ZADACI ZA VJEŽBU

MATEMATIKA 3E/R

Fourierov red

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1. Sljedeće parne, odnosno neparne funkcije, razvij u Fourierov red na intervalu $\langle -\pi, \pi \rangle$.

A. $f(x) = |x|$

$f(-x) = |-x| = |x|$ funkcija je parna

$b_n = 0$

$$a_0 = \frac{2}{L}\int_0^L f(x)dx = \frac{2}{\pi}\int_0^{\pi}|x|dx = \frac{2}{\pi}\frac{x^2}{2}\Big|_0^{\pi} = \frac{2}{\pi}\frac{\pi^2}{2} = \pi$$

$$a_n = \frac{2}{L}\int_0^L f(x)\cos\frac{n\pi x}{L}dx = \frac{2}{\pi}\int_0^{\pi}|x|\cos nxdx = \frac{2}{\pi}\int_0^{\pi}x\cos nxdx = \begin{vmatrix} u = x & dv = \cos nxdx \\ du = dx & v = \frac{1}{n}\sin nx \end{vmatrix} =$$

$$= \frac{2}{\pi}\left(\frac{x}{n}\sin nx\Big|_0^{\pi} - \frac{1}{n}\int_0^{\pi}\sin nxdx\right) = \frac{2}{\pi}\left(-\frac{1}{n}\frac{1}{n}(-\cos nx)\Big|_0^{\pi}\right) = \frac{2}{n^2\pi}(\cos n\pi - 1) = \frac{2}{n^2\pi}\left((-1)^n - 1\right)$$

Vrijedi: $$a_{2n} = 0$$ $$a_{2n+1} = \frac{-4}{(2n+1)^2\pi}$$

Rješenje:

$$S(x) = \frac{\pi}{2} + \sum_{n=1}^{\infty}\frac{-4}{(2n+1)^2\pi}\cos(2n+1)x = \frac{\pi}{2} - \frac{4}{\pi}\sum_{n=1}^{\infty}\frac{\cos(2n+1)x}{(2n+1)^2}$$

B. $f(x)=\sin ax$

$f(-x)=\sin(-ax)=-\sin ax=-f(x)$ funkcija je neparna

$a_0=0$
$a_n=0$

$$b_n=\frac{2}{L}\int_0^L f(x)\sin\frac{n\pi x}{L}=\frac{2}{\pi}\int_0^\pi \sin ax\sin nxdx=\frac{2}{\pi}\int_0^\pi \frac{\cos(a-n)x-\cos(a+n)x}{2}dx=$$
$$=\frac{1}{\pi}\left(\int_0^\pi \cos(a-n)xdx-\int_0^\pi \cos(a+n)xdx\right)=\frac{1}{\pi}\left(\frac{1}{a-n}\sin(a-n)x\,|_0^\pi-\frac{1}{a+n}\sin(a+n)x\,|_0^\pi\right)=$$
$$=\frac{1}{\pi}\left(\frac{1}{a-n}\sin(a-n)\pi-\frac{1}{a+n}\sin(a+n)\pi\right)=\frac{1}{\pi}\left(\frac{1}{a-n}(\sin(a\pi-n\pi))-\frac{1}{a+n}(\sin(a\pi+n\pi))\right)=$$
$$=\frac{1}{\pi}\left(\frac{1}{a-n}(\sin a\pi\cos n\pi-\cos a\pi\sin n\pi)-\frac{1}{a+n}(\sin a\pi\cos n\pi+\cos a\pi\sin n\pi)\right)=$$
$$=\frac{1}{\pi}\left(\frac{1}{a-n}\sin a\pi\cos n\pi-\frac{1}{a+n}\sin a\pi\cos n\pi\right)=\frac{1}{\pi}\left(\frac{1}{a-n}\sin a\pi(-1)^n-\frac{1}{a+n}\sin a\pi(-1)^n\right)=$$
$$=\frac{(-1)^n\sin a\pi}{\pi}\left(\frac{a+n-a+n}{a^2-n^2}\right)=\frac{(-1)^n\sin a\pi}{\pi}\frac{2n}{a^2-n^2}=\frac{(-1)^{n+1}\sin a\pi}{\pi}\frac{2n}{n^2-a^2}$$

Rješenje:

$$S(x)=\sum_{n=1}^{\infty}\frac{(-1)^{n+1}\sin a\pi}{\pi}\frac{2n}{n^2-a^2}\sin nx=\frac{2\sin\pi a}{\pi}\sum_{n=1}^{\infty}(-1)^{n+1}\frac{n\sin nx}{n^2-a^2}\text{, za } a\notin Z$$

Za $a\in Z$, rezultat je $S(x)=\sin ax$.

C. $f(x)=|\cos x|$

$f(-x)=|\cos(-x)|=|\cos x|=f(x)$ parna funkcija

$b_n=0$

$$a_0=\frac{2}{L}\int_0^L f(x)dx=\frac{2}{\pi}\int_0^{\pi}|\cos x|dx=\frac{2}{\pi}\left(\int_0^{\frac{\pi}{2}}\cos xdx+\int_{\frac{\pi}{2}}^{\pi}-\cos xdx\right)=\frac{2}{\pi}\left(\sin x\,|_0^{\frac{\pi}{2}}-\sin x\,|_{\frac{\pi}{2}}^{\pi}\right)=\frac{4}{\pi}$$

$$a_n=\frac{2}{L}\int_0^L f(x)\cos\frac{n\pi x}{L}dx=\frac{2}{\pi}\int_0^{\pi}|\cos x|\cos nxdx=\frac{2}{\pi}\left(\int_0^{\frac{\pi}{2}}\cos x\cos nxdx-\int_{\frac{\pi}{2}}^{\pi}\cos x\cos nxdx\right)=$$

$$=\frac{1}{\pi}\left(\int_0^{\frac{\pi}{2}}[\cos(n+1)x+\cos(n-1)x]dx-\int_{\frac{\pi}{2}}^{\pi}[\cos(n+1)x+\cos(n-1)x]dx\right)=$$

$$=\frac{1}{\pi}\left(\frac{1}{n+1}\sin(n+1)x\,|_0^{\frac{\pi}{2}}+\frac{1}{n-1}\sin(n-1)x\,|_0^{\frac{\pi}{2}}-\frac{1}{n+1}\sin(n+1)x\,|_{\frac{\pi}{2}}^{\pi}-\frac{1}{n-1}\sin(n-1)x\,|_{\frac{\pi}{2}}^{\pi}\right)=$$

