MATEMATIKA 3E

Zadaci za vježbu

Skalarna i vektorska polja – 1. dio

by Vedax/Rock n Rolla

2. Skalarna i vektorska polja – 1. dio

1. Izračunajte vrijednost gradijenta i apsolutnu vrijednost gradijenta za polje

$$f(x, y, z) = x^3 + y^3 + z^3 - 3xyz$$

u točki $A(2,1,1)$. U kojim je točkama grad f okomit na z-os, a u kojim se točkama poništava?

Rješenje:

$$\text{grad}\, f = \frac{\partial f}{\partial x}\vec{\imath} + \frac{\partial f}{\partial y}\vec{\jmath} + \frac{\partial f}{\partial z}\vec{k} = (3x^2 - 3yz)\vec{\imath} + (3y^2 - 3xz)\vec{\jmath} + (3z^2 - 3xy)\vec{k}$$

$$\text{grad}\, f\,\Big|_A = 9\vec{\imath} - 3\vec{\jmath} - 3\vec{k}$$

$$\left|\text{grad}\, f\,\Big|_A\right| = \sqrt{9^2 + (-3)^2 + (-3)^2} = \sqrt{81 + 9 + 9} = \sqrt{99} = 3\sqrt{11}$$

Ako su dva vektora okomita, kut $\varphi = 90°$, pa je skalarni produkt:

$$\vec{a} \cdot \vec{b} = |\vec{a}||\vec{b}|\cos\varphi \leftrightarrow \cos 90° = \frac{\vec{a} \cdot \vec{b}}{|\vec{a}||\vec{b}|} \leftrightarrow \vec{a} \cdot \vec{b} = 0$$

Za os z uzimamo jedinični vektor $\vec{k}$, pa imamo:

$$\text{grad}\, f \cdot \vec{k} = 0$$

$$\left((3x^2 - 3yz)\vec{\imath} + (3y^2 - 3xz)\vec{\jmath} + (3z^2 - 3xy)\vec{k}\right) \cdot \vec{k} = 0$$

$$3z^2 - 3xy = 0$$

$$z^2 = xy$$

Ako se poništava, vrijedi grad $f = \vec{0}$, odnosno:

$$3x^2 - 3yz = 0, \qquad 3y^2 - 3xz = 0, \qquad 3z^2 - 3xy,$$

iz čega proizlazi da je $x = y = z$.

2. Izračunajte gradijent skalarne funkcije $f = \vec{a} \cdot \vec{r}$, gdje je $\vec{a}$ konstantan vektor, a $\vec{r}$ radij vektor.

Rješenje:

Vrijedi da je $\vec{r} = x\vec{\imath} + y\vec{\jmath} + z\vec{k}$, a ako je vektor $\vec{a}$ konstanta vektor, pretpostavimo da je oblika $\vec{a} = a_1\vec{\imath} + a_2\vec{\jmath} + a_3\vec{k}$, gdje su koeficijenti uz jedinične vektore konstante.

Funkcija f jednaka je skalarnom produktu danih vektora:

$$f = \vec{a} \cdot \vec{r} = \left(a_1\vec{\imath} + a_2\vec{\jmath} + a_3\vec{k}\right)\left(x\vec{\imath} + y\vec{\jmath} + z\vec{k}\right) = a_1x + a_2y + a_3z$$

Pa je gradijent zadane skalarne funkcije jednak:

$$\operatorname{grad} f = \frac{\partial f}{\partial x}\vec{\imath} + \frac{\partial f}{\partial y}\vec{\jmath} + \frac{\partial f}{\partial z}\vec{k} = a_1\vec{\imath} + a_2\vec{\jmath} + a_3\vec{k} = \vec{a}$$

3. Izračunajte usmjerenu derivaciju polja $f(x, y, z) = xyz$ u točki $A(5,1,2)$, a u smjeru vektora $\overrightarrow{AB}$, gdje je $B(9,4,14)$.

