Vektorske funkcije skalarnog argumenta – rješenja i postupci zadataka za vježbu

1. a) $4x^2 + 9y^2 = 36 \to jed.elipse: b^2x^2 + a^2y^2 = a^2b^2 \to a = 3, b = 2 \to x = 3cost, y = 2sint$

$$\boldsymbol{\vec{r}(t) = 3cost\vec{\imath} + 2sint\vec{\jmath}}$$

b) $y = x^2 \to x = t \to y = t^2$

$$\boldsymbol{\vec{r}(t) = t\vec{\imath} + t^2\vec{\jmath}}$$

c) segment od točke A(1, 4, -2) do točke B(3, 9, 6)

uzmemo npr., da nam *t* ide od 0 do 1.

za *t* = 0 moramo dobiti točke $x_1 = 1, y_1 = 4, z_1 = -2$

za *t* = 1 moramo dobiti točke $x_2 = 3, y_2 = 9, z_2 = -6$

kada nešto pomnožimo s *t* = 0, dobit ćemo 0. da bi dobili x = 1, moramo imati 1 + a*t. da bi dobili y = 4 moramo imati 4 + b*t, a da bi dobili z = - 2, moramo imati – 2 +c*t.

i sad, iz druge točke moći ćemo odrediti koeficijente a, b i c. mora vrijediti sljedeće:

$$3 = 1 + a\,, \qquad 9 = 4 + b\,, \qquad 6 = -2 + c$$

odnosno, $a = 2$, $b = 5$ i $c = 8$

pa je parametrizacija: $x = 1 + 2t$, $y = 4 + 5t$ i $z = -2 + 8t$

pa je

$$\boldsymbol{\vec{r}(t) = (1 + 2t)\vec{\imath} + (4 + 5t)\vec{\jmath} + (-2 + 8t)\vec{k}}$$

btw, oni su uzeli da je $t \in [0,1]$, al vi ste mogli uzet bilo koji segment i radit s tim segmentom ☺

2. a) $\lim_{t\to 0} \vec{f}(t) = \left(\lim_{t\to 0} \frac{sint}{2t}\right)\vec{\imath} + (\lim_{t\to 0} e^{2t})\vec{\jmath} + \left(\lim_{t\to 0} \frac{t^2}{e^t}\right)\vec{k} = \left(\lim_{t\to 0} \frac{cost}{2}\right)\vec{\imath} + \vec{\jmath} = \boldsymbol{\frac{1}{2}\vec{\imath} + \vec{\jmath}}$

b) $\lim_{t\to 0} \vec{f}(t) = (\lim_{t\to 0} 3(t^2 - 1))\vec{\imath} + (\lim_{t\to 0} cost)\vec{\jmath} + \left(\lim_{t\to 0} \frac{t}{|t|}\right)\vec{k} = \boldsymbol{-3\vec{\imath} + \vec{\jmath} \pm \vec{k}}$

(za *k* se izračuna lijevi i desni limes, jedan je – 1, a drugi 1, pa je zato $\pm$ u igri)

3. valjda znate derivirat :D ovo je uber lagano. samo svaku komponentu posebno derivirate ☺

4. a) $\int_1^2 (\vec{\imath} + 2t\vec{\jmath})dt = \vec{\imath}\int_1^2 dt + 2\vec{\jmath}\int_1^2 dt = \boldsymbol{\vec{\imath} + 3\vec{\jmath}}$

b) $\int_1^2 \left(e^t\vec{\jmath} + e^{-t}\vec{k}\right)dt = \vec{\jmath}\int_0^1 e^t dt + \vec{k}\int_0^1 e^{-t}dt = \boldsymbol{(e - 1)\vec{\jmath} + \left(1 - \frac{1}{e}\right)\vec{k}}$

5. a)

$$\vec{f}(t) = \int \vec{f}'(t)dt = \frac{t^2}{2}\vec{\imath} + \sqrt{t^2+1}\vec{\jmath} + e^t(t-1)\vec{k} + \vec{c} =$$

$$= \frac{t^2}{2}\vec{\imath} + \sqrt{t^2+1}\vec{\jmath} + e^t(t-1)\vec{k} + a\vec{\imath} + b\vec{\jmath} + c\vec{k} = \left(\frac{t^2}{2} + a\right)\vec{\imath} + \left(\sqrt{t^2+1} + b\right)\vec{\jmath} + (e^t(t-1) + c)\vec{k}$$

