1. DOMAĆA ZADAĆA

1. a) $f(x+T)-f(x)=\cos\frac{1}{x+T}-\cos\frac{1}{x}=-2\sin\left(\frac{1}{2}\left(\frac{1}{x}+\frac{1}{x+T}\right)\right)\sin\left(\frac{1}{2}\left(\frac{1}{x}-\frac{1}{x+T}\right)\right)$

$\Rightarrow \frac{1}{x}+\frac{1}{x+T}=2k\pi \Rightarrow T=-\frac{2(k\pi x^2-x)}{2k\pi x-1} \neq \text{const.}$

b) $\cos x\cdot\cos 2x=\frac{1}{2}\left(\underset{T_1}{\cos 4x}+\underset{T_2}{\cos 2x}\right)$

$T_1=\frac{2\pi}{4},\quad T_2=\frac{2\pi}{2},\qquad T_1=\frac{\pi}{2},\ T_2=\pi \Rightarrow T=\pi$

---

2. zad. $f(x)=|\cos x|,\quad x\in\langle-\pi,\pi\rangle \to$ parna, $L=\pi$

$a_n=\frac{2}{\pi}\int_0^{\pi}|\cos x|\cdot\cos\frac{n\pi x}{\pi}\,dx=\frac{2}{\pi}\int_0^{\pi/2}\cos x\cos nx\,dx+\frac{2}{\pi}\int_{\pi/2}^{\pi}(-\cos x)\cos nx\,dx$

$=\frac{-4\cos\frac{n\pi}{2}}{\pi(n^2-1)}$

$a_{2n}=\frac{-4\cdot(-1)^n}{\pi(4n^2-1)},\qquad a_{2n+1}=0\qquad a_0=\frac{4}{\pi}$

$S(x)=\frac{2}{\pi}+\sum_{n\geq1}\frac{4\cdot(-1)^{n+1}}{\pi(4n^2-1)}\cos 2nx$

---

3. zad. $f(x)=x \Rightarrow$ neparna ; $x\in\langle-\pi,\pi\rangle$

$b_n=\frac{2}{\pi}\int_0^{\pi}x\sin nx\,dx=\frac{2\cdot(-1)^{n+1}}{n},$

$S(x)=\sum_{n\geq1}\frac{2(-1)^{n+1}}{n}\sin nx$

4. rez. $a_0 = \frac{1}{\pi}\int_{-\pi}^{\pi} e^x\,dx = \frac{2\,sh\,\pi}{\pi}$

$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} e^x \cos nx\,dx = \frac{2(-1)^n sh\,\pi}{\pi(n^2+1)}$

$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} e^x \sin nx\,dx = \frac{2n\,(-1)^{n+1} sh\,\pi}{\pi(n^2+1)}$

---

5. rez.

$a_0 = \frac{1}{\pi}\int_{-\pi}^{0}\left(-\frac{1}{2}\right) dx = \frac{1}{2}$

$a_n = \frac{1}{\pi}\int_{-\pi}^{0}\left(-\frac{1}{2}\right) \cos nx\,dx = 0$

$b_n = \frac{1}{\pi}\int_{-\pi}^{0}\left(-\frac{1}{2}\right) \sin nx\,dx = \frac{1}{n\pi}(\sin n\pi)^2$

$b_{2n} = 0;\quad b_{2n+1} = \frac{1}{(2n+1)\pi}$

$S(x) = \frac{1}{4} + \sum_{n\geq 0} \frac{1}{(2n+1)\pi} \sin(2n+1)x$

6. zad.

$$a_0 = \frac{2}{4\pi}\int_0^{4\pi} \cos\frac{x}{2}\, dx = 0$$

$$a_n = \frac{2}{4\pi}\int_0^{4\pi} \cos\frac{x}{2} \cos\frac{nx}{2}\, dx = (\text{formalno}) = \frac{2n \sin n\pi}{\pi(1-n^2)}$$

$$b_n = \frac{2}{4\pi}\int_0^{4\pi} \cos\frac{x}{2} \sin\frac{nx}{2}\, dx = \frac{2n \sin(n\pi)^2}{\pi(n^2-1)}$$

