6. Diferencijalni račun fja više var. 3.

6.1 Derivacija implicitno zadane fje

$f(x,y)=0 \rightarrow$ Da li je time određena fja φ $y=\varphi(x)$ takva da je $f[x,\varphi(x)]\equiv 0$

STAVAK 1.: stavak o implicitnoj fji

Neka je $\mathcal{U}$ otvoren skup u ravnini, i neka je $f:\mathcal{U}\to R$ fja klase C^1 (tj. neprekinuto dfb. po obje varijable), te $f(x_0,y_0)=0$. Ako je $\frac{\partial f}{\partial y}(x_0,y_0)\neq 0$ onda postoji jednoznačno određena fja $y=\varphi(x)$, C^1, def. na nekom otvorenom intervalu I oko x_0, takva da je $f(x,\varphi(x))\equiv 0$, $\forall x\in I$, $\varphi(x_0)=y_0$

Pri tome vrijedi $\varphi'(x_0)=-\dfrac{f'_x(x_0,y_0)}{f'_y(x_0,y_0)}$ ili drugačije pisano

$$\boxed{\frac{dy}{dx}(x_0)=-\frac{\frac{\partial f}{\partial x}(x_0,y_0)}{\frac{\partial f}{\partial y}(x_0,y_0)}}\ !$$

Dokaz - samo za konačnu formulu

$$f(x,y)=0 \Rightarrow df=\frac{\partial f}{\partial x}dx+\frac{\partial f}{\partial y}dy=0 \quad /:\frac{\partial f}{\partial y}\neq 0$$

$$\Rightarrow \frac{dy}{dx}=\frac{-\frac{\partial f}{\partial x}}{\frac{\partial f}{\partial y}}$$

Primjedba: S1 kaže da der. impl. fje možemo izračunati čak i onda kada samu fju ne možemo eksplicitno izraziti

Primjedba 2: S1 vrijedi i u mnogo općenitijem slučaju ! za fje: $f(x_1,\dots,x_n)$ koje imaju $n+1$ varijabli

→ knjiga TM 2 str 4

$$f(x_1,\dots x_n,u)=0 \qquad \frac{\partial u}{\partial x_i}(x_1^0,\dots,x_n^0)=-\frac{\frac{\partial f}{\partial x_i}(x_1^0,\dots x_n^0,u^0)}{\frac{\partial f}{\partial u}(x_1^0,\dots,x_n^0,u^0)\neq 0}, \quad i=1,2,\dots n$$

b)

$x, y, z) = 0 \Rightarrow \boxed{f(x, y, z(x,y)) \equiv 0}$

$z(x_0, y_0) = z_0$

$$\boxed{\frac{\partial z}{\partial x} = -\frac{\frac{\partial f}{\partial x}}{\frac{\partial f}{\partial z}} \qquad \frac{\partial z}{\partial y} = -\frac{\frac{\partial f}{\partial y}}{\frac{\partial f}{\partial z}}}$$

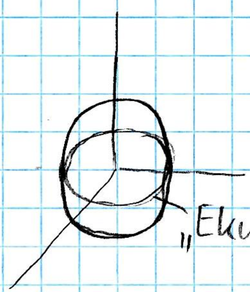
!!

① Ako je $x^2+y^2+z^2=1$ $\frac{\partial z}{\partial x}, \frac{\partial z}{\partial y}$, navesti podr. u kojem rj. vrijedi

1. implicitno

$$\underbrace{x^2+y^2+z^2-1}_{f(x,y,z)}=0$$

$$\left.\begin{aligned} \frac{\partial z}{\partial x} &= -\frac{\frac{\partial f}{\partial x}}{\frac{\partial f}{\partial z}} = -\frac{2x}{2z} = -\frac{x}{z} \\ \frac{\partial z}{\partial y} &= -\frac{\frac{\partial f}{\partial y}}{\frac{\partial f}{\partial z}} = -\frac{2y}{2z} = -\frac{y}{z} \end{aligned}\right\} z \neq 0$$



Dobiveni izrazi ne vrijede ako je uvjet $z \neq 0$ narušen, tj. $z=0$

$x^2+y^2+0=1$

$x^2+y^2=1$

Dobiveni izrazi vrijede na svim točkama

D osim na ekvatoru

② U paragrafu ④ izvedena je jdžba tang. ravn. Π u $T(x_0, y_0, z(x_0,y_0))$ za ekspl. zadanu plohu $z = z(x,y)$

$$\Pi_t \dots \quad z - z_0 = \left(\frac{\partial z}{\partial x}\right)_0 (x - x_0) + \left(\frac{\partial z}{\partial y}\right)_0 (y - y_0)$$

