2. školska zadaća iz Matematike 3E i 3R

Grupa A

04.11.2011.

1. (3 boda)

a) (**1b**) Iskažite teorem o prigušenju originala.

b) (**2b**) Odredite original funkcije

$$F(s) = \frac{2}{s^2 - 2s + 5} e^{-2s-1}.$$

2. (3 boda)

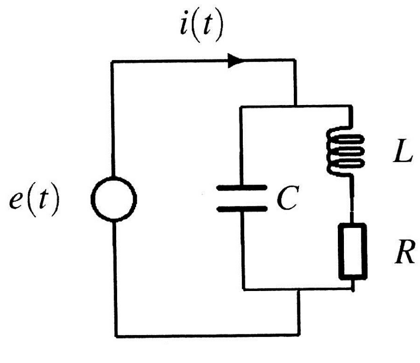
a) (**1b**) Iskažite teorem o deriviranju slike.

b) (**2b**) Odredite sliku funkcije

$$f(t) = t\cos(2t)e^{-t}.$$

3. (4 boda)

Pomoću Laplaceove transformacije izračunajte i skicirajte struju $i(t)$ strujnog kruga zadanog slikom uz priključeni napon $e(t) = \frac{1}{2}e^{-t}u(t)$.
($R = 2$, $L = 1$, $C = 1$)



A-12a

I_3

1) b) $$\frac{2}{s^2-2s+5}e^{-2s-1} = \frac{2}{(s-1)^2+2^2}e^{-2(s-1)}\cdot e^{-3} \circ\!\!-\!\!\bullet$$

$$\boxed{\frac{2}{s^2+2^2}e^{-2s} \circ\!\!-\!\!\bullet \sin(2(t-2))u(t-2)}$$

$$\circ\!\!-\!\!\bullet \boxed{e^{t-3}\cdot\sin(2(t-2))u(t-2)}$$

---

2) b) $f(t) \circ\!\!-\!\!\bullet F(s)$ $\qquad f(t)=\cos(2t)e^{-t}$

$$tf(t) \circ\!\!-\!\!\bullet -F'(s)$$

$$\cos(2t)e^{-t} \circ\!\!-\!\!\bullet \frac{s+1}{(s+1)^2+4} = \frac{s+1}{s^2+2s+5} = F(s)$$

$$t\cos(2t)e^{-t} \circ\!\!-\!\!\bullet -\left(\frac{s+1}{s^2+2s+5}\right)' =$$

$$= -\frac{s^2+2s+5-(s+1)(2s+2)}{(s^2+2s+5)^2} =$$

$$= -\frac{s^2+\cancel{2s}+5-2s^2-\cancel{2s}-2s-2}{(s^2+2s+5)^2} =$$

$$= -\frac{-s^2-2s+3}{(s^2+2s+5)^2} = \boxed{\frac{s^2+2s-3}{(s^2+2s+5)^2}}$$

3) $e(t) = \frac{1}{2} e^{-t} u(t) \circ\!\!-\!\!\circ \frac{1}{2} \cdot \frac{1}{s+1} = E(s)$ $\quad II_3$

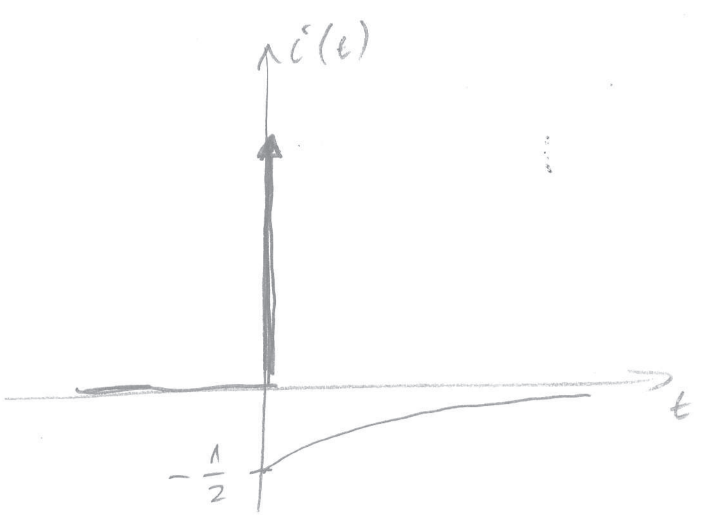
$$Z(s) = \frac{1}{\frac{1}{\frac{1}{Cs}} + \frac{1}{Ls+R}} = \frac{1}{s + \frac{1}{s+2}} =$$

