Zadaci za vježbu

Fourierov integral

by Vedax

53. str.

2. Prikaži Fourierovim integralom funkciju

$$f(x)=\begin{cases}1-\dfrac{x}{2}, & 0\le x\le 2,\\ 0, & x>2,\end{cases}$$

produživši je do parne funkcije na ***R***.

Ako funkciju produžujemo do parne, onda vrijedi:

$f(x)=f(-x)=1+\dfrac{x}{2}$, na intervalu $-2\le x\le 0$

Pošto smo funkciju produžili do parne, vrijedi da nam je $B(\lambda)=0$. Izračunajmo $A(\lambda)$.

$$A(\lambda)=\frac{2}{\pi}\int_0^{\infty} f(\varphi)\cos\lambda\varphi\, d\varphi=\frac{2}{\pi}\int_0^{2}\left(1-\frac{\varphi}{2}\right)\cos\lambda\varphi\, d\varphi=\frac{2}{\pi}\left(\int_0^{2}\cos\lambda\varphi\, d\varphi-\frac{1}{2}\int_0^{2}\varphi\cos\lambda\varphi\, d\varphi\right)=$$

$$=\left|\begin{matrix} u=\varphi & dv=\cos\lambda\varphi\, d\varphi\\ du=d\varphi & v=\dfrac{\sin\lambda\varphi}{\lambda}\end{matrix}\right|=\frac{2}{\pi}\left(\frac{\sin\lambda\varphi}{\lambda}\Bigg|_0^2-\frac{1}{2}\left(\frac{\varphi\sin\lambda\varphi}{\lambda}\Bigg|_0^2-\int_0^2\frac{\sin\lambda\varphi}{\lambda}d\varphi\right)\right)=$$

$$=\frac{2}{\pi}\left(\frac{\sin 2\lambda}{\lambda}-\frac{1}{2}\left(\frac{2\sin 2\lambda}{\lambda}+\frac{1}{\lambda^2}\cos\lambda\varphi\Bigg|_0^2\right)\right)=\frac{2}{\pi}\left(\frac{\sin 2\lambda}{\lambda}-\frac{\sin 2\lambda}{\lambda}-\frac{1}{2}\frac{1}{\lambda^2}(\cos 2\lambda-1)\right)=$$

$$=\frac{1-\cos 2\lambda}{\pi\lambda^2}=\frac{2\sin^2\lambda}{\pi\lambda^2}$$

Pa nam je Fourierov integral:

$$f(x)=\int_0^{\infty}\frac{2\sin^2\lambda}{\pi\lambda^2}\cos\lambda x\, d\lambda=\frac{2}{\pi}\int_0^{\infty}\frac{\sin^2\lambda}{\lambda^2}\cos\lambda x\, d\lambda$$

3. Prikaži u obliku Fourierovog integrala:

A. $f(x)=e^{-\alpha|x|}\sin\beta x$, $\alpha>0$

$f(-x)=e^{-\alpha|-x|}\sin(-\beta x)=-e^{-\alpha|x|}\sin\beta x=-f(x)$

Funkcija je neparna pa imamo $A(\lambda)=0$. Izračunajmo $B(\lambda)$.

$$B(\lambda)=\frac{2}{\pi}\int_0^\infty f(\varphi)\sin\lambda\varphi\,d\varphi=\frac{2}{\pi}\int_0^\infty e^{-\alpha|\varphi|}\sin\beta\varphi\sin\lambda\varphi\,d\varphi=\frac{2}{\pi}\int_0^\infty e^{-\alpha|\varphi|}\frac{\cos(\beta-\lambda)\varphi-\cos(\beta+\lambda)\varphi}{2}d\varphi=$$
$$=\frac{1}{\pi}\left(\int_0^\infty e^{-\alpha|\varphi|}\cos(\beta-\lambda)\varphi\,d\varphi-\int_0^\infty e^{-\alpha|\varphi|}\cos(\beta+\lambda)\varphi\,d\varphi\right)$$

