Rješenja i upute za prvu domaću zadaću iz Matematike 3 (R+E)

1. a) promotrite jednadžbu $f(x + T) - f(x) = 0$ uz pretpostavku da je T nepoznanica. Dobije se jedno rješenje

$$T = -\frac{2(k\pi x^2 - x)}{(2k\pi x - 1)} \neq const,$$

pa stoga nema perioda.

b) periodična s peeriodom $T = \pi$

2.
$$S(x) = \frac{2}{\pi} + \sum_{n \geq 1} \frac{4(-1)^{n+1}}{\pi(4n^2 - 1)} \cos 2nx.$$

3.
$$S(x) = \sum_{n \geq 1} \frac{2(-1)^{n+1}}{n} \sin nx.$$

4.
$$S(x) = \frac{sh\pi}{\pi} + \sum_{n \geq 1}[\frac{2(-1)^n sh\pi}{\pi(n^2 + 1)} \cos nx + \frac{2n(-1)^{n+1} sh\pi}{\pi(n^2 + 1)} \sin nx]$$

5.
$$S(x) = \frac{1}{4} + \sum_{n \geq 0} \frac{1}{(2n + 1)\pi} \sin(2n + 1)x.$$

6.
$$S(x) = \cos \frac{x}{2}.$$

7.
$$S(x) = \sin \frac{x}{3}.$$

8.

$$S(x) = \frac{2}{\pi} + \sum_{n \geq 1} \frac{4(-1)^n}{\pi(1 - 4n^2)} \cos 2nx.$$

9.

$$S(x) = \frac{1}{2} + \frac{1}{2} \cos 2x.$$

10.

$$S(x) = -\frac{1}{2} + \sum_{n \geq 0} \frac{-2}{\pi^2(2n + 1)^2} \cos(2n + 1)x.$$

11.

$$S(x) = 3 + \sum_{n \geq 1} \frac{12(-1)^{n+1}}{n\pi} \sin \frac{n\pi x}{3}.$$

12.

$$S(x) = \sum_{n \geq 0} \frac{16}{(2n + 1)^2 \pi^2} \cos \frac{(2n + 1)\pi x}{4}.$$

13.

$$S(x) = \sum_{n \geq 0} \frac{8(-1)^{n+1}}{n\pi} \sin \frac{n\pi x}{2}.$$

14.

$$S(x) = \sum_{n \geq 1} \frac{8n}{\pi(4n^2 - 1)} \sin nx.$$

15.

$$S(x) = \frac{\pi}{2} + \sum_{n \geq 0} \frac{-4}{(2n + 1)^2 \pi} \cos(2n + 1)x$$

16.

$$\sum_{n \geq 0} \frac{1}{(2n + 1)^2} = \frac{\pi^2}{8}.$$

17.

$$\sum_{n \geq 1} \frac{(-1)^n}{n^2} = -\frac{\pi^2}{12}.$$

18.

$$\sum_{n \geq 1} \frac{1}{(2n-1)^2} = \frac{\pi^2}{8}.$$

19.

$$c_n = \frac{4}{n\pi}, \ n \geq 1.$$

20.

$$a = \frac{3T}{2}.$$