$$= \frac{1}{\frac{s^2+2s+1}{s+2}} = \frac{s+2}{(s+1)^2}$$

$$I(s) = \frac{E(s)}{Z(s)} = \frac{\frac{1}{2} \cdot \frac{1}{\cancel{s+1}}}{\frac{s+2}{(s+1)^{\cancel{2}}}} = \frac{1}{2} \frac{s+1}{s+2} =$$

$$= \frac{1}{2}\left[\frac{s+2}{s+2} - \frac{1}{s+2}\right] = \frac{1}{2} - \frac{1}{2} \frac{1}{s+2} \circ\!\!-\!\!\circ$$

$$\circ\!\!-\!\!\circ \boxed{\frac{1}{2} \delta(t) - \frac{1}{2} e^{-2t} u(t) = i(t)}$$



2. školska zadaća iz Matematike 3E i 3R

Grupa B

04.11.2011.

1. (3 boda)

a) (**1b**) Iskažite teorem o pomaku originala.

b) (**2b**) Odredite original funkcije

$$F(s) = \frac{s+2}{s^2+4s+9} e^{-3s-4}.$$

2. (3 boda)

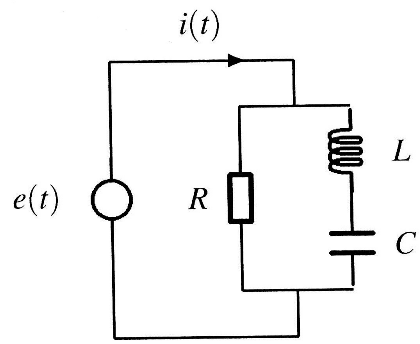
a) (**1b**) Iskažite teorem o deriviranju originala.

b) (**2b**) Odredite sliku funkcije $f(t)$, gdje je $f(t) = g''(t)$, a funkcija

$$g(t) = \sin(\sqrt{2}t)e^{3t}.$$

3. (4 boda)

Pomoću Laplaceove transformacije izračunajte i skicirajte struju $i(t)$ strujnog kruga zadanog slikom uz priključeni napon $e(t) = e^{-t}u(t)$.
($R = 2$, $L = 1$, $C = 1$)



$\boxed{B-12h}$ I_4

1) h)

$$\frac{s+2}{s^2+4s+9}e^{-3s-4} = \frac{s+2}{(s+2)^2+(\sqrt{5})^2}e^{-3(s+2)}e^{2}$$

$$\boxed{\frac{s}{s^2+(\sqrt{5})^2}e^{-3s} \circ\!\!-\!\!\bullet \cos(\sqrt{5}(t-3))u(t-3)}$$

$$\circ\!\!-\!\!\bullet \boxed{e^{-2t+2}\cdot\cos(\sqrt{5}(t-3))u(t-3)}$$

---

2) h) $g(t)=\sin(\sqrt{2}t)e^{3t} \circ\!\!-\!\!\bullet \dfrac{\sqrt{2}}{(s-3)^2+2} = G(s)$

$$f(t)=g''(t) \circ\!\!-\!\!\bullet s^2G(s)-sg(0)-g'(0) =$$

$$\boxed{\begin{aligned} &g'(t)=\cos(\sqrt{2}t)\sqrt{2}e^{3t}+\sin(\sqrt{2}t)e^{3t}\cdot 3 \\ &g'(0)=\sqrt{2} \quad , \quad g(0)=0 \end{aligned}}$$

$$= \frac{s^2\sqrt{2}}{(s-3)^2+2}-\sqrt{2} = \boxed{\sqrt{2}\left(\frac{s^2}{(s-3)^2+2}-1\right)}$$