Koristeći taj rez. izvesti jdžbu Π_t u $T(x_0, y_0, z_0)$ ako je ploha $z(x,y)$ zadana impl. jdžbom $F(x,y,z)=0$

$f(x,y,z)=0$

! može doći na ispit

$f(x,y,z(x,y))=0$

$z_0 = z(x_0,y_0)$

$$\Pi_t \dots \quad z - z_0 = \left(\frac{\partial z}{\partial x}\right)_0 (x - x_0) + \left(\frac{\partial z}{\partial y}\right)_0 (y - y_0)$$

$$-\frac{F_x'(x_0,y_0,z_0)}{F_z'(x_0,y_0,z_0)} \qquad -\frac{F_y'(x_0,y_0,z_0)}{F_z'(x_0,y_0,z_0)} \qquad \Big/ \cdot F_z'(x_0,y_0,z_0) \neq 0$$

$$\Pi_t \dots \Rightarrow \left(\frac{\partial F}{\partial x}\right)_0 (x - x_0) + \left(\frac{\partial F}{\partial y}\right)_0 (x - x_0) + \left(\frac{\partial F}{\partial z}\right)_0 (z - z_0) = 0$$

! ispitni
③ Jdžbom $z^2 - xy\ln z + \sin x - \cos y - \frac{1}{2} = 0$ implicitno je zadana fja $z = z(x,y)$, takva da je $z(\frac{\pi}{6}, 0) = 1$ (provjera $z^2 - 0 + \frac{1}{2} - 1 - \frac{1}{2} = 0$, $z^2 = 1 \Rightarrow z = 1$)

a) $\left(\frac{\partial z}{\partial x}\right)_T$

b) jdžba Π_t na plohu u točki $T(\frac{\pi}{6}, 0, 1)$

a) $\frac{\partial z}{\partial x} = -\frac{F_x'}{F_z'}$ gdje je $F(x,y,z) \equiv z^2 - xy\ln z + \sin x - \cos y - \frac{1}{2}$

$$\frac{\partial z}{\partial x} = -\frac{-y\ln z + \cos x}{2z - \frac{xy}{z}} \Rightarrow \left(\frac{\partial z}{\partial x}\right)_T = -\frac{0 + \frac{\sqrt{3}}{2}}{2 - 0} = -\frac{\sqrt{3}}{4}$$

b)

$$\left.\begin{aligned} F'_x &= -y\ln z + \cos x \\ F'_y &= x\ln z + \sin y \\ F'_z &= 2z - \frac{xy}{z} \end{aligned}\right\} \Rightarrow \left.\begin{aligned} F'_x(T) &= \frac{\sqrt{3}}{2} \\ F'_y(T) &= 0 \\ F'_z(T) &= 2 \end{aligned}\right\} \text{ u točki } T$$

$$\Pi_t \ldots \quad \frac{\sqrt{3}}{2}\left(x - \frac{\pi}{6}\right) + 0(y-c) + 2(z-1) = 0$$

$$\Pi_t \ldots \quad x\sqrt{3} + 4z - \frac{\pi\sqrt{3}}{6} - 4 = 0$$

(4) Za fj. Fja $z = z(x,y)$ zad. je impl. jednadžbom $x^3z^2 + yz^5 - 3 = 0$
Nađi dif z $(dz)_T$ u T ako je $T(1,2,1)$

$f'(x,y,z)$

$$\frac{\partial z}{\partial x} = -\frac{F'_x}{F'_z} \Rightarrow \left(\frac{\partial z}{\partial x}\right)_T = -\frac{3}{12}$$

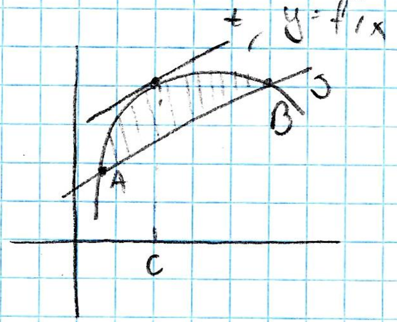
$$\boxed{(dz)_T = -\frac{3}{12}dx - \frac{1}{12}dy}$$

$$\frac{\partial z}{\partial y} = -\frac{F'_y}{F'_z} \Rightarrow \left(\frac{\partial z}{\partial y}\right)_T = -\frac{1}{12}$$