Vrijedi, zbog uvjeta $\vec{f}(0)$, sljedeće:

$$\frac{0^2}{2} + a = 1 \rightarrow a = 1$$

$$\sqrt{0^2+1} + b = 2 \rightarrow b = 1$$

$$e^0(0-1) + c = 3 \rightarrow c = 4$$

$$\boldsymbol{\vec{f}(t) = \left(\frac{t^2}{2} + 1\right)\vec{\imath} + \left(\sqrt{t^2+1} + 1\right)\vec{\jmath} + (e^t(t-1) + 4)\vec{k}}$$

(mislim da njima fali u rješenju kod jediničnog vektora **i**)

b)

$$\vec{f}(t) = \int \vec{f}'(t)dt = t\vec{\imath} + \frac{t^3}{3}\vec{\jmath} + \vec{c} =$$

$$= t\vec{\imath} + \frac{t^3}{3}\vec{\jmath} + a\vec{\imath} + b\vec{\jmath} + c\vec{k} = (t+a)\vec{\imath} + \left(\frac{t^3}{3} + b\right)\vec{\jmath} + c\vec{k}$$

Vrijedi, zbog uvjeta $\vec{f}(0)$, sljedeće:

$$0 + a = 0 \rightarrow a = 0$$

$$\frac{0^3}{3} + b = 1 \rightarrow b = 1$$

$$c = -1$$

$$\boldsymbol{\vec{f}(t) = t\vec{\imath} + \left(\frac{t^3}{3} + 1\right)\vec{\jmath} - \vec{k}}$$

(isto mislim da su zajebali, jer su napisali t^2 umjesto t^3…)

6. blah -.-

7. blaaah -.- :D

8. a) vektor smjera:

$$\vec{\boldsymbol{r}}'(\boldsymbol{t}=\boldsymbol{2}) = -\pi \sin t\pi\,\vec{\imath} + \pi\cos t\pi\,\vec{\jmath} + \vec{k} = \boldsymbol{\pi}\vec{\boldsymbol{j}} + \vec{\boldsymbol{k}}$$

tangenta: $\vec{\boldsymbol{r}}(\boldsymbol{u}) = \vec{r}(t=2) + \vec{r}'(t=2)\cdot u = \left(\vec{\boldsymbol{\imath}} + \boldsymbol{2}\vec{\boldsymbol{k}}\right) + \left(\boldsymbol{\pi}\vec{\boldsymbol{j}} + \vec{\boldsymbol{k}}\right)\cdot \boldsymbol{u}$

b) vektor smjera:

$$\vec{\boldsymbol{r}}'(\boldsymbol{t}=-\boldsymbol{1}) = \vec{b} + 2t\vec{c} = \vec{\boldsymbol{b}} - \boldsymbol{2}\vec{\boldsymbol{c}}$$

tangenta: $\vec{\boldsymbol{r}}(\boldsymbol{u}) = \vec{r}(t=-1) + \vec{r}'(t=-1)\cdot u = \left(\vec{\boldsymbol{a}} - \vec{\boldsymbol{b}} + \vec{\boldsymbol{c}}\right) + \left(\vec{\boldsymbol{b}} - \boldsymbol{2}\vec{\boldsymbol{c}}\right)\cdot \boldsymbol{u}$

c) $2 = 2t^2$, $0 = 1 - t$, $5 = 3 + 2t^2$ iz čega prozilazi da je $t = 1$

vektor smjera:

$$\vec{\boldsymbol{r}}'(\boldsymbol{t}=\boldsymbol{1}) = \boldsymbol{4}\vec{\boldsymbol{\imath}} - \vec{\boldsymbol{j}} + \boldsymbol{4}\vec{\boldsymbol{k}}$$

tangenta: $\vec{\boldsymbol{r}}(\boldsymbol{u}) = \vec{r}(t=1) + \vec{r}'(t=1)\cdot u = \left(\boldsymbol{2} + \boldsymbol{5}\vec{\boldsymbol{k}}\right) + \left(\boldsymbol{4}\vec{\boldsymbol{\imath}} - \vec{\boldsymbol{j}} + \boldsymbol{4}\vec{\boldsymbol{k}}\right)\cdot \boldsymbol{u}$

