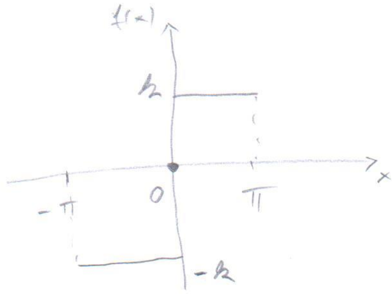
1.) $f(x) = k\,\mathrm{sgn}(e^x - 1)$



NEPARNA $\Rightarrow a_n = 0$

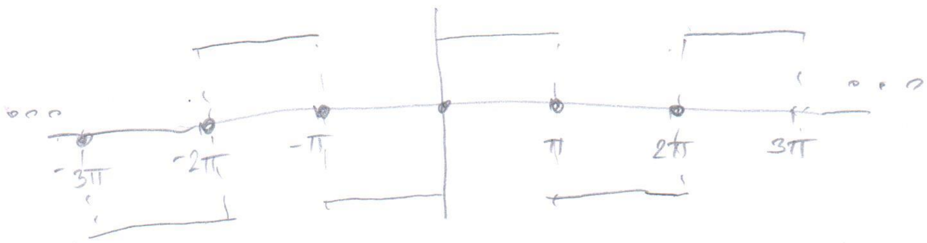
$$b_n = \frac{2}{2\pi}\int_{-\pi}^{\pi} f(x) \sin\frac{2n\pi x}{2\pi}\,dx$$

$$= \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\sin nx\,dx = \frac{2}{\pi}\int_{0}^{\pi} k \sin nx\,dx = \frac{2k}{\pi n}(-\cos nx)\Big|_{0}^{\pi}$$

$$= \frac{2k}{\pi n}\left(-\cos n\pi + 1\right) = \frac{2k}{\pi n}\left(1-(-1)^n\right)$$

$$= \begin{cases} \dfrac{4k}{\pi(2m-1)} & , n = 2m-1 \\ 0 & , n = 2m \end{cases}$$

$$\tilde{f}(x) = \sum_{n=1}^{\infty} b_n \sin nx = \sum_{m=1}^{\infty} \frac{4k}{\pi(2m-1)} \sin((2m-1)x)$$



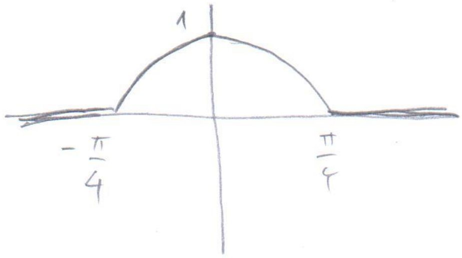
$x = \frac{\pi}{2} \Rightarrow \tilde{f}\left(\frac{\pi}{2}\right) = f\left(\frac{\pi}{2}\right) = k$

$$\parallel$$

$$\sum_{m=1}^{\infty} \frac{4k}{\pi(2m-1)} \sin\left((2m-1)\frac{\pi}{2}\right) = \sum_{m=1}^{\infty} \frac{4k}{(2m-1)\pi}(-1)^{m-1}$$

$$\Rightarrow \sum_{m=1}^{\infty} \frac{(-1)^{m-1}}{2m-1} = \frac{\pi}{4}$$

2.) $f(x) = \begin{cases} \cos 2x & , \ x \in \left(-\frac{\pi}{4}, \frac{\pi}{4}\right) \\ 0 & , \ \text{inače} \end{cases}$



f PARNA $\Rightarrow B(\lambda) = 0$

$$A(\lambda) = \frac{1}{\pi}\int_{-\infty}^{+\infty} f(x)\cos\lambda x\,dx = \frac{2}{\pi}\int_0^{\frac{\pi}{4}} \cos 2x \cos\lambda x\,dx$$

$$= \frac{1}{\pi}\int_0^{\frac{\pi}{4}} \left[\cos((2+\lambda)x) + \cos((2-\lambda)x)\right]dx$$

$$= \frac{1}{\pi}\left(\frac{\sin(2+\lambda)x}{2+\lambda}\Big|_0^{\pi/4} + \frac{\sin((2-\lambda)x)}{2-\lambda}\Big|_0^{\pi/4}\right)$$

$$= \frac{1}{\pi}\left(\frac{\sin(2+\lambda)\frac{\pi}{4}}{2+\lambda} + \frac{\sin(2-\lambda)\frac{\pi}{4}}{2-\lambda}\right)$$