$$=\frac{1}{\pi}\left(\frac{1}{n+1}\sin(n+1)\frac{\pi}{2}+\frac{1}{n-1}\sin(n-1)\frac{\pi}{2}-\left(\frac{1}{n+1}\sin(n+1)\pi-\frac{1}{n+1}\sin(n+1)\frac{\pi}{2}\right)-\right.$$

$$\left.-\left(\frac{1}{n-1}\sin(n-1)\pi-\frac{1}{n-1}\sin(n-1)\frac{\pi}{2}\right)\right)=$$

$$=\frac{1}{\pi}\left(\frac{1}{n+1}\sin(n+1)\frac{\pi}{2}+\frac{1}{n-1}\sin(n-1)\frac{\pi}{2}-\frac{1}{n+1}\sin(n+1)\pi+\frac{1}{n+1}\sin(n+1)\frac{\pi}{2}-\frac{1}{n-1}\sin(n-1)\pi+\frac{1}{n-1}\sin(n-1)\frac{\pi}{2}\right)=$$

$$=\frac{1}{\pi}\left(\frac{2}{n+1}\sin(n+1)\frac{\pi}{2}+\frac{2}{n-1}\sin(n-1)\frac{\pi}{2}\right)=\frac{2}{\pi(n^2-1)}\left((n-1)\sin(n+1)\frac{\pi}{2}+(n+1)\sin(n-1)\frac{\pi}{2}\right)$$

Vrijedi: $a_{2n}=\dfrac{2}{\pi(4n^2-1)}\left(2(-1)^{n+1}\right)=\dfrac{4(-1)^{n+1}}{\pi(4n^2-1)}$

$a_{2n+1}=0$

Rješenje:

$$S(x)=\frac{2}{\pi}+\sum_{n=0}^{\infty}\frac{4(-1)^{n+1}}{\pi(4n^2-1)}\cos 2nx=\frac{2}{\pi}+\frac{4}{\pi}\sum_{n=0}^{\infty}(-1)^{n+1}\frac{\cos 2nx}{4n^2-1}$$

2. Razvij u Fourierov red na intervalu $\langle -\pi, \pi \rangle$.

A. $f(x)=e^{x}$

$f(-x)=e^{-x}=\frac{1}{e^{x}}$ funkcija nije ni parna ni neparna

$$a_0=\frac{2}{T}\int_a^b f(x)dx=\frac{2}{2\pi}\int_{-\pi}^{\pi}e^{x}dx=\frac{2}{\pi}\left(\frac{e^{\pi}-e^{-\pi}}{2}\right)=\frac{2sh\pi}{\pi}$$

$$a_n=\frac{2}{T}\int_a^b f(x)\cos\frac{2n\pi x}{T}dx=\frac{2}{2\pi}\int_{-\pi}^{\pi}e^{x}\cos nxdx$$

$$\int e^{x}\cos nxdx=\begin{vmatrix} u=e^{x} & dv=\cos nxdx \\ du=e^{x}dx & v=\frac{1}{n}\sin nx \end{vmatrix}=\frac{e^{x}}{n}\sin nx-\frac{1}{n}\int e^{x}\sin nxdx=\begin{vmatrix} u=e^{x} & dv=\sin nxdx \\ du=e^{x}dx & v=-\frac{1}{n}\cos nx \end{vmatrix}=$$

$$=\frac{e^{x}}{n}\sin nx-\frac{1}{n}\left(-\frac{e^{x}}{n}\cos nx-\int -e^{x}\cos nxdx\right)=\frac{e^{x}}{n}\sin nx+\frac{e^{x}}{n^{2}}\cos nx-\int e^{x}\cos nxdx$$

$$2\int e^{x}\cos nxdx=\frac{e^{x}}{n}\sin nx+\frac{e^{x}}{n^{2}}\cos nx\Leftrightarrow\int e^{x}\cos nxdx=\frac{\frac{e^{x}}{n}\sin nx+\frac{e^{x}}{n^{2}}\cos nx}{2}=\frac{ne^{x}\sin nx+e^{x}\cos nx}{2n^{2}}$$

$$a_n=\frac{1}{2n^{2}\pi}\left(ne^{x}\sin nx\,|_{-\pi}^{\pi}+e^{x}\cos nx\,|_{-\pi}^{\pi}\right)=\frac{1}{2n^{2}\pi}\left(e^{\pi}\cos n\pi-e^{-\pi}\cos n\pi\right)=\frac{\cos n\pi}{n^{2}\pi}sh\pi=\frac{(-1)^{n}sh\pi}{n^{2}\pi}$$

$$b_n=\frac{2}{T}\int_a^b f(x)\sin\frac{2n\pi x}{T}dx=\frac{2}{2\pi}\int_{-\pi}^{\pi}e^{x}\sin nxdx$$

$$\int e^{x}\sin nxdx=\begin{vmatrix} u=e^{x} & dv=\sin nxdx \\ du=e^{x}dx & v=-\frac{1}{n}\cos nx \end{vmatrix}=-\frac{e^{x}}{n}\cos nx+\frac{1}{n}\int e^{x}\cos nxdx=\begin{vmatrix} u=e^{x} & dv=\cos nxdx \\ du=e^{x}dx & v=\frac{1}{n}\sin nx \end{vmatrix}=$$

$$=-\frac{e^{x}}{n}\cos nx+\frac{1}{n}\left(\frac{e^{x}}{n}\sin nx-\int e^{x}\sin nxdx\right)=\frac{e^{x}}{n^{2}}\sin nx-\frac{e^{x}}{n}\cos nx-\int e^{x}\sin nxdx$$

$$2\int e^{x}\sin nxdx=\frac{e^{x}}{n^{2}}\sin nx-\frac{e^{x}}{n}\cos nx\Leftrightarrow\int e^{x}\sin nxdx=\frac{\frac{e^{x}}{n^{2}}\sin nx-\frac{e^{x}}{n}\cos nx}{2}=\frac{e^{x}\sin nx-ne^{x}\cos nx}{2n^{2}}$$

$$b_n=\frac{1}{2n^{2}\pi}\left(e^{x}\sin nx\,|_{-\pi}^{\pi}-ne^{x}\cos nx\,|_{-\pi}^{\pi}\right)=\frac{1}{2n^{2}\pi}\left(-ne^{\pi}\cos n\pi+ne^{-\pi}\cos n\pi\right)=\frac{(-1)^{n+1}sh\pi}{n\pi}$$

