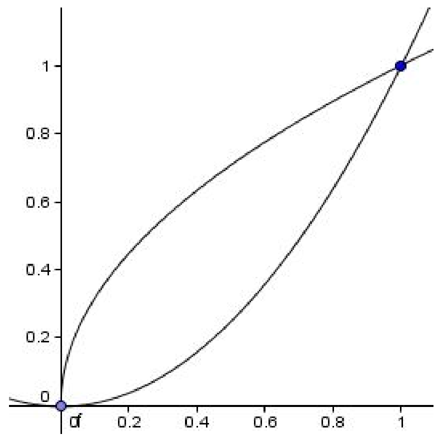
1



$$\begin{aligned}
\iint_D (x^2+y)dxdy &= \int_0^1 dx \int_{x^2}^{\sqrt{x}} (x^2+y)dy \\
&= \int_0^1 x^2y + \frac{y^2}{2}\bigg|_{x^2}^{\sqrt{x}} dx \\
&= \int_0^1 x^{\frac{5}{2}} - x^4 + \frac{x}{2} - \frac{x^4}{2} dx \\
&= \frac{2x^{\frac{7}{2}}}{7} + \frac{x^2}{4} - \frac{3x^5}{10}\bigg|_0^1 \\
&= \frac{33}{140}
\end{aligned}$$

2

a) $\iint_D f(x,y)dxdy = f(x_0,y_0)\cdot\mu(D)$, $(x_0,y_0)\in D$, $\mu(D)$ je površina skupa D.

b)

$$\begin{aligned}
V &= \iint_D \sqrt{5-x^2-y^2}dxdy \\
&= \int_0^{2\pi} d\varphi \int_0^1 r\sqrt{5-r^2}dr \\
&= \begin{vmatrix} u = 5-r^2 \\ du = -2rdr \end{vmatrix} \\
&= 2\pi \int_5^4 \sqrt{u}\frac{-du}{2} \\
&= -\pi\frac{2}{3}u^{\frac{3}{2}}\Big|_5^4 \\
&= -\pi\frac{2}{3}(8-\sqrt{125}) \\
&= \frac{2\pi}{3}(5\sqrt{5}-8)
\end{aligned}$$

Baza valjka je krug polumjera 1, pa je njegova površina π.

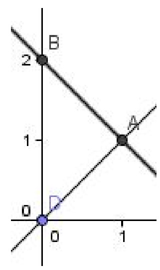
$$h = \frac{V}{\pi} = \frac{2}{3}(5\sqrt{5}-8)$$

3

Prvo nađemo jednadžbu ravnine koja prolazi točkama A, B i C.

Uvrstimo zadane koordinate u determinantu $\begin{vmatrix} x - x_a & y - y_a & z - z_a \\ x_b - x_a & y_b - y_a & z_b - z_a \\ x_c - x_a & y_c - y_a & z_c - z_a \end{vmatrix} = 0$

Dobijemo $x + y + 2z - 2 = 0$.



$$
\begin{aligned}
\iiint_V x\,dx\,dy\,dz &= \int_0^1 x\,dx \int_x^{-x+2} dy \int_0^{\frac{2-x-y}{2}} dz \\
&= \int_0^1 x\,dx \int_x^{-x+2} \frac{2-x-y}{2} dy \\
&= \int_0^1 x\,dx \left(y - \frac{xy}{2} - \frac{y^2}{4} \right) \Bigg|_{y=x}^{y=-x+2} \\
&= ... \\
&= \int_0^1 (x^3 - 2x^2 + x) dx \\
&= ... \\
&= \frac{1}{12}
\end{aligned}
$$

