① a) $\operatorname{grad} f(r) = f'(r) \cdot \frac{\vec{r}}{r}$

b) $\operatorname{div}\left(\frac{\vec{r}}{r^2}\right) = ?$

$$\frac{\vec{r}}{r^2} = \frac{x\vec{i} + y\vec{j} + z\vec{k}}{x^2+y^2+z^2} = \underbrace{\frac{x}{x^2+y^2+z^2}}_{a_1}\vec{i} + \underbrace{\frac{y}{x^2+y^2+z^2}}_{a_2}\vec{j} + \underbrace{\frac{z}{x^2+y^2+z^2}}_{a_3}\vec{k}$$

$$\operatorname{div}\left(\frac{\vec{r}}{r^2}\right) = \frac{\partial a_1}{\partial x} + \frac{\partial a_2}{\partial y} + \frac{\partial a_3}{\partial z} =$$

$$= \frac{r^2 - x \cdot 2x}{r^4} + \frac{r^2 - y \cdot 2y}{r^4} + \frac{r^2 - z\, 2z}{r^4} =$$

$$= \frac{3r^2 - 2x^2 - 2y^2 - 2z^2}{r^4} = \frac{3r^2 - 2r^2}{r^4} = \frac{1}{r^2}$$

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② $\Delta(x^2 + xy + xz^2) = \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}\right)(x^2 + xy + xz^2) =$

$$= 2 + 0 + 2x = 2x + 2$$

③ $\vec{r}(t) = (3\cos t, 5\sin t, 4\cos t)$, $t \in (0, 2\pi)$

$$l = \int_K ds = \int_0^{2\pi} \sqrt{9\sin^2 t + 25\cos^2 t + 16\sin^2 t}\, dt =$$

$$= \int_0^{2\pi} \sqrt{25}\, dt = 5 \int_0^{2\pi} dt = 10\pi$$

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④ $\int_\Gamma \frac{x}{y} dx + \ln x\, dy + z\, dz$, $A(1,1,1)$, $B(2,3,4)$

$$\frac{x - x_1}{x_2 - x_1} = \frac{y - y_1}{y_2 - y_1} = \frac{z - z_1}{z_2 - z_1} = t$$

$$\frac{x-1}{2-1} = \frac{y-1}{3-1} = \frac{z-1}{4-1} = t$$

$x - 1 = t$

$x = t + 1$

$y = 2t + 1$

$$x-1=t$$
$$x=t+1$$
$$y=2t+1$$
$$z=3t+1\quad,\ t\in[0,1]$$

$$dx=dt$$
$$dy=2dt$$
$$dz=3dt$$

$$I=\int_0^1 \frac{t+1}{2t+1}\cdot dt+\ln(t+1)\cdot 2dt+(3t+1)\cdot 3dt=$$

$$=\int_0^1\left[\frac{t+1}{2t+1}+9t+3+2\ln(t+1)\right]dt=\dots=6+\frac{\ln 3}{4}+\ln 16$$

c) $$p(x,y,z)=\int_{x_0}^{x}(2x+yz)\,dx+\int_{y_0}^{y}x_0 z\,dy+\int_{z_0}^{z}(x_0y_0+2z)\,dz=$$

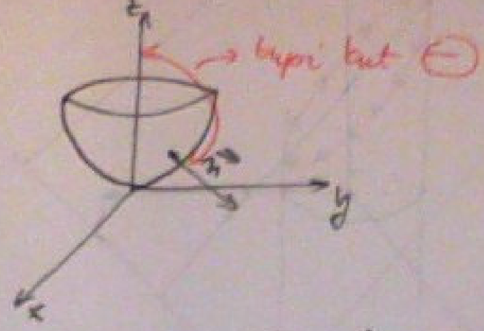
$$=\int_0^x(2x+yz)\,dx+\int_0^y 0\cdot z\,dy+\int_0^z(0\cdot 0+2z)\,dz=$$

$$=x^2\Big|_0^x+yzx\Big|_0^x+z^2\Big|_0^z+C=$$

$$=x^2+xyz+z^2+C$$

$$I=p(1,2,2)-p(1,0,1)=1+4+4-(1+0+1)=$$
$$=7$$

(6) $\iint_{S^+} z\,dx\,dy$, $S \ldots z = x^2+y^2$, $z \in [0,1]$

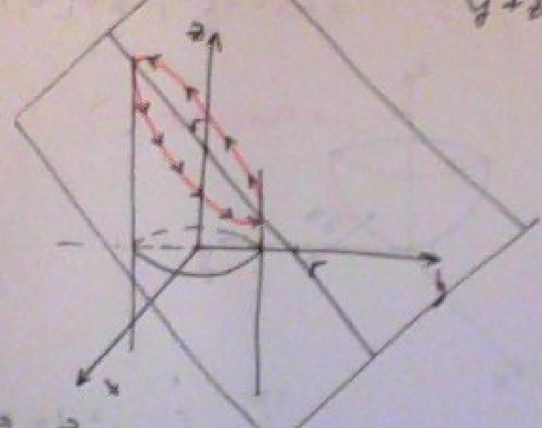


$$I = -\iint (x^2+y^2)\,dx\,dy = -\int_0^{2\pi} d\varphi \int_0^1 r \cdot r^2\,dr =$$

$$= -2\pi \cdot \frac{r^4}{4}\Big|_0^1 = -\cancel{2}\pi \cdot \frac{1}{\cancel{4}_2} = -\frac{\pi}{2}$$

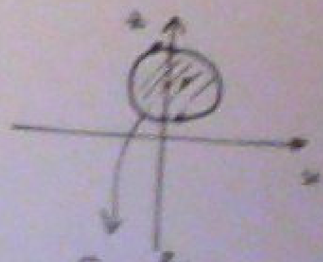
(7) a) $\oint_k a_1\,dx + a_2\,dy + a_3\,dz = \iint_{S^+} \text{rot}\,\vec{a}\,d\vec{S}$

b) $\oint_c z\,dx + 2\,dy - x\,dz = ?$ $\quad c \ldots \; x^2+y^2=9$, $y+z=5$



$x^2+(5-z)^2=9$

$x^2+(z-5)^2=9$



$P = r^2\pi =$
$= 9\pi$

$$\text{rot}\,\vec{a} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ z & 2 & -x \end{vmatrix} = -\vec{j}(-1-1) = 2\vec{j}$$

$$\oint_c = \iint_{S^+} 2\,dx\,dz = 2\iint_{S^+} dx\,dz = 2 \cdot 9\pi = 18\pi$$