Rješenje:

$$S(x)=\frac{1}{2}\frac{2sh\pi}{\pi}+\sum_{n=1}^{\infty}\left(\frac{(-1)^n sh\pi}{n^2\pi}\cos nx+\frac{(-1)^{n+1} sh\pi}{n\pi}\sin nx\right)=\frac{2}{\pi}sh\pi\left(\frac{1}{2}+\sum_{n=1}^{\infty}\frac{(-1)^n}{n^2+1}(\cos nx-n\sin nx)\right)$$

B. $f(x)=x^3$

$f(-x)=-x^3=-f(x)$ funkcija je neparna

$a_0=a_n=0$

$$b_n=\frac{2}{L}\int_0^L f(x)\sin\frac{n\pi x}{L}=\frac{2}{\pi}\int_0^\pi x^3\sin nxdx$$

$$\int x^3\sin nxdx=\begin{vmatrix} u=x^3 & dv=\sin nxdx \\ du=3x^2dx & v=-\frac{1}{n}\cos nx \end{vmatrix}=-\frac{x^3}{n}\cos nx+\frac{3}{n}\int x^2\cos nxdx=\begin{vmatrix} u=x^2 & dv=\cos nxdx \\ du=2xdx & v=\frac{1}{n}\sin nx \end{vmatrix}=$$

$$=-\frac{x^3}{n}\cos nx+\frac{3}{n}\left(\frac{x^2}{n}\sin nx-\frac{2}{n}\int x\sin nxdx\right)=-\frac{x^3}{n}\cos nx+\frac{3x^2\sin nx}{n^2}-\frac{6}{n^2}\int x\sin nxdx=$$

$$=\begin{vmatrix} u=x & dv=\sin nxdx \\ du=dx & v=-\frac{1}{n}\cos nx \end{vmatrix}=-\frac{x^3}{n}\cos nx+\frac{3x^2\sin nx}{n^2}-\frac{6}{n^2}\left(-\frac{x\cos nx}{n}+\frac{1}{n}\int\cos nxdx\right)=$$

$$=-\frac{x^3}{n}\cos nx+\frac{3x^2\sin nx}{n^2}+\frac{6x\cos nx}{n^3}-\frac{6\sin nx}{n^4}$$

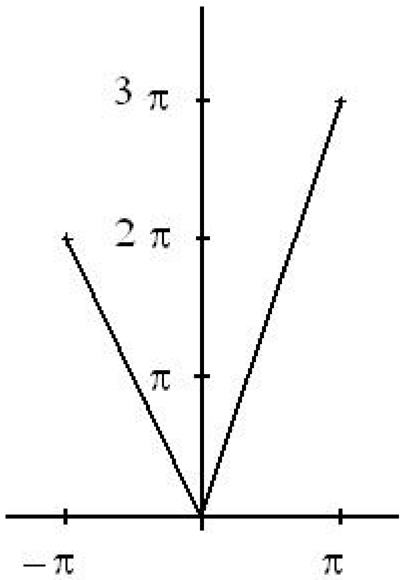
$$b_n=\frac{2}{\pi}\left(-\frac{x^3}{n}\cos nx\,|_0^\pi+\frac{3x^2\sin nx}{n^2}\,|_0^\pi+\frac{6x\cos nx}{n^3}\,|_0^\pi-\frac{6\sin nx}{n^4}\,|_0^\pi\right)=\frac{2}{\pi}\left(-\frac{x^3}{n}\cos nx\,|_0^\pi+\frac{6x\cos nx}{n^3}\,|_0^\pi\right)=$$

$$=\frac{2}{\pi}\left(-\frac{\pi^3\cos n\pi}{n}+\frac{6\pi\cos n\pi}{n^3}\right)=\frac{2}{\pi}\left(\frac{6\pi(-1)^n}{n^3}-\frac{\pi^3(-1)^n}{n}\right)=\frac{2\pi(-1)^n}{\pi}\left(\frac{6}{n^3}-\frac{\pi^2}{n}\right)=(-1)^n\left(\frac{12}{n^3}-\frac{2\pi^2}{n}\right)$$

Rješenje:

$$S(x)=\sum_{n=1}^{\infty}(-1)^n\left(\frac{12}{n^3}-\frac{2\pi^2}{n}\right)\sin nx$$

c. $f(x)=\begin{cases}-2x, & -\pi<x<0\\ 3x, & 0<x<\pi\end{cases}$.



Funkcija nije ni parna ni neparna (što vidimo iz slike jer nije ni simetrična s obzirom na ishodište ni s obzirom na y-os).

$$a_0=\frac{2}{T}\int_a^b f(x)dx=\frac{2}{2\pi}\left(\int_{-\pi}^{0}-2xdx+\int_0^{\pi}3xdx\right)=\frac{2}{2\pi}\left(-2\frac{x^2}{2}\Big|_{-\pi}^{0}+3\frac{x^2}{2}\Big|_0^{\pi}\right)=\frac{2}{2\pi}\left[-\left(0^2-(-\pi)^2\right)+\frac{3}{2}\left(\pi^2-0^2\right)\right]=$$

$$=\frac{1}{\pi}\left(\pi^2+\frac{3}{2}\pi^2\right)=\frac{5\pi}{2}$$

$$a_n=\frac{2}{T}\int_a^b f(x)\cos\frac{2n\pi x}{T}dx=\frac{2}{2\pi}\left(\int_{-\pi}^{0}-2x\cos nxdx+\int_0^{\pi}3x\cos nxdx\right)=\frac{1}{\pi}\left(\int_{-\pi}^{0}-2x\cos nxdx+\int_0^{\pi}3x\cos nxdx\right)=$$

$$=\left|\begin{matrix}u=x & dv=\cos nxdx\\ du=dx & v=\frac{1}{n}\sin nx\end{matrix}\right|=\frac{1}{\pi}\left[-2\left(\frac{x\sin nx}{n}\Big|_{-\pi}^{0}-\frac{1}{n}\int_{-\pi}^{0}\sin nxdx\right)+3\left(\frac{x\sin nx}{n}\Big|_0^{\pi}-\frac{1}{n}\int_0^{\pi}\sin nxdx\right)\right]=$$

$$=\frac{1}{\pi}\left(\frac{2}{n}\int_{-\pi}^{0}\sin nxdx-\frac{3}{n}\int_0^{\pi}\sin nxdx\right)=\frac{1}{\pi}\left(-\frac{2}{n^2}\cos nx\Big|_{-\pi}^{0}+\frac{3}{n^2}\cos nx\Big|_0^{\pi}\right)=\frac{1}{\pi}\left(\frac{2}{n^2}\left((-1)^n-1\right)+\frac{3}{n^2}\left((-1)^n-1\right)\right)$$

