Rješenja 3. domaće zadaće MAT3E/R

v1.0

Primjenom Laplaceove transformacije riješi sljedeće diferencijalne jednadžbe

1. $y'' - 2y' - 3y = 3te^{2t},\ y(0) = 1,\ y'(0) = 0$

   Rješenje:

$$y(t) \circ\!\!-\!\!\bullet\ Y(S)$$
$$y'(t) \circ\!\!-\!\!\bullet\ s \cdot Y(s) - y(0) = s \cdot Y(s) - 1$$
$$y''(t) \circ\!\!-\!\!\bullet\ s(s \cdot Y(s) - 1) - y'(0) = s^2 \cdot Y(s) - s$$
$$te^{2t} \circ\!\!-\!\!\bullet\ \frac{1}{(s-2)^2}$$
$$s^2 \cdot Y(s) - s - 2s \cdot Y(s) + 2 - 3Y(s) = \frac{3}{(s-2)^2}$$
$$Y(s)(s^2 - 2s - 3) = s - 2 + \frac{3}{(s-2)^2}$$
$$Y(s) = \frac{s}{(s-3)(s+1)} - \frac{2}{(s-3)(s+1)} + \frac{3}{(s-2)^2(s-3)(s+1)}$$
$$Y(s) = \frac{s(s-2)^2 - 2(s-2)^2 + 3}{(s-2)^2(s-3)(s+1)} = \frac{s(s^2-4s+4) - 2(s^2-4s+4) + 3}{(s-2)^2(s-3)(s+1)}$$
$$= \frac{s^3 - 4s^2 + 4s - 2s^2 + 8s - 8 + 3}{(s-2)^2(s-3)(s+1)} = \frac{s^3 - 6s^2 + 12s - 5}{(s-2)^2(s-3)(s+1)} = \frac{As+B}{(s-2)^2} + \frac{C}{s-3} + \frac{D}{s+1}$$
$$s^3 - 6s^2 + 12s - 5 = (As+B)(s-3)(s+1) + C(s-2)^2(s+1) + D(s-3)(s-2)^2$$

Uvrštavanje tipičnih vrijednosti s u jednadžbe.

$$s = -1 \Rightarrow -1 - 6 - 12 - 5 = 0 + 0 + D(-4) \cdot 9$$
$$-24 = -36D \Rightarrow D = \frac{2}{3}$$
$$s = 3 \Rightarrow 27 - 54 + 36 - 5 = 0 + 4C + 0 \Rightarrow 4 = 4C \Rightarrow C = 1$$
$$s = 2 \Rightarrow 8 - 24 + 24 - 5 = (2A+B) \cdot (-1) \cdot 3 + 0 + 0 \Rightarrow 3 = -3(2A+B) \Rightarrow 2A + B = -1$$
$$s = 1 \Rightarrow 1 - 6 + 12 - 5 = (A+B) \cdot (-2) \cdot 2 + 1 \cdot (-1)^2 \cdot 2 + \frac{2}{3} \cdot (-2)(-1)^2$$
$$2 = -4(A+B) + 2 - \frac{4}{3} \Rightarrow 4(A+B) = -\frac{4}{3} \Rightarrow A + B = -\frac{1}{3}$$
$$B = -\frac{1}{3} - A \Rightarrow 2A - \frac{1}{3} - A = -1 \Rightarrow A = -1 + \frac{1}{3} = -\frac{2}{3} \Rightarrow B = -\frac{1}{3} + \frac{2}{3} = \frac{1}{3}$$

Uvrštavanje dobivenih vrijednosti nazad u jednadžbu.

$$Y(s) = -\frac{2}{3}\frac{s}{(s-2)^2} + \frac{1}{3}\frac{1}{(s-2)^2} + \frac{1}{s-3} + \frac{2}{3}\frac{1}{s+1}$$
$$\frac{s}{(s-2)^2} = \frac{s+2-2}{(s-2)^2} = \frac{s-2}{(s-2)^2} + \frac{2}{(s-2)^2} = \frac{1}{s-2} + \frac{2}{(s-2)^2}$$
$$Y(s) = -\frac{2}{3}\frac{1}{s-2} - \frac{4}{3}\frac{1}{(s-2)^2} + \frac{1}{3}\frac{1}{(s-2)^2} + \frac{1}{s-3} + \frac{2}{3}\frac{1}{s+1}$$
$$Y(s) = -\frac{2}{3}\frac{1}{s-2} - \frac{1}{(s-2)^2} + \frac{1}{s-3} + \frac{2}{3}\frac{1}{s+1}$$

I konačno rješenje nakon inverzne transformacije:

$$y(t) = -\frac{2}{3}e^{2t} - te^{2t} + e^{3t} + \frac{2}{3}e^{-t}$$

Sve eventualne pogreške molim da javite isključivo privatnom porukom.