3) $e(t) = e^{-t}u(t) = \dfrac{1}{s+1} = E(s)$

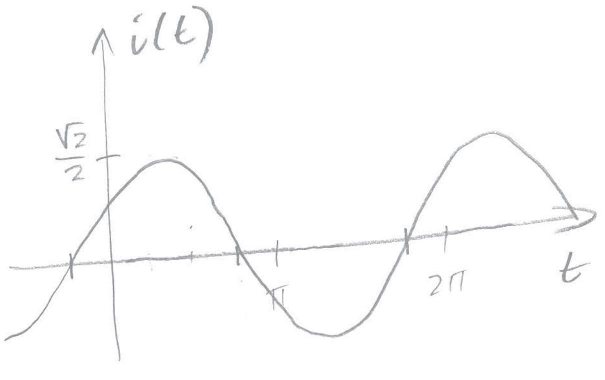
$$Z(s) = \frac{1}{\dfrac{1}{R} + \dfrac{1}{Ls + \dfrac{1}{Cs}}} = \frac{1}{\dfrac{1}{2} + \dfrac{1}{s + \dfrac{1}{s}}} =$$

$$= \frac{1}{\dfrac{1}{2} + \dfrac{1}{\dfrac{s^2+1}{s}}} = \frac{1}{\dfrac{1}{2} + \dfrac{s}{s^2+1}} = \frac{1}{\dfrac{s^2+1+2s}{2(s^2+1)}} =$$

$$= \frac{2(s^2+1)}{(s+1)^2}$$

$$I(s) = \frac{E(s)}{Z(s)} = \frac{\dfrac{1}{\cancel{s+1}}}{\dfrac{2(s^2+1)}{(s+1)^{\cancel{2}}}} = \frac{1}{2}\,\frac{s+1}{s^2+1} =$$

$$= \frac{1}{2}\left(\frac{s}{s^2+1} + \frac{1}{s^2+1}\right) \circ\!\!-\!\!\circ \boxed{\frac{1}{2}(\cos t + \sin t) = i(t)}$$



$$= \frac{1}{2}\left(\sin\left(t + \frac{\pi}{2}\right) + \sin t\right) =$$

$$= \sin\left(\frac{2t + \frac{\pi}{2}}{2}\right)\cos\left(\frac{\frac{\pi}{2}}{2}\right) =$$

$$= \sin\left(t + \frac{\pi}{4}\right)\cos\left(\frac{\pi}{4}\right) =$$

$$= \frac{\sqrt{2}}{2}\sin\left(t + \frac{\pi}{4}\right)$$

2. školska zadaća iz Matematike 3E i 3R

Grupa A

04.11.2011.

1. (3 boda)

a) (**1b**) Definirajte Laplaceovu transformaciju.

b) (**2b**) Odredite original funkcije

$$F(s) = \frac{s+2}{s(s+4)+7} e^{-2(s+1)}.$$

2. (3 boda)

a) (**1b**) Iskažite teorem o integriranju slike.

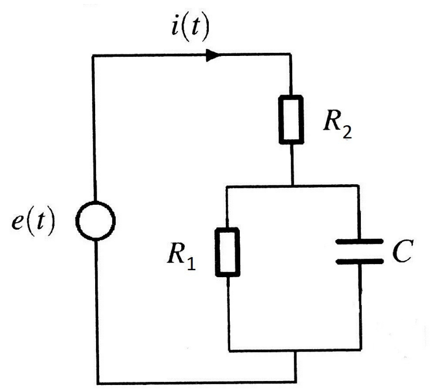
b) (**2b**) Korištenjem Laplaceove transformacije izračunajte integral

$$\int_0^\infty \frac{\sin t}{t}\, dt.$$

3. (4 boda)

Pomoću Laplaceove transformacije izračunajte i skicirajte struju $i(t)$ strujnog kruga zadanog slikom uz priključeni napon $e(t) = \int_0^t e^{3\tau} e^{-(t-\tau)}\, d\tau$.