6.2 Integrali ovisni o parametru
→ vježbe

6.3 Teorem o srednjoj vrijednosti

dz MA1



$t \parallel s$

$y = f(x) \to$ dfb na (a,b) postoji c iz (a,b) takav da je

$$\frac{f(b) - f(a)}{b-a} = f'(c)$$

$$f(b) - f(a) = f'(c)(b-a)$$

STAVAK ③ Lagrangeov SSV za fje više var

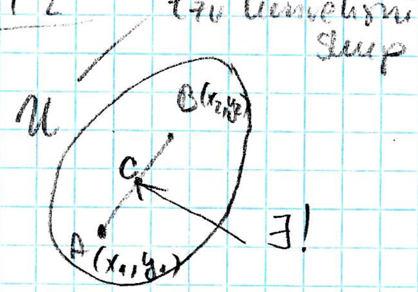
Neka je $f: \mathcal{U} \to \mathbb{R}$ dfb fja def na otvorenom skupu $\mathcal{U} \subseteq \mathbb{R}^n$, te $\vec{a}, \vec{b} \in \mathcal{U}$ takvi da je spojnica točaka $\vec{a}, \vec{b}$ također u $\mathcal{U}$. Onda na toj spojnici postoji točka $\vec{c}$ takva da je:

$$\boxed{f(\vec{b}) - f(\vec{a}) = \nabla f(\vec{c}) \cdot (\vec{b} - \vec{a})}$$

! TEOR. PIT

gdje je $\nabla f = \text{grad} f = \left(\frac{\partial f}{\partial x_1}, \dots, \frac{\partial f}{\partial x_n}\right)$ gradijent fje f a umnožak „·" je skalarni produkt dvaju vektora u $\mathbb{R}^n$

n=2



Postoji još dva teorema koji su poopćenja poznatih stavaka iz MAT 1

S④ Neka je $\mathcal{U}$ otvoren i konveksan skup u $\mathbb{R}^n$ te neka su $f, g: \mathcal{U} \to \mathbb{R}$ dfb fje

(1) Ako je $\text{grad } f \equiv \vec{0}$ na $\mathcal{U}$ (tj. $\text{grad} f(\vec{x}) = \vec{0}, \forall \vec{x} \in \mathcal{U}$) onda je f konstanta

(2) Ako je $\text{grad } f = \text{grad } g$, onda se f i g razlikuju za konstantu (tj. postoji c td za sve $\vec{x} \in \mathcal{U}$ $f(\vec{x}) = g(\vec{x}) + c$)

Dokaz (1)

Iz S③

$$f(\vec{b}) - f(\vec{a}) = \underbrace{\text{grad} f(\vec{c})}_{= \vec{0} \text{ po pretp.}} (\vec{b} - \vec{a})$$

$$\Rightarrow f(\vec{b}) - f(\vec{a}) = 0$$

$$f(\vec{b}) = f(\vec{a}) \Rightarrow f = \text{konst na } \mathcal{U}$$

QED(1)

Dokaz (2)

$$\text{grad} f = \text{grad}\, g$$

$$\text{grad} f - \text{grad}\, g = \vec{0}$$

$$\text{grad}(f - g) = \vec{0} \Rightarrow \text{iz (1) } f - g = \text{const.}$$

QED(2)

Primjedba

S④ vrijedi i općenitije, tj $\mathcal{U}$ ne mora biti konveksan, nego je dovoljno da je povezan

DZ. str. 14 → pročitati

6.4 Derivacije višeg reda i Schwartzov teorem

DEF ① Parcijalne derivacije višeg reda

Opetovanim parc. der. ukoliko te derivacije postoje dobivamo više parc derivacija.

Pr. $z = z(x, y)$

$$z''_{xx} = \frac{\partial^2 z}{\partial x^2} := \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right)$$

$$z''_{xy} = \frac{\partial^2 z}{\partial x \partial y} := \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)$$

$$z'''_{xyy} = \frac{\partial^3 z}{\partial x \partial y^2} := \frac{\partial}{\partial y}\left(\frac{\partial^2 z}{\partial x \partial y}\right)$$