2. $y'' + 4y = 3\sin 2t,\ y(0) = 2,\ y'(0) = -1$
   Rješenje:

$$y(t) \circ\!\!-\!\!\bullet\ Y(S)$$
$$y'(t) \circ\!\!-\!\!\bullet\ s \cdot Y(s) - y(0) = s \cdot Y(s) - 2$$
$$y''(t) \circ\!\!-\!\!\bullet\ s(s \cdot Y(s) - 2) - y'(0) = s^2 \cdot Y(s) - 2s + 1$$
$$f(t) = 3\sin 2t \circ\!\!-\!\!\bullet\ F(s)$$

$$s^2 \cdot Y(s) - 2s + 1 + 4Y(s) = F(s)$$
$$Y(s)(s^2 + 4) = 2s - 1 + F(s)$$
$$Y(s) = \frac{2s}{s^2+4} - \frac{1}{s^2+4} + \frac{F(s)}{s^2+4}$$
$$\frac{1}{s^2+4} \cdot F(s) \circ\!\!-\!\!\bullet\ \frac{1}{2}\sin 2t * f(t) = \int_0^t \sin 2\tau \cdot 3\sin 2(t-\tau)d\tau = \frac{3}{2}\int_0^t \sin 2\tau \cdot \sin 2(t-\tau)d\tau$$
$$= \frac{3}{2}\int_0^t \frac{\cos(2\tau - 2t + 2\tau)}{2}d\tau - \frac{3}{2}\int_0^t \frac{\cos(2\tau + 2t - 2\tau)}{2}d\tau = \frac{3}{4}\int_0^t \cos(4\tau - 2t)d\tau - \frac{3}{4}\int_0^t \cos(2t)d\tau$$
$$= \frac{3}{4}\frac{\sin(4\tau - 2t)}{4}\Big|_0^t - \frac{3}{4}\cos(2t)\cdot\tau\Big|_0^t = \frac{3}{16}\sin(4t-2t) - \frac{3}{16}\sin(-2t) - \frac{3}{4}t\cos(2t) = \frac{3}{8}\sin(2t) - \frac{3}{4}t\cos(2t)$$

   I konačno rješenje nakon inverzne transformacije:

$$y(t) = 2\cos(2t) - \frac{1}{2}\sin(2t) + \frac{3}{8}\sin(2t) - \frac{3}{4}t\cos(2t) = (2 - \frac{3}{4}t)\cos(2t) - \frac{1}{8}\sin(2t)$$

3. $y'' + 2y' + 2y = e^t,\ y(0) = 0,\ y'(0) = 0$
   Rješenje:

$$y(t) \circ\!\!-\!\!\bullet\ Y(S)$$
$$y'(t) \circ\!\!-\!\!\bullet\ s \cdot Y(s) - y(0) = s \cdot Y(s)$$
$$y''(t) \circ\!\!-\!\!\bullet\ = s^2 \cdot Y(s)$$
$$e^t \circ\!\!-\!\!\bullet\ \frac{1}{s-1}$$

$$s^2 \cdot Y(s) + 2s \cdot Y(s) + sY(s) = \frac{1}{s-1}$$
$$Y(s) \cdot (s^2 + 2s + 2) = \frac{1}{s-1}$$
$$Y(s) = \frac{1}{(s-1)(s^2+2s+2)}$$
$$\frac{1}{(s-1)(s^2+2s+2)} = \frac{A}{s-1} + \frac{Bs+C}{s^2+2s+2}$$
$$1 = A(s^2+2s+2) + (Bs+C)(s-1) \Rightarrow 1 = As^2 + 2As + 2A + Bs^2 - Bs + Cs - C$$
$$1 = s^2(A+B) + s(2A - B + C) + 2A - C \Rightarrow$$
$$A + B = 0 \Rightarrow A = -B$$

$$2A - B + C = 0 \Rightarrow -2B - B + C = 0 \Rightarrow C = 3B$$
$$2A - C = 1$$
$$-2B - 3B = 1 \Rightarrow B = -\frac{1}{5},\ A = \frac{1}{5},\ C = -\frac{3}{5}$$

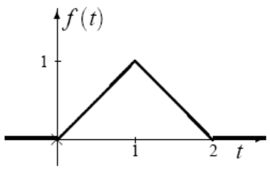
Uvrštavanje dobivenih vrijednosti nazad u jednadžbu.

$$Y(s) = \frac{1}{5}\frac{1}{s-1} - \frac{1}{5}\frac{s}{s^2+2s+2} - \frac{3}{5}\frac{1}{s^2+2s+2} = \frac{1}{5}\frac{1}{s-1} - \frac{1}{5}\frac{s}{(s-1)^2+1} - \frac{3}{5}\frac{s}{(s+1)^2+1}$$
$$= \frac{1}{5}\frac{1}{s-1} - \frac{1}{5}\frac{s+1-1}{(s+1)^2+1} - \frac{3}{5}\frac{1}{(s+1)^2+1} = \frac{1}{5}\frac{1}{s-1} - \frac{1}{5}\frac{s+1}{(s+1)^2+1} - \frac{2}{5}\frac{1}{(s+1)^2+1}$$