Rješenje:

Označimo vektor $\overrightarrow{AB}$ sa $\vec{a}$ (radi lakšeg zapisa). Odredimo taj vektor:

$$\vec{a} = (9-5)\vec{\imath} + (4-1)\vec{\jmath} + (14-2)\vec{k} = 4\vec{\imath} + 3\vec{\jmath} + 12\vec{k}$$

Usmjerena derivacija skalarnog polja je $\left(\vec{a}_0 = \frac{\vec{a}_0}{|\vec{a}|}\right)$:

$$\frac{\partial f}{\partial \vec{a}} = \vec{a}_0 \operatorname{grad} f = \frac{4\vec{\imath} + 3\vec{\jmath} + 12\vec{k}}{\sqrt{4^2 + 3^2 + 12^2}}\left(yz\vec{\imath} + xz\vec{\jmath} + xy\vec{k}\right) = \frac{\left(4\vec{\imath} + 3\vec{\jmath} + 12\vec{k}\right)\left(yz\vec{\imath} + xz\vec{\jmath} + xy\vec{k}\right)}{13}$$

U točki A, onda je jednaka:

$$\left.\frac{\partial f}{\partial \vec{a}}\right|_A = \frac{\left(4\vec{\imath} + 3\vec{\jmath} + 12\vec{k}\right)\left(2\vec{\imath} + 10\vec{\jmath} + 5\vec{k}\right)}{13} = \frac{8 + 30 + 60}{13} = \frac{98}{13}$$

4. Zadano je vektorsko polje $\vec{a} = x^2\vec{\imath} + xyz\vec{\jmath} + z^2\vec{k}$, te vektori $\vec{s} = \vec{\imath} + \vec{\jmath} + \vec{k}$ i $\vec{b} = \sqrt{3}\left(2\vec{\imath} + \vec{\jmath} + 2\vec{k}\right)$.

a) Izračunajte $\frac{\partial \vec{a}}{\partial \vec{s}}$.

b) Odredite točku T u prostoru, za koju vrijedi $\frac{\partial \vec{a}}{\partial \vec{s}} = \vec{b}$.

Rješenje:

a) Kao prvo, vrijedi $\vec{s}_0 = \frac{\vec{s}}{|\vec{s}|} = \frac{\vec{\imath}+\vec{\jmath}+\vec{k}}{\sqrt{3}} = \frac{1}{\sqrt{3}}\vec{\imath} + \frac{1}{\sqrt{3}}\vec{\jmath} + \frac{1}{\sqrt{3}}\vec{k} = s_{01}\vec{\imath} + s_{02}\vec{\jmath} + s_{03}\vec{k}$.

Zatim, usmjerena derivacija vektorskog polja je:

$$\frac{\partial \vec{a}}{\partial \vec{s}} = \left(s_{01}\frac{\partial}{\partial x} + s_{02}\frac{\partial}{\partial y} + s_{03}\frac{\partial}{\partial z}\right)\vec{a} = \frac{1}{\sqrt{3}}\left(\frac{\partial}{\partial x} + \frac{\partial}{\partial y} + \frac{\partial}{\partial z}\right)\left(x^2\vec{\imath} + xyz\vec{\jmath} + z^2\vec{k}\right) =$$

$$= \frac{1}{\sqrt{3}}\left(2x\vec{\imath} + (yz + xz + xy)\vec{\jmath} + 2z\vec{k}\right)$$

b) Vrijedi:

$$\frac{1}{\sqrt{3}}\left(2x\vec{\imath} + (yz + xz + xy)\vec{\jmath} + 2z\vec{k}\right) = \sqrt{3}\left(2\vec{\imath} + \vec{\jmath} + 2\vec{k}\right)$$

$$2x\vec{\imath} + (yz + xz + xy)\vec{\jmath} + 2z\vec{k} = 6\vec{\imath} + 3\vec{\jmath} + 6\vec{k}$$

$$2x = 6, \qquad yz + xz + xy = 3, \qquad 2z = 6$$

Iz prve i zadnje jednadžbe dobijemo da je $x = 3$ i $z = 3$, pa kad to uvrstimo u drugu jednadžbu dobijemo $y = -1$.

Točka T za koju vrijedi $\frac{\partial \vec{a}}{\partial \vec{s}} = \vec{b}$ je $T(3, -1, 3)$.

