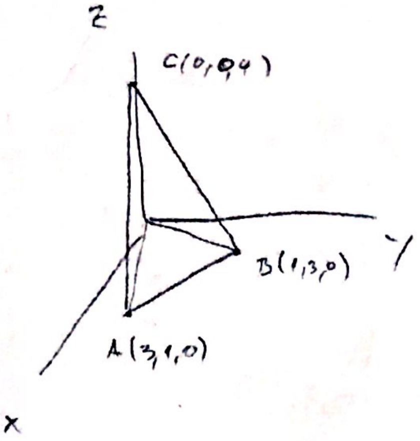
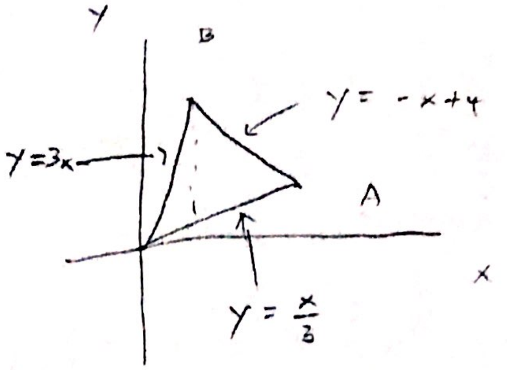
1.) $\iiint_V f(x,y,z)\,dx\,dy\,dz$





π... $\vec{n} = \vec{CA} \times \vec{CB} = (3,1,-4) \times (1,3,-4) = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 3 & 1 & -4 \\ 1 & 3 & -4 \end{vmatrix}$

$= \vec{i}(-4+12) - \vec{j}(-12+4) + \vec{k}(9-1)$

$= 8\vec{i} + 8\vec{j} + 8\vec{k} \sim \vec{i} + \vec{j} + \vec{k}$

π... $x + y + (z-4) = 0 \Rightarrow \boxed{z = 4-x-y}$

$$\iiint_V f(x,y,z)\,dx\,dy\,dz = \int_0^1 dx \int_{\frac{x}{3}}^{3x} dy \int_0^{4-x-y} f(x,y,z)\,dz + \int_1^3 dx \int_{x/3}^{4-x} dy \int_0^{4-x-y} f(x,y,z)\,dz$$

Scanned by CamScanner

2.) b) $\vec{f}(x,y,z) = xy\vec{i} - xz^2\vec{j} + z^3\vec{k}$

$$\vec{n} = \frac{\nabla \varphi}{\|\nabla \varphi\|}\Bigg|_T \quad , \quad \nabla\varphi = yz\,\vec{i} + xz\,\vec{j} + xy\,\vec{k}$$

$$\nabla\varphi(1,1,1) = \vec{i} + \vec{j} + \vec{k}$$

$$\Rightarrow \vec{n} = \frac{1}{\sqrt{3}}(\vec{i} + \vec{j} + \vec{k})$$

$$\frac{\partial \vec{f}}{\partial \vec{n}} = (\vec{n}\cdot\nabla)\vec{f} = \frac{1}{\sqrt{3}}\left(\frac{\partial}{\partial x} + \frac{\partial}{\partial y} + \frac{\partial}{\partial z}\right)\vec{f}$$

$$= \frac{1}{\sqrt{3}}\Big((y+x)\vec{i} - (z^2 + 2xz)\vec{j} + 3z^2\vec{k}\Big)$$

$$\frac{\partial \vec{f}}{\partial \vec{n}}\Bigg|_T = \frac{1}{\sqrt{3}}\left(2\vec{i} - 3\vec{j} + 3\vec{k}\right)$$

3.) b.) $\nabla\left[\nabla\times(3\vec{a}\cdot\vec{r})\vec{r}+3\vec{r}\right]$

$$= \nabla\left[3\nabla(\vec{a}\cdot\vec{r})\times\vec{r}+3(\vec{a}\cdot\vec{r})(\nabla\times\vec{r})+3\vec{r}\right]$$

$$= \nabla\left[3\vec{a}\times\vec{r}+0+3\vec{r}\right]$$

$$= 3\left[\nabla\cdot(\vec{a}\times\vec{r})+\nabla\vec{r}\right] = 3\left((\nabla,\vec{a},\vec{r})+3\right)$$

$$= 3\Big(\underbrace{-(\vec{a},\nabla,\vec{r})}_{=-\vec{a}\cdot\underbrace{(\nabla\times\vec{r})}_{=0}}+3\Big) = \underline{\underline{9}}$$