I konačno rješenje nakon inverzne transformacije:

$$y(t) = \frac{1}{5}e^t - \frac{1}{5}e^{-t}\cos t - \frac{2}{5}e^{-t}\sin t$$

4. $y' + y = f(t),\ y(0) = 1$



Rješenje:

$$f(t) = \begin{cases} t & \text{ako } t \in (0,1) \\ -t+2 & \text{ako } t \in (1,2) \\ 0 & \text{inače} \end{cases}$$

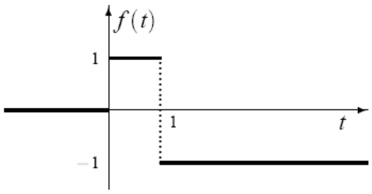
$$f(t) = t \cdot u(t) - t \cdot u(t-1) + (-t+2) \cdot u(t-1) - (-t+2) \cdot u(t-2)$$
$$= t \cdot u(t) - t \cdot u(t-1) - (t-2) \cdot u(t-1) + (t-2) \cdot u(t-2)$$
$$= t \cdot u(t) - t \cdot u(t-1) - (t-1-1) \cdot u(t-1) + (t-2) \cdot u(t-2)$$
$$= t \cdot u(t) - t \cdot u(t-1) - (t-1) \cdot u(t-1) + u(t-1) + (t-2) \cdot u(t-2)$$
$$= t \cdot u(t) - (t-1) \cdot u(t-1) - (t-1) \cdot u(t-1) + (t-2) \cdot u(t-2)$$
$$= t \cdot u(t) - 2(t-1) \cdot u(t-1) + (t-2) \cdot u(t-2)$$

$$y(t) \circ\!\!-\!\!\bullet\ Y(s)$$
$$y'(t) \circ\!\!-\!\!\bullet\ s \cdot Y(s) - y(0) = s \cdot Y(s) - 1$$
$$s \cdot Y(s) - 1 + Y(s) = \frac{1}{s^2} - \frac{2e^{-s}}{s^2} + \frac{e^{-2s}}{s^2}$$
$$(s+1)Y(s) = 1 + \frac{1}{s^2} - \frac{2e^{-s}}{s^2} + \frac{e^{-2s}}{s^2}$$
$$Y(s) = \frac{1}{s+1} + \frac{1}{s^2(s+1)} - \frac{2e^{-s}}{s^2(s+1)} + \frac{e^{-2s}}{s^2(s+1)}$$

$$\frac{1}{s^2(s+1)} \circ\!\!-\!\!\bullet\ e^{-t} + t - 1$$
$$y(t) = e^{-t} + e^{-t} + t - 1 - 2(e^{-(t-1)} + t - 1 - 1)u(t-1) + (e^{-(t-2)} + t - 2 - 1)u(t-2)$$
$$= 2e^{-t} + t - 1 - 2(e^{1-t} + t - 2)u(t-1) + (e^{2-t} + t - 3)u(t-2)$$

Sve eventualne pogreške molim da javite isključivo privatnom porukom.

5. $y'' + y = f(t),\ y(0) = 0,\ y'(0) = 0$



Rješenje:

$$f(t) = \begin{cases} 1 & \text{ako } t \in (0,1) \\ -1 & \text{ako } t \in (1,\infty) \\ 0 & \text{inače} \end{cases}$$

$$f(t) = u(t) - u(t-1) - u(t-1) = u(t) - 2u(t-1)$$

$$y(t) \circ\!\!-\!\!\bullet\ Y(s)$$

$$y'(t) \circ\!\!-\!\!\bullet\ sY(s) - y(0) = sY(s)$$

$$y''(t) \circ\!\!-\!\!\bullet\ s^2Y(s) - y'(0) = s^2Y(s)$$

$$s^2Y(s) + Y(s) = \frac{1}{s} - \frac{2e^{-s}}{s}$$

$$Y(s) = \frac{1}{s} \cdot \frac{1}{s^2+1} - \frac{1}{s}\frac{2e^{-s}}{s^2+1}$$

$$\frac{1}{s(s^2+1)} = \frac{A}{s} + \frac{Bs+C}{s^2+1} \Rightarrow \frac{1}{s} - \frac{s}{s^2+1}$$

$$1 = As^2 + A + Bs^2 + Cs = s^2(A+B) + Cs + A$$

$$A + B = 0 \Rightarrow B = -1,\ C = 0,\ A = 1$$

$$Y(s) = \frac{1}{s} - \frac{s}{s^2+1} - \frac{2e^{-s}}{s} + \frac{2se^{-s}}{s^2+1}$$