① Naći sve parc. der. 2. reda ako je $z = x^y$

$z'_x = y x^{y-1}$

$z'_y = x^y \ln x$

$z''_{xx} = \frac{\partial}{\partial x}(z'_x) = \frac{\partial}{\partial x}(y x^{y-1}) = y(y-1)x^{y-2}$

$$\left.\begin{aligned} z''_{xy} &= \frac{\partial}{\partial y}(z'_x) = \frac{\partial}{\partial y}(y x^{y-1}) = x^{y-1} + y \cdot x^{y-1} \ln x \\ z''_{yx} &= \frac{\partial}{\partial x}(z'_y) = \frac{\partial}{\partial x}(x^y \ln x) = y \cdot x^{y-1} \cdot \ln x + x^y \frac{1}{x} \end{aligned}\right\} =$$

$z''_{yy} = \frac{\partial}{\partial y}(z'_y) = \frac{\partial}{\partial y}(x^y \ln x) = x^y \ln^2 x$

Vidimo da je u zadatku 1 $z''_{xy} = z''_{yx}$ tj izrazi za "mješovite" druge der. ne ovise o redoslijedu deriviranja.

5 ⑤ SCHVARTZ-ov Teorem

Neka je $z = f(x,y)$ domena od $f = M$, $T_0 \in M$.
Ako u nekom okolišu T_0 postoje derivacije:
f'_x, f'_y, f''_{xy}, f''_{yx}, pri čemu su f''_{xy} i f''_{yx} neprekidne u T_0 onda je:

$$\boxed{f''_{xy}(x_0,y_0) = f''_{yx}(x_0,y_0)}$$!!

Primjedba

Vrijedi i općenitije, uz neke uvjete koje ćemo smatrati ispunj.
Više parc. der. ne ovise o redoslijedu deriviranja.

npr

1) $f'''_{xyz} = f'''_{xzy} = f'''_{yxz} = f'''_{yzx} = f'''_{zxy} = f'''_{zyx}$

2) $f^{IV}_{xxyz} = f^{IV}_{xyxz} = f^{IV}_{zxxy} = \ldots$ $\quad \frac{4!}{2!} = 12$ mogućnosti

DZ - vježba

za vj.

) $z = z(x,y)$ zadana je impl. jdžbom $x^3 + y^3 + x + y + z + e^z - 1 = 0$

naći $\frac{\partial^2 z}{\partial x \partial y}$ u $T(1,-1,0)$

$$F(x,y,z) = x^3 + y^3 + x + y + z + e^z - 1$$

$$\Rightarrow \frac{\partial z}{\partial x} = -\frac{F'_x}{F'_z} = -\frac{3x^2+1}{1+e^z}$$

$$\frac{\partial z}{\partial y} = -\frac{3y^2+1}{1+e^z}$$

$$\frac{\partial^2 z}{\partial x \partial y} = \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right) = \frac{\partial}{\partial y}\left(-\frac{3x^2}{1+e^z}\right) = \ldots = \frac{3x^2+1}{(1+e^z)^2} \cdot \frac{\partial z}{\partial y} = -\frac{(3x^2+1)(3y^2+1)}{(1+e^z)^3}$$

$$\Rightarrow \left(\frac{\partial^2 z}{\partial x \partial y}\right)_T = -2$$

$\boxed{6.5}$ Drugi diferencijal fje

Diferencijali višeg reda

U paragrafu ④ definiran je 1. dif funkcije $z = z(x,y)$ u $T(x,y)$:

$$dz := \frac{\partial z}{\partial x} dx + \frac{\partial z}{\partial y} dy$$

DEF ② Drugi dif od $z = z(x,y)$ definiramo kao $d(dz)$, pri čemu dif dx i dy smatramo konstantama.

$$\boxed{d^2 z := d(dz)} \; !!$$

Općenitije vrijedi

$$d^n z := d(d^{n-1} z), \; n \geq 2$$

Zadaci

ISPITNI, ali baš baš !!

① Izvesti formulu za $d^2 z$, ako fja $z = z(x,y)$ zadovoljava uvjete Schwartzovog teorema.