($R_1 = 2$, $R_2 = 1$, $C = \frac{1}{2}$)



$\boxed{A - 13h}$ $\underline{I}_1$

1) h) $$\frac{s+2}{s(s+4)+7} e^{-2(s+1)} = \frac{s+2}{s^2+4s+7} e^{-2(s+1)} =$$

$$= \frac{s+2}{(s+2)^2+(\sqrt{3})^2} e^{-2(s+2)} \cdot e^2 \circ\!-\!\circ$$

$$\boxed{\frac{s}{s^2+(\sqrt{3})^2} e^{-2s} \circ\!-\!\circ \cos(\sqrt{3}(t-2))\,u(t-2)}$$

$$\circ\!-\!\circ \boxed{e^{-2t+2} \cos(\sqrt{3}(t-2))\,u(t-2)}$$

2) l) $$I = \int_0^\infty \frac{\sin t}{t}\,dt, \quad F(s) = \int_0^\infty e^{-st} f(t)\,dt$$

$$f(t) = \frac{\sin t}{t}, \quad I = F(0), \quad g(t) = \sin t$$

$$f(t) = \frac{\sin t}{t} = \frac{g(t)}{t} \circ\!-\!\circ \int_s^\infty G(s)\,ds = \int_s^\infty \frac{1}{s^2+1}\,ds =$$

$$= \operatorname{arctg} s \Big|_s^\infty = \frac{\pi}{2} - \operatorname{arctg} s = F(s)$$

$$I = F(0) = \frac{\pi}{2} - \operatorname{arctg} 0 = \boxed{\frac{\pi}{2}}$$

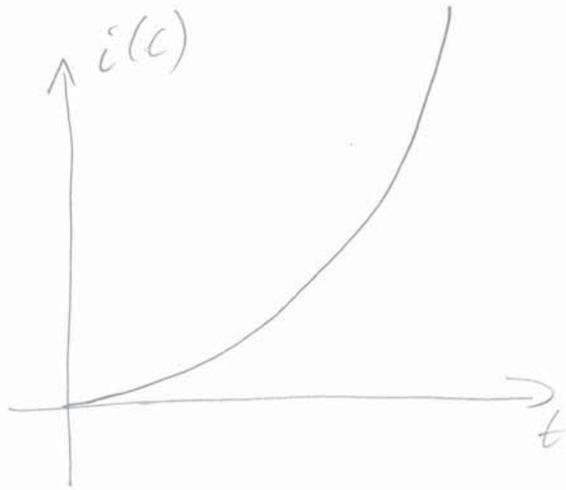
$\underline{\text{II}}_1$

3) $$e(t)=\int_0^t e^{3\tau}e^{-(t-\tau)}\,d\tau = e^{3t} * e^{-t} \circ\!\!-\!\!\circ \frac{1}{s-3}\cdot\frac{1}{s+1}=E(s)$$

$$Z(s)=\frac{1}{\frac{1}{R_1}+\frac{1}{\frac{1}{Cs}}}+R_2=\frac{1}{\frac{1}{2}+\frac{1}{2}s}+1=\frac{2}{s+1}+1=\frac{s+3}{s+1}$$

$$I(s)=\frac{E(s)}{Z(s)}=\frac{\frac{1}{s-3}\cdot\cancel{\frac{1}{s+1}}}{\frac{s+3}{\cancel{s+1}}}=\frac{1}{s^2-3^2}=\frac{1}{3}\cdot\frac{3}{s^2-3^2}\circ\!\!-\!\!\circ$$

$$\circ\!\!-\!\!\circ\ \boxed{\frac{1}{3}\,sh(3t)\,u(t)=i(t)}$$



2. školska zadaća iz Matematike 3E i 3R

Grupa B

04.11.2011.

