PZI 08/09

1.

I. $x-2y=3$ → RAVNINA

$\frac{x-3}{2}=y$

II. $z^2-y=1$ → OVALNA CIJEV

NEKAKVA ŠTO SE PROTEŽU DUŽ x-OSI

$y=z^2-1$

$z=t$

$y=t^2-1$

$x=3+2t^2-2$

→ PRESJEK JE NEKA MALO RASTEGNUTA PARABOLA U xz RAVNINI

$T(3+2t^2-2,\ t^2-1,\ t)=A(3,0,1)$

$3+2t^2-2=3 \qquad t^2-1=0 \qquad t=1$

2.

$$\nabla\left(\frac{1}{\vec{a}\cdot\vec{r}}\right)=-\frac{1}{(\vec{a}\cdot\vec{r})^2}\,\nabla(\vec{a}\cdot\vec{r}) \qquad \nabla(f(r))=\frac{df}{dr}\cdot\nabla r$$

$$=-\frac{\vec{a}}{(\vec{a}\cdot\vec{r})^2}$$

3.

Presjek $x^2=3y \qquad 2xy=9z \qquad x=t$

$y=\frac{t^2}{3}$

$$z=\frac{2\cdot t\frac{t^2}{3}}{9}=\frac{2t^3}{27}$$

$A(0,y_A,z_A) \qquad B(3,y_B,z_B)$

$t\,[0,3]$

$$ds=\sqrt{(t')^2+\left(\frac{t^2}{3}'\right)^2+\left(\frac{2t^3}{27}'\right)^2}\,dt$$

$$ds=\sqrt{1+\frac{4t^2}{9}+\left(\frac{6t^2}{27}\right)^2}$$

$$ds=\sqrt{1+\frac{4}{9}t^2+\frac{4}{81}t^4}\,dt$$

$$ds=\frac{1}{9}\sqrt{81+36t^2+4t^4}$$

$$\int_0^3 ds=\frac{1}{9}\int_0^3\sqrt{(2t^2+9)^2}\,dt$$

$$\frac{1}{9}\int_0^3(2t^2+9)\,dt=\frac{1}{9}\left[\frac{2}{3}t^3\Big|_0^3+9\cdot t\Big|_0^3\right]$$

$$=\frac{1}{9}\left[\frac{2\cdot 9^3}{3}+9\cdot 3\right]$$

$$=2+3=5$$

4.

a) $\operatorname{rot} \vec{a} = 0$

b) $$f(x,y,z) = \frac{1}{xyz}\left(yz\,\vec{i} + xz\,\vec{j} + xy\,\vec{k}\right) = \frac{1}{x}\vec{i} + \frac{1}{y}\vec{j} + \frac{1}{z}\vec{k}$$

$$\begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ \frac{1}{x} & \frac{1}{y} & \frac{1}{z} \end{vmatrix} = 0$$

c) potencijal:

$$T(x_0, y_0, z_0) = (1,1,1)$$

$$\int_{x_0}^{x} f(x,y,z)\,dx + \int_{y_0}^{y} f(x_0,y,z)\,dy + \int_{z_0}^{z} f(x_0,y_0,z)\,dz$$

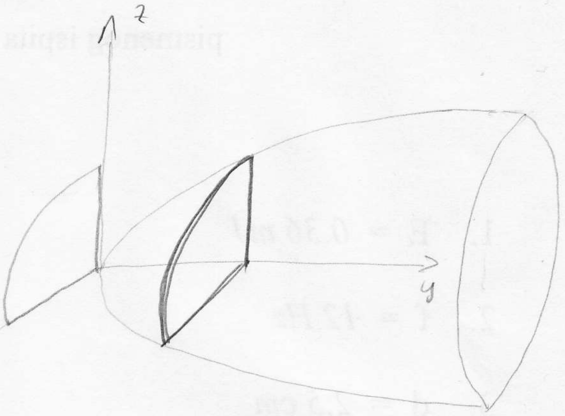
$$= \int_1^x \frac{1}{x}\,dx + \int_1^y \frac{1}{y}\,dy + \int_1^z \frac{1}{z}\,dz = \ln x - \cancel{\ln 1} + \ln y - \ln 1 + \ln z + \ln 1$$