I konačno rješenje nakon inverzne transformacije:

$$y(t) = 1 - \cos t - 2u(t-1) + 2\cos(t-1)u(t-1)$$

6. Primjenom Laplaceove transformacije rijesi integralnu jednadžbu: $y(t) = t + 2\int_0^t \cos(t-u)y(u)du$

Rješenje:

$$y(t) = t + 2\cos t * y(t)$$

$$Y(s) = \frac{1}{s^2} + 2\frac{s}{s^2+1}Y(s)$$

$$Y(s)\left(1 - \frac{2s}{s^2+1}\right) = \frac{1}{s^2}$$

$$Y(s) = \frac{s^2+1}{s^2(s^2-2s+1)} = \frac{s^2+1}{s^2(s-1)^2} = \frac{s^2}{s^2(s-1)^2} + \frac{1}{s^2(s-1)^2} = \frac{1}{(s-1)^2} + \frac{1}{s^2(s-1)^2}$$

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$$\frac{1}{s^2(s^2-2s+1)} = \frac{As+B}{s^2} + \frac{Cs+D}{s^2-2s+1}$$

$$1 = (As+B)(s^2-2s+1) + (Cs+D)s^2$$
$$1 = As^3 - 2As^2 + As + Bs^2 - 2Bs + B + Cs^3 + Ds^2$$
$$1 = s^3(A+C) + s^2(-2A+B+D) + s(A-2B) + B$$
$$A + C = 0 \Rightarrow C = -2$$
$$-2A + B + D = 0 \Rightarrow D = 3$$
$$A - 2B = 0 \Rightarrow A = 2$$
$$B = 1$$

$$\frac{2s}{(s-1)^2} = \frac{2(s-1+1)}{(s-1)^2} = \frac{2(s-1)}{(s-1)^2} + \frac{2}{(s-1)^2}$$

$$Y(s) = \frac{1}{(s-1)^2} + \frac{2}{s} + \frac{1}{s^2} - \frac{2}{s-1} - \frac{2}{(s-1)^2} + \frac{3}{(s-1)^2}$$
$$= \frac{2}{s} + \frac{1}{s^2} - \frac{2}{s-1} + \frac{2}{(s-1)^2}$$

I konačno rješenje nakon inverzne transformacije:

$$y(t) = 2 + t - 2te^t + 2te^t$$

7. Primjenom Laplaceove transformacije rijesi integralnu jednadžbu: $y(t) = t + \frac{1}{6}\int_0^t (t-u)^3 y(u)du$

Rješenje:

$$y(t) = t + \frac{1}{6}t^3 * y(t)$$
$$Y(s) = \frac{1}{s^2} + \frac{1}{6}\cdot\frac{6}{s^4}\cdot Y(s)$$
$$Y(s)\left(1 - \frac{1}{s^4}\right) = \frac{1}{s^2}$$
$$Y(s)\frac{s^4-1}{s^4} = \frac{1}{s^2}$$
$$Y(s) = \frac{s^2}{s^4-1} = \frac{s^2}{(s^2-1)(s^2+1)}$$
$$\frac{s^2}{(s^2-1)(s^2+1)} = \frac{As+B}{s^2-1} + \frac{Cs+D}{s^2+1} \Rightarrow \frac{1}{2}\frac{1}{s^2-1} + \frac{1}{2}\frac{1}{s^2+1}$$
$$s^2 = (As+B)(s^2+1) + (Cs+D)(s^2-1)$$
$$s^2 = As^3 + As + Bs^2 + B + Cs^3 - Cs + Ds^2 - D$$
$$s^2 = s^3(A+C) + s^2(B+D) + s(A-C) + B - D$$
$$A + C = 0 \Rightarrow A = -C = 0$$
$$B + D = 1 \Rightarrow 2B = 1 \Rightarrow B = \frac{1}{2},\ D = 12,\ A = C = 0$$
$$A - C = 0 \Rightarrow A = C = 0$$

Sve eventualne pogreške molim da javite isključivo privatnom porukom.

$$B - D = 0 \Rightarrow B = D$$

$$Y(s) = \frac{1}{2}\frac{1}{s^2-1} + \frac{1}{2}\frac{1}{s^2+1}$$

$$y(t) = \frac{1}{2}\sinh t + \frac{1}{2}\sin t = \frac{1}{2}(\frac{e^t - e^{-t}}{2}) + \frac{1}{2}\sin t = \frac{1}{4}(e^t - e^{-t} + 2\sin t)$$

8. Primjenom Laplaceove transformacije rijesi integralno-diferencijalnu jednadžbu:
   $y''(t) + 3y(t) + 2\int_0^t (t-u)y(u)du = 0,\ y(0) = 0,\ y'(0) = 1$