1. (3 boda)

a) (**1b**) Izračunajte po definiciji Laplaceov transformat funkcije $f(t) = e^{\alpha t}$, gdje je $\alpha \in \mathbb{R}$.

b) (**2b**) Odredite original funkcije

$$F(s) = \frac{\sqrt{2}}{s^2 - 6s + 11} e^{-s}.$$

2. (3 boda)

a) (**1b**) Iskažite teorem o integriranju originala.

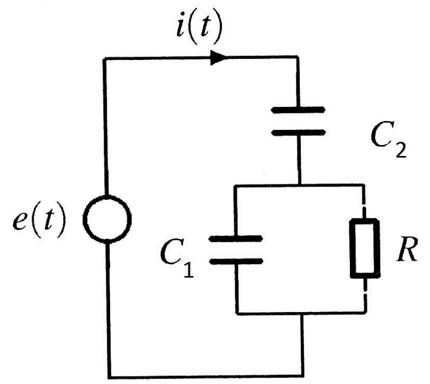
b) (**2b**) Odredite sliku funkcije

$$f(t) = \int_0^t \mathrm{sh}(2t) e^{-3t}\, dt.$$

3. (4 boda)

Pomoću Laplaceove transformacije izračunajte i skicirajte struju $i(t)$ strujnog kruga zadanog slikom uz priključeni napon $e(t) = 2e^{-2t} u(t)$.

($C_1 = 1$, $C_2 = 1$, $R = \frac{1}{2}$)



$\boxed{B-13b}$ $\underline{I}_2$

1) b) $\frac{\sqrt{2}}{s^2-6s+11}e^{-s} = \frac{\sqrt{2}}{(s-3)^2+(\sqrt{2})^2}e^{-(s-3)} \cdot e^{-3}$ ○—●

$\boxed{\frac{\sqrt{2}}{s^2+(\sqrt{2})^2}e^{-s} \circ\!\!-\!\!\bullet \sin(\sqrt{2}(t-1))\,u(t-1)}$

○—● $\boxed{e^{3t-3} \cdot \sin(\sqrt{2}(t-1))\,u(t-1)}$

---

2) b) $\int_0^t sh(2t)e^{-3t}$ ●—○ $\frac{\frac{2}{(s+3)^2-4}}{s} = \boxed{\frac{2}{s(s^2+6s+5)}}$

$$\boxed{\begin{array}{l} sh(2t)e^{-3t} \bullet\!\!-\!\!\circ \frac{2}{(s+3)^2-4} \\ sh(2t) \bullet\!\!-\!\!\circ \frac{2}{s^2-4} \end{array}}$$

II_2

3) $e(t) = 2e^{-2t}u(t) \circ\!\!-\!\!\circ \dfrac{2}{s+2} = E(s)$

$$Z(s) = \frac{1}{\frac{1}{\frac{1}{C_1 s}} + \frac{1}{R}} + \frac{1}{C_2 s} = \frac{1}{s+2} + \frac{1}{s} =$$

$$= \frac{s+s+2}{s(s+2)} = \frac{2s+2}{s(s+2)}$$

$$I(s) = \frac{E(s)}{Z(s)} = \frac{\frac{2}{\cancel{s+2}}}{\frac{2s+2}{s(\cancel{s+2})}} = \frac{2s}{2s+2} = \frac{s}{s+1} =$$

$$= \frac{s+1}{s+1} - \frac{1}{s+1} = 1 - \frac{1}{s+1} \circ\!\!-\!\!\circ \boxed{\delta(t) - e^{-t}u(t) = i(t)}$$