$$\int e^{-\alpha|\varphi|}\cos(\beta-\lambda)\varphi\,d\varphi=\left|\begin{matrix} u=e^{-\alpha|\varphi|} & dv=\cos(\beta-\lambda)\varphi\,d\varphi \\ du=-\alpha e^{-\alpha|\varphi|}d\varphi & v=\dfrac{\sin(\beta-\lambda)\varphi}{\beta-\lambda}\end{matrix}\right|=$$
$$=\frac{e^{-\alpha|\varphi|}\sin(\beta-\lambda)\varphi}{\beta-\lambda}+\frac{\alpha}{\beta-\lambda}\int e^{-\alpha|\varphi|}\sin(\beta-\lambda)\varphi\,d\varphi=\left|\begin{matrix} u=e^{-\alpha|\varphi|} & dv=\sin(\beta-\lambda)\varphi\,d\varphi \\ du=-\alpha e^{-\alpha|\varphi|}d\varphi & v=-\dfrac{\cos(\beta-\lambda)\varphi}{\beta-\lambda}\end{matrix}\right|=$$
$$=\frac{e^{-\alpha|\varphi|}\sin(\beta-\lambda)\varphi}{\beta-\lambda}+\frac{\alpha}{\beta-\lambda}\left(-\frac{e^{-\alpha|\varphi|}\cos(\beta-\lambda)\varphi}{\beta-\lambda}-\frac{\alpha}{\beta-\lambda}\int e^{-\alpha|\varphi|}\cos(\beta-\lambda)\varphi\,d\varphi\right)=$$
$$=\frac{e^{-\alpha|\varphi|}\sin(\beta-\lambda)\varphi}{\beta-\lambda}-\frac{\alpha e^{-\alpha|\varphi|}\cos(\beta-\lambda)\varphi}{(\beta-\lambda)^2}-\frac{\alpha^2}{(\beta-\lambda)^2}\int e^{-\alpha|\varphi|}\cos(\beta-\lambda)\varphi\,d\varphi$$
$$\left(1+\frac{\alpha^2}{(\beta-\lambda)^2}\right)\int e^{-\alpha|\varphi|}\cos(\beta-\lambda)\varphi\,d\varphi=\frac{e^{-\alpha|\varphi|}\sin(\beta-\lambda)\varphi}{\beta-\lambda}-\frac{\alpha e^{-\alpha|\varphi|}\cos(\beta-\lambda)\varphi}{(\beta-\lambda)^2}$$

$$\int_0^\infty e^{-\alpha|\varphi|}\cos(\beta-\lambda)\varphi\,d\varphi=\frac{(\beta-\lambda)e^{-\alpha|\varphi|}\sin(\beta-\lambda)\varphi\Big|_0^\infty-\alpha e^{-\alpha|\varphi|}\cos(\beta-\lambda)\varphi\Big|_0^\infty}{\alpha^2+(\beta-\lambda)^2}=$$
$$=\frac{\alpha}{\alpha^2+(\beta-\lambda)^2}$$

Analogno tome, izračunamo da je $\int_0^\infty e^{-\alpha|\varphi|}\cos(\beta+\lambda)\varphi\,d\varphi$ jednak:

$$\int_0^\infty e^{-\alpha|\varphi|}\cos(\beta+\lambda)\varphi\, d\varphi = \frac{(\beta+\lambda)e^{-\alpha|\varphi|}\sin(\beta+\lambda)\varphi\Big|_0^\infty - \alpha e^{-\alpha|\varphi|}\cos(\beta+\lambda)\varphi\Big|_0^\infty}{\alpha^2+(\beta+\lambda)^2} =$$

$$= \frac{\alpha}{\alpha^2+(\beta+\lambda)^2}$$

Pa imamo:

$$B(\lambda) = \frac{1}{\pi}\left(\frac{\alpha}{\alpha^2+(\beta-\lambda)^2} - \frac{\alpha}{\alpha^2+(\beta+\lambda)^2}\right) = \frac{1}{\pi}\frac{\alpha^3+\alpha\beta^2+2\alpha\beta\lambda+\alpha\lambda^2-\alpha^3-\alpha\beta^2+2\alpha\beta\lambda-\alpha\lambda^2}{\left(\alpha^2+(\beta-\lambda)^2\right)\left(\alpha^2+(\beta+\lambda)^2\right)} =$$

$$= \frac{4\alpha\beta}{\pi}\frac{\lambda}{\left(\alpha^2+(\beta-\lambda)^2\right)\left(\alpha^2+(\beta+\lambda)^2\right)}$$