Vrijedi:
$$a_{2n}=0$$
$$a_{2n+1}=\frac{1}{\pi}\left(-\frac{4}{(2n+1)^2}-\frac{6}{(2n+1)^2}\right)=-\frac{10}{(2n+1)^2\pi}$$

$$b_n=\frac{2}{T}\int_a^b f(x)\sin\frac{2n\pi x}{T}dx=\frac{2}{2\pi}\left(\int_{-\pi}^{0}-2xdx\sin nxdx+\int_0^{\pi}3x\sin vxdx\right)=\frac{1}{\pi}\left(\int_{-\pi}^{0}-2xdx\sin nxdx+\int_0^{\pi}3x\sin vxdx\right)=$$

$$=\begin{vmatrix} u=x & dv=\sin nxdx \\ du=dx & v=-\frac{1}{n}\cos nx \end{vmatrix}=\frac{1}{\pi}\left[-2\left(-\frac{x\cos nx}{n}\Big|_{-\pi}^{0}+\frac{1}{n}\int_{-\pi}^{0}\cos nxdx\right)+3\left(-\frac{x\cos nx}{n}\Big|_{0}^{\pi}+\frac{1}{n}\int_0^{\pi}\cos nxdx\right)\right]=$$

$$=\frac{1}{\pi}\left(\frac{2}{n}x\cos nx\Big|_{-\pi}^{0}-\frac{3}{n}x\cos nx\Big|_0^{\pi}\right)=\frac{1}{\pi}\left(\frac{2}{n}\pi(-1)^n-\frac{3}{n}\pi(-1)^n\right)=\frac{(-1)^{n+1}}{n}$$

Rješenje:

$$S(x)=\frac{5\pi}{4}+\sum_{n=0}^{\infty}-\frac{10}{(2n+1)^2\pi}\cos(2n+1)x+\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{n}\sin nx=\frac{5\pi}{4}-\frac{10}{\pi}\left(\cos x+\frac{\cos 3x}{3^2}+\frac{\cos 5x}{5^2}+\ldots\right)+$$

ili napisano kao u knjizi

$$S(x)=\frac{5\pi}{4}-\frac{10}{\pi}\left(\cos x+\frac{\cos 3x}{3^2}+\frac{\cos 5x}{5^2}+\ldots\right)+\left(\sin x-\frac{\sin 2x}{2}+\frac{\sin 3x}{x}+\ldots\right)$$

D. $f(x)=\begin{cases} -\dfrac{1}{2}, & -\pi < x < 0 \\ 0, & 0 < x < \pi \end{cases}$

Funkcija nije ni parna ni neparna.

$$a_0 = \frac{2}{T}\int_a^b f(x)dx = \frac{2}{2\pi}\int_{-\pi}^{0} -\frac{1}{2}dx = -\frac{1}{2\pi}x\,|_{-\pi}^{0} = \frac{1}{2}$$

$$a_n = \frac{2}{T}\int_a^b f(x)\cos\frac{2n\pi x}{T}dx = \frac{2}{2\pi}\int_{-\pi}^{0} -\frac{1}{2}\cos nx dx = -\frac{1}{2\pi}\int_{-\pi}^{0}\cos nx dx = 0$$

$$b_n = \frac{2}{T}\int_a^b f(x)\sin\frac{2n\pi x}{T}dx = \frac{2}{2\pi}\int_{-\pi}^{0} -\frac{1}{2}\sin nx dx = -\frac{1}{2\pi}\int_{-\pi}^{0}\sin nx dx = \frac{1}{2n\pi}\left(1-(-1)^n\right)$$

Vrijedi: $\begin{aligned} b_{2n} &= 0 \\ b_{2n+1} &= \frac{1}{(2n+1)\pi} \end{aligned}$

Rješenje:

$$S(x) = \frac{1}{4} + \sum_{n=0}^{\infty}\frac{1}{(2n+1)\pi}\sin(2n+1)x = \frac{1}{4} + \frac{1}{\pi}\sum_{n=0}^{\infty}\frac{\sin(2n+1)x}{(2n+1)}$$

3. Sljedeće periodičke funkcije razvij na intervalu duljine njihovog perioda.

A. $f(x)=\sin\frac{x}{2}$

Pogledajte zadatak **1. B**. Tamo je zadana funkcija oblika $f(x)=\sin ax$ i dobili smo da je rješenje jednako $S(x)=\frac{2\sin \pi a}{\pi}\sum_{n=1}^{\infty}(-1)^{n+1}\frac{n\sin nx}{n^2-a^2}$ za $a\notin Z$. Nama je $a=\frac{1}{2}$, pa ćemo samo uvrstiti u rješenje taj razlomak:

$$S(x)=\frac{2\sin\frac{\pi}{2}}{\pi}\sum_{n=1}^{\infty}(-1)^{n+1}\frac{n\sin nx}{n^2-\left(\frac{1}{2}\right)^2}=\frac{2}{\pi}\sum_{n=1}^{\infty}(-1)^{n+1}\frac{n\sin nx}{n^2-\frac{1}{4}}=\frac{2}{\pi}\sum_{n=1}^{\infty}(-1)^{n+1}\frac{4n\sin nx}{4n^2-1}$$

Rješenje:

$$S(x)=\frac{8}{\pi}\sum_{n=1}^{\infty}(-1)^{n+1}\frac{n\sin nx}{4n^2-1}$$

4. Razvij u Fourierov red perioda $2L$ funkciju zadanu na intervalu $\langle -L, L\rangle$.

A. $f(x) = |x| - 5, \quad -2 < L < 2$

Funkcija je parna.

$$b_n = 0$$

$$a_0 = \frac{2}{L}\int_0^L f(x)dx = \frac{2}{2}\int_0^2 (|x| - 5)dx = \int_0^2 xdx - 5\int_0^2 dx = \frac{x^2}{2}\Big|_0^2 - 5x\Big|_0^2 = 2 - 10 = -8$$

$$a_n = \frac{2}{L}\int_0^L f(x)\cos\frac{n\pi x}{L}dx = \frac{2}{2}\int_0^2 (|x| - 5)\cos\frac{n\pi x}{2}dx = \int_0^2 x\cos\frac{n\pi x}{2}dx - 5\int_0^2 \cos\frac{n\pi x}{2}dx =$$

$$= \left| \begin{matrix} u = x & dv = \cos\frac{n\pi x}{2}dx \\ du = dx & v = \frac{2}{n\pi}\sin\frac{n\pi x}{2} \end{matrix} \right| = \frac{2x}{n\pi}\sin\frac{n\pi x}{2}\Big|_0^2 - \frac{2}{n\pi}\int_0^2 \sin\frac{n\pi x}{2}dx + \frac{10}{n\pi}\sin\frac{n\pi x}{2}\Big|_0^2 = -\frac{2}{n\pi}\int_0^2 \sin\frac{n\pi x}{2}dx =$$

$$= \frac{4}{n^2\pi^2}\cos\frac{n\pi x}{2}\Big|_0^2 = \frac{4}{n^2\pi^2}\left((-1)^n - 1\right)$$