4.

$$\oint_{\Gamma} \sqrt{x^2+y^2}\, ds$$

$$x^2+y^2=cx \,, \quad c>0$$

$$\left(x-\frac{c}{2}\right)^2+y^2=\frac{c^2}{4}$$

$$x=\frac{c}{2}+\frac{c}{2}\cos t \,, \quad y=\frac{c}{2}\sin t \,, \quad t\in[0,2\pi].$$

$$ds=\sqrt{\dot{x}^2+\dot{y}^2}=\sqrt{\frac{c^2}{4}\sin^2 t+\frac{c^2}{4}\cos^2 t}=\frac{c}{2}.$$

$$\oint_{\Gamma}\sqrt{x^2+y^2}\,dS=\int_0^{2\pi}\sqrt{\frac{c^2}{4}+\frac{c^2}{2}\cos t+\frac{c^2}{4}\cos^2 t+\frac{c^2}{4}\sin^2 t}\cdot\frac{c}{2}\,dt=$$

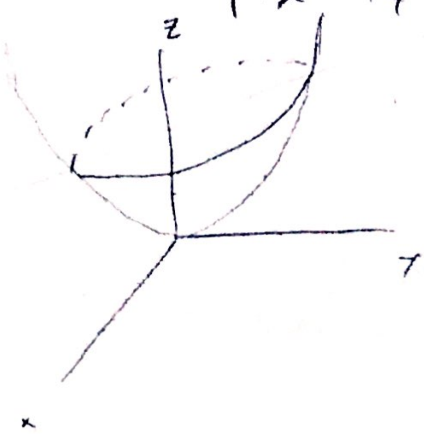
$$\frac{c^2}{4}\int_0^{2\pi}\sqrt{2+2\cos t}\,dt=\frac{c^2\sqrt{2}}{4}\int_0^{2\pi}\sqrt{1+\cos t}\,dt=\frac{c^2\sqrt{2}}{4}\int_0^{2\pi}\sqrt{2\cos^2\frac{t}{2}}\,dt$$

$$=\frac{c^2}{2}\int_0^{2\pi}\left|\cos\frac{t}{2}\right|dt=\frac{c^2}{2}\int_0^{\pi}\cos\frac{t}{2}\,dt-\frac{c^2}{2}\int_{\pi}^{2\pi}\cos\frac{t}{2}\,dt=$$

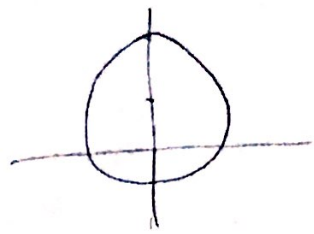
$$=c^2\sin\frac{t}{2}\Big|_0^{\pi}-c^2\sin\frac{t}{2}\Big|_{\pi}^{2\pi}=c^2(1-0)-c^2(0-1)=2c^2$$

Scanned by CamScanner

5.) $$\iint_S \frac{y\,dS}{\sqrt{4x^2+4y^2+1}}, \quad z = x^2+y^2, \quad z = 2y+3.$$



$$\Omega_{xy} \ldots \quad x^2+y^2 = 2y+3$$
$$x^2+(y-1)^2 = 4$$



$$dS = \sqrt{1+\left(\frac{\partial z}{\partial x}\right)^2+\left(\frac{\partial z}{\partial y}\right)^2} = \sqrt{1+4x^2+4y^2}$$

$$\iint_S \frac{y\,dS}{\sqrt{4x^2+4y^2+1}} = \int_{\Omega_{xy}} y\,dx\,dy = \left| \begin{array}{l} x = r\cos\varphi \\ y = 1+r\sin\varphi \\ |J| = r \end{array} \right|$$

$$= \int_0^{2\pi} d\varphi \int_0^2 r(1+r\sin\varphi)\,dr = \int_0^{2\pi} \left(\frac{r^2}{2}+\frac{r^3}{3}\sin\varphi\right)\Big|_0^2 d\varphi$$

$$= \int_0^{2\pi} \left(2+\frac{8}{3}\sin\varphi\right) d\varphi = \underline{\underline{4\pi}}$$