I zapišemo kao Fourierov integral:

$$f(x) = \int_0^\infty \frac{4\alpha\beta}{\pi}\frac{\lambda}{\left(\alpha^2+(\beta-\lambda)^2\right)\left(\alpha^2+(\beta+\lambda)^2\right)}\sin\lambda x\, d\lambda = \frac{4\alpha\beta}{\pi}\int_0^\infty \frac{\lambda\sin\lambda x\, d\lambda}{\left(\alpha^2+(\beta-\lambda)^2\right)\left(\alpha^2+(\beta+\lambda)^2\right)}$$

B. $f(x)=\cos x$, $0\le x\le \pi$

Funkcija je parna.

$B(\lambda)=0$

$$A(\lambda)=\frac{2}{\pi}\int_0^{\infty} f(\varphi)\cos\lambda\varphi d\varphi=\frac{2}{\pi}\int_0^{\pi}\cos\varphi\cos\lambda\varphi d\varphi=\frac{1}{\pi}\int_0^{\pi}(\cos(1+\lambda)\varphi+\cos(1-\lambda)\varphi)d\varphi=$$

$$=\frac{1}{\pi}\left(\int_0^{\pi}\cos(1+\lambda)\varphi\, d\varphi+\int_0^{\pi}\cos(1-\lambda)\varphi\, d\varphi\right)=\frac{1}{\pi}\left(\frac{1}{1+\lambda}\sin(1+\lambda)\varphi\Big|_0^{\pi}+\frac{1}{1-\lambda}\sin(1-\lambda)\varphi\Big|_0^{\pi}\right)=$$

$$=\frac{1}{\pi}\frac{(1-\lambda)\sin(\pi+\lambda\pi)+(1+\lambda)\sin(\pi-\lambda\pi)}{1-\lambda^2}=\frac{1}{\pi}\frac{(1-\lambda)(-\sin(\lambda\pi))+(1+\lambda)\sin(\lambda\pi)}{1-\lambda^2}=$$

$$=\frac{1}{\pi}\frac{-\sin(\lambda\pi)+\lambda\sin(\lambda\pi)+\sin(\lambda\pi)+\lambda\sin(\lambda\pi)}{1-\lambda^2}=\frac{1}{\pi}\frac{2\lambda\sin(\lambda\pi)}{1-\lambda^2}$$

Pa je Fourierov integral jednak:

$$f(x)=\int_0^{\infty}\frac{1}{\pi}\frac{2\lambda\sin(\lambda\pi)}{1-\lambda^2}\cos\lambda x\, d\lambda=\frac{2}{\pi}\int_0^{\infty}\frac{\lambda\sin(\lambda\pi)}{1-\lambda^2}\cos\lambda x\, d\lambda$$

4. Funkciju

$$f(x)=\begin{cases} x, & x\in[-1,1], \\ 0, & x\notin[-1,1] \end{cases}$$

Razvij u Fourierov integral.

Funkcija je neparna, jer je simetrična s obzirom na ishodište.

$A(\lambda)=0$

$$B(\lambda)=\frac{2}{\pi}\int_0^{\infty} f(\varphi)\sin\lambda\varphi\,d\varphi=\frac{2}{\pi}\int_0^{1}\varphi\sin\lambda\varphi\,d\varphi=\begin{vmatrix} u=\varphi & dv=\sin\lambda\varphi\,d\varphi \\ du=d\varphi & v=-\dfrac{\cos\lambda\varphi}{\lambda}\end{vmatrix}=$$

$$=\frac{2}{\pi}\left(-\frac{\varphi\cos\lambda\varphi}{\lambda}\Bigg|_0^1+\frac{1}{\lambda}\int_0^1\cos\lambda\varphi\,d\varphi\right)=\frac{2}{\pi}\left(-\frac{\cos\lambda}{\lambda}+\frac{1}{\lambda^2}\sin\lambda\varphi\Bigg|_0^1\right)=\frac{2}{\pi}\left(-\frac{\cos\lambda}{\lambda}+\frac{1}{\lambda^2}\sin\lambda\right)=$$

