1+ap}$

$a(x + yy') = y - xy'$

$(x+ay)y' = y - ax$ $\quad xp' = -\cancel{a}\dfrac{p^2+1}{\cancel{a}(p + \frac{1}{a})}$ $\quad b$

$y' = \dfrac{y-ax}{x+ay}$ $\quad \displaystyle\int\frac{p+b}{p^2+1}\,dp = -\int\frac{dx}{x}$ isto kao Zad 18.

$y' = \dfrac{\cancel{x}(\frac{y}{x} - a)}{\cancel{x}(1 + a\frac{y}{x})}$ $\quad b\cdot\arctan p = \ln\dfrac{c}{x\sqrt{p^2+1}}$

$p = \dfrac{y}{x}$ $\quad y' = p + xp'$ $\quad \boxed{\dfrac{1}{a}\arctan\dfrac{y}{x} = \ln\dfrac{c}{\sqrt{x^2+y^2}}}$

Zad 21 $\alpha = \frac{\pi}{3}$, familija srednjih kružnica :

$$x^2+y^2=C$$

$x^2+y^2=C$

1) $y' \updownarrow \frac{y'-\tan\alpha}{1+y'\tan\alpha}$ 2) $y' \updownarrow \frac{y'+\tan\alpha}{1-y'\tan\alpha}$

$\cancel{2}x + \cancel{2}yy' = 0$

$y' = -\frac{x}{y}$ $y' \updownarrow \frac{y'-\sqrt{3}}{1+\sqrt{3}y'}$

Dva rješenja, drugo za vježbu :)

$\frac{y'-\sqrt{3}}{1+\sqrt{3}y'} = -\frac{x}{y}$

$y'y - \sqrt{3}y = -x - \sqrt{3}xy'$

$y'(y+x\sqrt{3}) = \sqrt{3}y - x$

$y' = \frac{\sqrt{3}y-x}{y+x\sqrt{3}} = \frac{x(\sqrt{3}\frac{y}{x}-1)}{x(\frac{y}{x}+\sqrt{3})}$ / $u=\frac{y}{x}$ $y'=u+u'x$

$u+xu' = \frac{\sqrt{3}u-1}{u+\sqrt{3}} \Rightarrow xu' = \frac{\sqrt{3}u-1-\sqrt{3}u-u^2}{u+\sqrt{3}}$

$xu' = -\frac{u^2+1}{u+\sqrt{3}} \Rightarrow \int \frac{u+\sqrt{3}}{u^2+1}\,du = -\int\frac{dx}{x}$

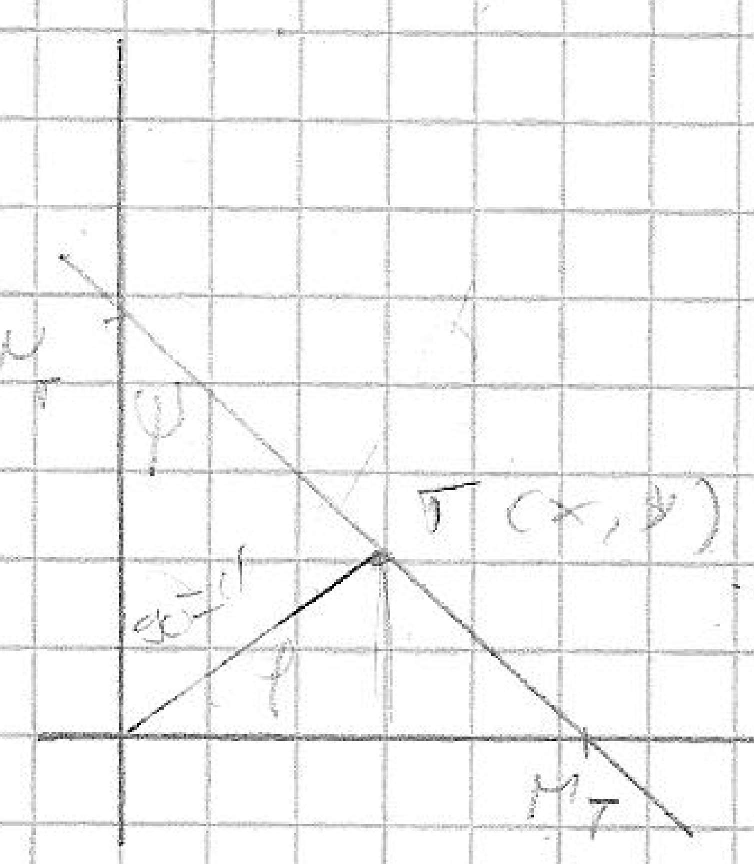
$\underbrace{\frac{1}{2}\int\frac{d(u^2+1)}{u^2+1}}_{\ln\sqrt{u^2+1}} + \underbrace{\sqrt{3}\int\frac{du}{u^2+1}}_{\sqrt{3}\arctan u} = \ln\frac{C}{x}$