$b_1 = 0$, $b_n = 0$, $n \neq 1$

$a_1 = 1$, $a_n = 0$, $n \neq 1$

$S(x) = \cos\frac{x}{2}$

---

7. zad. $f(x) = \sin\frac{x}{3}$ $T = 6\pi$

$$a_0 = \frac{2}{6\pi}\int_0^{6\pi} \sin\frac{x}{3}\, dx = 0 \qquad a_n = \frac{2}{6\pi}\int_0^{6\pi} \sin\frac{x}{3} \cos\frac{nx}{3}\, dx = 0,\ n \neq 1$$

$b_n = 0$, $n \neq 1$ $b_1 = 1$

---

8. zad. $f(x) = |\cos x|$ $T = \pi$, parna $L = \frac{\pi}{2}$

$$a_n = \frac{2}{\frac{\pi}{2}}\int_0^{\pi/2} \cos x \cos\frac{n\pi x}{\frac{\pi}{2}}\, dx = \frac{4\cdot(-1)^n}{\pi(1-4n^2)}$$

zad $f(x) = \cos^2 x$ parna, $T = \pi$, $b_n = 0$

$$a_n = \frac{2}{\frac{\pi}{2}} \int_0^{\pi/2} \cos^2 x \cdot \cos \frac{n\pi x}{\frac{\pi}{2}}\, dx = \frac{4}{\pi} \int_0^{\pi/2} \left(\frac{1+\cos 2x}{2} \right) \cdot \cos 2n x\, dx$$

$$a_n = \frac{\sin n\pi}{\pi(n-n^3)} \qquad n=0,\ n=1? \qquad a_0 = \frac{4}{\pi} \int_0^{\pi/2} \cos^2 x\, dx = 1$$

$$a_1 = \frac{4}{\pi} \int_0^{\pi/2} \cos^2 x \cdot \cos 2x\, dx = \frac{1}{2}$$

$$\Rightarrow S(x) = \frac{1}{2} + \frac{1}{2} \cos 2x$$

---

10. zad $f(x) = |x| - 1$ $\quad -1 \le x < 1$ parna $\Rightarrow b_n = 0$

$$a_n = \frac{2}{1} \int_0^1 (x-1) \cdot \cos \frac{n\pi x}{1}\, dx = \frac{2\left[(-1)^n - 1\right]}{n^2 \pi^2} \qquad n=0?$$

$$a_0 = -1, \qquad a_{2n} = 0, \quad a_{2n+1} = \frac{-2}{\pi^2 (2n+1)^2}$$

$$\Rightarrow S(x) = -\frac{1}{2} + \sum_{n \ge 0} \frac{-2}{\pi^2 (2n+1)^2} \cdot \cos (2n+1)\pi x .$$

---

11. zad

$$a_0 = \frac{1}{3} \int_{-3}^{3} (2x+3)\, dx = 6$$

$$a_n = \frac{1}{3} \int_{-3}^{3} (2x+3) \cdot \cos \frac{2n\pi x}{6} = 0$$

$$b_n = \frac{1}{3} \int_{-3}^{3} (2x+3) \sin \frac{2n\pi x}{6} = \frac{-12 \cdot (-1)^{n+1}}{n\pi}$$

$$S(x) = \frac{a_0}{2} + \sum_{n \ge 1} b_n \cdot \sin \frac{n\pi x}{3}$$

zad $b_n = 0$, $a_n = \frac{2}{4}\int_0^4 (2-|x|)\cos\frac{n\pi x}{4}\,dx = \frac{8}{n^2\pi^2}\left[1-(-1)^n\right]$, $n>0$

$a_0 = 0$, $a_{2n} = 0$, $a_{2n+1} = \frac{8}{(2n+1)^2\cdot\pi^2}\cdot 2 = \frac{16}{(2n+1)^2\pi^2}$

$$\Rightarrow S(x) = \sum_{n\geq 0} \frac{16}{(2n+1)^2\cdot\pi^2}\cdot\cos\frac{(2n+1)\pi x}{4}.$$