Scanned by CamScanner

6.) $\oiint_S x^3\,dy\,dz + \frac{y^3}{4}\,dx\,dz + \frac{3z^3}{4}\,dx\,dy$

S ... vanjska strana $x^2 + \frac{y^2}{4} + \frac{3}{4}z^2 = 1$

$$\operatorname{div}\vec{f} = 3x^2 + \frac{3}{4}y^2 + \frac{9}{4}z^2$$

$$\oiint_S \vec{f}\cdot d\vec{S} = \iiint_V \operatorname{div}\vec{f}\,dV = \iiint_V \left(3x^2 + \frac{3}{4}y^2 + \frac{9}{4}z^2\right)dV$$

$x = r\sin\theta\cos\varphi$

$y = 2r\sin\theta\sin\varphi$

$z = \frac{2}{\sqrt{3}}r\cos\theta$

$|J| = \frac{4}{\sqrt{3}}r^2\sin\theta$

$$3x^2 + \frac{3}{4}y^2 + \frac{9}{4}z^2 = 3r^2\sin^2\theta\cos^2\varphi + 3r^2\sin^2\theta\sin^2\varphi + 3r^2\cos^2\theta = 3r^2$$

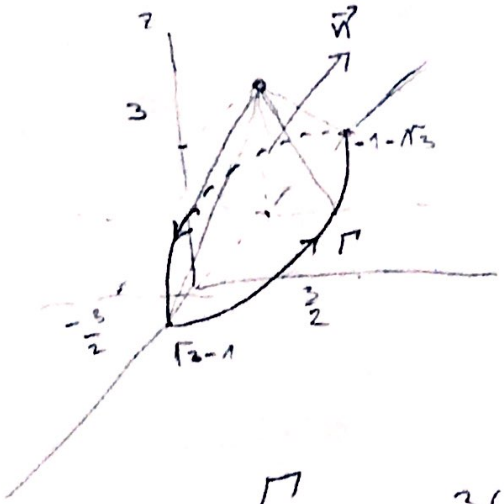
$$\iiint_V \operatorname{div}\vec{f}\,dV = \int_0^{2\pi} d\varphi \int_0^{\pi}\int_0^{1} \frac{4}{\sqrt{3}}r^2\sin\theta\cdot 3r^2\,d\theta\,dr$$

$$= \frac{12}{\sqrt{3}}\cdot 2\pi \int_0^{\pi}\sin\theta\,d\theta \int_0^1 r^4\,dr$$

$$= \frac{24\pi}{\sqrt{3}}\underbrace{(-\cos\theta)\Big|_0^{\pi}}_{=2}\ \frac{r^5}{5}\Big|_0^1$$

$$= \frac{48\pi}{5\sqrt{3}}$$

7.) $\vec{v}(x,y,z) = \text{rot}\left(xz^2\vec{i} + x\vec{j} + y^3\vec{k}\right)$



$$\iint_S \text{rot}\,\vec{f}\cdot d\vec{S}$$

$$= \oint_\Gamma \vec{f}\cdot d\vec{r}$$

$\Gamma \ldots \quad 3(x+1)^2 + 4y^2 = 9 \quad /:12$

$$\left(\frac{x+1}{2}\right)^2 + \left(\frac{y}{\sqrt{3}}\right)^2 = \frac{3}{4}$$

$x = -1 + \sqrt{3}\cos t$

$y = \frac{1}{2}\sin t$

$z = 0 \qquad , t \in [0, 2\pi]$

$dx = -\sqrt{3}\sin t\, dt$

$dy = \frac{3}{2}\cos t\, dt$

$dz = 0$

$$\oint_\Gamma \vec{f}\cdot d\vec{r} = \oint_\Gamma xz^2dx + xdy + y^3dz$$

$$= \int_0^{2\pi} 0\cdot dx + (\sqrt{3}\cos t - 1)\cdot\frac{1}{2}\cos t\,dt + y^3\cdot 0$$

$$= \frac{\sqrt{3}}{2}\int_0^{2\pi}\cos^2 t\,dt - \underbrace{\frac{1}{2}\int_0^{2\pi}\cos t\,dt}_{=0}$$

$$= \frac{\sqrt{3}}{2}3\int_0^{2\pi}\left(\frac{1}{2} + \frac{1}{2}\cos 2t\right)dt$$

$$= 3\frac{\sqrt{3}}{4}\cdot 2\pi + 3\cdot\frac{\sqrt{3}}{4}\int_0^{2\pi}\cos 2t\,dt$$

$$= 3\frac{\pi\sqrt{3}}{2} + 3\cdot\frac{\sqrt{3}}{8}\sin 2t\Big|_0^{2\pi} = \frac{3\pi\sqrt{3}}{2}$$