$\sqrt{3}\arctan\frac{y}{x} = \ln\frac{C}{x\cdot\sqrt{\frac{y^2}{x^2}+1}} = \ln\frac{C}{\sqrt{y^2+x^2}}$

$y = r\cdot\sin\varphi$, $x = r\cdot\cos\varphi$

$\sqrt{3}\varphi = \ln\frac{C}{r}$ $\boxed{r = Ce^{-\sqrt{3}\varphi}}$

Zad 22.



$\boxed{2\varphi=\psi}$

$\varphi = \arctan\left|\frac{y}{x}\right|$

$\psi = \arctan\left|\frac{M_T}{N_T}\right|$

$= \arctan\left(\left|\frac{x-\frac{y}{y'}}{y-y'x}\right|\right)$

$= \arctan\left|\frac{xy'-y}{y'(y-xy')}\right|$

$= \arctan\left|-\frac{1}{y'}\right|$

$2\arctan\left|\frac{y}{x}\right| = \arctan\left|-\frac{1}{y'}\right| \quad /\tan$

$\frac{y}{x}=p$

$\tan 2x = \frac{2\tan x}{1-\tan^2 x}$

$\tan\left[2\arctan p\right] = \left|-\frac{1}{y'}\right|$

$\frac{1}{y'} = \frac{2p}{1-p^2}$ $\qquad$ $\frac{1}{y'} = \frac{2p}{p^2-1}$

$y' = \frac{1-p^2}{2p}$ $\qquad$ $y' = \frac{p^2-1}{2p}$

$y'=p+xp'$

$p+xp' = \frac{1-p^2}{2p}$ $\qquad$ $p+xp' = \frac{p^2-1}{2p}$

$xp' = \frac{1-p^2-2p^2}{2p}$ $\qquad$ $xp' = \frac{p^2-1-2p^2}{2p}$

$xp' = \frac{1-3p^2}{2p}$ $\qquad$ $xp' = -\frac{p^2+1}{2p}$

$\int\frac{2p}{3p^2-1}\,dp = -\int\frac{dx}{x}$ $\qquad$ $\int\frac{2p}{p^2+1}\,dp = -\int\frac{dx}{x}$

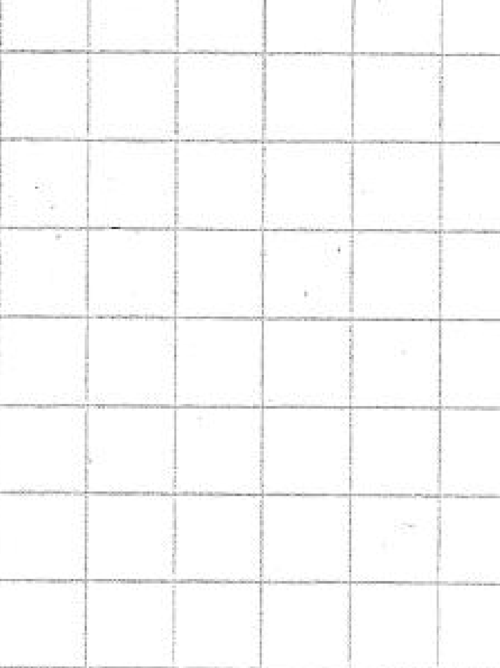
$\frac{1}{3}\ln(3p^2-1) = \ln\frac{C}{x}$ $\qquad$ $\ln(p^2+1) = \ln\frac{C}{x}$