Vrijedi: $$a_{2n} = 0, \qquad a_{2n+1} = \frac{-8}{(2n+1)^2\pi^2}$$

Rješenje:

$$S(x) = -4 - \frac{8}{\pi^2}\sum_{n=0}^{\infty}\frac{1}{(2n+1)^2}\cos\frac{(2n+1)\pi x}{2}$$

5. Sljedeće funkcije definirane na intervalu $\langle 0,\pi\rangle$ razvij u Fourierov red po sinus funkcijama.

A. $f(x)=x^2$

$a_0=a_n=0$

$$b_n=\frac{2}{L}\int_0^L f(x)\sin\frac{n\pi x}{L}dx=\frac{2}{\pi}\int_0^\pi x^2\sin nxdx=\begin{vmatrix} u=x^2 & dv=\sin nxdx \\ du=2xdx & v=-\frac{1}{n}\cos nx\end{vmatrix}=\frac{2}{\pi}\left(-\frac{x^2\cos nx}{n}|_0^\pi+\frac{2}{n}\int_0^\pi x\cos nxdx\right)=$$

$$=\frac{2}{\pi}\left(-\frac{\pi^2\cos n\pi}{n}+\frac{2}{n}\int_0^\pi x\cos nxdx\right)=\begin{vmatrix} u=x & dv=\cos nxdx \\ du=dx & v=\frac{1}{n}\sin nx\end{vmatrix}=\frac{2}{\pi}\left(-\frac{\pi^2(-1)^n}{n}+\frac{2}{n}\left(\frac{x\sin nx}{n}|_0^\pi-\frac{1}{n}\int_0^\pi \sin nxdx\right)\right)$$

$$=\frac{2}{\pi}\left(-\frac{\pi^2(-1)^n}{n}-\frac{2}{n^2}\int_0^\pi \sin nxdx\right)=\frac{2}{\pi}\left(-\frac{\pi^2(-1)^n}{n}+\frac{2}{n^3}\cos nx|_0^\pi\right)=\frac{2}{\pi}\left(-\frac{\pi^2(-1)^n}{n}+\frac{2}{n^3}\left((-1)^n-1\right)\right)$$

Vrijedi:
$$b_{2n}=-\frac{\pi}{n}$$
$$b_{2n+1}=\frac{2}{\pi}\left(\frac{\pi^2}{2n+1}-\frac{4}{(2n+1)^3}\right)=\frac{2\pi}{2n+1}-\frac{8}{(2n+1)^3\pi}$$

Rješenje:

$$S(x)=\sum_{n=1}^\infty-\frac{\pi}{n}\sin 2nx+\sum_{n=0}^\infty\left(\frac{2\pi}{2n+1}-\frac{8}{(2n+1)^3\pi}\right)\sin(2n+1)x=$$

$$=2\pi\sum_{n=1}^\infty-\frac{\sin 2nx}{2n}+2\pi\sum_{n=0}^\infty\frac{\sin(2n+1)x}{2n+1}-\frac{8}{\pi}\sum_{n=0}^\infty\frac{\sin(2n+1)x}{(2n+1)^3}=$$

$$=2\pi\left(-\frac{\sin 2x}{2}-\frac{\sin 4x}{4}-\frac{\sin 6x}{6}-...\right)+2\pi\left(\sin x+\frac{\sin 3x}{3}+\frac{\sin 5x}{5}+...\right)-\frac{8}{\pi}\left(\sin x+\frac{\sin 3x}{3^3}+\frac{\sin x}{5^3}+...\right)=$$

$$=2\pi\left(\sin x-\frac{\sin 2x}{2}+\frac{\sin 3x}{3}-\frac{\sin 4x}{4}+...\right)-\frac{8}{\pi}\left(\sin x+\frac{\sin 3x}{3^3}+\frac{\sin x}{5^3}+...\right)$$

6. Sljedeće funkcije definirane na intervalu $\langle 0,\pi\rangle$ razvij u Fourierov red po kosinus funkcijama.

A. $f(x)=x^3$

$b_n=0$

$$a_0=\frac{2}{L}\int_0^L f(x)=\frac{2}{\pi}\int_0^\pi x^3dx=\frac{2}{\pi}\frac{x^4}{4}|_0^\pi=\frac{\pi^3}{2}$$

$$a_n=\frac{2}{L}\int_0^L f(x)\cos\frac{n\pi x}{L}=\frac{2}{\pi}\int_0^\pi x^3\cos nxdx$$

$$\int x^3\cos nxdx=\begin{vmatrix} u=x^3 & dv=\cos nxdx \\ du=3x^2dx & v=\frac{1}{n}\sin nx\end{vmatrix}=\frac{x^3}{n}\sin nx-\frac{3}{n}\int x^2\sin nxdx=\begin{vmatrix} u=x^2 & dv=\sin nxdx \\ du=2xdx & v=-\frac{1}{n}\cos nx\end{vmatrix}=$$

$$=\frac{x^3}{n}\sin nx-\frac{3}{n}\left(-\frac{x^2}{n}\cos nx+\frac{2}{n}\int x\cos nxdx\right)=\frac{x^3}{n}\sin nx+\frac{3x^2\cos nx}{n^2}-\frac{6}{n^2}\int x\cos nxdx=$$

$$=\begin{vmatrix} u=x & dv=\cos nxdx \\ du=dx & v=\frac{1}{n}\sin nx\end{vmatrix}=\frac{x^3}{n}\sin nx+\frac{3x^2\cos nx}{n^2}-\frac{6}{n^2}\left(\frac{x\sin nx}{n}-\frac{1}{n}\int\sin nxdx\right)=$$

$$=\frac{x^3}{n}\sin nx+\frac{3x^2\cos nx}{n^2}-\frac{6x\sin nx}{n^3}-\frac{6\cos nx}{n^4}$$

$$a_n=\frac{2}{\pi}\left(\frac{x^3}{n}\sin nx|_0^\pi+\frac{3x^2\cos nx}{n^2}|_0^\pi-\frac{6x\sin nx}{n^3}|_0^\pi-\frac{6\cos nx}{n^4}|_0^\pi\right)=\frac{2}{\pi}\left(\frac{3x^2\cos nx}{n^2}|_0^\pi-\frac{6\cos nx}{n^4}|_0^\pi\right)=$$

$$=\frac{2}{\pi}\left(\frac{3\pi^2\cos n\pi}{n^2}-\frac{6(\cos n\pi-1)}{n^4}\right)=\frac{2}{\pi}\left(\frac{3\pi^2(-1)^n}{n^2}-\frac{6((-1)^n-1)}{n^4}\right)$$

