VEKTOR SMJERA TANGENTE

$\boldsymbol{r}(t) = x(t)\boldsymbol{i} + y(t)\boldsymbol{j} + z(t)\boldsymbol{k}$
$\boldsymbol{a}(t) = a_1\boldsymbol{i} + a_2\boldsymbol{j} + a_3\boldsymbol{k}$
$\boldsymbol{a}(t) = \log_{t\to 0}\boldsymbol{r}(t) = \log_{t\to 0} x(t)\,\boldsymbol{i} + \log_{t\to 0} y(t)\,\boldsymbol{j} + \log_{t\to 0} z(t)\,\boldsymbol{k}$
$\boldsymbol{s}(t) = \mathbf{r}'(\mathrm{t}) = x'(t)\boldsymbol{i} + y'(t)\boldsymbol{j} + z'(t)\boldsymbol{k}$

JEDNADŽBA TANGENTE NA KRIVULJU

$\boldsymbol{t}(t) = \mathbf{r}(\mathrm{t}) + \mathrm{u}\big(\mathbf{s}(\mathrm{t})\big) = \mathbf{r}(\mathrm{t}) + \mathbf{r}'(\mathrm{t})$

GRADIJENT SKALARNOG POLJA

$\varphi(x, y, z) => derivabilno\ skalarno\ polje$

$$grad\ \varphi = \nabla\varphi = \left(\frac{\partial}{\partial x}\boldsymbol{i} + \frac{\partial}{\partial y}\boldsymbol{j} + \frac{\partial}{\partial z}\boldsymbol{k}\right)\varphi$$

LAPLACEOV OPERATOR

$$\Delta f = \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}\right)f$$

$\to harmonijska\ fja{:}\ \Delta f = 0$

VEKTOR SMJERA NORMALE NA PLOHU JE GRAD U TOČKI

$\boldsymbol{n} = (\nabla f)_T$

VEKTOR SMJERA NAJBRŽEG RASTA

$\nabla f(r) = f'(r)\boldsymbol{r_0}$

USMJERENA DERIVACIJA SKALARNOG POLJA

$$\frac{\partial f}{\partial \boldsymbol{s}} = \nabla f\boldsymbol{s_0} = (\boldsymbol{s_0}\nabla)f = \left(s_{0x}\frac{\partial}{\partial x} + s_{0y}\frac{\partial}{\partial y} + s_{0z}\frac{\partial}{\partial z}\right)f$$

USMJERENA DERIVACIJA RADIJALNOG POLJA

$\boldsymbol{r} = x\boldsymbol{i} + y\boldsymbol{j} + z\boldsymbol{k}$

$$\frac{\partial f(r)}{\partial \boldsymbol{s}} = f'(\boldsymbol{s_0}\,\boldsymbol{r})$$

STRUJNICE VEKTORSKOG POLJA

$\boldsymbol{v}(x, y, z) = v_1(x, y, z)\boldsymbol{i} + v_2(x, y, z)\boldsymbol{j} + v_3(x, y, z)\boldsymbol{k}$

$$\frac{dx}{v_1} = \frac{dy}{v_2} = \frac{dz}{v_3}$$

DIVERGENCIJA VEKTORSKOG POLJA

$$div\ \boldsymbol{v} = \frac{\partial v_1}{\partial x} + \frac{\partial v_2}{\partial y} + \frac{\partial v_3}{\partial z}$$

$\text{solenoidno} -> \ div\ \boldsymbol{v} = 0$

ROTACIJA VEKTORSKOG POLJA

$$\mathrm{rot}\,\boldsymbol{v} = \begin{vmatrix} \boldsymbol{i} & \boldsymbol{j} & \boldsymbol{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ v_1 & v_2 & v_3 \end{vmatrix}$$

$rot\ \boldsymbol{r} = 0$

$\to potencijalno\ polje{:}\ \ rot\ \boldsymbol{v} = 0$

POTENCIJAL VEKTORSKOG POLJA

$$p(x,y,z) = \int_{x_0}^{x} v_1(t,y,z)\,dt + \int_{y_0}^{y} v_2(x_0,t,z)\,dt + \int_{z_0}^{z} v_3(x_0,y_0,t)\,dt + c$$

$$\int_{(x_0,y_0,z_0)}^{(x_1,y_1,z_1)} v_1\,dx + v_2\,dy + v_3\,dz = p(x_1,y_1,z_1) - p(x_0,y_0,z_0)$$

KRIVULJNI INTEGRAL SKALARNOG POLJA

$\int_\Gamma f\,ds = \int_a^b f(x(t),y(t),z(t))\sqrt{(x'(t))^2 + (y'(t))^2 + (z'(t))^2}\,dt$

$\rightarrow duljina\ luka\ krivulje\ f\big(x(t),y(t),z(t)\big) = 1$

GREENOV TEOREM

Neka je $\Omega \subset R^2$ otvoren i povezan skup omeđen pozivitno orijentiranom jednostavno zatvorenom krivuljom Γ, te neka je $\boldsymbol{f}(x,y) = f_1(x,y)\,\boldsymbol{i} + f_2(x,y)\boldsymbol{j}$ glatko vektorsko polje. Tada vrijedi Greenova formula

$$\oint_\Gamma f_1(x,y)dx + f_2(x,y)dy = \iint_\Omega \left[\frac{\partial f_2(x,y)}{\partial x} - \frac{\partial f_1(x,y)}{\partial y}\right] dxdy$$

Dokaz! Dovoljno je pokazati da vrijede sljedeće jednakosti:

$$\oint_\Gamma f_1(x,y)dx = -\iint_\Omega \left[\frac{\partial f_1(x,y)}{\partial y}\right] dxdy$$

$$\oint_\Gamma f_2(x,y)dy = \iint_\Omega \left[\frac{\partial f_2(x,y)}{\partial x}\right] dxdy$$

TEOREM O DIVERGENCIJI

Neka je ∑ zatvorena ploha koja omeđuje tijelo V, te neka je $\boldsymbol{f}(x,y,z) = f_1(x,y,z)\,\boldsymbol{i} + f_2(x,y,z)\boldsymbol{j} + f_3(x,y,z)\boldsymbol{k}$ glatko vektorsko polje definirano na nekom podskupo od $R^3$ koje sadrži plohu ∑ tada je

$$\iint_\Sigma \boldsymbol{f} \cdot d\boldsymbol{S} = \iiint_V div\,\boldsymbol{f}\,dV = \iiint_V div\,\boldsymbol{f}\,dxdydz$$

STOKESOV TEOREM

Neka je ∑ orijentabilna ploha u $R^3$ čiji je rub određen jednostavnom zatvorenom krivuljom Γ, te neka je $\boldsymbol{f} = f_1(x,y,z)\boldsymbol{i} + f_2(x,y,z)\boldsymbol{j} + f_3(x,y,z)\boldsymbol{k}$ glatko vektorsko polje definirano na nekom skupu od $R^3$ koje sadrži plohu ∑. Tada vrijedi

$$\oint_\Gamma \boldsymbol{f} \cdot d\boldsymbol{r} = \iint_\Sigma rot\boldsymbol{f} \cdot d\boldsymbol{S}$$

Pri čemu je **n** jedinični vektor na plohu ∑ koji je usmjeren tako da je, gledano s vrha vektora **n**, krivulja Γ orijentirana pozitivno.