$\sqrt[3]{3p^2-1} = \frac{C}{x} \quad /\cdot x$ $\qquad$ $\frac{y^2}{x^2}+1 = \frac{C}{x} \quad /\cdot x^2$

$\boxed{x\sqrt[3]{1-3\frac{y^2}{x^2}} = C}$ $\qquad$ $y^2+x^2 = Cx \qquad C=2c$

$x^2-2cx+y^2=0$

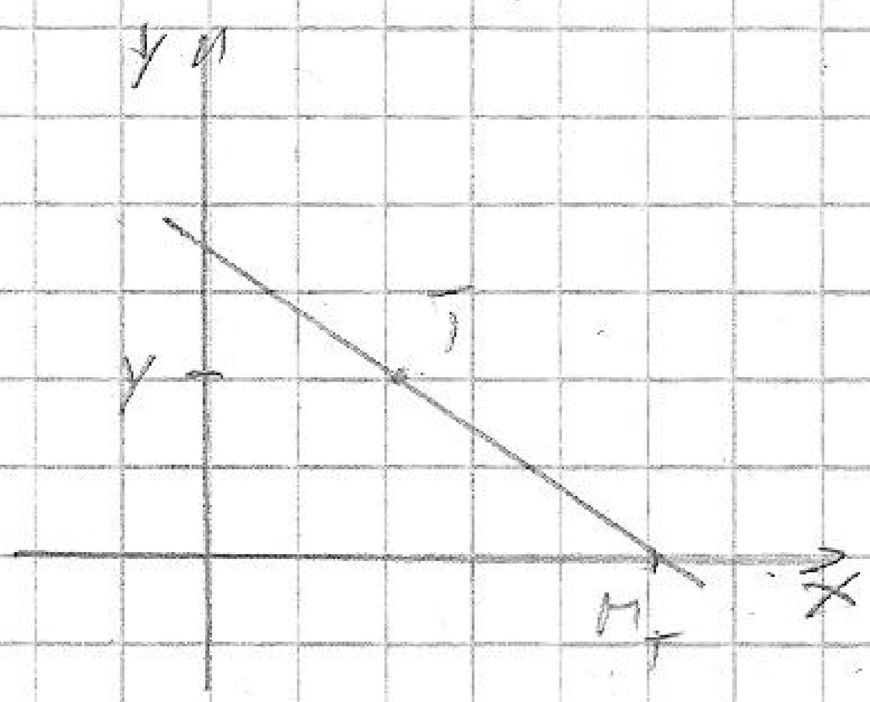
$(x-c)^2+y^2=c^2$



Ne znam kako se u rješenju ne vrijedi u svakoj točki

odnosno došao samo do jednog oblika :)

Zad 23 / Zad 24   23. samo poseban slučaj od 24.



$|M_T| = y^2$

$x - \frac{y}{y'} = \pm y^2$

$xy' - y \pm y'y^2 = 0$

$y'(x \pm y^2) = y$

$y' = \frac{y}{x \pm y^2} \Rightarrow xy' \pm y^2 y' = y$

$x = x(y)$

$\frac{x}{x'} \pm \frac{y^2}{x'} = y \Rightarrow x' - \frac{x}{y} = \pm y$ linearna po x i x'

$f(y)$ $g(y)$

$x' - \frac{x}{y} = 0$ $x = C(y) \cdot y$

$\int \frac{dx}{x} = \int \frac{dy}{y}$ $C'y + \cancel{C} - \cancel{C} = \pm y$ $x = C_1 y \pm y^2 \quad /:y$