Vrijedi:
$$a_{2n}=\frac{3\pi}{2n^2}$$
$$a_{2n+1}=\frac{2}{\pi}\left(-\frac{3\pi^2}{(2n+1)^2}+\frac{12}{(2n+1)^4}\right)=-\frac{6\pi}{(2n+1)^2}+\frac{24}{\pi(2n+1)^4}$$

Rješenje:

$$S(x)=\frac{\pi^3}{4}+\sum_{n=1}^\infty\frac{3\pi}{2n^2}\cos 2nx+\sum_{n=0}^\infty\left(-\frac{6\pi}{(2n+1)^2}+\frac{24}{\pi(2n+1)^4}\right)\cos(2n+1)x$$

B. $f(x)=\pi-2x$

$b_n=0$

$$a_0=\frac{2}{L}\int_0^L f(x)=\frac{2}{\pi}\int_0^{\pi}(\pi-2x)dx=\frac{2}{\pi}\left(\pi\int_0^{\pi}dx-2\int_0^{\pi}xdx\right)=2x\,|_0^{\pi}-\frac{4}{\pi}\frac{x^2}{2}\,|_0^{\pi}=2\pi-2\pi=0$$

$$a_n=\frac{2}{L}\int_0^L f(x)\cos\frac{n\pi x}{L}=\frac{2}{\pi}\int_0^{\pi}(\pi-2x)\cos nxdx=\frac{2}{\pi}\left(\pi\int_0^{\pi}\cos nxdx-2\int_0^{\pi}x\cos nxdx\right)=$$

$$=\begin{vmatrix} u=x & dv=\cos nxdx \\ du=dx & v=\frac{1}{n}\sin nx \end{vmatrix}=\frac{2}{\pi}\left(\pi\frac{1}{n}\sin nx\,|_0^{\pi}-2\left(\frac{x\sin nx}{n}\,|_0^{\pi}-\frac{1}{n}\int_0^{\pi}\sin nxdx\right)\right)=\frac{4}{n\pi}\int_0^{\pi}\sin nxdx=$$

$$=-\frac{4}{n^2\pi}\cos nx\,|_0^{\pi}=-\frac{4}{n^2\pi}\left((-1)^n-1\right)$$

Vrijedi: $$\begin{aligned} a_{2n}&=0 \\ a_{2n+1}&=\frac{8}{(2n+1)^2\pi} \end{aligned}$$

Rješenje:

$$S(x)=\sum_{n=0}^{\infty}\frac{8}{(2n+1)^2\pi}\cos(2n+1)x$$

8. Koristeći razvoj funkcije $f(x)=|x|$ u Fourierov red na intervalu $[-1,1]$, izračunaj sumu reda

$$1+\frac{1}{3^2}+\frac{1}{5^2}+\frac{1}{7^2}+\dots$$

$f(x)=|x|$ je parna funckija

$b_n=0$

$$a_0=\frac{2}{L}\int_0^L f(x)dx=2\int_0^1|x|dx=2\frac{x^2}{2}\Big|_0^1=2\frac{1}{2}=1$$

$$a_n=\frac{2}{L}\int_0^L f(x)\cos\frac{n\pi x}{L}dx=2\int_0^1|x|\cos nxdx=2\int_0^1 x\cos nxdx=\begin{vmatrix} u=x & dv=\cos nxdx \\ du=dx & v=\frac{1}{n}\sin nx\end{vmatrix}=$$

$$=2\left(\frac{x}{n}\sin nx\Big|_0^1-\frac{1}{n}\int_0^1\sin nxdx\right)=2\left(\frac{\sin n}{n}+\frac{1}{n^2}\cos nx\Big|_0^1\right)=2\left(\frac{\sin n}{n}+\frac{1}{n^2}(\cos n-1)\right)$$

$$a_{k\pi}=2\left(\frac{\sin k\pi}{k\pi}+\frac{1}{k^2\pi^2}(\cos k\pi-1)\right)=\frac{2(\cos k\pi-1)}{k^2\pi^2}$$

$$S(x)=\frac{1}{2}+\sum_{k=1}^{\infty}\frac{2(\cos k\pi-1)}{k^2\pi^2}\cos k\pi x=\frac{1}{2}+2\sum_{k=1}^{\infty}\frac{(-1)^k-1}{k^2\pi^2}\cos k\pi x$$

Za parne k – ove vrijednosti će biti jednake 0, što znači da ćemo gledati samo $2k$ - 1 vrijednosti.

$$S(x)=\frac{1}{2}+2\sum_{k=1}^{\infty}\frac{(-1)^{2k-1}-1}{(2k-1)^2\pi^2}\cos k\pi x$$

Uzmemo da je x = 0 (to jest, neki broj iz intervala $[-1,1]$, obično uvrštavamo „okrugle“ brojeve), pa imamo:

$$S(0)=\frac{1}{2}+2\sum_{k=1}^{\infty}\frac{(-1)^{2k-1}-1}{(2k-1)^2\pi^2}\cos 0=\frac{1}{2}+2\sum_{k=1}^{\infty}\frac{(-1)^{2k-1}-1}{(2k-1)^2\pi^2}=\frac{1}{2}+2\sum_{k=1}^{\infty}\frac{-2}{(2k-1)^2\pi^2}=\frac{1}{2}-4\sum_{k=1}^{\infty}\frac{1}{(2k-1)^2\pi^2}$$

Vrijedi: $f(x)=S(x)$, pa imamo:

$$f(x)=\frac{1}{2}-4\sum_{k=1}^{\infty}\frac{1}{(2k-1)^2\pi^2}\Leftrightarrow 0=\frac{1}{2}-4\sum_{k=1}^{\infty}\frac{1}{(2k-1)^2\pi^2}\Leftrightarrow -\frac{1}{2}=-4\sum_{k=1}^{\infty}\frac{1}{(2k-1)^2\pi^2}$$

Rješenje:

$$\sum_{k=1}^{\infty}\frac{1}{(2k-1)^2}=\frac{\pi^2}{8}$$

10. Funkciju $f(x)=x^2$, $x\in[0,1]$, razvij u Fourierov red po kosinus funkcijama i pomoću dobivenog razvoja izračunaj sumu reda $\sum_{k=1}^{\infty}\frac{(-1)^k}{k^2}$.