5. Izračunajte usmjerenu derivaciju radij vektora $\vec{r}$ u smjeru zadanog vektora $\vec{a}$.

Rješenje:

Vrijedi sljedeće:

$$\vec{r} = x\vec{\imath} + y\vec{\jmath} + z\vec{k}$$

$$\vec{a}_0 = \frac{\vec{a}}{|\vec{a}|} = a_{01}\vec{\imath} + a_{02}\vec{\jmath} + a_{03}\vec{k}$$

Pa imamo:

$$\frac{\partial\vec{r}}{\partial\vec{a}} = \left(a_{01}\frac{\partial}{\partial x} + a_{02}\frac{\partial}{\partial y} + a_{03}\frac{\partial}{\partial z}\right)\left(x\vec{\imath} + y\vec{\jmath} + z\vec{k}\right) = a_{01}\vec{\imath} + a_{02}\vec{\jmath} + a_{03}\vec{k} = \vec{a}_0$$

6. Izračunajte $\operatorname{div}\vec{a}$ i $\operatorname{rot}\vec{a}$ ako je $\vec{a} = xz\vec{\imath} + y\vec{k}$.

Rješenje:

$$\operatorname{div}\vec{a} = \frac{\partial}{\partial x}(xz) + \frac{\partial}{\partial y}(0) + \frac{\partial}{\partial z}(y) = z$$

$$\operatorname{rot}\vec{a} = \begin{vmatrix} \vec{\imath} & \vec{\jmath} & \vec{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ xz & 0 & y \end{vmatrix} = \vec{\imath}\left(\frac{\partial}{\partial y}(y) - \frac{\partial}{\partial z}(0)\right) - \vec{\jmath}\left(\frac{\partial}{\partial x}(y) - \frac{\partial}{\partial z}(xz)\right) + \vec{k}\left(\frac{\partial}{\partial x}(0) - \frac{\partial}{\partial y}(xz)\right) =$$

$$= \vec{\imath} + x\vec{\jmath}$$

7. Neka je $\vec{r} = x\vec{\imath} + y\vec{\jmath} + z\vec{k}$, $r = |\vec{r}| = \sqrt{x^2 + y^2 + z^2}$ i $\vec{r}_0 = \frac{\vec{r}}{r}$. Izračunajte:

a) $\operatorname{grad} r$,
b) $\operatorname{div} \vec{r}$,
c) $\operatorname{rot} \vec{r}$,
d) $\operatorname{grad} f(r)$, gdje je $f(r)$ derivabilna funkcija,
e) $\operatorname{div}(f(r)\vec{r})$,
f) $\operatorname{rot}(f(r)\vec{r})$,
g) $\operatorname{div} \vec{r}_0$
h) $\frac{\partial f(r)}{\partial \vec{s}}$, gdje je $\vec{s}$ zadani vektor,
i) $\frac{\partial (f(r)\vec{r})}{\partial \vec{s}}$.

Rješenje:

a)

$$\operatorname{grad} r = \frac{\partial}{\partial x}\left(\sqrt{x^2 + y^2 + z^2}\right)\vec{\imath} + \frac{\partial}{\partial y}\left(\sqrt{x^2 + y^2 + z^2}\right)\vec{\jmath} + \frac{\partial}{\partial y}\left(\sqrt{x^2 + y^2 + z^2}\right)\vec{k} =$$

$$= \frac{x}{\sqrt{x^2 + y^2 + z^2}}\vec{\imath} + \frac{x}{\sqrt{x^2 + y^2 + z^2}}\vec{\jmath} + \frac{x}{\sqrt{x^2 + y^2 + z^2}}\vec{k} = \frac{x}{r}\vec{\imath} + \frac{x}{r}\vec{\jmath} + \frac{x}{r}\vec{k} = \frac{1}{r}\left(x\vec{\imath} + y\vec{\jmath} + z\vec{k}\right) =$$

$$= \frac{\vec{r}}{r} = \vec{r}_0$$

b)

$$\operatorname{div} \vec{r} = \frac{\partial}{\partial x}(x) + \frac{\partial}{\partial y}(y) + \frac{\partial}{\partial z}(z) = 1 + 1 + 1 = 3$$

c)

$$\operatorname{rot} \vec{r} = \begin{vmatrix} \vec{\imath} & \vec{\jmath} & \vec{k} \\ \dfrac{\partial}{\partial x} & \dfrac{\partial}{\partial y} & \dfrac{\partial}{\partial z} \\ x & y & z \end{vmatrix} = 0 \cdot \vec{\imath} + 0 \cdot \vec{\jmath} + 0 \cdot \vec{k} = \vec{0}$$

d) (teorem 2., str. 27)

$$\operatorname{grad} f(r) = f'(r) \operatorname{grad} r = f'(r)\vec{r}_0$$

e) (teorem 7. (2), str. 33)

$$\operatorname{div}(f(r)\vec{r}) = \vec{r}\,\operatorname{grad} f(r) + f(r)\operatorname{div}\vec{r} = \vec{r}f'(r)\vec{r}_0 + 3f(r) =$$