$x_H = C \cdot y$ $C' = \pm 1$ $\boxed{\frac{x}{y} \pm y = C}$

$C = \pm y + C_1$

23. ... $T(-2, 1)$ $\pm y^2 + x - Cy = 0$

$\pm 1 - 2 - C = 0$

$C = \pm 1 - 2$

$C_1 = -3 \quad C_2 = -1$

$$\boxed{\begin{aligned} -y^2 + x + 3y &= 0 \\ y^2 + x + y &= 0 \end{aligned}}$$

Ja mislim da su oba rješenja točna

Dodatak

Zad 1: 10. $\quad (2a-x)y^2 = x^3 \quad /\frac{d}{dx}$

$$-y^2 + 2yy'(2a-x) = 3x^2$$

$$-y^2 + 2yy'\frac{x^3}{y^2} = 3x^2$$

$$-y^2 + \frac{2x^3}{y}y' = 3x^2$$

$$-y^2 + \frac{2x^3}{-yy'} = 3x^2 \quad / \cdot -yy'$$

$$+y^3y' + 2x^3 = -3x^2yy' \quad / : y^3$$

$$y' + 2\left(\frac{x}{y}\right)^3 = -3\left(\frac{x}{y}\right)^2 y'$$

$$y'\left(1+3\left(\frac{x}{y}\right)^2\right) = -2\left(\frac{x}{y}\right)^3$$

$$y' = \frac{-2\left(\frac{x}{y}\right)^3}{1+3\left(\frac{x}{y}\right)^2} = \frac{-2\left(\frac{x}{y}\right)}{\left(\frac{y}{x}\right)^2\left(3+\left(\frac{y}{x}\right)^2\right)} = \frac{-2}{\left(\frac{y}{x}\right)\left(\left(\frac{y}{x}\right)^2+3\right)}$$

$$p = \frac{y}{x} \qquad y' = p + xp'$$

$$p + xp' = \frac{-2}{p(p^2+3)} \Rightarrow xp' = \frac{-2 - p^2(p^2+3)}{p(p^2+3)}$$

$$\int \frac{p(p^2+3)}{2+p^2(p^2+3)}\,dp = -\int\frac{dx}{x} = \ln\frac{C}{x}$$

$$\int \frac{p(p^2+3)}{2+p^2(p^2+3)}\,dp = \left| \begin{array}{l} t = p^2+3 \\ dt = 2p\,dp \end{array}\right| = \int \frac{p\,\frac{dt}{2p}\,t}{2+(t-3)t} = \frac{1}{2}\int\frac{dt}{t^2-3t+2} = \frac{1}{2}\int\frac{dt}{(t-2)(t-1)}$$

$$\frac{1}{(t-2)(t-1)} = \frac{(t-1)-(t-2)}{(t-2)(t-1)} = \frac{1}{t-2} - \frac{1}{t-1}$$

$$\frac{1}{2}\int\frac{dt}{t-2} - \frac{1}{2}\int\frac{dt}{t-1} = \frac{1}{2}\left[\ln(t-2) - \ln(t-1)\right] = \frac{1}{2}\ln\frac{p^2+1}{p^2+2}$$

$$\ln\frac{p^2+1}{p^2+2} = 2\ln\frac{C}{x} \Rightarrow \frac{\frac{y^2}{x^2}+1}{\frac{y^2}{x^2}+2} = \frac{C^2}{x^2}$$