$$= \ln xyz + C$$

$$\ln a \cdot b = \ln a + \ln b$$

d) $P(B) - P(A) = \ln(2 \cdot 3 \cdot 4) - \ln(1 \cdot 1 \cdot 1) = \ln 24$ //

5. $\iint_S dS \qquad y = x^2 + z^2 \qquad 0 \le x \le 1 \qquad 0 \le y \le 1 \qquad 0 \le z \le 1$



$$\iint_{\Omega xz} \sqrt{1+4(x^2+y^2)} = \int_0^{\frac{\pi}{2}} d\varphi \int_0^1 r\cdot\sqrt{1+4r^2}$$

$$\left|\begin{array}{l} 1+4r^2 = t \\ 8r\,dr = dt \\ r\,dr = \frac{dt}{8} \\ 1 \to 5 \end{array}\right|$$

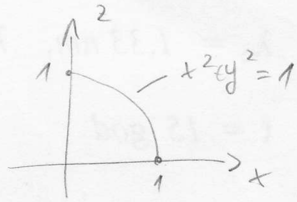
$$= \frac{\pi}{2} \int_1^5 \sqrt{t}\;\frac{dt}{8} = \frac{\pi}{\cancel{16}}\cdot\frac{\cancel{2}}{3}\cdot t^{\frac{3}{2}}\Big|_1^5$$

$$= \frac{\pi}{24}\left(5\sqrt{5}-1\right)$$

$x = r\cos\varphi$

$y = r\sin\varphi$

$|J| = r$



$$dS = \sqrt{1+\left(\frac{\partial y}{\partial x}\right)^2+\left(\frac{\partial y}{\partial z}\right)^2}$$

$$dS = \sqrt{1+4x^2+4y^2}$$

6. $\vec{a} = x^2\vec{i} + y^2\vec{j} + z\vec{k}$

S je rub tijela → dakle S je već zatvorena jer tijelo nije "šuplje"

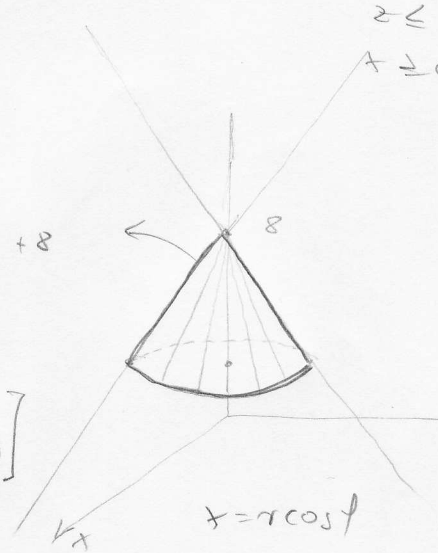
$\operatorname{div}\vec{a} = 2(x+y+1)$

$$V = \left\{(x,y,z)\in\mathbb{R}^3 \;(z-8)^2 \ge x^2+y^2,\; z\ge 4,\; z\le 8,\; x\ge 0\right\}$$

$$I = 2\iiint_V (x+y+z)\,dx\,dy\,dz =$$

$z = -\sqrt{x^2+y^2}+8$

$$= 2\cdot\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} d\varphi \int_0^4 dr \int_4^{-\sqrt{r^2}+8} r\cdot\left(r\cos\varphi + r\sin\varphi + 1\right)dz$$



$$= 2\cdot\int d\varphi \int_0^4 dr\left[\left(r^2\cos\varphi + r^2\sin\varphi + r\right)\cdot\left(8 - r - 4\right)\right]$$