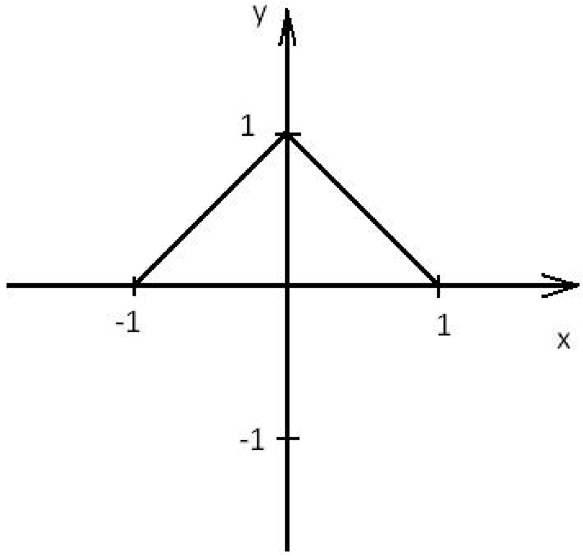
$$=\frac{2}{\pi}\frac{\sin\lambda-\lambda\cos\lambda}{\lambda^2}$$

Pa je Fourierov integral jednak:

$$f(x)=\int_0^{\infty}\frac{2}{\pi}\frac{\sin\lambda-\lambda\cos\lambda}{\lambda^2}\sin\lambda x\,d\lambda=\frac{2}{\pi}\int_0^{\infty}\frac{\sin\lambda-\lambda\cos\lambda}{\lambda^2}\sin\lambda x\,d\lambda$$

5. Funkciju

$$f(x)=\begin{cases}1-x, & x\in(0,1),\\ 1+x, & x\in(-1,0),\\ 0, & \text{inace}\end{cases}$$



Iz slike vidimo da je funkcija parna, pa računamo samo $A(\lambda)$.

$$A(\lambda)=\frac{2}{\pi}\int_0^{\infty} f(\varphi)\cos\lambda\varphi\,d\varphi=\frac{2}{\pi}\int_0^1(1-\varphi)\cos\lambda\varphi\,d\varphi=\frac{2}{\pi}\left(\int_0^1\cos\lambda\varphi\,d\varphi-\int_0^1\varphi\cos\lambda\varphi\,d\varphi\right)=$$

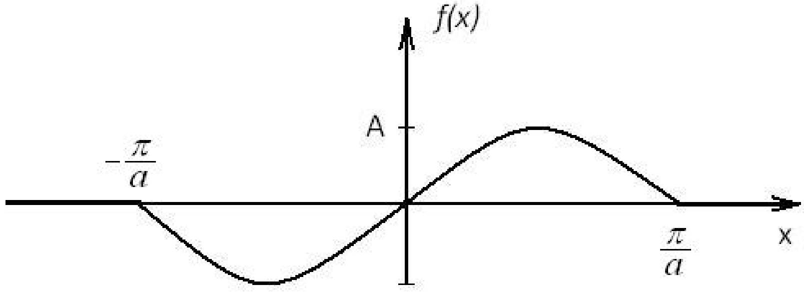
$$=\left|\begin{matrix}u=\varphi & dv=\cos\lambda\varphi\,d\varphi\\ du=d\varphi & v=\dfrac{\sin\lambda\varphi}{\lambda}\end{matrix}\right|=\frac{2}{\pi}\left(\frac{\sin\lambda\varphi}{\lambda}\Bigg|_0^1-\left(\frac{\varphi\sin\lambda\varphi}{\lambda}\Bigg|_0^1-\frac{1}{\lambda}\int_0^1\sin\lambda\varphi\,d\varphi\right)\right)=$$

$$=\frac{2}{\pi}\left(\frac{\sin\lambda}{\lambda}-\frac{\sin\lambda}{\lambda}-\frac{1}{\lambda^2}\cos\lambda\varphi\Bigg|_0^1\right)=\frac{2}{\pi}\frac{1-\cos\lambda}{\lambda^2}$$

Pa je Fourierov integral jednak:

$$f(x)=\int_0^{\infty}\frac{2}{\pi}\frac{1-\cos\lambda}{\lambda^2}\cos\lambda x\,d\lambda=\frac{2}{\pi}\int_0^{\infty}\frac{1-\cos\lambda}{\lambda^2}\cos\lambda x\,d\lambda$$

7. Funkciju $f(x)$ zadanu slikom prikaži u obliku Fourierovog integrala.



Imamo da je $f(x) = A\sin ax$ (jer je $\omega = \frac{2\pi}{T} = \frac{2\pi}{\frac{2\pi}{a}} = a$).