$$= \frac{1}{\pi}\left(\frac{\sin\left(\frac{\lambda\pi}{4}+\frac{\pi}{2}\right)}{2+\lambda} + \frac{\sin\left(\frac{\pi}{2}-\frac{\lambda\pi}{4}\right)}{2-\lambda}\right)$$

$$= \frac{1}{\pi}\left(\frac{1}{2+\lambda}\cos\frac{\lambda\pi}{4} + \frac{1}{2-\lambda}\cos\frac{\lambda\pi}{4}\right)$$

$$= \frac{1}{\pi}\cos\frac{\lambda\pi}{4}\cdot\frac{2-\cancel{\lambda}+2+\cancel{\lambda}}{4-\lambda^2} = \frac{4}{\pi(4-\lambda^2)}\cos\frac{\lambda\pi}{4}$$

$$\tilde{f}(x) = \frac{4}{\pi}\int_0^{+\infty} \frac{1}{4-\lambda^2}\cos\frac{\lambda\pi}{4}\cos\lambda x\,d\lambda$$

$$\Rightarrow \tilde{f}(0) = 1 = \frac{4}{\pi}\int_0^{+\infty}\frac{1}{4-\lambda^2}\cos\frac{\lambda\pi}{4}\,d\lambda$$

$$\Rightarrow \int_0^{+\infty}\frac{1}{4-\lambda^2}\cos\frac{\lambda\pi}{4}\,d\lambda = \frac{\pi}{4}$$

3.) $$\boxed{f(t) \circ\!\!-\!\!\bullet F(s)} := \frac{s-1}{(s^2-2s+4)^2}$$

$$\Rightarrow \quad \frac{f(t)}{t} \circ\!\!-\!\!\bullet \int_s^\infty F(x)\,dx = \frac{1}{2}\int_s^\infty \frac{2x-2}{(x^2-2x+4)^2}\,dx = -\frac{1}{2}\,\frac{1}{x^2-2x+4}\Big|_s^\infty$$

$$= \frac{1}{2\sqrt{3}}\,\frac{\sqrt{3}}{(s-1)^2+3} \bullet\!\!-\!\!\circ \frac{1}{2\sqrt{3}}\sin(t\sqrt{3})\,e^t u(t)$$

$$\Rightarrow \quad \underline{\underline{f(t) = \frac{t}{2\sqrt{3}}\sin(t\sqrt{3})\,e^t u(t)}}$$

4.) $\begin{cases} x' = -2x + y \\ y' = -3y \end{cases}$

$x(0) = 1, \quad y(0) = 2$

R:

$$\begin{cases} sX(s) - 1 = -2X(s) + Y(s) \\ sY(s) - 2 = -3Y(s) \end{cases}$$

$$(s+2)X(s) - Y(s) = 1$$
$$(s+3)Y(s) = 2 \quad \Rightarrow \quad Y(s) = \frac{2}{s+3}$$

$$(s+2)X(s) = 1 + \frac{2}{s+3} = \frac{s+5}{s+3}$$

$$X(s) = \frac{s+5}{(s+2)(s+3)} = \frac{A}{s+2} + \frac{B}{s+3} = \frac{A(s+3) + B(s+2)}{(s+2)(s+3)}$$

$$s+5 = (A+B)s + 3A + 2B$$

$$\left.\begin{array}{l} A+B = 1 \\ 3A + 2B = 5 \end{array}\right\} \Rightarrow \boxed{\begin{array}{l} B = -2 \\ A = 3 \end{array}}$$

$$X(s) = \frac{3}{s+2} - \frac{2}{s+3}$$

$$\Rightarrow \boxed{\begin{array}{l} x(t) = 3e^{-2t} - 2e^{-3t} \\ y(t) = 2e^{-3t} \end{array}}$$

5.) $\begin{cases} y'' + 4y = 8u(t-2\pi) \\ y(0) = 3 \\ y'(0) = 0 \end{cases}$

$\mathcal{L}$: $s\left(sY(s) - y(0)\right) - y'(0) + 4Y(s) = 8\dfrac{e^{-2\pi s}}{s}$

$$s^2 Y(s) - 3s + 4Y(s) = \frac{8e^{-2\pi s}}{s}$$

$$(s^2+4)Y(s) = 3s + \frac{8e^{-2\pi s}}{s}$$

$$Y(s) = \underbrace{\frac{3s}{s^2+4}}_{\substack{\bullet \\ | \\ \circ \\ 3\cos 2t\, u(t)}} + \frac{8e^{-2\pi s}}{s(s^2+4)}$$