$x = r\cos\varphi$

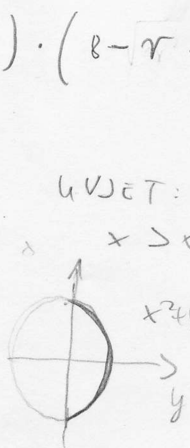
$y = r\sin\varphi$

$z = z$

$= \;\vdots$

UVJET:

$x > 0$



7.

$$\oint_c \vec V\, d\vec r \qquad \vec V = -3y\,\vec i + 3x\,\vec j + z^4\,\vec k$$

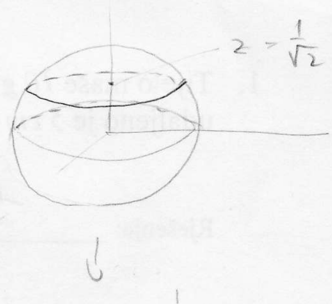
c je presječnica

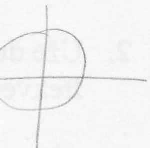
$$2x^2+2y^2+z^2=1$$

$$z=\frac{1}{\sqrt2} \quad \oplus \text{ orjentirana}$$

$$\operatorname{rot}\vec V = \begin{vmatrix} \vec i & \vec j & \vec k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ -3y & 3x & z^4 \end{vmatrix}\begin{matrix} \vec i & \vec j \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} \\ -3y & 3x \end{matrix} = 0+0+3k-(-3k) = 6\vec k$$

$$\oint_c = \iint_S 6\,dx\,dy = 6\int_0^{2\pi} d\varphi \int_0^{\frac12} r\,dr = 12\pi\cdot\frac18 = \frac{3\pi}{2}$$





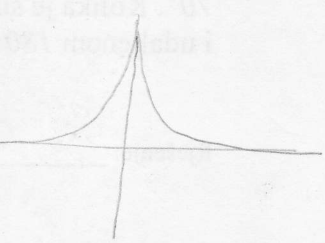
$$2x^2+2y^2+\left(\frac{1}{\sqrt2}\right)^2 = 1$$

$$2x^2+2y^2 = \frac12 \quad /:2$$

$$x^2+y^2 = \left(\frac12\right)^2$$

8. $f(x)=e^{-|x|}$

$$A(\lambda) = \frac{2}{\pi}\int_0^\infty e^{-\varphi}\cos\lambda\varphi\, d\varphi \quad \left| \begin{matrix} e^{-\varphi}=u & du=-e^{-\varphi} \\ dv=\cos\lambda\varphi & v=\frac{1}{\lambda}\sin\lambda\varphi \end{matrix} \right|$$



$$= \frac{2}{\pi}\left[e^{-\varphi}\frac{1}{\lambda}\sin\lambda\varphi \Big|_0^\infty + \frac{1}{\lambda}\int_0^\infty e^{-\varphi}\sin\lambda\varphi \right] \quad \left| \begin{matrix} e^{-\varphi}=u & du=-e^{-\varphi} \\ dv=\sin\lambda\varphi & v=-\frac{1}{\lambda}\cos\lambda\varphi \end{matrix} \right|$$

$$= \frac{2}{\pi}\left[0+(-)\frac{1}{\lambda^2}e^{-\varphi}\cos\lambda\varphi \Big|_0^\infty - \frac{1}{\lambda^2}\int_0^\infty e^{-\varphi}\cos\lambda\varphi\, d\varphi \right]$$

$$I = \int_0^\infty e^{-\varphi}\cos\lambda\varphi\, d\varphi$$

$$= \frac{2}{\pi}\left[-\frac{1}{\lambda^2} - \frac{1}{\lambda^2} I = I \right]$$

$$= \frac{2}{\pi}\left[\frac{-\frac{1}{\lambda^2}}{\frac{1}{\lambda^2}+\frac{\lambda^2}{\lambda^2}} = I \right] = \frac{2}{\pi}\cdot\frac{1}{(1+\lambda^2)}$$