4

$$\begin{aligned}
x-2 &= r\sin\theta\cos\varphi \\
y &= r\sin\theta\sin\varphi \\
z &= r\cos\varphi \\
J &= r^2\sin\theta
\end{aligned}$$

$$\begin{aligned}
\iiint_V \frac{y^2}{\sqrt{(x-2)^2+y^2+z^2}}dxdydz &= \int_0^{2\pi} d\varphi \int_0^{\pi} d\theta \int_0^1 r^2\sin\theta \frac{r^2\sin^2\theta\sin^2\varphi}{r}dr \\
&= \int_0^{2\pi} \sin^2\varphi d\varphi \int_0^{\pi} \sin^3\theta d\theta \int_0^1 r^3 dr \\
&= \int_0^{2\pi} \frac{1-\cos 2\varphi}{2} d\varphi \int_0^{\pi} (1-\cos^2\theta)\sin\theta d\theta \int_0^1 r^3 dr \\
&= \int_0^{2\pi} \frac{1-\cos 2\varphi}{2} d\varphi \int_1^{-1} (u^2-1) du \int_0^1 r^3 dr \\
&= ... \\
&= \frac{\pi}{3}
\end{aligned}$$

5

$$V = \iint_D dxdy \int_{x^2+y^2}^{x+y} dz$$

Nađimo područje D tako da riješimo sustav $x^2 + y^2 = x + y$. Dobije se $\left(x - \frac{1}{2}\right)^2 + \left(y - \frac{1}{2}\right)^2 = \frac{1}{2}$. Iz ovoga vidimo da je područje D krug sa središtem u točki $\left(\frac{1}{2}, \frac{1}{2}\right)$ polumjera $\frac{1}{\sqrt{2}}$.

$$\begin{aligned} x &= r\cos\varphi + \frac{1}{2} \\ y &= r\sin\varphi + \frac{1}{2} \\ z &= z \end{aligned}$$

Integral prelazi u:

$$\begin{aligned} \int_0^{2\pi} d\varphi \int_0^{\frac{1}{\sqrt{2}}} rdr \int_{r^2+r(\cos\varphi+\sin\varphi)+\frac{1}{2}}^{r(\cos\varphi+\sin\varphi)+1} dz &= \int_0^{2\pi} d\varphi \int_0^{\frac{1}{\sqrt{2}}} \left(\frac{1}{2} - r^2\right) dr \\ &= ... \\ &= \frac{\pi}{8} \end{aligned}$$

6

a) $\mathbf{r}(t) = x(t)\mathbf{i} + y(t)\mathbf{j} + z(t)\mathbf{k}$

$$\begin{aligned}
\mathbf{r}'(t) &= \lim_{h\to 0} \frac{\mathbf{r}(t+h) - \mathbf{r}(t)}{h} \\
&= \lim_{h\to 0} \frac{x(t+h)\mathbf{i} + y(t+h)\mathbf{j} + z(t+h)\mathbf{k} - x(t)\mathbf{i} - y(t)\mathbf{j} - z(t)\mathbf{k}}{h} \\
&= \lim_{h\to 0} \frac{x(t+h) - x(t)}{h}\mathbf{i} + \lim_{h\to 0} \frac{y(t+h) - y(t)}{h}\mathbf{j} + \lim_{h\to 0} \frac{z(t+h) - z(t)}{h}\mathbf{k} \\
&= x'(t)\mathbf{i} + y'(t)\mathbf{j} + z'(t)\mathbf{k}
\end{aligned}$$

b)

$$y(t) = t$$
$$z(t) = 1 - 2t$$
$$x(t) = \sqrt{1 - t^2}$$
$$\left.\begin{array}{c} y(t_0) = \frac{\sqrt{3}}{2} \\ z(t_0) = 1 - \sqrt{3} \\ x(t_0) = \frac{1}{2} \end{array}\right\} \Rightarrow t_0 = \frac{\sqrt{3}}{2}$$
$$y'(t_0) = 1$$
$$z'(t_0) = -2$$
$$x'(t_0) = -\sqrt{3}$$

Konačna jednadžba tangente: $\dfrac{x - \frac{1}{2}}{-\sqrt{3}} = \dfrac{y - \frac{\sqrt{3}}{2}}{1} = \dfrac{z - 1 + \sqrt{3}}{-2}$.