LAPLACEOV TRANSFORMAT

$$F(s) := \int_0^\infty e^{-st} f(t)\,dt \quad , t > 0$$

$f(t) \circ\!\!-\!\!\bullet F(s) \rightarrow F(s)$ se pridružuje

$F(s) \bullet\!\!-\!\!\circ f(t) \rightarrow f(t)$ se pridružuje

LAPLACEOV INTEGRAL

$$\int_0^\infty e^{-st} f(t)\,dt := \lim_{M \to \infty} \int_0^M e^{-st} f(t)\,dt$$

APSOLUTNA VRIJEDNOST KOMPLEKSNOG BROJA e^z

$$z = x + iy$$

$$e^z = e^{x+iy} = e^x \cdot e^{iy} = e^x(\cos y + i\sin y)$$

$$|e^z| = |e^x|\,|\cos y + i\sin y| = e^x\sqrt{\cos^2 y + \sin^2 y} = e^x = e^{\mathrm{Re}\,z}$$

$$|e^{i\alpha}| = 1$$

LINEARNOST LAPLACEOVE TRANSFORMACIJE

$$\begin{matrix} f(t) \circ\!\!-\!\!\bullet F(s) \\ g(t) \circ\!\!-\!\!\bullet G(s) \end{matrix} \rightarrow \alpha f(t) + \beta g(t) \circ\!\!-\!\!\bullet \alpha F(s) + \beta G(s)$$

<u>dokaz</u>

$$\mathcal{L}(\alpha f + \beta g) = \int_0^\infty e^{-st}(\alpha f(t) + \beta g(t))\,dt$$

$$= \alpha \int_0^\infty e^{-st} f(t)\,dt + \beta \int_0^\infty e^{-st} g(t)\,dt$$

$$= \alpha \mathcal{L}(f) + \beta \mathcal{L}(g)$$

ORIGINAL

f je original ako (dovoljni uvjeti):

1. $f(t)=0$ za $t<0$
2. f je po dijelovima neprekidna
3. f je eksponencijalnog rasta, tj. postoje konstante $M>0$, $a>0$ takve da za $t>0$ vrijedi

$$|f(t)| \le M e^{at}$$

EKSPONENT RASTA

$\boxed{a_0} ==$ infimum svih konstanti a

f je eksponencijalnog rasta ako za neku konstantu a postoji limes $\lim_{t\to\infty} e^{-at}|f(t)|$. $a_0 \le a$

$-at == 0 \Rightarrow a_0$

SVOJSTVA LAPLACEOVE TRANSFORMACIJE

1) MNOŽENJE VARIJABLE KONSTANTOM

$$f(t) \circ\!\!-\!\!\bullet F(s)$$

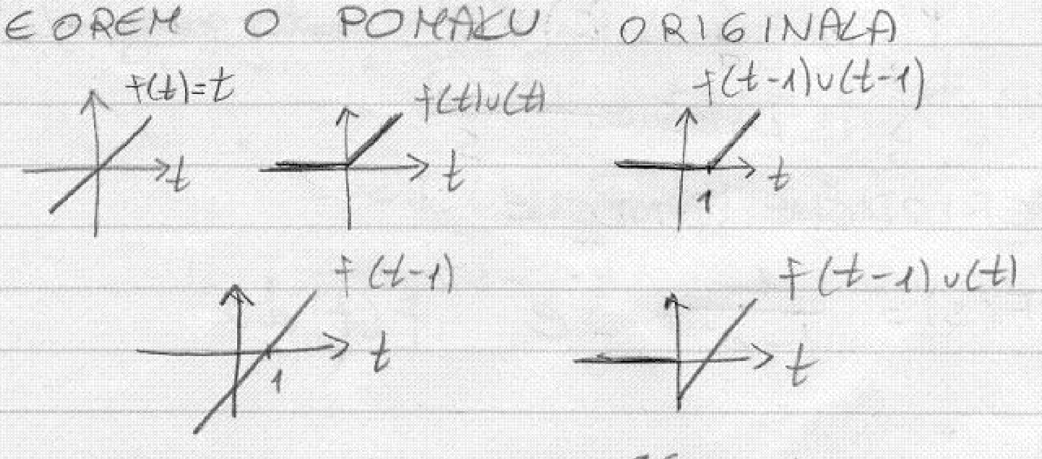
$$f(at) \circ\!\!-\!\!\bullet \frac{1}{a} F\left(\frac{s}{a}\right)$$

$$F(bs) \bullet\!\!-\!\!\circ \frac{1}{b} f\left(\frac{t}{b}\right)$$

2) TEOREM O PRIGUŠENJU

$$e^{-at} f(t) \circ\!\!-\!\!\bullet F(s+a)$$

3) TEOREM O POMAKU ORIGINALA

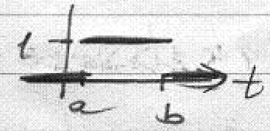


f → funkcija
u → do kud traje prigušenje

$$f(t-a)u(t-a) \circ\!\!-\!\!\bullet e^{-as} F(s)$$

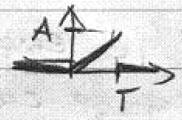
GATE FUNKCIJA

$$g_{[a,b]}(t) = \begin{cases} 1, & a \le t \le b \\ 0, & \text{inače} \end{cases}$$



$$g_{[a,b]}(t) = u(t-a) - u(t-b) \qquad 0<a<b$$

$$g_{[a,b]}(t) \circ\!\!-\!\!\bullet \frac{e^{-sa}}{s} - \frac{e^{-sb}}{s}$$



$f(t) = \frac{A}{T} t$

pr. $f(t) = \begin{cases} 3, & 0<t<2 \\ -1, & 2<t<4 \\ 0, & t>4 \end{cases}$

$$f(t) = 3\, g_{[0,2]}(t) - g_{[2,4]}(t)$$

DERIVIRANJE ORIGINALA

$$f'(t) \circ\!\!-\!\!\bullet sF(s) - f(0)$$

$$f^{(n)}(t) \circ\!\!-\!\!\bullet s^n F(s) - s^{n-1} f(0) - s^{n-2} f'(0) - \dots - f^{(n-1)}(0)$$

DERIVIRANJE SLIKE

$$(-t) f(t) \circ\!\!-\!\!\bullet F'(s)$$

$$(-t)^n f(t) \circ\!\!-\!\!\bullet F^{(n)}(s)$$

$$t^n f(t) \circ\!\!-\!\!\bullet (-1)^n F^{(n)}(s)$$

INTEGRIRANJE SLIKE

$$f(t) \circ\!\!-\!\!\bullet F(s)$$

Ako je $\frac{f(t)}{t}$ original $\Rightarrow$ $\frac{f(t)}{t} \circ\!\!-\!\!\bullet \int_s^{\infty} F(s)\,ds$

INTEGRIRANJE ORIGINALA

Ako je $f(t)$ original, tada je i

$\varphi(t) := \int_0^t f(\tau)\,d\tau$ također original

$$\int_0^t f(\tau)\,d\tau \circ\!\!-\!\!\bullet \frac{F(s)}{s}$$

SLIKA PERIODIČNE FUNKCIJE

$$F(s) = \frac{1}{1-e^{-sT}} \int_0^T e^{-st} f(t)\,dt$$