$$= \left[\vec{r}_0 = \frac{\vec{r}}{r} \leftrightarrow \vec{r}_0\vec{r} = \frac{\vec{r}\vec{r}}{r} = \frac{|\vec{r}|\cdot|\vec{r}|\cos 0}{r} = \frac{r\cdot r}{r} = r\right] =$$

$$= rf'(r) + 3f(r)$$

f) (teorem 10. (2), str. 35)

$$\operatorname{rot}(f(r)\vec{r}) = \operatorname{grad} f(r)\times\vec{r} + f(r)\operatorname{rot}\vec{r} = f'(r)\vec{r}_0\times\vec{r} + f(r)\cdot\vec{0} =$$

$$= \left[\vec{r}_0\times\vec{r} = \frac{\vec{r}\times\vec{r}}{r} = \frac{|\vec{r}|\cdot|\vec{r}|\sin 0}{r} = 0\right] = \vec{0}$$

g)

$$\operatorname{div}\vec{r}_0 = \operatorname{div}\left(\frac{x}{r}\vec{\imath} + \frac{x}{r}\vec{\jmath} + \frac{x}{r}\vec{k}\right) = \frac{\partial}{\partial x}\left(\frac{x}{r}\right) + \frac{\partial}{\partial y}\left(\frac{y}{r}\right) + \frac{\partial}{\partial z}\left(\frac{z}{r}\right) = \frac{1}{r} + \frac{1}{r} + \frac{1}{r} = \frac{3}{r}$$

h)

$$\frac{\partial f(r)}{\partial\vec{s}} = \vec{s}_0\operatorname{grad} f(r) = \vec{s}_0 f'(r)\vec{r}_0 = (\vec{s}_0\cdot\vec{r}_0)f'(r)$$

gdje je $\vec{s}_0 = \frac{\vec{s}}{|\vec{s}|}$.

i)

$$\frac{\partial(f(r)\vec{r})}{\partial\vec{s}} = \left(s_{01}\frac{\partial}{\partial x} + s_{02}\frac{\partial}{\partial y} + s_{03}\frac{\partial}{\partial z}\right)(f(r)\vec{r}) =$$

$$= s_{01}\frac{\partial}{\partial x}\Big(f(r)\big(x\vec{\imath} + y\vec{\jmath} + z\vec{k}\big)\Big) + s_{02}\frac{\partial}{\partial y}\Big(f(r)\big(x\vec{\imath} + y\vec{\jmath} + z\vec{k}\big)\Big) + s_{03}\frac{\partial}{\partial z}\Big(f(r)\big(x\vec{\imath} + y\vec{\jmath} + z\vec{k}\big)\Big) =$$

$$= s_{01}\left[\frac{f'(r)x}{r}\big(x\vec{\imath} + y\vec{\jmath} + z\vec{k}\big) + f(r)\vec{\imath}\right] + s_{02}\left[\frac{f'(r)y}{r}\big(x\vec{\imath} + y\vec{\jmath} + z\vec{k}\big) + f(r)\vec{\jmath}\right] +$$

$$+ s_{03}\left[\frac{f'(r)z}{r}\big(x\vec{\imath} + y\vec{\jmath} + z\vec{k}\big) + f(r)\vec{k}\right] =$$

$$= f(r)\big(s_{01}\vec{\imath} + s_{02}\vec{\jmath} + s_{03}\vec{k}\big) + f'(r)\big(x\vec{\imath} + y\vec{\jmath} + z\vec{k}\big)\left(\frac{s_{01}x}{r} + \frac{s_{02}y}{r} + \frac{s_{03}z}{r}\right) =$$

$$= \vec{s}_0 f(r) + \vec{r}(\vec{s}_0 \cdot \vec{r}_0) f'(r)$$

8. Zadano je vektorsko polje $\vec{a} = x^2\vec{\imath} + xyz\vec{\jmath} + z^2\vec{k}$, te vektori $\vec{s} = \vec{\imath} + \vec{\jmath} + \vec{k}$. Izračunajte

$$\frac{\partial(\operatorname{rot}\vec{a})}{\partial\vec{s}}.$$

Rješenje:

$$\operatorname{rot}\vec{a} = \begin{vmatrix} \vec{\imath} & \vec{\jmath} & \vec{k} \\ \dfrac{\partial}{\partial x} & \dfrac{\partial}{\partial y} & \dfrac{\partial}{\partial z} \\ x^2 & xyz & z^2 \end{vmatrix} = -xy\vec{\imath} + yz\vec{\jmath}$$