---

13. zad

$$a_0 = \frac{2}{4}\int_{-2}^{2}(2x-1)\,dx = -2$$

$$a_n = \frac{2}{4}\int_{-2}^{2}(2x-1)\cos\frac{2n\pi x}{4}\,dx = 0, \quad n\geq 1$$

$$b_n = \frac{2}{4}\int_{-2}^{2}(2x-1)\sin\frac{2n\pi x}{4}\,dx = \frac{-8\cdot(-1)^n}{n\pi} \Rightarrow$$

$$S(x) = \sum_{n\geq 1}\frac{8}{n\pi}\cdot(-1)^{n+1}\cdot\sin\frac{n\pi x}{2}$$

---

14. zad f neparna $\Rightarrow b_n = \frac{2}{\pi}\int_0^{\pi}\cos x\cdot\sin\frac{n\cancel{\pi}x}{\cancel{\pi}}\,dx =$

$$= \frac{2}{\pi}\int_0^{\pi}\cos x\cdot\sin nx\,dx = \frac{2}{\pi}\int_0^{\pi}\frac{\sin(n+1)x + \sin(n-1)x}{2}\,dx =$$

$$= \frac{2n\cdot\left[1+(-1)^n\right]}{\pi\cdot(n^2-1)}, \quad n\neq 1, \quad b_{2n+1} = 0, \quad b_{2n} = \frac{2\cdot 4n}{\pi(4n^2-1)}$$

$$b_1 = 0 \Rightarrow \cos x \sim \sum_{n\geq 1}\frac{4\cdot 2n}{\pi(4n^2-1)}\cdot\sin nx$$

15. zad $a_n = \frac{2}{\pi}\int_0^{\pi} x\cdot\cos\frac{n\pi x}{\pi}\,dx = (n\geq 1) = \frac{2\left[(-1)+(-1)^n\right]}{n^2\cdot\pi}$

$a_0 = \pi$, $a_{2n} = 0$, $a_{2n+1} = \frac{-4}{(2n+1)^2\cdot\pi}$ $\Rightarrow$

$x \sim \frac{\pi}{2} + \sum_{n\geq 0} \frac{-4}{(2n+1)^2\pi}\cdot\cos(2n+1)x$

---

16. zad f parna $a_n = \frac{2}{1}\int_0^{1} x\cdot\cos\frac{n\pi x}{1}\,dx = (n\geq 1) = \frac{2\left[(-1)^n-1\right]}{n^2\cdot\pi^2}$

$a_0 = 1$, $a_{2n} = 0$, $a_{2n+1} = \frac{-4}{(2n+1)^2\cdot\pi^2}$ $\Rightarrow$

• $|x| \sim \frac{1}{2} + \sum_{n\geq 0}\frac{-4}{(2n+1)^2\pi^2}\cdot\cos(2n+1)\cdot\pi\cdot x$, $x = 0 \Rightarrow$

$\frac{4}{\pi^2}\sum_{n\geq 0}\frac{1}{(2n+1)^2} = \frac{1}{2} \Rightarrow \sum_{n\geq 0}\frac{1}{(2n+1)^2} = \frac{\pi^2}{8}$

---

17. zad $a_0 = \frac{2}{1}\int_0^1 x^2\cdot dx = \frac{2}{3}$, $a_n = \frac{2}{1}\int_0^1 x^2\cos\frac{n\pi x}{1}\,dx =$

$= (2.\text{ put. P.I.}) = \frac{4\cdot(-1)^n}{n^2\pi^2} \Rightarrow$

$x^2 \sim \frac{1}{3} + \sum_{n\geq 1}\frac{4(-1)^n}{n^2\pi^2}\cdot\cos n\pi x$, $x = 0 \Rightarrow$

$-\frac{1}{3} = \frac{4}{\pi^2}\sum_{n\geq 1}\frac{(-1)^n}{n^2} \Rightarrow \sum_{n\geq 1}\frac{(-1)^n}{n^2} = -\frac{\pi^2}{12}$