$$= \frac{2}{\pi(1+\lambda^2)}$$

$$f(x) = \frac{2}{\pi}\int_0^\infty \frac{\cos\lambda x}{(1+\lambda^2)}\, dx$$

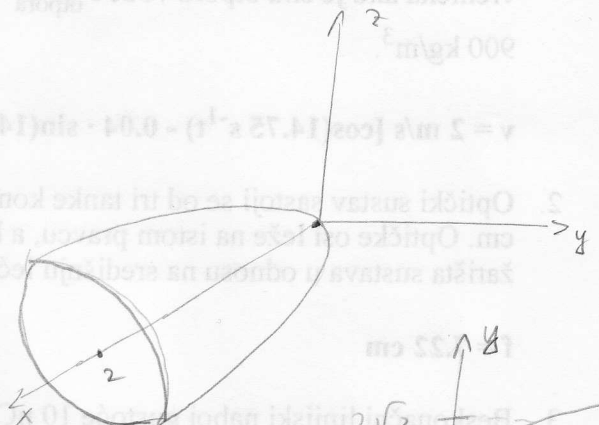
(9)

c) $f(t) = (t-1)^2 \, u(t-1) = \frac{2!}{s^3} \cdot e^{-s}$

a) $f(t) = t^n \longrightarrow F(s) = \int_0^{\infty} e^{-st} t^n \, dt \quad \ldots$

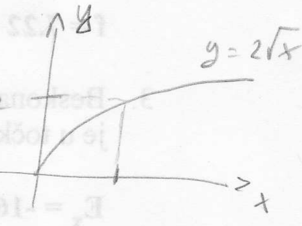
(10)

$$\int_0^2 dx \int_0^{2\sqrt{x}} dy \int_0^{\sqrt{(4x-y^2)/2}} x \, dz$$



$$z = \sqrt{2x - \frac{y^2}{2}} \; \Big| ^2$$

$$z^2 = 2x - \frac{y^2}{2}$$



$$\frac{z^2}{2} + \frac{y^2}{4} = x \longrightarrow u \; z \quad z = 0 \rightarrow xy \text{ ravnina}$$

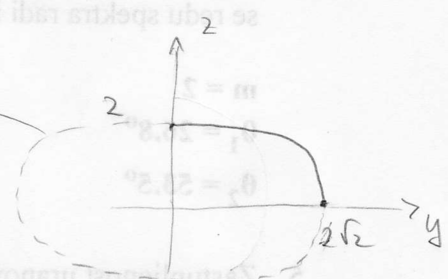
imamo $y^2 = 4x$

$y = 2\sqrt{x}$

za $x = 2$

$$\frac{z^2}{2} + \frac{y^2}{4} = 2 \; \Big| : 2$$

$$\frac{z^2}{4} + \frac{y^2}{8} = 1$$



$y = 2\sqrt{2} \cos\varphi$

$z = 2 \sin\varphi$

$|J| = 4\sqrt{2}\, r$

$$\int_0^{\frac{\pi}{2}} d\varphi \int_0^1 dr \int_{\sqrt{2r^2}}^{2} 4\sqrt{2}\, r \cdot x \, dx =$$

$$= \frac{\pi}{\cancel{2}} \cdot \cancel{4}\sqrt{2} \int_0^1 r \int_{\sqrt{2}\,r}^{2} x \, dx =$$

$$= 2\pi\sqrt{2} \int_0^1 r \cdot (2 - r^2) \, dr$$

$$= 2\pi\sqrt{2} \left[r^2 \Big|_0^1 - \frac{1}{3} r^3 \Big|_0^1 \right] =$$

$$= 2\pi\sqrt{2} \left[1 - \frac{1}{3} \right] = \frac{4\pi}{3} \sqrt{2} \quad //$$