   Rješenje:

$$y''(t) + 3y(t) + 2t * y(t)$$
$$y(t) \circ\!\!-\!\!\!-\!\!\bullet\ Y(s)$$
$$y'(t) \circ\!\!-\!\!\!-\!\!\bullet\ sY(s) - y(0) = sY(s)$$
$$y''(t) \circ\!\!-\!\!\!-\!\!\bullet\ s^2Y(s) - y'(0) = s^2Y(s) - 1$$

$$s^2Y(s) - 1 + 3Y(s) + 2\frac{1}{s^2}Y(s) = 0$$
$$Y(s)\left(s^2 + 3 + \frac{3}{s^2}\right) = 1$$
$$Y(s)\left(\frac{s^4 + 3s^2 + 2}{s^2}\right) = 1$$
$$Y(s) = \frac{s^2}{s^4 + 3s^2 + 2} = \frac{s^2}{s^4 + 3s^2 + 3 - 1} = \frac{s^2}{(s^4 - 1) + 3(s^2 + 1)}$$
$$= \frac{s^2}{(s^2-1)(s^2+1) + 3(s^2+1)} = \frac{s^2}{(s^2+1)(s^2 - 1 + 3)} = \frac{s^2}{(s^2+1)(s^2+2)}$$
$$\frac{s^2}{(s^2+1)(s^2+2)} = \frac{As + B}{s^2+1} + \frac{Cs + D}{s^2+2}$$
$$s^2 = (As + B)(s^2 + 2) + (Cs + D)(s^2 + 1)$$
$$s^2 = As^3 + 2As + Bs^2 + 2B +^C s^3 + Cs + Ds^2 + D$$
$$s^2 = s^3(A + C) + s^2(B + D) + s(2A + C) + 2B + D$$
$$A + C = 0 \Rightarrow A = -C = 0$$
$$B + D = 1 \Rightarrow B = 1 - D \Rightarrow B = 1 + 2B \Rightarrow B = -1$$
$$2A + C = 0 \Rightarrow 2A = -C = 0$$
$$2B + D = 0 \Rightarrow D = -2B \Rightarrow D = 2$$

$$Y(s) = -\frac{1}{s^2+1} + \frac{2}{s^2+2} = -\frac{1}{s^2+1} + \frac{2}{\sqrt{2}} \cdot \frac{\sqrt{2}}{s^2 + (\sqrt{2})^2}$$

I konačno rješenje nakon inverzne transformacije:

$$y(t) = -\sin t + \frac{2}{\sqrt{2}}\sin(\sqrt{2}t)$$
$$y(t) = -\sin t + \sqrt{2}\sin(\sqrt{2}t)$$

Sve eventualne pogreške molim da javite isključivo privatnom porukom.

9. Riješi sustav diferencijalnih jednadžbi:

$$f(t) = \begin{cases} x' + 5x - 2y = e^t \\ y' - x + 6y = e^{-2t} \end{cases} \quad x(0) = y(0) = 0$$

Rješenje:

$$y(t) \circ\!\!-\!\!\bullet\, Y(s) \qquad x(t) \circ\!\!-\!\!\bullet\, X(s)$$

$$y'(t) \circ\!\!-\!\!\bullet\, s \cdot Y(s) - y(0) = s \cdot Y(s) \qquad x'(t) \circ\!\!-\!\!\bullet\, s \cdot X(s) - x(0) = s \cdot X(s)$$

$$e^t \circ\!\!-\!\!\bullet\, \frac{1}{s-1} \qquad e^{-2t} \circ\!\!-\!\!\bullet\, \frac{1}{s+2}$$

$$\begin{cases} s \cdot X(s) + 5X(s) - 2Y(s) = \frac{1}{s-1} \\ s \cdot Y(s) - X(s) + 6Y(s) = \frac{1}{s+2} \end{cases}$$

$$\begin{cases} X(s) \cdot (s+5) - 2Y(s) = \frac{1}{s-1} \\ -X(s) + Y(s) \cdot (s+6) = \frac{1}{s+2} \end{cases}$$

$$sI - A = \begin{bmatrix} s+5 & -2 \\ -1 & s+6 \end{bmatrix}$$

$$C = \begin{bmatrix} \frac{1}{s-1} \\ \frac{1}{s+2} \end{bmatrix}$$

$$\begin{bmatrix} X(s) \\ Y(s) \end{bmatrix} = \frac{1}{detA}(sI - A)^{-1} \cdot C$$

$$\begin{bmatrix} X(s) \\ Y(s) \end{bmatrix} = \frac{1}{(s+5)(s+6) - (-2)(-1)} \cdot \begin{bmatrix} s+5 & -2 \\ -1 & s+6 \end{bmatrix} \cdot \begin{bmatrix} \frac{1}{s-1} \\ \frac{1}{s+2} \end{bmatrix}$$

$$= \frac{1}{s^2 + 6s + 5s + 30 - 2} \cdot \begin{bmatrix} \frac{s+6}{s-1} + \frac{1}{s+2} \\ \frac{2}{s-1} + \frac{s+5}{s+2} \end{bmatrix}$$