$$d^2 z := d(dz) = d\left(\frac{\partial z}{\partial x} dx + \frac{\partial z}{\partial y} dy\right) = \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x} dx + \frac{\partial z}{\partial y} dy\right) dx +$$

$$\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x} dx + \frac{\partial z}{\partial y} dy\right) \cdot dy =$$

$$= \left(\frac{\partial^2 z}{\partial x^2} dx + \frac{\partial^2 z}{\partial y \partial x} dy\right) \cdot dx + \left(\frac{\partial^2 z}{\partial x \partial y} \cdot dx + \frac{\partial^2 z}{\partial y^2} dy\right) \cdot dy =$$

$$= \frac{\partial^2 z}{\partial x^2} (dx)^2 + \underbrace{\frac{\partial^2 z}{\partial y \partial x} dx dy + \frac{\partial^2 z}{\partial x \partial y} dx dy}_{\text{schwartz}} + \frac{\partial^2 z}{\partial y^2} (dy)^2 =$$

$$d^2 z = z''_{xx} (dx)^2 + 2 \cdot z''_{xy} + z''_{yy} (dy)^2$$

Možemo pisati → simbolički kvadrat

$$d^2 z = dz^{[2]} = \left(\frac{\partial}{\partial x} dx + \frac{\partial}{\partial y} dy\right)^{[2]} z$$

stvarni kvadrat od dz bio bi $(dz)^2 = \left(\frac{\partial z}{\partial x}\right)^2 (dx)^2 + 2 \cdot \frac{\partial z}{\partial x} \frac{\partial z}{\partial y} \cdot dx dy + \left(\frac{\partial z}{\partial y}\right)^2 (dy)^2$

② Izvesti formulu za d^3z ako je $z = z(x,y)$, i ako je poznat izraz za d^2z

$$d^3z = d(d^2z) = d\left[z''_{xx}(dx)^2 + 2z''_{xy}dxdy + z''_{yy}(dy)^2\right] =$$

$$= \frac{\partial}{\partial x}\left[z''_{xx}(dx)^2 + 2z''_{xy}dxdy + z''_{yy}(dy)^2\right] + \frac{\partial}{\partial y}\left[z''_{xx}(dx)^2 + 2z''_{xy}dxdy + z''_{yy}(dy)^2\right]$$

$$= \left[z'''_{xxx}(dx)^2 + 2z'''_{xyx}dxdy + z'''_{yyx}(dy)^2\right]dx + \left[z'''_{xxy}(dx)^2 + 2z'''_{xyy}dxdy + z'''_{yyy}(dy)^2\right]dy =$$

$$= z'''_{xxx}(dx)^3 + \underbrace{2z'''_{xyx}(dx)^2dy + z'''_{xxy}(dx)^2dy}_{=\text{schw}} + \underbrace{z'''_{yyx}(dy)^2dx + 2z'''_{xyy}dx(dy)^2}_{=\text{schw}} + z'''_{yyy}(dy)^3 =$$

$$= \boxed{d^3z = z'''_{xxx}(dx)^3 + 3z'''_{xxy}(3x)^2dy + 3z'''_{xyy}dx(dy)^2 + z'''_{yyy}(dy)^3}$$!!

Možemo pisati

simbolički (x i y u indeksima) $x^2 = xx$ $x^3 = xxx$ itd

$$d^3z = (dz)^{[3]} = (z'_x dx + z'_y dy)^{[3]}$$

Općenito vrijedi

$$d^nz = (dz)^{[n]}$$

- vrijedi i za fje n varijabli
- formalni zapis n-tog dif gdje je n simbolična n-ta potencija u smislu da je npr.

$$\left(dx\frac{\partial}{\partial x}\right)^{[2]}_z \cdot \left(dy\frac{\partial}{\partial y}\right)^{[3]}_z = \frac{\partial^5 z}{\partial x^2 \partial y^3}(dx)^2(dy)^3$$

③ Naći $d^2z(T)$ ako je $z = x^2 + \cos^2 y$, $T(0, \pi)$.
Kakav je predznak izraza $d^2z(T)$ u ovisnosti o diferencijalima dx i dy.

$$z''_{xx} = 2$$
$$z''_{xy} = 0 \qquad z'_y = 2\cos y(-\sin y) = -\sin 2y$$
$$z''_{yy} = -2\cos 2y \qquad (z''_{xx})_T = 2, (z''_{xy})_T = 0, (z''_{yy})_T = -2$$

$$\Rightarrow (d^2z)_T = 2(dx)^2 + 2\cdot 0\, dx dy + (-2)(dy)^2 \Rightarrow (d^2z)_T = 2(dx)^2 - 2(dy)^2$$

b) za $(dx, dy) = (1, 0) \Rightarrow (d^2z)_T = +$
za $(dx, dy) = (0, 1) \Rightarrow (d^2z)_T = -$ $\Rightarrow$ dakle $(d^2z)_T$ je izraz promjenjivog predznaka, a takvi izr. se zovu TZV

INDEFINITNE FORME

④ za vj.