$$\vec{s}_0 = \frac{\vec{s}}{|\vec{s}|} = \frac{\vec{\imath} + \vec{\jmath} + \vec{k}}{\sqrt{3}} = \frac{1}{\sqrt{3}}\vec{\imath} + \frac{1}{\sqrt{3}}\vec{\jmath} + \frac{1}{\sqrt{3}}\vec{k}$$

$$\frac{\partial(\operatorname{rot}\vec{a})}{\partial\vec{s}} = \frac{1}{\sqrt{3}}\left(\frac{\partial}{\partial x} + \frac{\partial}{\partial y} + \frac{\partial}{\partial z}\right)(-xy\vec{\imath} + yz\vec{\jmath}) = \frac{1}{\sqrt{3}}[(-x-y)\vec{\imath} + (y+z)\vec{\jmath}]$$

9. Izračunajte usmjerenu derivaciju divergencije polja $\vec{v} = x^2\vec{\imath} - 2yz\vec{\jmath}$ u smjeru vektora $\vec{a} = \vec{\imath} - \vec{k}$.

Rješenje:

$$\operatorname{div}\vec{v} = \frac{\partial}{\partial x}(x^2) + \frac{\partial}{\partial y}(-2yz) + \frac{\partial}{\partial z}(0) = 2x - 2z$$

$$\vec{a}_0 = \frac{\vec{a}}{|\vec{a}|} = \frac{\vec{\imath} - \vec{k}}{\sqrt{2}} = \frac{1}{\sqrt{2}}\vec{\imath} - \frac{1}{\sqrt{2}}\vec{k}$$

$$\frac{\partial(\operatorname{div}\vec{v})}{\partial\vec{s}} = \left(\frac{1}{\sqrt{2}}\frac{\partial}{\partial x} + 0 \cdot \frac{\partial}{\partial y} - \frac{1}{\sqrt{2}}\frac{\partial}{\partial z}\right)\operatorname{grad}(\operatorname{div}\vec{v}) = \left(\frac{1}{\sqrt{2}}\frac{\partial}{\partial x} - \frac{1}{\sqrt{2}}\frac{\partial}{\partial z}\right)\left(2\vec{\imath} - 2\vec{k}\right) =$$

$$= \frac{2}{\sqrt{2}} + \frac{2}{\sqrt{2}} = \frac{4}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = 2\sqrt{2}$$

10. Izračunajte

$$\frac{\partial(\vec{r}\ln r)}{\partial\vec{r}}\bigg|_T$$

ako je $T(2,3,5)$, a $\vec{r}$ radij vektor i r je njegova duljina.

Rješenje:

$$\frac{\partial(\vec{r}\ln r)}{\partial\vec{r}} = \left(\frac{x}{r}\frac{\partial}{\partial x} + \frac{y}{r}\frac{\partial}{\partial y} + \frac{z}{r}\frac{\partial}{\partial z}\right)(\vec{r}\ln r) = \frac{x}{r}\left(\frac{\partial}{\partial x} + \frac{\partial}{\partial y} + \frac{\partial}{\partial z}\right)\left(x\ln r\,\vec{\imath} + y\ln r\,\vec{\jmath} + z\ln r\,\vec{k}\right) =$$

$$= \frac{x}{r}\left(\frac{\partial}{\partial x} + \frac{\partial}{\partial y} + \frac{\partial}{\partial z}\right)\left(\left(x\ln\sqrt{x^2+y^2+z^2}\right)\vec{\imath} + \left(y\ln\sqrt{x^2+y^2+z^2}\right)\vec{\jmath} + \left(z\ln\sqrt{x^2+y^2+z^2}\right)\vec{k}\right)$$