9. Ako uzmemo da je $x = at$, onda kvadriranjem dobijemo $y = (at)^2 = a^2t^2 = bt^2$

pa imamo: $\boldsymbol{y} = \boldsymbol{b}\left(\frac{\boldsymbol{x}}{\boldsymbol{a}}\right)^{\boldsymbol{2}} = \frac{\boldsymbol{b}}{\boldsymbol{a}^2}\boldsymbol{x}^{\boldsymbol{2}}$

10.

$$\vec{r}(t) = t\vec{\imath} + (1+t^2)\vec{\jmath}$$

$$\vec{r}'(t) = \vec{\imath} + 2t\vec{\jmath}$$

a) ako su okomiti:

$$cos\varphi = 0 = \frac{\vec{r}(t)\cdot\vec{r}'(t)}{|\vec{r}(t)|\cdot|\vec{r}'(t)|} \rightarrow \vec{r}(t)\cdot\vec{r}'(t) = \boldsymbol{0}$$

$$[t\vec{\imath} + (1+t^2)\vec{\jmath}]\cdot[\vec{\imath} + 2t\vec{\jmath}] = t + (2t + 2t^3) = 0$$

$$2t^3 + 3t = 0 \rightarrow t = 0$$

S obzirom da je $x(t) = t$, onda je $x = 0$, pošto je $y(t) = (1+t^2)$, onda je $y = 1$.

$$\boldsymbol{T}(\boldsymbol{0},\boldsymbol{1})$$

b) ako su istog smjera, onda je $\vec{r}'(t) = \vec{r}''(t)$

$$\vec{\imath} + 2t\vec{\jmath} = 2\vec{\jmath} \rightarrow t = 1$$

$$\boldsymbol{T}(\boldsymbol{1},\boldsymbol{2})$$

c) ako su suprotnog smjera, onda je $\vec{r}'^{(t)} = -\vec{r}''(t)$

$$\vec{\imath} + 2t\vec{\jmath} = -2\vec{\jmath} \to t = -1$$

$$\boldsymbol{T(-1, 2)}$$

11. a) $y^2 = x - 1$. uzmemo da je $y = t$, onda je $x = 1 + y^2 = 1 + t^2$

pa je

$$\boldsymbol{\vec{r}(t) = (1 + t^2)\vec{\imath} + t\vec{\jmath}}$$

b) trenutno ne znam ;D

c) $y^2 = x^3, y \leq 0 \to y = -\sqrt{x^3}$

uzmemo da je $x = t$, pa je onda $y = -\sqrt{t^3}$

$$\boldsymbol{\vec{r}(t) = t\vec{\imath} - \sqrt{t^3}\vec{\jmath}}$$

d) $y^3 = x^2$

Uzmemo da je $x = t^3$, pa je onda $y^3 = t^6 \to y = t^2$

$$\boldsymbol{\vec{r}(t) = t^3\vec{\imath} + t^2\vec{\jmath}}$$

12. $\vec{r}_1(t) = e^t\vec{\imath} + 2\sin\left(t + \frac{\pi}{2}\right)\vec{\jmath} + (t^2 - 2)\vec{k}$, $\vec{r}_2(t) = u\vec{\imath} + 2\vec{\jmath} + (u^2 - 3)\vec{k}$

nađemo prvo sjecište – izjednačimo komponente

$$e^t = u$$

$$2\sin\left(t + \frac{\pi}{2}\right) = 2$$

$$t^2 - 2 = u^2 - 3$$

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također vrijedi da su prve jednake (u točki u kojoj se sijeku krivulje):

$$e^t = 1$$

$$2\sin\left(t + \frac{\pi}{2}\right) = 2$$

$$2t = 0$$

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zaključujemo: $t = 0 \to u = 1$

pa je točka presjecišta $\boldsymbol{T(1,2,-2)}$

odredimo vektore smjera prve i druge krivulje:

$$\vec{r}_1'(t=0) = e^t\vec{\imath} + 2\cos\left(t+\frac{\pi}{2}\right)\vec{\jmath} + 2t\vec{k} = \vec{\imath}$$

$$\vec{r}_2'(u=1) = \vec{\imath} + 2u\vec{k} = \vec{\imath} + 2\vec{k}$$

vrijedi:

$$\boldsymbol{\cos\varphi} = \frac{\vec{r}_1'(t=0)\cdot\vec{r}_2'(u=1)}{|\vec{r}_1'(t=0)|\cdot|\vec{r}_2'(u=1)|} = \frac{1}{\sqrt{5}} = \boldsymbol{\frac{\sqrt{5}}{5}}$$