Iz slike vidimo da je neparna, pa računamo samo $A(\lambda)$.

$$A(\lambda) = \frac{2}{\pi}\int_0^\infty f(\varphi)\cos\lambda\varphi\, d\varphi = \frac{2}{\pi}\int_0^{\frac{\pi}{a}} A\sin ax\cos\lambda\varphi\, d\varphi = \frac{2A}{\pi}\left(\int_0^{\frac{\pi}{a}} \frac{\cos(a-\lambda)\varphi - \cos(a+\lambda)\varphi}{2}\, d\varphi\right) =$$

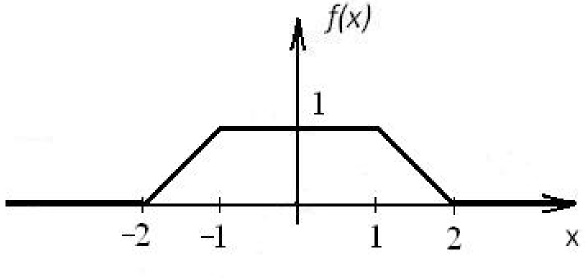
$$= \frac{A}{\pi}\left(\int_0^{\frac{\pi}{a}} \cos(a-\lambda)\varphi\, d\varphi - \int_0^{\frac{\pi}{a}} \cos(a+\lambda)\varphi\, d\varphi\right) = \frac{A}{\pi}\left(\frac{\sin(a-\lambda)\varphi}{a-\lambda}\Bigg|_0^{\frac{\pi}{a}} - \frac{\sin(a+\lambda)\varphi}{a+\lambda}\Bigg|_0^{\frac{\pi}{a}}\right) =$$

$$= \frac{A}{\pi}\left(\frac{\sin(a-\lambda)\frac{\pi}{a}}{a-\lambda} - \frac{\sin(a+\lambda)\frac{\pi}{a}}{a+\lambda}\right) = \frac{A}{\pi}\left(\frac{\sin\left(\pi - \lambda\frac{\pi}{a}\right)}{a-\lambda} - \frac{\sin\left(\pi + \lambda\frac{\pi}{a}\right)}{a+\lambda}\right) = \frac{A}{\pi}\frac{2a\sin\frac{\lambda\pi}{a}}{a^2-\lambda^2}$$

Pa je Fourierov integral jednak:

$$f(x) = \int_0^\infty \frac{A}{\pi}\frac{2a\sin\frac{\lambda\pi}{a}}{a^2-\lambda^2}\cos\lambda x\, d\lambda = \frac{2Aa}{\pi}\int_0^\infty \frac{\sin\frac{\lambda\pi}{a}}{a^2-\lambda^2}\cos\lambda x\, d\lambda$$

8. Funkciju $f(x)$ zadanu slikom prikaži u obliku Fourierovog integrala.



Iz slike vidimo da je funkcija parna. Ona je oblika

$$f(x)=\begin{cases} -x+2, & x\in(1,2), \\ 1, & x\in(0,1) \end{cases}$$

Računamo samo $A(\lambda)$.