18. zad $T = 2\pi$ neparna, $a_n = 0$

$$b_n = \frac{2}{\pi}\int_0^{\pi} \underbrace{\operatorname{sgn}(\sin x)}_{1} \sin\frac{n\pi x}{\pi}\,dx = \frac{2}{\pi}\int_0^{\pi}\sin nx\,dx =$$

$$= \frac{2}{n\pi}\left[1-(-1)^n\right] \Rightarrow b_{2n} = 0,\quad b_{2n+1} = \frac{4}{(2n+1)\pi} \Rightarrow$$

$$\operatorname{sgn}(\sin x) \sim \sum_{n\geq 0}\frac{4}{(2n+1)\pi},\ = \sum_{n\geq 1}\frac{4}{(2n-1)\pi}$$

$$\Rightarrow \sum_{n=1}^{\infty}\frac{16}{\pi^2\cdot(2n-1)^2} = \frac{2}{2\pi}\int_{-\pi}^{\pi}|\operatorname{sgn}(\sin x)|^2\,dx = \frac{2}{\pi}\int_0^{\pi}dx = 2$$

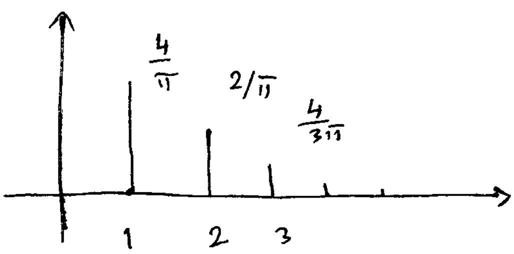
$$\Rightarrow \sum_{n\geq 1}\frac{1}{(2n-1)^2} = \frac{\pi^2}{8}$$

---

19. zad $T = 4$

$$b_n = \frac{2}{2}\int_0^2 x^2 \sin\frac{n\pi x}{2}\,dx = \int_0^2 x^2 \sin\frac{n\pi x}{2}\,dx =$$

$$= \frac{-4}{n\pi} \qquad c_n = |b_n| = \frac{4}{n\pi},\ n\geq 1$$



...

20.2d) $$a_0 = \frac{2}{T}\int_0^a A\cdot dx + \frac{2}{T}\int_{T-a}^{T} A\, dx = \frac{4A\cdot a}{T}$$

$$a_2 = \frac{2}{T}\int_0^a A\cdot\cos\frac{4\pi x}{T}\,dx + \frac{2}{T}\int_{T-a}^{T} A\cdot\cos\frac{4\pi x}{T} = \frac{A\sin\left(\frac{4a\pi}{T}\right)}{2\pi} + \frac{A\sin\left(\frac{4a\pi}{T}\right)}{2\pi}$$

$$b_2 = \frac{2}{T}\int_0^a A\cdot\sin\frac{4\pi x}{T}\,dx + \frac{2}{T}\int_{T-a}^{T} A\cdot\sin\frac{4\pi x}{T} = \frac{A\cdot\sin\left(\frac{2a\pi}{T}\right)^2}{\pi} - \frac{A\cdot\sin\left(\frac{2a\pi}{T}\right)^2}{\pi} = 0$$

$$\Rightarrow \quad , \quad c_2 = \frac{A}{\pi}\left|\sin\frac{4a\pi}{T}\right|$$

$$a_1 = \frac{2}{T}\int_0^a A\cdot\cos\frac{2\pi x}{T}\,dx + \frac{2}{T}\int_{T-a}^{T} A\cdot\cos\frac{2\pi x}{T}\,dx = \frac{2A}{\pi}\sin\frac{2a\pi}{T}$$

$b_1 = 0 \Rightarrow$ $c_1 = 2c_2$ uvjet $\Rightarrow \frac{2A}{\pi}\sin\frac{2a\pi}{T} = 2\cdot\frac{A}{\pi}\sin\frac{4a\pi}{T} \Rightarrow$

$\Rightarrow \sin\frac{2a\pi}{T}\left(2\cos\frac{2a\pi}{T} - 1\right) = 0 \Rightarrow \frac{2a\pi}{T} = k\pi \Rightarrow a = \frac{kT}{2}$ NE zbog $0 < a < T/2$

$\frac{2a\pi}{T} = \frac{\pi}{3} \Rightarrow T = \frac{2a}{3}$; $a = \frac{3T}{2}$