Rješenja drugog međuispita iz Matematike 3E
27.11.2008.

1. (3 boda)

$$\int_0^1 dy \int_{\frac{y^2}{2}}^{1-\sqrt{1-y^2}} f(x,y)\,dx + \int_0^1 dy \int_{1+\sqrt{1-y^2}}^{2} f(x,y)\,dx + \int_1^2 dy \int_{\frac{y^2}{2}}^{2} f(x,y)\,dx$$

2. (3 boda)
Pomaknute eliptičke koordinate:

$$\begin{aligned} x &= r\cos\varphi \\ y &= 3 + 2r\sin\varphi \\ dxdy &= 2r \end{aligned}$$

$$\iint_D \sqrt{4x^2+y^2-6y+17}\,dx\,dy = \cdots = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} d\varphi \int_0^1 4r\sqrt{r^2+2}\,dr = \cdots = \frac{4\pi}{3}\left(3\sqrt{3}-2\sqrt{2}\right)$$

3. (5 bodova)
a) (**1b**)

$$J = \begin{vmatrix} \frac{\partial x}{\partial u} & \frac{\partial x}{\partial v} \\ \frac{\partial y}{\partial u} & \frac{\partial y}{\partial v} \end{vmatrix}$$

b) (**1b**)

$$J = \cdots = u + 4uv - 2v^2$$

c) (**2b**)

$$x = \frac{u+v}{2}, y = \frac{u-v}{2}, J = \cdots = -\frac{1}{2}$$

$$\iint_D f(x,y)\,dx\,dy = \iint_{D'} f\left(x(u,v), y(u,v)\right)|J|\,du\,dv = \frac{1}{2}\int_0^3 du \int_{-1}^1 f\left(\frac{u+v}{2}, \frac{u-v}{2}\right) dv$$

d) (**1b**)

$$r = \sqrt{5}$$

4. (4 boda)
Normala na ravninu BCD je $\mathbf{n} = 2\mathbf{i} + \mathbf{j} + 2\mathbf{k}$.
Jednadzba ravnine je $z = 1 - x - \frac{y}{2}$.

$$\iiint_V x\,dV = \int_0^1 x\,dx \int_0^{2-x} dy \int_0^{1-x-\frac{y}{2}} dz = \cdots = \frac{1}{48}$$

5. (4 boda)
Cilindrične koordinate:

$$\begin{aligned} x &= r\cos\varphi \\ y &= r\sin\varphi \\ z &= z \\ dxdydz &= r d\varphi dr dz \end{aligned}$$

Presjek stošca i paraboloida: $x^2 + y^2 = \frac{4}{9}$, $z = \frac{2}{3}$.
Stožac $z = r$ i paraboloid $z = 2 - 3r^2$.

$$\iiint_V dV = \int_0^{2\pi} d\varphi \int_0^{\frac{2}{3}} r\,dr \int_r^{2-3r^2} dz = \cdots = \frac{32}{81}\pi$$

6. (6 bodova)

a) (**2b**)

$$\mathbf{r}'(t) = \frac{-\pi t \sin \pi t - \cos \pi t}{t^2}\mathbf{i} + 2t \cos \pi t^2 \mathbf{j} + 3t^2 \mathbf{k}$$

$$\mathbf{r}'(1) = \cdots = \mathbf{i} - 2\mathbf{j} + 3\mathbf{k}$$

b) (**2b**)

$$\cos \varphi = \frac{|(\mathbf{i} - 2\mathbf{j} + 3\mathbf{k}) \cdot \mathbf{i}|}{|\mathbf{i} - 2\mathbf{j} + 3\mathbf{k}| \cdot |\mathbf{i}|} = \cdots = \frac{1}{\sqrt{14}}$$

c) (**2b**)

$$\begin{aligned} x &= 1 + \cos t \\ y &= \sin t \\ & t \in [0, 2\pi] \end{aligned}$$