ZI-09

① $f(x,y,z) = \sqrt{2}\cdot z^2 - 2xy$

$\vec{s} = \vec{i} - \vec{j} - \sqrt{2}\vec{k}$

$A(-1,2,1)$

$B(1,3,2)$

$$\vec{s}_0 = \frac{\vec{i}-\vec{j}-\sqrt{2}\vec{k}}{\sqrt{4}} = \frac{1}{2}\vec{i} - \frac{1}{2}\vec{j} - \frac{\sqrt{2}}{2}\vec{k}$$

$$\frac{\partial f}{\partial \vec{s}} = \vec{s}_0 \cdot \operatorname{grad} f = \left(\frac{1}{2}\vec{i} - \frac{1}{2}\vec{j} - \frac{\sqrt{2}}{2}\vec{k}\right)\cdot\left(-2y\vec{i} - 2x\vec{j} + 2\sqrt{2}z\vec{k}\right) =$$

$$= -y + x - 2z = 0$$

$$x - y - 2z = 0 \quad (1)$$

$$\frac{x-x_A}{x_B-x_A} = \frac{y-y_A}{y_B-y_A} = \frac{z-z_A}{z_B-z_A} = t$$

$$\frac{x+1}{1+1} = \frac{y-2}{3-2} = \frac{z-1}{2-1} = t$$

$$\frac{x+1}{2} = \frac{y-2}{1} = \frac{z-1}{1} = t$$

$$x = 2t-1$$

$$x = 2t-1$$
$$y = t+2 \quad (2)$$
$$z = t+1$$

$(2) \to (1)$

$$2t - 1 - t - 2 - 2(t+1) = 0$$

$$\boxed{t = -5}$$

$$\boxed{x = -11} \qquad \boxed{T(-11,-3,-4)}$$

$$\boxed{y = -3}$$

$$\boxed{z = -4}$$

(2) $$\int_\Gamma \frac{x}{\sqrt[4]{x^2+y^2}}\,ds \,, \quad \Gamma \ldots \quad r = 1+\cos\varphi \,, \quad \varphi \in [0, 2\pi]$$

$$\left.\begin{aligned} x &= r\cos\varphi \\ y &= r\sin\varphi \end{aligned}\right\} x^2+y^2 = r^2$$

$$x = (1+\cos\varphi)\cos\varphi$$

$$ds = \sqrt{r^2+(r')^2}\,d\varphi$$

$$ds = \sqrt{1+2\cos\varphi+\cos^2\varphi+\sin^2\varphi}\,d\varphi$$

$$ds = \sqrt{1+1+2\cos\varphi}\,d\varphi$$

$$ds = \sqrt{2+2\cos\varphi}\,d\varphi$$

$$ds = \sqrt{2}\sqrt{1+\cos\varphi}\,d\varphi$$

$$I = \int_0^{2\pi} \frac{(1+\cos\varphi)\cos\varphi}{\sqrt[4]{r^2}} \cdot \sqrt{2}\sqrt{1+\cos\varphi}\,d\varphi =$$

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$$= \sqrt{2}\int_0^{2\pi} \frac{\cos\varphi\,(1+\cos\varphi)\sqrt{1+\cos\varphi}}{\sqrt{r}}\,d\varphi =$$

$$= \sqrt{2}\int_0^{2\pi} \frac{\cos\varphi\,(1+\cos\varphi)\cancel{\sqrt{1+\cos\varphi}}}{\cancel{\sqrt{1+\cos\varphi}}}\,d\varphi =$$

$$= \sqrt{2}\underbrace{\int_0^{2\pi}\cos\varphi\,d\varphi}_{0} + \sqrt{2}\int_0^{2\pi}\cos^2\varphi\,d\varphi =$$

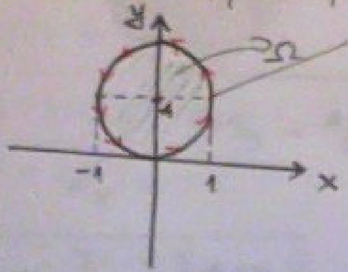
$$= \sqrt{2}\int_0^{2\pi}\frac{1+\cos 2\varphi}{2}\,d\varphi = \frac{\sqrt{2}}{2}\cdot 2\pi + \frac{\sqrt{2}}{2}\underbrace{\int_0^{2\pi}\cos 2\varphi\,d\varphi}_{0} =$$

$$= \pi\sqrt{2}$$

(3) a) $\oint_{\gamma} \vec{f}\,d\vec{r} = \oint_{\Gamma} f_1\,dx + f_2\,dy = \iint_{\Omega}\left(\frac{\partial f_2}{\partial x} - \frac{\partial f_1}{\partial y}\right) dx\,dy$

b) $\oint_c (\sin x + y)\,dx + (x^2+\cos y)\,dy = ?$

c … $A(0,1) \Rightarrow$ kružnica, $r = 1$ $\quad x^2+(y-1)^2=1$



$P_\Omega = r^2\pi = \pi$

$$\oint_c = \iint_\Omega (2x-1)\,dx\,dy = 2\iint_\Omega x\,dx\,dy - \underbrace{\iint_\Omega dx\,dy}_{\pi} =$$

$$= 2\int_{-1}^{1} x\,dx \int_{1-\sqrt{1-x^2}}^{1+\sqrt{1-x^2}} dy - \pi = 2\int_{-1}^{1} x\left(\cancel{1}+\sqrt{1-x^2}-\cancel{1}+\sqrt{1-x^2}\right)dx - \pi =$$