$$X(s) = \frac{\frac{s+6}{s-1} + \frac{1}{s+2}}{s^2 + 11s + 28}$$

$$Y(s) = \frac{\frac{2}{s-1} + \frac{s+5}{s+2}}{s^2 + 11s + 28}$$

$$X(s) = \frac{s+6}{(s+7)(s+4)(s-1)} + \frac{1}{(s+7)(s+4)(s-1)}$$

$$\frac{s+6}{(s+7)(s+4)(s-1)} = \frac{A}{s+7} + \frac{B}{s+4} + \frac{C}{s-1}$$

$$s + 6 = A(s+4)(s-1) + B(s+7)(s-1) + C(s+7)(s+4)$$

$$za\ s = -4 \Rightarrow 2 = 0 + B \cdot 3 \cdot (-5) + \Longrightarrow B = -\frac{2}{15}$$

$$za\ s = -7 \Rightarrow -1 = A \cdot (-3) \cdot (-8) + 0 + 0 \Rightarrow A = -\frac{1}{24}$$

$$za\ s = 1 \Rightarrow 7 = 0 + 0 + C \cdot 8 \cdot 5 \Rightarrow C = \frac{7}{40}$$

$$\frac{1}{(s+7)(s+4)(s+2)} = \frac{A}{s+7} + \frac{B}{s+4} + \frac{C}{s+2}$$
$$1 = A(s+4)(s+2) + B(s+7)(s+2) + C(s+7)(s+4)$$
$$za\ \ s = -4 \Rightarrow 1 = 0 + B \cdot 3 \cdot (-2) + 0 \Rightarrow B = -\frac{1}{6}$$
$$za\ \ s = -7 \Rightarrow 1 = A \cdot (-3) \cdot (-5) + 0 + 0 \Rightarrow A = -\frac{1}{15}$$
$$za\ \ s = -2 \Rightarrow 1 = 0 + 0 + C \cdot 5 \cdot 2 \Rightarrow C = \frac{1}{10}$$

$$\begin{aligned} X(s) &= -\frac{1}{24}\frac{1}{s+7} - \frac{2}{15}\frac{1}{s+4} + \frac{7}{40}\frac{1}{s-1} - \frac{1}{15}\frac{1}{s+7} - \frac{1}{6}\frac{1}{s+4} + \frac{1}{10}\frac{1}{s+2} \\ &= \frac{1}{s+7}\left(-\frac{1}{24} - \frac{1}{15}\right) + \frac{1}{s+4}\left(-\frac{2}{15} - \frac{1}{6}\right) + \frac{7}{40}\frac{1}{s-1} + \frac{1}{10}\frac{1}{s+2} \\ &= \frac{-5-8}{120}\frac{1}{s+7} + \frac{-16-20}{120}\frac{1}{s+4} + \frac{21}{120}\frac{1}{s-1} + \frac{12}{120}\frac{1}{s+2} \end{aligned}$$

Jedno rješenje:

$$\begin{aligned} x(t) &= -\frac{13}{120}e^{-7t} - \frac{36}{120}e^{-4t} + \frac{21}{120}e^{t} + \frac{12}{120}e^{-2t} \\ &= \frac{1}{120}e^{-7t}(-13 - 36e^{3t} + 21e^{8t} + 12e^{5t}) \end{aligned}$$

$$Y(s) = \frac{2}{(s+7)(s+4)(s-1)} + \frac{s+5}{(s+7)(s+4)(s+2)}$$
$$\frac{2}{(s+7)(s+4)(s-1)} = \frac{A}{s+4} + \frac{B}{s+7} + \frac{C}{s-1}$$
$$2 = A(s+7)(s-1) + B(s+4)(s-1) + C(s+4)(s+7)$$
$$za\ \ s = -7 \Rightarrow 2 = 0 + B \cdot (-3) \cdot (-8) + 0 \Rightarrow B = -\frac{1}{12}$$
$$za\ \ s = -4 \Rightarrow 2 = A \cdot 3 \cdot (-5) + 0 + 0 \Rightarrow A = -\frac{2}{15}$$
$$za\ \ s = 1 \Rightarrow 2 = 0 + 0 + C \cdot 5 \cdot 8 \Rightarrow C = \frac{1}{20}$$
$$\frac{s+5}{(s+7)(s+4)(s+2)} = \frac{A}{s+4} + \frac{B}{s+7} + \frac{C}{s+2}$$
$$s+5 = A(s+7)(s+2) + B(s+4)(s+2) + C(s+4)(s+7)$$
$$za\ \ s = -7 \Rightarrow -2 = 0 + B \cdot (-3) \cdot (-5) + 0 \Rightarrow B = \frac{2}{15}$$
$$za\ \ s = -4 \Rightarrow 1 = A \cdot 3 \cdot (-2) + 0 + 0 \Rightarrow A = -\frac{1}{6}$$
$$za\ \ s = -5 \Rightarrow 3 = 0 + 0 + C \cdot 2 \cdot 5 \Rightarrow C = \frac{3}{10}$$