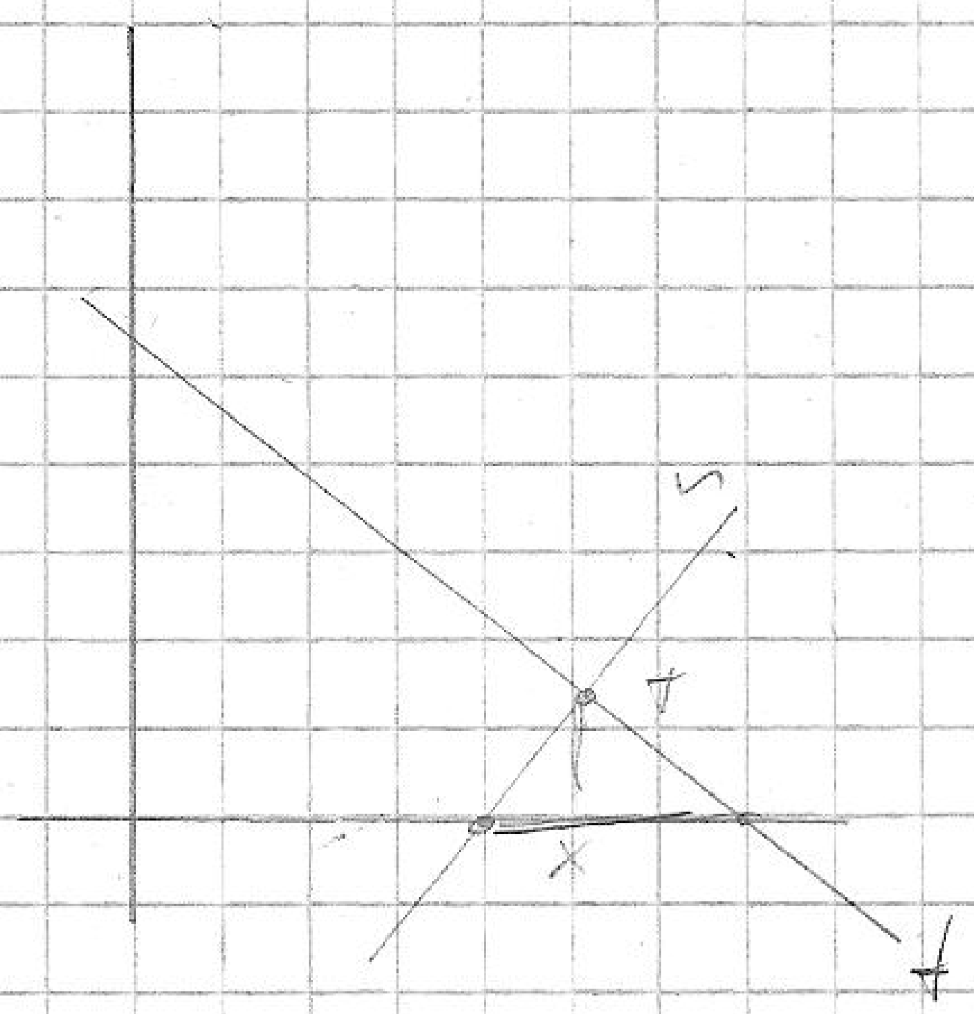
$$\frac{x^2+y^2}{y^2+2x^2} = \frac{C^2}{x^2}$$

$$\boxed{x^2+y^2 = \frac{1}{x^2}C^2(2x^2+y^2)}$$

Ja ne uspjevam
dobiti kao oni
? možeš li
naći grešku ??

Dodatek

Zad 12



$$\left| M_T - M_N \right| = 2x$$

$$\left| \cancel{x} - \frac{y}{y'} - \cancel{x} - yy' \right| = 2x$$

$$\left| \frac{y}{y'} + yy' \right| = 2x$$

$$\left| y + yy'^2 \right| = 2x|y'|$$

$$y + yy'^2 = \pm 2xy'$$

$$yy'^2 \pm 2xy' + y = 0$$

$$yy'^2 - 2xy' + y = 0 \qquad yy'^2 + 2xy' + y = 0$$

$$y' = \frac{2x \pm \sqrt{4x^2 - 4y^2}}{2y}$$

$$y' = \frac{x}{y} \pm \sqrt{\left(\frac{x}{y}\right)^2 - 1} \qquad p = \frac{y}{x} \quad y' = p + xp'$$

$$p + xp' = \frac{1}{p} \pm \sqrt{\frac{1}{p^2} - 1} = \frac{1 \pm p\sqrt{\frac{1}{p^2} - 1}}{p} = \frac{1 \pm \sqrt{1 - p^2}}{p}$$

$$xp' = \frac{(1 - p^2) \pm \sqrt{1 - p^2}}{p}$$

$$\int \frac{p\,dp}{(1 - p^2) \pm \sqrt{1 - p^2}} = \int \frac{dx}{x} = \ln Cx$$

$$\int \frac{p\,dp}{(1 - p^2) \pm \sqrt{1 - p^2}} \quad \begin{vmatrix} t^2 = 1 - p^2 \\ 2t\,dt = -2p\,dp \end{vmatrix} \quad = \int \frac{-t\,dt}{t^2 \pm t} = -\int \frac{\cancel{t}\,dt}{\cancel{t}(t \pm 1)} =$$

$$= -\ln(t \pm 1) = \ln Cx$$

$$t \pm 1 = \frac{1}{Cx} \qquad \sqrt{1 - p^2} \pm 1 = \frac{1}{Cx}$$

$$\sqrt{1 - \frac{y^2}{x^2}} \pm 1 = \frac{1}{Cx}$$

$$\frac{1}{x}\sqrt{x^2 - y^2} \pm 1 = \frac{1}{Cx} \quad / \cdot x$$

$$\boxed{\sqrt{x^2 - y^2} \pm x = K}$$