$$b_n=0$$

$$a_0=\frac{2}{L}\int_0^L f(x)dx=2\int_0^1 x^2dx=2\frac{x^3}{3}\Big|_0^1=\frac{2}{3}$$

$$a_n=\frac{2}{L}\int_0^L f(x)\cos\frac{n\pi x}{L}dx=2\int_0^1 x^2\cos(n\pi x)dx=\begin{vmatrix} u=x^2 & dv=\cos(n\pi x)dx \\ du=2xdx & v=\frac{1}{n\pi}\sin(n\pi x)\end{vmatrix}=$$

$$=2\left(\frac{x^2\sin(n\pi x)}{n\pi}\Big|_0^1-\frac{2}{n\pi}\int_0^1 x\sin(n\pi x)dx=-\frac{2}{n\pi}\int_0^1 x\sin(n\pi x)dx\right)=\begin{vmatrix} u=x & dv=\sin(n\pi x)dx \\ du=dx & v=-\frac{1}{n\pi}\cos(n\pi x)\end{vmatrix}=$$

$$=2\left(-\frac{2}{n\pi}\left(-\frac{x\cos(n\pi x)}{n\pi}\Big|_0^1-\int_0^1-\frac{1}{n\pi}\cos(n\pi x)dx\right)\right)=-\frac{4}{n\pi}\left(-\frac{\cos n\pi}{n\pi}+\frac{1}{n^2\pi^2}\sin(n\pi x)\Big|_0^1\right)=$$

$$=\frac{4(-1)^n}{n^2\pi^2}$$

$$S(x)=\frac{1}{3}+\sum_{n=1}^{\infty}\frac{4(-1)^n}{n^2\pi^2}\cos(n\pi x)$$

$$S(0)=\frac{1}{3}+\sum_{n=1}^{\infty}\frac{4(-1)^n}{n^2\pi^2}\cos(0)=\frac{1}{3}+\frac{4}{\pi^2}\sum_{n=1}^{\infty}\frac{(-1)^n}{n^2}$$

$$f(x)=S(x)$$

$$f(0)=\frac{1}{3}+\frac{4}{\pi^2}\sum_{n=1}^{\infty}\frac{(-1)^n}{n^2}$$

$$0=\frac{1}{3}+\frac{4}{\pi^2}\sum_{n=1}^{\infty}\frac{(-1)^n}{n^2}$$

$$\frac{4}{\pi^2}\sum_{n=1}^{\infty}\frac{(-1)^n}{n^2}=-\frac{1}{3}$$

$$\sum_{n=1}^{\infty}\frac{(-1)^n}{n^2}=-\frac{\pi^2}{12}$$

Rješenje:

$$\sum_{n=1}^{\infty}\frac{(-1)^n}{n^2}=-\frac{\pi^2}{12}$$

11. Izračunaj sumu reda $1+\frac{1}{3^2}+\frac{1}{5^2}+\dots$ koristeći razvoj u red po kosinus funkcijama funkcije f koja je u intervalu $\langle 0,\pi\rangle$ dana izrazom $f(x)=1-\frac{2x}{\pi}$.

$$b_n=0$$

$$a_0=\frac{2}{L}\int_0^L f(x)dx=\frac{2}{\pi}\int_0^\pi\left(1-\frac{2x}{\pi}\right)dx=\frac{2}{\pi}\left(\int_0^\pi dx-\frac{2}{\pi}\int_0^\pi xdx\right)=\frac{2}{\pi}\left(x\,|_0^\pi-\frac{2}{\pi}\frac{x^2}{2}\,|_0^\pi\right)=\frac{2}{\pi}(\pi-\pi)=0$$

$$a_n=\frac{2}{L}\int_0^L f(x)\cos\frac{n\pi x}{L}dx=\frac{2}{\pi}\int_0^\pi\left(1-\frac{2x}{\pi}\right)\cos nxdx=\frac{2}{\pi}\left(\int_0^\pi\cos nxdx-\frac{2}{\pi}\int_0^\pi x\cos nxdx\right)=\begin{vmatrix} u=x & dv=\cos nxdx\\ du=dx & v=\frac{1}{n}\sin nx\end{vmatrix}=$$
$$=\frac{2}{\pi}\left(-\frac{2}{\pi}\left(\frac{x\sin nx}{n}\,|_0^\pi-\frac{1}{n}\int_0^\pi\sin nxdx\right)\right)=-\frac{4}{n^2\pi^2}\cos nx\,|_0^\pi=-\frac{4}{n^2\pi^2}\left((-1)^n-1\right)$$

Vrijedi:
$$a_{2n}=0$$
$$a_{2n+1}=\frac{8}{(2n+1)^2\pi^2}$$

$$S(x)=\sum_{n=1}^\infty\frac{8}{(2n+1)^2\pi^2}\cos(2n+1)x$$
$$S(0)=\sum_{n=1}^\infty\frac{8}{(2n+1)^2\pi^2}\cos 0=\sum_{n=1}^\infty\frac{8}{(2n+1)^2\pi^2}$$

$$f(x)=S(x)$$

$$f(0)=\sum_{n=1}^\infty\frac{8}{(2n+1)^2\pi^2}$$
$$1=\sum_{n=1}^\infty\frac{8}{(2n+1)^2\pi^2}$$
$$\frac{\pi^2}{8}=\sum_{n=1}^\infty\frac{1}{(2n+1)^2}$$

Rješenje:

$$\sum_{n=1}^\infty\frac{1}{(2n+1)^2}=\frac{\pi^2}{8}$$

12. Izračunaj sumu reda $\sum_{n=1}^{\infty}\frac{(-1)^{n-1}}{2n-1}$ pomoću razvoja funkcije $y=\frac{\pi}{4}-\frac{x}{2}$ na intervalu $\langle 0,\pi\rangle$ u red po sinus funkcijama.