$$A(\lambda)=\frac{2}{\pi}\int_0^\infty f(\varphi)\cos\lambda\varphi\,d\varphi=\frac{2}{\pi}\left(\int_0^1\cos\lambda\varphi\,d\varphi+\int_1^2(-\varphi+2)\cos\lambda\varphi\,d\varphi\right)=$$

$$=\frac{2}{\pi}\left(\frac{\sin\lambda\varphi}{\lambda}\Bigg|_0^1-\int_1^2\varphi\cos\lambda\varphi\,d\varphi+2\int_1^2\cos\lambda\varphi\,d\varphi\right)=\left|\begin{matrix} u=\varphi & dv=\cos\lambda\varphi\,d\varphi \\ du=d\varphi & v=\dfrac{\sin\lambda\varphi}{\lambda}\end{matrix}\right|=$$

$$=\frac{2}{\pi}\left(\frac{\sin\lambda}{\lambda}-\left(\frac{\varphi\sin\lambda\varphi}{\lambda}\Bigg|_1^2-\frac{1}{\lambda}\int_1^2\sin\lambda\varphi\,d\varphi\right)+\frac{2\sin\lambda\varphi}{\lambda}\Bigg|_1^2\right)=$$

$$=\frac{2}{\pi}\left(\frac{\sin\lambda}{\lambda}-\left(\frac{2\sin2\lambda-\sin\lambda}{\lambda}+\frac{\cos\lambda\varphi}{\lambda^2}\Bigg|_1^2\right)+\frac{2\sin2\lambda-2\sin\lambda}{\lambda}\right)=$$

$$=\frac{2}{\pi}\left(\frac{\sin\lambda}{\lambda}-\left(\frac{2\sin2\lambda-\sin\lambda}{\lambda}+\frac{\cos2\lambda-\cos\lambda}{\lambda^2}\right)+\frac{2\sin2\lambda-2\sin\lambda}{\lambda}\right)=$$

$$=\frac{2}{\pi}\left(\frac{\sin\lambda}{\lambda}-\frac{2\sin2\lambda-\sin\lambda}{\lambda}-\frac{\cos2\lambda-\cos\lambda}{\lambda^2}+\frac{2\sin2\lambda-2\sin\lambda}{\lambda}\right)=$$

$$=\frac{2}{\pi}\frac{\cos\lambda-\cos2\lambda}{\lambda^2}$$

Pa je Fourierov integral jednak:

$$f(x)=\int_0^\infty\frac{2}{\pi}\frac{\cos\lambda-\cos2\lambda}{\lambda^2}\cos\lambda x\,d\lambda=\frac{2}{\pi}\int_0^\infty\frac{\cos\lambda-\cos2\lambda}{\lambda^2}\cos\lambda x\,d\lambda$$

9. Odredi sinusni i kosinusni spektar funkcije

$$f(x)=\begin{cases}\dfrac{A}{T}x, & 0\le x\le T,\\ 0 & x<0 \text{ ili } x>T\end{cases}$$

$$A(\lambda)=\frac{1}{\pi}\int_0^{\infty} f(\varphi)\cos\lambda\varphi\,d\varphi=\frac{A}{T\pi}\int_0^{T}\varphi\cos\lambda\varphi\,d\varphi=\begin{vmatrix} u=\varphi & dv=\cos\lambda\varphi\,d\varphi\\ du=d\varphi & v=\dfrac{\sin\lambda\varphi}{\lambda}\end{vmatrix}=$$

$$=\frac{A}{T\pi}\left(\frac{\varphi\sin\lambda\varphi}{\lambda}\bigg|_0^T-\frac{1}{\lambda}\int_0^T\sin\lambda\varphi\,d\varphi\right)=\frac{A}{T\pi}\left(\frac{T\sin\lambda T}{\lambda}+\frac{\cos\lambda\varphi}{\lambda^2}\bigg|_0^T\right)=$$

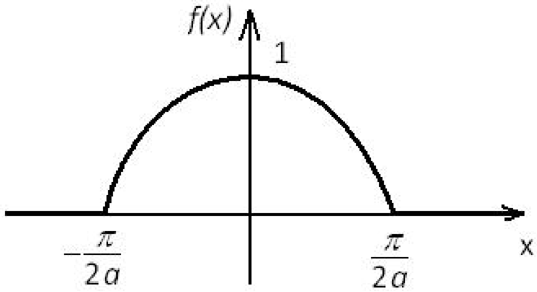
$$=\frac{A}{T\pi}\left(\frac{T\sin\lambda T}{\lambda}+\frac{\cos\lambda T}{\lambda^2}-\frac{1}{\lambda^2}\right)$$