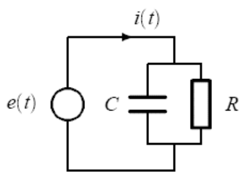
$$Y(s) = -\frac{2}{15}\frac{1}{s+4} - \frac{1}{12}\frac{1}{s+7} + \frac{1}{20}\frac{1}{s-1} - \frac{1}{6}\frac{1}{s+4} + \frac{2}{15}\frac{1}{s+7} + \frac{3}{10}\frac{1}{s+2}$$

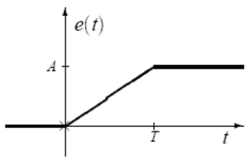
$$= \frac{1}{s+7}\Big(-\frac{1}{12}-\frac{2}{15}\Big)+\frac{1}{s+4}\Big(-\frac{2}{15}-\frac{1}{6}\Big)+\frac{1}{20}\frac{1}{s-1}+\frac{3}{10}\frac{1}{s+2}$$
$$= \frac{-10+16}{120}\frac{1}{s+7}-\frac{16+20}{120}\frac{1}{s+4}+\frac{1}{20}\frac{1}{s-1}+\frac{3}{10}\frac{1}{s+2}=\frac{6}{120}\frac{1}{s+7}-\frac{36}{120}\frac{1}{s+4}+\frac{6}{120}\frac{1}{s-1}+\frac{36}{120}\frac{1}{s+2}$$

Drugo rješenje:

$$y(t)=\frac{6}{120}e^{-7t}-\frac{36}{120}e^{-4t}+\frac{6}{120}e^{t}+\frac{36}{120}e^{-2t}=\frac{1}{120}e^{-7t}(6-36e^{3t}+6e^{8t}+36e^{5t})$$

10. Primjenom Laplaceove transformacije odredi i skiciraj struju $i(t)$ električnog kruga zadanog prema slici ako je priključen zadani napon $e(t)$.





Rješenje:

$$Z(s)=\frac{\frac{R}{Cs}}{R+\frac{1}{Cs}}=\frac{R}{RCs+1}$$

$$e(t)=\begin{cases}\frac{A}{T}t & \text{ako } t\in(0,T)\\ A & \text{ako } t\in(T,\infty)\\ 0 & \text{inače}\end{cases}$$

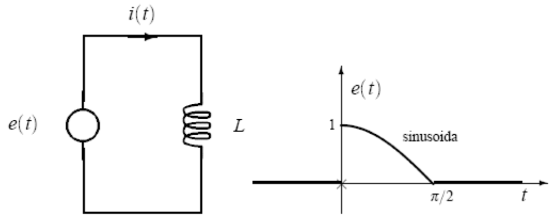
$$e(t)=\frac{A}{T}t\cdot u(t)-\frac{A}{T}\cdot u(t-T)+A\cdot u(t-T)=\frac{A}{T}t\cdot u(t)-\frac{A}{T}(t-T+T)\cdot u(t-T)+A\cdot u(t-T)$$
$$=\frac{A}{T}t\cdot u(t)-\frac{A}{T}(t-T)\cdot u(t-T)-A\cdot u(t-T)+A\cdot u(t-T)$$

$$E(s)=\frac{A}{T}\frac{1}{s^2}-\frac{A}{T}\frac{1}{s^2}e^{-Ts}$$

$$I(s)=\frac{E(s)}{Z(s)}=\frac{RCs+1}{R}\Big(\frac{A}{T}\frac{1}{s^2}-\frac{A}{T}\frac{1}{s^2}e^{-Ts}\Big)=\Big(Cs+\frac{1}{R}\Big)\Big(\frac{A}{T}\frac{1}{s^2}-\frac{A}{T}\frac{1}{s^2}e^{-Ts}\Big)$$
$$=\frac{AC}{T}1s-\frac{AC}{T}\frac{1}{s}e^{-Ts}+\frac{1}{R}\frac{A}{T}\frac{1}{s^2}-\frac{A}{T}\frac{1}{R}\frac{1}{s^2}e^{-Ts}$$

$$i(t)=\frac{AC}{T}-\frac{AC}{T}\cdot u(t-T)+\frac{A}{RT}t-\frac{A}{RT}(t-T)\cdot u(t-T)=\frac{1}{T}\Big(AC-AC\cdot u(t-T)+\frac{A}{R}t-\frac{A}{R}(t-T)\cdot u(t-T)\Big)$$

11. Primjenom Laplaceove transformacije odredi i skiciraj struju $i(t)$ električnog kruga zadanog prema slici ako je priključen zadani napon $e(t)$.