Parcijalne derivacije:

$$\frac{\partial}{\partial x}\left(x\ln\sqrt{x^2+y^2+z^2}\right)\bigg|_T = \ln\sqrt{x^2+y^2+z^2} + \frac{x^2}{\sqrt{x^2+y^2+z^2}} = \ln\sqrt{38} + \frac{4}{\sqrt{38}}$$

$$\frac{\partial}{\partial x}\left(y\ln\sqrt{x^2+y^2+z^2}\right)\bigg|_T = \frac{xy}{\sqrt{x^2+y^2+z^2}} = \frac{6}{\sqrt{38}}$$

$$\frac{\partial}{\partial x}\left(z\ln\sqrt{x^2+y^2+z^2}\right)\bigg|_T = \frac{xz}{\sqrt{x^2+y^2+z^2}} = \frac{10}{\sqrt{38}}$$

$$\frac{\partial}{\partial y}\left(x\ln\sqrt{x^2+y^2+z^2}\right)\bigg|_T = \frac{xy}{\sqrt{x^2+y^2+z^2}} = \frac{6}{\sqrt{38}}$$

$$\frac{\partial}{\partial y}\left(y\ln\sqrt{x^2+y^2+z^2}\right)\bigg|_T = \ln\sqrt{x^2+y^2+z^2} + \frac{y^2}{\sqrt{x^2+y^2+z^2}} = \ln\sqrt{38} + \frac{9}{\sqrt{38}}$$

$$\frac{\partial}{\partial y}\left(z\ln\sqrt{x^2+y^2+z^2}\right)\bigg|_T = \frac{yz}{\sqrt{x^2+y^2+z^2}} = \frac{15}{\sqrt{38}}$$

$$\frac{\partial}{\partial z}\left(x\ln\sqrt{x^2+y^2+z^2}\right)\bigg|_T = \frac{xz}{\sqrt{x^2+y^2+z^2}} = \frac{10}{\sqrt{38}}$$

$$\frac{\partial}{\partial z}\left(y\ln\sqrt{x^2+y^2+z^2}\right)\bigg|_T = \frac{yz}{\sqrt{x^2+y^2+z^2}} = \frac{15}{\sqrt{38}}$$

$$\frac{\partial}{\partial z}\left(z\ln\sqrt{x^2+y^2+z^2}\right)\Big|_T = \ln\sqrt{x^2+y^2+z^2}+\frac{z^2}{\sqrt{x^2+y^2+z^2}} = \ln\sqrt{38}+\frac{25}{\sqrt{38}}$$

Pa imamo da je:

$$\frac{\partial(\vec{r}\ln r)}{\partial\vec{r}}\Big|_T =$$

$$= \left(\ln\sqrt{38}+\frac{4}{\sqrt{38}}+\frac{6}{\sqrt{38}}+\frac{10}{\sqrt{38}}\right)\vec{\imath}+\left(\frac{6}{\sqrt{38}}+\ln\sqrt{38}+\frac{9}{\sqrt{38}}+\frac{15}{\sqrt{38}}\right)\vec{\jmath}+\left(\frac{10}{\sqrt{38}}+\frac{15}{\sqrt{38}}+\ln\sqrt{38}+\frac{25}{\sqrt{38}}\right)\vec{k} =$$

$$= \left(\ln\sqrt{38}+\frac{20}{\sqrt{38}}\right)\vec{\imath}+\left(\ln\sqrt{38}+\frac{30}{\sqrt{38}}\right)\vec{\jmath}+\left(\ln\sqrt{38}+\frac{50}{\sqrt{38}}\right)\vec{k}$$

11. Izračunajte usmjerenu derivaciju polja $\vec{v}=\operatorname{grad}(r^2\ln r)$ u smjeru vektora $\vec{k}$, u točki $T(\sqrt{3},0,1)$. Pri tome je $\vec{r}$ radij vektor a r njegova duljina.

Rješenje:

$$\vec{v}=\operatorname{grad}(r^2\ln r)=\operatorname{grad}\left[(x^2+y^2+z^2)\ln\sqrt{x^2+y^2+z^2}\right]=$$

$$=\left(2x\ln\sqrt{x^2+y^2+z^2}+x\right)\vec{\imath}+\left(2y\ln\sqrt{x^2+y^2+z^2}+y\right)\vec{\jmath}+\left(2z\ln\sqrt{x^2+y^2+z^2}+z\right)\vec{k}$$

$$\frac{\partial\vec{v}}{\partial\vec{k}}=\frac{\partial\vec{v}}{\partial z}=\frac{2xz}{x^2+y^2+z^2}\vec{\imath}+\frac{2yz}{x^2+y^2+z^2}\vec{\jmath}+\left(2\ln\sqrt{x^2+y^2+z^2}+\frac{2z^2}{x^2+y^2+z^2}+1\right)\vec{k}$$

$$\frac{\partial\vec{v}}{\partial\vec{k}}\Big|_T=\frac{\sqrt{3}}{2}\vec{\imath}+\left(2\ln 2+\frac{3}{2}\right)\vec{k}$$