$$B(\lambda)=\frac{1}{\pi}\int_0^{\infty} f(\varphi)\sin\lambda\varphi\,d\varphi=\frac{A}{T\pi}\int_0^{T}\varphi\sin\lambda\varphi\,d\varphi=\begin{vmatrix} u=\varphi & dv=\sin\lambda\varphi\,d\varphi\\ du=d\varphi & v=-\dfrac{\cos\lambda\varphi}{\lambda}\end{vmatrix}=$$

$$=\frac{A}{T\pi}\left(-\frac{\varphi\cos\lambda\varphi}{\lambda}\bigg|_0^T+\frac{1}{\lambda}\int_0^T\cos\lambda\varphi\,d\varphi\right)=\frac{A}{T\pi}\left(-\frac{T\cos\lambda T}{\lambda}+\frac{\sin\lambda\varphi}{\lambda^2}\bigg|_0^T\right)=$$

$$=\frac{A}{T\pi}\left(\frac{\sin\lambda T}{\lambda^2}-\frac{T\cos\lambda T}{\lambda}\right)$$

10. Funkciju zadanu slikom razviju u Fourierov integral, a zatim pomoću tog prikaza odredi vrijednost integrala $\int_0^{\infty} \frac{\cos\frac{t\pi}{2a}}{a^2-t^2}\,dt$.



Funkcija je parna, i vrijedi da je $f(x)=\cos ax$.

$$A(\lambda)=\frac{2}{\pi}\int_0^{\infty} f(\varphi)\cos\lambda\varphi\,d\varphi=\frac{2}{\pi}\int_0^{\frac{\pi}{2a}}\cos a\varphi\cos\lambda\varphi\,d\varphi=$$

$$=\frac{2}{\pi}\int_0^{\frac{\pi}{2a}}\frac{\cos(a+\lambda)\varphi+\cos(a-\lambda)\varphi}{2}\,d\varphi=\frac{1}{\pi}\left(\int_0^{\frac{\pi}{2a}}\cos(a+\lambda)\varphi\,d\varphi+\int_0^{\frac{\pi}{2a}}\cos(a-\lambda)\varphi\,d\varphi\right)=$$

$$=\frac{1}{\pi}\left(\frac{\sin(a+\lambda)\varphi}{a+\lambda}\Bigg|_0^{\frac{\pi}{2a}}+\frac{\sin(a-\lambda)\varphi}{a-\lambda}\Bigg|_0^{\frac{\pi}{2a}}\right)=\frac{1}{\pi}\left(\frac{\sin\left(\frac{\pi}{2}+\frac{\lambda\pi}{2a}\right)}{a+\lambda}+\frac{\sin\left(\frac{\pi}{2}-\frac{\lambda\pi}{2a}\right)}{a-\lambda}\right)=$$

$$=\frac{1}{\pi}\left(\frac{\cos\frac{\lambda\pi}{2a}}{a+\lambda}+\frac{\cos\frac{\lambda\pi}{2a}}{a-\lambda}\right)=\frac{1}{\pi}\,\frac{(a-\lambda)\cos\frac{\lambda\pi}{2a}+(a+\lambda)\cos\frac{\lambda\pi}{2a}}{a^2-\lambda^2}=\frac{2a\cos\frac{\lambda\pi}{2a}}{\pi\left(a^2-\lambda^2\right)}$$

$$f(x)=\int_0^{\infty}\frac{2a\cos\frac{\lambda\pi}{2a}}{\pi\left(a^2-\lambda^2\right)}\cos\lambda x\,d\lambda=\frac{2a}{\pi}\int_0^{\infty}\frac{\cos\frac{\lambda\pi}{2a}}{a^2-\lambda^2}\cos\lambda x\,d\lambda$$

Uzmemo da je $\lambda=t$ i da je $x=0$. Tada nam je $f(0)=1$ i $\cos 0=1$ pa imamo sljedeće:

$$1=\frac{2a}{\pi}\int_0^{\infty}\frac{\cos\frac{t\pi}{2a}}{a^2-t^2}\,dt\text{ , odnosno: }\int_0^{\infty}\frac{\cos\frac{t\pi}{2a}}{a^2-t^2}\,dt=\frac{\pi}{2a}\text{ .}$$