$a_0 = a_n = 0$

$$b_n=\frac{2}{L}\int_0^L f(x)\sin\frac{n\pi x}{L}dx=\frac{2}{\pi}\int_0^{\pi}\left(\frac{\pi}{4}-\frac{x}{2}\right)\sin nxdx=\frac{2}{\pi}\left(\frac{\pi}{4}\int_0^{\pi}\sin nxdx-\frac{1}{2}\int_0^{\pi}x\sin nxdx\right)=\begin{vmatrix} u=x & dv=\sin nxdx \\ du=dx & v=-\frac{1}{n}\cos nx\end{vmatrix}=$$

$$=\frac{2}{\pi}\left(-\frac{\pi}{4n}\cos nx\,|_0^{\pi}-\frac{1}{2}\left(-\frac{x\cos nx}{n}\,|_0^{\pi}-\int_0^{\pi}-\frac{1}{n}\cos nxdx\right)\right)=\frac{2}{\pi}\left(-\frac{\pi\left((-1)^n-1\right)}{4n}+\frac{1}{2}\frac{\pi(-1)^n}{n}\right)=$$

$$=-\frac{\left((-1)^n-1\right)}{2n}+\frac{(-1)^n}{n}=\frac{-(-1)^n+1+2(-1)^n}{2n}=\frac{(-1)^n+1}{2n}$$

Vrijedi: $a_{2n}=\frac{1}{2n}$

$a_{2n+1}=0$

$$S(x)=\sum_{n=1}^{\infty}\frac{\sin 2nx}{2n}$$

Eh, ovo stvaaaarno ne znam kako pretvoriti pa da dobijem sumu zadanog reda ☹

13. Funkciju $f(x)=x-|x-\pi|,\ x\in[0,2\pi]$ razvij u Fourierov red po kosinus funkcijama.

$b_n=0$

$$a_0=\frac{2}{L}\int_0^L f(x)dx=\frac{2}{2\pi}\int_0^{2\pi}(x-|x-\pi|)dx=\frac{1}{\pi}\left(\int_0^{2\pi}xdx-\int_0^{2\pi}|x-\pi|dx\right)=\frac{1}{\pi}\left(\frac{x^2}{2}\Big|_0^{2\pi}-\left(-\int_0^{\pi}(x-\pi)dx+\int_{\pi}^{2\pi}(x-\pi)dx\right)\right)=$$

$$=\frac{1}{\pi}\left(2\pi^2+\int_0^{\pi}xdx-\pi\int_0^{\pi}dx-\int_{\pi}^{2\pi}xdx+\pi\int_{\pi}^{2\pi}dx\right)=\frac{1}{\pi}\left(2\pi^2+\frac{\pi^2}{2}-\pi^2-2\pi^2+\frac{\pi^2}{2}+\pi^2\right)=\pi$$

$$a_n=\frac{2}{L}\int_0^L f(x)\cos\frac{n\pi x}{L}dx=\frac{2}{2\pi}\int_0^{2\pi}(x-|x-\pi|)\cos\frac{nx}{2}dx=\frac{1}{\pi}\left(\int_0^{2\pi}x\cos\frac{nx}{2}dx-\int_0^{2\pi}|x-\pi|\cos\frac{nx}{2}dx\right)=$$

$$=\left|\begin{matrix}u=x & dv=\cos\frac{nx}{2}dx\\ du=dx & v=\frac{2}{n}\sin\frac{nx}{2}dx\end{matrix}\right|=\frac{1}{\pi}\left(\frac{2x}{n}\sin\frac{nx}{2}\Big|_0^{2\pi}-\frac{2}{n}\int_0^{2\pi}\sin\frac{nx}{2}dx-\left(-\int_0^{\pi}(x-\pi)\cos\frac{nx}{2}dx+\int_{\pi}^{2\pi}(x-\pi)\cos\frac{nx}{2}dx\right)\right)=$$

$$=\frac{1}{\pi}\left(\frac{4}{n^2}\cos\frac{nx}{2}\Big|_0^{2\pi}+\int_0^{\pi}x\cos\frac{nx}{2}dx-\pi\int_0^{\pi}\cos\frac{nx}{2}dx-\int_{\pi}^{2\pi}x\cos\frac{nx}{2}dx+\pi\int_{\pi}^{2\pi}\cos\frac{nx}{2}dx\right)=$$

$$=\frac{1}{\pi}\left(\frac{4}{n^2}\left((-1)^n-1\right)+\frac{2x}{n}\sin\frac{nx}{2}\Big|_0^{\pi}-\frac{2}{n}\int_0^{\pi}\sin\frac{nx}{2}dx-\pi\frac{2}{n}\sin\frac{nx}{2}\Big|_0^{\pi}-\frac{2x}{n}\sin\frac{nx}{2}\Big|_{\pi}^{2\pi}+\frac{2}{n}\int_{\pi}^{2\pi}\sin\frac{nx}{2}dx+\pi\frac{2}{n}\sin\frac{nx}{2}\Big|_{\pi}^{2\pi}\right)=$$

$$=\frac{1}{\pi}\left(\frac{4}{n^2}\left((-1)^n-1\right)+\frac{2\pi}{n}\sin\frac{n\pi}{2}+\frac{4}{n^2}\cos\frac{nx}{2}\Big|_0^{\pi}-\frac{2\pi}{n}\sin\frac{n\pi}{2}+\frac{2\pi}{n}\sin\frac{n\pi}{2}-\frac{4}{n^2}\cos\frac{nx}{2}\Big|_{\pi}^{2\pi}-\frac{2\pi}{n}\sin\frac{n\pi}{2}\right)=$$

$$=\frac{1}{\pi}\left(\frac{4}{n^2}\left((-1)^n-1\right)+\frac{4}{n^2}\left(\cos\frac{n\pi}{2}-1\right)-\frac{4}{n^2}\left(\cos n\pi-\cos\frac{n\pi}{2}\right)\right)=$$

$$=\frac{1}{\pi}\left(\frac{4}{n^2}\left((-1)^n-1\right)+\frac{4}{n^2}\cos\frac{n\pi}{2}-\frac{4}{n^2}-\frac{4}{n^2}(-1)^n+\frac{4}{n^2}\cos\frac{n\pi}{2}\right)=$$

$$=\frac{1}{\pi}\left(-\frac{4}{n^2}+\frac{8}{n^2}\cos\frac{n\pi}{2}-\frac{4}{n^2}\right)=\frac{1}{\pi}\left(\frac{8}{n^2}\cos\frac{n\pi}{2}-\frac{8}{n^2}\right)$$

$$S(x)=\frac{\pi}{2}+\sum_{n=1}^{\infty}\frac{1}{\pi}\left(\frac{8}{n^2}\cos\frac{n\pi}{2}-\frac{8}{n^2}\right)\cos\frac{nx}{2}=\frac{\pi}{2}+\sum_{n=1}^{\infty}\frac{8}{n^2\pi}\left(\cos\frac{n\pi}{2}-1\right)\cos\frac{nx}{2}$$

Rješenje:

$$S(x)=\frac{\pi}{2}+\sum_{n=1}^{\infty}\frac{8}{n^2\pi}\left(\cos\frac{n\pi}{2}-1\right)\cos\frac{nx}{2}$$