$$\frac{1}{s(s^2+4)} = \frac{A}{s} + \frac{Bs+C}{s^2+4} = \frac{A(s^2+4) + Bs^2 + Cs}{s(s^2+4)}$$

$$1 = (A+B)s^2 + Cs + 4A$$

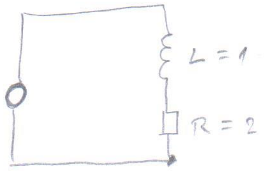
$\Rightarrow \quad 4A = 1 \Rightarrow A = \frac{1}{4}, \quad C = 0, \quad B = 1 - A = \frac{3}{4}$

$$\frac{1}{s(s^2+4)} = \frac{1}{4s} + \frac{3}{4(s^2+4)} = \frac{1}{4s} + \frac{3}{8}\,\frac{2}{s^2+4} \;\bullet\!\!-\!\!\circ\; \frac{1}{4}u(t) + \frac{3}{8}\cos 2t\, u(t)$$

$\Rightarrow \quad \dfrac{e^{-2\pi s}}{s(s^2+4)} \;\bullet\!\!-\!\!\circ\; \dfrac{1}{4}u(t-2\pi) + \dfrac{3}{8}\underbrace{\cos 2(t-2\pi)}_{=\cos 2t}u(t-2\pi)$

$\Rightarrow \quad y(t) = 3\cos 2t\, u(t) + \dfrac{1}{4}u(t-2\pi) + \dfrac{3}{8}\cos 2t\, u(t-2\pi)$

6c)



$$e(t) = (-t+3)\,g_{[0,3]}(t) = (-t+3)\mu(t) - (-t+3)\mu(t-3)$$

$$= -t\mu(t) + 3\mu(t) + (t-3)\mu(t-3)$$

$$\circ\!-\!\bullet\ -\frac{1}{s^2} + \frac{3}{s} + \frac{e^{-3s}}{s}$$

$$Z(s) = R + Ls = 2 + s$$

$$I(s) = \frac{E(s)}{Z(s)} = \frac{1}{s+2}\left(-\frac{1}{s^2} + \frac{3}{s} + \frac{e^{-3s}}{s}\right)$$

$$= -\frac{1}{s^2(s+2)} + \frac{3}{s(s+2)} + \frac{e^{-3s}}{s(s+2)}$$

$$\frac{1}{s(s+2)} = \frac{A}{s} + \frac{B}{s+2} = \frac{A(s+2) + Bs}{s(s+2)} \Rightarrow 1 = 2A + s(A+B)$$

$$\Rightarrow A = \frac{1}{2},\ B = -\frac{1}{2}$$

$$\frac{1}{s(s+2)} = \frac{1}{2s} - \frac{1}{2(s+2)} \ \bullet\!-\!\circ\ \frac{1}{2}\mu(t) - \frac{1}{2}e^{-2t}\mu(t)$$

$$\frac{e^{-3s}}{s(s+2)} \ \bullet\!-\!\circ\ \frac{1}{2}\mu(t-3) - \frac{1}{2}e^{-2(t-3)}\mu(t-3)$$

$$\frac{1}{s^2(s+2)} = \frac{A}{s} + \frac{B}{s^2} + \frac{C}{s+2} = \frac{A(s^2+2s) + B(s+2) + Cs^2}{s^2(s+2)}$$

$$\Rightarrow \quad \underline{1 = (A+C)s^2 + (2A+B)s + 2B}$$

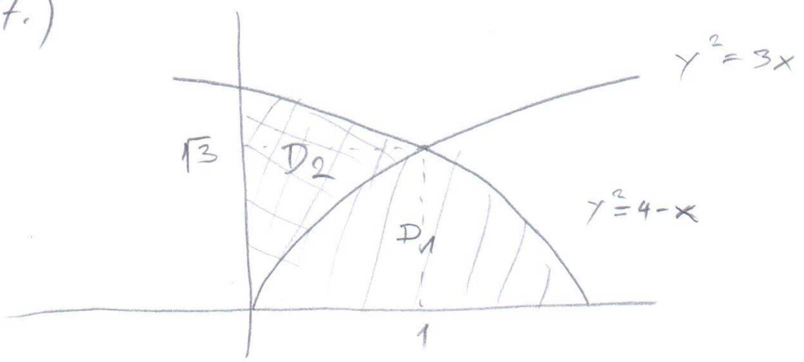
$$\Rightarrow \ 2B = 1 \quad 2A = -B \quad C = -A$$
$$B = 1/2 \quad A = -1/4 \quad C = 1/4$$