d^2f, $f(x,y) = x^2 + 3y^2 - 2xy$, i ispitati predznak tog diferenc

$$\Rightarrow d^2f = 2(dx)^2 - 4dxdy + 6(dy)^2$$

$\text{sgn}(d^2f) \to ?$

$$2((dx)^2 - 2dxdy + 3(dy)^2) \qquad d^2f = 2(dx - dy)^2 + 4(dy)^2$$

$$2[(dx)^2 - 2dydx + (dy)^2] + 4(dy)^2 \qquad \boxed{>0}$$

za $(dx, dy) \neq (0, 0)$

tj. d^2f je pozitivno definitna forma

ISPIT

5 a) Napisati formulu za d^2u, ako je $u = u(x, y, z)$

b) $d^2u(T)$ za fju

$u = xy + z^2$, $T(1, 0, -1)$, te ispitati predznak (definitnost) dobivenog izraza

a) $d^2u = (du)^{[2]} = [u'_x dx + u'_y dy + u_z dz]^{[2]} =$

$= u''_{xx}(dx)^2 + u''_{yy}(dy)^2 + u''_{zz}(dz)^2 + 2 \cdot [u''_{xy}dxdy + u''_{xz}dxdz + u''_{yz}dydz] =$

b) $d^2u \Rightarrow u'_x = y$, $u'_y = x$, $u'_z = 2z$

$, u''_{xx} = 0$, $u''_{yy} = 0$ $, u''_{zz} = 2$, $u''_{xy} = 1$, $u''_{xz} = 0$, $u''_{yz} = 0$

$d^2u = 0(dx)^2 + 0(dy)^2 + 2(dz)^2 + 2[1 \cdot dxdy + 0 \cdot dxdz + 0 \cdot dydz]$

$= 2(dz)^2 + 2dxdy \Rightarrow (d^2u)_{(T)} = 2(dz)^2 + 2dxdy$ - nemamo gdje uvstiti

$$\left.\begin{aligned} &\text{za } (dx, dy, dz) = (0, 0, dz) = 2(dz)^2 = (d^2u)_T \\ &\text{za } (dx, -dx, 0) = -2(dx)^2 = (d^2u)_T \end{aligned}\right\} \text{ indefinitna forma}$$

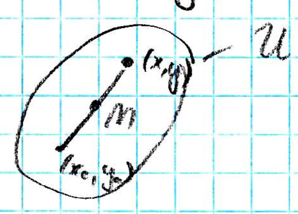
dakle d^2u je indefinitna

6.6 TAYLOR-ova FORMULA za fje dviju varijabli

STAVAK ⑥ Neka je $f(x,y)$ definirana na otvorenom skupu $\mathcal{U} \subseteq \mathbb{R}^2$, i ima neprekinute sve parc. der. do uključivo reda $n+1$ (tj. f je klase C^{n+1}). Neka je $(x_0,y_0) \in \mathcal{U}$. Onda vrijedi Taylorova formula:

$$\boxed{\begin{aligned} f(x,y) &= f(x_0,y_0) + \frac{1}{1!}\left[(f'_x)_0(x-x_0) + (f'_y)_0(y-y_0)\right] + \\ &+ \frac{1}{2!}\left[(f''_x)_0(x-x_0)^2 + 2(f''_{xy})_0(x-x_0)(y-y_0) + (f''_{yy})_0(y-y_0)^2\right] + \\ &+ \dots + \frac{1}{n!}\left[(x-x_0)\frac{\partial}{\partial x} + (y-y_0)\frac{\partial}{\partial y}\right]^{[n]} f(x_0,y_0) + R_n(x,y) \end{aligned}}$$

Pojašnjenje - gdje je $R_n(x,y)$ tzv. n-ti ostatak, koji sadrži derivacije reda $n+1$, u točki M koja leži na spojnici točaka (x_0,y_0), (x,y)



Dokaz → str 21, 22, bolji studenti

Drukčije pisano $f(x,y) = \underbrace{T_n(x,y)}_{\text{TAYLOROV POLINOM n-tog STUPNJA}} + R_n(x,y)$ - n-ti LAGRANGEOV ostatak

Gdje je $R_n = \frac{1}{(n+1)!}\left[(x-x_0)\frac{\partial}{\partial x} + (y-y_0)\frac{\partial}{\partial y}\right]^{[n+1]} f(T_c)$, gdje je T_c točka na spojnici $\overline{T_0 T}$