Rješenje:

$$e(t) = \begin{cases} \cos t & \text{ako } t \in (0, \frac{\pi}{2}) \\ 0 & \text{inače} \end{cases}$$

$$\begin{aligned} e(t) &= \cos t \cdot u(t) - \cos t \cdot u\left(t - \frac{\pi}{2}\right) = \cos t \cdot u(t) - \cos\left(t - \frac{\pi}{2} + \frac{\pi}{2}\right) \cdot u\left(t - \frac{\pi}{2}\right) \\ &= \cos t \cdot u(t) - \left[\cos\left(t - \frac{\pi}{2}\right)\cos\left(\frac{\pi}{2}\right)u\left(t - \frac{\pi}{2}\right) - \sin\left(t - \frac{\pi}{2}\right)\sin\left(\frac{\pi}{2}\right)u\left(t - \frac{\pi}{2}\right)\right] \\ &= \cos t \cdot u(t) + \sin\left(t - \frac{\pi}{2}\right)u\left(t - \frac{\pi}{2}\right) \end{aligned}$$

$$E(s) = \frac{s}{s^2+1} + \frac{1}{s^2+1}e^{-\frac{\pi}{2}s}$$

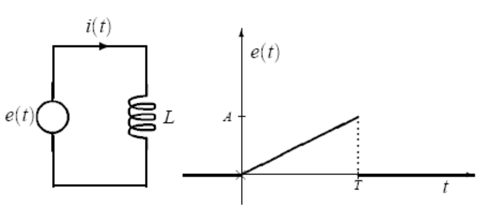
$$\begin{aligned} I(s) &= \frac{E(s)}{Z(s)} = \frac{1}{Ls}\left(\frac{s}{s^2+1} + \frac{1}{s^2+1}e^{-\frac{\pi}{2}s}\right) \\ &= \frac{1}{L}\frac{1}{s^2+1} + \frac{1}{L}\frac{1}{s(s^2+1)}e^{-\frac{\pi}{2}s} \end{aligned}$$

$$\begin{aligned} \frac{1}{s(s^2+1)} &= \frac{A}{s} + \frac{Bs+C}{s^2+1} \\ 1 &= As^2 + A + Bs^2 + Cs \\ A + B = 0 &\Rightarrow B = -1 \Rightarrow C = 0 \Rightarrow A = 1 \end{aligned}$$

$$I(s) = \frac{1}{L}\frac{1}{s^2+1} + \frac{1}{L}\frac{1}{s}e^{-\frac{\pi}{2}s} - \frac{1}{L}\frac{s}{s^2+1}e^{-\frac{\pi}{2}s}$$

$$i(t) = \frac{1}{2}\sin t + \frac{1}{L} \cdot u\left(t - \frac{\pi}{2}\right) - \frac{1}{L}\cos\left(t - \frac{\pi}{2}\right) \cdot u\left(t - \frac{\pi}{2}\right) = \frac{1}{2}\sin t + \frac{1}{L} \cdot u\left(t - \frac{\pi}{2}\right) - \frac{1}{L}\sin t \cdot u\left(t - \frac{\pi}{2}\right)$$

12. Primjenom Laplaceove transformacije odredi i skiciraj struju $i(t)$ električnog kruga zadanog prema slici ako je priključen zadani napon $e(t)$.



Sve eventualne pogreške molim da javite isključivo privatnom porukom.

Rješenje:

$$e(t) = \begin{cases} \frac{A}{T}t & \text{ako } t \in (0, T) \\ 0 & \text{inače} \end{cases}$$

$$e(t) = \frac{A}{T}t \cdot u(t) - \frac{A}{T}t \cdot u(t-T) = \frac{A}{T}t \cdot u(t) - \frac{A}{T}(t-T+T) \cdot u(t-T) = \frac{A}{T}t \cdot u(t) - \frac{A}{T}(t-T) \cdot u(t-T) - A \cdot u(t-T)$$

$$E(s) = \frac{A}{T}\frac{1}{s^2} - \frac{A}{T}\frac{1}{s^2}e^{-Ts} - \frac{A}{s}e^{-Ts}$$

$$\begin{aligned} I(s) = \frac{E(s)}{Z(s)} &= \frac{1}{LS}\left(\frac{A}{T}\frac{1}{s^2} - \frac{A}{T}\frac{1}{s^2}e^{-Ts} - \frac{A}{s}e^{-Ts}\right) \\ &= \frac{A}{LT}\frac{1}{s^3} - \frac{A}{LT}\frac{1}{s^3}e^{-Ts} - \frac{A}{L}\frac{1}{s^2}e^{-Ts} \end{aligned}$$

$$i(t) = \frac{1}{2}\frac{A}{LT}t^2 - \frac{1}{2}\frac{A}{LT}(t-T)^2u(t-T) - \frac{A}{L}(t-T)u(t-T) = \frac{1}{T}\left(\frac{A}{2L}t^2 - \frac{A}{2L}(t-T)^2u(t-T) - \frac{AT}{L}(t-T)u(t-T)\right)$$

Sve eventualne pogreške molim da javite isključivo privatnom porukom.