$$\Rightarrow \frac{1}{s^2(s+2)} = -\frac{1}{4s} + \frac{1}{2s^2} + \frac{1}{4(s+2)} \ \bullet\!-\!\circ\ -\frac{1}{4}\mu(t) + \frac{1}{2}t\mu(t) + \frac{1}{4}e^{-2t}\mu(t)$$

$$i(t) = \frac{1}{4}\mu(t) - \frac{1}{2}t\mu(t) - \frac{1}{4}e^{-2t}\mu(t) + \frac{3}{2}\mu(t) - \frac{3}{2}e^{-2t}\mu(t)$$
$$+ \frac{1}{2}\mu(t-3) - \frac{1}{2}e^{-2(t-3)}\mu(t-3)$$

$$= \left(\frac{7}{4} - \frac{1}{2}t - \frac{7}{4}e^{-2t}\right)\mu(t) + \left(\frac{1}{2} - \frac{1}{2}e^{-2(t-3)}\right)\mu(t-3)$$

7.)



SLOBODAN IZBOR: D1 ILI D2 PODRUČJE

$$I_{D_1} = \int_0^{\sqrt{3}} dy \int_{\frac{y^2}{3}}^{4-y^2} y\,dx = \int_0^{\sqrt{3}} xy \Big|_{\frac{y^2}{3}}^{4-y^2} dy = \int_0^{\sqrt{3}} \left(4y - y^3 - \frac{y^3}{3}\right) dy$$

$$= \int_0^{\sqrt{3}} \left(4y - \frac{4}{3}y^3\right) dy = \left(2y^2 - \frac{1}{3}y^4\right)\Big|_0^{\sqrt{3}} = 2\cdot 3 - \frac{1}{3}\cdot 9$$

$$= 6 - 3 = \underline{\underline{3}}$$

$$V_{D_1} = 10\, I_{D_1} = \underline{\underline{30}}$$

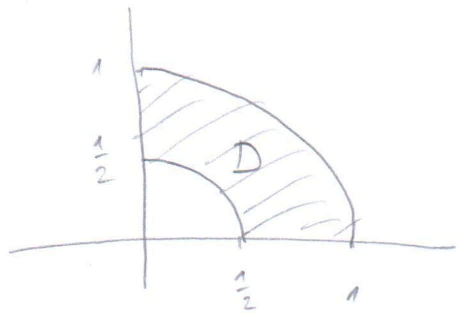
$\boxed{\text{ALTERNATIVNO:}}$

$$I_{D_2} = \int_0^1 dx \int_{\sqrt{3x}}^{\sqrt{4-x}} y\,dy = \int_0^1 \frac{y^2}{2}\Big|_{\sqrt{3x}}^{\sqrt{4-x}} dx = \int_0^1 \left(\frac{4-x}{2} - \frac{3x}{2}\right) dx$$

$$= \int_0^1 \frac{4-4x}{2}\, dx = \left(2x - x^2\right)\Big|_0^1 = 2 - 1 = \underline{\underline{1}}$$

$$V_{D_2} = 10\, I_{D_2} = \underline{\underline{10}}$$

8.)



$$y=\sqrt{\tfrac14-x^2} \qquad y=\sqrt{1-x^2}$$
$$y^2+x^2=\tfrac14 \qquad y^2+x^2=1$$

$$\sqrt{\tfrac14-\min\{x^2,\tfrac14\}} = \begin{cases}\sqrt{\tfrac14-x^2} & , 0\le x\le \tfrac12\\ 0 & , \tfrac12< x\le 1\end{cases}$$

$$I=\int_0^{\frac{\pi}{2}} d\varphi \int_{\frac12}^{1} r\, r^5\, dr = \int_0^{\frac{\pi}{2}} d\varphi \int_{\frac12}^{1} r^6\, dr = \int_0^{\frac{\pi}{2}} \frac{r^7}{7}\Big|_{\frac12}^{1} d\varphi$$

$$=\frac{\pi}{2}\cdot\left(\frac17-\frac{1}{7\cdot 2^7}\right)=\frac{\pi}{14}\left(1-\frac{1}{2^7}\right)$$

$$=\frac{\pi}{14}\left(1-\frac{1}{128}\right)=\underline{\underline{\frac{\pi}{14}\ \frac{127}{128}}}$$