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$$= 4\int_{-1}^{1} x\sqrt{1-x^2}\,dx - \pi = \left[\begin{array}{c} 1-x^2 = t \\ -2x\,dx = dt \\ x\,dx = -\frac{dt}{2} \end{array}\right] =$$

$$= -2\underbrace{\int_0^0 \sqrt{t}\,dt}_{0} - \pi = -\pi$$

4 a) $\vec{a} = e^{xyz}(yz\vec{i} + xz\vec{j} + xy\vec{k})$

$$\operatorname{rot}\vec{a} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ e^{xyz}yz & e^{xyz}xz & e^{xyz}xy \end{vmatrix} =$$

$$\cancel{= \vec{i}\left(e^{xyz}y + e^{xyz}x^2y^2z^2 - e^{xyz}\right.}$$

$$= \vec{i}\left(e^{xyz}x^2yz + e^{xyz}x - e^{xyz}x^2yz - e^{xyz}x\right) +$$
$$- \vec{j}\left(e^{xyz}xy^2z + e^{xyz}y - e^{xyz}xy^2z - e^{xyz}y\right) +$$
$$+ \vec{k}\left(e^{xyz}xyz^2 + e^{xyz}z - e^{xyz}xyz^2 - e^{xyz}z\right) =$$
$$= \vec{0}$$

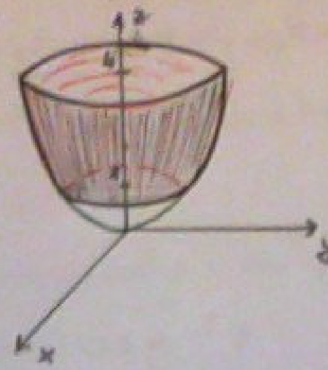
$$p(x,y,z) = \int_{x_0=0}^{x} e^{xyz}yz\,dx + \underbrace{\int_{y_0=0}^{y} e^{x_0yz}x_0z\,dy}_{0} + \underbrace{\int_{z_0=0}^{z} e^{x_0y_0z}x_0y_0\,dz}_{0} =$$

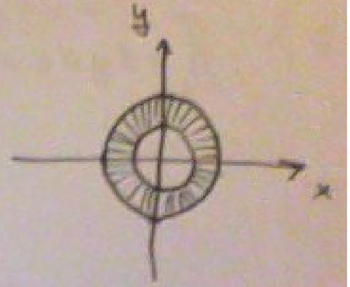
$$= yz\int_0^x e^{xyz}dx = \cancel{yz}\,\frac{1}{\cancel{yz}}\,e^{xyz}\Big|_0^x + C =$$

$$= e^{xyz} - e^0 + C = e^{xyz} - 1 + C = e^{xyz} + K$$

b) s obzirom da je ovo zatvorena krivulja i iz def. potencijalnosti slijedi da je 0 rješenje
Poč. točka = završna točka !!!

(5) $\iint_S (x^2+y^2)\,dS$, $S \ldots \; z = x^2+y^2$, $1 \le z \le 4$





$$dS = \sqrt{1+\left(\frac{\partial z}{\partial x}\right)^2+\left(\frac{\partial z}{\partial y}\right)^2}\,dx\,dy$$

$$dS = \sqrt{1+4x^2+4y^2}\,dx\,dy$$

$$dS = \sqrt{1+4(x^2+y^2)}\,dx\,dy$$

$$I = \int_0^{2\pi} d\varphi \int_1^2 r\cdot r^2\cdot\sqrt{1+4r^2}\,dr =$$

$$= \int_0^{2\pi} d\varphi \int_1^2 r^3\sqrt{1+4r^2}\,dr = \left[\begin{array}{ll} 1+4r^2 = t \,|\,' & r^2 = \frac{t-1}{4} \\ 8r\,dr = dt & \\ r\,dr = \frac{dt}{8} & \end{array}\right] =$$

$$= \int_0^{2\pi} d\varphi \int_5^{17} \frac{t-1}{4}\sqrt{t}\,\frac{dt}{8} =$$

$$= \frac{1}{32}\int_0^{2\pi} d\varphi \int_5^{17} (t\sqrt{t}-\sqrt{t})\,dt = \ldots =$$

$$= \frac{\pi}{16}\left[\frac{2}{5}\left(17^2\sqrt{17}-25\sqrt{5}\right)-\frac{2}{3}\left(17\sqrt{17}-5\sqrt{5}\right)\right]$$